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Goals and Optimal Decisions in Earthquake Prediction

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GOALS AND OPTIMAL DECISIONS IN EARTHQUAKE PREDICTION

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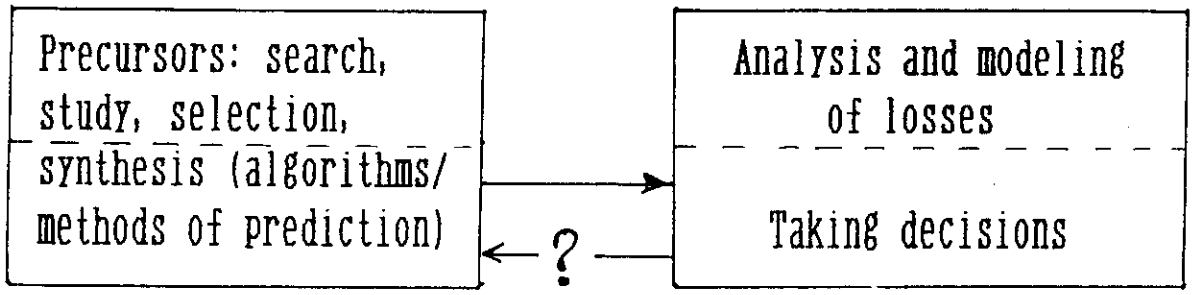
Prediction problem on the whole

Geophysical part

"Economical" part
(or goals on research stage)



Problem: modeling and optimization of losses

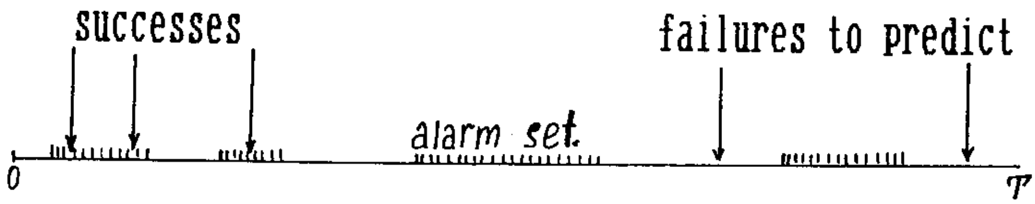
Goals: to understand
- how are synthesized precursors best ?
- which statistical properties of geophysical information J(t) should be analyzed for prediction?

The simplest optimization problem in prediction

Prediction policy:

$$\pi(t \mid \mathcal{J}_t) = \begin{cases} 1 & \text{alarm} \\ 0 & \text{no alarm} \end{cases} \text{ in } (t, t+\Delta) \text{ with probablity } \begin{matrix} p(\mathcal{J}_t) \\ 1 - p(\mathcal{J}_t) \end{matrix}$$

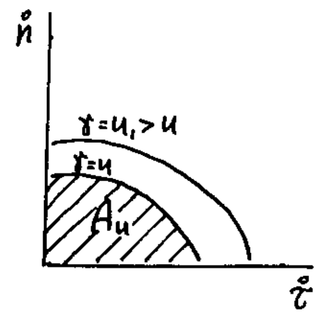
(The typical prediction policy: $p(J_t) = 1$ or 0)
J is geophysical information at moment t:
(state of geophysical fields in some intervals $(t-t_i,\ t-h_i)$,
catalogs of events etc.)



Long-term characteristics (errors) of prediction:

rate of failure-to-predict $\quad \mathring{n} = \dfrac{\#\{\text{failures-to-predict}\}}{\#\{\text{all strong events}\}}, \quad T \gg 1$

