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COLLEGE ON SOIL PHYSICS

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"Heat Transfer in Soils and Soil Temperature - I"

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These are preliminary lecture notes, intended only for distribution to participants.

HEAT TRANSFER IN SOILS AND SOIL TEMPERATURE – I

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We know that various biological and chemical activities are expressions of heat transfer. Such phenomena may continue in sufficient amounts if certain values of temperature be kept around. Nitrification begins favourably after the soil temperature of the order of 10-12°C. The optimum temperature for plant growth varies widely. In fact the amount of heat flow in soils affects germination. Even the thermal properties of the seed[7] vary with temperature (Fig.1). While designing roads and

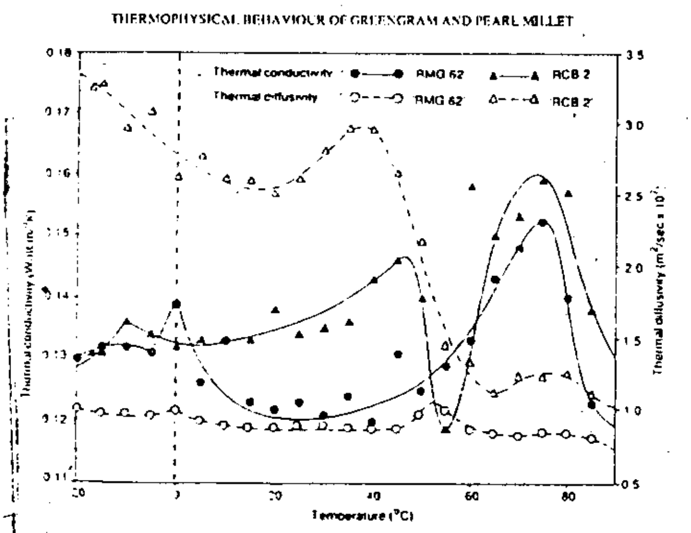

Fig 1 Relation of thermal conductivity and thermal diffusivity of greengram and pearl millet with temperature

buildings, laying pipe lines and underground cables, the need for the thermal characteristics of the soil is felt. Through such data one recognises why and how the annual temperature variation penetrates into the soil at larger depths than diurnal variation. The study of soil temperature tells us what values of maximum and minimum temperature may occur in the dry soil and how these will be modified when the moisture and other environmental conditions are altered. Besides, the insulative nature of dry sand enables one to use it as a solar sand collector and to cut down heat dissipation from heat stores.

1.0 Heat Balance at the Soil Surface[2,4,6]

As there exists plant cover, topography is uneven, there might be different adjoining locations of land and moisture, therefore"soil surface" loses its meaning as a mathematical surface. we term it as an active surface, which means that it is a dynamically active region at which solar insolation is transferred. It is at the active surface that heat exchange with the surface air and the underlying soil layers occurs. The temperature oscillations involve a fairly deep soil layer (whose thickness is determined by the thermophysical coefficients and the period of the heat wave), and the lowest is the constant temperature level. This depth is not reached by the heat wave (Fig.2).

