INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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SMR/697-3

RESEARCH WORKSHOP ON CONDENSED MATTER PHYSICS
(21 June - 3 September 1993)

WORKING PARTY ON SMALL SEMICONDUCTOR STRUCTURES
(2 - 13 August 1993)

MAGNETOTUNNELLING AND CHAOS - II

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These are preliminary lecture notes, intended only for distribution to participants

Lecture 2

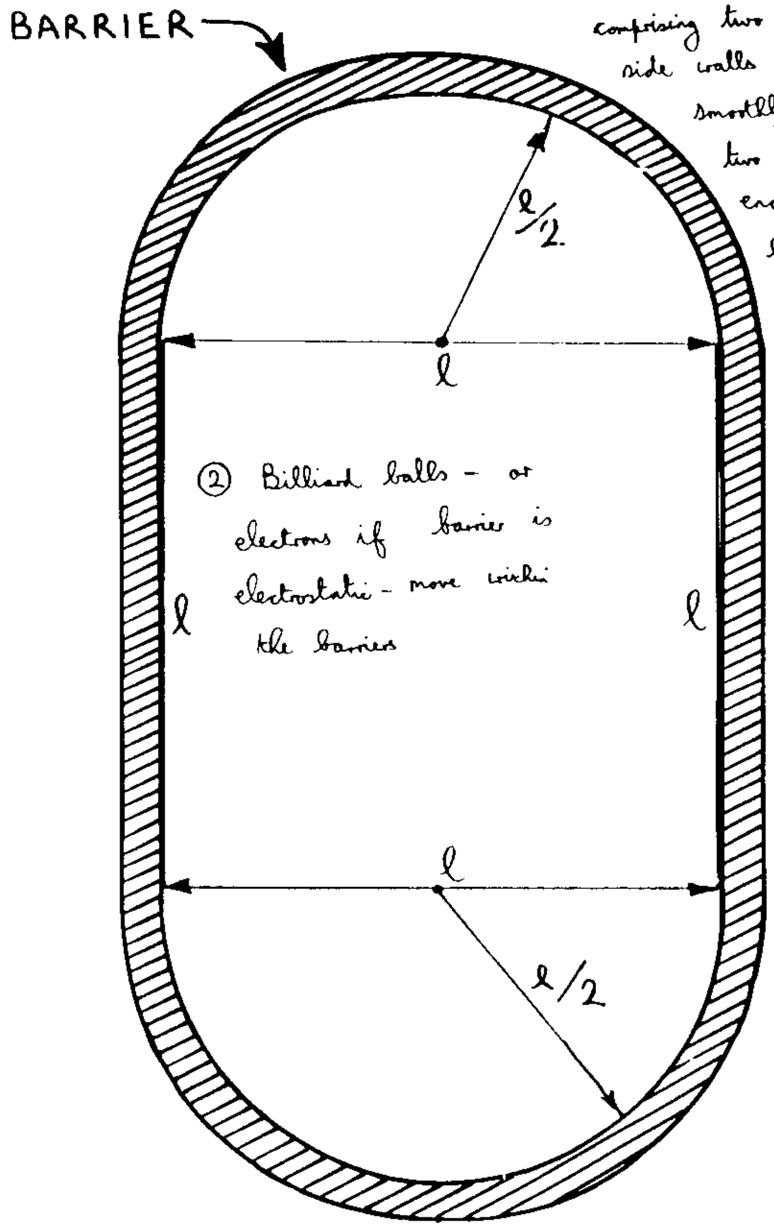
1) Introduction to classical chaos. 'Billiard' stadia. Chaos in the H-atom.

2) Quantum properties of classically chaotic systems. Energy level statistics. Garton-Tompkins oscillations in the absorption spectra of chaotic hydrogenic atoms. The role of periodic orbits.

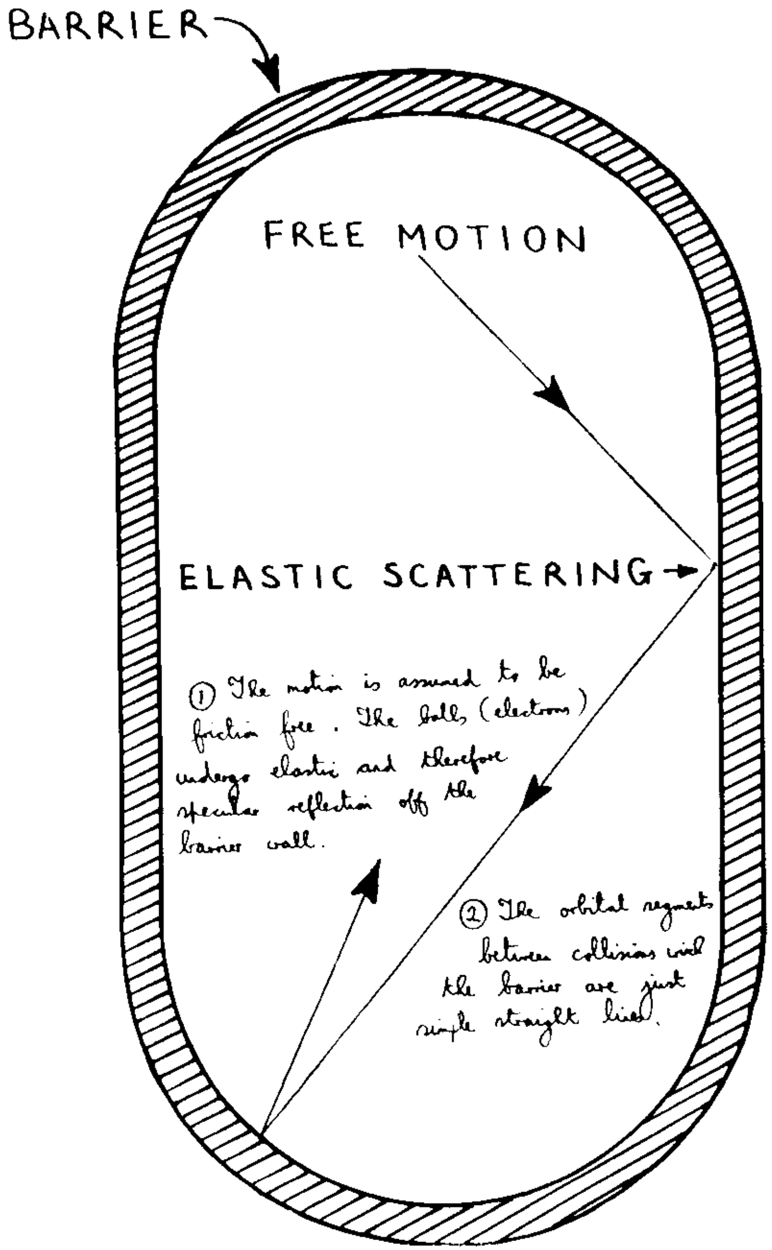
3) Chaotic classical motion in a potential well with a tilted magnetic field. Probing the corresponding quantum energy level pattern using resonant magnetotunnelling spectroscopy. Dramatic change of the quasi-periodic resonances in the current-voltage characteristics with tilt angle. Semiclassical analysis of the origin and evolution of the resonant structure. Links with the Garton-Tompkins oscillations.

STADIUM OF BUNIMOVICH

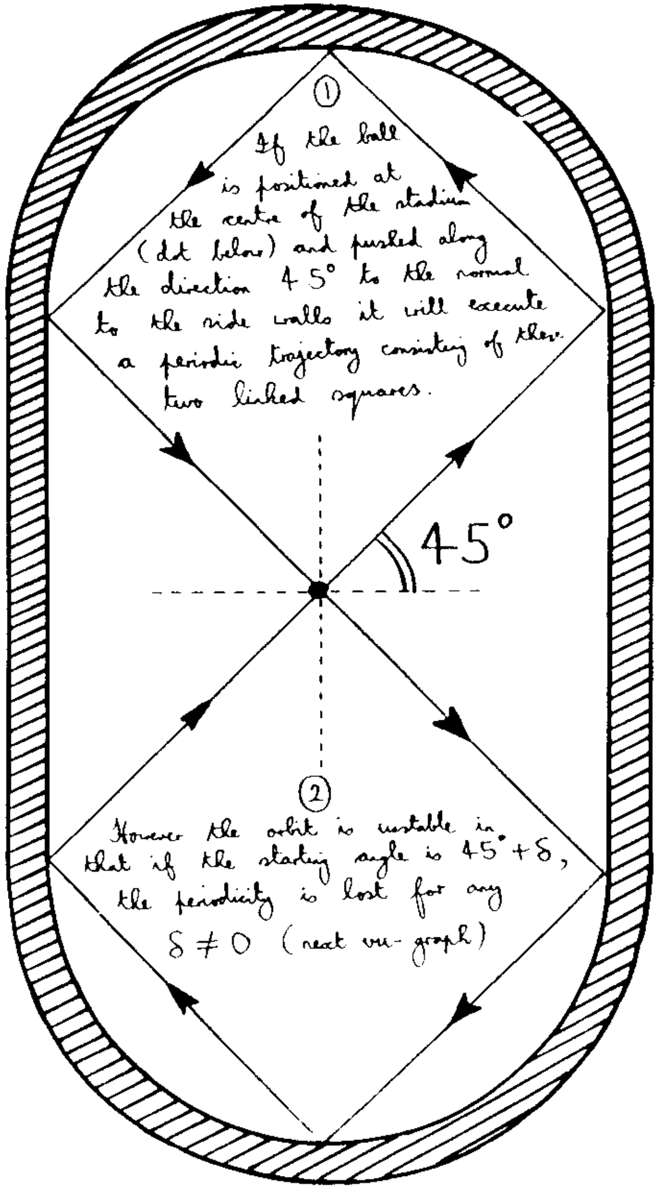
① Is a 2D 'billiard' table comprising two parallel side walls of length ℓ smoothly joined to two semi-circular ends of radius $\ell/2$.



STADIUM OF BUNIMOVICH

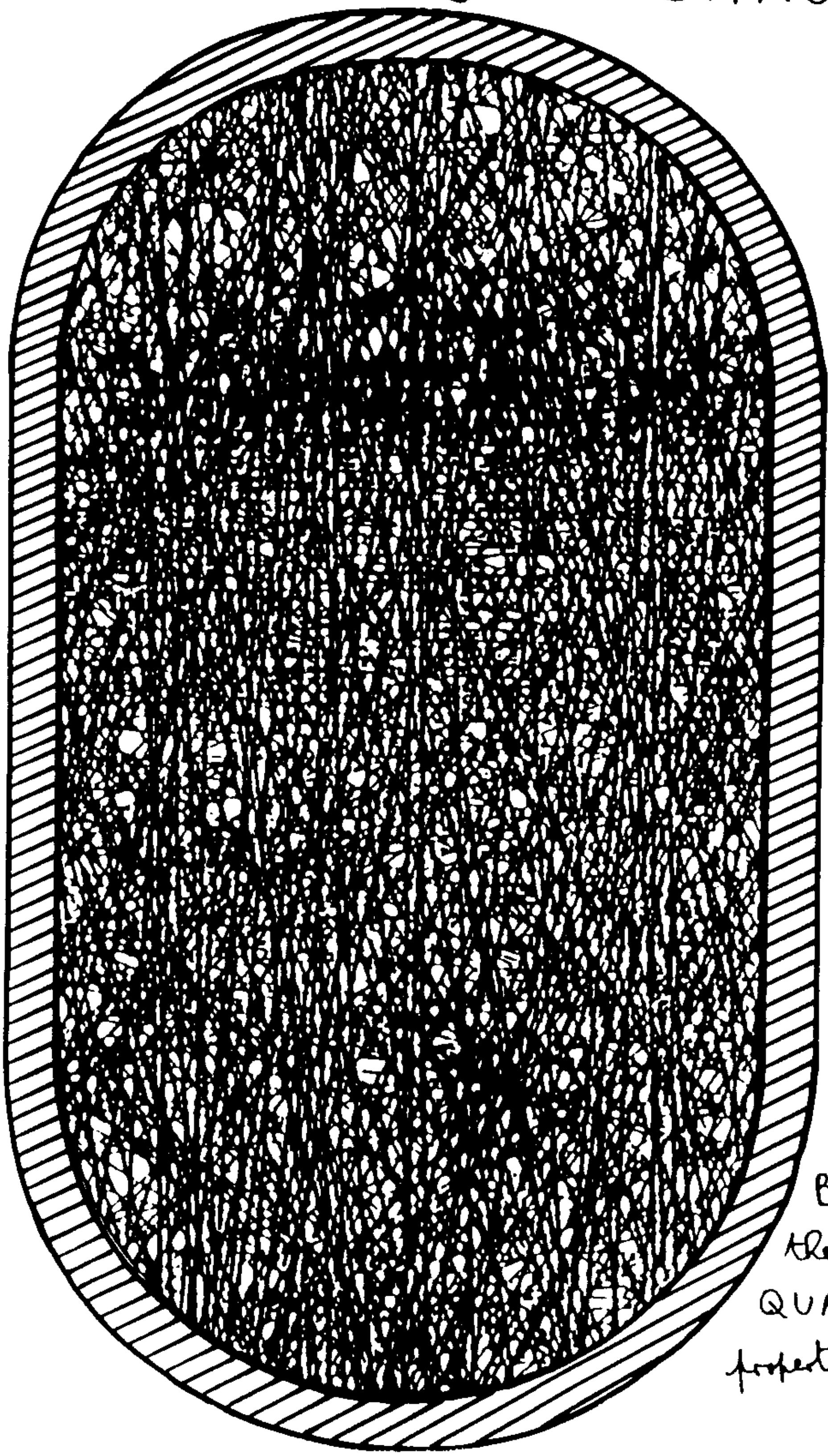


STADIUM OF BUNIMOVICH



STADIUM OF BUNIMOVICH

10^4 BOUNCES = CHAOS !



①

For an initial angle of, ray, 46°, the electron (ball) orbit deviates from the 'double square' path and eventually uniformly fills the interior of the stadium.

②

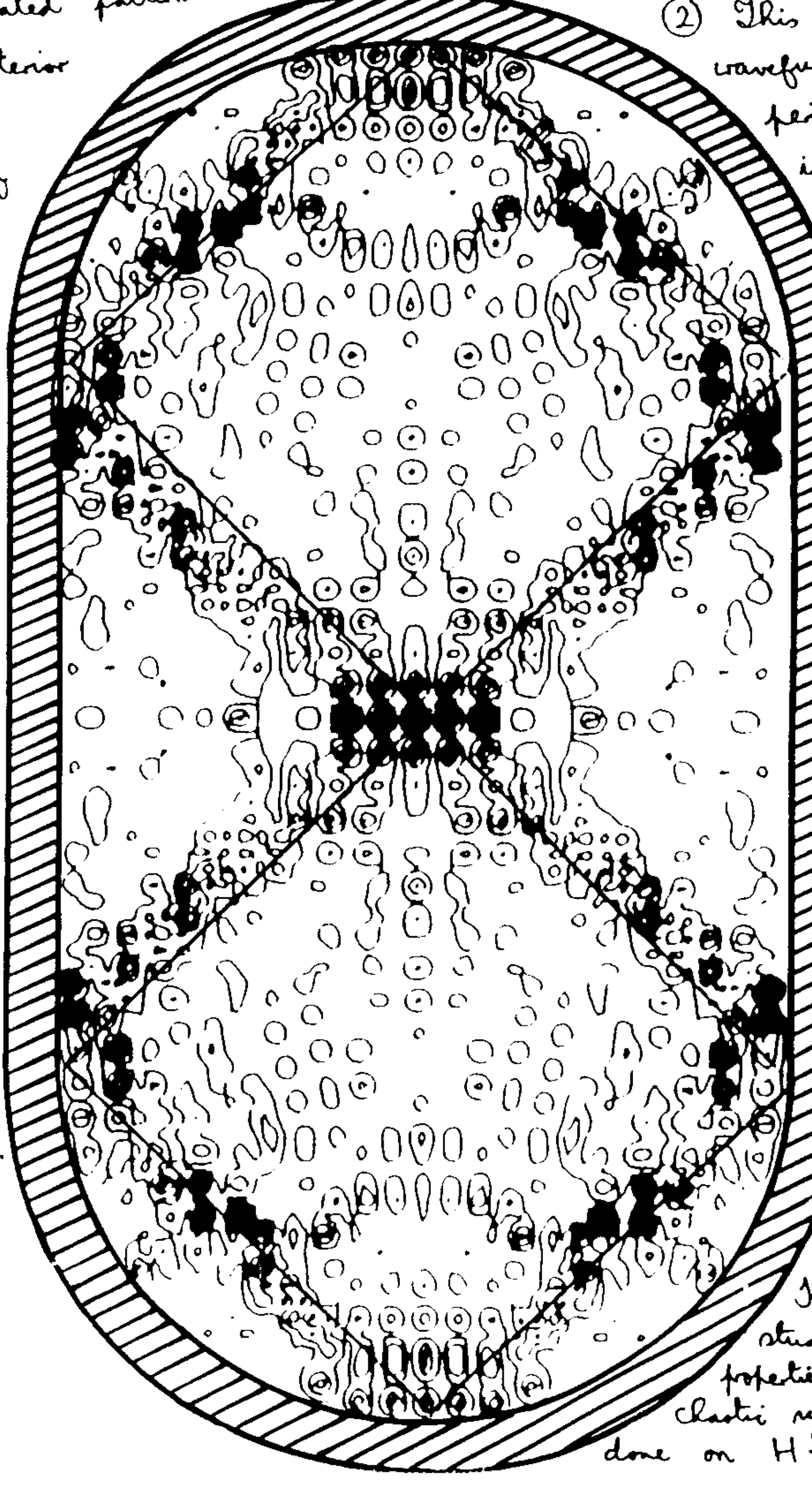
This extreme instability of the periodic orbits (and aperiodic orbits) with respect to infinitesimal changes in the starting conditions is the DEFINING CHARACTERISTIC of classical chaos.

— || —

But what about the corresponding QUANTUM properties?

STADIUM OF BUNIMOVICH

HELLER (1984)
PRL 53, 1515,



① The complicated pattern within the interior shows the contour plot of the probability density $|\psi|^2$ for an excited QUANTUM state of the stadium. Note that the islands of high probability density (black) lie close to the periodic CLASSICAL path we discussed earlier. (c.f. high n states of a non-chaotic simple harmonic oscillator).

② This 'scarring' of the wavefunction by the periodic classical orbits is one common feature of the eigenfunctions of a classically chaotic system. We'll look at the properties of the eigenvalues with reference to the H-atom (next)

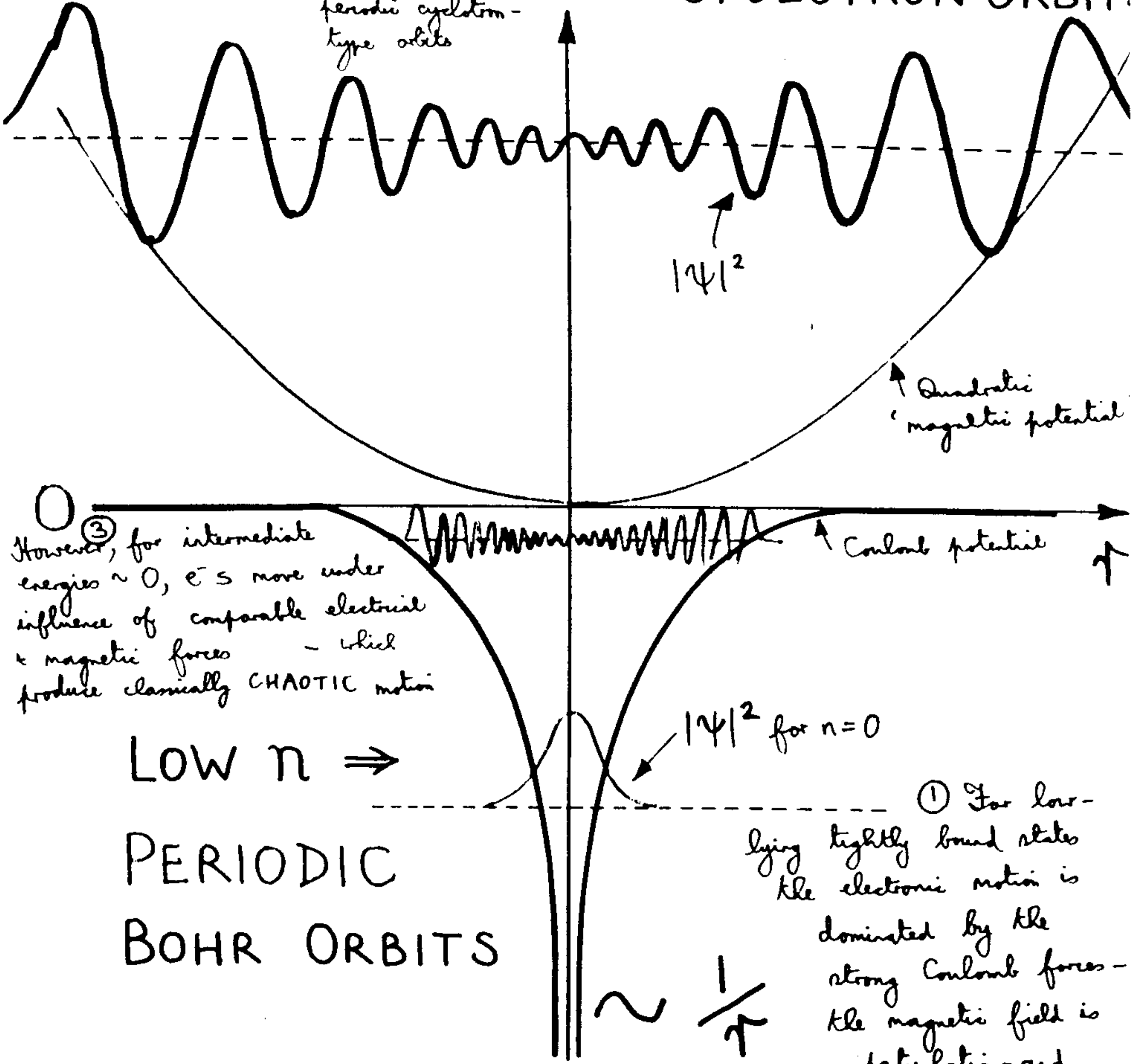
③ Some experiments have been done on the transport properties of chaotic & regular semiconductor stad (e.g.) Marcus et al. PRL 69, 506, (1992

However most expt^l studies of quantum properties of classically chaotic systems have been done on H-atoms —

H-ATOM IN HIGH $\underline{B}$-FIELD

② For energies well into the continuum the electron spends most of its time away from the nucleus where the magnetic potential varies rapidly & the Coulomb force is now the perturbation. The electron executes periodic cyclotron-type orbits

CONTINUUM ⇒
PERIODIC
CYCLOTRON ORBIT.



③ However, for intermediate energies ~ 0, e^-s move under influence of comparable electrical & magnetic forces – which produce classically CHAOTIC motion

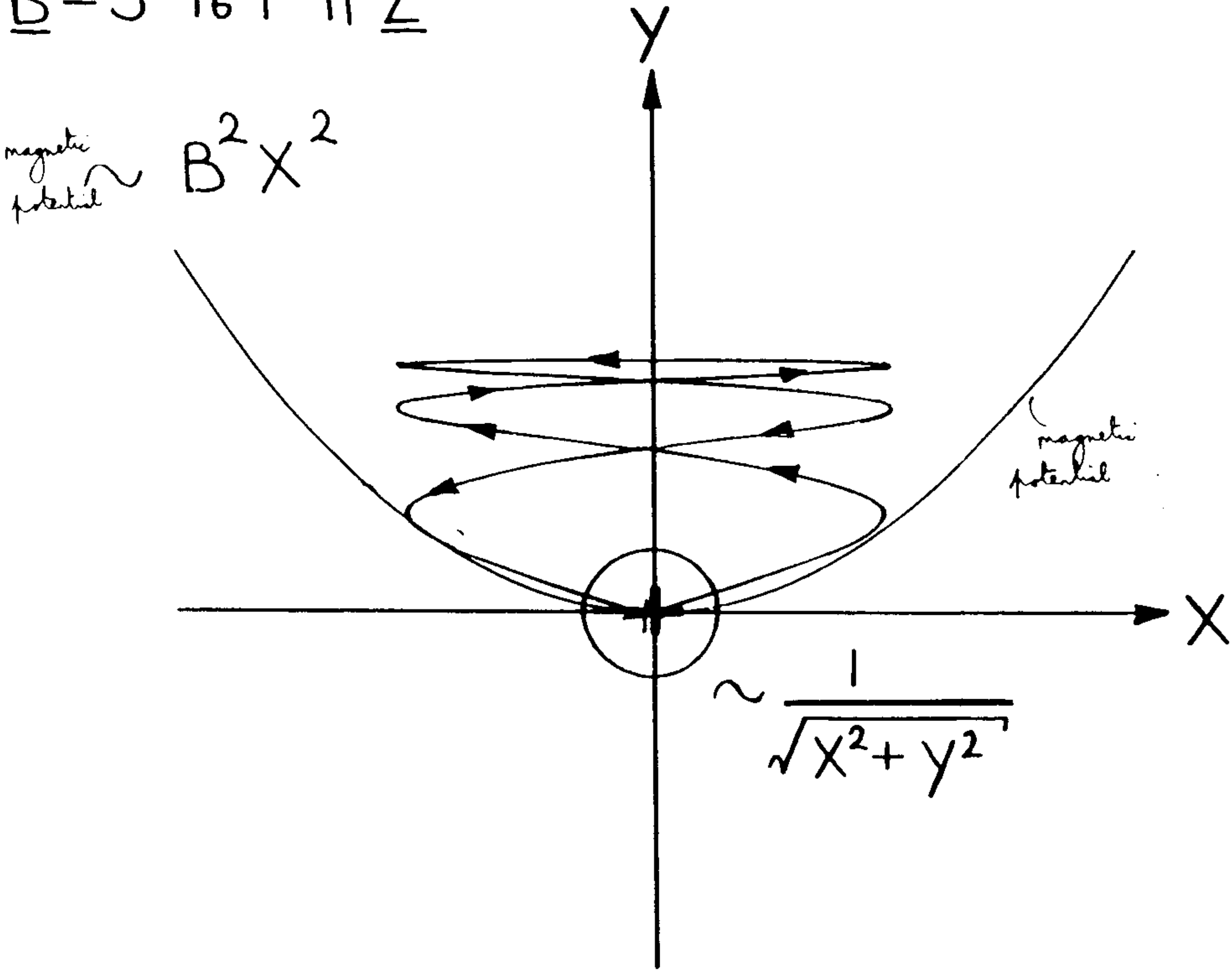
LOW n ⇒

PERIODIC
BOHR ORBITS

① For low-lying tightly bound states the electronic motion is dominated by the strong Coulomb forces – the magnetic field is a perturbation – and the electron describes periodic Bohr-type orbits

EXCITED 2D H-ATOM IN A HIGH MAGNETIC FIELD:— PERIODIC MOTION

$\underline{B} = 5{\cdot}96\,T \parallel \underline{Z}$

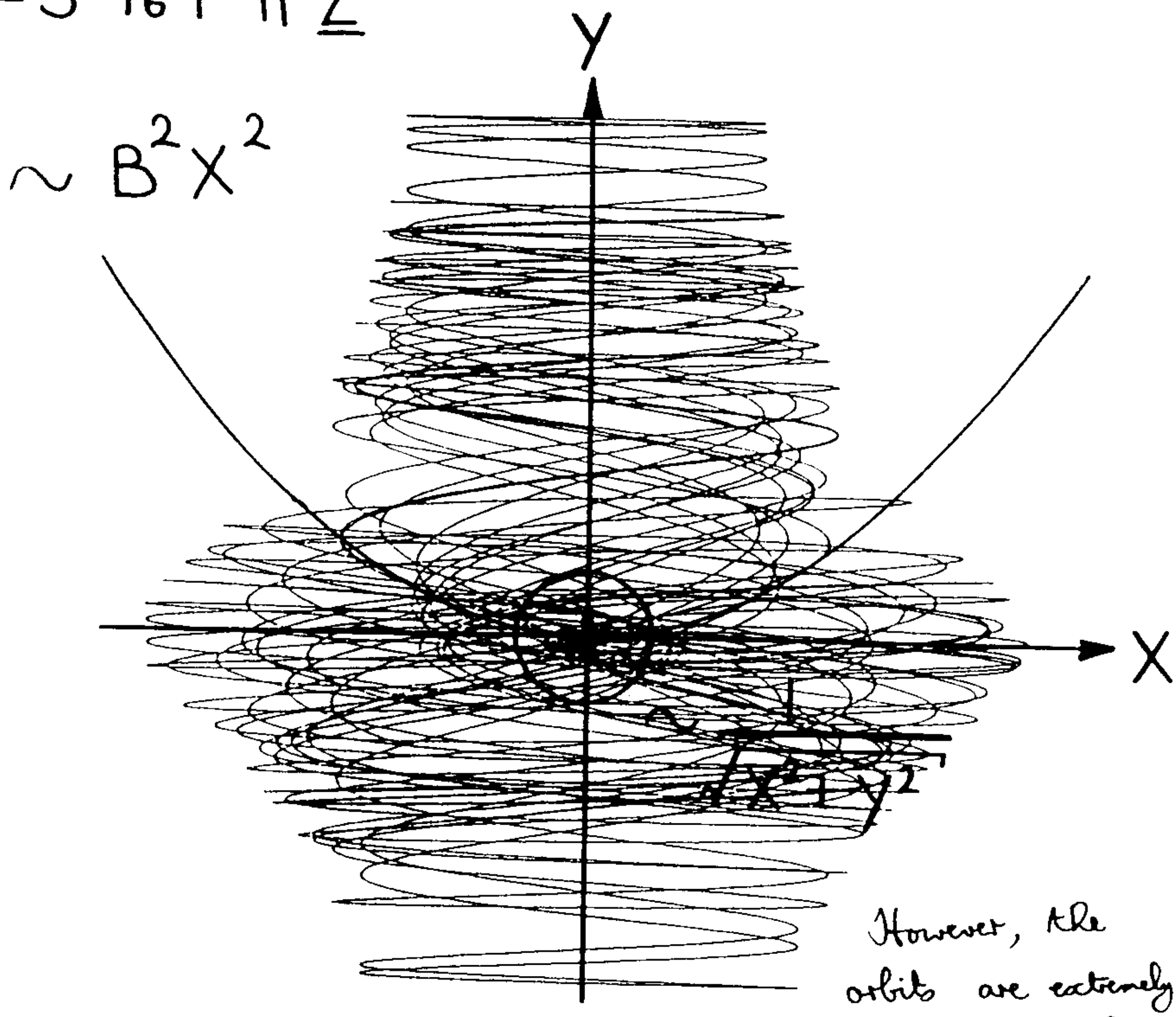


DU & DELOS (1988) Phys. Rev. A $\underline{38}$, 1896, (1988)

Considered the classical motion in a 2D H-atom (in the X-Y plane with high $\underline{B} \parallel \underline{Z}$. They showed that even in the classically chaotic regime (energy ~0, the ionization potential) PERIODIC orbits do exist as shown above.

