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**"System Sizing"**

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**These are preliminary lecture notes, intended only for distribution to participants.**

## SYSTEM SIZING

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### 1. INTRODUCTION

System sizing is the process of determining the cheapest combination of array size and storage capacity that will meet the load requirements with an acceptable level of security over the expected lifetime of the installation.

By "cheapest" is meant the system with the lowest life cycle cost (LCC). This takes into account not only the initial capital outlay for procuring, transporting and installing the equipment but also the discount rate, the inflation rate and the costs of operation, maintenance, insurance, repair and replacement.

Security of supply, sometimes referred to as "availability", is the percentage of a specified period of years over which the system is likely to meet all load demands. It has a very important bearing on the capital cost, which rises steeply as the 100% mark is approached. For domestic standalone systems, a security level of 95% or even lower may be considered sufficient but for professional applications like telecommunications a level of 99.99% or more is generally required.

Because the solar radiation varies and, in many cases, the load demand cannot be accurately predicted, sizing is necessarily an approximate calculation, based on probabilities. Most system designers have their own methods and computer software for sizing, the details of which are kept confidential for commercial reasons. However, to give some idea of the process, a logical step-by-step procedure for sizing a standalone PV/battery system is set out in the next section. It is assumed that, for simplicity, the system does not have a maximum power point tracker and there is no back-up. A model calculation, following this procedure, is given in Section 3.

### 2. PROCEDURE

- 1) Make a list of all the appliances it is desired to supply from the system, quoting the operating voltage and power demand of each and the average daily duty cycle (h/day) for each month of the year.
- 2) Group the appliances by type (ac or dc) and operating voltage. Total the power demands in each group.
- 3) Decide on the nominal voltage of the dc bus. In dc systems, this should be the same as the voltage of the biggest load. Loads at other voltages can then be supplied through converters. In ac or mixed ac/dc systems, the dc bus voltage should be compatible with the input voltage of the selected inverter.
- 4) Taking the month of January first, multiply the power demand of each appliance by the average daily duty cycle to give the mean daily energy requirements in Wh/day. If an appliance draws its current through an inverter or converter, its energy requirement must be divided by the appropriate conversion efficiency to allow for power conditioning losses.

- 5) Divide the mean daily energy requirements calculated in Step 4 by the nominal dc bus voltage and add to give the total mean daily load in Ah/day. (Some adjustment of the Ah load figures may be necessary in cases where constant power loads are operating at night, because the actual dc bus voltage is governed by the battery voltage and this will fall below the nominal 2V/cell as the battery discharges. Such loads will therefore draw more current at night than during the day).
- 6) Divide the total mean daily load determined in Step 5 by factors to allow for losses in the battery and cables. Battery losses depend on the charge efficiency (0.7 to 0.9) and the proportion of PV-generated electricity that is stored before use, a figure that can be estimated from the daily load profile in winter. Cables should be chosen to keep cable losses to less than 2.5%, so a factor of 0.975 is appropriate here.
- 7) Carry out Steps 4, 5 and 6 for the other months of the year and, from the resulting data, calculate the annual mean daily load, corrected for losses in the power conditioning equipment, battery and cables.
- 8) From meteorological data for the site, take the monthly mean daily irradiation figures in kWh/m<sup>2</sup> per day at three tilt angles, including the angle of latitude.
- 9) For each month, divide the mean daily load (Ah/day) by the mean daily irradiation to give the design current required from the array, i.e. the current at an irradiance of 1000W/m<sup>2</sup>.
- 10) Select the largest array design current for each tilt angle. This represents in each case the worst combination of load demand and irradiation.
- 11) Choose the smallest array design current from the three values selected in Step 10. The optimum tilt angle is that corresponding to this minimum.
- 12) Determine the highest ambient temperature the array is likely to experience, on a probability based on the required level of security. Then add the estimated temperature rise in the selected type of module to give the estimated maximum operating temperature. (For modern Tedlar/foil-backed modules mounted in an open rack, the temperature rise will be about 25°C but, if the array is roof-mounted, it will run somewhat hotter).
- 13) From the I-V characteristic of the selected module at this temperature and an irradiance of 1000W/m<sup>2</sup>, determine the voltage at which maximum power is delivered. Choose a working voltage just below this, to allow for variations from the nominal performance.
- 14) Calculate the number of series-connected modules required to produce the dc bus voltage, thus :-

$$\text{No. of modules in a series string} = \frac{\text{dc bus voltage} + \text{diode voltage drop}}{\text{module working voltage}}$$

- 15) From the same I-V characteristic, find the current output at the

working voltage and  $1000\text{W/m}^2$ . (This current is approximately proportional to the irradiance, so it may be expressed in terms of A per  $\text{kW/m}^2$ . It is not significantly affected by changes of temperature below the maximum operating temperature).