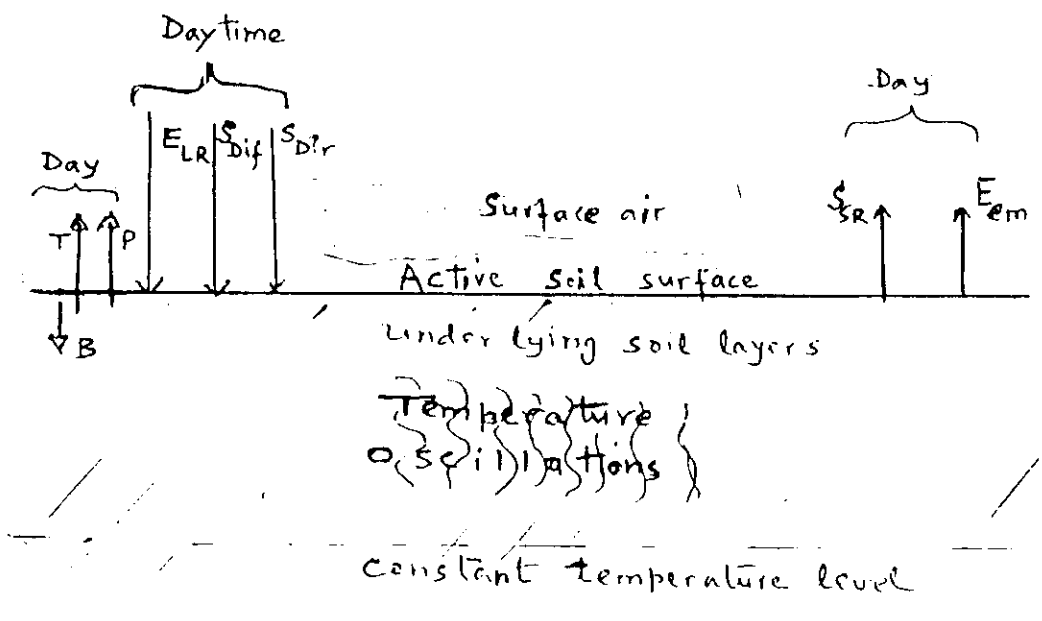

Fig.2. Heat balance at the soil surface

The chief source of all thermal effects on soil surface is the direct short-wave solar radiation. Let us call the intensity of this flux as S_{Dir} (cal min^{-1} cm^{-2}). The next contribution comes from the solar radiation and is termed as diffuse radiation S_{Dif}. This reaches the active surface after scattering by dust particles and air components. In addition to these we also have long-wave radiation flux E_{LR} from the atmosphere. Now we consider the part given out by the surface. Energy is mainly reflected as short-wave radiation by an amount, say, S_{SR}. Also, part of the long-wave radiation is emitted by the active surface into the atmosphere, say, E_{em}. The sum of radiation fluxes at the active surface during the day time is (Fig.2)

$$R_{Day} = S_{Dir} + S_{Dif} + E_{LR} - S_{SR} - E_{em} \qquad (1)$$

At night, $S_{Dir} = S_{Dif} = S_{SR} = 0$

therefore, the radiation balance is expressed by

$$R_{Night} = E_{LR} - E_{em}$$

The amount of flux remaining after all losses into the active surface should be shared by (i) P - the quantity spent on heat exchange with surface air, (ii) T - an amount utilised in evaporation and (iii) H - an amount accounted for the heat transmitted to the underlying soils by conduction. In summer, during the day, P and T are directed from the soil surface to the atmosphere and B from the soil surface inwards. At night in summer, the directions are correspondingly reversed.

P and T are defined as

$$P = A\ C_p\ \left(\frac{d\theta}{dx}\right)_{x=0} \qquad (2)$$

$$T = A\ L\ \left(\frac{dq}{dx}\right)_{x=0} \qquad (3)$$

Here, A is the coefficient of turbulent thermal diffusivity, C_p is the specific heat of air and L is the latent heat of vaporization of water.

The heat flux H into the soil is expressed as

$$H = C\ \Delta\theta + \lambda\ \left(\frac{d\theta}{dx}\right)_{x=x_1} \qquad (4)$$

where C is the heat capacity of the surface layer of thickness 0-x with base 1 cm^2, $\Delta\theta$ is the temperature variation on this layer per unit time, and λ is the thermal conductivity of the soil below this layer $x_1 \rightarrow \infty$.

