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RESEARCH WORKSHOP ON CONDENSED MATTER PHYSICS
(21 June - 3 September 1993)

WORKING PARTY ON SMALL SEMICONDUCTOR STRUCTURES
(2 - 13 August 1993)

PERSISTENT CURRENT OF A WIGNER CRYSTAL-RING

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Persistent Current of a One-Dimensional Wigner Crystal-Ring.

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Kharkov, Ukraine

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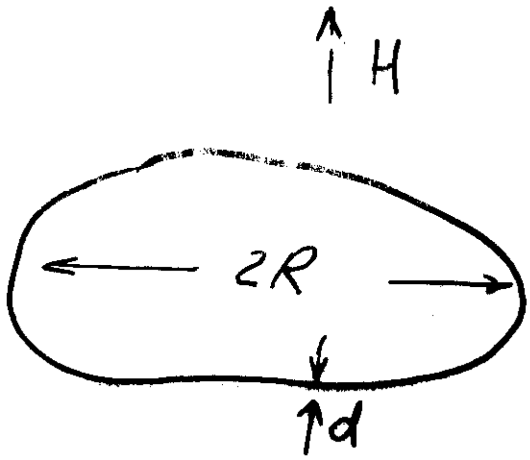
S.M. Girvin – Indiana University
USA.

M. Jonson – Chalmers, Sweden

Aharonov - Bohm Effect in Conductors



$\Phi = H \cdot S \; ; \; \Phi_0 = h/e$

$R \simeq 1\mu m$

$d \simeq 100 \mathring{A}$

M. Büttiker, Y. Imry, R. Landauer
Phys. Lett. 96A, 365 ([illegible])

Persistent Current

$$I(\Phi) = -\frac{\partial F}{\partial \Phi}$$ – Periodic function of Φ with period Φ_0

$$\boxed{I_{max}(T) \simeq \frac{e}{\tau_c} \exp\left\{-\left(\frac{T}{T_0}\right)^n\right\}}$$ for - ring.

$$T_0 \simeq \hbar/\tau_c \; ; \quad n = \begin{cases} 1, & \ell \gg L \\ 0.5, & \ell \ll L \end{cases}$$

τ_c – circulation time

$$\tau_c \simeq \begin{cases} L/v_F & \ell \gg L \\ L^2/D & \ell \ll L \end{cases} \; ; \quad D \simeq \frac{v_F \ell}{3}$$

Diffusive and Ballistic Rings

I. Metallic rings

$d \gg \lambda_F$ Many transverse modes
(s-d ring)

$L \gg \ell$ Diffusive transport of electrons

$r_B \cdot n^{1/3} \gg 1$ Ideal gas of electrons (mean field)

II. Gate-produced rings in 2-D semiconducting layers.

1. D. Mailly, C. Chapelier, A. Benoit
   PRL, 70, 2020 (1993)

2. J. Liu, W.X. Gao, K. Ismail, K.Y. Lee, Y.H. Hong, S. Washburn
   PRB (unpublished)

1. $d \simeq \lambda_B$ (one or few modes of transverse motion; nearly 1-D ring)

2. $\ell > L$ Ballistic transport of electrons

3. $r_B \cdot n^{1/2} \equiv \alpha^2 < 1$ (Electronic correlations should be important)

What is the role of interelectronic Coulomb correlations?

$$I_{exp}(T=0) \simeq \frac{eV_F}{L}$$ 1-D ballistic ring. (free electrons)

In a diffusive ring Coulomb interaction of electrons increase the persistent current

1. V. Ambegaokar, U. Eckern
   PRL, <u>65</u>, 381 (1990)

2. A. Schmid, PRL, <u>66</u>, 80, (1991)

Conclusion → Coulomb interaction is not essential in semiconducting ring.

Is this conclusion correct?

