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Modeling of Seismicity

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Abstract: The modeling of seismicity, i.e. of earthquake sequences in contrast with individual events, is an important component of the earthquake prediction problem. Observations provide a hint toward a physical basis for premonitory patterns and suggest the existence of precursory phenomena. The possibility of producing sufficiently long model "catalogs" would permit the testing of statistical significance. Moreover, it would be possible to adjust the numerical parameterization of premonitory patterns found in advance of the few strong earthquakes that have been observed. Recently, methods and ideas of nonlinear science found applications to models of seismicity and to methods of earthquake prediction. We present here a brief review of different approaches to modeling of seismicity.

Introduction. One of the principal directions in earthquake prediction studies is the search for precursors of strong earthquakes based on anomalous patterns observed in moderate seismicity. The principal source of observational data are "catalogs," which are listings of earthquake events according to their size (i.e. magnitude or energy), and the time and location of occurrence. (For comparison purposes, "catalogs" are also produced from numerical models of earthquakes.) Patterns, then, have a decidedly statistical and often ephemeral meaning. Numerous seismicity patterns have been suggested as precursory phenomena (Mogi, 1985; Keilis-Borok, 1990a; Ma *et al.*, 1990; Bonin, 1991). This includes different types of activation, quiescence—including combinations of activation and

quiescence—anomalous aftershock and foreshock activity, seismic migration, and spatio-temporal concentration of moderate earthquakes before a strong earthquake.

Unfortunately, none of these phenomena taken individually are reliable indicators, and few of them have been tested in forward prediction. Moreover, in many seismically active regions, reliable seismicity catalogs with sufficient numbers and kinds of seismicity patterns for the identification of precursory phenomena are available for too short a time period, thus excluding any statistical tests. Therefore, real progress in this area is impossible without an adequate model of the seismotectonic process, one that simulates both the occurrence and interaction of strong and moderate earthquakes.

The following are among the principal features of the lithosphere that should be incorporated into a model for it to be regarded as adequate:

- Interaction of the processes of different physical origin, and of different spatial and temporal scales;
- Hierarchical block or possibly "fractal" structure; and
- Self-similarity in space, time, and energy.

The traditional approach to modeling is based on one *specific* tectonic fault and, often, one strong earthquake in order to reproduce certain pre- and/or post-seismic phenomena (relevant to this specific earthquake). In contrast, the recently developed class of the slider-block and cellular automata models treats the seismotectonic process in the most abstract way, in order to reproduce *general* universal properties of seismicity, first of all, the Gutenberg-Richter frequency of occurrence law, starting from a homogeneous lattice of simple threshold elements.

The specific and general approaches have their respective advantages and disadvantages. The first approach, which takes into account detailed information on the local geotectonic environment, usually misses universal properties of a series of events in a system of interacting faults. The second approach may be considered to be a zero-order approximation to reality. However, the importance of this approach and, in general, the importance of the application of the methods of theoretical physics and nonlinear science to the earthquake prediction problem lies in the possibility of establishing generic analogs with problems in other sciences, and to elaborate a new language for the description of seismicity patterns on the basis of the well-developed lexicon of nonlinear science.

The presence of a large number of different nonlinear mechanisms relevant to the seismotectonic process suggests the applicability of the general approach of nonlinear science to complex dissipative systems (Keilis-Borok, 1990b), unveiling the universal patterns due to self-organization rather than investigating the numerous details of the specific mechanisms involved.

It seems, therefore, that an adequate model of seismicity should incorporate the universal features of self-organized nonlinear systems, as well as the specific geometry of interacting tectonic faults. In the following sections, we will review some of the most important features in modeling seismicity and earthquake prediction and go on to discuss their overall significance.

Earthquake sequences. Earthquake sequences in real catalogs manifest some general features despite the different tectonic structures and levels of seismicity found in various seismic regions.

The sequence of earthquakes is apparently stationary; no noticeable trend has been discerned in the level of seismicity during the 100 or so years of detailed studies of world-wide seismicity. There is, in addition, a considerable stochastic component in the earthquake sequence.