Thus, the heat balance equation for a bare soil surface in summer in day time is

$$S_{Dir} + S_{Dif} - S_{SR} + E_{LR} - E_{em} = C\Delta\theta + \lambda \left(\frac{\Delta\theta}{\Delta x}\right)_{x=x_1} - \rho C_p \left(\frac{\Delta\theta}{\Delta x}\right)_{x=0} - \rho L \left(\frac{\Delta\theta}{\Delta x}\right)_{x=0}$$

..........(5)

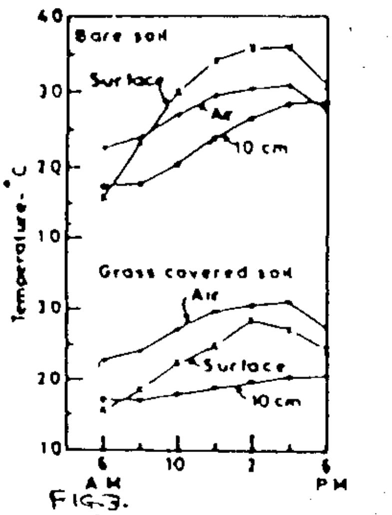

FIG.3.
Soil temperature in bare and grass covered soils [redacted]

The problem is far complicated for a surface if partially covered by vegetation. For summer in day time

$$[\ S_{Dir} + S_{Dif} - S_{SR} + E_{LR} - E_{em} + C\Delta\theta + \lambda\ \frac{\Delta\theta}{\Delta x} - AC_p\ \frac{\Delta\theta}{\Delta x} - AL\ \frac{\Delta q}{\Delta x}\ ] +$$

$$[S_{Dir} + S_{Dif} + E_{LR} - \sigma_{leaf} T^4_{leaf} - S^{leaf}_{SR} - a_{airleaf}\Delta\theta_{airleaf} - rX\Delta\theta\ ] = 0$$

.........(6)

Here, σ_{leaf} is the radiation constant of the leaf, S^{leaf}_{SR}= leaf reflectivity, $a_{airleaf}$ = heat transport coefficient, X is the mass transfer coefficient and r is the numerical factor dependent upon moisture content.

2.0. Temperature Regime of Soils[2,4,6]

To investigate the temperature field in soil, we must find the equation $\theta = f(x,y,z,t)$, where θ is the soil temperature, t is the time and x, y, z are spatial cartesian coordinates. While determining the temperature field, we assume that the soil is isotropic. This means that θ is assumed to vary with depth x and the horizontal surfaces parallel to the soil surface are considered isothermal. Therefore, $\frac{\delta\theta}{\delta y} = \frac{\delta\theta}{\delta z} = 0$ and we are only left with $\frac{\delta\theta}{\delta x}$ or $\theta = f(x,t)$. We shall also assume that heat transfer in soil occurs through the mechanism of conduction only, as in a continuous solid and homogeneous body. As such, it is the Fick law connecting heat flux density ($\vec{H}$) to the temperature gradient through direct proportionality that holds :

$$H = -\lambda \frac{\delta\theta}{\delta x} \qquad (7)$$

where λ is the coefficient of thermal conductivity (cal/cm/sec/°C) or (W/m/K) of the soil. In fact, the values of various thermophysical coefficients are required to write the differential equation. These form the basis through which we can obtain solutions to the energetics problem of finding, evaluating and analysing the temperature field in a soil. In the preceding section we have formulated the boundary conditions. It is the heat balance equation used as the boundary condition that provides the most common approach to the problem.

Let $-(\frac{\delta H}{\delta x})$ be the rate of change of heat flux density with depth (x). Then, in the volume dV (=1x1x dx), heat stored in unit time will be $-(\frac{\delta H}{\delta x})dx$. Also if one defines C as the volumetric heat capacity (cal/cm^3/°c) ,then $C(\frac{\delta\theta}{\delta t})dx$ is the amount of heat stored in the soil. On equating the amount of heat stored in the soil found above by (a) the difference in heat flux density and (b) by the temperature change caused, one gets :-

$$-(\frac{\delta H}{\delta x})\, dx = C\, (\frac{\delta\theta}{\delta t})\, dx$$

$$-(\frac{\delta H}{\delta x}) = C\, (\frac{\delta\theta}{\delta t}) \qquad (8)$$