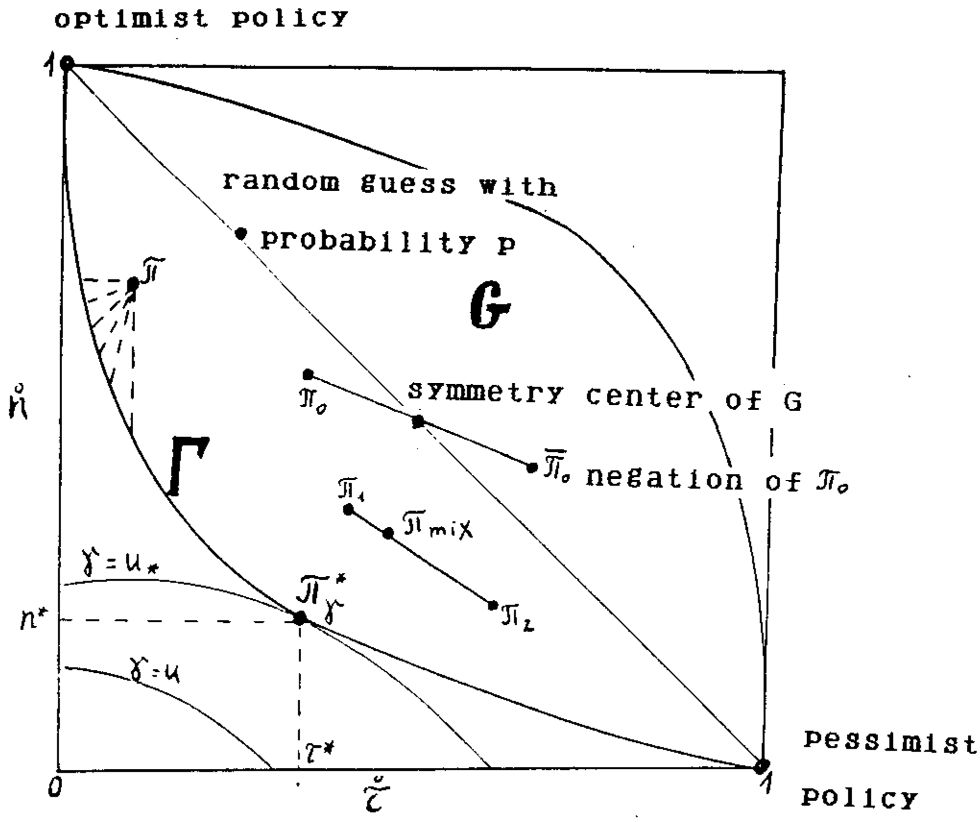
rate of alarm time $\quad \mathring{\tau} = \dfrac{\text{mes}\{\text{alarm sets}\}}{T}, \quad T \gg 1$



"Loss" function $\quad \gamma = f(\mathring{n}, \mathring{\tau})$
$A_u = \{\mathring{n}, \mathring{\tau} : \gamma < u\}$ is convex

Prediction goal: to minimize γ

ERRORS SET G FOR ALL POLICIES BASED ON J(t).



(i) G is convex: $\pi_{mix} = \begin{cases} \pi_1 \\ \pi_2 \end{cases}$ with probability $\begin{matrix} p \\ 1-p \end{matrix}$

(ii) G is set of the best policies based on J(t)

(iii) $\min_{\pi} \gamma = f(\mathring{n}^*, \mathring{\tau}^*) = u_*$

The diagram **Γ** characterizes the limit capabilities of information J(t) in earthquake prediction

⇓

Estimation of **Γ** is geophysical part of prediction problem

Examples: $(\mathring{n}, \mathring{\tau})$ for different prediction methods.

STRUCTURE OF γ-OPTIMAL POLICY, $\pi^*_\gamma(t)$

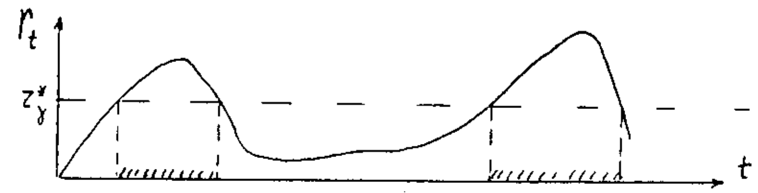
HAZARD FUNCTION (conditional intensity of events):

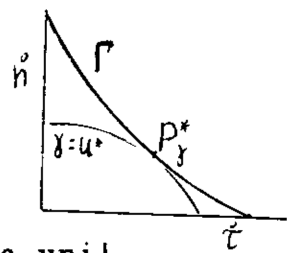
$$r_t = \Pr\{ n(t) > 0 \mid J(t) = u \} / \Delta \doteq r(u)$$

where

$$n(t) = \#\{ \text{events in } (t, t+\Delta) \}$$

If $\{ n(t), J(t) \}$ is stationary ergodic flows then
- optimal alarm set (//////////) is as follows:





where

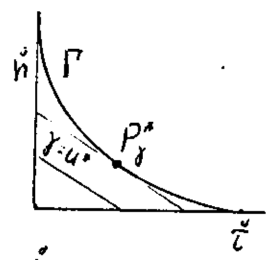
$$r^*_\gamma = -\lambda \frac{d\mathring{n}}{d\mathring{\tau}} (P^*_\gamma)$$