- 16) For each month of the year, multiply the current determined in Step 15 by the mean daily irradiation ( $\text{kWh/m}^2$  per day) at the optimum tilt angle to give the gross mean daily output of the module string in Ah/day. Reduce this figure by, say, 5% to allow for array losses due to module mismatch, blocking diodes, shading, dirt and degradation.
- 17) From these monthly figures, calculate the net annual mean daily output of the module string.
- 18) Divide the annual mean daily load by the net annual mean daily output of a module string to give the minimum number of strings required for the array.
- 19) Using this minimum number, calculate the array output for each month, thus :-  

$$\text{Monthly array output (Ah)} = \frac{\text{Net mean daily output/string}}{\text{x No. of strings x No. of days in the month}}$$
- 20) Compare the monthly array outputs with the corresponding load demands and work out the surplus or deficit for each month. From these figures, determine the total summer surplus and winter deficit.
- 21) Calculate the battery capacity required for seasonal storage, thus :  

$$\text{Battery capacity (Ah)} = \frac{\text{Winter deficit (Ah)}}{\text{Max. depth of discharge x temp. corr. factor}}$$

The temperature correction factor allows for the lower capacity of the battery during the winter. As the capacity loss of a lead-acid battery below  $25^\circ\text{C}$  is about 0.6% per degree C, a factor of 0.9 will cater for temperatures down to about  $8^\circ\text{C}$ .
- 22) Check that this capacity is sufficient to cater for the longest period of cloudy days likely to be experienced, with a probability based on the required level of security. If it is not, increase the capacity accordingly.
- 23) Divide the dc bus voltage by the nominal voltage of the selected battery to give the number of batteries in a series string.
- 24) Divide the capacity calculated in Step 21 by the  $C_{100}$  capacity rating of the battery and round up to give the number of battery strings in parallel.
- 25) Estimate the life cycle cost of the system.
- 26) Repeat Steps 19 to 25, assuming more than the minimum number of module strings, to determine the cheapest combination of array and battery.

### 3.1 The problem

### 3.2 Load and irradiation data

Table II shows, as an example, the load calculations for December.

Table I LOAD AND IRRADIATION DATA

Month	Mean daily load Ah/day	Mean daily irradiation kWh/m <sup>2</sup> per day		
		30°	Latitude	60°
January	35.0	3.05	3.21	3.38
February	33.9	3.70	3.80	3.78
March	33.1	4.49	4.49	4.19
April	32.3	5.47	5.35	4.65
May	31.5	5.97	5.72	4.70
June	31.5	6.30	5.98	4.74
July	31.5	6.70	6.38	5.12
August	31.5	6.62	6.44	5.48
September	32.3	5.82	5.81	5.34
October	33.1	4.62	4.75	4.71
November	34.0	3.74	3.94	4.13
December	34.9	2.73	2.89	3.07
Annual mean	33.7	4.94	4.90	4.44

Table II      LOAD CALCULATION FOR DECEMBER

[illegible]

### 3.3 Number of storage days

From the irradiation statistics for Messina and taking into account the required 95% level of security, the number of storage days, i.e. the maximum number of consecutive cloudy days over which the battery would be required to meet the load demand with no contribution from the array, is taken as 4.

### 3.4 Determination of optimum tilt

Table III shows the array design currents for the three tilt angles, obtained by dividing the mean daily load by the mean daily irradiation for each month. The largest currents are all in the month of December, indicating that this is when the worst combination of load demand and irradiation occurs. The smallest of these three values is at a tilt of 60°. So this is the optimum tilt angle.

Table III ARRAY DESIGN CURRENTS

Month	Array design current A per kW/m <sup>2</sup>		
	30°	Latitude	60°
January	11.47	10.90	10.35
February	9.16	8.92	8.97
March	7.37	7.37	7.90
April	5.90	6.04	6.95
May	5.28	5.51	6.70
June	5.00	5.27	6.64
July	4.70	4.94	6.15
August	4.76	4.89	5.75
September	5.55	5.56	6.05
October	7.16	6.97	7.03
November	9.09	8.63	8.23
December	12.78	12.08	11.37

### 3.5 Maximum operating cell temperature

The maximum ambient temperature at Stromboli, based on a 95% probability, is expected to be 27°C and the temperature rise of the selected module, a commercial 12V, 50Wp ( $\pm 10\%$ ) crystalline silicon type, 25°C, assuming open-rack mounting. So the estimated maximum operating cell temperature is  $(27 + 25) = 52^\circ\text{C}$ .