Since H is given by Fick law,

$$-\frac{\delta}{\delta x}\left(-\lambda\frac{\delta\theta}{\delta x}\right) = C\frac{\delta\theta}{\delta t}$$

$$\frac{\delta\theta}{\delta t} = \frac{\lambda}{C}\frac{\delta^2\theta}{\delta x^2} = \alpha\frac{\delta^2\theta}{\delta x^2} \qquad (9)$$

α is termed as thermal diffusivity (m^2s^{-1} or cm^2s^{-1}).

The above equation is the differential equation for $\theta(x,t)$ which must be obeyed by any possible temperature variation caused by heat transport. To obtain an introductory discussion of temperature variation, we assume that the temperature varies as a pure harmonic function of time around an average value of temperature at all depths.

The surface temperature (x=0) can be written as

$$\theta\ (0,t) = \theta_{av} + A_{\theta_o}\ \mathrm{Sin}\ \omega\ t \qquad (10)$$

Here, θ_{av} is the average temperature in the soil. this is assumed to be the same at all depths. A_{θ_o} is the amplitude of the periodic wave at the surface and ω is the angular frequency. For diurnal variation $\omega = 2\pi\ /\ 24\times60\times60\ s^{-1}$. The temperature at any arbitrary depth would similarly be

$$\theta\ (x,t) = \theta_{av} + A\,\theta_x \mathrm{Sin}\ [\omega t + \phi(x)] \qquad (11)$$

The amplitude and phase terms can be determined by using the above in the differential equation. Thus, we get the temperature variation in the soil at depth as

$$\theta\ (x,t) = \theta_{av} + A_{\theta_o}\ \exp\ \left(\frac{-x}{d}\right)\ \mathrm{Sin}\ \left(\ \omega t - \frac{x}{d}\ \right) \qquad (12)$$

Here, d is the damping depth. On comparing the equation of temperature at the surface we find that in case of the later, the amplitude is smaller by a factor of $\exp\left(\frac{-x}{d}\right)$ as also a phase shift of $\left(\frac{-x}{d}\right)$ occurs. Thus, at depths, temperature diminishes and maximum value will be attained at different times. The constant d is determined through the thermal characteristic of the soil and the frequency of temperature variation: $d = (2\lambda/\omega c)^{1/2}$ (13). For annual variation it is $(365)^{1/2} = 19$ times larger than diurnal

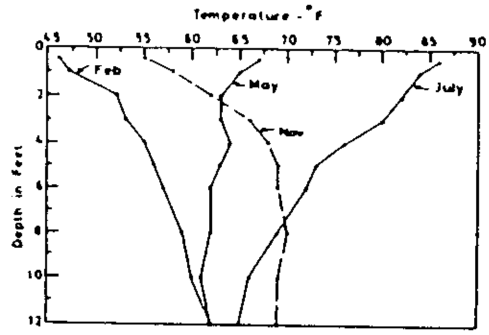

Fig.4. Seasonal variations in soil temperature at various depths.([redacted])

variation. The diurnal variation becomes unperceptible at the depth of 30–50 cm. depending on the thermal characteristic of the soil and the annual variation not below 10 m.

Now, we write the expression for heat flux density H (cal/cm^2/s) for a sinusoidal variation :

$$H(x,t) = -\lambda\left(\frac{\delta\theta}{\delta x}ax\right) + A\theta_o(\lambda C\omega)^{1/2}\exp\left(\frac{-x}{d}\right)\,\mathrm{Sin}\left(\omega t+\phi_o-\frac{x}{d}+\frac{\pi}{4}\right)$$

..........(13)

$(\lambda C)^{1/2}$ is called the heat storage coefficient ($Wm^{-2}s^{1/2}$ $°C^{-1}$) β of the soil.

