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SMR/697-6

RESEARCH WORKSHOP ON CONDENSED MATTER PHYSICS
(21 June - 3 September 1993)

WORKING PARTY ON SMALL SEMICONDUCTOR STRUCTURES
(2 - 13 August 1993)

" STOCHASTIC APPROACH TO THE THEORY OF SHOT NOISES IN DOUBLE BARRIER RESONANT TUNNELING STRUCTURES "

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STOCHASTIC

Stochastic approach to

Shot Noise in Double Barrier

Resonant Tunneling Systems

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1. Introduction

2. Single barrier tunneling $\Rightarrow$ full shot noise

3. DB tunneling – coherent process

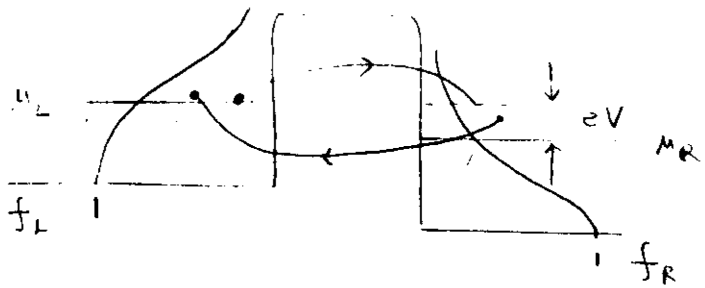
4. DB tunneling – incoherent process

5. Discussion

1. Introduction

Noises in an electronic device

a) Thermal noise



$k_BT \gg eV$

The charge carrier passing through the junction with large thermal fluctuation

S_i = the noise power density $= 4k_BT\,G$

G = the conductance of the junction.

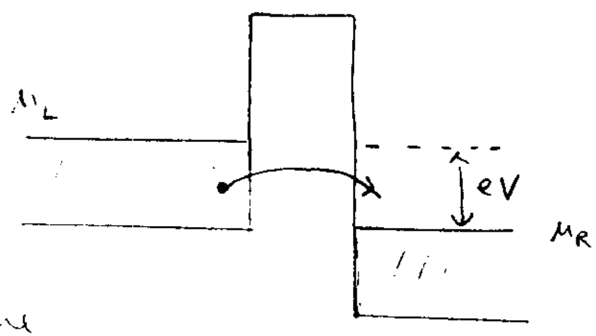
[illegible]

The noise power spectrum $S_I(\omega) \propto 1/\omega$ for $\omega < 10^3$ Hz. The origin of this noise is not clear, but may be due to fluctuations in the resistance within the junction or by the changing of traps and scattering centers

c) shot noises

$k_B T \ll eV$

$\frac{1}{\tau_{sc}} \ll \omega \ll \frac{1}{\tau}$

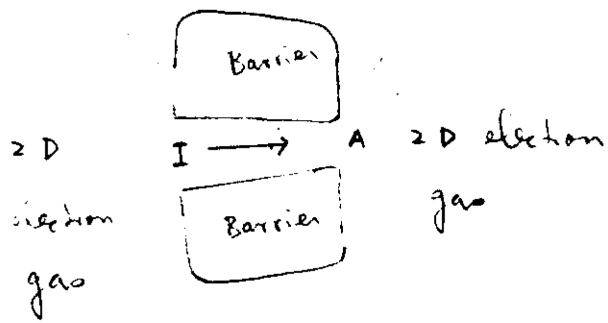


τ = average time between electron shots $\sim 10^{-12}$ sec

Transport is dominated by random sequence of discrete shots from left electrode to right

The random nature of the electron shots will give rise a finite shot noise.

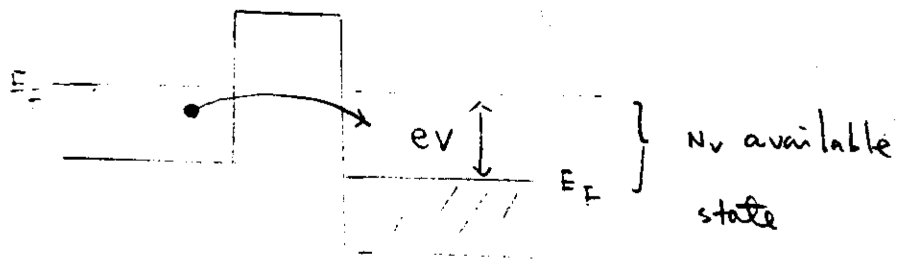
However for a ballistic channel



electrons passing through the channel emerge at the region A one by one. There is no randomness in the sequence of electron shots

$\Rightarrow$ the noise power density $\equiv 0$

2. Single Barrier Tunneling – Full shot noise



For each state, the i-th electron on the left tunnels through the barrier and emerges to the right of the barrier at time t_i, the next electron ($i+1$-th) emerges to the right at t_{i+1}

The time sequence $\{t_i\}$ is random without constraint

$\langle t_{i+1} - t_i \rangle = \tau$

$\gamma = \frac{1}{\tau}$ = tunneling rate

τ = the average time interval between two tunneling events

current for this particular state $i = \frac{e}{\tau}$.

Since there are N_v available state, the total tunneling current $I = N_v i = N_v e/\tau$

$\gamma = \frac{1}{\tau}$ = probability for one tunneling event to occur per unit time

$\frac{dt}{\tau} = \gamma dt$ = the probability of one tunneling event in dt.

$\because$ No correlation between successive tunneling electrons. We can use a stochastic description to study the probability distribution function $P(n,t)$