Strong Coulomb Correlations

$$\alpha^2 = \frac{r_B}{a} \ll 1 \qquad r_B \simeq \frac{\varepsilon_0 \hbar^2}{me^2}$$

$\frac{E_{kin}}{E_{coul.}} \simeq \alpha^2 \ll 1 \Rightarrow$ Electrons form a Wigner Crystal

Persistent current is just a rotation of the Wigner Crystal as a whole

If we shift the crystal by period $\underline{a}$ during some time we produce the current which corresponds to the full rotation of a ~~single~~ electron. $\Rightarrow I = \hbar v_F / L$

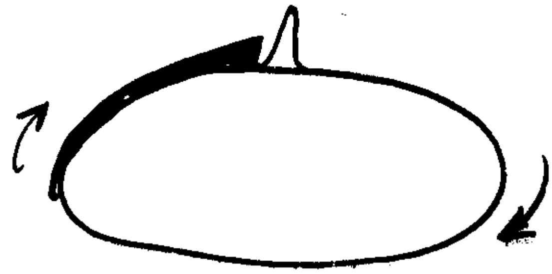
$$V_\varphi \simeq \frac{e}{mc} A = \frac{\hbar}{Lm}\frac{\varphi}{\varphi_0} \qquad \text{At } \varphi = \varphi_0 \quad V_\varphi = \frac{\hbar}{mL}$$

$$E = MV_\varphi^2 = \frac{L}{a} m V_\varphi^2 = \frac{\hbar^2}{maL} = \frac{\hbar V_F}{L}\ ; \quad V_F = \frac{\hbar}{ma}$$

At $T \gtrsim MV_\varphi^2 = T_0$ we destroy decompensation of clockwise and counterclockwise rotations and current will disapear

$$T = \hbar V_F /$$

Persistent Current in the WC Ring containing a Barrier.



A rotation of the WC is acompanied with a crystal deformation

U_{pin} – Pinning Energy

$U = ms^2 \frac{a}{L}$ – Deformation Energy

Strong pinning

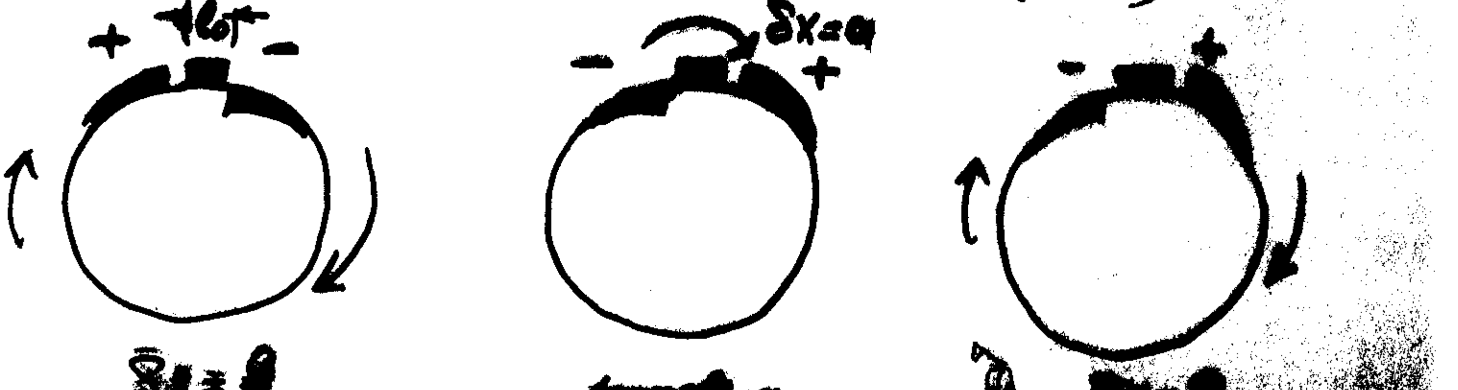
A. Larkin, P. Lee
PR [illegible], 1596 (1978)

$$\frac{U_{pin}}{U} \gg 1$$

Three steps of the rotation

There are two parts of WC: 1) a pinned part with size $\boxed{l_0 = \frac{ms^2 a}{V_{pin}} \ll L}$