Against this stationary stochastic background different regular patterns appear. The best known of these is the Gutenberg-Richter frequency of occurrence law

$$\log N(M) = \alpha - \beta M$$

where $N(M)$ is the distribution function of earthquakes above magnitude M (Gutenberg and Richter, 1944; Ishimoto and Iida, 1939). This can be interpreted as an indicator of self-

similarity of seismicity in magnitude (energy). The constant β is usually close to 1, which roughly corresponds to the uniform distribution of the total source area of earthquakes over a wide range of the source sizes. It should be noted, however, that the Gutenberg-Richter law is valid only within a certain range of magnitudes, with the lower cutoff at the magnitude below 3 (Aki, 1987) and the upper cutoff (depending on the seismic region) close to magnitude 9 for world-wide seismicity. There is also a noticeable break in this law around the magnitude 6–6.5, separating the weaker intra-crustal earthquakes from the larger earthquakes that rupture the entire seismogenic zone (Scholz, 1990, Ch. 4). Also, the Gutenberg-Richter law is not applicable to the strongest earthquakes on a single fault, due to the existence of *characteristic earthquakes* with the magnitude related to the fault geometry (Schwartz *et al.*, 1981; Schwartz and Coppersmith, 1984).

Other types of self-similarity include the Omori law (Omori, 1895) for the temporal distribution (self-similarity in time) of the number $n(t)$ of aftershocks of a strong earthquake

$$n(t) = c/(1+t)^p \quad ,$$

where p is near 1, and the fractal spatial distribution of the epicenters (self-similarity in space). See Kagan and Knopoff (1980), Kagan (1991), Takayasu (1990, p. 31), and Turcotte (1992, Ch. 4) for a discussion of these different empirical scaling laws.

Another type of regular behavior is the *migration* of earthquakes along tectonic structures (Mogi, 1968; Anderson, 1975; Lehner *et al.*, 1981). A useful way to think of this is that during some interval of time, say ten years, the majority of earthquakes in a given tectonic region are localized to some subset of this region. Then, in the following interval of time, the geographic center of earthquake activity has seemingly moved, giving rise to an apparent systematic migration.

Earthquakes often appear in *clusters*—see e.g. Rice and Gu (1983)—both in time and in space. One common clustering pattern is a main shock followed by a series of aftershocks. Foreshock activity is not so clearly expressed as aftershock sequences since the number of foreshocks of strong earthquakes is usually small or zero while aftershock series of such earthquakes can contain hundreds of events. Finally, there are doublets of strong earthquakes and "swarms" of earthquakes that cannot be separated into main shock, foreshocks and aftershocks because they have similar magnitudes.

The concept of the ***seismic cycle***, originating with Reid (1910), implies characteristic time intervals between the strongest earthquakes in a certain region, with periods of post-seismic relaxation and inter-seismic stress accumulation between them – see Scholz (1990, Ch. 5) for a review. However, the actual time intervals are not equal and can deviate considerably from the average characteristic period which can vary from tens to hundreds of years. One of the most famous examples of a comparatively short seismic cycle is the Parkfield area of California where moderate earthquakes have recurred approximately every 22 years (Bakun and McEvily, 1984). Nevertheless, the earthquake that according to this periodicity was expected around 1988, never occurred.

Numerous premonitory seismicity patterns have been found in earthquake cat logs. These include different types of activation—the general increase of activity, "swarms" of earthquakes, bursts of aftershocks, see Keilis-Borok *et al.* (1980)—and of quiescence or "seismic gap" (Wyss and Habermann, 1988; Ogata, 1992). None of these premonitory patterns is reliable enough by itself and different combinations have been suggested for intermediate-term earthquake prediction. One particular combination of quiescence, in the area of a future strong earthquake and activation, in the surrounding areas, is called the "doughnut pattern" (Mogi, 1969). More complicated combinations were suggested by Keilis-Borok and Rotwain (1990) and Keilis-Borok and Kossobokov (1990) based on pattern recognition methods. These premonitory patterns appear with a characteristic time scale of 1–5 years (intermediate-term prediction) and a characteristic space scale of hundreds of kilometers.