$P(n,t)$ = the probability distribution of n tunneling events between $(0,t)$. It satisfies

$$p(n,t+dt) = p(n-1,t)\gamma dt + p(n,t)(1-\gamma dt)$$

$$\frac{p(n,t+dt)-p(n,t)}{dt} = \gamma\left[p(n-1,t)-p(n,t)\right]$$

$$\frac{d\,p(n,t)}{dt} = \gamma\left[p(n-1,t)-p(n,t)\right]$$

It is straight forward to show

$$p(n,t) = e^{-\gamma t}(\gamma t)^n \frac{1}{n!}$$

Poisson distribution function

with

$$\sum_{n=0}^{\infty} p(n,t) = 1$$

For each state the noise power spectrum due to the current fluctuation is the Fourier transform of

$$S(\tau) = \langle \delta i(t+\tau)\delta i(t)\rangle \quad , \quad \delta i = i - \langle i\rangle$$

$$S(\omega) = \int_{-\infty}^{\infty} S(\tau)\, e^{-i\omega\tau}\, d\tau$$

$$\langle i\rangle = i_0 = \frac{e}{\tau}$$

It is straight forward to show

$$S(\omega) = \lim_{T\to\infty} \frac{1}{2T}\left\langle \left|\int_{-T}^{T} \delta i(t)\, e^{i\omega t}\, dt\right|^2 \right\rangle$$

$$+ \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |\tau|\, S(\tau)\, e^{-i\omega\tau}\, d\tau$$

$$\Downarrow$$

$$0 \qquad S(\tau) \sim e^{-\tau/\tau_0}$$

For low frequency $\omega \sim 0$

$$S(0) = \lim_{T\to\infty} \frac{1}{2T} \left\langle \left| \int_{-T}^{T} \delta i(t)\,dt \right|^2 \right\rangle$$

$$\int_{-T}^{T} \delta i(t)\,dt = 2\int_0^T [i(t) - i_0]\,dt$$

$$= 2eN(T) - 2eN_0 \qquad \left(eN_0 = T i_0 = e\frac{T}{\tau}\right)$$

$$eN(T) = \int_0^T i(t)\,dt$$

$$S(0) = \lim_{T\to\infty} \frac{2e^2}{T} \langle N^2(T) - N_0^2 \rangle$$

$N_0 = \langle N(T)\rangle$ = average tunneling events from 0 to $T = \frac{T}{\tau}$

$$\langle N^2(T)\rangle = \sum_{n=0}^{\infty} P(n,T)\, n^2$$

$$= \sum_{n=0}^{\infty} e^{-\gamma T} (\gamma T)^n \frac{1}{n!} (n^2)$$

$$\langle N^2(T)\rangle = \sum_{n=0}^{\infty} e^{-\gamma T} \frac{(\gamma T)^n}{n!} \left[(\gamma T)^2 + (\gamma T) \right]$$

$$= \left[\left(\frac{T}{\tau}\right)^2 + \left(\frac{T}{\tau}\right) \right] \sum_{n=0}^{\infty} P(n,T)$$

$$= \left(\frac{T}{\tau}\right)^2 + \left(\frac{T}{\tau}\right) = N_0^2 + N_0$$

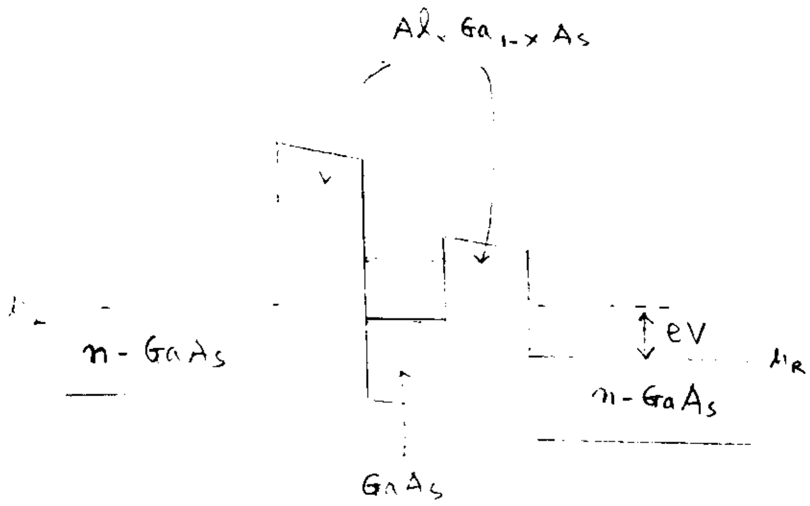
$$S(0) = \lim_{T\to\infty} \frac{2e^2}{T} N_0 = 2e^2 \frac{1}{\tau} = 2e\,i$$

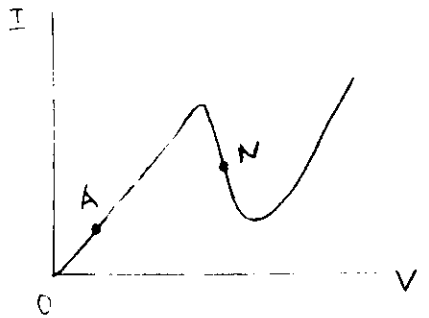
The total noise power density for N_v available tunneling states

$$S_T(\omega \sim 0) = N_v S(0) = 2e\left(N_v \frac{e}{\tau}\right) = 2eI$$

The full shot noise

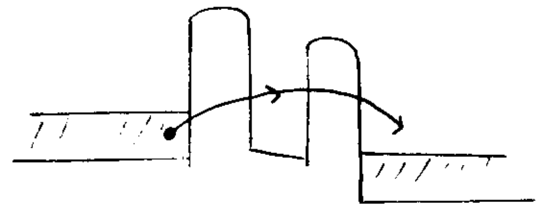
3. Double barrier resonant tunneling structure





shot noise power density due to current-fluctuation

$$S = 2eI\cdot K$$



usual or conventional expectation for coherent process is a full shot noise (K=1) because an electron sees the two barriers as a single junction.

However Experiments by Li et al (Tsui's group) Phys. Rev. B41, 8388 (1990) indicate that K<1 or the shot noise is suppressed in symmetrical barrier structures.

Therefore they claim in their paper that the tunneling process must be incoherent.