λ is number of events per time unit

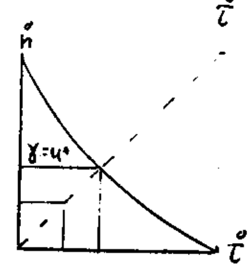
EXAMPLES

1. Linear losses : $\gamma = \alpha \lambda \mathring{n} + \beta \mathring{\tau} \quad \Rightarrow \quad r^*_\gamma = \beta / \alpha$

α is losses for one failure-to-predict
β is losses for alarm time unit in g
γ is total losses of prediction per time unit

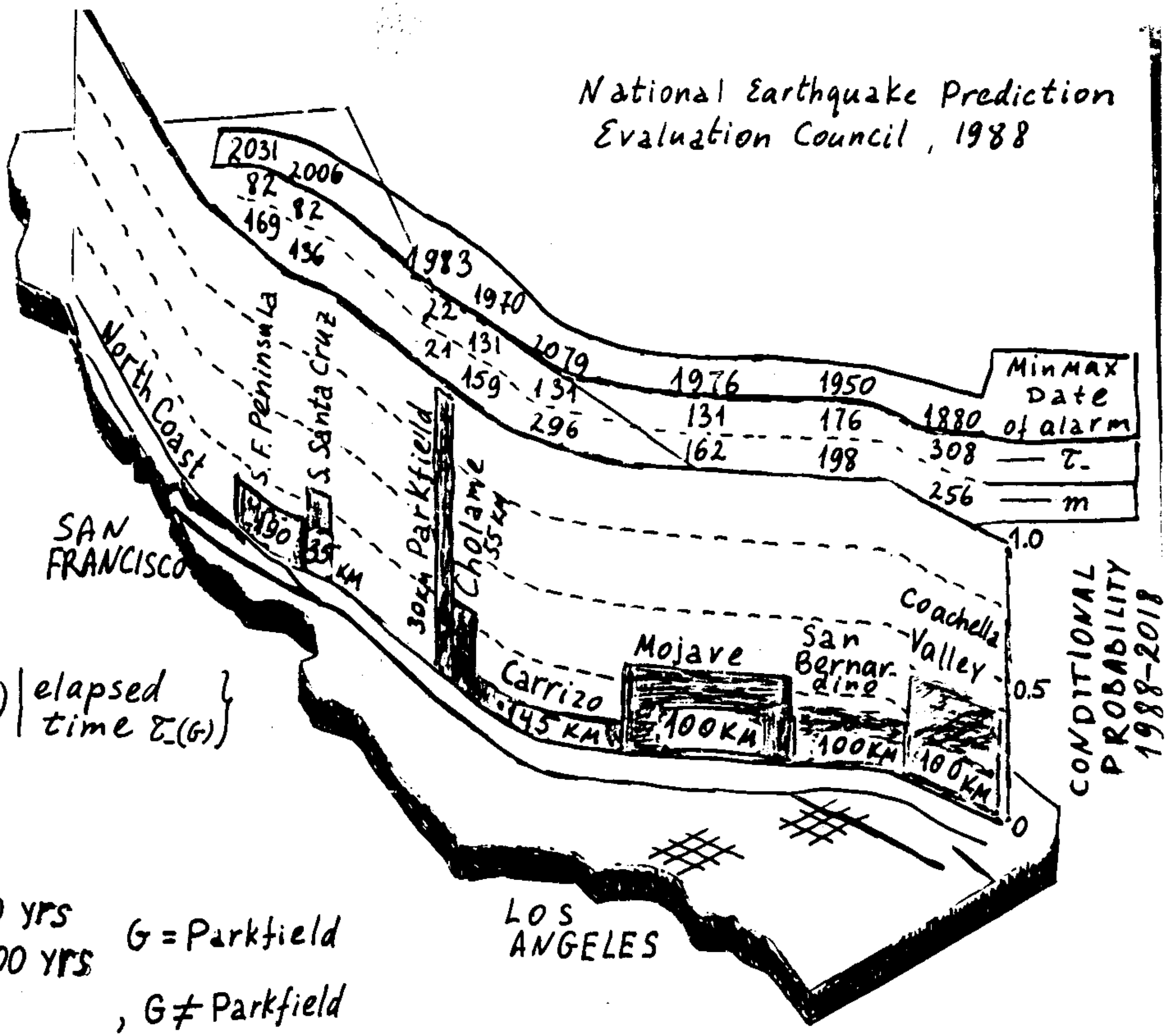


2. Minimax policy: $\gamma \Rightarrow \min$
   where $\gamma = \max(\mathring{n}, \mathring{\tau})$ $\Leftrightarrow$ $\mathring{n} = \mathring{\tau} \Rightarrow \min_{\pi}$