EXCITED 2D H-ATOM IN A HIGH MAGNETIC FIELD:— CHAOTIC MOTION

$\underline{B} = 5 \cdot 96\,T \parallel \underline{Z}$



DU & DELOS (1988)

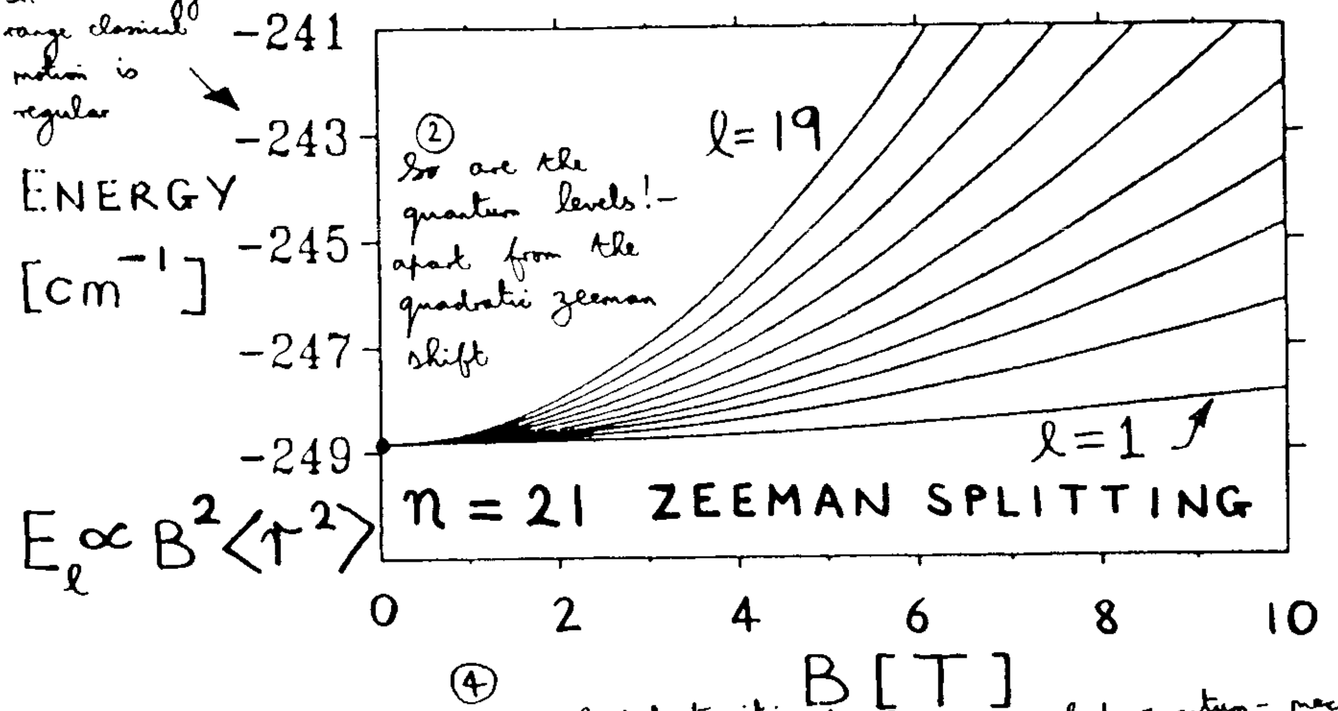
However, the orbits are extremely sensitive to small changes in the starting velocity.

Here $\frac{\Delta V}{V_{periodic}} = 10^{-6}$!

So Du & Delos demonstrated that for E~0 (excited states) the H-atom becomes classically chaotic in the presence of a high B-field — but what about the QUANTUM properties — let's have a look at the B-dependence of the energy levels.

EFFECT OF A MAGNETIC FIELD ON $n = 21$ & $43 \rightarrow 46$ ENERGY LEVELS OF A HYDROGEN ATOM

① In this energy range classical motion is regular

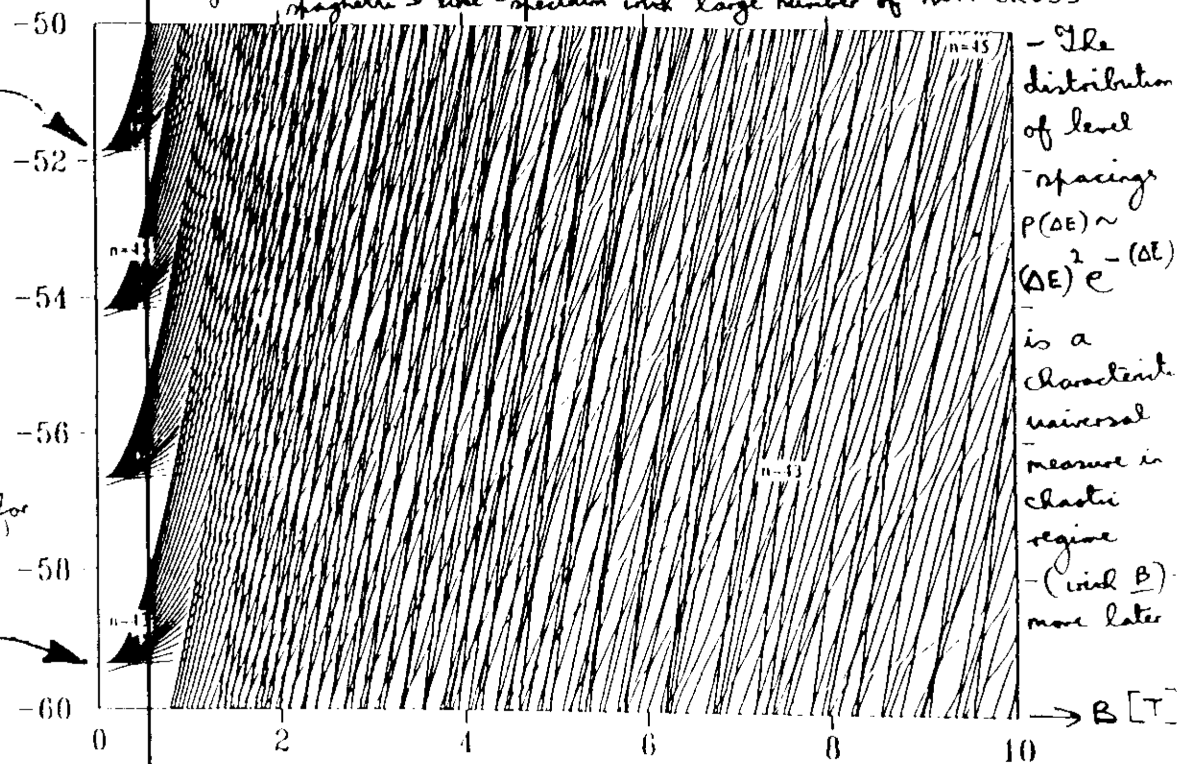


② So are the quantum levels! – apart from the quadratic Zeeman shift

$E_\ell \propto B^2 \langle r^2 \rangle$

④ The classical transition to chaos is marked quantum-mechanically by the loss of regular Zeeman spectrum & onset of complicated 'spaghetti' like spectrum with large number of ANTI-CROSSINGS

$n = 46$



$n = 43$

③ In this energy range classical motion is regular for $B \leq 0.75\,T$ & chaotic at higher B

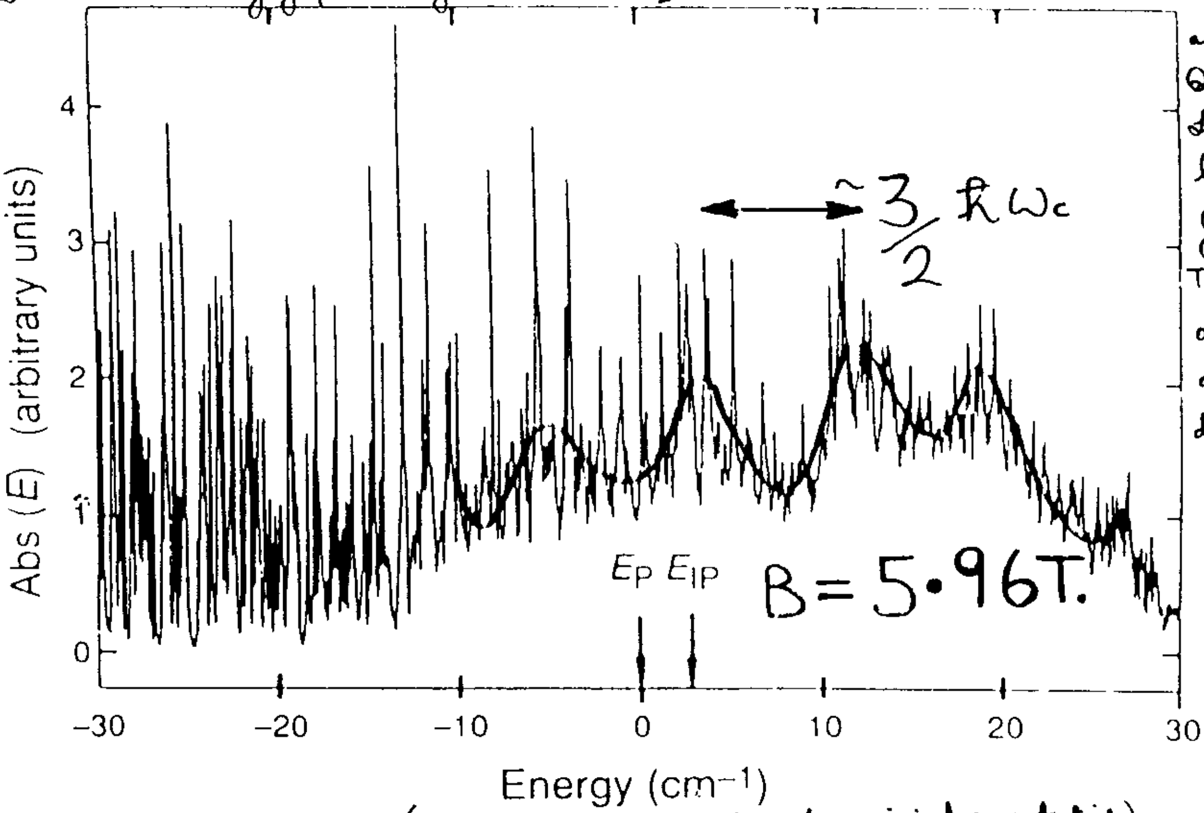
– The distribution of level spacings $P(\Delta E) \sim (\Delta E)^2 e^{-(\Delta E)}$ is a characteristic universal measure in chaotic regime – (with B) more later

PERIODIC MOTION ← | → 'n MIXING' & CHAOS FOR $n \gtrsim 43$ & ENERGY ≈ 0

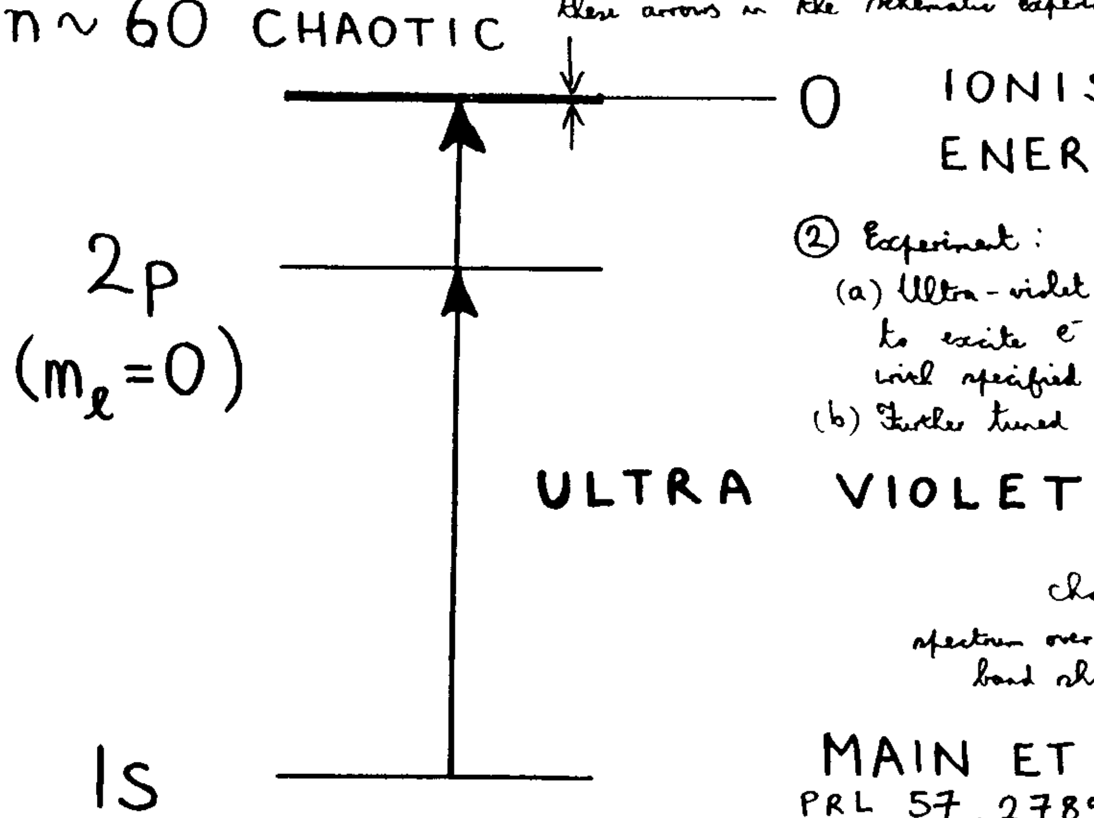
GARTON - TOMPKINS OSCILLATIONS

(Astrophys. J. 158, 839, (1969))

① The optical absorption spectrum reflects the complexity of the energy level pattern - EXCEPT there is a clear underlying periodicity with $\Delta E \sim \frac{3}{2}\hbar\omega_c$ - hence the oscillations are referred to as Quasi-Landau level or Garton-Tompkins oscillations after the discoverers



(very narrow range close to ionization potential)
Range of x-axis corresponds to these arrows in the schematic experimental set up.



② Experiment:

(a) Ultra-violet illumination used to excite e^- from 1s to 2p state with specified m_ℓ

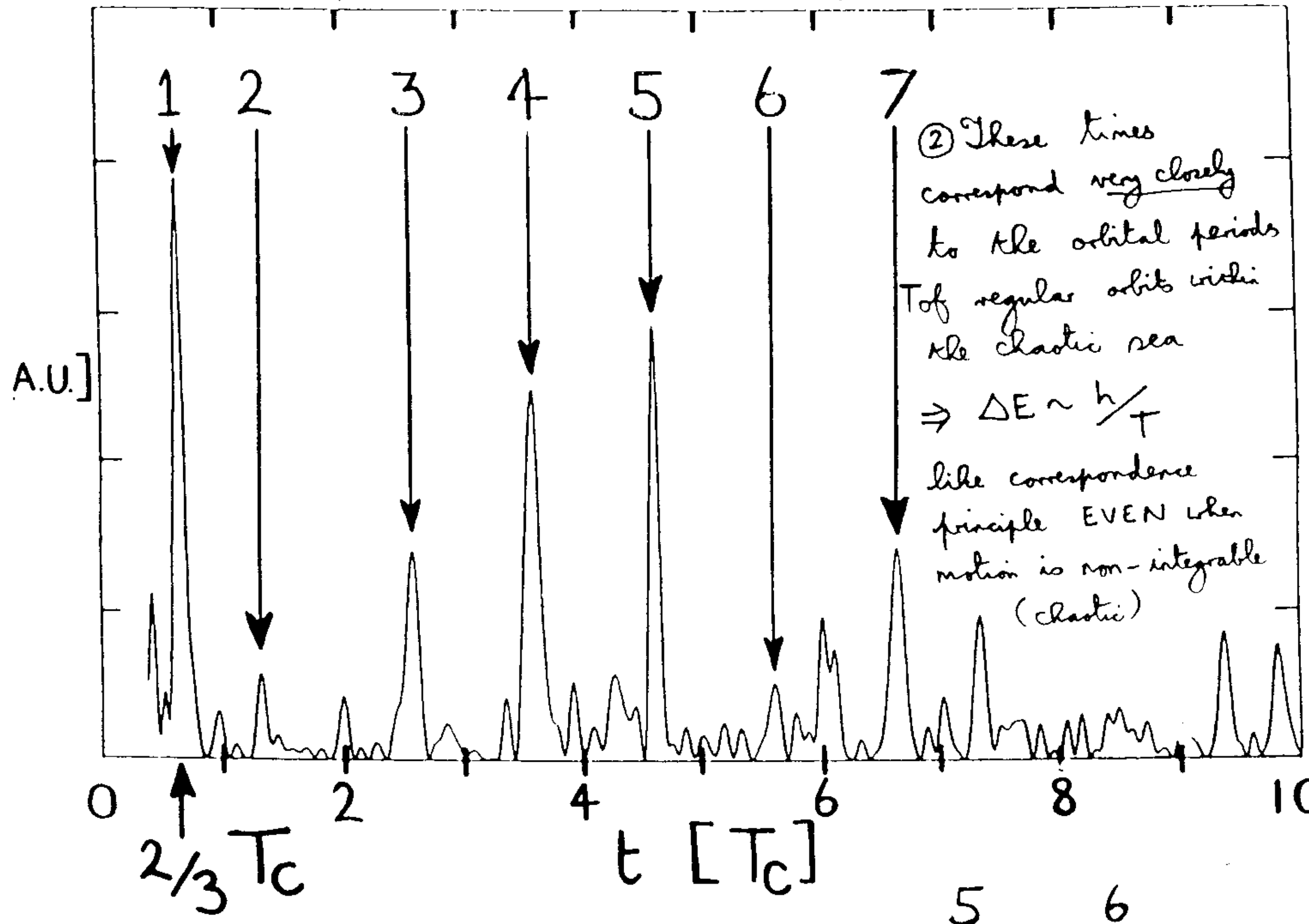
(b) Further tuned excitation from 2p to narrow energy band close to E=0

Chaotic domain - absorption spectrum over this narrow energy band shown above

MAIN ET AL. (1986)
PRL 57, 2789

FOURIER TRANSFORM

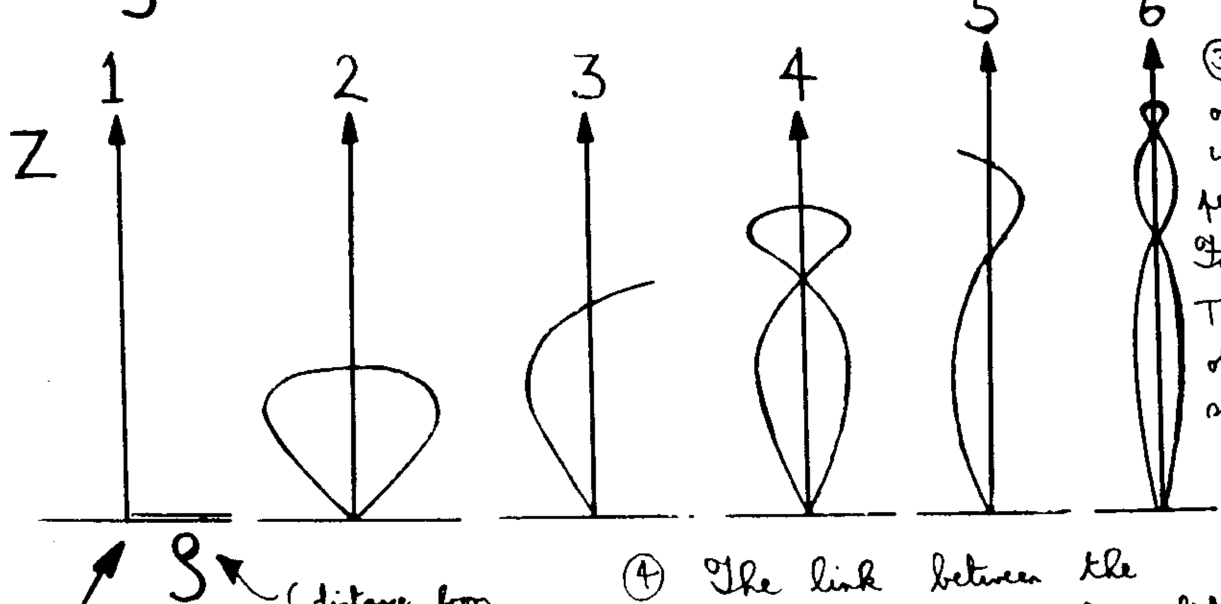
① Reveals additional periodic structure (lots!) with different characteristic times.



② These times correspond very closely to the orbital periods T of regular orbits within the chaotic sea

$\Rightarrow \Delta E \sim h/T$

like correspondence principle EVEN when motion is non-integrable (chaotic)



③ Periodic orbits linked with numbered peaks in Fourier Transform of absorption spectrum.

ρ (distance from nucleus in x-y plane)

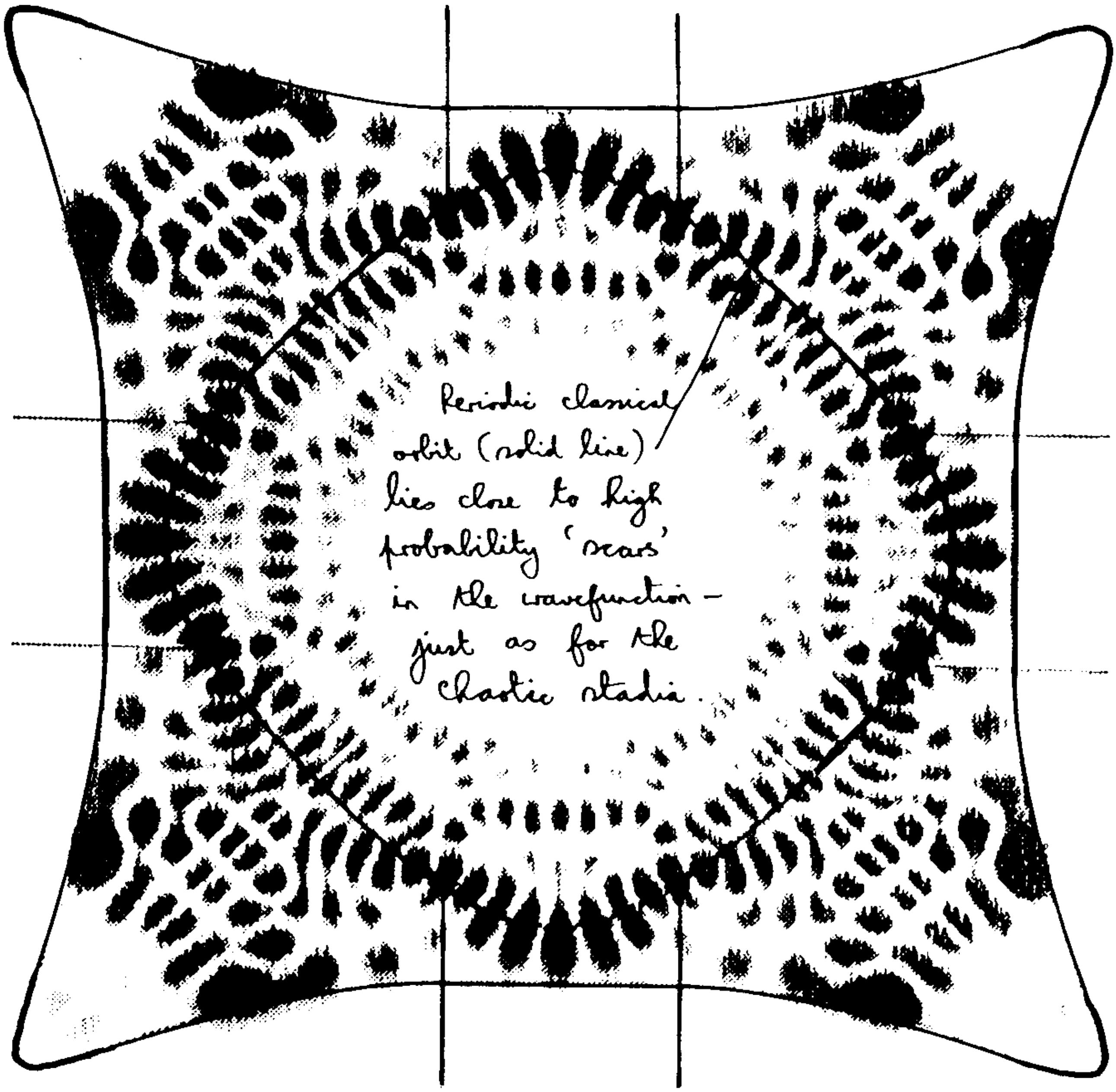
GARTON – TOMPKINS ORBIT

④ The link between the periods of unstable regular orbits & the separation of quasi-periodically spaced energy levels is central in the next section – Resonant tunnelling in a tilted $\underline{B}$-field.

