INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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H4.SMR/709-23

Second Workshop on Non-Linear Dynamics
and Earthquake Prediction

22 November - 10 December 1993

Definition of self-organized criticality (SOC)

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Definition of self-organized criticality (SOC)

1.) A natural system in a marginally stable state, when perturbed it will evolve back to a state of marginal stability.

2.) "Energy" is added continuously. "Energy" is lost in discrete events which have fractal frequency-magnitude statistics.

Examples: 1.) The earth's lithosphere
2.) The earth's atmosphere

CS 16

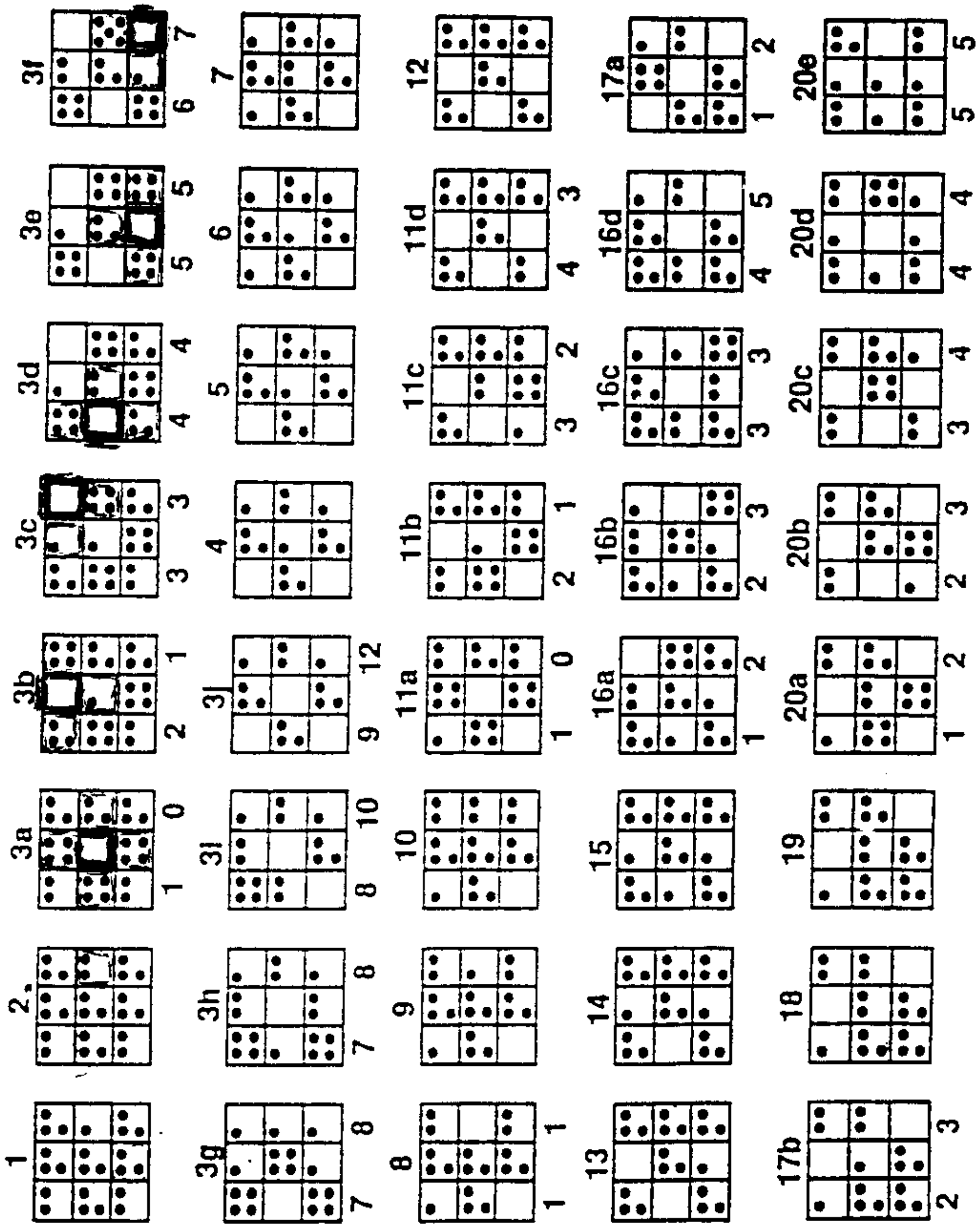
Cellular Automata model for SOC

1.) Consider a square grid of n^2 boxes

2.) Particles are randomly added to boxes.