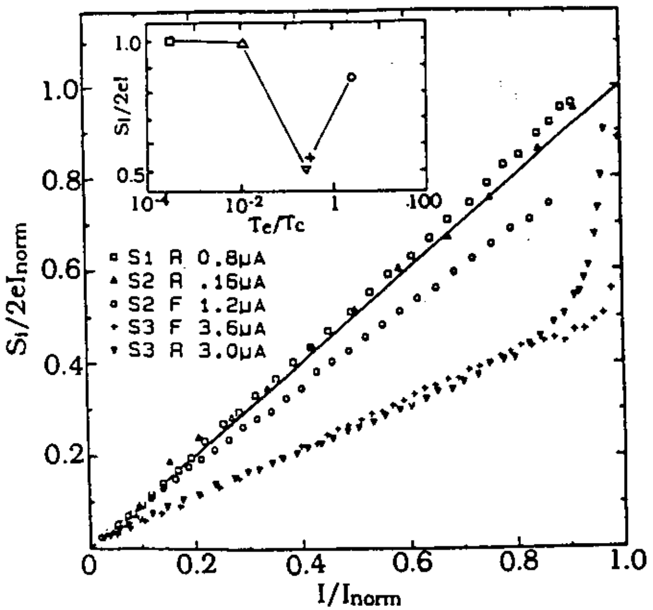
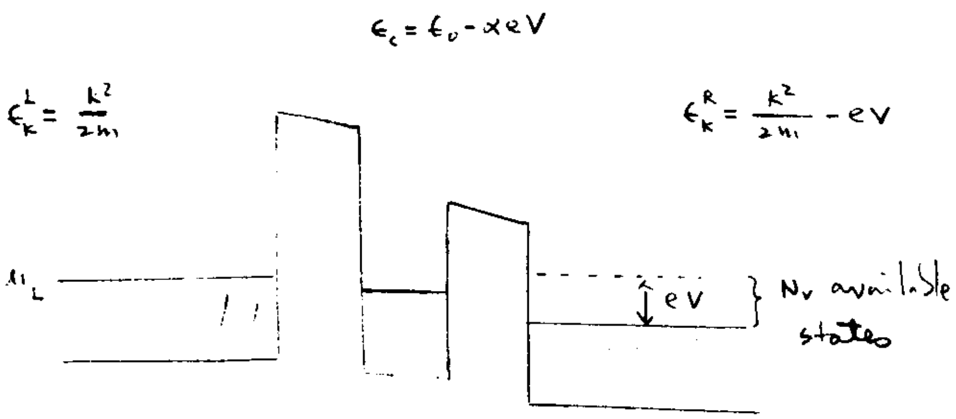


FIG. 3. $S_i/2eI_{\rm norm}$ vs $I/I_{\rm norm}$ in the resonant-tunneling region for three different samples. $I_{\rm norm}$ is a normalization factor (≈corresponding peak current). $S1\,R\,0.8\ \mu$A in the legend stands for sample 1, reverse bias, $I_{\rm norm}=0.8\ \mu$A. The solid line represents the theoretical full shot noise. The data were averaged between 2.5 and 5 kHz for sample 2 and between 5 and 10 kHz for samples 1 and 3. Inset: $S_i/2eI$ vs T_e/T_c, where T_e/T_c is calculated self-consistently as described in Ref. 12 and the data points are connected as a guide to the eye.

DBRT - coherent tunneling (Phys. Rev. B43 4534 (1991))



The electron in each k state tunnels through the barrier independently. We adopt the 1-D Hamiltonian

$$H = \sum_k \epsilon_k^L a_k^+ a_k + \epsilon_c c^+ c + \sum_k \epsilon_k^R b_k^+ b_k$$

$$+ \sum_k \left(T_{Lk} c^+ a_k + T_{Lk}^* a_k^+ c \right) + \sum_k \left(T_{Rk} c^+ b_k + b_k^+ c \right)$$

The tunneling matrices T_{Lk} and T_{Rk} depend on V & Barrier profile.

The density matrix of the system

$$\rho_0 = e^{-\beta \sum_k (\epsilon_k^L - \mu_L) a_k^\dagger a_k}\, \rho_c\, e^{-\beta \sum_p (\epsilon_p^R - \mu_R) b_p^\dagger b_p}$$

$$\mu_L - \mu_R = eV$$

ρ_c is the density matrix for the quantum well electrons, which are in nonequilibrium state, and needs to be determined.

Quantum statistical average

$$\hat O(t) = \hat O[q_1(t), q_2(t) \cdots q_N(t)] \Leftarrow \text{Observable}$$

$$\langle \hat O(t) \rangle = \mathrm{Tr}\, \hat O(t)\, \hat\rho_0(0)$$

$\{|q_\alpha(t)\rangle\}$ form a complete set

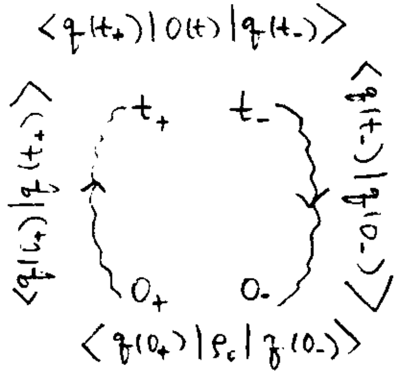
$$\hat q(t)|q_\alpha(t)\rangle = q_\alpha(t)|q_\alpha(t)\rangle$$

According to Feynman and Vernon (Ann. Phys. 24 118 (1963))

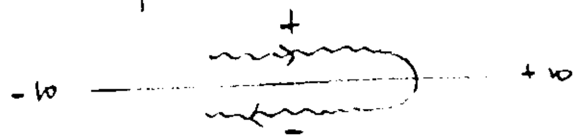
$$\langle \hat O(t) \rangle = \int [dq(t_+)][dq(t_-)] \langle q(t_+)|\hat O(t)|q(t_-)\rangle$$

$$\langle q(t_-)|q(0_-)\rangle \langle q(0_-)|\rho_0|q(0_+)\rangle \langle q(0_+)|q(t_+)\rangle$$

$$[dq(t_\pm)] = dq(t_\pm)\, dq(0_\pm)$$



quantum interference



Define the Lagrangian

$$L = \sum_k a_k^+ (i\partial_t) a_k + \sum_k b_k^+ (i\partial_t) b_k + c^+ (i\partial_t) c - H$$

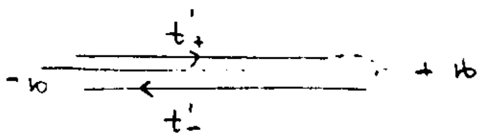
In terms of Feynman path integral over the close time path

(Z.B. Su et al Phys. Rev. B37 9810 (1988))

$$\langle \hat{O}(t) \rangle = \int [da_k^+][da_k][db_k^+][db_k][dc^+][dc]$$

$$\hat{O}(t) \exp\left[i \int_P dt' L(t') \right]$$

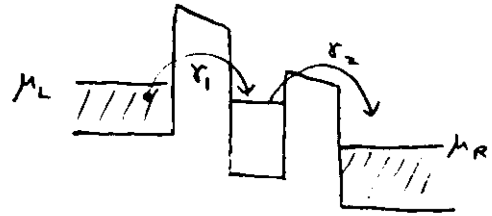
$$\int_P dt' = \int_{-\infty}^{\infty} dt'_+ + \int_{+\infty}^{-\infty} dt'_-$$



The nonequilibrium Green's function (steady state)

inside the quantum well

$$G_{\substack{r\\a}}(\omega) = \frac{1}{\omega - \varepsilon_c \pm i\gamma}$$



$$\gamma = \gamma_1 + \gamma_2$$

$$\gamma_1 = \sum_k |T_{Lk}|^2 \pi \delta(\omega - \varepsilon_k^L)$$

= the tunneling rate from the left electrode to the quantum well

$$\gamma_2 = \sum_k |T_{Rk}|^2 \pi \delta(\omega - \varepsilon_k^R)$$

= the tunneling rate from the quantum well to the right electrode.