H-ATOM IN UNIFORM $\underline{B}$ - FIELD

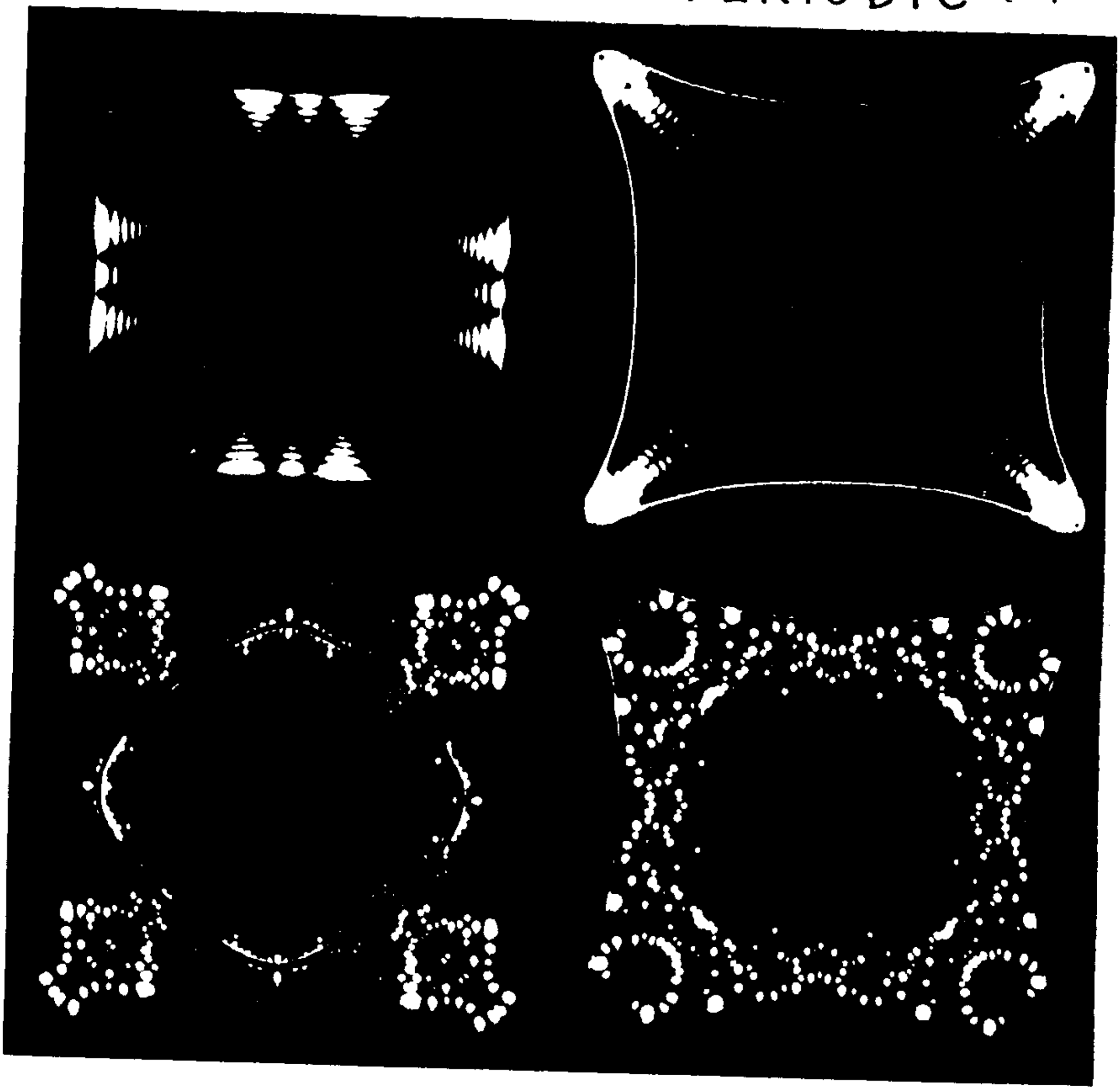
$|\psi|^2$ FOR 116^{TH} EXCITED STATE

Plot of probability density ⇒ black = high probability



WINTGEN et al. P.R.L. $\underline{63}$, 1467, (1989) ① The probability density plots are smooth in the classically regular (integrable) domain – but much more fragmented when the classical motion is chaotic. Nevertheless many of these fragments of high probability density hug the periodic classical orbits and collectively 'scar' the wavefunction.

CLASSICALLY PERIODIC (top two)



CLASSICALLY STABLE.
(bottom two)

② White = high probability.

EVEN WHEN HYDROGEN ATOM IS CLASSICALLY CHAOTIC, QUANTUM ABSORPTION PEAKS ARE OBSERVED & RELATE TO

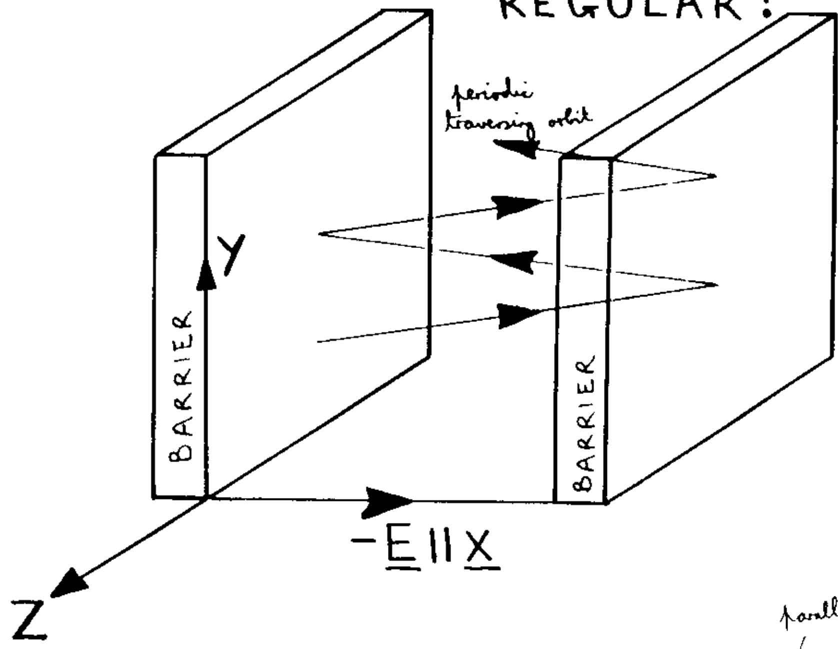
UNSTABLE <u>BUT</u> PERIODIC

CLASSICAL ORBITS.

AMIDST A CHAOTIC SEA.

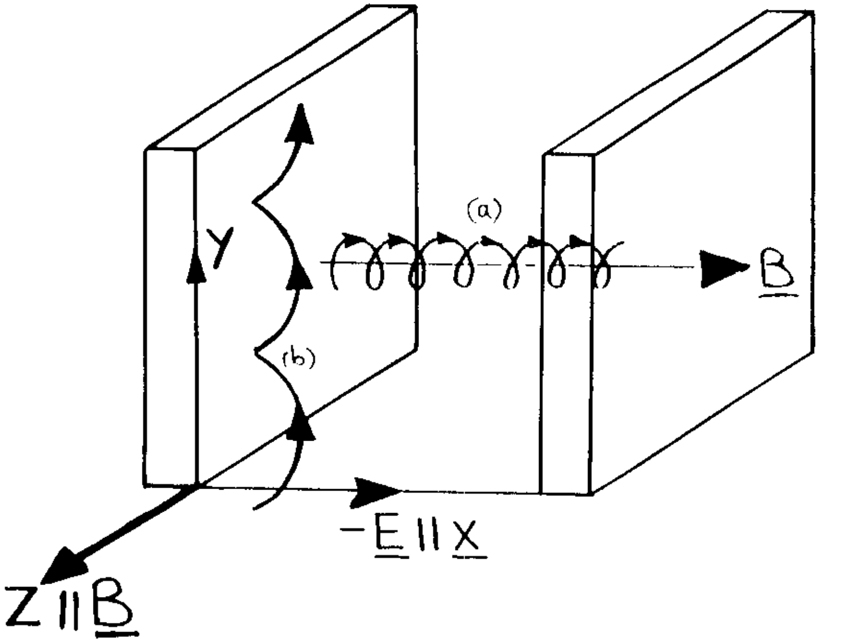
HOW DOES THIS RELATE TO MAGNETOTUNNELLING IN WIDE-WELL DOUBLE-BARRIER STRUCTURES ?

MOTION SEPARABLE & REGULAR!



① Consider two parallel potential barriers which lie parallel to the Y-Z plane and have a uniform electric field oriented along the normal to the barrier interfaces

② We saw in 1st lecture that classical motion between the barriers is separable – the electrons execute traversing orbits and are specularly reflected from the surface of each barrier in turn. There is a constant time for periodic return to the LH (emitter) barrier



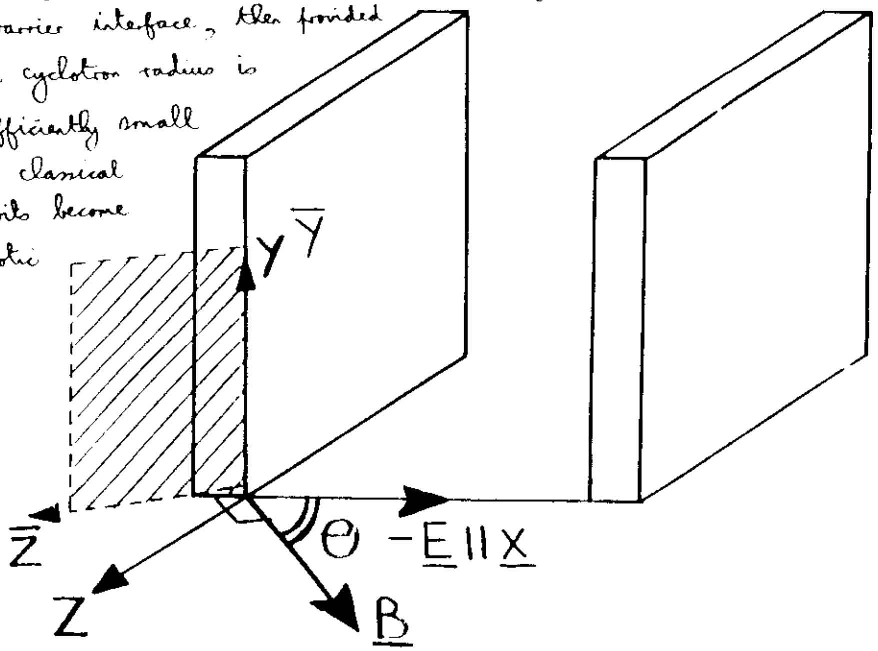
① If we apply a strong B - field either along the normal to the barrier interface or in the plane of the barrier the motion is still regular & consists either of

(a) Traversing orbits with additional helical 'corkscrew' motion about B

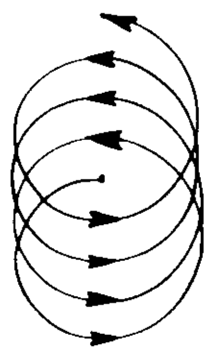
(c) Skipping orbits along the barrier interfaces (which become traversing if B is reduced to small value as we saw in 1st lecture).

TILTED B = CHAOS !

① If we tilt B to be at an angle θ to normal to the barrier interface, then provided the cyclotron radius is sufficiently small the classical orbits become chaotic



$V_d = E \sin\theta / B$



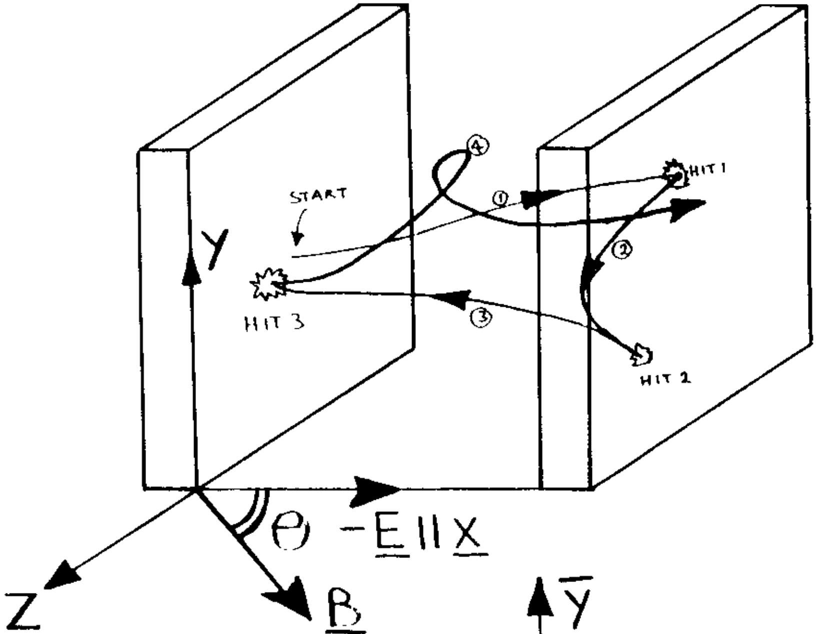
CYCLOID

CONSTANT ACCELERATION

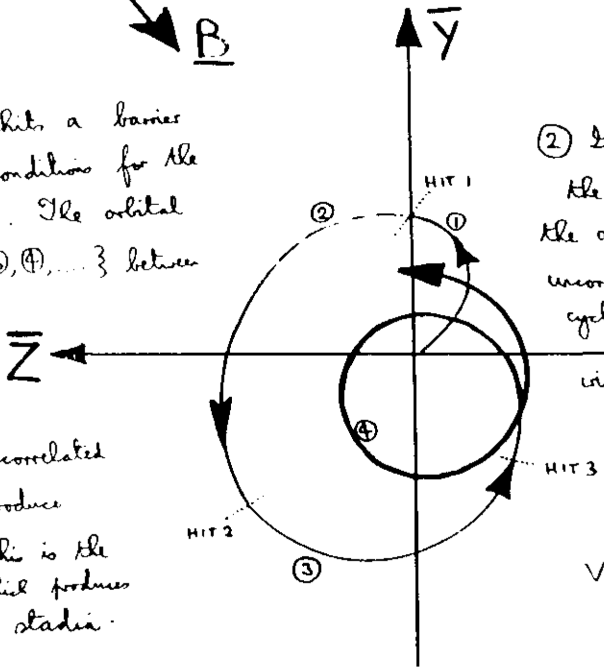
$\propto E\cos\theta$

② Motion between hits on the barriers is still regular & separable though & comprises constant acceleration along B and a cycloidal drift in the orthogonal ($\bar{Y}-\bar{Z}$) plane. So where does the chaos come from?

TILTED $\underline{B}$ = CHAOS!



① Each time e⁻ hits a barrier the boundary conditions for the orbital motion are reset. The orbital segments {①,②,③,④,....} between hits, even though individually entirely predictable, rapidly become uncorrelated and collectively produce chaotic paths. This is the same mechanism which produces chaos in billiard stadia.



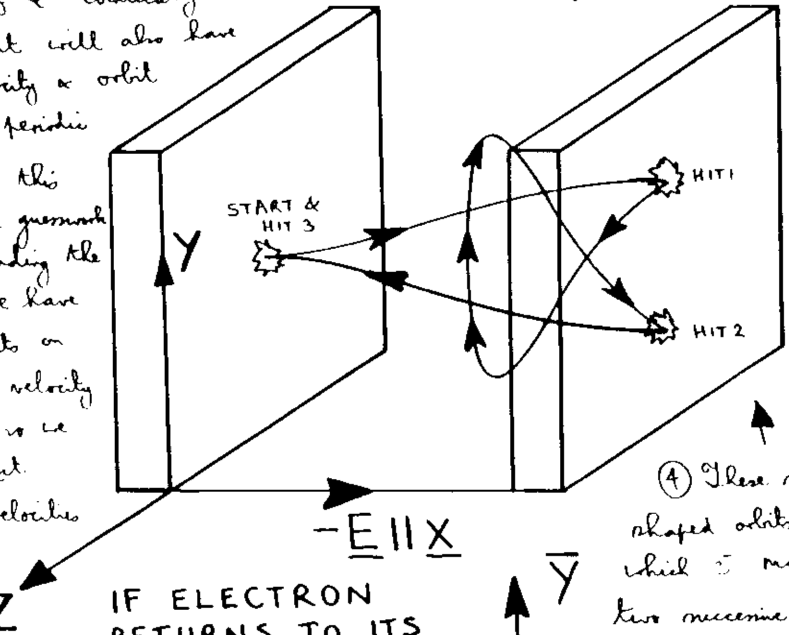
② In projection on the $\bar{Y} - \bar{Z}$ plane the orbit comprises uncorrelated cycloidal drifts with distinct radii, but the same drift velocity

$$V_d = \frac{E \sin\theta}{B}$$

① An infinite number of periodic orbits do, however, exist with different topologies specified by the sequence of hits on the two barriers

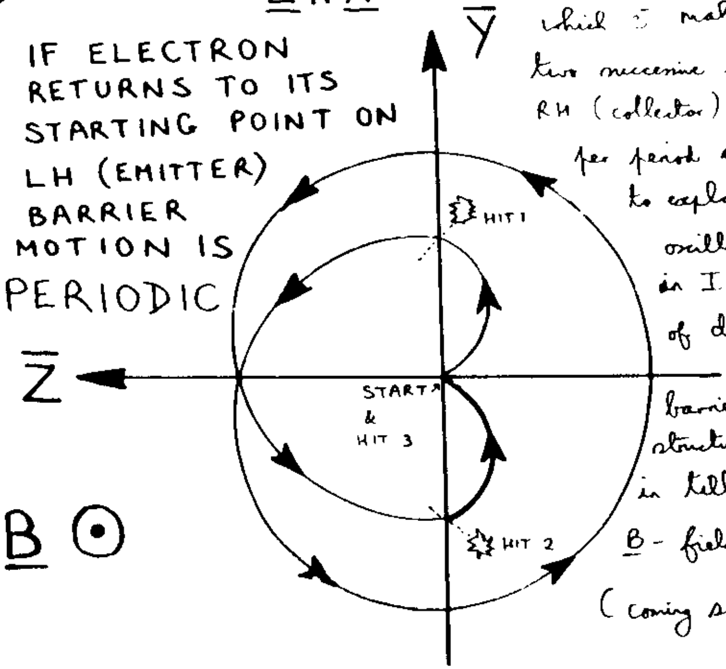
② Because motion between hits is simple & analytical it's easy to show that if e⁻ eventually returns to same point (starting point) on emitter barrier it will also have same velocity & orbit will be periodic

③ Great! This takes the guesswork out of finding the orbits – we have 2 constraints on 3 starting velocity components so we can predict starting velocities required for periodic orbits of arbitrary topology.

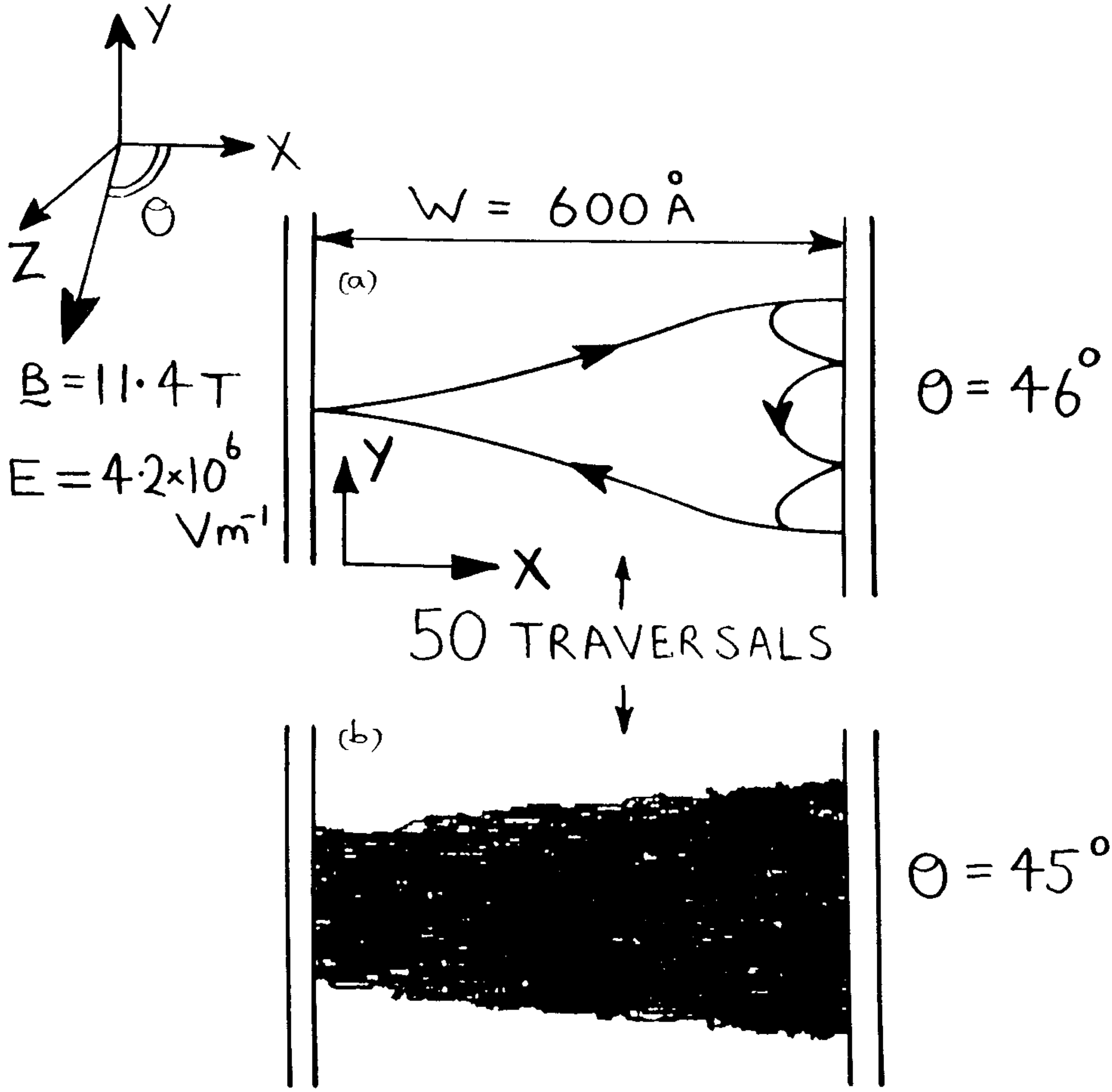


④ These star-shaped orbits in which e⁻ makes two successive hits on RH (collector) barrier per period crucial to explaining oscillations in I(V) plot of double-barrier structure in tilted $\underline{B}$-field (coming soon!)

IF ELECTRON RETURNS TO ITS STARTING POINT ON LH (EMITTER) BARRIER MOTION IS PERIODIC



STABILITY OF CLASSICAL ORBITS



(a) Shows a periodic orbit in a 600 Å well for $\theta = 46^\circ$. Electron makes 3 hits on collector per period and traverses orbit > 50 times without us seeing appreciable deviation

(b) Reducing θ by just one degree means that orbit becomes aperiodic over time. Take for 50 traversals of periodic orbit (a). Orbits show instability characteristic of chaotic system (sensitivity to initial and system conditions)

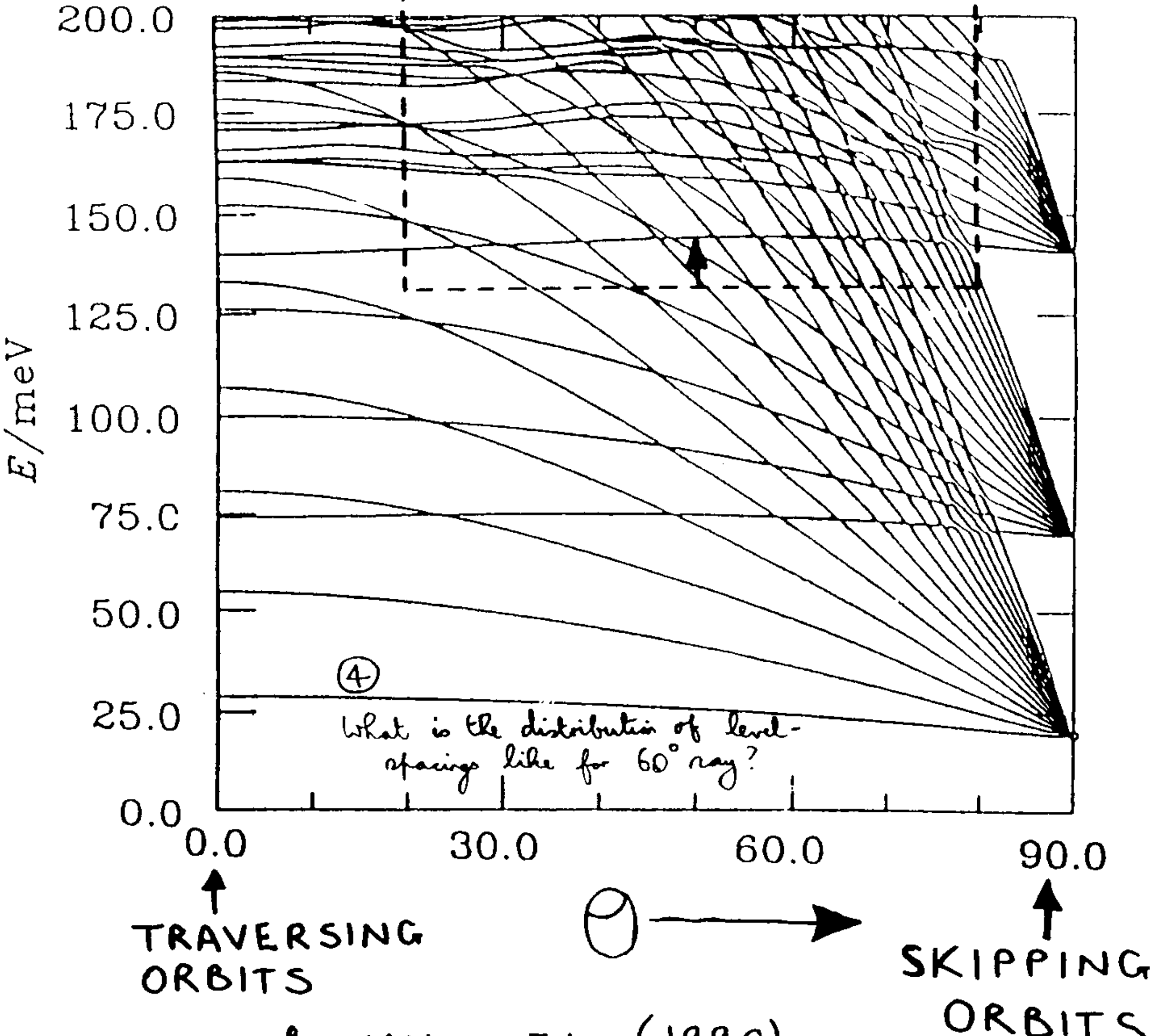
① What about the energy level pattern – how does this change as a function of tilt angle θ?

ENERGY LEVELS OF A 150 Å QUANTUM WELL IN A TILTED 15 T B-FIELD

② For θ ≈ 0° or 90° where classical motion is regular so are energy levels.