2) an unpinned part $\rightarrow (L - 2l_0)$

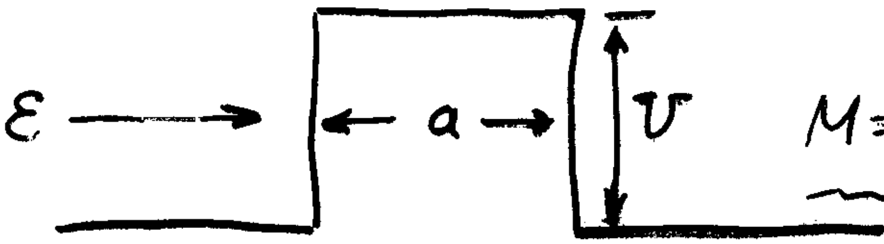


$\delta x =$ [illegible]
rotation + deformation

tunneling
through the barrier

[illegible]
[illegible]

Tunneling through the ellastic deformed configuration



$M = Nm = \frac{L}{a} m$

$U = ms^2 \frac{a}{L}$; $E_0 = \frac{\hbar^2}{2Ma^2} =$ [illegible] $\equiv T_0$ $\quad V_F =$ [illegible]

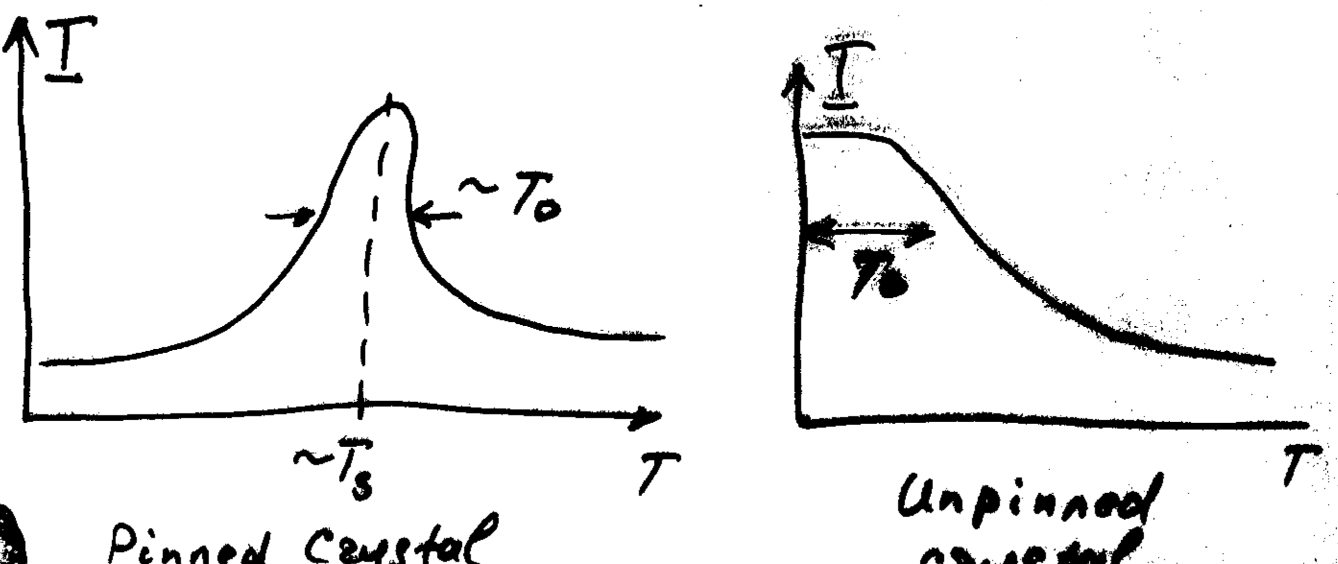
$E_0 / U = (v/s)^2 \equiv \alpha^2 \ll 1$.

$$W(\varepsilon, T) \simeq \exp - \left\{ \frac{a}{\hbar} \sqrt{2M(U - \varepsilon)} + \frac{\varepsilon}{T} \right\}$$