What we note from the above equation is that heat flux is also a harmonic function of time with $\pi/4$ phase advanced as compared with the temperature variation at a given depth. This phase shift of $\pi/4$ between H and θ corresponds to a time shift of 3 hours for diurnal variation and 1.5 months for annual variation. As an example, the maximum temperature at a surface with grass is approximately 13 hrs.. The maximum heat flux density in a homogeneous soil does not occur at the time of maximum insolation, but earlier. This is due to the interaction of various components in the energy balance at the surface.

3.0. The Influence of Soil Thermal Properties on the Temperature Regime[2,4,6] :-

We have noticed that the heat flux density in the soil is proportional to $(\lambda C\omega)^{1/2}$. The values of coefficients for sand are λ = 0.0042 cgs units and c = 0.5 cgs units. For peat, λ = 0.0007 and c = 0.75. For the sand, we get d = 15.2 cm. and for peat, d = 5.1 cm.. The surface amplitude of heat flux is inversely proportional to $\beta = (\lambda C)^{1/2}$. We get β_{sand} = 0.04 cgs units and β_{peat} = 0.02. Therefore, we conclude that the surface amplitude for peat is twice that of sand. We should also note here that the decrease of amplitude is more rapid in the peat soil having a smaller value of d.

The above example shows clearly how the thermal characteristics of the soil influence the temperature and heat flux values in the soil. We shall now consider this subject in detail. In fact, it is necessary to know a set of four thermophysical coefficients: λ, C, α and β. This set gives us a comprehensive representation of the thermal properties of the soil.

The thermal conductivity (λ) characterises the soil from its ability to conduct heat and is the amount of heat transferred by the soil per unit time through unit area under unit temperature gradient.

The volumetric specific heat (C) characterises the soil from the viewpoint of its capacity to heat up or cool down. It is the amount of heat necessary to change the temperature of one cubic centimeter of soil by one degree.

The thermal diffusivity (α) is numerically equal to the thermal conductivity of a soil sample with unit volumetric specific heat.

The heat storage coefficient (β) characterises the soil from the viewpoint of its storage qualities and is numerically equal to the thermal conductivity of the soil sample whose thermal diffusivity is equal to unity.

Since the soil has a capillary-porous structure, heat exchange in it should be considered to take place by a combination of the following processes :-

(i) Thermal conduction over the mass of individual granules of solid soil skeleton;

(ii) heat conduction from particle to particle at the contact points;

(iii) the molecular heat conduction through the water and air present in the pores.

(iv) convection by the gas or liquid in the pores; &

(v) radiation from particle to particle.

Using the analogy of thermal conductivity we may express the coefficients of convective and radiative heat transfer λ_{conv} and λ_{rad} as well as a coefficient of moisture conductivity λ_{mois}. We can write the respective fluxes in the form of Ficks equation(7) as

$$H_{cond} = \lambda_{cond} \frac{\delta\theta}{\delta x} \qquad (14)$$

Each of the coefficients λ_{cond}, λ_{conv}, λ_{rad}, λ_{mois} is a complicated funtion of various parameters characterising the thermophysical properties and state of the soil. We give here in brief the factors affacting the thermophysical characteristics of the soil.

Conduction The conductivity of the soil depends on the configuration of the constituent phases, packing density or porosity, the nature of contact between the particles, the dimensions of the particles, temperature, water content etc.. We shall be taking this topic in detail in the next section.

Convection The natural convection in soil may occur due to pressure difference. Under the influence of this, the flow of gas or liquid may occur accompanied by the flow of heat. Analysis shows that in natural thick layers and in the presence of small temperature drops, convection does not occur at all. For layers up to 0.5 cm. in thickness convection is not observed up to a temperature drop of 100 ℃. As such the transport of heat in soils by the mechanism of convection may be ignored in natural conditions. For particles and size of soil pores of ~ 0.1–0.2 mm. convection amounts to 0.15–0.3% of the total heat transfer. Only for very large particles and pores of ~ 3.0 mm. does this percentage reach 5%.