The two principal mechanisms involved in the seismotectonic process are tectonic loading, with characteristic rate of a few cm/yr, and the elastic stress accumulation and

redistribution, with characteristic rate of ≈ 1 km/sec. In the typical time scale (10-100 yrs) of earthquake prediction studies, the first of these mechanisms can be considered to be a uniform rate of motion, and the second to be an instantaneous stress drop.

At the same time, there are several nonlinear mechanisms of the different physical nature that develop in the time scales intermediate between the two extremes, overlapping with the time scales of the premonitory seismicity patterns. This includes the spatial heterogeneity and hierarchical block structure of the lithosphere, different types of nonlinear rheology of the fault zones and friction along the fault planes, gravitational and thermodynamic processes, physical-chemical and phase transitions, fluid migration and stress corrosion. It is quite possible that these (and maybe some other, still unknown) mechanisms are responsible for the premonitory seismicity patterns characterized by the intermediate time scale and long-range spatial correlations in moderate seismicity preceding the strong earthquakes.

Elastic rebound theory. One important ingredient in the modeling of seismicity is based on the so-called elastic rebound theory (Reid, 1910) which emerged in the aftermath of the great San Francisco earthquake of 1906. According to this theory, elastic stress in a seismically active region accumulates due to some external source, e.g. movement of tectonic plates, and is released when the stress exceeds the strength of the medium. In the simplest case (constant rate of stress accumulation, fixed strength and residual stress) this model produces a periodic sequence of earthquakes of equal magnitude. This links the elastic rebound theory with the concepts of the seismic cycle and of characteristic earthquakes.

If only strength or residual stress is fixed in this model, we have the so-called "time-predictable" model (the time interval until the next earthquake is defined by the magnitude of the previous one) and the "slip-predictable" model (the magnitude of an expected earthquake increases with the elapsed time). Although a model of this type is used for long-term prediction (Nishenko and Buland, 1987), real sequences of strong earthquakes are fundamentally more complicated (Thatcher, 1990; Scholz, 1990, Ch. 5). In particular, the elastic rebound model suggests that a strong earthquake should be followed by a period of quiescence, whereas in reality a strong earthquake is followed by a period of activation and sometimes by another earthquake of comparable magnitude. Simple deterministic nonlinear models for repetitive seismicity containing some of the attributes of "chaos" were developed by Newman and Knopoff (1982a,1982b) and by Knopoff and Newman (1983).

To incorporate the post-seismic activity following a strong earthquake, Elsasser (1969) suggested an additional viscous mechanism due to the interaction of the elastic upper crust with the asthenosphere and upper mantle. Yet another way to incorporate post-seismic activity is to include viscous interaction into rheology of the fault plane. A three-dimensional model of this type, with inhomogeneity in the distribution of the model parameters along the fault plane, was investigated by Mikumo and Miyatake (1983). These concepts were further developed to include the Maxwellian visco-elastic rheology as well as horizontal inhomogeneity (Rice, 1980; Lehner *et al.*, 1981; Li and Rice, 1987; Ben-Zion *et al.*, 1993). The aseismic (creeping) part in these models satisfies the constitutive equation

$$\sigma_{ij} = (K - \frac{2}{3}\mu)\epsilon_{kk}\delta_{ij} + 2\mu(\epsilon_{ij} - \epsilon_{ij}^{cr}) \quad .$$

where K and μ are elastic moduli, and the total strain ϵ is represented as a sum of the elastic strain ϵ^{el} and the inelastic creep strain ϵ^{cr}, i.e. $\epsilon_{ij} = \epsilon_{ij}^{el} + \epsilon_{ij}^{cr}$. In particular, Ben-Zion *et al.* (1993) consider the Parkfield sequence as a part of the great 1857 earthquake cycle and argue that, due to the relaxation mechanism, the frequency and magnitude of the earthquake in the sequence should actually decrease in time. Note that the presence of a threshold for failure introduces strong nonlinearity into these otherwise linear models.