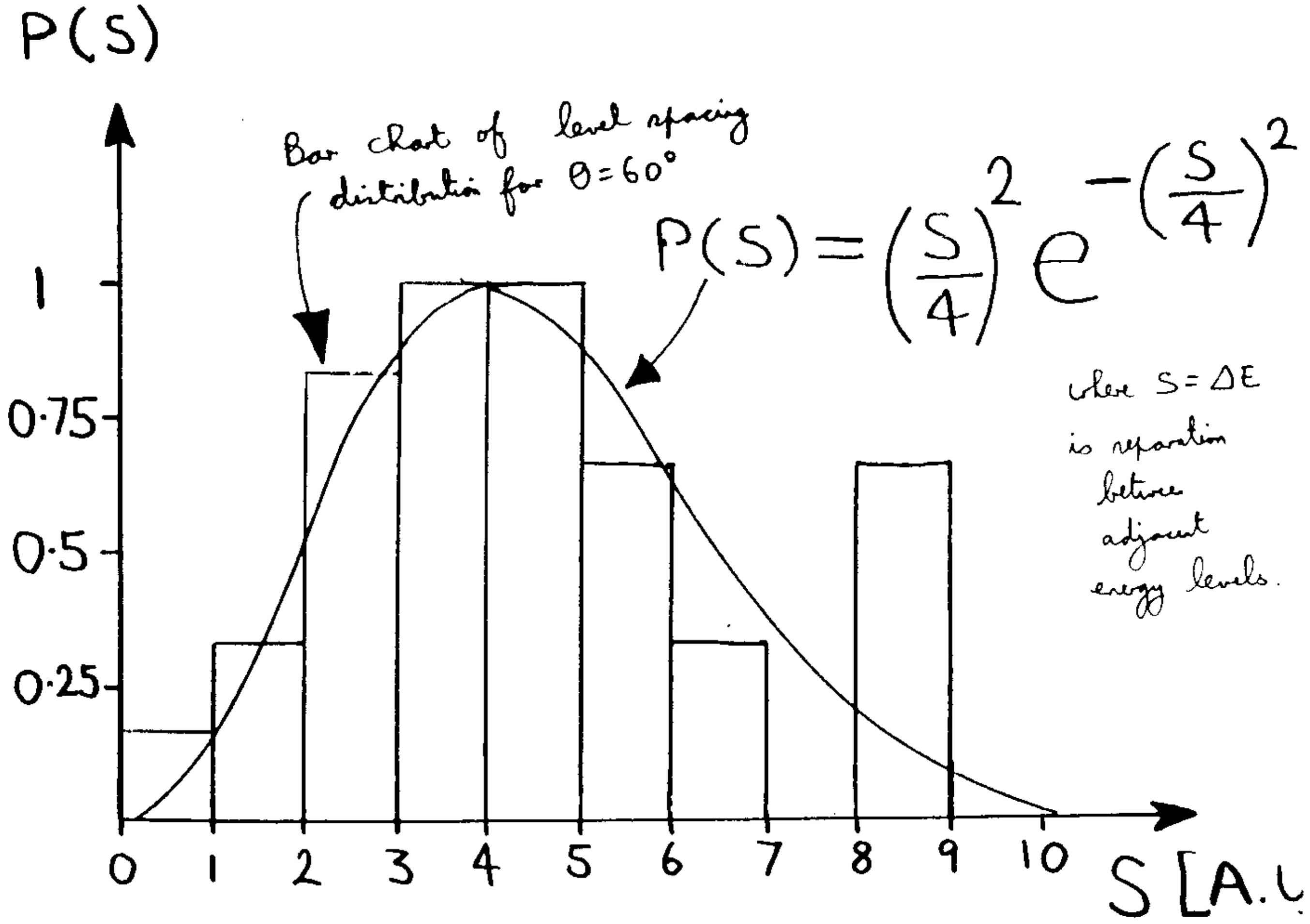
③ However for intermediate tilt angles a energy ≳ 125 meV adjacent levels interact strongly & are messy, c.f. chaotic H-atom.

20° CHAOS ? 80°



MARX & KUMMEL (1990)
Springer Series in Solid-State Sciences, $\frac{101}{(1992)}$ High Magnetic fields in Semiconductor Physics III, Springer-Verlag, p 180.

DISTRIBUTION OF LEVEL SPACINGS FOR 60° TILT



$$P(S) = \left(\frac{S}{4}\right)^2 e^{-\left(\frac{S}{4}\right)^2}$$

① Energy level spacings, 'unfolded' so that the mean separation is 4 (1 is more usual!), follow a Wigner-type distribution.

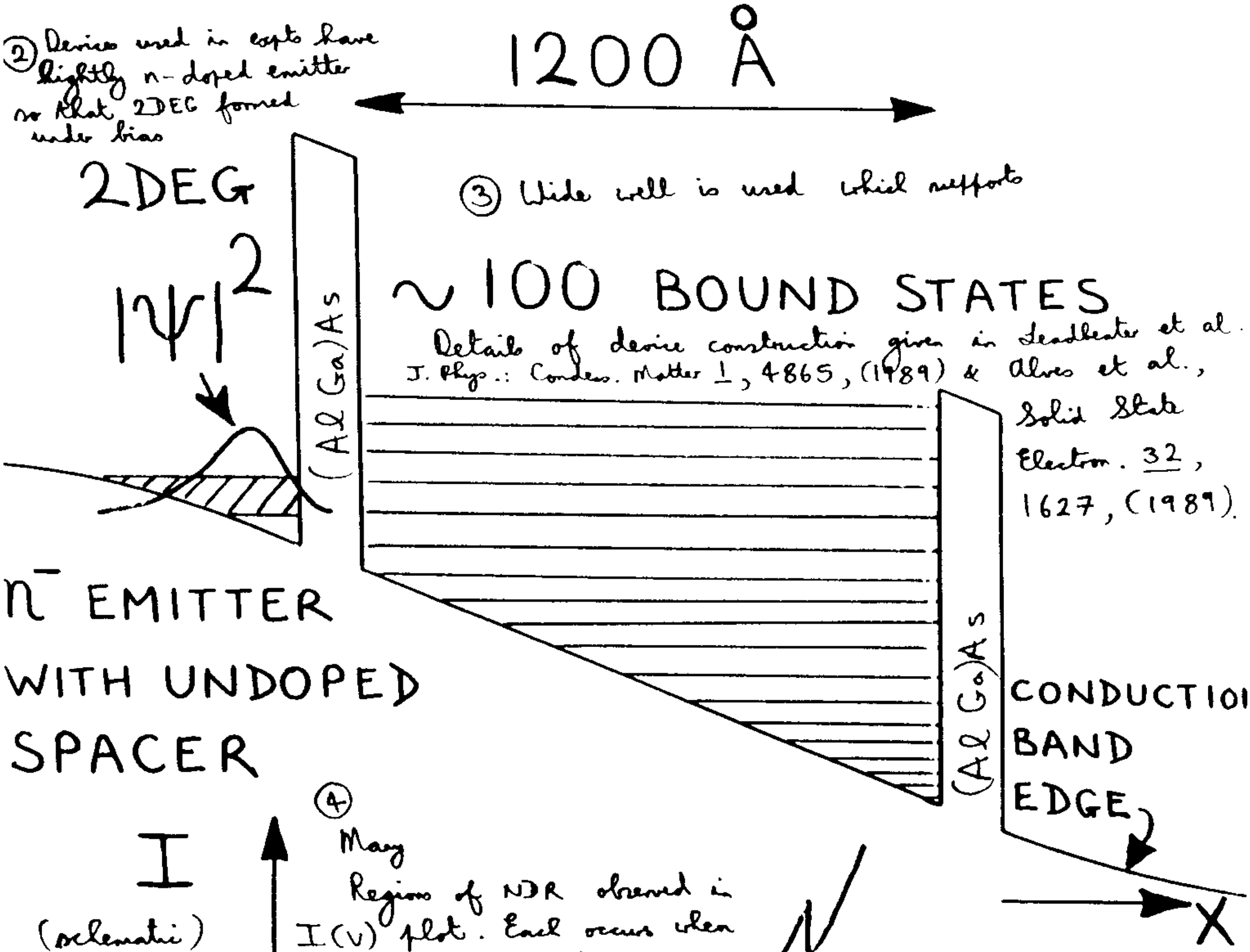
② This same distribution applies to the eigenvalues of a complex Hermitian matrix with random elements and is the universal, defining, characteristic of a quantum chaotic system which does not possess time-reversal symmetry (Berry, "The Bakerian Lecture", Proc. R. Soc. Lond. A, 413, 183, (1987) & references therein.)

INVESTIGATION OF LEVEL PATTERN USING RESONANT MAGNETOTUNNELLING SPECTROSCOPY

① We can probe the energy level spectrum by incorporating the double barriers in a resonant tunnelling diode

② Devices used in expts have lightly n-doped emitter so that 2DEG formed under bias

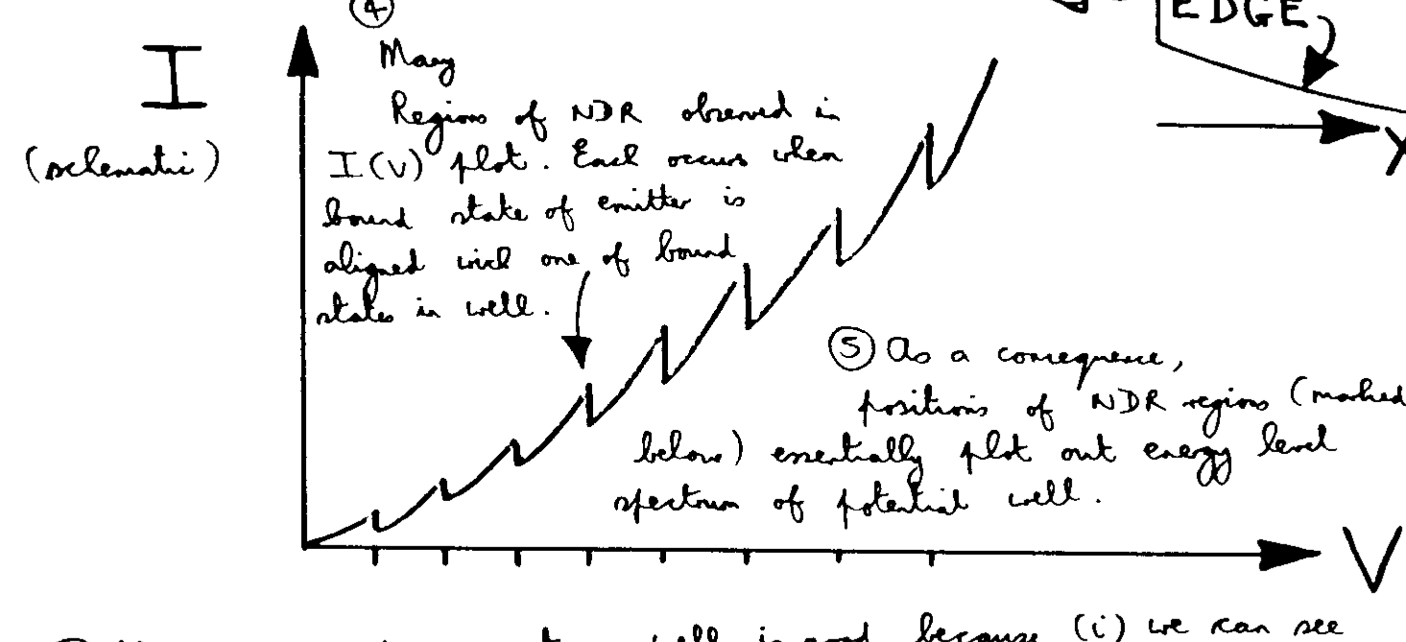
③ Wide well is used which supports



Details of device construction given in Leadbeater et al. J. Phys.: Condens. Matter 1, 4865, (1989) & Alves et al., Solid State Electron. 32, 1627, (1989).

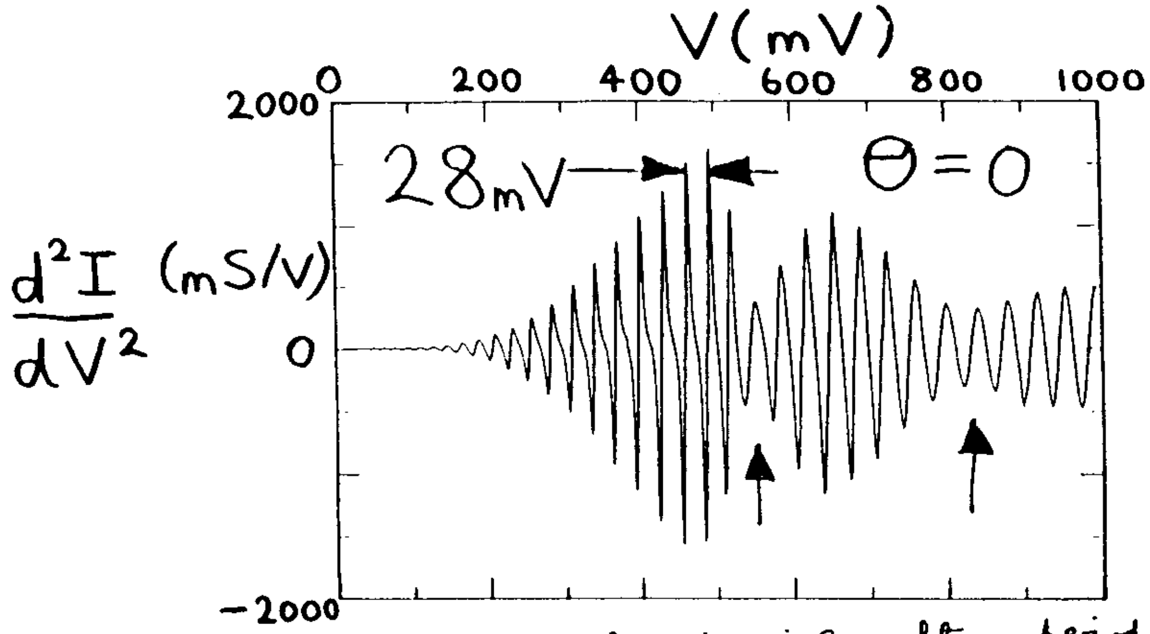
④ Many Regions of NDR observed in I(V) plot. Each occurs when bound state of emitter is aligned with one of bound states in well.

⑤ As a consequence, positions of NDR regions (marked below) essentially plot out energy level spectrum of potential well.



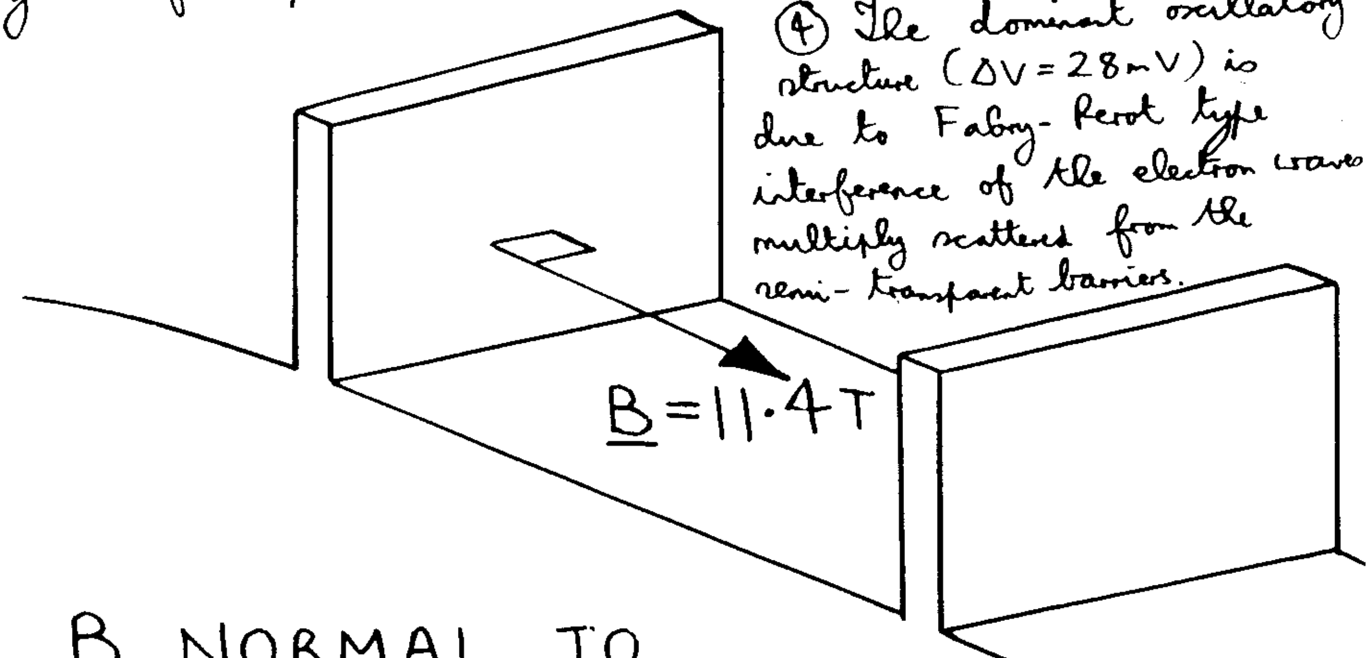
⑥ Having a wide quantum well is good because (i) we can see a large number of energy levels, (ii) a semiclassical analysis is appropriate but bad because NDR regions are closely-spaced & relatively poorly resolved in I(V) — we can enhance it by taking derivative plots though:—

① Experimental d^2I/dV^2 plot for the resonant tunnelling mode.
$\underline{B} = 11{\cdot}4\,T$ is applied along normal to barrier interfaces (see below).
② In this geometry $\underline{B}$ doesn't significantly perturb motion normal to the barriers & d^2I/dV^2 is essentially identical to when $\underline{B} = 0$.



③ Quasi-periodic oscillations observed with voltage period $\Delta V \approx 28\,mV$
The beating with minima indicated by vertical arrows above originates from quantum interference above the RH collector barrier.

④ The dominant oscillatory structure ($\Delta V = 28\,mV$) is due to Fabry-Perot type interference of the electron waves multiply scattered from the semi-transparent barriers.



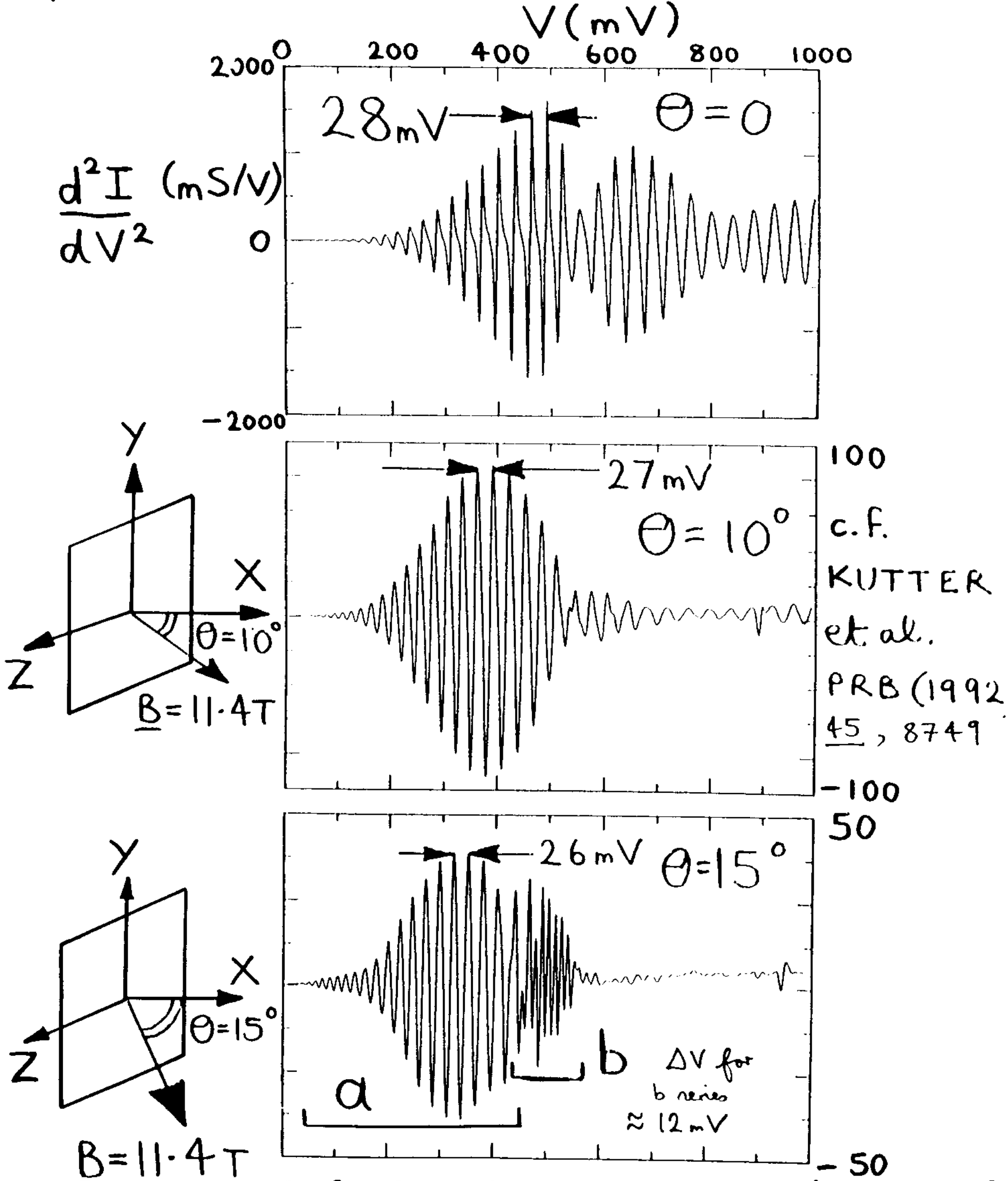
$\underline{B}$ NORMAL TO BARRIER INTERFACES

($\Theta = 0$)

⑤ How does the resonant structure in d^2I/dV^2 evolve as $\underline{B}$ is tilted?? ⇒

① As Θ increased from 0° to 10° resonant peaks evolve smoothly (
& voltage period drops slightly

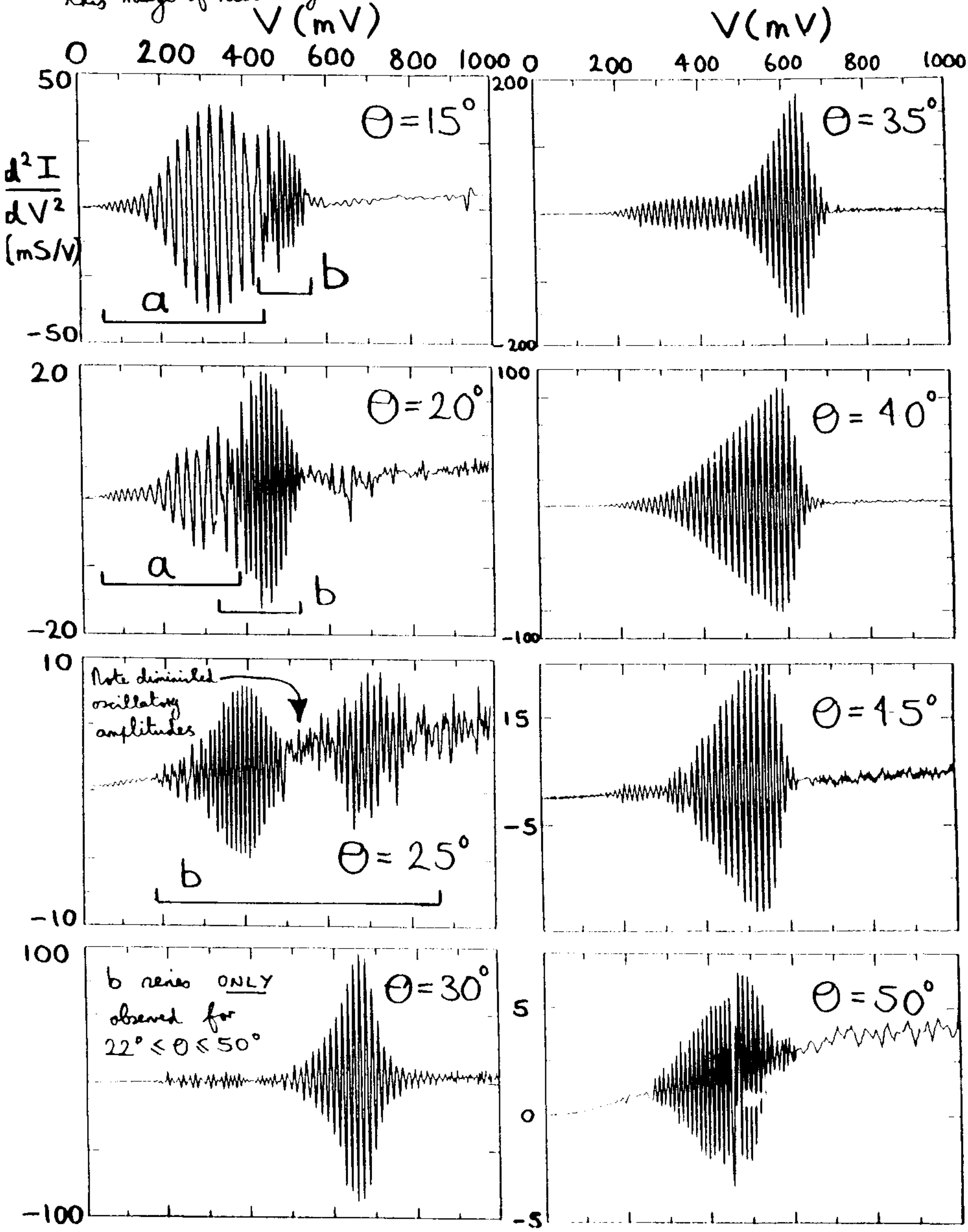
② Voltage period continues to decrease as Θ increased to 15°. Most important though is sudden reduction of ΔV by factor ~2 seen for $V \approx 450$ mV. Two series of oscillations are therefore seen – labelled (a) & (b).



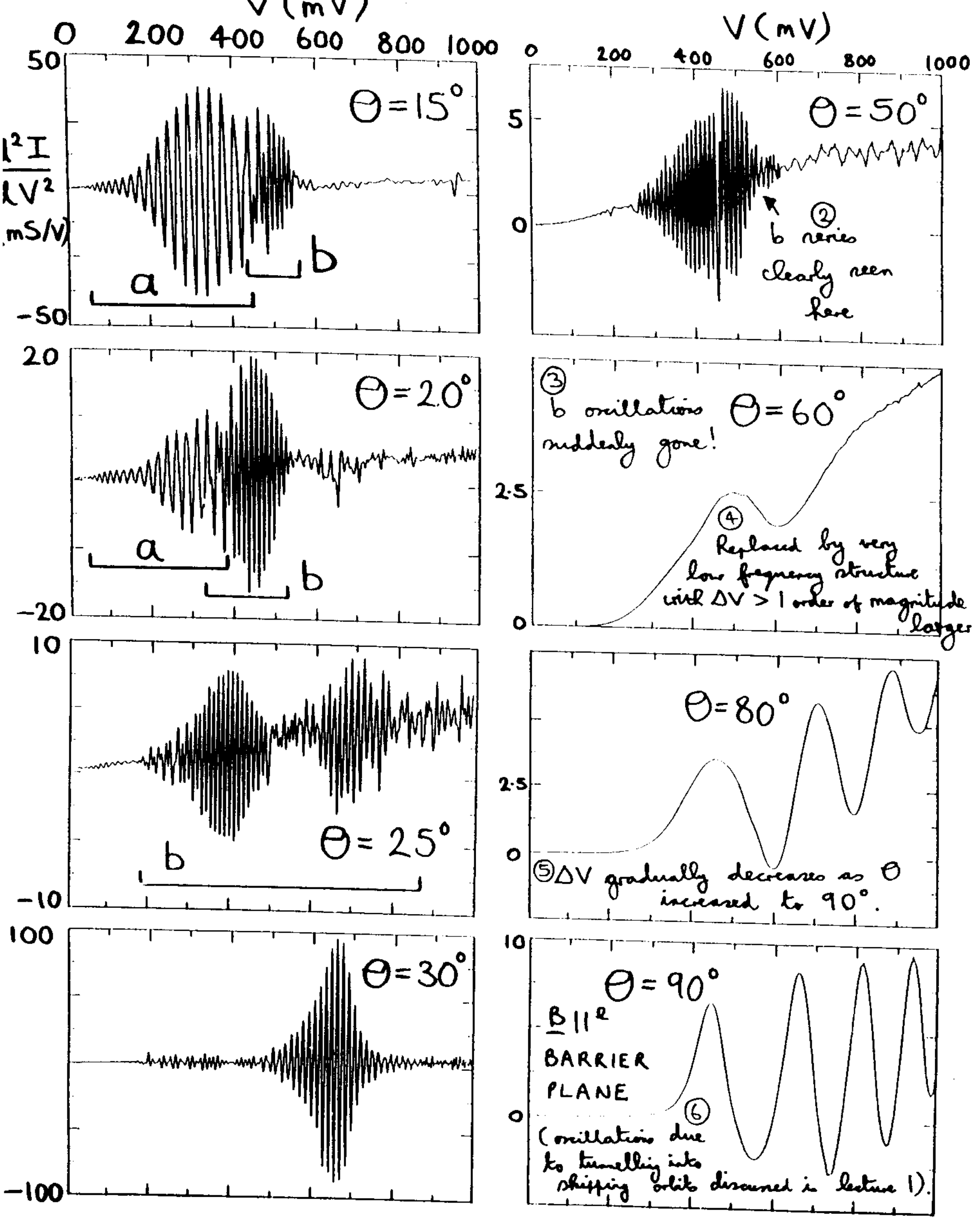
③ What happens to series a & b if Θ is further increased?