The distribution Green function

$$iG^<(t_1 - t_2) = -\langle c^+(t_2) c(t_1) \rangle$$

$$G^{<}(\omega) = -F(\omega)\left[G_r(\omega) - G_a(\omega)\right]$$

the distribution function for the electrons in the resonant level

$$F(\omega) = \frac{\gamma_1 f_L(\omega) + \gamma_2 f_R(\omega)}{\gamma_1 + \gamma_2}$$

$$f_L(\omega) = \frac{1}{e^{\beta(\omega-\mu_L)}+1}, \quad f_R = \frac{1}{e^{\beta(\omega-\mu_R)}+1}$$

The conduction electron's Green functions in the left and right electrodes are well known and need not be listed.

The current operator from left to the quantum well

$$\hat{i}_L = -ie\left[H, \sum_k a_k^+ a_k\right], \quad \hat{i}_R = -ie\left[H, \sum_k b_k^+ b_k\right]$$

The terminal current $\hat{i} = \frac{1}{2}(\hat{i}_R + \hat{i}_L) \rightarrow$ Ramo-shockley theorem

$$\bar{i} = \langle \hat{i} \rangle = e\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2} = \frac{e}{\tau}$$

$$\hat{i}(t) = -\frac{ie}{2}\sum_k\left[T_{Lk}c^+a_k - T_{Lk}^* a_k^+ c + T_{Rk} b_k^+ c - T_{Rk}^* c^+ b_k\right]$$

Noise power density due to current fluctuation

$$S(t-t') = \langle \delta\hat{i}(t)\delta\hat{i}(t')\rangle$$

$$\delta\hat{i}(t) = \hat{i}(t) - \langle i\rangle$$

$$S(\omega) = e\bar{i}\left[1 + \left(1 - \frac{4\gamma_1\gamma_2}{(\gamma_1+\gamma_2)^2}\right)\frac{4\gamma^2}{\omega^2+4\gamma^2}\right]$$

Since there are N_v available states

$$S_T(\omega \sim 0) = 2eI\left[1 - \frac{2\gamma_1\gamma_2}{(\gamma_1+\gamma_2)^2}\right]$$

$$I = N_v \bar{i}$$

Here $\omega \ll \gamma$ $(\sim 10^{11}/\text{sec})$

The shot noise power density is suppressed by the factor

$$\frac{S_T(0)}{2eI} = 1 - \frac{2\gamma_1\gamma_2}{(\gamma_1+\gamma_2)^2} = K$$

which is structure and bias dependent.

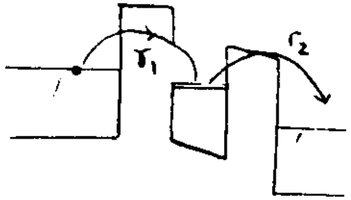
$K = 0.5$ if $\gamma_1 = \gamma_2$ (a symmetric double barrier)

$K = 1$ if $\gamma_1 \to \infty$ (or $\gamma_2 \to \infty$) $\Rightarrow$ a single barrier

$0.5 \leq K \leq 1$ $\Leftarrow$ the result of this study

Experiments (Li et al PR <u>B41</u>, 8388 (1990))

$K < 1$



	$\left(\frac{\gamma_1}{\gamma_2}\right)$
□	3×10^{-3}
△	1×10^{-2}
○	4
▽ +	0.7

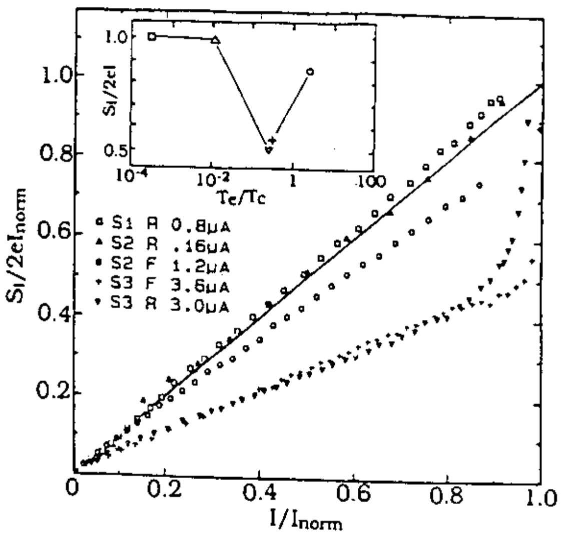
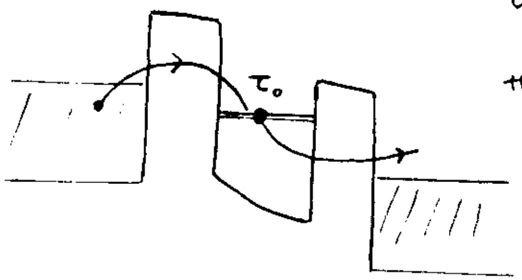


FIG. 3. $S_i/2eI_{norm}$ vs I/I_{norm} in the resonant-tunneling region for three different samples. I_{norm} is a normalization factor (≈corresponding peak current). $S1\,R$ 0.8 μA in the legend stands for sample 1, reverse bias, $I_{norm}=0.8$ μA. The solid line represents the theoretical full shot noise. The data were averaged between 2.5 and 5 kHz for sample 2 and between 5 and 10 kHz for samples 1 and 3. Inset: $S_i/2eI$ vs T_e/T_c, where T_e/T_c is calculated self-consistently as described in Ref. 12 and the data points are connected as a guide to the eye.

$$S_T(0) = 2eIK$$

$$K = 1 - \frac{2\left(\frac{\gamma_1}{\gamma_2}\right)}{\left[1+\left(\frac{\gamma_1}{\gamma_2}\right)\right]^2}$$

The physical reason why the full shot noise is suppressed



$\tau_0 =$ life time of the resonant state

time for the j-th electron to emerge from the right barrier $= t_j$

$\langle t_{j+1} - t_j \rangle = \tau =$ the average time interval for two successful tunneling events.