CN algorithm (Keilis-Borok, Rotwain) gives: $\mathring{n} = \mathring{\tau} = 20 - 25\%$

CONDITIONAL PROBABILITY OF MAJOR EARTHQUAKES ALONG SEGMENTS OF THE SAN ANDREAS FAULT 1988-2018



Log-Normal model $F(x)$ with

$m_G = 22 - 296$ yer

$\frac{\sigma_G}{m_G} = 0.24 - 0.6$

$$R = Pr\{t_i \in (1988, 2018) \mid \text{elapsed time } \tau_-(G)\}$$

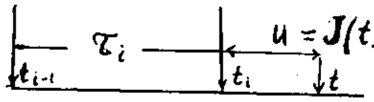
$$R = \begin{cases} > 0.9, & \tau_- < 800 \text{ yrs} \quad G = \text{Parkfield} \\ \simeq 0.5, & \tau_- > 4000 \text{ yrs} \\ < 0.45, & \tau_- > 0, \quad G \neq \text{Parkfield} \end{cases}$$

Conclusions by R-test:
Parkfield is the most dangerous region at any moment of time

EARTHQUAKE PREDICTION ON SAN ANDREAS FAULT
(National Earthquake Prediction Evaluation Council):

$R = \Pr\{\text{events within } (t,\ t+30\text{years}) \mid J(t)\}$

where $J(t)$ = (time elapsed since the last event)



1^o Value of R_t cannot be considered as a lucky choice for comparison of different segments;

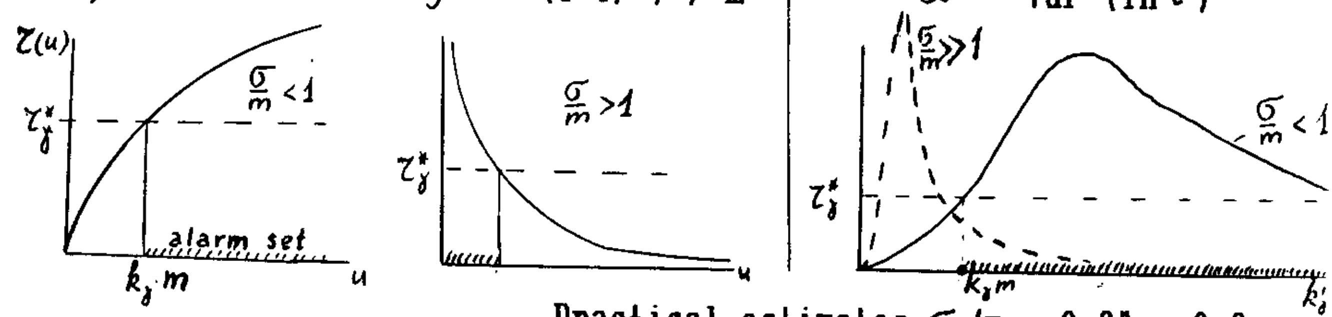
R $\leqslant .45$ constantly for all segments except Parkfield
$\geqslant .90$ in Parkfield for $0 < u < 800$ years

2^o If $F(u) = \Pr\{\tau_i < u\}$ is interevent time distribution then hazard function

$$r(u) = \frac{F'(u)}{1 - F(u)}, \quad u > 0$$

<u>EXAMPLES of F(u):</u> $E\tau = m$, $Var\ \tau = \sigma$

Gamma	Weibull	Log-Normal
$F'(u) = c\, u^{\alpha-1} e^{-\beta u}$	$F(u) = 1 - e^{-(\beta u)^\alpha}$	$F(u) = \Phi(1/\alpha \cdot \ln(u/m) + \alpha/2)$
$\beta = \alpha / m$	$\beta = \Gamma(1+1/\alpha) / m$	$\alpha^2 = Var(\ln\tau)$



Practical estimates $\sigma/m = 0.24 - 0.6$

If prediction goals are fixed (γ), then optimal decision for alarm is defined for the whole interevent time, i.e. R dynamics is not required.