3.) When a box has four particles it is unstable and the four particles are redistributed to the four adjacent boxes. Redistributions from edge boxes result in the loss of one particle from the grid, redistributions from corner boxes result in the loss of two particles.

4.) After a redistribution further boxes may be unstable and additional redistributions are required.



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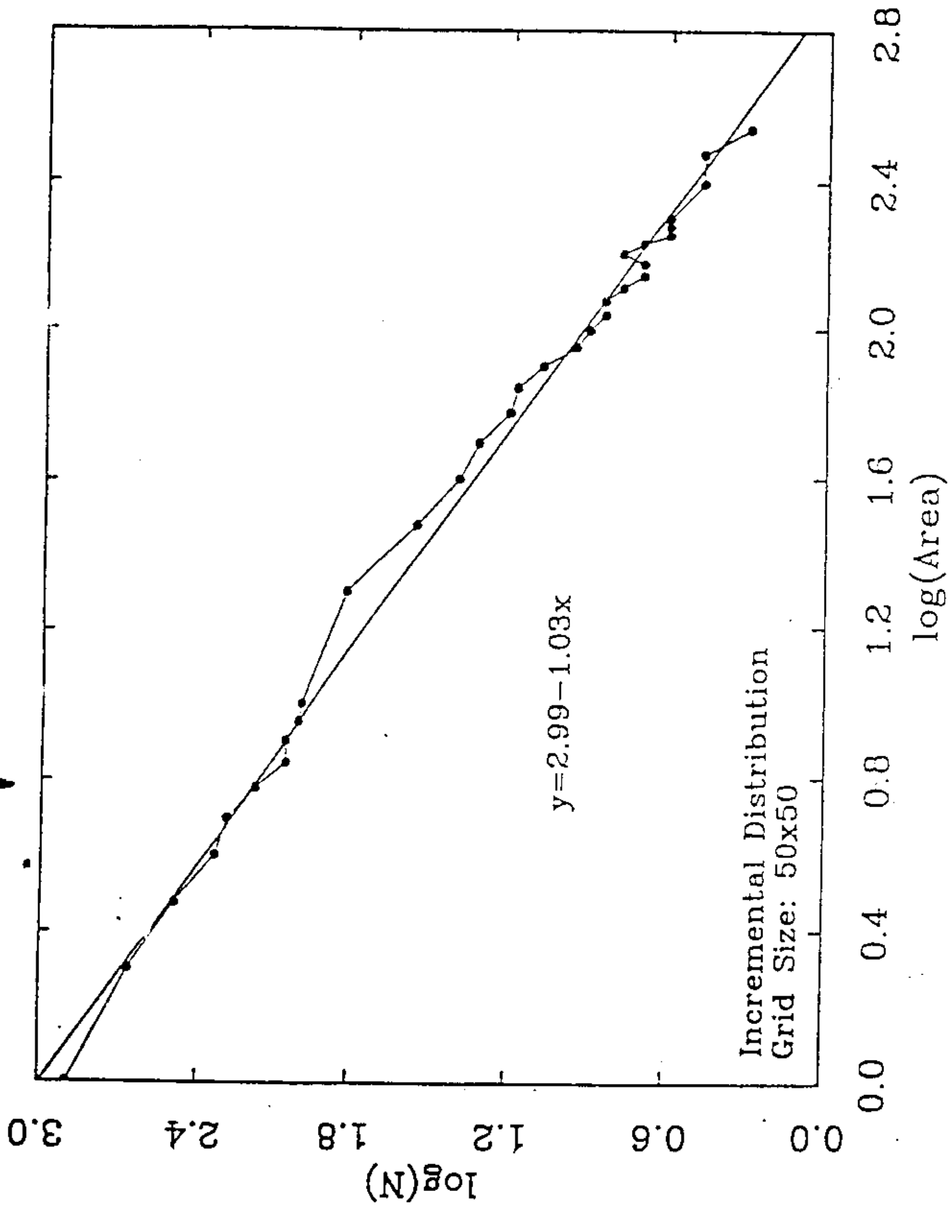
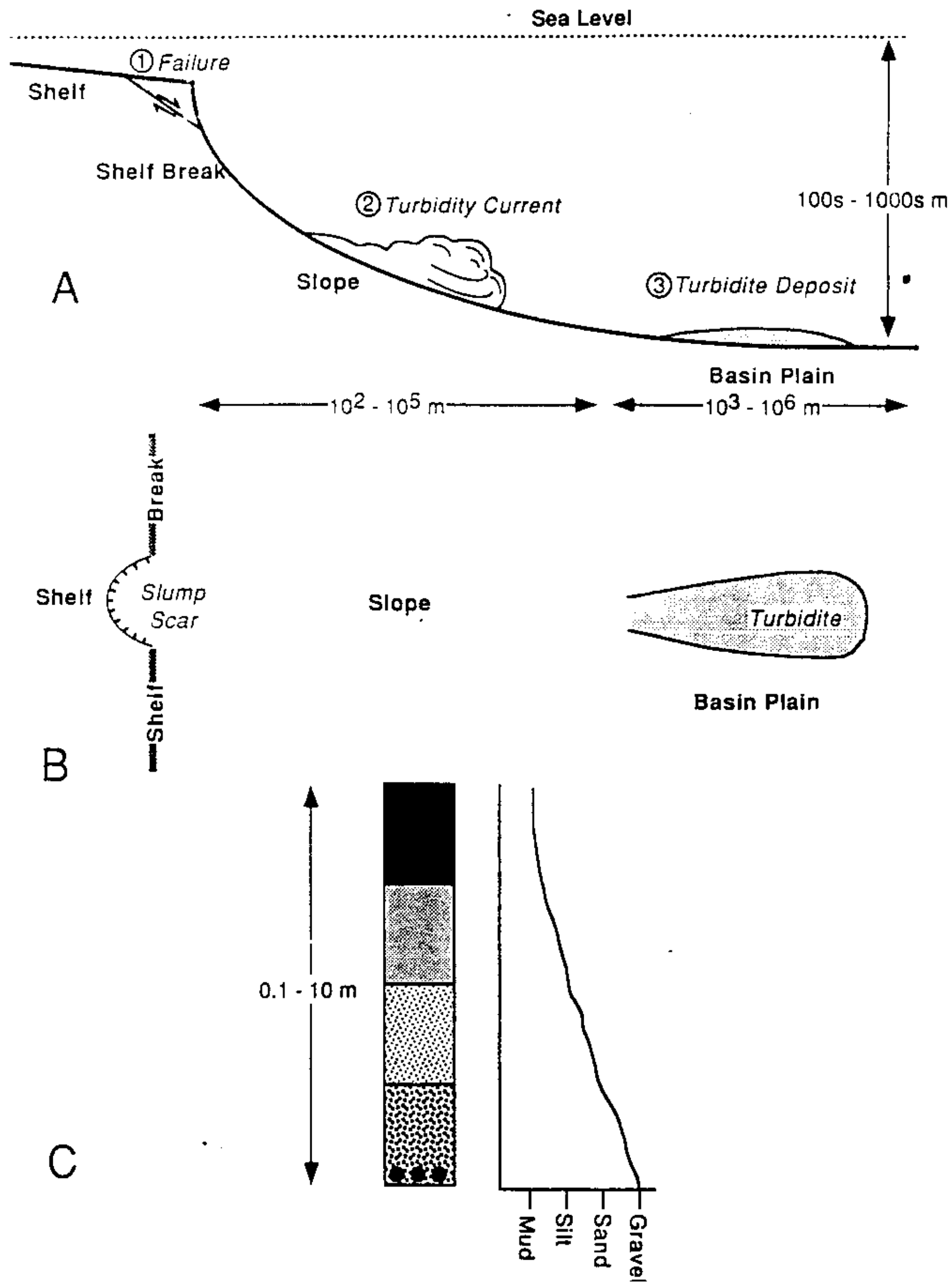
F 18