The time sequence $[t_1, t_2, \cdots, t_j, t_{j+1}, \cdots t_N]$ $= \{t_j\}$ is still a random sequence but subject to the constraint $\langle t_{j+1} - t_j \rangle = \tau > \tau_0$

$\Rightarrow$ Pauli Exclusion principle.

For shot noise

$$S_I(0) = 2eIK$$

$$K = 1 - \frac{\tau_0}{\tau}$$

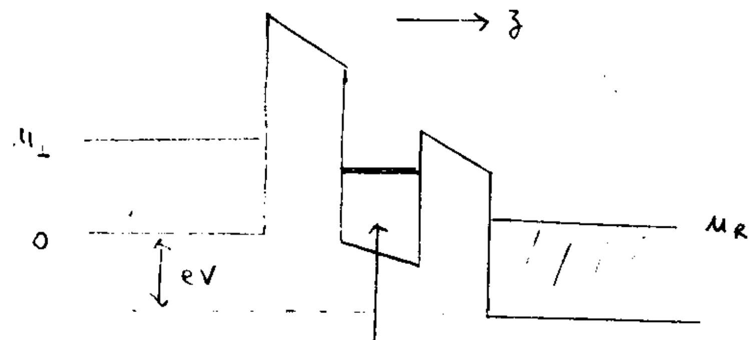
$$\tau = \frac{1}{\gamma_1} + \frac{1}{\gamma_2} = \frac{\gamma_1 + \gamma_2}{\gamma_1 \gamma_2}$$

$$\tau_0 = \frac{2}{\gamma_1 + \gamma_2}$$

$$K = 1 - \frac{\tau_0}{\tau} = 1 - \frac{2\gamma_1\gamma_2}{(\gamma_1 + \gamma_2)^2}.$$

Noise is suppressed because of Pauli Exclusion principle.

4 DBRT – a coherent or sequential process



quantum well ⇐ 2D electron band

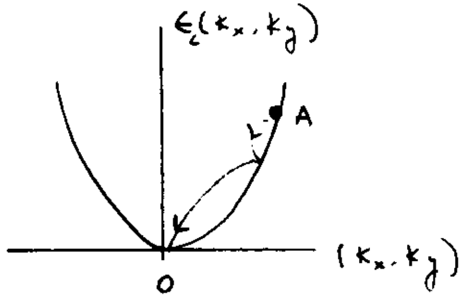
↓

$$\epsilon_c(k_x,k_y) = E_r + \frac{1}{2m}(k_x^2+k_y^2)$$

coherent tunneling ⇒ each state tunnels through the barrier independently. For example from the emitter state $\varepsilon^L(k_x,k_y,k_z)$ to the Q.W. state $\varepsilon_c(k_x\ k_y)$ to the collector state $\varepsilon^R(k_x,k_y,k_z')$

46 4717 (1992)

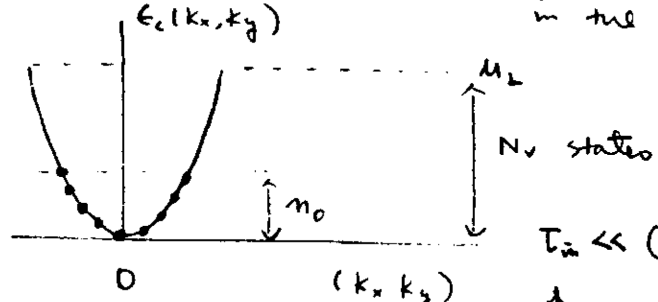
sequential tunneling, PRB (August 15, 1992)



when an electron enters the quantum well, it occupies the state at A.

subsequently the electron will lost its energy by scattering with phonons and other electrons to occupy the lowest available energy state

At equilibrium there are N_v available states with n_0 electrons occupy the lowest n_0 states in the Q.W.



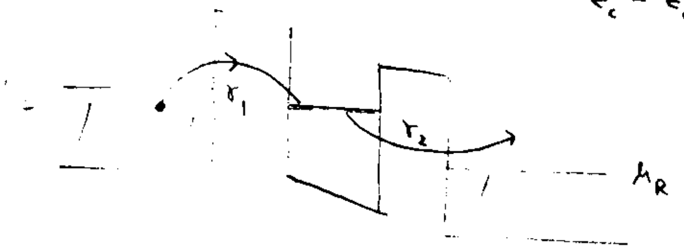
$T_{in} \ll (1/\gamma)$ ← life time of Resonance level

↑ inelastic scattering time

Eventually some of the n_0 electrons would tunnel to the collector.

Stochastic approach to incoherent tunneling

$$\epsilon_c = \epsilon_c(k_x, k_y)$$



$$\gamma_1 = \sum_k |T_{Lk}|^2 \pi\, \delta(\epsilon_c - \varepsilon_k^L)$$ = tunneling rate from the emitter to the QW

$$\gamma_2 = \sum_k |T_{Rk}|^2 \pi\, \delta(\epsilon_c - \varepsilon_k^R)$$ = tunneling rate from the QW to the collector

Within the time interval $(0, t)$

$n_1(t)$ electrons tunnel from the emitter to QW

$n_2(t)$ electrons tunnel from the QW to collector

$e\,\partial_t n_1(t) = i_1(t)$ = the emitter current

$e\,\partial_t n_2(t) = i_2(t)$ = the collector current

$i(t) = \frac{1}{2}[i_1(t) + i_2(t)]$ is the terminal current

Define two random variables $[N(t), n(t)]$

$N(t) = n_1(t) + n_2(t)$ = total number of tunneling events between $(0, t)$

$n(t) = n_1(t) - n_2(t) + n_0$ = total number of electrons staying in the QW at t.

n_0 = electron number in QW at equilibrium ($t \to \infty$)