### 3.6 Module working voltage

Fig.1 shows the I-V characteristic of the selected module, at 52°C and an irradiance of 1000W/m<sup>2</sup>. We can see that maximum power is delivered at 16.0V. A suitable working voltage, bearing in mind the manufacturer's tolerance of  $\pm 10\%$  on the power output, is 13.5V.

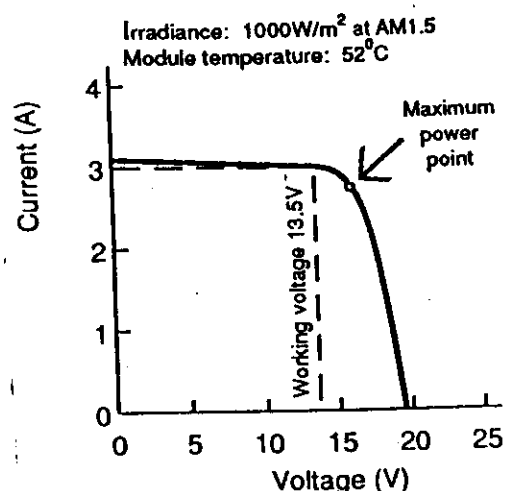


Fig.1 MODULE I-V CHARACTERISTIC

### 3.7 Array sizing

For a dc bus voltage of 48V,

$$\begin{aligned} \text{No. of modules per string} &= \frac{48 + 1 \text{ (allowance for blocking diode)}}{13.5} \\ &= 3.63, \text{ rounded up to } 4 \end{aligned}$$

The output current at the working voltage is 3.0A, so, at a mean daily irradiation of  $G \text{ kWh/m}^2$ , each module string will deliver  $3G \text{ Ah/day}$  gross.

This output must be reduced by 5% to allow for array losses. Therefore, the net mean daily output of each string

$$= (3 \times 0.95)G \text{ Ah/day}$$

$$= 2.85G \text{ Ah/day}$$

Table IV lists the monthly mean daily outputs from the module string at the optimum tilt of  $60^\circ$ , calculated from the above expression and the irradiance data in Table I.

Dividing the annual mean daily load by the annual mean daily output from each string, we get :-

$$\begin{aligned} \text{Minimum number of module strings} &= \frac{33.7}{12.7} \\ &= 2.65 \end{aligned}$$

- rounded up to 3 strings.

### 3.8 Battery sizing

Table V lists the surplus or deficit for each month by comparing the load demand with the 3-string array outputs at  $60^\circ$  tilt.

The battery capacity required to cater for the total winter deficit of 501 Ah, allowing for a maximum depth of discharge of 80% and a temperature correction factor of 0.9 is :

$$\frac{501}{0.8 \times 0.9} \text{ Ah}$$

$$= 696 \text{ Ah, rounded up to } 700 \text{ Ah}$$

The number of storage days this capacity will cater for in January, when the daily load is heaviest ( $35 \text{ Ah/day}$ ) is :

$$\frac{700 \times 0.8 \times 0.9}{35} = 14 \text{ days}$$

Table IV MODULE STRING OUTPUTS

Month	Mean daily output from a module string Ah / day
January	9.63
February	10.77
March	11.94
April	13.25
May	13.40
June	13.51
July	14.59
August	15.62
September	15.22
October	13.42
November	11.77
December	8.75
Mean	12.70

Table V MONTHLY OUTPUT/LOAD BALANCE  
- 3-MODULE STRINGS

Month	Load Demand Ah	Array Output Ah	Surplus (+) or Deficit (-) Ah
January	1085	896	-189
February	949	905	-44
March	1026	1110	+84
April	969	1193	+224
May	977	1246	+269
June	945	1216	+271
July	977	1357	+380
August	977	1453	+476
September	969	1370	+401
October	1026	1248	+222
November	1020	1059	+39
December	1082	814	-268
Total summer surplus			+2366
Total winter deficit			-501

The selected lead-acid battery has a  $C_{100}$  rating of 100Ah and a nominal voltage of 12V.

$$\text{Therefore, the number of batteries in a series string} = \frac{48}{12} = 4$$

$$\text{and the number of battery cells in parallel} = \frac{700}{100} = 7$$

The sizing from the initial calculation (Option 1) is therefore :

Array : 3 strings of 4 50Wp modules = 600 Wp

Battery : 7 strings of 4 12V 100Ah batteries = 33.6 kWh

### 3.9 Effect of larger array

Table VI MONTHLY OUTPUT/LOAD BALANCE  
- 4-MODULE STRINGS

Now let us see the effect of increasing the number of module strings from 3 to 4. Table VI shows the monthly output/load balance for this configuration at 60° tilt.