CS3

Number of failures vs their area

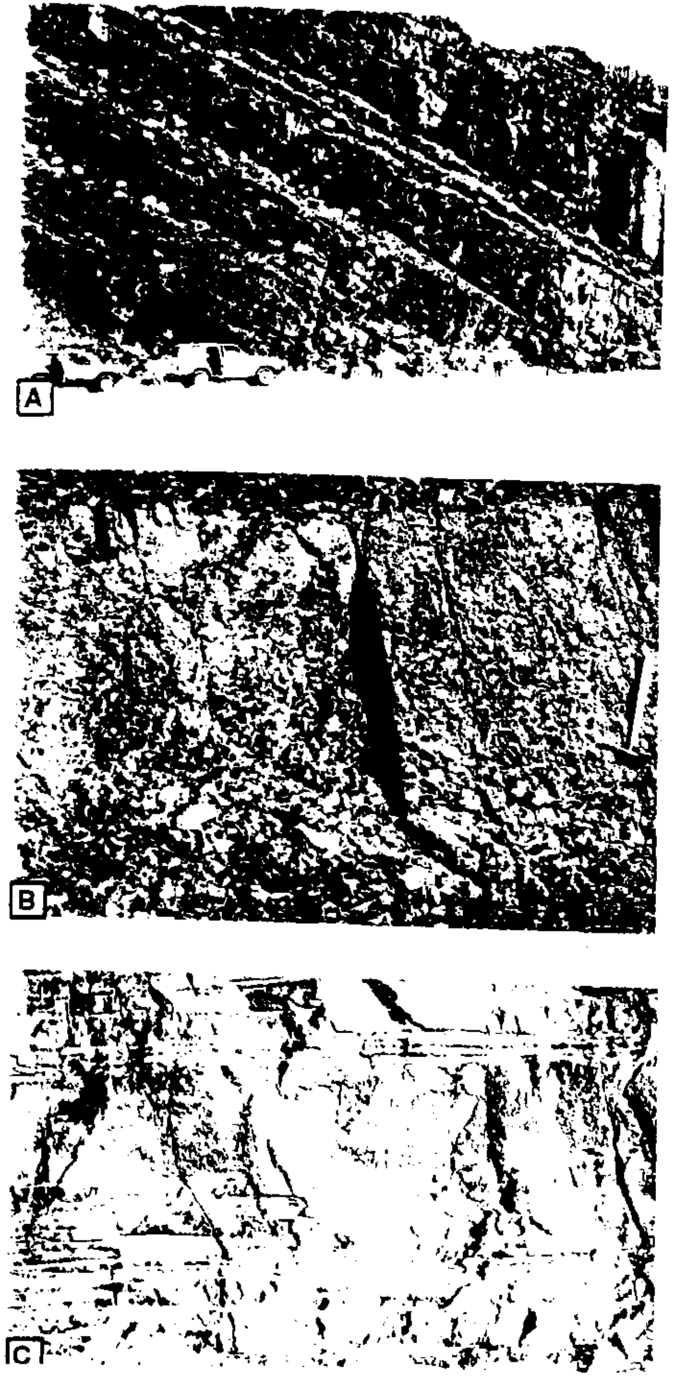


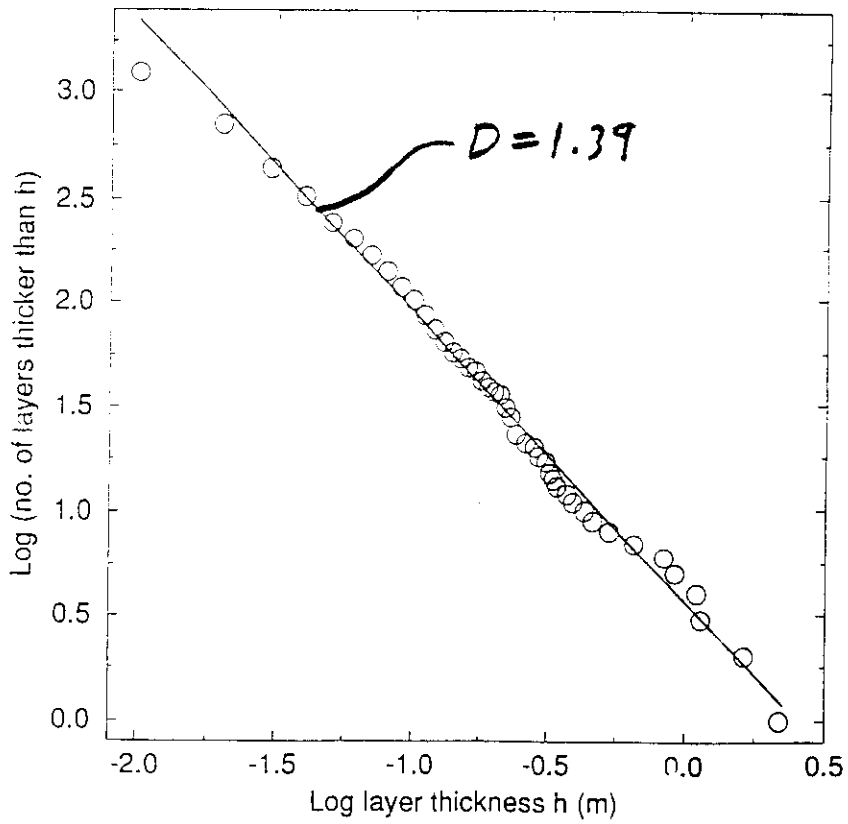