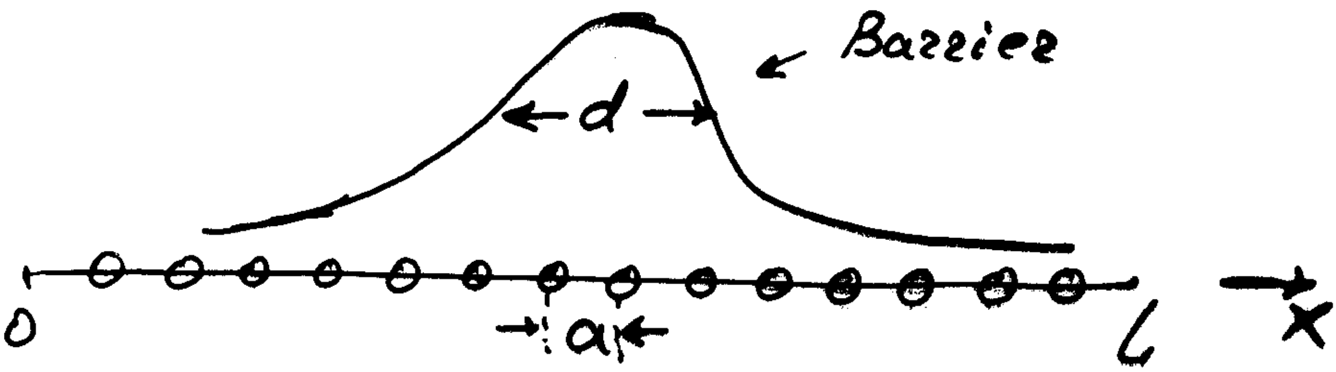
$$W(\varepsilon_{max}, T) \simeq \exp - \left\{ \frac{U}{T} + \frac{T}{E_0} \right\}$$

$$\boxed{T_{max} = \sqrt{U E_0} = \frac{\hbar s}{L} \equiv T_s \gg T_0}$$

$$\delta T \simeq T_{max} \left(\frac{T_{max}}{U} \right)^{1/2} \ll T_{max}.$$



Formulation of the Problem



$$a \ll d \ll L$$

We will use the longwavelength approximation.

Pinning Energy $V(u)$

Displacement of electrons $u(x)$

$V(u) = V(u+a)$ periodic function can be expanded in Fourier series

If $d \gg a$ we can use only a first harmonics (others will be exponentially smaller

$$\boxed{V(u) = Const + V\cos\frac{2\pi u(x=0)}{a} \times \delta(x)}$$

L. I. Glazman, I. M. Ruzin, B. I. Shklovskii PR B45, 8584 (1992)

Long wavelength Approximation

$$\varphi(x) = 2\pi \frac{u(x)}{a}$$ Dimensionless displacement

$$\mathcal{L} = \frac{ma^2}{8\pi^2}\left\{\dot{\varphi}^2 - s^2(\varphi')^2\right\} - V_p\,\delta(x)\cos(\varphi)$$

s - sound velocity

$$\mathcal{L}_\Phi = \frac{\hbar}{L}\frac{\Phi}{\varphi_0}\dot{\varphi} \qquad A_\varphi = \frac{\Phi}{L};\ \varphi_0 = \frac{\hbar c}{e}$$

Free Energy

$$F = -T\ln\left\{\int D\varphi \exp\left(-S_E[\varphi]/\hbar\right)\right\}$$

$$S_E(\varphi) = \int_{-\beta/2}^{\beta/2} d\tau \int_0^L dx\, \mathcal{L}[\varphi(i\tau,x)] + i\pi n N\hbar$$

$$\varphi_n(\tau+\beta, x) = \varphi_n(\tau,x) + 2\pi n$$

$$\boxed{I = -\frac{\partial F}{\partial \Phi}}$$

Quasiclassic limit

$$\alpha = \sqrt{\frac{\partial_B}{a}} \ll 1.$$

We can use the saddle point approximation to calculate path integral.