Conclusions for minimax policy

- prediction results ($\dot{n}, \overset{o}{\tau}$) and normalized time threshold (k) of alarm are in weak dependence with type of distribution F when $\sigma/m < 1$;
- 5 out of 8 segments of San Andreas fault should be in state of (minimax) alarm

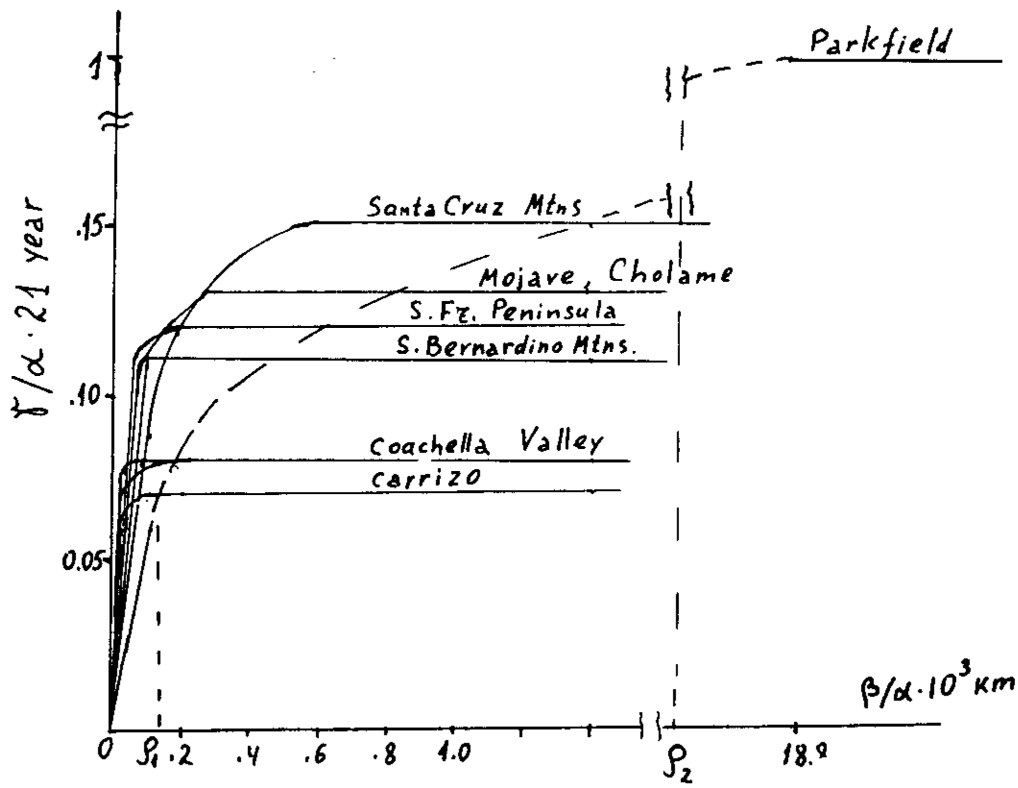
Conclusions for losses: $\gamma = \alpha_G \lambda_G \overset{o}{n} + \beta |G| \overset{o}{\tau}$