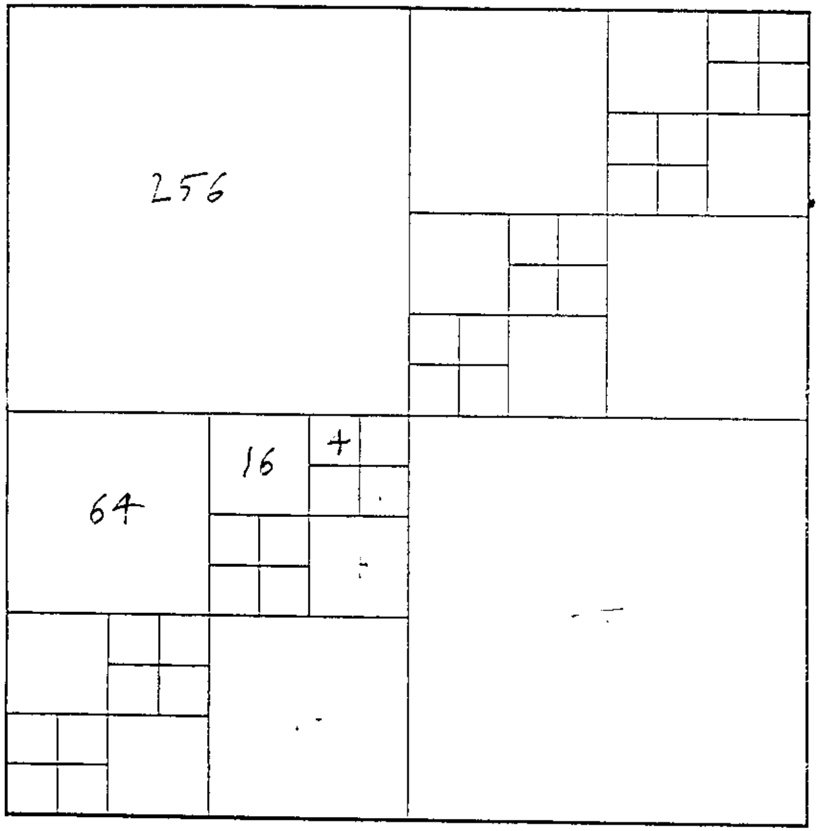
FS18c

FIGURE 4

Rothman, Grotzinger and Flemings (1993)
Interbedded turbidites
Kingston Peak Formation at Sperry Wash