Rate-dependent and state-dependent friction. A model with a rate-dependent and state-dependent friction law, based on laboratory experiments using rock samples, was introduced by Dieterich (1972) and further developed and studied by Ruina (1983), Tse and Rice (1986), and others. The model defines the dependence of the friction coefficient μ ($\tau = \mu\sigma$, where τ and σ are the tangent and normal stress components) on the slip velocity V and state variable θ according to

$$\mu = \mu_0 + a \ln\left(\frac{V}{V^*}\right) + b\theta,$$

$$\frac{d\theta}{dt} = -\frac{V}{L}\left[\theta + \ln\left(\frac{V}{V^*}\right)\right] \quad .$$

Here V^*, μ_0, a, b, L are constants. For $a > b$ ("velocity strengthening") the model always gives stable sliding, and, for $a < b$ ("velocity weakening"), instability appears when the stiffness is below a critical value namely, $-(a-b)\sigma/L$—see Gu *et al.* (1984). The model gives an adequate description of preseismic, coseismic and postseismic slip on a fault, especially when, as in Tse and Rice (1986), transition from velocity weakening to velocity strengthening with depth is included. See also Rice (1993) where the slip is allowed to vary along the strike, as well as with the depth, and an additional viscous damping term is added to account for the seismic radiation. Rice and Gu (1983) suggested that this friction law, together with relaxation processes in the lower lithosphere and asthenosphere, could be a possible mechanism for post-seismic activation effects. Lorenzetti and Tullis (1989) discussed possible implications of this model to short-term prediction based on preseismic slip measurements. Marone *et al.* (1991) suggest an opposite depth distribution of $a - b$ in the friction law (i.e. strengthening in the upper 3-5 km and weakening in the seismogenic layer) in order to explain earthquake afterslip at faults with a thick sedimentary cover.

The principal problem in this modeling is the applicability of the complicated friction law, derived from laboratory experiments on flat surfaces of homogeneous rock samples, to real fault zones that are neither homogeneous nor flat. The parameters in this friction law are empirical, and it is not clear how to scale them properly for real faults. The behavior of the system with this friction law is very sensitive to small variation in the values of the parameters—in the presence of noise, they may become virtually unpredictable.

Spatial heterogeneity. Another direction in the modeling of complex earthquake sequences takes into account the spatial inhomogeneity of the strength distribution in the fault plane. The key concepts here are barriers, asperities, and characteristic earthquakes (Aki, 1984). Asperities and barriers represent strong patches in the fault plane, while the difference is in their relation to the earthquake source. *Asperities* are strong patches on the stress-free background (due to preslip and foreshocks) and break during the earthquake (Kanamori and Stewart, 1978). *Barriers* appear as strong patches that do not allow further propagation of a fracture (Das and Aki, 1977; Aki, 1979). The interpretation of barriers in terms of the geometry of tectonic faults was suggested by King and Nabelek (1985) and by King (1986). In particular, King (1986) suggested the existence of "soft" barriers where a seismic rupture terminates due to the absence of accumulated stress. Both asperities and barriers suggest the possible recurrence of earthquakes with a preferred source size, i.e. *characteristic earthquakes* (Schwartz *et al.*, 1981).

Stress corrosion. Stress corrosion, or static fatigue (Anderson and Grew, 1977; Atkinson, 1984) is often considered to be one of the possible mechanisms for the time delay in the seismotectonic process. In this mechanism, especially in the presence of active fluids (Rhebinder and Shtchukin, 1972), the fractures in the stressed material grow and propagate quasi-statically under stresses that are substantially below the brittle fracture threshold and the effective strength of the material can be reduced by several orders of magnitude. This mechanism was suggested in Das and Scholz (1981), Newman and Knopoff (1982a, 1982b), Knopoff and Newman (1983), and Yamashita and Knopoff (1987) to explain aftershock sequences. In Yamashita and Knopoff (1989) and Sornette *et al.* (1992), the stress corrosion mechanism was included in a model of foreshock activity. Gabrielov and Keilis-Borok (1983) considered geometrical patterns of the stress corrosion in inhomogeneously stressed medium as a possible mechanism for the spatial inhomogeneity of strength responsible for precursory phenomena, such as the doughnut pattern.