Extreme trajectories we get from the equation.

$$\boxed{\begin{gathered}\ddot{\varphi} + s^2\varphi'' = \frac{8\pi^2}{ma^2} V_p \, \delta(x) \cos(\varphi) \\ \varphi(\tau = +\beta/2, x) = \varphi(\tau = -\beta/2, x) + 2\pi n\end{gathered}}$$

Persistent Current of WC with no Barrier.

$\underset{\sim}{V_P = 0}$

$$\varphi_n(\tau) = \frac{2\pi n}{\beta}\tau$$

By Calcullation of the action of the extreme trajectory we can find

$$\boxed{I_{WC} = (-1)^N 2I_0 \begin{cases} \frac{1}{\pi}\arctan\left[\tan\left(\pi\frac{\varphi}{\varphi_0}\right)\right], & T \ll T_0 \\ \frac{T}{T_0} e^{-T/T_0} \sin\left(2\pi\frac{\varphi}{\varphi_0}\right), & T \gtrsim T_0 \end{cases}}$$

$$I_0 = \frac{eV_F}{L}; \quad T_0 = \frac{\hbar V_F}{L}$$

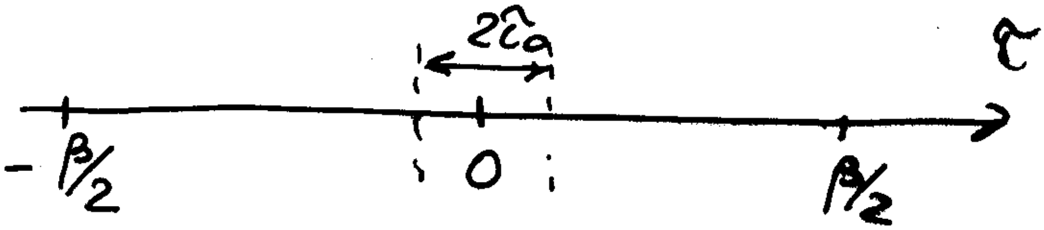
$$V_F = \hbar/ma$$

We got the same result as for free electron gas.

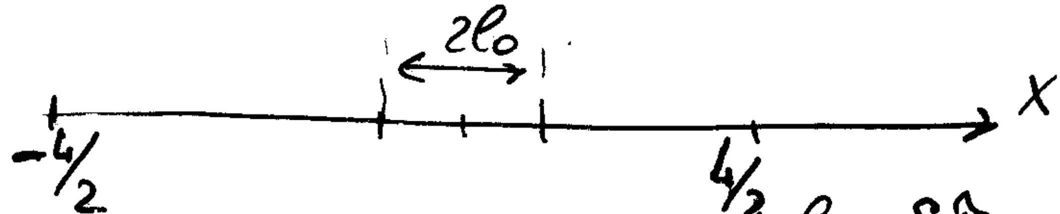
Persistent Current in the Case of a Strong Pinning.

$$V_p \gg \alpha^{-1} \cdot T_s \qquad \alpha^{-1} = s/v_F$$

$$T_s = \hbar s / L$$



A.I. Larkin, P.A. Lee PR B17, 1596 (1978)