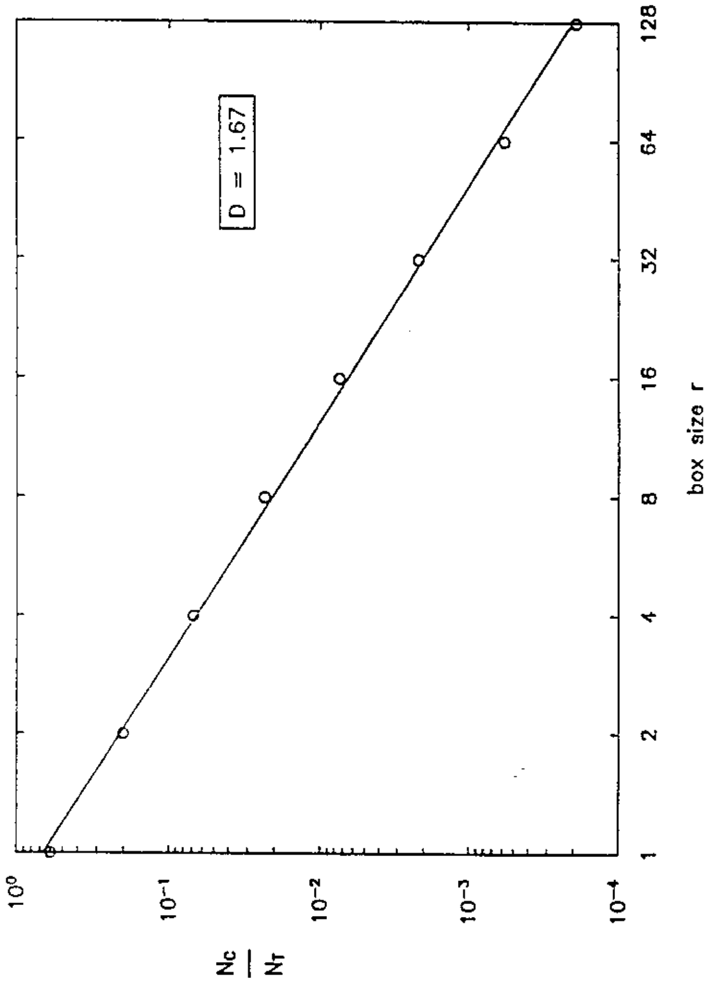


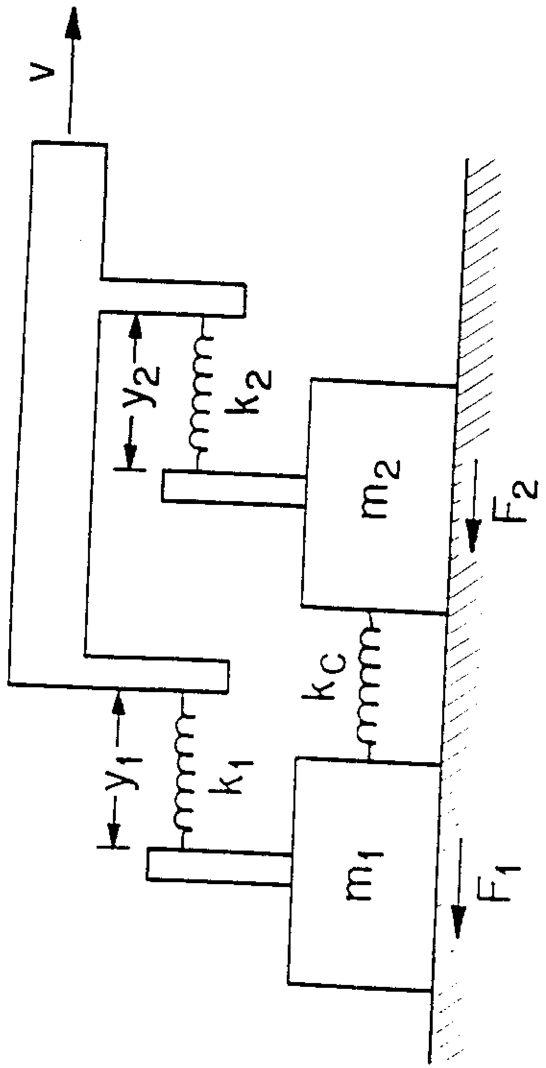




Cellular-automata model
Fractal distribution of box sizes

Instability





Equations of Motion

$$m\ddot{y}_1 + ky_1 + \alpha k(y_1 - y_2) = F$$

$$m\ddot{y}_2 + ky_2 + \alpha k(y_2 - y_1) = \beta F$$

Static/Dynamic Friction

$$F = \begin{cases} F_s & |\dot{y}| = 0 \\ F_d & |\dot{y}| \neq 0 \end{cases}$$

Non-dimensionalization

$$\tau = t(k/m)^{1/2}, \quad Y = \frac{ky}{F_s}, \quad \phi = \frac{F_s}{F_d}$$

Non-dimensional form (Static/Dynamic)