① Boundary between a and b series shift to lower V with increasing θ (< 22°)

② b series dominant for $22° \leq \theta \leq 50°$. Voltage period hardly changes over this range of tilt angles.



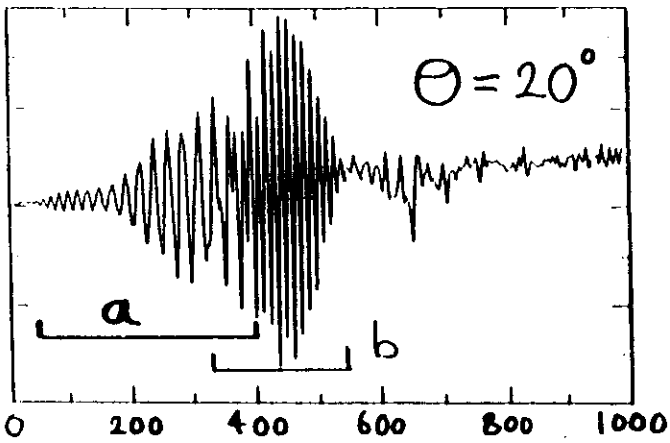
① There is a very dramatic change in the resonant structure as θ is increased from 50° to 60° – b series disappears to be replaced by extremely low frequency modulation

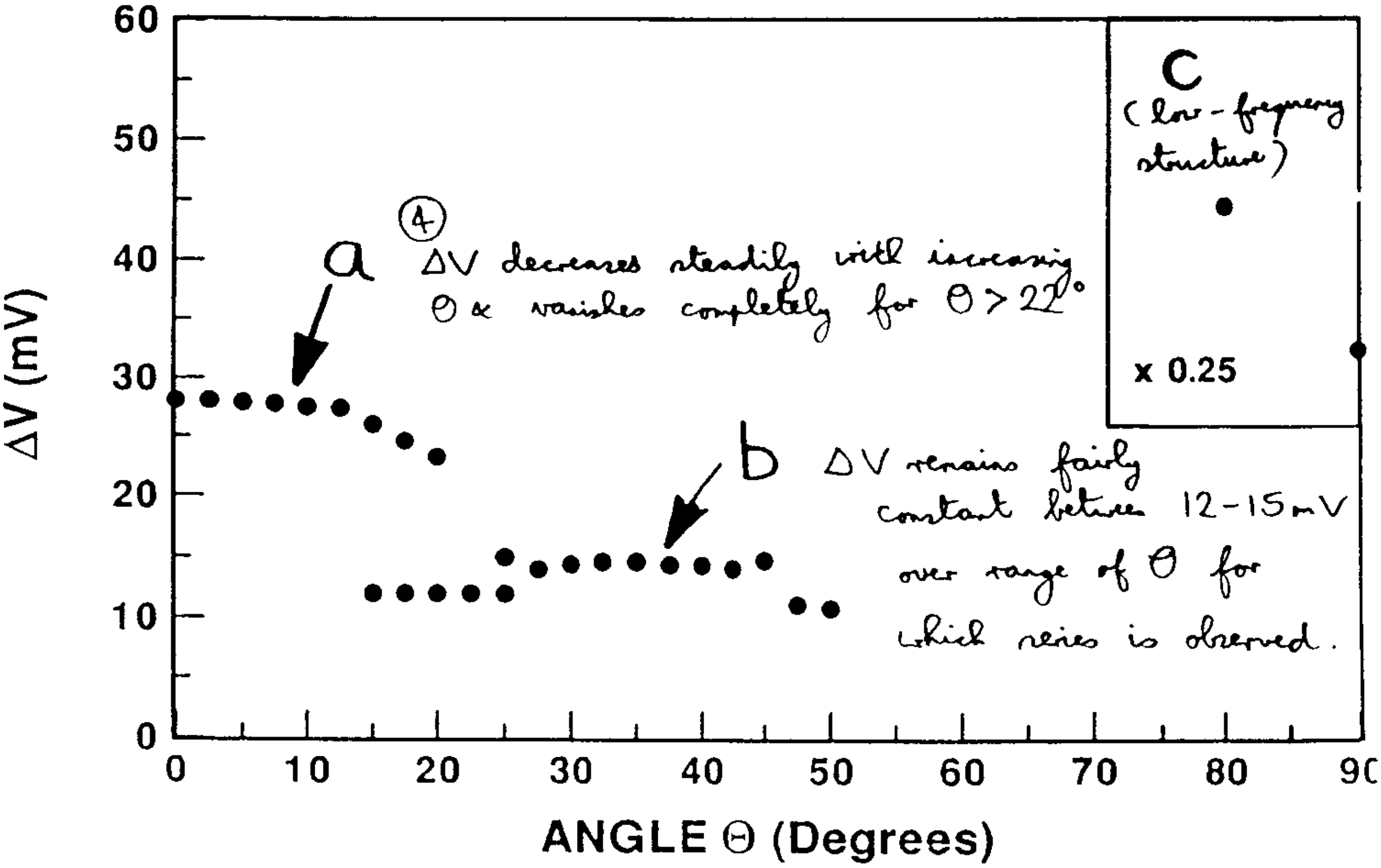


① Would now like to summarise how (a) the voltage period (b) the voltage ranges of the a & b series change with Θ.

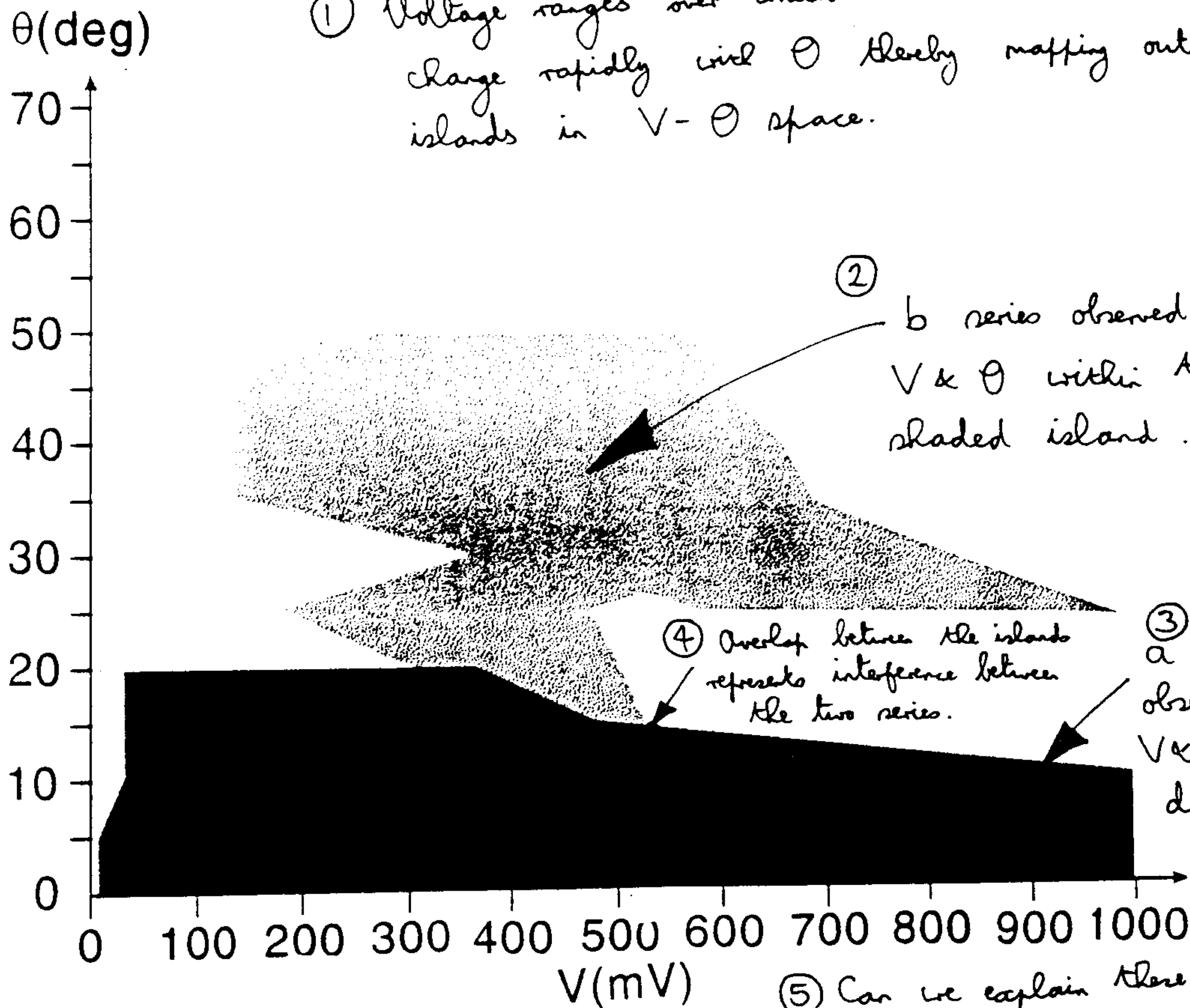
② Will later compare these key features of the evolution with the prediction of a semiclassical model based on the chaotic dynamics within the potential well.

③ First let's see how ΔV for the a & b series (illustrated right) changes with Θ :—





① Voltage ranges over which a & b series observed change rapidly with θ thereby mapping out these islands in V-θ space.



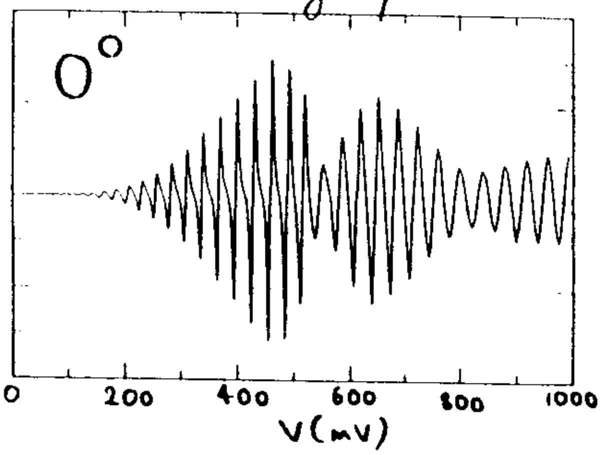
② b series observed for all V & θ within this lighter shaded island.

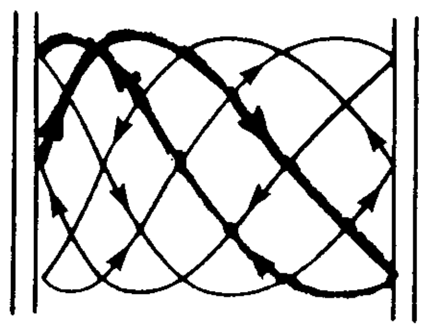
③ a series observed for V & θ within darker island

⑤ Can we explain these shapes??

(2) Even though oscillatory structure is very different for these cases they are a good starting point because the motion is regular & separable.

(3) Because resonant structure originates from tunnelling into energy levels in the well with high quantum numbers, Correspondence principle is applicable.





$\tau = \cdot 27$ ps

$\Downarrow$

$\Delta V = 38$ mV

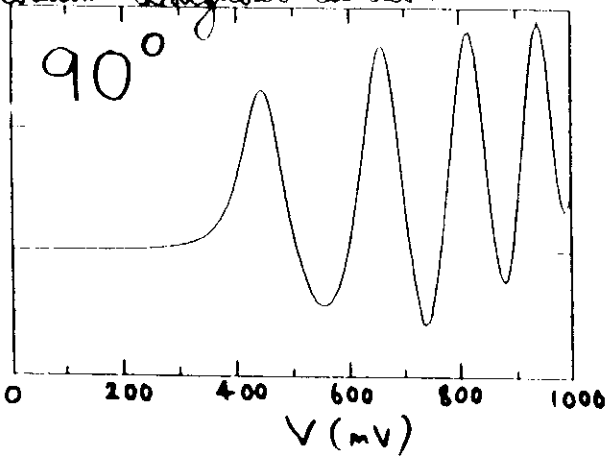
(5) For $\theta = 0°$ classical orbits are helical shown above in projection. We predict $\Delta V = 38$ mV. This is slightly larger than the observed value of ≈ 28 mV because we neglect the effect of nonparabolicity (which raises e^- mass).

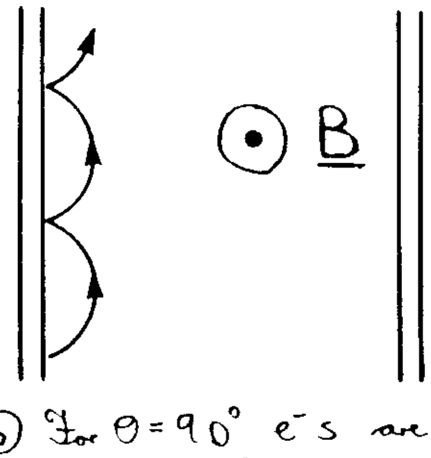
CORRESPONDENCE PRINCIPLE $\Rightarrow$

$$\Delta E = h/\tau = ef(V)\Delta V$$

(4) Correspondence principle enables us to relate voltage period ΔV to periodic time τ for return to the emitter barrier of electrons in classical orbits in the well. Function $f(V)$ depends on potential distribution throughout the device.

$\approx 1/2$ for entire range of applied bias voltage.



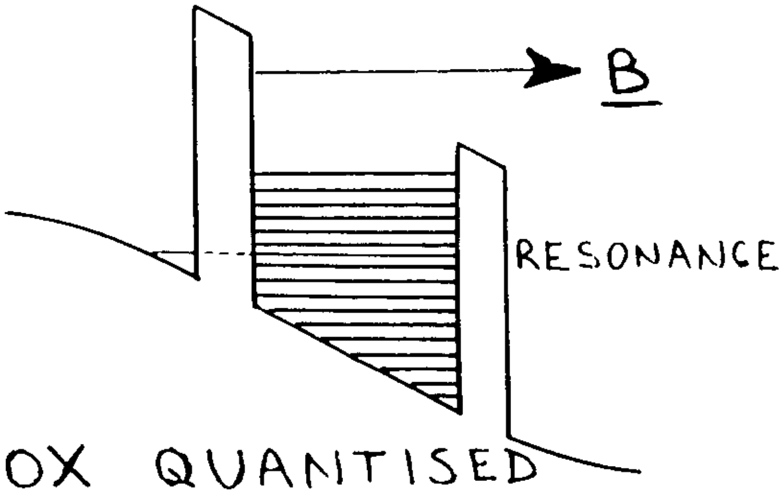


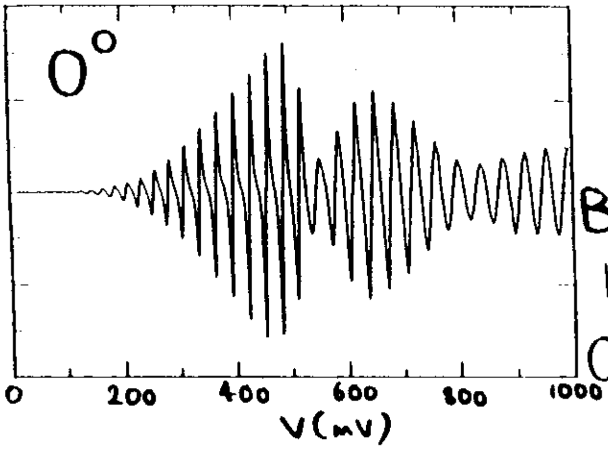
$\tau = 0{\cdot}09$ ps

$\Downarrow$

$\Delta V = 120$ mV

(6) For $\theta = 90°$ e^-s are forced to skip along barrier interface, as described in 1st lecture. Consequently both the orbital path length between hits on the emitter barrier and the return time τ are $\approx \frac{1}{3}$ the value for $\theta = 0° \Rightarrow$ which explains the increased ΔV values in the in-plane geometry.





BOX QUANTISED LEVELS INDEPENDENT OF B.

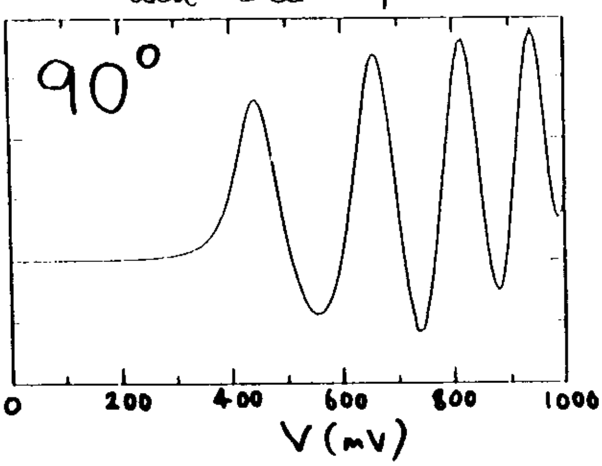
CORRESPONDENCE PRINCIPLE $\Rightarrow$

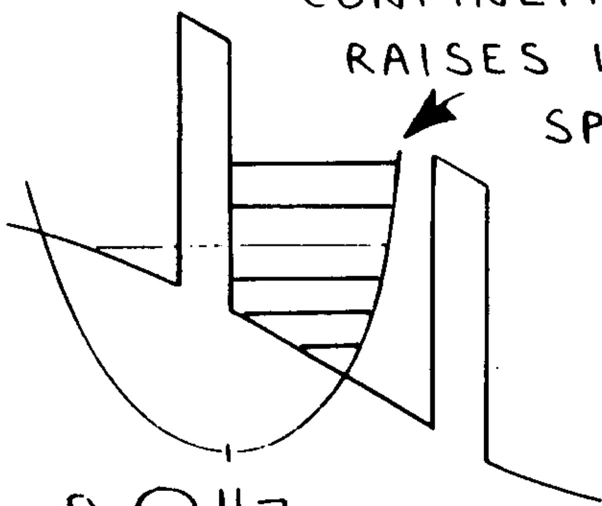
$$\Delta E = h/\tau = e f(V) \Delta V$$

$$\approx 1/2$$

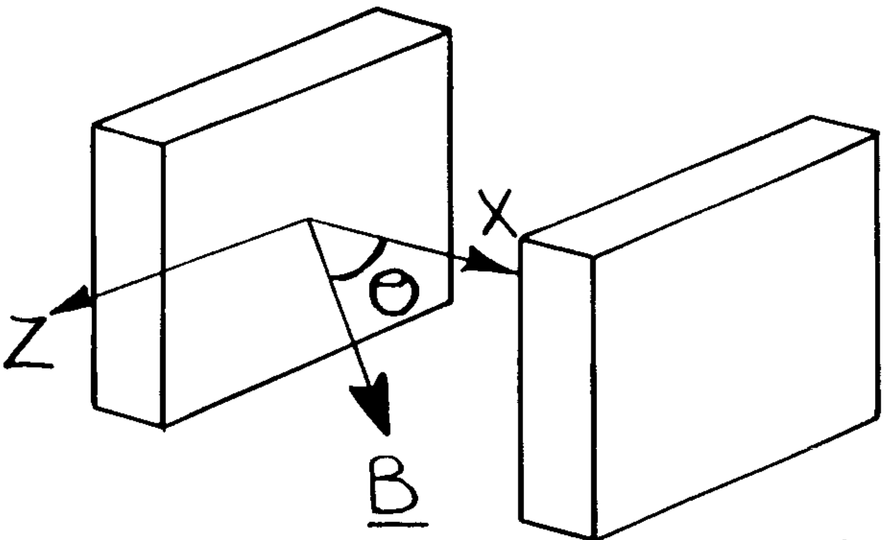
① In a quantum picture ΔV is greater for 90° than for 0° because the 1D magnetic potential (in the Landau gauge) squeezes the wavefunctions in the well thereby increasing the energy levels and their separation.

MAGNETIC CONFINEMENT RAISES LEVEL SPACING





$\underline{B} \odot \parallel z$



② Can we give a similar explanation at the intermediate tilt angles when, as we've seen, the classical motion is chaotic?

CHAOTIC

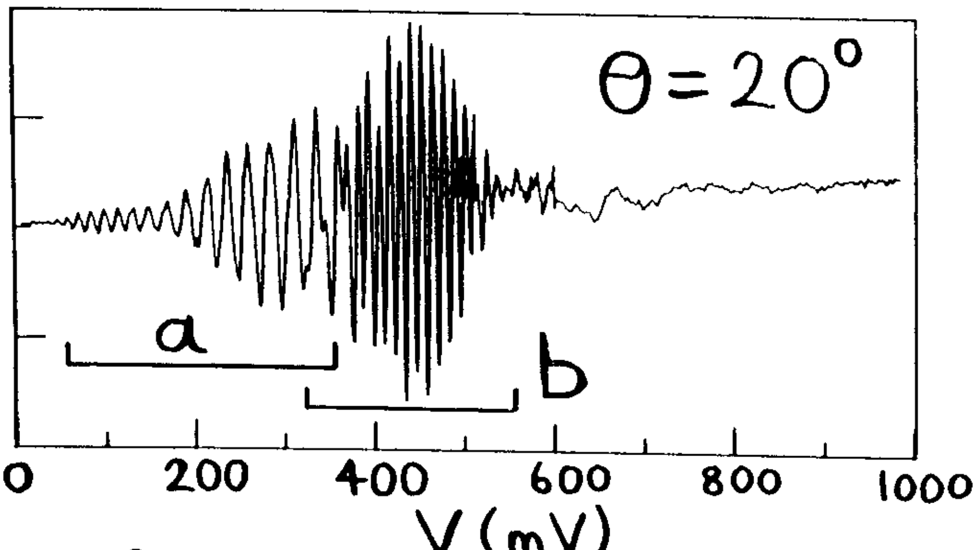
0° ⟷ 90°

SEPARABLE

NON – CHAOTIC

① We've seen that we can use a simple semiclassical model to give a quantitative explanation for the ΔV values when the classical motion in the potential well is regular.

① Consider $\theta = 20^\circ$. Can we come up with a model which accounts for
(a) The origin of the a & b resonances
(b) Their distinct voltage periods ΔV
(c) Their voltage ranges. ?? If so we can then change θ & see if the observed evolution of the resonant structure is also reproduced.

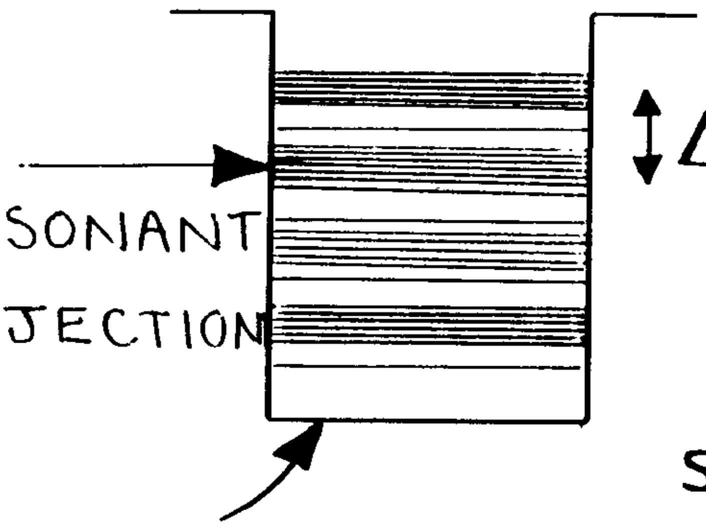


② We have some help in our quest because in the chaotic regime the Gutzwiller trace formula may be applied. (Gutzwiller, "Chaos in Classical and Quantum Mechanics", Springer, New York, 1990)

CHAOTIC REGIME:

GUTZWILLER TRACE FORMULA

③ The formula asserts that the individual energy levels within the well will tend to bunch periodically & that the separation between adjacent level clusters is inversely related to the periods of the unstable PERIODIC orbits within the chaotic sea (c.f. quasi Landau level resonances in the H-atom)



RESONANT INJECTION

$$\Delta E = h/T$$

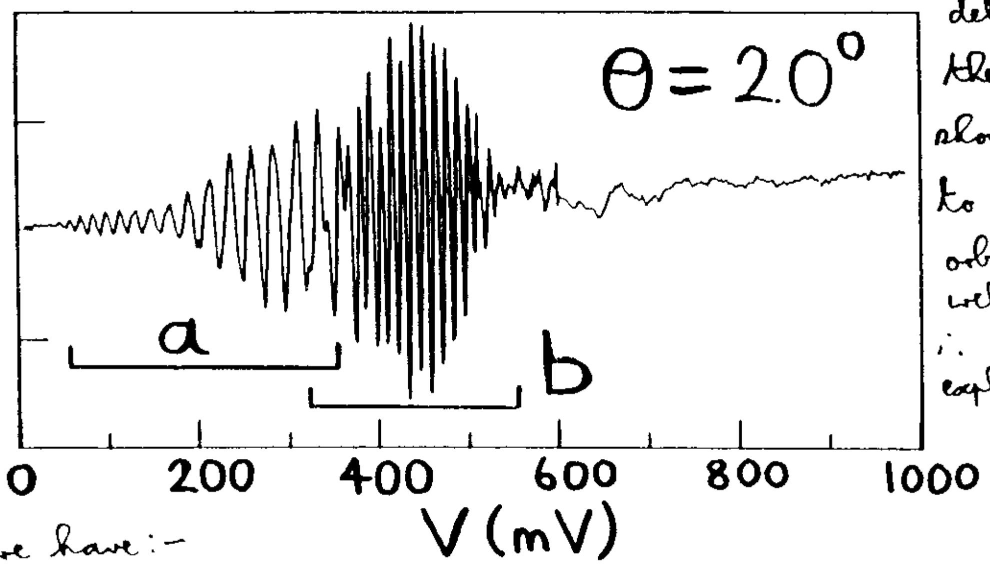
POTENTIAL WELL

PERIOD OF SHORTER REGULAR ORBITS

④ The formula is clearly analogous to the Correspondence principle, but is concerned with the spacing of level clusters rather than individual levels.

① Suppose that [illegible] broadens the levels sufficiently that we cannot resolve individual levels within the clusters, but we <u>can</u> resolve the clusters themselves.

② We would expect to see resonances in I(V) & derivatives due to resonant injection into the level clusters — & further, their separation which determines the ΔV values should relate to the periodic orbits of the well. We should $\therefore$ be able to explain the resonant structure by finding these orbits & seeing how they change with θ.