Slider-block models and self-organized criticality. In contrast with the aforementioned models, a number of models composed of "masses and springs" or of cellular automata suggest the possibility of apparently chaotic earthquake sequences with a power law distribution of sizes in a spatially homogeneous medium due to self-organizing processes in a system of interacting elements (blocks, faults, etc.). The first class of these models, the slider-block models originally proposed by Burridge and Knopoff (1967), have been studied by Cao and Aki (1986), Takayasu and Matsuzaki (1988), Carlson and Langer (1989), Carlson *et al.* (1991), and others. In these models a linear system of rigid blocks connected by springs to adjacent blocks and to a driving slab and interacting with a stable surface according to a specified friction law.

In the original paper by Burridge and Knopoff, the model was shown to reproduce such important properties of seismicity as the Gutenberg-Richter law and, with the inclusion of additional viscous elements, aftershock activity. Cao and Aki (1986) considered a system of blocks with a rate-dependent and state-dependent friction law in order to reproduce premonitory quiescence. Carlson and Langer (1989) found a bimodal population of earthquakes in their model. While the small earthquakes obey a power law distribution, the strongest (runaway) events appear much more often than the extrapolation of the power law established for the small earthquakes would suggest. They associated this phenomenon with the concept of characteristic earthquakes. Shaw *et al.* (1992) reproduced activation and concentration patterns for small events before a strong earthquake in their model catalog. Carlson (1991), Huang *et al.* (1992), and Narkounskaia *et al.* (1992) considered a two-dimensional variant of the slider-block model.

Bak *et al.* (1987, 1988) suggested a simple cellular automaton-type ("sandpile") model represented by a lattice of threshold elements with random loading and a simple deterministic rule of stress release and nearest-neighbor redistribution. A sequence of consecutive breaks in the stress redistribution phase of the model was called an *avalanche*. The model is mathematically equivalent to a variant of the slider-block model in the limit of zero-mass blocks, and the avalanches can be interpreted as the earthquakes in the Burridge-Knopoff model. The sandpile model demonstrates an important property of *self-organized criticality*: from any initial state it evolves to a critical state characterized by a power law distribution of the avalanche sizes and two-point correlations. The applications of this model and its different variations and modifications can be found in Bak and Tang (1989), Ito and Matsuzaki (1990), Nakanishi (1991), Brown *et al.* (1991), Lomnitz-Adler *et al.* (1992), Vasconcelos *et al.* (1992), and Olami *et al.* (1992). See Ito (1992) for a review.

These models are concerned, in particular, with the power law distribution of earthquake sizes and, in general, with the chaotic character of a simple, homogeneous, and often deterministic, system. Different macroscopic effects due to changes in the local interaction rules, and phase transition phenomena according to variation of parameters were also investigated.

Although these models are rather abstract and oversimplified, some important features of seismicity can be understood in these models, and the influence of different types of interaction on the model catalog can be easily verified. It is important also as a possibility to establish analogies between the problems of predictability in solid Earth geophysics and other sciences.

Hierarchical and fractal structures. Models of crack nucleation based on the hierarchical block structure of the Earth's lithosphere were suggested in Allègre *et al.* (1982), Knopoff and Newman (1983), Smalley *et al.* (1985), Narkunskaya and Schnirman (1990), Molchanov *et al.*(1990), Newman and Gabrielov (1991), Gabrielov and Newman (1991), and Tumarkin and Shnirman (1992). All of these models explicitly introduce fractures of several scales and apply renormalization group methods to study interrelations between different scales. The condition for failure sometimes appears in these models as a critical phenomenon. In Smalley *et al.* (1985) and Newman and Gabrielov (1991), this approach explains the apparent low strength of fault zones— however, a critical point for failure does

not emerge. Narkunskaya and Schnirman (1990) suggested a precursory pattern "upward bend of the frequency law" for major failures based on an analytic and numerical study of their model. This pattern has been later found in catalogs of seismicity for several regions.