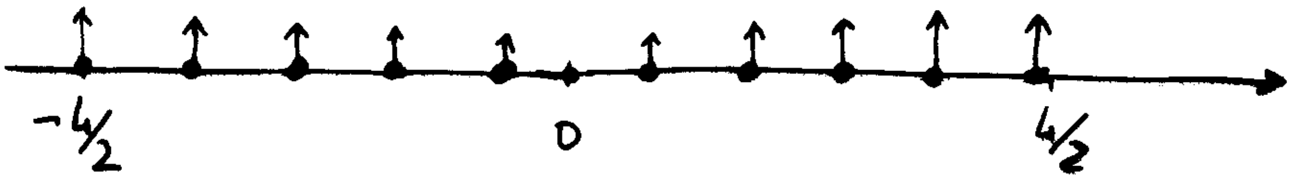


$l_0 = s\tau_0$

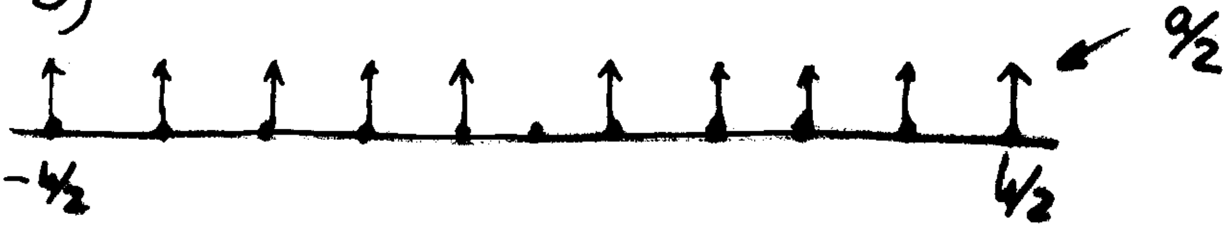
$$l_0 = \frac{m s^2 a}{V_p} \ll L$$

For $|x| > l_0$; $|\tau| > \tau_0$ we can use the free elasticity equation to find the trajectory part, corresponding to the elastic relaxation.

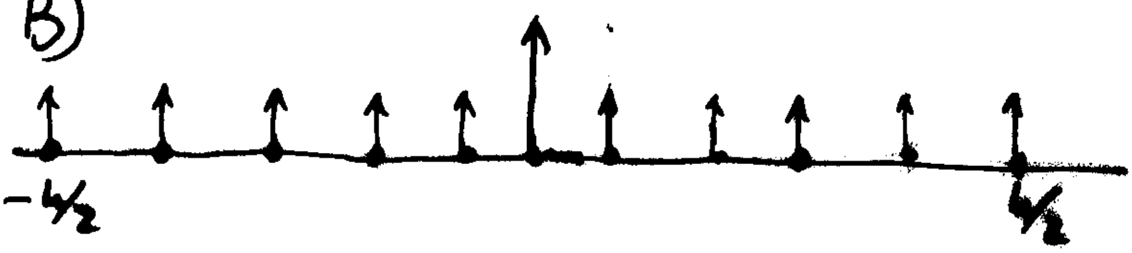
A) $\uparrow\varphi$ $-\beta/2 < \tau < 0$ $(-\tau_0)$