③ Therefore we have:-

- FOUND PERIODIC ORBITS IN POTENTIAL WELL WITH CONSTRAINTS:

1) STARTING VELOCITIES CONSISTENT WITH THOSE IN EMITTER CONTACT

2) $T \lesssim \pi \tau_{LO} \sim 0{\cdot}6\,\mathrm{ps}$

LO phonon emission time

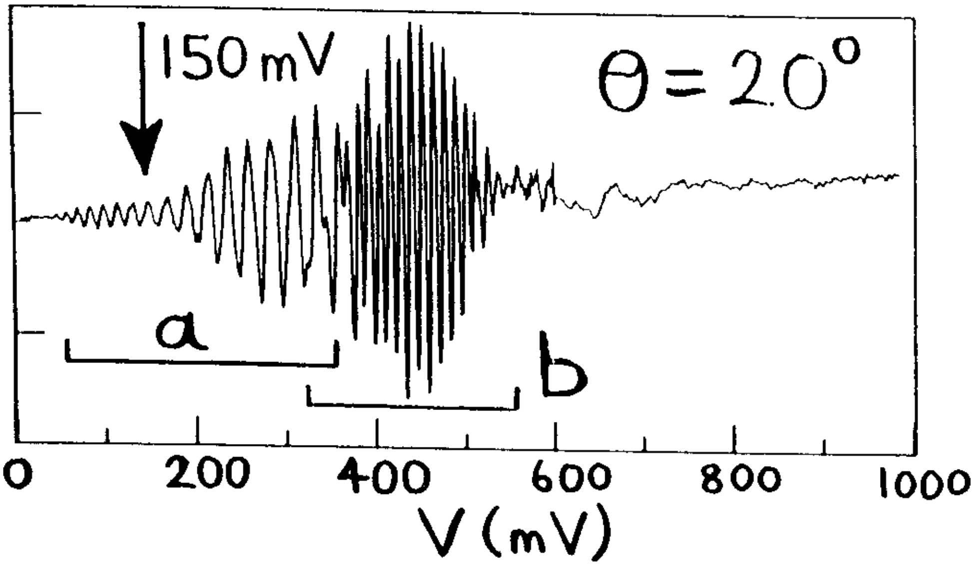
cut-off time ⇒ if period exceeds 0·6 ps we won't see any associated resonant structure.

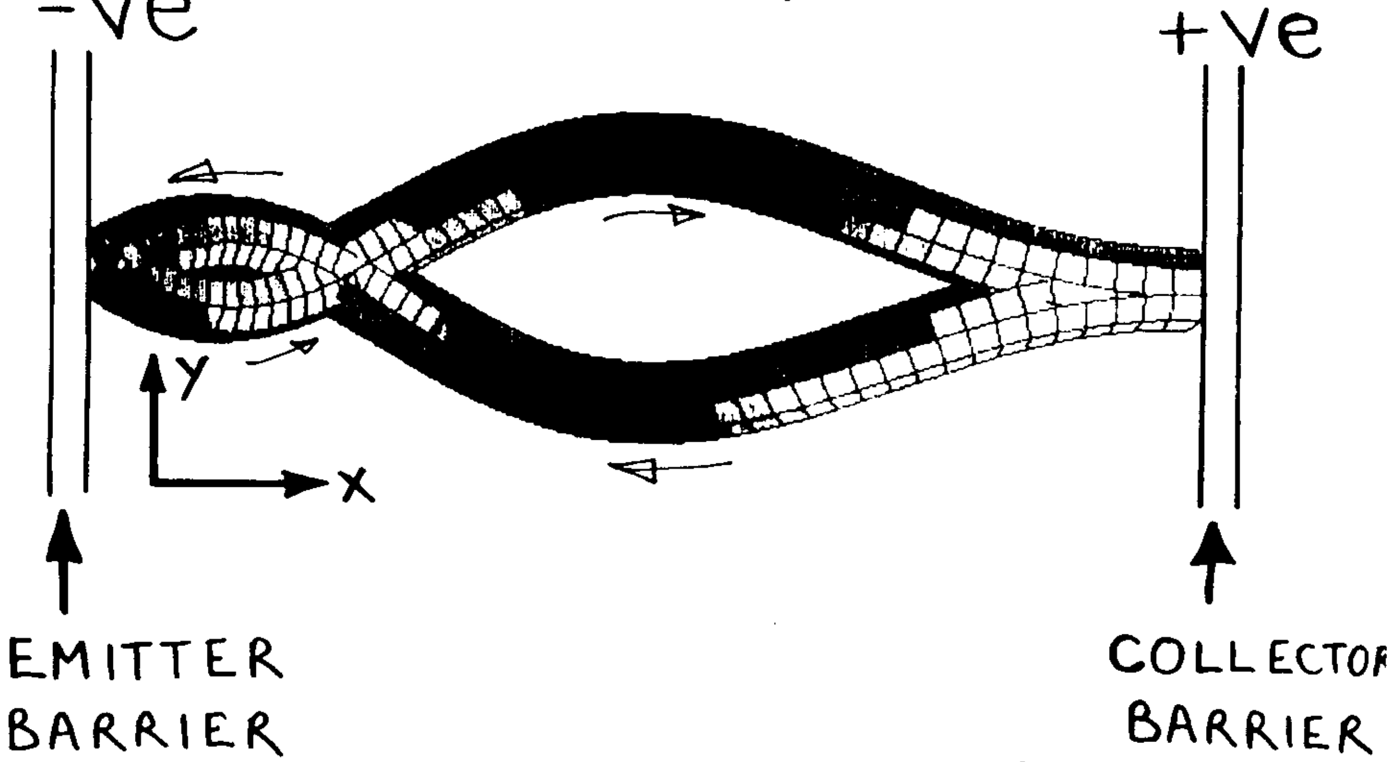
in order to resolve the clusters their spacing ΔE must satisfy

$$\left(\Delta E = \frac{h}{T} >> \frac{\hbar}{\tau_{LO}} \right)$$

① So what are the allowed periodic ( ) orbits which satisfy these constraints??

② Consider first low voltages & in particular $V = 150$ mV.

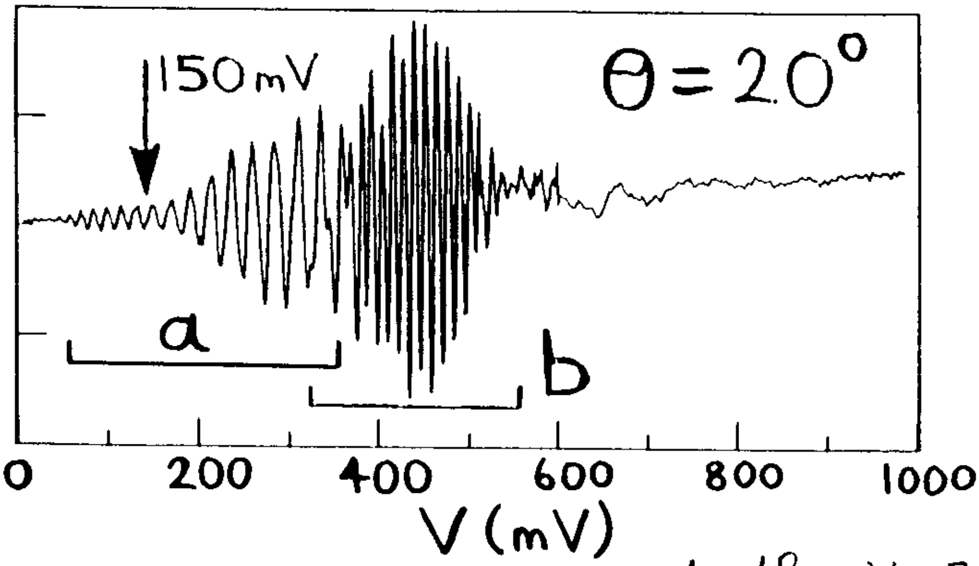




③ One type of allowed orbit is the periodic traversing orbit shown above. The 3D tube is intended to give some information about the orbital curvature. The electron trajectories pass right through the centre of the tube.

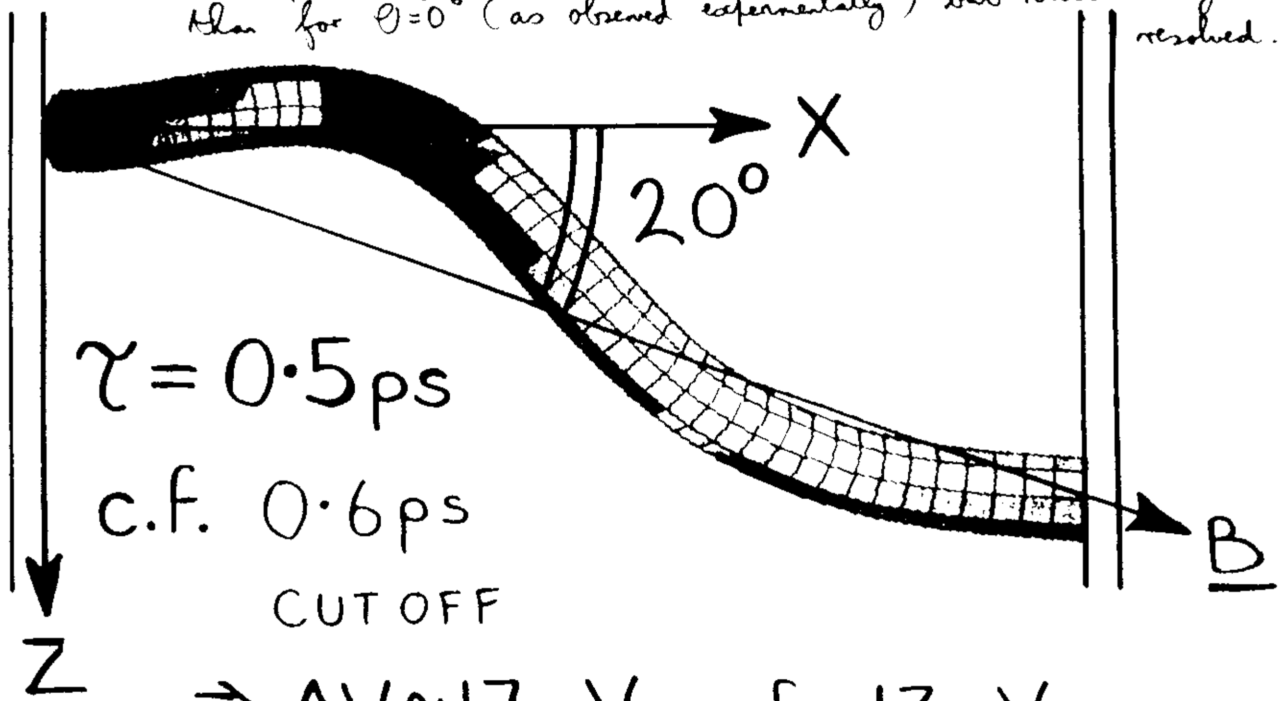
④ Electrons in these orbits hit alternately with the two barriers — so they are closely related to the helical traversing orbits for $\theta = 0^{\circ}$. However, there is a difference ⇒

THE PERIOD OF TRAVERSING ORBITS INCREASE WITH $\theta \Rightarrow$ WHICH EXPLAINS WHY ΔV IS OBSERVED TO DECREASE



① If we look at the orbit projected onto the X − Z plane in which the magnetic field lies we see that the electron tends to rotate about $\underline{B}$ (as expected).

② The orbital path length $\approx 2W/\cos\theta$ and period $T = 0{\cdot}5\,ps$ are ∴ longer than for the same voltage when $\theta = 0°$. The trace formula tells us that the period ΔV of the associated resonant structure will be smaller than for $\theta = 0°$ (as observed experimentally) but still clearly resolved.



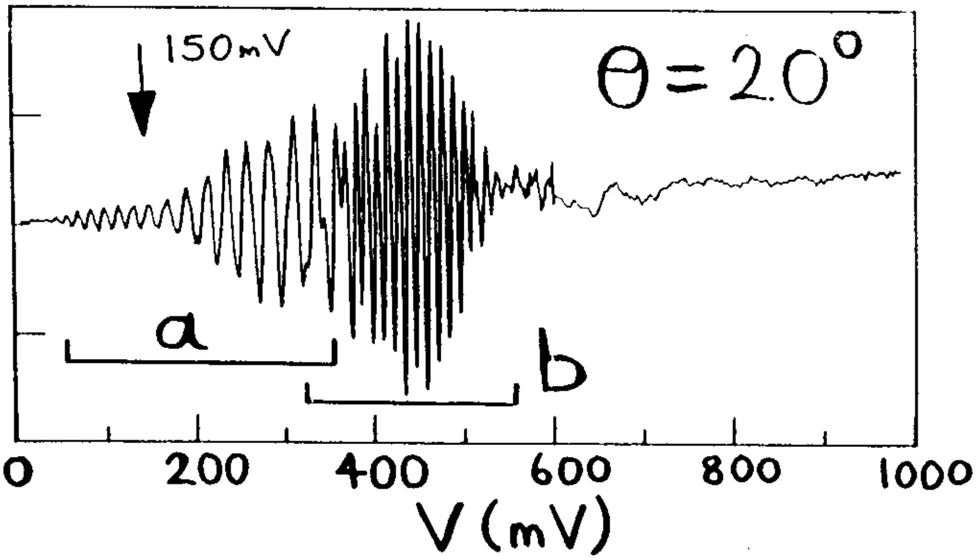
$\tau = 0{\cdot}5\,ps$

c.f. $0{\cdot}6\,ps$

CUTOFF

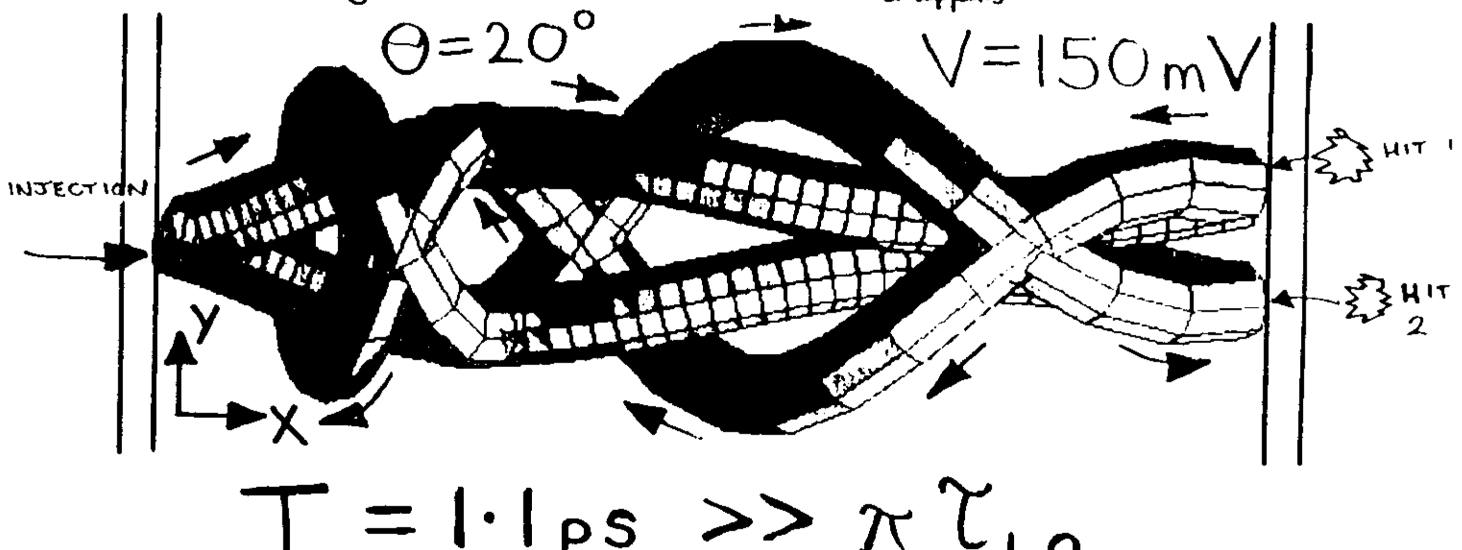
$\Rightarrow \Delta V \approx 17\,mV$ c.f. $13\,mV$ EXPERIMENT

① In addition to the traversing orbits for V = 150 mV, the electrons can also be injected into periodic orbits with a totally different topology (below). Electrons in these orbits make two successive collisions on the collector barrier per period.

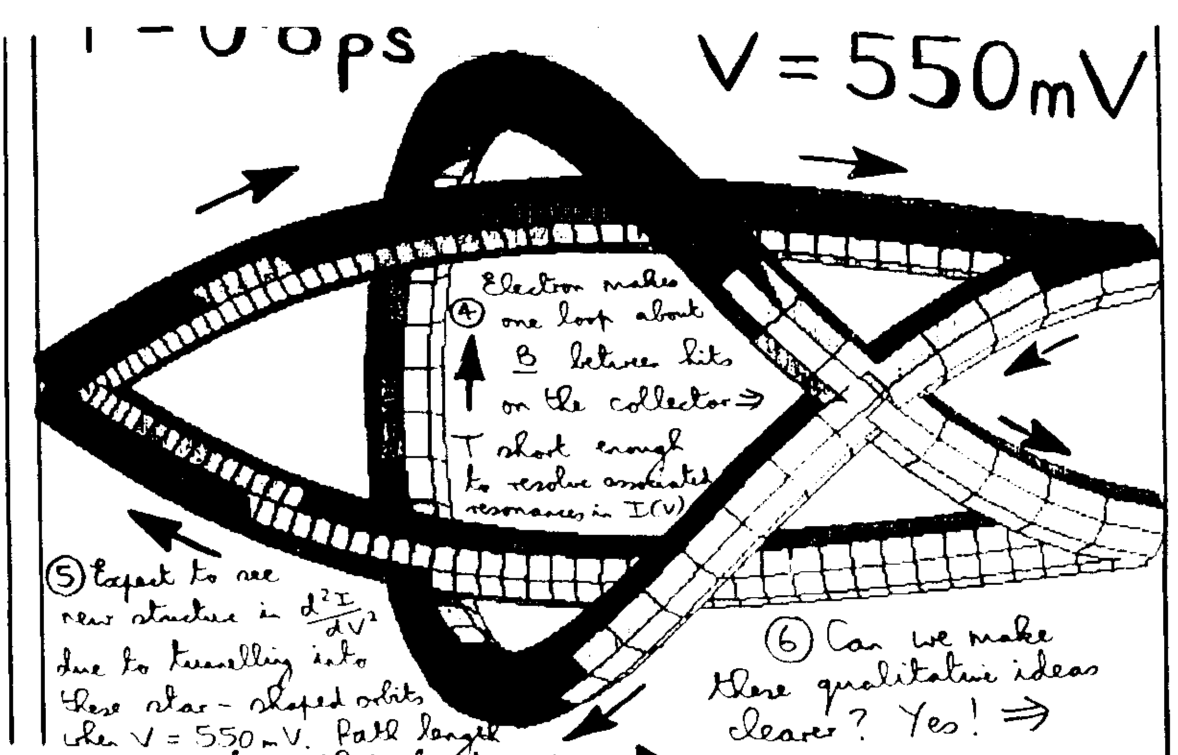


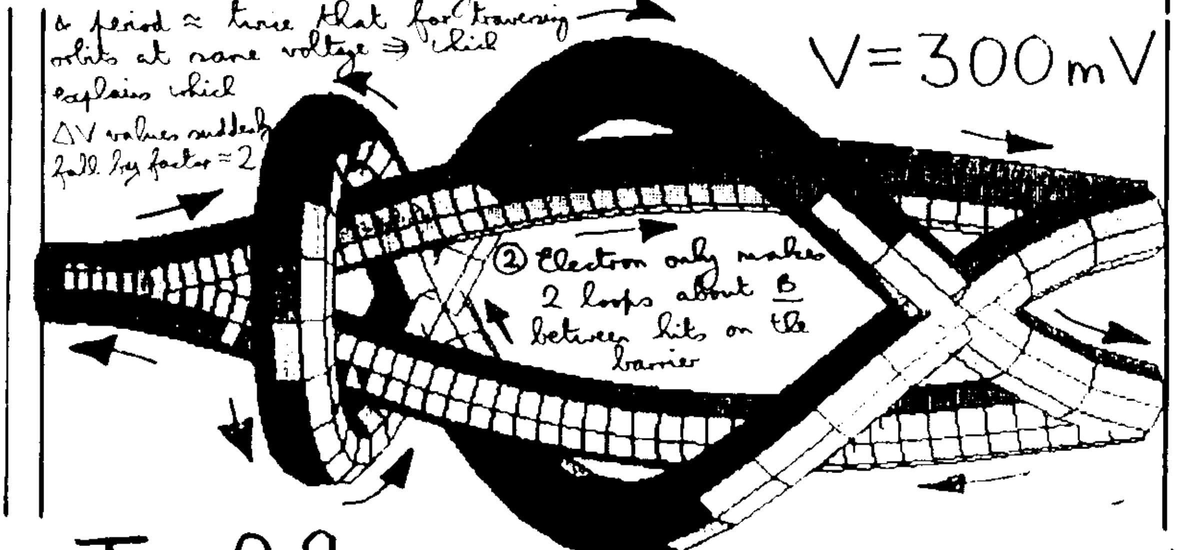
② After the first collision with the RH (collector) barrier the electron embarks on a twisting path, making three complete revolutions about the axis of $\underline{B}$ before returning to make the second hit on the collector.

③ Due to the twisting orbital path, the period T = 1·1 ps is longer than the cut-off time (0·6 ps) so we do not see any associated resonances in I(V). However, see what happens when we increase V ⇒



$$T = 1{\cdot}1\,\mathrm{ps} \gg \pi \tau_{LO}$$

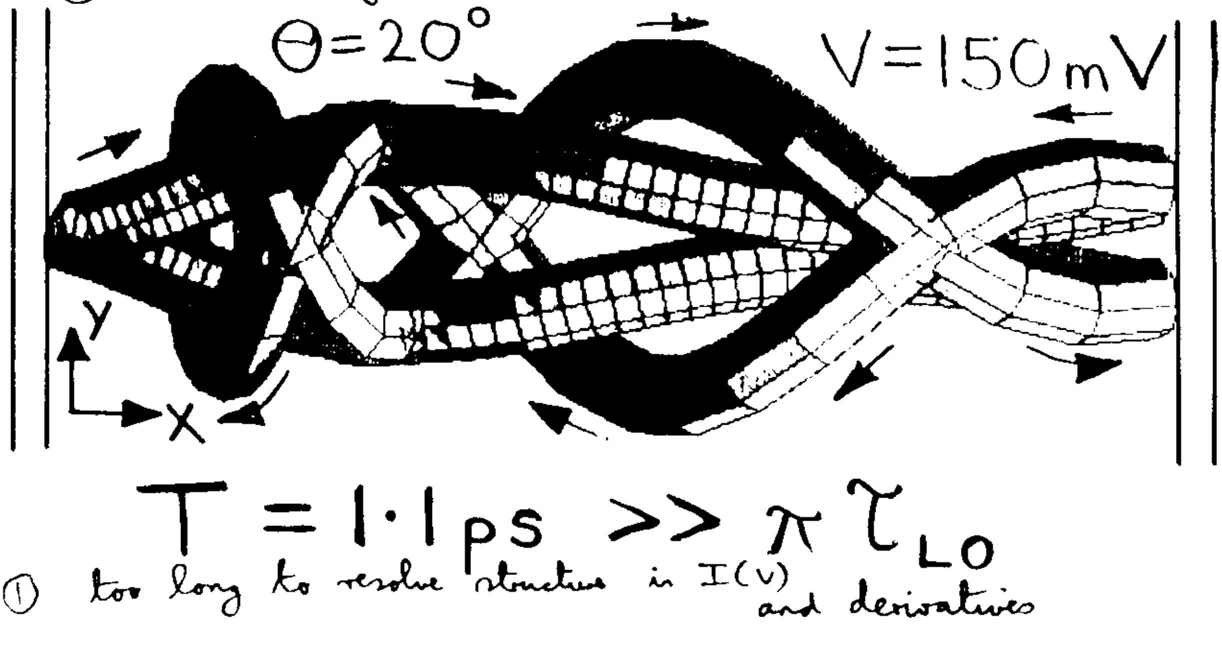




increasing V
topology of orbits unchanged but period T falls

$T = 0{\cdot}9\,ps$

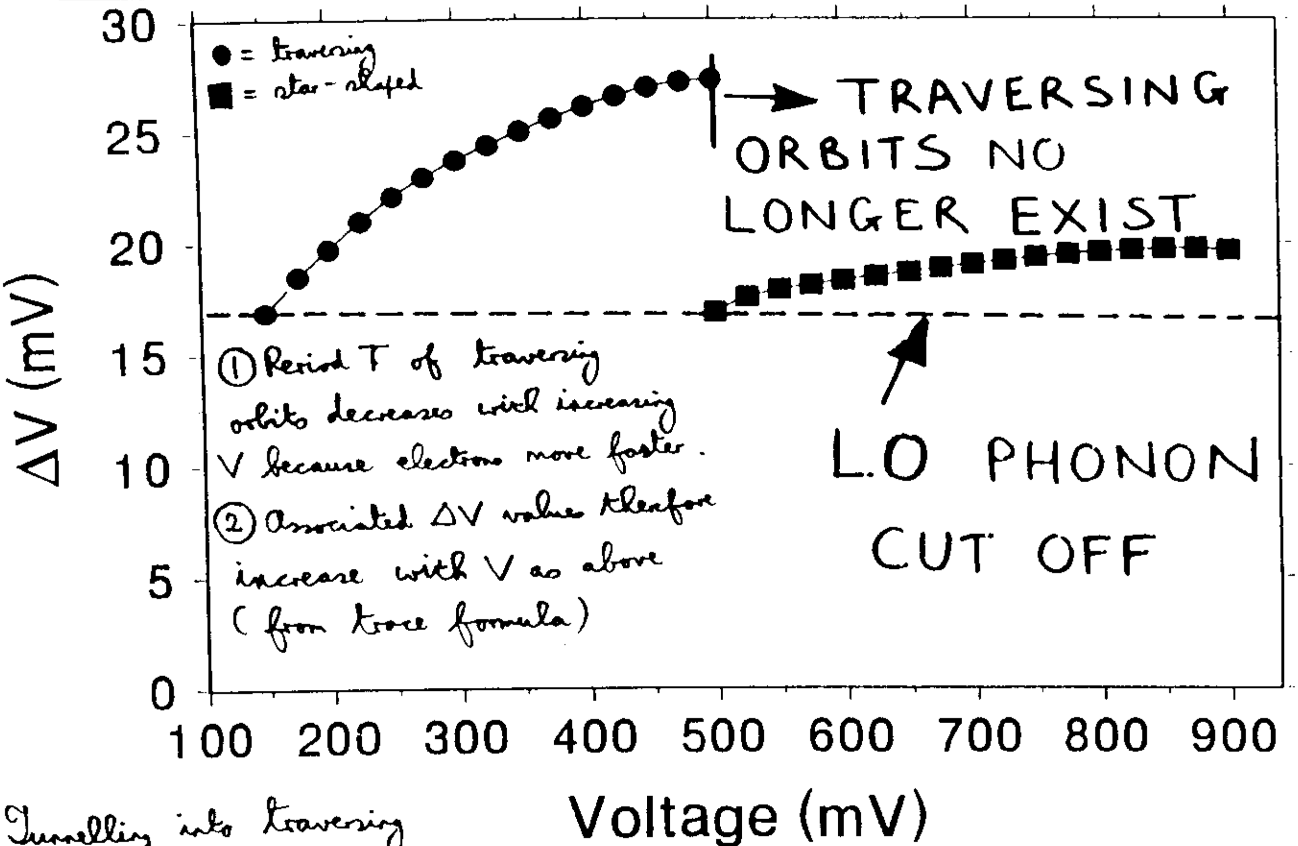
③ (still too long to resolve oscillatory structure)