Define

$$\int_0^t i(t')dt' = \frac{1}{2}\int_0^t [i_1(t') + i_2(t')]dt'$$

$$= \frac{e}{2}\{n_1(t) + n_2(t)] = \frac{e}{2}N(t)$$

the measured current

$$I = \lim_{t\to\infty} \frac{1}{t}\int_0^t \langle i(t')\rangle = \lim_{t\to\infty} e\langle N(t)\rangle / 2t .$$

$$\delta N(t) = N(t) - \langle N(t)\rangle = N(t) - N_0$$

the noise power density

$$S(\omega \sim 0) = \lim_{t\to\infty} \frac{2}{t}\langle |\int_0^t \delta i(t')dt'|^2\rangle$$

$$= e^2 \lim_{t\to\infty} \frac{1}{2t}\langle |N(t) - \langle N(t)\rangle|^2\rangle .$$

The evolutions of the random variables $[N(t), n(t)]$ are governed by the following stochastic equations

$$\begin{pmatrix} N(t+\Delta t) \\ n(t+\Delta t) \end{pmatrix} = \left\{ \begin{array}{l} \begin{pmatrix} N(t)+1 \\ n(t)+1 \end{pmatrix} \\ \begin{pmatrix} N(t)+1 \\ n(t)-1 \end{pmatrix} \\ \begin{pmatrix} N(t) \\ n(t) \end{pmatrix} \end{array} \right\} \text{ with probability } \begin{array}{l} p\Delta t \\ q\Delta t \\ 1-(p+q)\Delta t \end{array}$$

$p = [N_v - n(t)]\gamma_1$ = effective tunneling probability per unit time from emitter to QW.

$\Rightarrow$ Pauli-exclusion principle

$q = n(t)\gamma_2$ = effective tunneling probability per unit time from the QW to collector

$$N(t) = n_1(t) + n_2(t), \quad n(t) = n_1(t) - n_2(t) + n_0 .$$

The probability distribution function $P(N,n,t)$ satisfies

$$P(N,n,t+\Delta t) = P(N-1,n-1,t)\,\Delta t\,\gamma_1\,[N_v-(n-1)]$$
$$+\ P(N-1,n+1,t)\,\Delta t\,\gamma_2\,(n+1)$$
$$+\ P(N,n,t)\left[1-\Delta t\,\gamma_1(N_v-n)-\Delta t\,n\gamma_2\right]$$

or

$$\partial_t P(N,n,t) = -\left[\gamma_1(N_v-n)+\gamma_2 n\right]P(N,n,t)$$
$$+\ \gamma_1\,(N_v-n+1)\,P(N-1,n-1,t)$$
$$+\ \gamma_2\,(n+1)\,P(N-1,n+1,t)$$

The initial distribution function $(t=0)$

$N=0$, $P(N,n,t=0) = \delta_N\, P_0(n)$, $\delta_N = \delta_{N,0}$

$P_0(n)$ is chosen as $P_0(n) = \sum_N P(N,n,t\to\infty)$

The equilibrium distribution function.

such that $n_0 = \sum_n n P_0(n) = \sum_{N n} n P(N,n,t=\infty)$

Define the following reduced distribution function,

$$P(n,t) = \sum_N P(N,n,t)$$
$$Q(n,t) = \sum_N N P(N,n,t)$$
$$S(n,t) = \sum_N N^2 P(N,n,t)$$

and their generating functions

$$P(z,t) = \sum_n z^n P(n,t)$$
$$Q(z,t) = \sum_n z^n Q(n,t)$$
$$S(z,t) = \sum_n z^n S(n,t)$$

and

$$P(z=1,t) = \sum_{nN} P(N,n,t) = 1$$
$$Q(z=1,t) = \sum_{nN} N P(N,n,t) = \langle N(t)\rangle$$
$$S(z=1,t) = \sum_{nN} N^2 P(N,n,t) = \langle N^2(t)\rangle$$

$$\partial_t P(N,n,t) = -[\gamma_1(N_v - n) + \gamma_2 n] P[N,n,t]$$
$$+ \gamma_1 [N_v - n + 1] P(N-1, n-1, t)$$
$$+ \gamma_2 [n+1] P(N-1, n+1, t)$$

$$\Big\updownarrow \sum_{n,N} z^n , \qquad P(z,t) = \sum_{N,n} z^n P(N,n,t)$$

$$[\partial_t + N_v \gamma_1 (1-z)] P(z,t) + (\gamma_1 z + \gamma_2)(z-1)\partial_z P(z,t) = 0 \qquad (1)$$

similarly we can also obtain

$$[\partial_t + \gamma_1 N_v (1-z)] Q(z,t) + (\gamma_1 z + \gamma_2)(z-1)\partial_z Q(z,t)$$
$$= \gamma_1 N_v z P(z,t) + (\gamma_2 - \gamma_1 z^2)\partial_z P(z,t) \qquad (2)$$

$$[\partial_t + \gamma_1 N_v (1-z)] S(z,t) + (\gamma_1 z + \gamma_2)(z-1)\partial_z S(z,t)$$
$$= 2\gamma_1 N_v z Q(z,t) + 2(\gamma_2 - \gamma_1 z^2)\partial_z Q(z,t)$$
$$+ \gamma_1 N_v z P(z,t) + (\gamma_2 - \gamma_1 z^2)\partial_z P(z,t) \qquad (3)$$

$$\left.\begin{aligned} Q(z,t) &= \sum_{Nn} z^n N P(N,n,t) = \langle N(t)\rangle \\ S(z,t) &= \sum_{nN} z^n N^2 P(N,n,t) = \langle N^2(t)\rangle \end{aligned}\right\rangle z=1$$

$$[\partial_t + N_v \gamma_1 (1-z)] P(z,t) + (\gamma_1 z + \gamma_2)(z-1)\partial_z P(z,t) = 0$$

Use the Laplace transform

$$P(z,s) = \int_0^\infty e^{-st} P(z,t)\, dt$$

$$\int_0^\infty e^{-st} \partial_t P(z,t)\, dt = -P(z,0) + sP(z,s)$$

$$P(z,0) = \sum_{Nn} z^n P(N,n,t=0) = \sum_n z^n P_0(n) = P_0(z)$$

We obtain

$$[s + \gamma_1 N_v (1-z)] P(z,s) + (\gamma_1 z + \gamma_2)(1-z)\partial_z P(z,s)$$
$$= P_0(z)$$