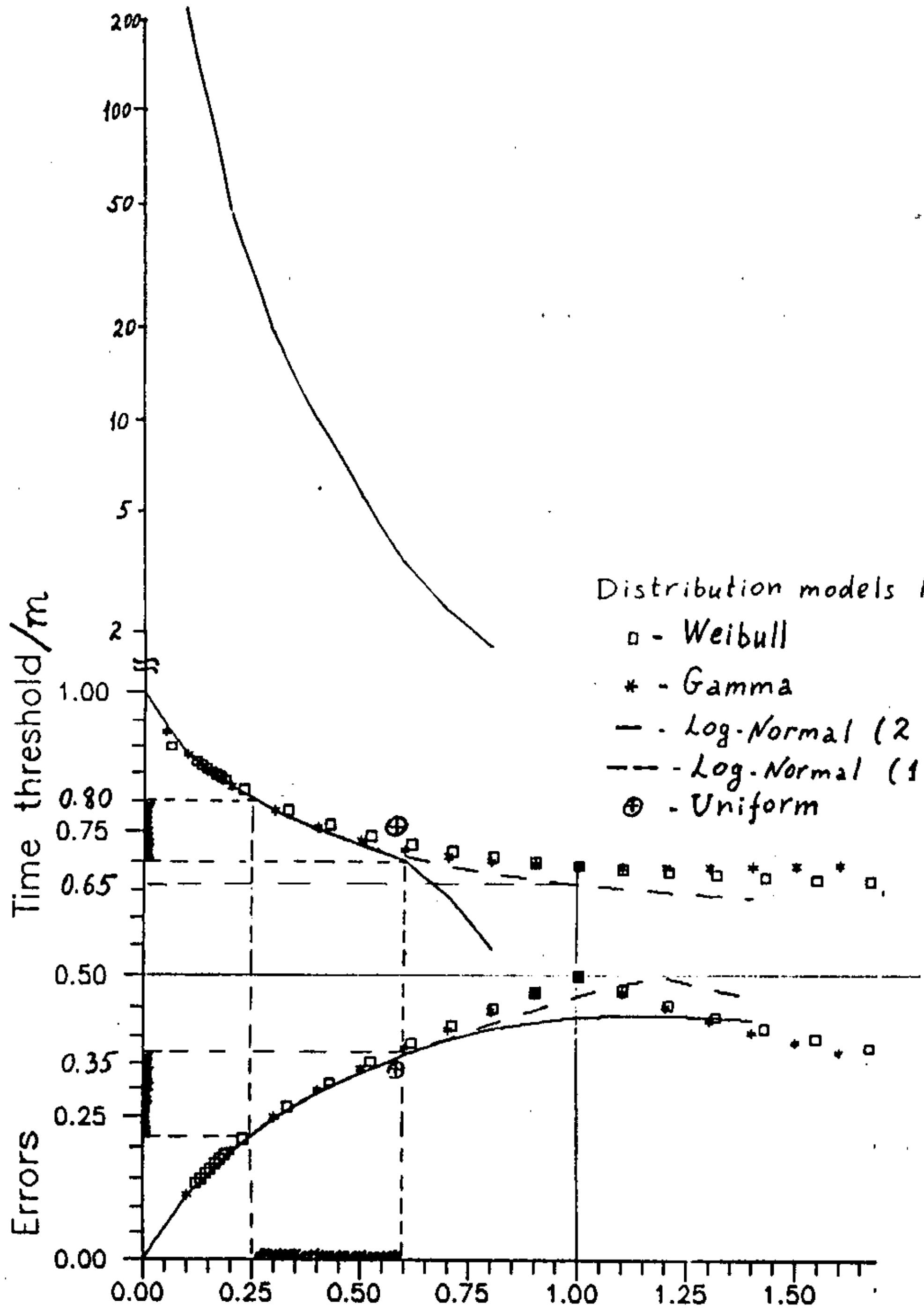
i) If $\alpha_G = \alpha$

$$\gamma(\text{Parkfield}) = \begin{cases} \min_i \gamma(G_i) \\ \max_i \gamma(G_i) \end{cases} \quad \text{for} \quad \begin{matrix} \beta/\alpha < \rho_1 \\ \beta/\alpha > \rho_2 \end{matrix}$$

ii) If $\alpha_G = \alpha \cdot m_G = \alpha / \lambda_G$ (natural model in insurance) then

$$\gamma(\text{Parkfield}) = \min_i \gamma(G_i) \quad \text{for any } \beta/\alpha$$





Parameters of the minimax strategies

MULTIPHASE ALARM MODEL

$A = \{A_i\}$ – phases (types) of alarm;

$\alpha = \{\alpha_i\}$ – prevention losses:

alarm A_i prevents losses α_i in case of strong event;

$\beta = \{\beta_i\}$ – cost of alarm maintenance:

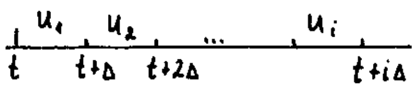
A_i requires expenses β_i per time unit

$C = [C_{ij}]$ – cost matrix for alarm phase change:

$A_i \to A_j$ requires immediate expenses $C_{ij} \geq 0$; $C_{ii} = 0$;

$C_{ij} = \infty$ if alarms transition $A_i \to A_j$ is not permitted.