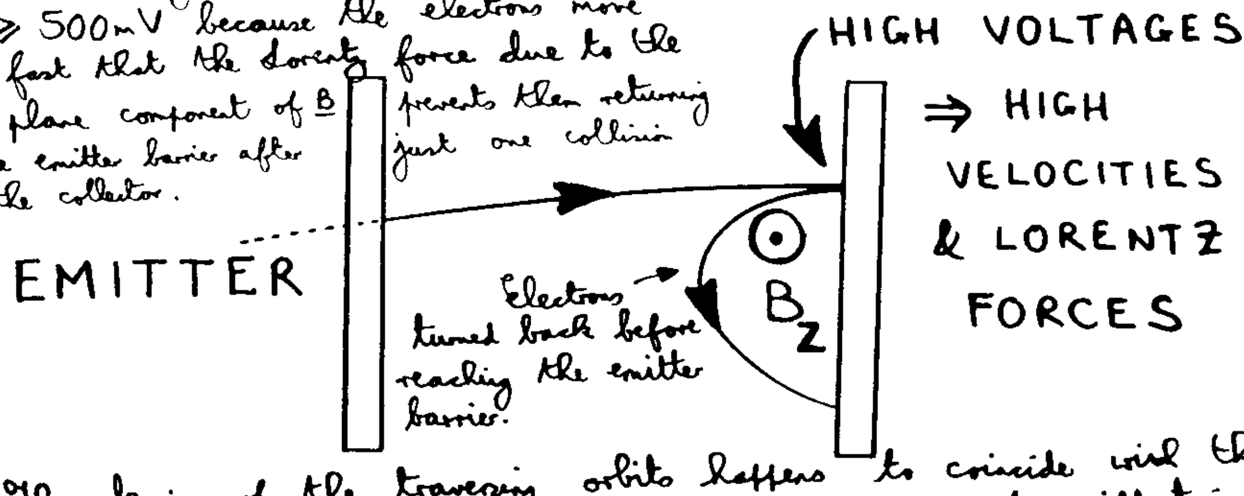
$T = 1{\cdot}1\,ps \gg \pi\tau_{LO}$

① too long to resolve structure in I(V) and derivatives

VOLTAGE PERIODICITIES CALCULATED FOR PERIODIC TRAVERSING & STAR-SHAPED ORBITS ($\theta = 20°$)

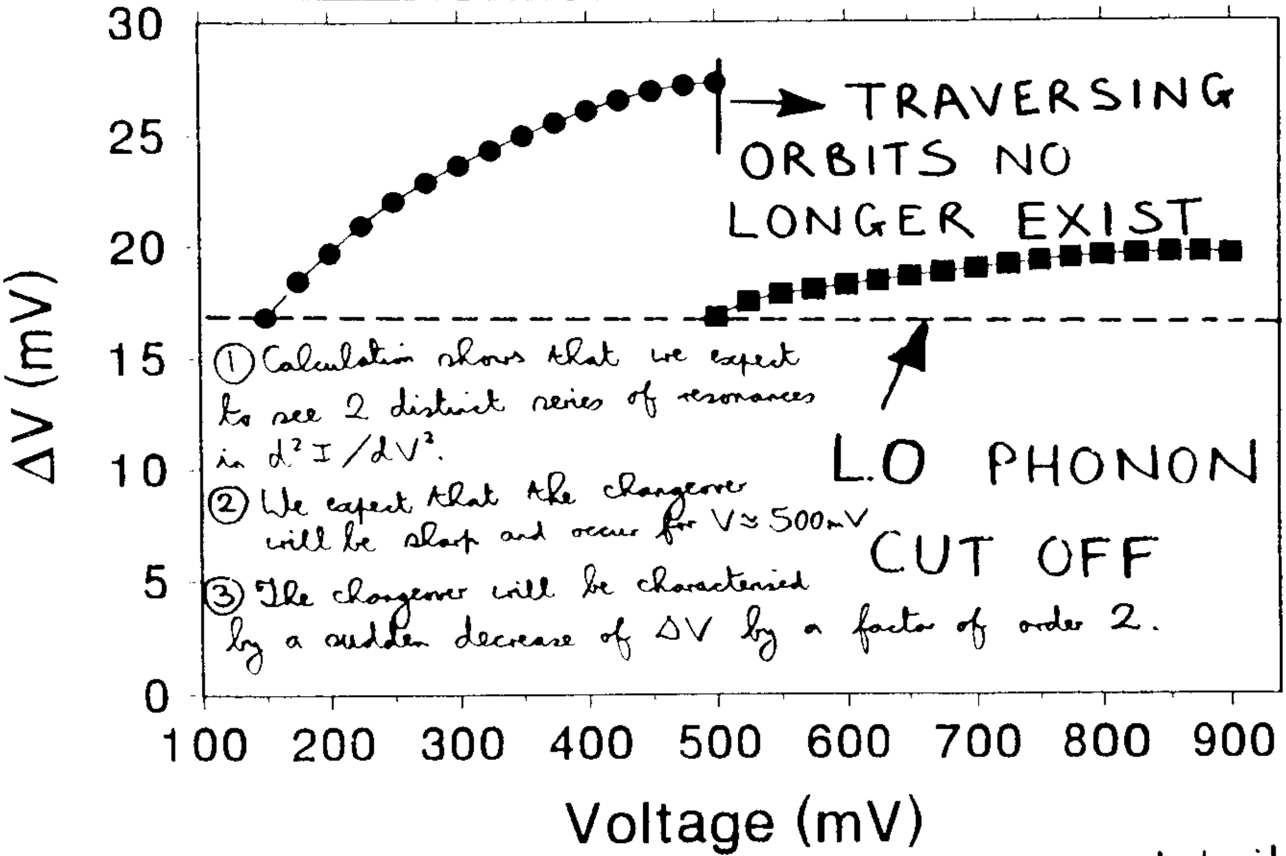


③ Tunnelling into traversing orbits no longer occurs for $V \geq 500\,mV$ because the electrons move so fast that the Lorentz force due to the in-plane component of $\underline{B}$ prevents them returning to the emitter barrier after just one collision on the collector.



④ The demise of the traversing orbits happens to coincide with the threshold voltage for the observation of a new series of oscillations in d^2I/dV^2 associated with the periodic star-shaped orbits. Although these orbits still exist at lower voltages, their period T is too long for the resonances to be resolved.

VOLTAGE PERIODICITIES CALCULATED FOR PERIODIC TRAVERSING & STAR-SHAPED ORBITS ($\theta = 20°$)

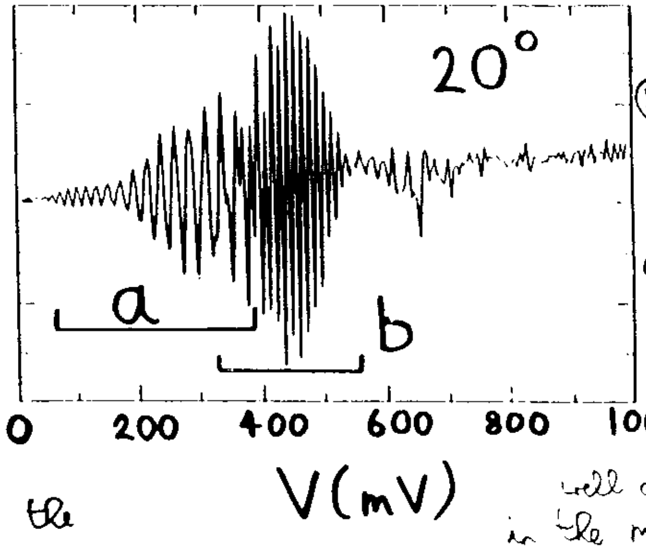


④ These predictions are in good agreement with experiment and provide strong evidence that the a series originate from tunnelling into periodic traversing orbits and the b series is associated with the periodic star-shaped orbits.

EXPERIMENT:

$\Delta V_a = 24$ mV

$\Delta V_b = 12$ mV



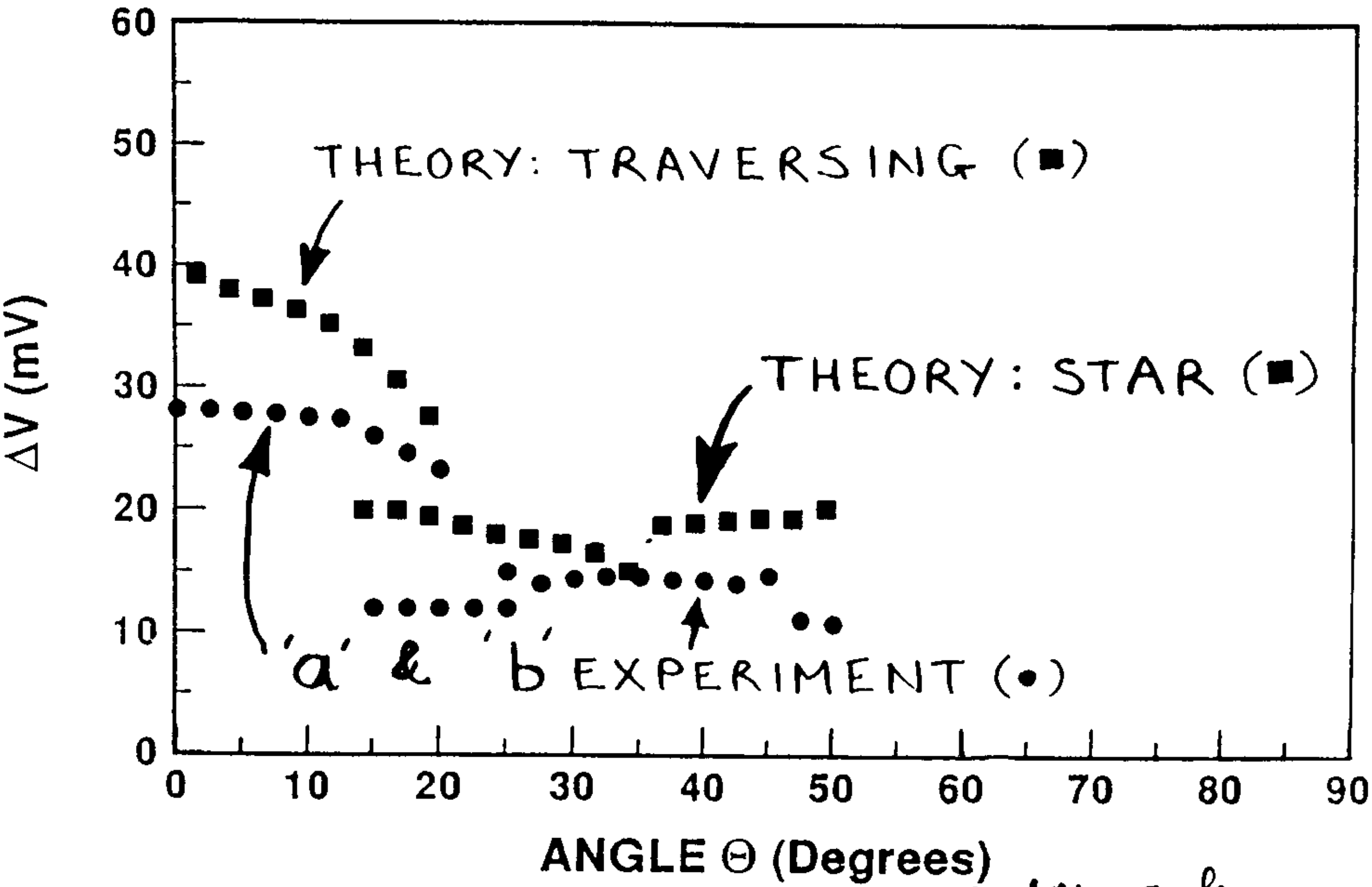
⑤ Note: the theory overestimates the changeover voltage somewhat – probably because the effects of conduction band nonparabolicity in the potential well are not included in the model.

⑥ If we now change θ does the model explain the observed variation of the ΔV values & voltage ranges of the two series??

1) First let's look at the variation of ΔV with Θ. The experimental values are shown by the filled circles below

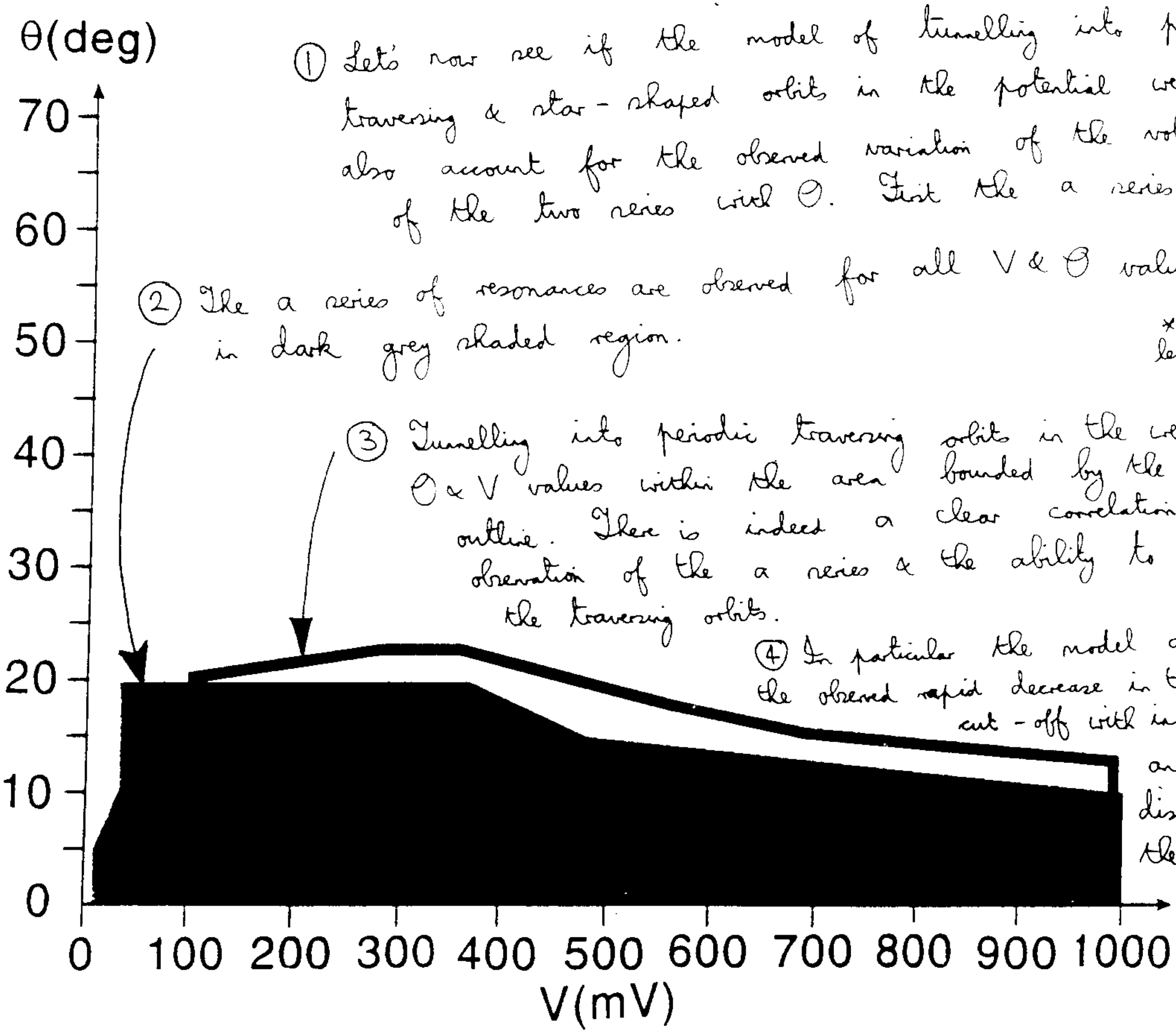
② The values predicted for tunnelling into periodic traversing and star-shaped orbits are shown by filled squares.

③ The model accounts for the observed decrease of ΔV for the a series of oscillations with increasing Θ – together with the disappearance of this series when $\Theta \gtrsim 20°$



④ The model also accounts for the range of tilt angles $12.5° \leq \Theta \leq 50°$ over which the b series is observed and gives a reasonable prediction for the ΔV values ⇒ which change weakly with Θ.

⑤ In general we slightly overestimate the observed ΔV values. Again this is because the model does not include the effects of conduction band nonparabolicity in the potential well. Non-parabolicity increases the effective mass of the electrons. This would reduce the predicted separation of the level clusters, thereby bringing the predicted ΔV values into better agreement with experiment.



① Let's now see if the model of tunnelling into periodic traversing & star-shaped orbits in the potential well can also account for the observed variation of the voltage ranges of the two series with Θ. First the a series:

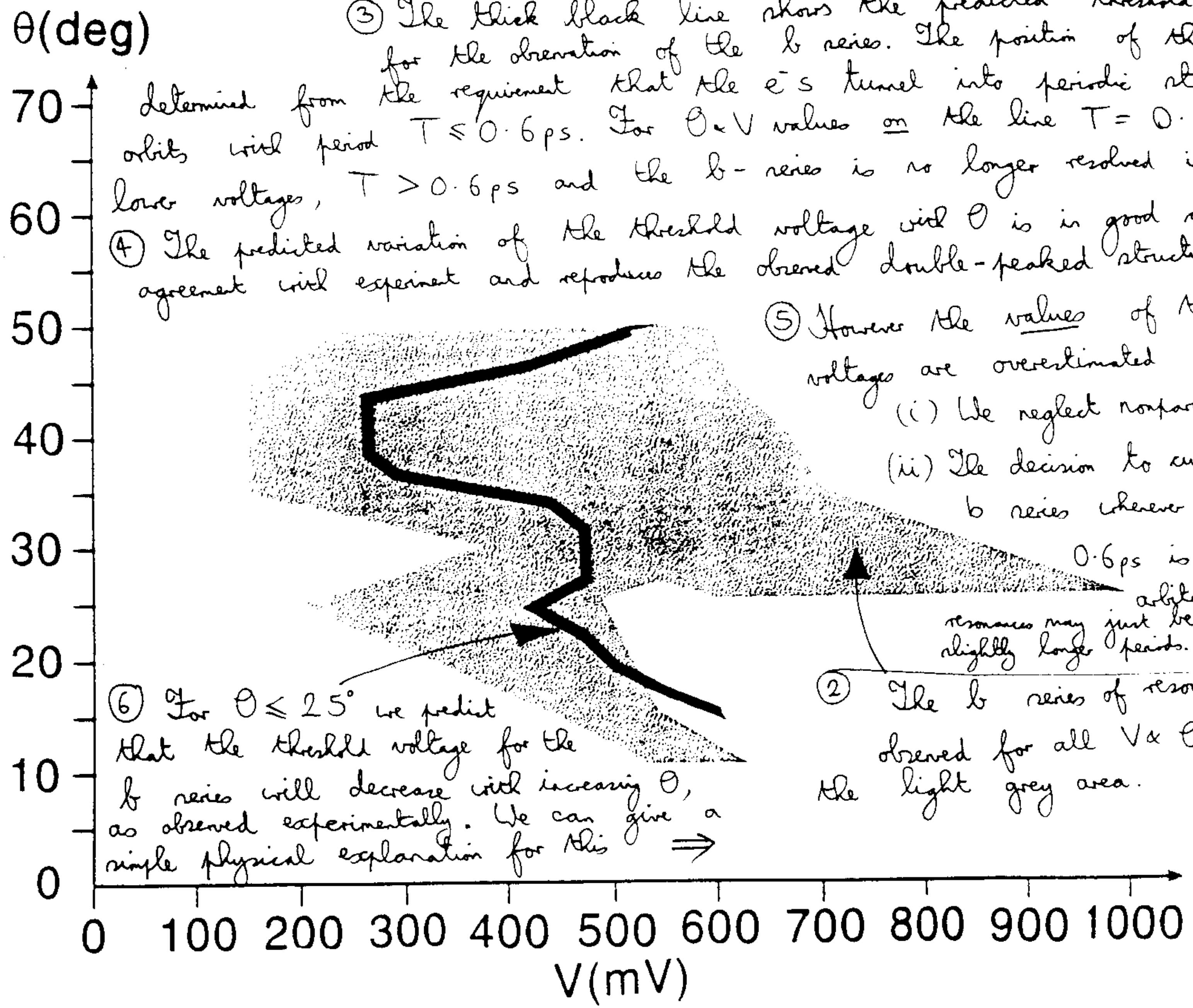
② The a series of resonances are observed for all V & Θ values within in dark grey shaded region.

③ Tunnelling into periodic traversing orbits in the well* occurs for Θ & V values within the area bounded by the thick black outline. There is indeed a clear correlation between the observation of the a series & the ability to tunnel into the traversing orbits.

* with periods less than 0.6ps

④ In particular the model accounts for the observed rapid decrease in the high-voltage cut-off with increasing Θ and the eventual disappearance of the a series for $\Theta \geq 20°$

① What about the b series??



② The b series of resonances is observed for all V & θ values within the light grey area.

③ The thick black line shows the predicted threshold voltages for the observation of the b series. The position of the line is determined from the requirement that the e^-s tunnel into periodic star-shaped orbits with period $T \leqslant 0 \cdot 6$ps. For θ & V values <u>on</u> the line $T = 0 \cdot 6$ps. For lower voltages, $T > 0 \cdot 6$ps and the b-series is no longer resolved in d^2I/dV^2.

④ The predicted variation of the threshold voltage with θ is in good qualitative agreement with experiment and reproduces the observed double-peaked structure.

⑤ However the <u>values</u> of the threshold voltages are overestimated because:

(i) We neglect nonparabolicity in the well

(ii) The decision to cut off the b series whenever T exceeds 0·6ps is somewhat arbitrary ⇒ some resonances may just be seen for slightly longer periods.

⑥ For $\theta \leqslant 25°$ we predict that the threshold voltage for the b series will decrease with increasing θ, as observed experimentally. We can give a simple physical explanation for this ⇒

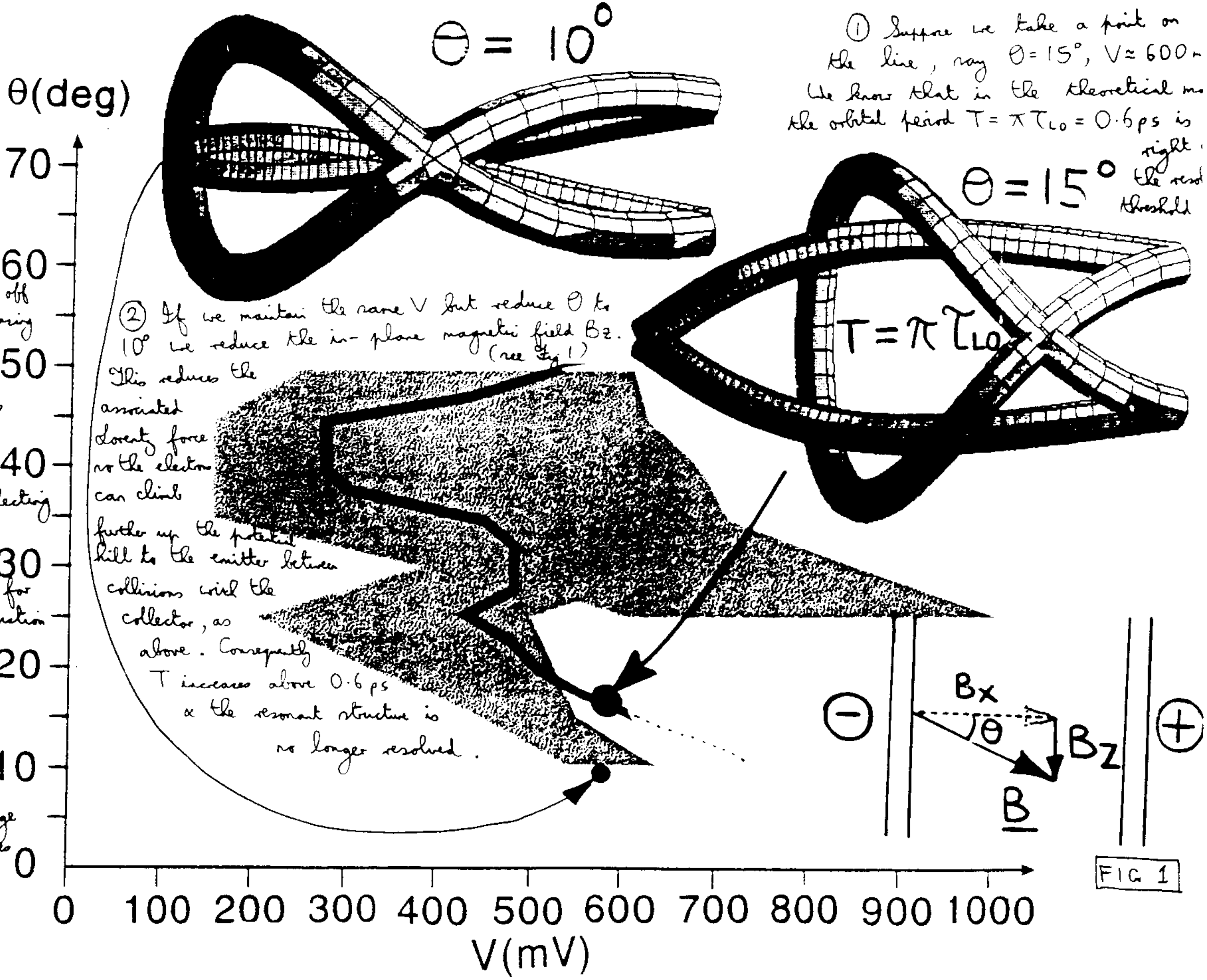


FIG 1

① Suppose we take a point on the line, say $\theta = 15°$, $V \simeq 600$ m[V]. We know that in the theoretical mo[del] the orbital period $T = \pi \tau_{LO} = 0.6$ ps is right [at] the reso[nance] threshold

② If we maintain the same V but reduce θ to 10° we reduce the in-plane magnetic field B_z. (see Fig 1) This reduces the associated Lorentz force so the electron can climb further up the potential hill to the emitter between collisions with the collector, as above. Consequently T increases above 0·6 ps & the resonant structure is no longer resolved.

③ However, we can once more reduce T to below this cut-off value by increasing the voltage.

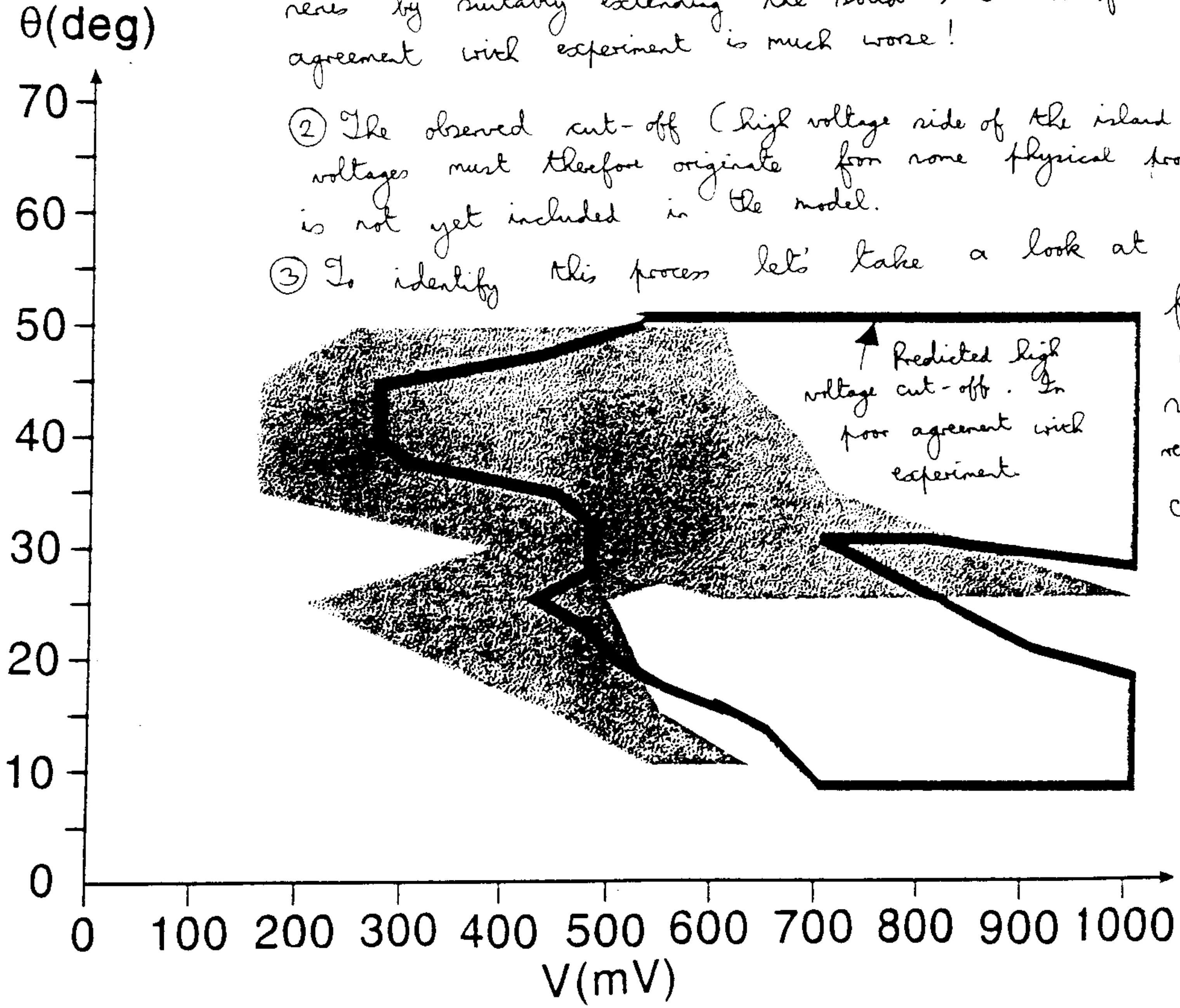
④ This increases the speed of the electrons & restores the deflecting Lorentz force by compensating for the previous reduction of B_z.