Radiation At relatively low temperatures and for samples not having large sized granules, λ_{rad} is also negligibly small. At soil temperatures up to 60^{o} C the radiative flux density is 0.5×10^{-5} cal/cm^2/sec. For pores of size 0.1 mm. and 1.00 mm. the heat flux due to radiation will be 0.2% to 2.5% respectively of the total flux due to all types of heat transfer. In anamously aggregated soil, when the size reaches 6 mm., H_{rad} may be ~ 7%.

Mass Transfer Moisture flow in soil occurs both as vapour and liquid. The moisture moved carries heat with it, which modifies the conditions of heat transfer and also affects the value of thermal coefficient of the soil. As long as the soil temperature does not exceed 50 ℃, the heat transport by vapour flow is not more than 10% of the total. The effect of liquid flow is even smaller. The average value of $\lambda_{vap} \approx 0.137 \times 10^{-3}$ cgs units. The measured values of the soil's thermal conductivity are 20 to 40 times higher than λ_{vap}. For natural soils which are not highly eroded, nor are anomalously wet or overheated, it is reasonable to neglect the effect of mass transfer on the values of thermophysical characteristics of the soil.

3.1. Thermal Conductivity of Soils[2,4,6] :-

Let us first consider the factors which influence the magnitude of equivalent thermophysical characteristics of the soil.

a) Structure and influence of aggregation - The mutual positions of soil particles and pores determine the system and, as such, may give highly diversified values of thermal conductivity. As an example, we consider the idealized situation where solid grains are assumed to be spheres packed symmetrically. A cubic packing will result into a loose system and a tetrahedral packing will yield a compact system. With the varied size of particles and

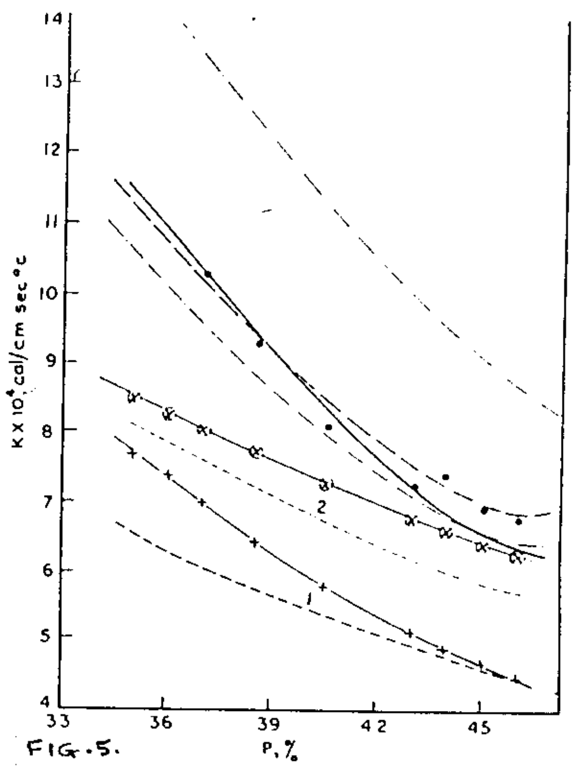

FIG.5.

random packing the complexity of the problem increases. In the figure we have shown how the conductivity values may vary if we consider different structures of the soil, other factors remaining the same.

An increase in the size of particles and pores should interfere overall heat transfer. When soil aggregates are crushed, the number of contacts between the many resulting particles sharply increase. These contacts are loose and the interstitial air phase is less conductive; this results in the decrease of effective conductivity. On the other hand, experiments have shown a rise of 50% in λ_e of enlarged granules. For soil containing many large paricles like sand, the thermal conductivity increases as compared with fine textured clay.