LOSS FUNCTION



prediction losses for policy in time intervals Δ_i

$S_\pi(t) = u_1 + q\,u_2 + q^2 u_3 + \dots$ – total discounted losses in (t, ∞) for π

$q = \exp(-k\Delta)$ – discount factor

$k > 0$ – coefficient of efficiency for capital investment

GOALS of PREDICTION

to minimize the total discounted expected losses

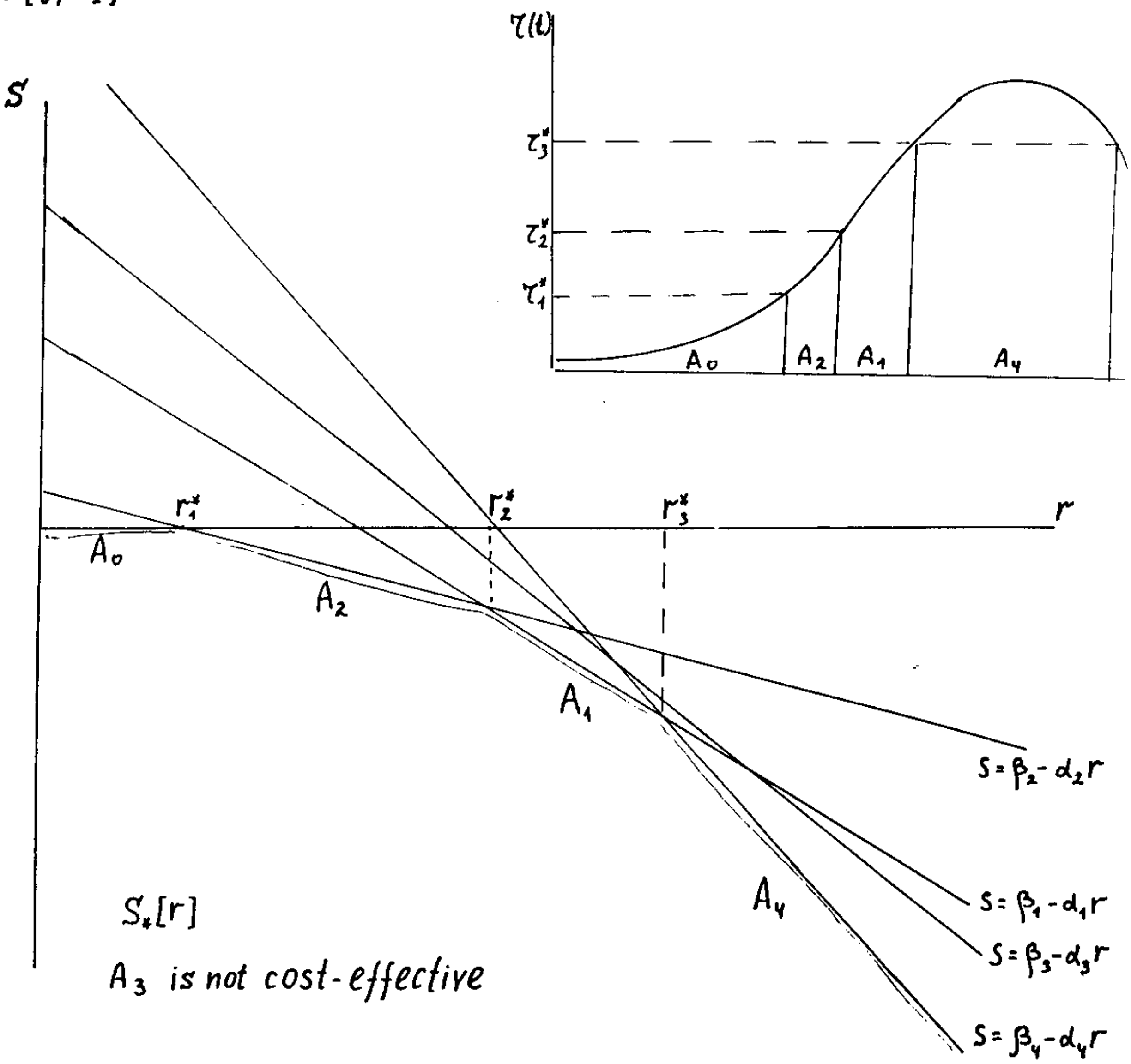
$$E\,S_\pi \Rightarrow \min_\pi$$

Unordered multiphase alarm without initial cost:

$$C_{ij} = 0\,, \quad \varepsilon_{ij} = 1$$

$$S_*^q(u,\mu) = \min_j \left(\beta_j - \alpha_j r(u)\right)\cdot\frac{\Delta}{1-q} = S_*[r]\cdot k^{-1}$$

i.e. global optimal prediction policy π_q^*, $q\in(0,1)$ coincides with local (in time) optimal policy π_0^* and does not depend on $q\in[0,1]$.