⑤ Consequently, as we reduce θ, the threshold voltage for the b-series increases.

① If we now show the predicted (high voltage) cut-off voltages for the b-series by suitably extending the solid line we find that the agreement with experiment is much worse!

② The observed cut-off (high voltage side of the island below) voltages must therefore originate from some physical process which is not yet included in the model.

③ To identify this process let's take a look at our model for the origin of the b-series of resonances in closer detail.



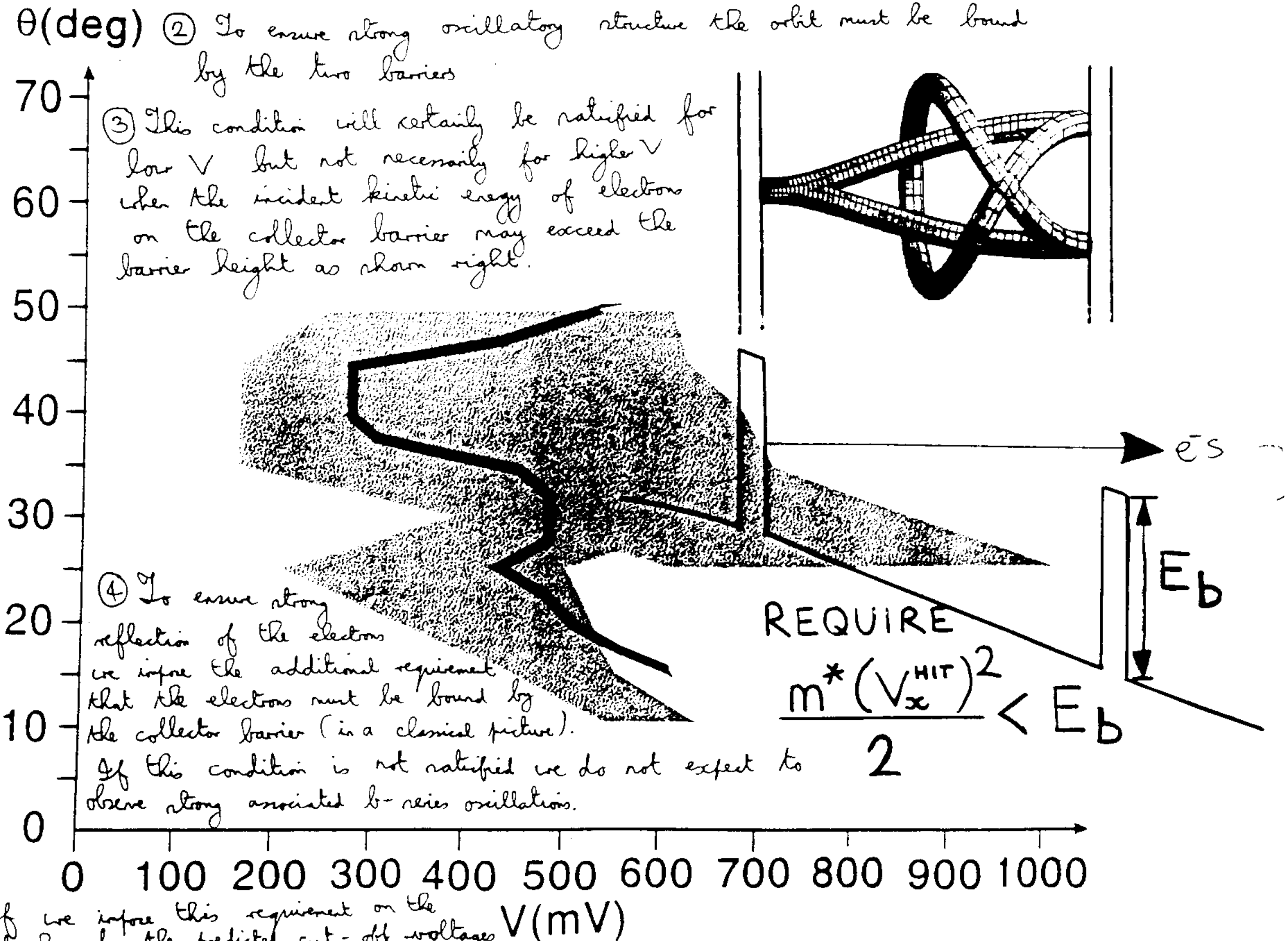
① We believe that the b - series of resonances is associated with the periodic star - shaped orbits shown in the 3D plot below

② To ensure strong oscillatory structure the orbit must be bound by the two barriers

③ This condition will certainly be satisfied for low V but not necessarily for higher V when the incident kinetic energy of electrons on the collector barrier may exceed the barrier height as shown right.

④ To ensure strong reflection of the electrons we impose the additional requirement that the electrons must be bound by the collector barrier (in a classical picture).

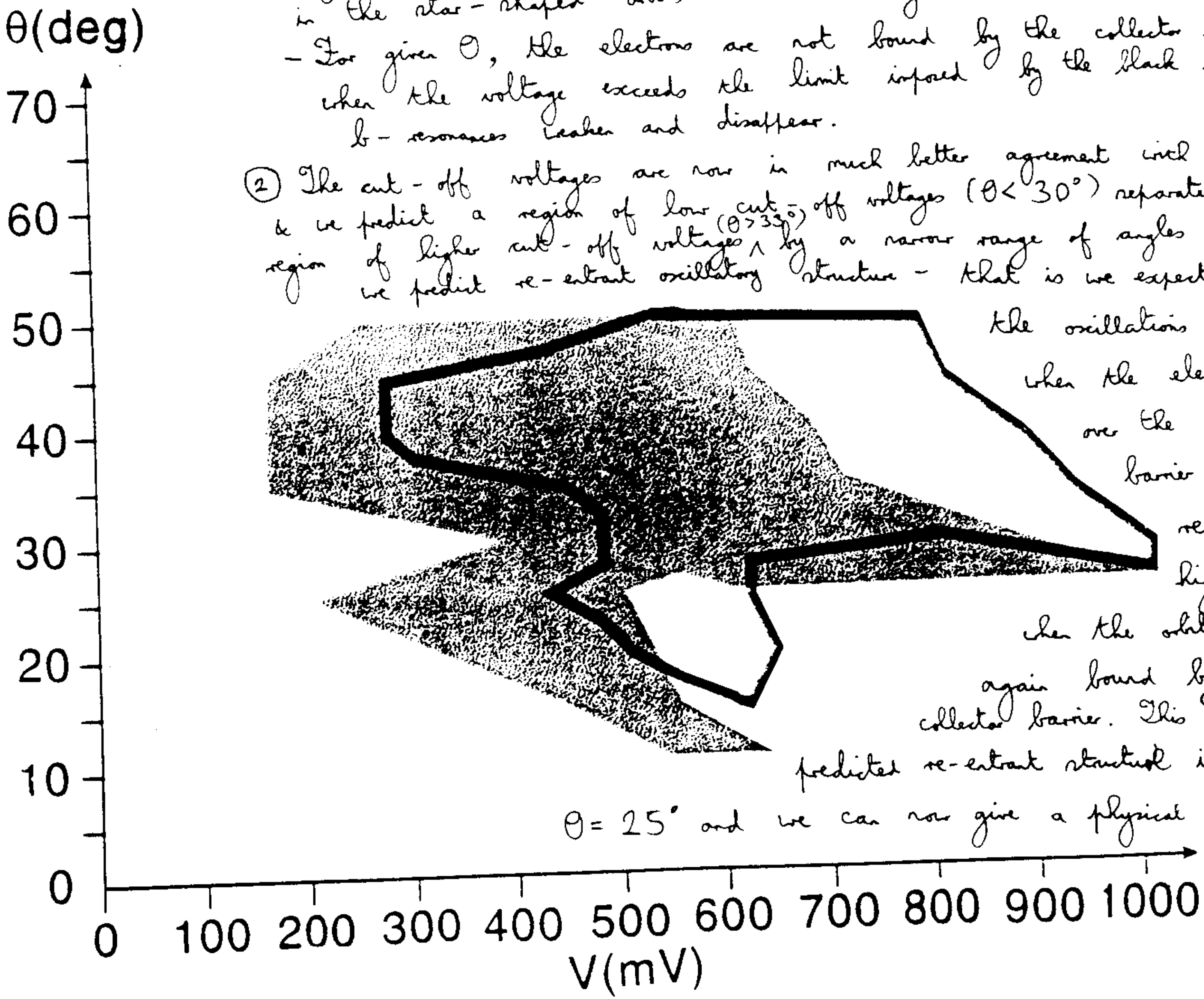
If this condition is not satisfied we do not expect to observe strong associated b - series oscillations.

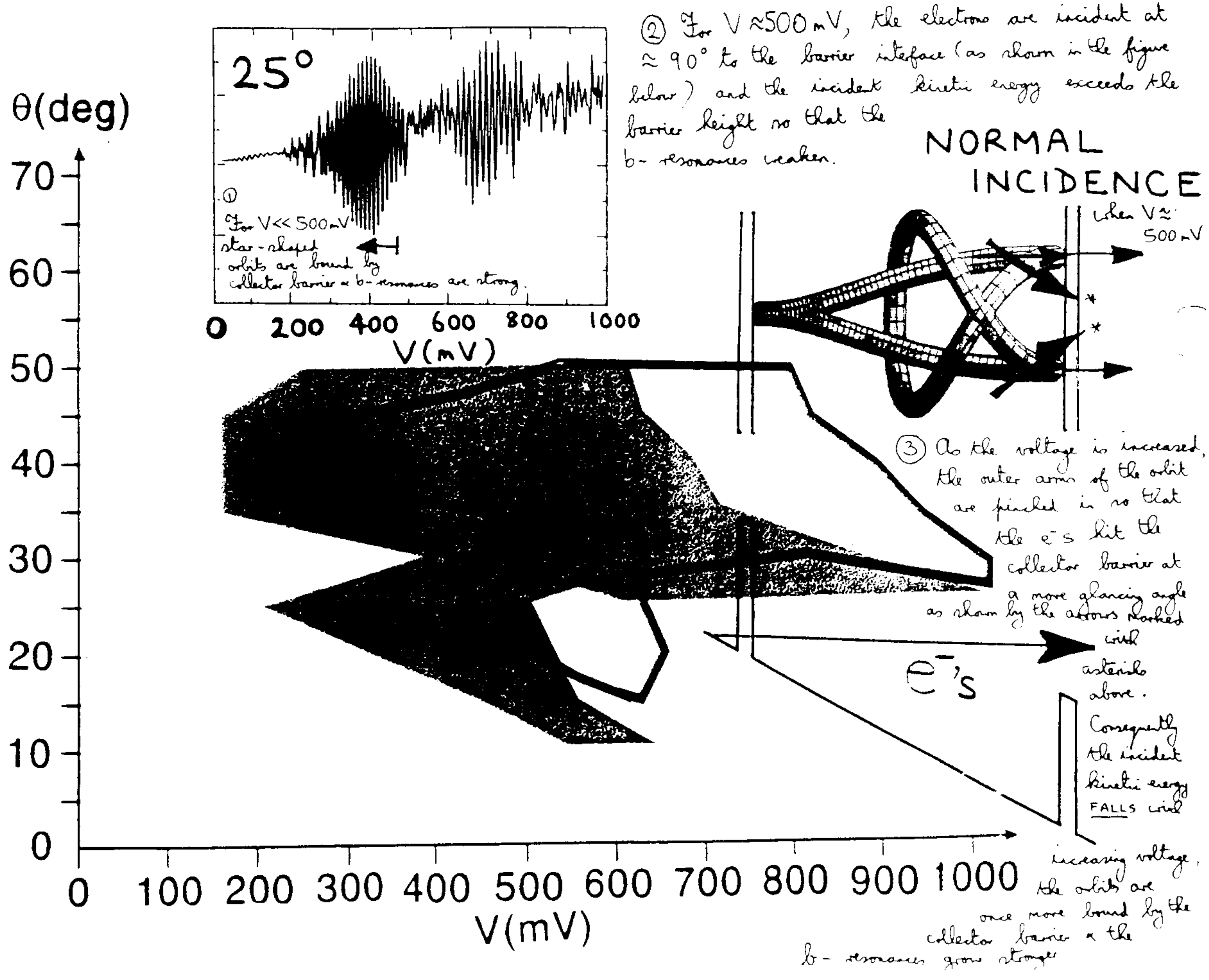


⑤ If we impose this requirement on the orbits how do the predicted cut - off voltages change ??

① The new continuation of the thick black line shows the high cut-off voltages imposed by the requirement that the electrons in the star-shaped orbits are bound by the collector barrier.
– For given θ, the electrons are not bound by the collector barrier when the voltage exceeds the limit imposed by the black line – and the b-resonances weaken and disappear.

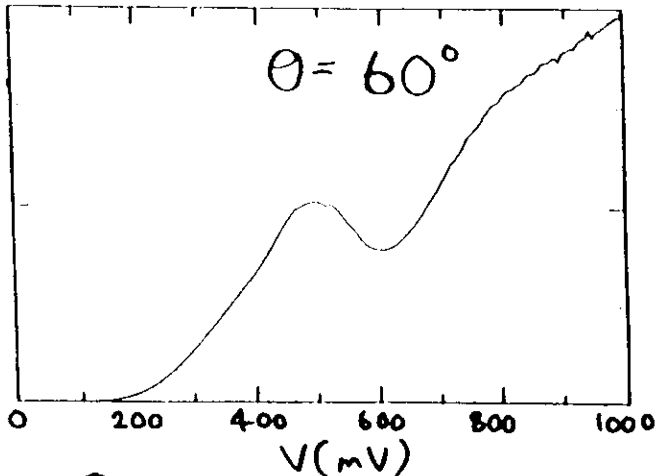
② The cut-off voltages are now in much better agreement with experiment & we predict a region of low cut-off voltages ($\theta < 30°$) separated from a region of higher cut-off voltages ($\theta > 33°$) by a narrow range of angles over which we predict re-entrant oscillatory structure – that is we expect that the oscillations will disappear when the electrons pass over the top of the barrier, but later reappear at higher voltages when the orbits are once again bound by the collector barrier. This type of predicted re-entrant structure is observed for $\theta = 25°$ and we can now give a physical explanation for it. ⇒



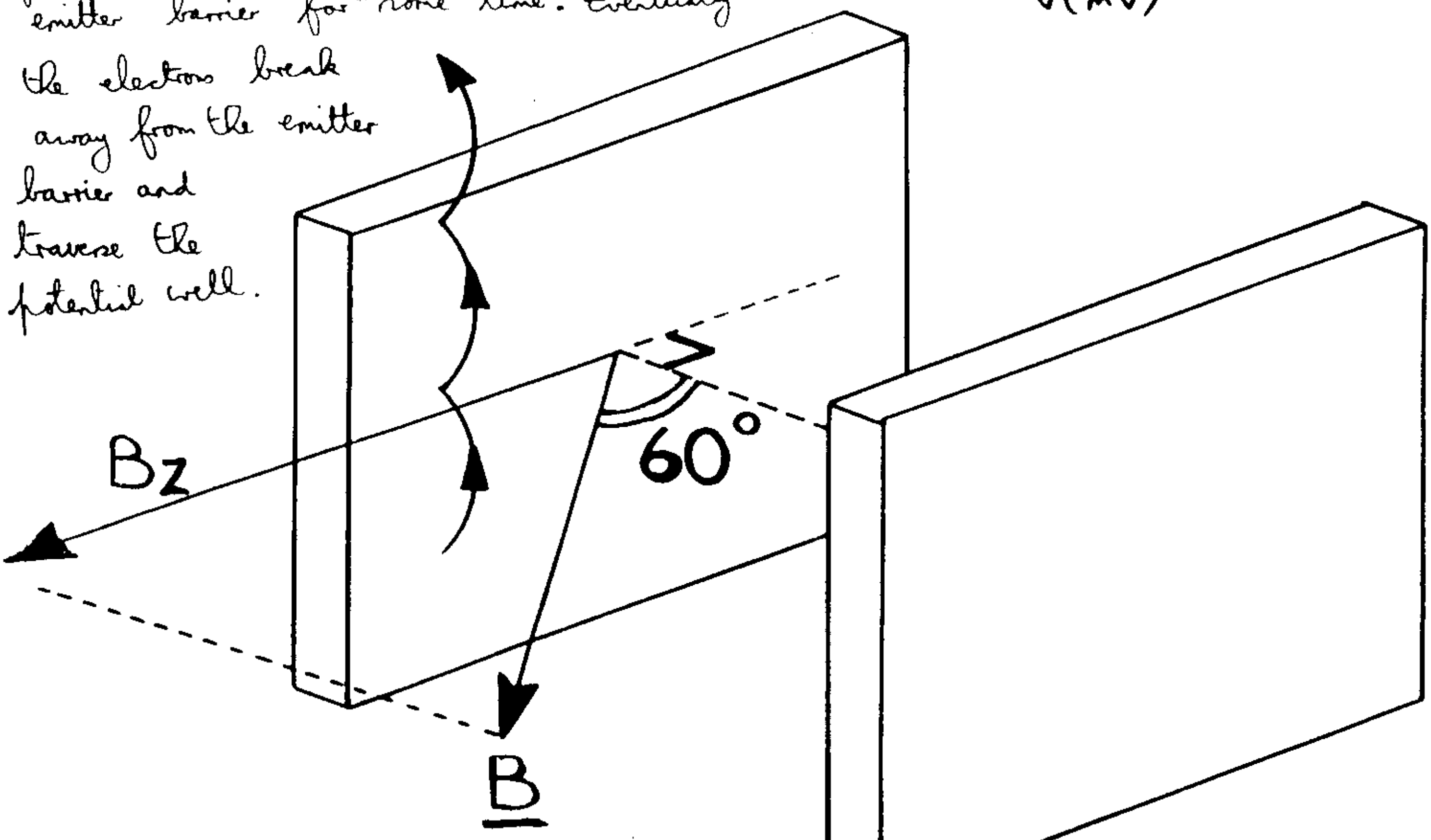


25°
0
200
400
600
800
1000
V(mV)
θ(deg)
70
60
50
40
30
20
10
0
0
100
200
300
400
500
600
700
800
900
1000
V(mV)
e⁻'s
① For V<<500mV star-shaped orbits are bound by collector barrier & b-resonances are strong.
② For V ≈500mV, the electrons are incident at ≈ 90° to the barrier interface (as shown in the figure below) and the incident kinetic energy exceeds the barrier height so that the b-resonances weaken.
NORMAL INCIDENCE
when V ≈ 500mV
③ As the voltage is increased, the outer arms of the orbit are pinched in so that the e⁻s hit the collector barrier at a more glancing angle as shown by the arrows marked with asterisks above. Consequently the incident kinetic energy FALLS with increasing voltage, the orbits are once more bound by the collector barrier & the b-resonances grow stronger

① The theory predicts that the b-series of oscillations will disappear for $\theta \gtrsim 50^\circ$ because the star-shaped orbits no longer exist. Why do they disappear? What are they replaced by and can the new orbits explain the sudden dramatic increase in the voltage periodicity illustrated below for $\theta = 60^\circ$?

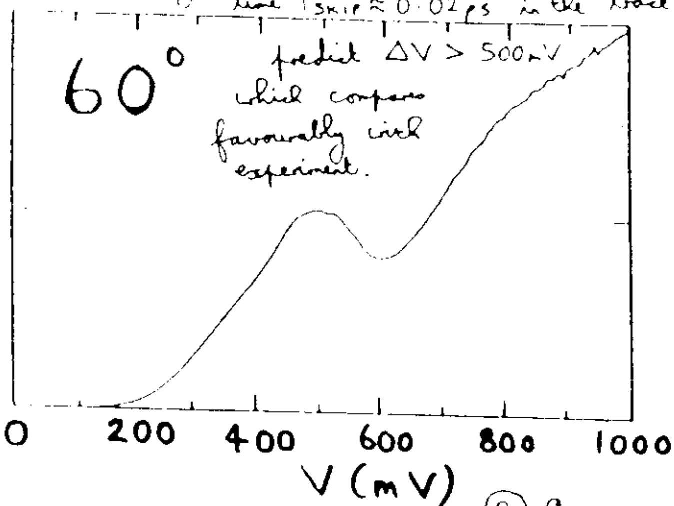


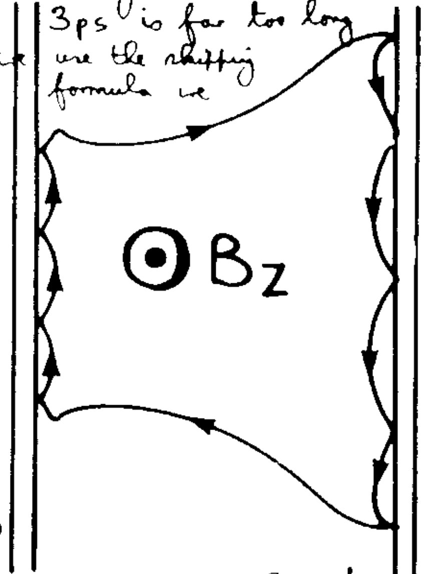
② The key to understanding the sudden transition in ΔV is to realise that by the time the tilt angle is increased to 60°, the in-plane component of the magnetic field has become so large that after injection into the well the electrons are forced to skip very rapidly along the emitter barrier for some time. Eventually the electrons break away from the emitter barrier and traverse the potential well.



③ The skipping times (for return to the emitter) are very much shorter than the periods of the star-shaped orbits ($\approx 0 \cdot 02$ ps compared with $0 \cdot 5$ ps!) so that the oscillatory periodicity is greatly increased (see next vu-graph).

① After injection into potential well, the electrons in periodic orbits for $\Theta = 60^\circ$
(i) Make rapid skips along the emitter barrier before traversing the well, (ii) then skip down the collector barrier before once more traversing the well and returning to their injection point. The total orbital period ~ 3ps is far too long to resolve any associated structure. However, if we use the skipping time $T_{SKIP} \approx 0.02\,ps$ in the trace formula we predict $\Delta V > 500\,mV$ which compares favourably with experiment.



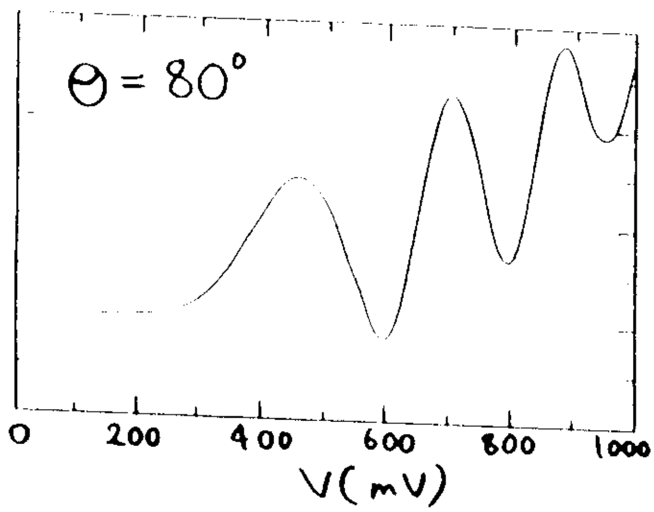


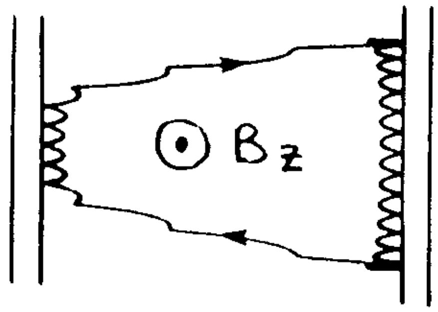
$T_{SKIP} = 0.02\,ps$

$\Downarrow$

$\Delta V = 540\,mV$

② As we further increase Θ two things happen:
(i) the electron makes more skips along the emitter barrier before breaking away and crossing the well – when $\Theta = 90^\circ$, of course, the electron never breaks away



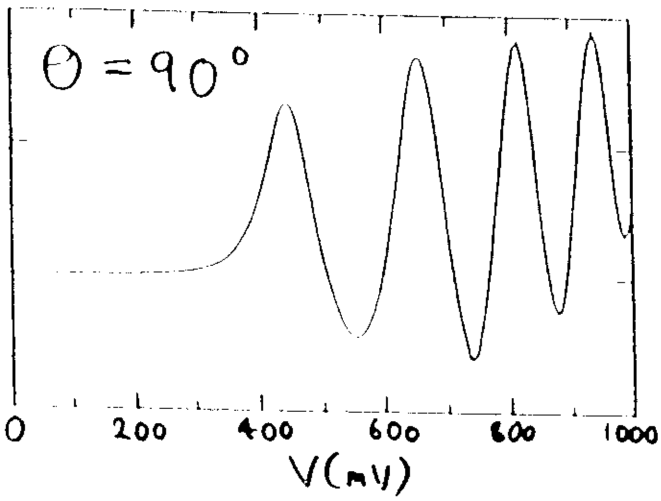


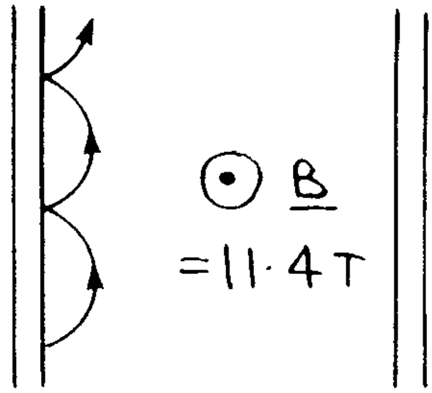
$T_{SKIP} = 0.07\,ps$

$\Downarrow$

$\Delta V = 154\,mV$

(ii) The skipping time T_{SKIP} increases as shown, thereby providing a reasonable quantitative account of the observed decrease of ΔV.





$T_{SKIP} = 0.09\,ps$

$\Downarrow$

$\Delta V = 120\,mV$

all orbits above shown projected onto the X–Y plane.

SUMMARY

- CLASSICAL MOTION OF e^-'S IN A POTENTIAL WELL IS CHAOTIC IN A TILTED $\underline{B}$-FIELD
- HAVE PROBED CORRESPONDING QUANTUM ENERGY LEVELS USING RESONANT MAGNETOTUNNELING SPECTROSCOPY
- $I(V)$ CURVE & DERIVATIVES CONTAIN COMPLICATED RESONANT STRUCTURE WHICH CHANGES DRAMATICALLY WITH $\underline{B}$-FIELD TILT ANGLE θ
- ORIGIN & EVOLUTION OF THIS RESONANT STRUCTURE DUE TO TUNNELING INTO PERIODIC TRAVERSING, STAR-SHAPED & SKIPPING-TYPE ORBITS WITHIN CHAOTIC SEA OF POTENTIAL WELL
- RESONANT STRUCTURE ANALOGOUS TO QUASI LANDAU LEVEL OSCILLATIONS IN ABSORPTION SPECTRUM OF CHAOTIC H-ATOM (HIGH B REGIME)