In this case, there is no winter deficit, so the battery capacity is determined by that required to sustain the load over the 4 storage days in winter, namely :

$$\frac{4 \times 35.0}{0.8 \times 0.9} = 194 \text{ Ah}$$

- rounded up to 200 Ah

This could be provided by two 100 Ah battery strings.

Month	Load Demand Ah	Array Output Ah	Surplus (+) or Deficit (-) Ah
January	1085	1195	+110
February	949	1207	+258
March	1026	1480	+454
April	969	1591	+622
May	977	1661	+684
June	945	1621	+676
July	977	1809	+832
August	977	1937	+960
September	969	1827	+858
October	1026	1664	+638
November	1020	1412	+392
December	1082	1085	+3
Total summer surplus			+6487
Total winter deficit			nil

The sizing from the second calculation (Option 2) is therefore :

Array : 4 strings of 4 50Wp modules = 800 Wp

Battery : 2 strings of 4 12V 100Ah batteries = 9.6 kWh



### 3.10 Life cycle cost analysis

The life cycle cost is calculated as follows :-

$$LCC = C + M + R - S$$

where :  
C = capital cost  
M = operation and maintenance cost  
R = repair and replacement cost  
S = salvage value at end of life

All these costs are expressed in terms of their present worth value.

The formula for the single present worth, P, of a sum of money, F, to be paid in a given year, N, at a net discount rate,  $l$  (i.e. nominal discount rate minus the rate of inflation) is :-

$$P = \frac{F}{(1 + l)^N}$$

The formula for the uniform present worth, P, of a sum, A, spent or received annually over a period of N years at a net discount rate,  $l$ , is :

$$P = \frac{A(1 - (1 + l)^{-N})}{l}$$

The "present worth factor" for a single payment is  $P/F$  and, for an annual payment,  $P/A$ .

The LCC of Options 1 and 2 are calculated in Table VII overleaf. The following assumptions are made in the analysis :-

Module price	£ 3/Wp
Array installation cost	£ 0.4/Wp
Battery price	£ 90/kWh
Battery housing & installation cost	£ 5/kWh
Array life	20 years
Battery life	8 years
Annual operation & maintenance cost	1% of initial capital cost
Net discount rate	5%

The analysis shows that, on the basis of these assumptions, Option 2 is the cheaper. Increasing the array size further would simply increase the overall cost, as the battery would still have to cater for the 4 storage days.

The preferred sizing is therefore :-  
Array : 800 Wp at 60° tilt  
Battery : 9.6 kWh

Over an average year, the array would produce surplus energy amounting to some 300 kWh. Rather than waste it, the householder could use it to run more appliances (e.g. a water heater) during the summer months.

Table VII LIFE CYCLE COST ANALYSIS

Item	Option 1					Option 2				
	Single present worth year	Uniform present worth year	Cost £	Present worth factor	Present worth £	Single present worth year	Uniform present worth year	Cost £	Present worth factor	Present worth £
<b>1. Capital equipment</b>										
Array @ £3/W <sub>p</sub> .....	0		1800	1	1800	0		2400	1	2400
Array installation .....	0		240	1	240	0		320	1	320
Battery bank & housing @ £95 / kWh .....	0		3192	1	3192	0		912	1	912
Inverter .....	0		1800	1	1800	0		1800	1	1800
Other components .....	0		500	1	500	0		500	1	500
Installation .....	0		750	1	750	0		750	1	750
Sub-total (equipment & instaln.) .....			8282		8282			6682		6682
<b>2. Operation &amp; Maintenance</b>										
Labour: yearly inspection .....		20	80	12.46	997		20	67	12.46	835
<b>3. Repair &amp; Replacement</b>										
Battery bank @ £90 / kWh ....	8		3024	0.677	2047	8		864	0.677	585
Battery bank @ £90 / kWh ....	16		3024	0.458	1385	16		864	0.458	396
Rebuild inverter .....	10		900	0.614	553	10		900	0.614	553
<b>4. Salvage</b>										
20% of original cost of equipment .....	20		1656	0.377	624	20		1336	0.377	504
<b>Life Cycle Cost (Items 1+2+3-4)</b>	12640					8547				