For $z=1$, $\quad P(z=1,s) = \frac{1}{s} P_0(z=1) = \frac{1}{s}$

put $z = 1-\epsilon$, $(\epsilon \ll 1)$

$$P(z=1-\epsilon, s) = P(z=1,s) + \partial_z P(z=1,s)(-\epsilon)$$
$$+ \frac{1}{2}\partial_z^2 P(z=1,s)\epsilon^2 + \cdots$$

$$\partial_z P(z=1-\epsilon, s) = \partial_z P(z=1,s) + \partial_z^2 P(z=1,s)(-\epsilon)$$

$$(s+\gamma_1 N_v \epsilon)\left[P(z=1,s) - \epsilon\, \partial_z P(z=1,s) + \frac{\epsilon^2}{2}\partial_z^2 P(z=1,s)\right]$$

$$-(\gamma_1+\gamma_2-\gamma_1\epsilon)\,\epsilon\left[\partial_z P(z=1,s) - \epsilon\,\partial_z^2 P(z=1,s)\right]$$

$$= P_0(z=1) - \epsilon\,\partial_z P_0(z=1) + \frac{\epsilon^2}{2}\partial_z^2 P_0(z=1).$$

$$P_0(z) = \sum_{nN} z^n P(N,n,t=\infty), \quad P_0(z=1)=1$$

$$\partial_z P_0(z=1) = \langle n(t=\infty)\rangle = \langle n\rangle_0 = n_0$$

$$\partial_z^2 P_0(z=1) = \langle n(n-1)\rangle_0$$

$$A + B\epsilon + C\epsilon^2 = 0$$

A : $\quad P(z=1,s) = \frac{1}{s}$

B : $\quad [s+\gamma_1+\gamma_2]\,\partial_z P(z=1,s) = n_0 + \gamma_1 N_v P(z=1,s)$

$$= n_0 + \frac{\gamma_1 N_v}{s}$$

$$\partial_z P(z=1,s) = \left(n_0 - \frac{\gamma_1 N_v}{\gamma_1+\gamma_2}\right)\frac{1}{s+\gamma_1+\gamma_2} + \frac{\gamma_1 N_v}{\gamma_1+\gamma_2}\,\frac{1}{s}$$

C : $\quad \partial_z^2 P(z=1,s) = \left[\langle n(n-1)\rangle_0 - \frac{\gamma_1 N_v \gamma_2 (N_v-1)}{(\gamma_1+\gamma_2)^2}\right]$

$$\times \frac{1}{s+2(\gamma_1+\gamma_2)} + \frac{\gamma_1 N_v \gamma_2 (N_v-1)}{(\gamma_1+\gamma_2)^2}\,\frac{1}{s}$$

The inverse Laplace transform

$$f(t) = \frac{1}{2\pi i}\int_{-\infty+ic}^{\infty+ic} ds\, f(s)\, e^{st}$$

We obtain

$$P(z=1,t) = 1$$

$$\partial_z P(z=1,t) = \left(n_0 - \frac{\gamma_1 N_v}{\gamma_1+\gamma_2}\right)e^{-(\gamma_1+\gamma_2)t} + \frac{\gamma_1 N_v}{\gamma_1+\gamma_2}$$

$$= \langle n(t)\rangle = n_0 \quad \text{at } t=0$$

$$= \frac{\gamma_1 N_v}{\gamma_1+\gamma_2} = n_0 \quad \text{at } t=\infty$$

n_0 is the equilibrium carrier no. inside the QW.

and is chosen as the initial occupation no. inside the quantum well.

$$\partial_z P(z=1,t) = \frac{\gamma_1 N_v}{\gamma_1+\gamma_2} = n_0$$

$$\partial_z^2 P(z=1,t) = \left[\langle n(n-1)\rangle_0 - \frac{\gamma_1 N_v \gamma_1 (N_v-1)}{(\gamma_1+\gamma_2)^2}\right]$$

$$\times e^{-2(\gamma_1+\gamma_2)t} + \frac{\gamma_1 N_v \gamma_1 (N_v-1)}{(\gamma_1+\gamma_2)^2}$$

$$\because \langle n(n-1)\rangle_0 = \partial_z^2 P(z=1, t=0) = \frac{\gamma_1 N_v \gamma_1 (N_v-1)}{(\gamma_1+\gamma_2)^2}$$

$$\partial_z^2 P(z=1,t) = \frac{\gamma_1 N_v \gamma_1 (N_v-1)}{(\gamma_1+\gamma_2)^2}$$

Now let us go back to the equation

$$(\partial_t + \gamma_1 N_v (1-z)) Q(z,t) = (\gamma_1 z + \gamma_2)(z-1)\partial_z Q(z,t)$$

$$= \gamma_1 N_v z P(z,t) + (\gamma_2 - \gamma_1 z^2)\partial_z P(z,t)$$

Here $Q(z,t) = \sum_{N n} N z^n P(N,n,t)$

set $z = 1-\epsilon$, $Q(z=1-\epsilon,t) = Q(1,t) - \epsilon\partial_z Q(z=1,t)$

$$(\partial_t + \gamma_1 N_v \epsilon)\left[Q(1,t) - \epsilon\partial_z Q(z=1,t)\right] - (\gamma_1+\gamma_2)\epsilon$$

$$\times \partial_z Q(z=1,t)$$

$$= \gamma_1 N_v (1-\epsilon)\left[P(1,t) - \epsilon\partial_z P(z=1,t)\right]$$

$$+ (\gamma_2 - \gamma_1 + 2\gamma_1\epsilon)\left[\partial_z P(z=1,t) - \epsilon\partial_z^2 P(z=1,t)\right]$$

$$D + F\epsilon = 0$$

$$D: \quad \partial_t Q(z=1,t) = \gamma_1 N_v P(z=1,t) + (\gamma_2-\gamma_1)\partial_z P(z=1,t)$$

$$= \gamma_1 N_v + (\gamma_2-\gamma_1)\frac{\gamma_1 N_v}{\gamma_1+\gamma_2}$$

$$= \frac{2\gamma_1\gamma_2 N_v}{\gamma_1+\gamma_2}$$

$$\underbrace{Q(z=1,t) - Q(z=1,0)}_{} = \frac{2\gamma_1\gamma_2 N_v t}{\gamma_1+\gamma_2} = \langle N(t)\rangle$$

$$Q(z=1,0) = 0$$

F : $[\partial_t + (r_1+r_2)]\,\partial_z Q(z=1,t) = r_1 N_v\, Q(z=1,t)$

$$+ r_1 N_v P(z=1,t) + (r_1 N_v - 2r_1)\,\partial_z P(z=1,t)$$

$$+ (r_2 - r_1)\,\partial_z^2 P(z=1,t)$$

$$= r_1 N_v \frac{2 r_1 r_2 N_v t}{r_1+r_2} + r_1 N_v + (r_1 N_v - 2r_1)\frac{r_1 N_v}{r_1+r_2}$$

$$+ (r_2 - r_1)\frac{r_1 N_v r_1 (N_v - 1)}{(r_1+r_2)^2}$$

$$= 2 r_2 (r_1+r_2) n_0^2 t + \alpha$$

$$n_0 = \langle n(t=\infty)\rangle = \langle n(t=0)\rangle = \frac{r_1 N_v}{r_1+r_2}$$