OPTIMAL PREDICTION

If

$$\pi(t) = f[J(t), \pi(s), s<t]$$

$$Pr\{n(t)>0 \mid J(t)=u, J(s), s<t\} = r(u)\Delta$$

$$Pr\{J(t+\Delta) \mid J(t)=u, J(s), s<t\} = P_{u,v}$$

(Markov Property)

then

i) the optimal expected losses $S_*(U,i)$ under the initial conditions $J(t) = u$ and $\pi(t-\Delta) = i$ are given as solution of the following equation

$$(*) \quad S(u,i) = \min_j \left(c_{ij} + \beta_j \Delta - r(u)\cdot\Delta\cdot\alpha_j + q \sum_v P_{u,v} S(v,j)\right) \equiv T_q S$$

ii) the equation $S = T_q S$ has a unique solution S_*^q :

$$S_0^q = 0 \; ; \; S_{n+1}^q = T_q S_n^q \longrightarrow S_*^q \; :$$

$$\max | S_n^q - S_*^q | < L q^n (1-q)^{-1}$$

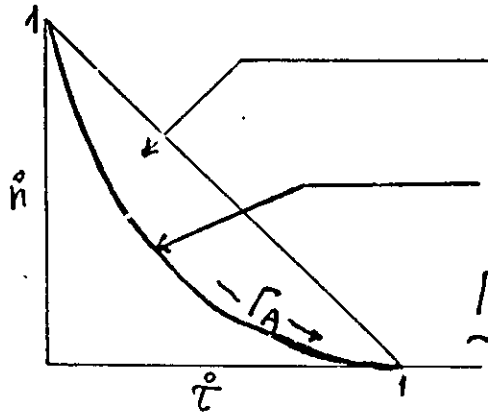
OPTIMAL DECISION:

$\pi(t) = A_j$ under condition $J(t) = u$, $\pi(t-\Delta) = i$
if the index j relizes the minimum in (*)

iii) if $q = 1$ (no discount factor) then S_N^q is optimal expected losses in the interval $(0, N\Delta)$.

PREDICTION ALGORITHMS COMPARED

p - internal parameters of the algorithm *A (or precursor)*

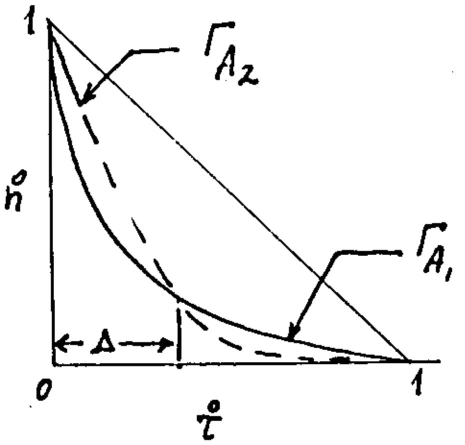


$\pi_A(p)$ - strategies of the algorithm

$\mathcal{T}_A(p)$ - best strategies of the algorithm

Γ_A - curve line of errors characterises predictive ability of the algorithm *(or precursor)*

(estimate of Γ_A , dim p, strong events statistics) - empiric characteristic feature of predictive ability of the algorithm.



Δ- area where A_1 is better than A_2

- Conclusion does not depend on the choic losses.

Conclusions

1. At the research stage of prediction it is necessary to estimate errors diagrams $(\hat{n}, \hat{\tau})$ proceeding from various types of information $J(t)$.
   Estimating of $(\hat{n}, \hat{\tau})$ errors for different prediction algorithms is to be used for this problem.

2. The task of seismostatistics in general case is to estimate the hazard function $r(u)$ and the transition probability matrix $[p_{u,v}] = \mathcal{P}$ for information J on sequential time intervals. ($r(u)$ and $\mathcal{P}$ depend not on time but on the discrete state of seismic process!)
   Aki's method of $r(u)$ estimating for a set of precursors overestimates $r(u)$, as soon as it is based on the assumption of precursors conditional independency.
   To analyze P systematic descriptions of strong shock preparation processes are very important.

3. There is only one hazard function threshold r^* in the simplest prediction problem, here we can hope for the prediction stability.
   In real prediction problems there should be several hazard function thresholds. In complicate cases we must know $r(u)$ and $p_{u,v}$ in details.

   So compromises are desirable.

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