King (1983) suggested that the kinematic incompatibility of the motion of lithospheric blocks was the source of fractal structure in the lithosphere in King (1983). Fractality is a general pattern of finite brittle strain in different materials (Turcotte, 1986 and 1990; Sornette *et al.*, 1990; Sornette, 1991; King and Sammis, 1992).

Interaction of tectonic faults. There are a few models of seismicity where the interaction of tectonic faults is taken into account. One is the fluctuation model due to Rundle (1988) where earthquakes are treated as small thermodynamic fluctuations in the steady tectonic loading process in an elastic medium with embedded fault patches. Another is the block model (Gabrielov *et al.*, 1990) where a seismically active region is modeled as a system of rigid blocks of arbitrary geometry separated by thin layers that represent fault zones. In Gabrielov *et al.* (1990), an algorithm known as *CN* for intermediate-term earthquake prediction is successfully applied to a model catalog. More recently, Yamashita and Knopoff (1992) suggested interaction in a system of faults as a mechanism for the activation-quiescence pattern.

Discussion. The physical mechanism for earthquakes and other influence on subsequent events ("aftereffects") is still not fully understood. Adequate modeling of these aftereffects is important for the earthquake prediction problem, particularly because abnormal post-seismic activation is one of the intermediate-term premonitory patterns. Many of the models of seismicity in earthquake prediction studies associate premonitory patterns with the processes in the Earth's lithosphere on an intermediate time scale (i.e. between tectonic loading and elastic stress drop). These processes have been studied, mainly in laboratory experiments, and are not yet well understood for the solid Earth. A fundamental question is whether and what type of scaling may exist between laboratory samples and real tectonic faults. Most of the existing models of seismicity do not include the spatial distribution of earthquakes or are restricted to a single fault plane. At the same time, many of the premonitory seismicity patterns, as well as earthquake aftereffects, are observed far away from the fault where a major earthquake takes place. This means that the modeling of the interaction of the processes in different tectonic faults is important for understanding the sequence of events leading up to an earthquake.

Two important problems make it difficult to include real fault geometry in models of seismicity. First, to produce a long time model catalog with properties that stay unchanged in time, we need a stationary process in the model. Stationarity is an important ingredient of most earthquake prediction methods, providing the possibility to transfer the previously observed patterns to possible future events. At the same time, the underlying tectonics cannot be stationary, due to simple geometric considerations (McKenzie and Morgan, 1969; King, 1983). Tectonic faults tend to grow in time. The kinematic instability of fault junctions leads to the creation and growth of complicated fractal structures—such as "morphostructural nodes" (Alekseevskaya *et al.*, 1977)—around existing junctions and, eventually, to the emergence of new major faults. This non-stationarity appears as a fundamental challenge to the modeling of seismicity based on the actual fault's geometry.

Second, fault systems have a hierarchical fractal structure, and premonitory seismicity patterns are usually based on the properties of weak and moderate earthquakes that appear in the lower levels of this hierarchy. It is impossible in practical terms to handle several levels of this hierarchy in an explicit way. So, the problem that emerges is how to combine the available information on the geometry of principal faults with what is essentially statistical data on the fault system as a whole.

The challenge ahead is to develop an adequate language of description for the seismotectonic process. The language of continuum mechanics cannot describe the combination of continuous and discrete features of the seismotectonic process, namely its self-similar multi-scale spatial and temporal nature. However, the language of statistical physics and

nonlinear science can describe complicated universal phenomena, but does not accommodate the specific geometry of individual tectonic faults. What is required, therefore, is a synthesis of these traditional and new approaches.

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