(b) <u>Chemical-mineralogical composition</u> — Different soils have difference in the chemical mineralogical composition of their skeletons. They have silicon, feldspar, quartz, mica, metal oxides or marble, granite, schists, limestone etc.. Therefore, the thermal conductivity of the solid phase (λ_s) becomes a crucial factor. The conduction of heat through interstitial medium also occurs. Therefore, the value of the thermal conductivity of the fluid phase (λ_f) is also an effective factor.

(c) <u>Temperature</u> - A change in the temperature affects the thermal process in the soil pore. The thermal conductivity of soil increases with the rise in temperature. For calculating the rise in thermal conductivity of soils per degree, the following

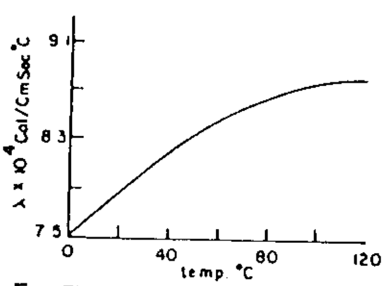

Fig.6 Thermal Conductivity with temperature

empirical formula is also used :

$$\delta\theta = \frac{\delta\lambda}{\lambda\delta\theta} = 0.65\ [2.45\ \rho + 1] + 9.5(D-0.66)^{1.1} \qquad (15)$$

where ρ is the specific weight and D is the particle size.

(d) Porosity and Specific Weight - Experimental study on a variety of dispersed media has shown that porosity of the medium strongly affects the thermophysical characteristics. There is an appreciable increase in equivalent conductivity due to an increase in specific weight. Specific weight and porosity are related through a simple expression. It is instructive to have expressions of conductivity estimation in terms of the porosity of the sample. It has been found that the conductivity of the soil increases as porosity decreases. This is because

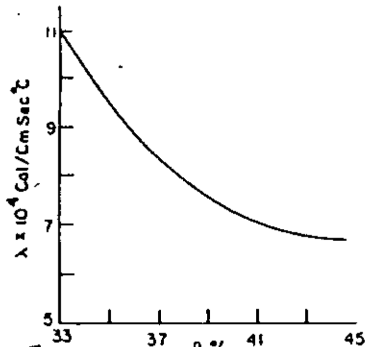

Fig.7.
Thermal Conductivity with porosity

non-conducting air in the pores is replaced by a fraction of good conductor solids.

(e) Soil Water - Water content is the most influential factor in modifying the thermal coefficient of the soil. The physical reasoning for this is that, when soils are wetted, air, a poor

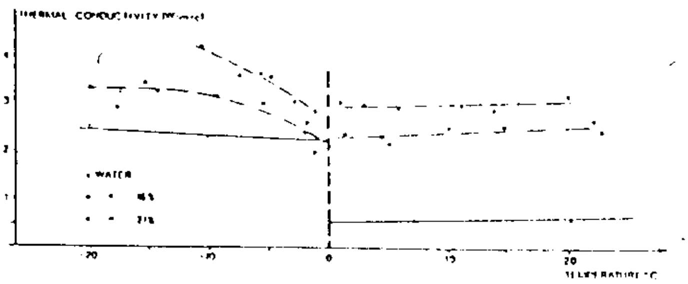

Fig.8. Effective thermal conductivity of sand, at two water contents, versus temperature.

conductor is removed and replaced by water, a good conductor. For sandy soils, over the possible range of water content variation, the effective conductivity may vary by a factor of 5 or even 10.

Like thermal conductivity, other thermal characteristics α, β and C also depend on the above factors. It may be noted that vegetation, mulches, irrigation and drainage also change the thermal characteristics of the soil surface. The dark soil will absorb more energy. Red and yellow soil will show a more rapid temperature rise than white soil. The thermal conductivity of different soils follow the order sand > loam > clay > peat.