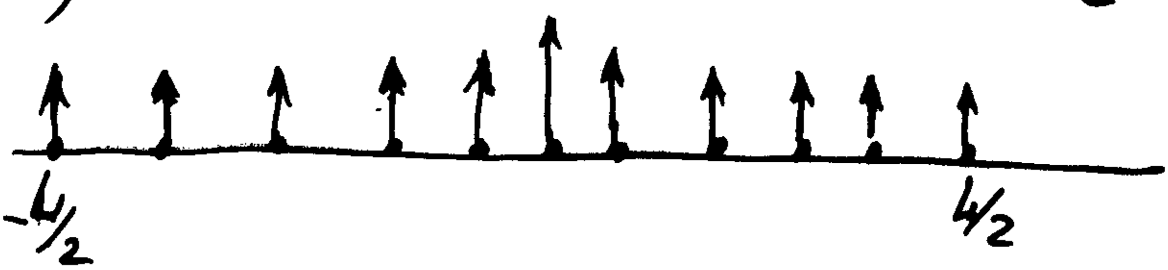
B) $\tau = -0$ $(-\tau_0)$



B) $\tau = +0$ $(+\tau_0)$



C) (τ_0) $0 < \tau < \beta/2$



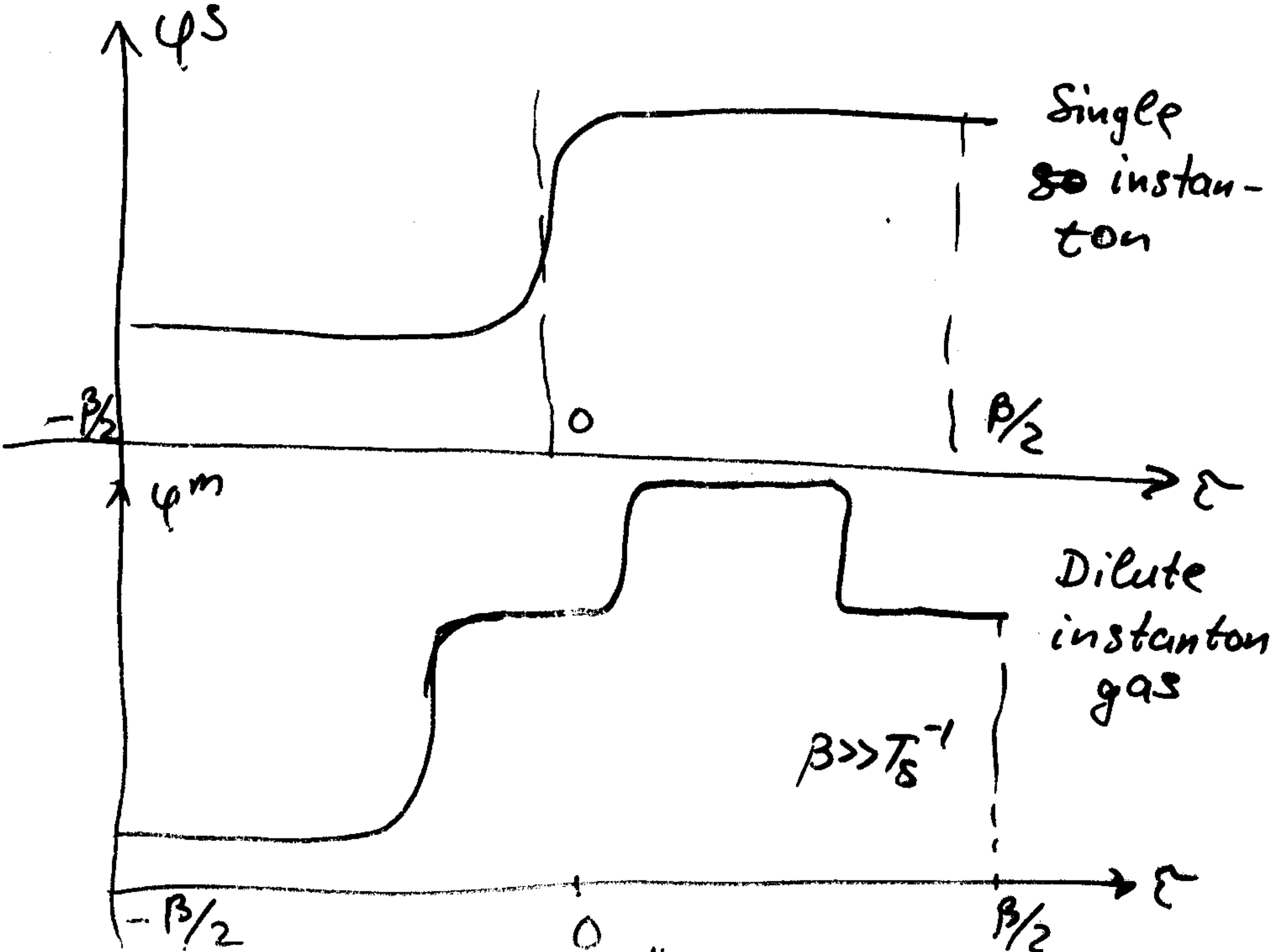
Instanton solutions for elastic relaxation

for $n = \pm 1$, $T \simeq T_s$

$$\boxed{\varphi_\sigma^s(x,\tau) = 2\sigma \arctan\left[\coth\left(\frac{\pi|x|}{\hbar s\beta}\right)\tan\frac{\pi\tau}{\hbar\beta}\right] \qquad \sigma = \pm 1}$$

for $T \ll T_s$

$$\boxed{\varphi_\sigma^{(m)} = \sum_e \varphi^s_{\sigma_e}(\tau-\tau_e) \qquad \sigma = \sum_e \sigma_e = \pm 1}$$