The above equation can be solved by using the Laplace transform

$$(s + r_1 + r_2)\,\partial_z Q(z=1,s) = 2 r_2 (r_1+r_2) n_0^2 \frac{1}{s^2} + \frac{\alpha}{s}$$

$$\partial_z Q(z=1,s) = 2 r_2 (r_1+r_2) n_0^2 \frac{1}{s^2(s+r_1+r_2)} + \frac{\alpha}{s(s+r_1+r_2)}$$

$$\partial_z Q(z=1,s) = 2 r_2 n_0^2 \left[\frac{1}{s^2} - \frac{1}{(r_1+r_2)}\left(\frac{1}{s} - \frac{1}{s+r_1+r_2}\right)\right]$$

$$+ \frac{\alpha}{r_1+r_2}\left(\frac{1}{s} - \frac{1}{s+r_1+r_2}\right)]$$

$$\partial_z Q(z=1,t) = 2 r_2 n_0^2 t - \frac{2 r_2 n_0^2}{(r_1+r_2)}\left[1 - e^{-(r_1+r_2)t}\right]$$

$$+ \frac{\alpha}{r_1+r_2}\left[1 - e^{-(r_1+r_2)t}\right]$$

Since we are interested only the long time behavior $t \to \infty$

$$\partial_z Q(z=1,t) \simeq 2 r_2 n_0^2 t - \frac{2 r_2 n_0^2 - \alpha}{r_1+r_2}$$

$$\alpha = \frac{r_2 (r_2 - r_1) n_0}{r_1 + r_2} + 2 r_2 n_0^2$$

$$\partial_z Q(z=1,t) = 2 r_2 n_0^2 t + \frac{r_2 (r_2 - r_1)}{r_1 + r_2} n_0$$

Finally let us examine the equation satisfied by $S(z,t) = \sum_{nN} z^n N^2 P(N,n,t)$

For $z=1$ $\quad S(z=1,t) = \langle N^2(t)\rangle$

$$\partial_t S(z=1,t) = \gamma_1 N_v P(z=1,t) + (\gamma_2-\gamma_1)\partial_z P(z=1,t)$$

$$+ 2\gamma_1 N_v Q(z=1,t) + 2(\gamma_2-\gamma_1)\partial_z Q(z=1,t)$$

$$\begin{cases} P(z=1,t) = 1 \\ \partial_z P(z=1,t) = \dfrac{\gamma_1 N_v}{\gamma_1+\gamma_2} = n_0 = \langle n(t)\rangle \\ Q(z=1,t) = \langle N(t)\rangle = \dfrac{2\gamma_1\gamma_2 N_v t}{\gamma_1+\gamma_2} = 2\gamma_2 n_0 t \\ \partial_z Q(z=1,t) = 2\gamma_2 n_0 t + \dfrac{\gamma_2(\gamma_2-\gamma_1)}{\gamma_2+\gamma_1} n_0 \end{cases}$$

$$\partial_t S(z=1,t) = \frac{4\gamma_2(\gamma_1^2+\gamma_2^2)}{(\gamma_1+\gamma_2)^2} n_0 + 8\gamma_2^2 n_0^2 t$$

$$S(z=1,t) = 4\gamma_2^2 n_0^2 t^2 + \frac{2(\gamma_1^2+\gamma_2^2)}{(\gamma_1+\gamma_2)^2} 2\gamma_2 n_0 t$$

$$S(z=1,t) = \langle N^2(t)\rangle$$

$$= \langle N(t)\rangle^2 + \frac{2(\gamma_2^2+\gamma_1^2)}{(\gamma_1+\gamma_2)^2}\langle N(t)\rangle$$

$$\langle N(t)\rangle = 2\gamma_2 n_0 t$$

the tunneling current

$$I = \frac{e}{2}\lim_{t\to\infty}\frac{\langle N(t)\rangle}{t} = e\gamma_2 n_0$$

$$= \frac{e\gamma_1\gamma_2 N_v}{(\gamma_1+\gamma_2)}$$

the noise power density

$$S(\omega\to 0) = e^2 \lim_{t\to\infty}\frac{1}{2t}\langle |N(t) - \langle N(t)\rangle|^2\rangle$$

$$= e^2\lim_{t\to\infty}\frac{1}{2t}\left[\langle N^2(t)\rangle - \langle N(t)\rangle^2\right]$$

$$= \frac{e^2}{2}\lim_{t\to\infty}\frac{\langle N(t)\rangle}{t}\left[1 + \frac{(\gamma_1-\gamma_2)^2}{(\gamma_1+\gamma_2)^2}\right]$$

$$S(w \sim 0) = eI \left[1 + \frac{(\gamma_1+\gamma_2)^2 - 4\gamma_1\gamma_2}{(\gamma_1+\gamma_2)^2} \right]$$

$$= 2eI \left[1 - \frac{2\gamma_1\gamma_2}{(\gamma_1+\gamma_2)^2} \right]$$

In sequential tunneling the noise power density is identical to that of coherent tunneling.

5. Discussion

1) The coherent tunneling across a single junction with out structure gives the full shot noise power density

$$S_i(w \sim 0) = 2eI$$

2) For a DBRTS, the shot noises calculated for both coherent and sequential (incoherent) tunneling are identical

$$S_i(w \sim 0) = 2eI \left[1 - \frac{2\gamma_1\gamma_2}{(\gamma_1+\gamma_2)^2} \right]$$

This result is in agreement with experiments