Persistent Current

$$S_{extr} = \int_{|\tau|>\tau_0} d\tau \int_{|x|>\ell_0} dx\, L\{\varphi_{extr}\}/\hbar$$

$$\boxed{I_{wc}(T=0) \simeq (-1)^N \frac{es}{L} \left(\frac{T_s}{\alpha V_0}\right)^{1/\alpha} \sin(2\pi\, \varphi/\varphi_0)}$$

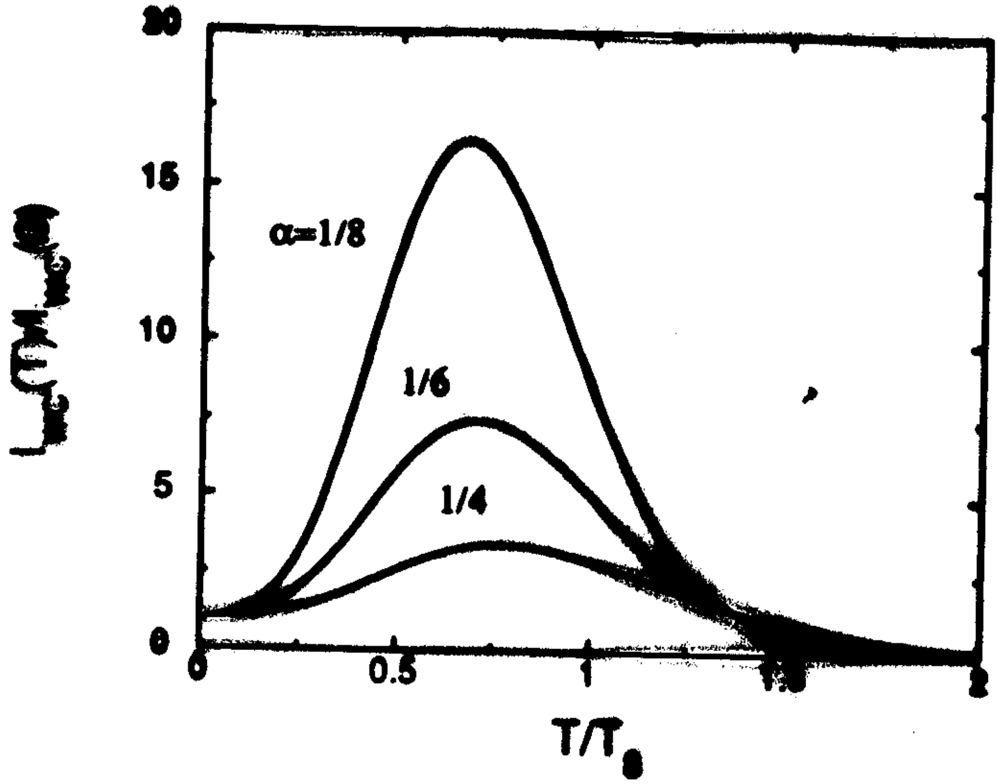
$$\frac{I_{wc}(T)}{I_{wc}(0)} = \frac{T}{T_s} \exp\left[\frac{1}{\alpha} f(T/T_s)\right] \qquad T \ll \alpha V_0$$

$$f(x) = \frac{\pi}{2} x - \ln\left[\frac{\sinh(\pi x)}{x}\right]$$

$$I_{wc}(T = T_{max}) = \max I_{wc}.$$

$$T_{max} = 0.5\, T_s \qquad \alpha \ll 1.$$

Figure 2.



Conclusions

- Persistent Current in a Ballistic 1-D Ring does not influenced by strong Coulomb correlations

- Incorporation of potential Barrier in a Ring results in qualitative change of the magnitude and temperature dependence of the persistent current

- Experiments with a gate controlled barrier can be used for detecting and investigation the properties of the Wigner Crystal.