(1) Failure conditions:

$$Y_1+\alpha(Y_1-Y_2) = 1$$

$$Y_2+\alpha(Y_2-Y_1) = \beta$$

(2) Equations of motion during slip:

$$\ddot{Y}_1+Y_1+\alpha(Y_1-Y_2) = 1/\phi$$

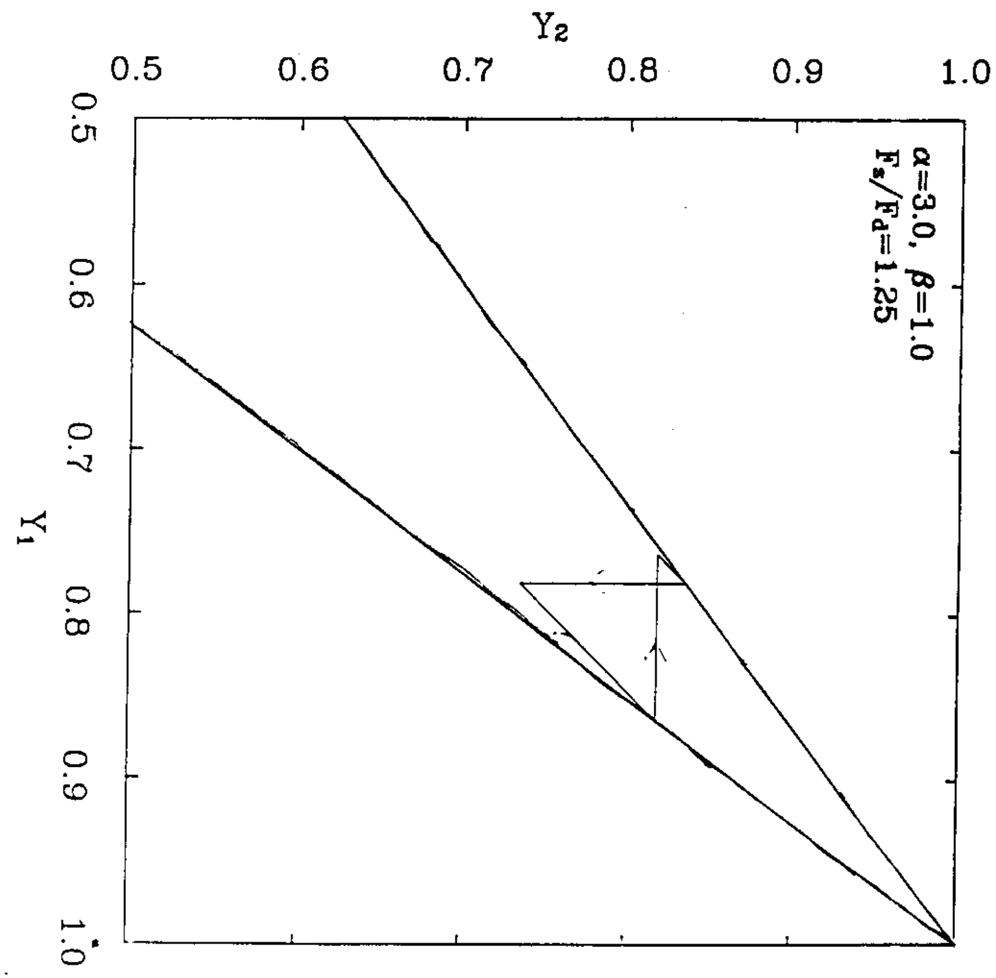
$$\ddot{Y}_2+Y_2+\alpha(Y_2-Y_1) = \beta/\phi$$

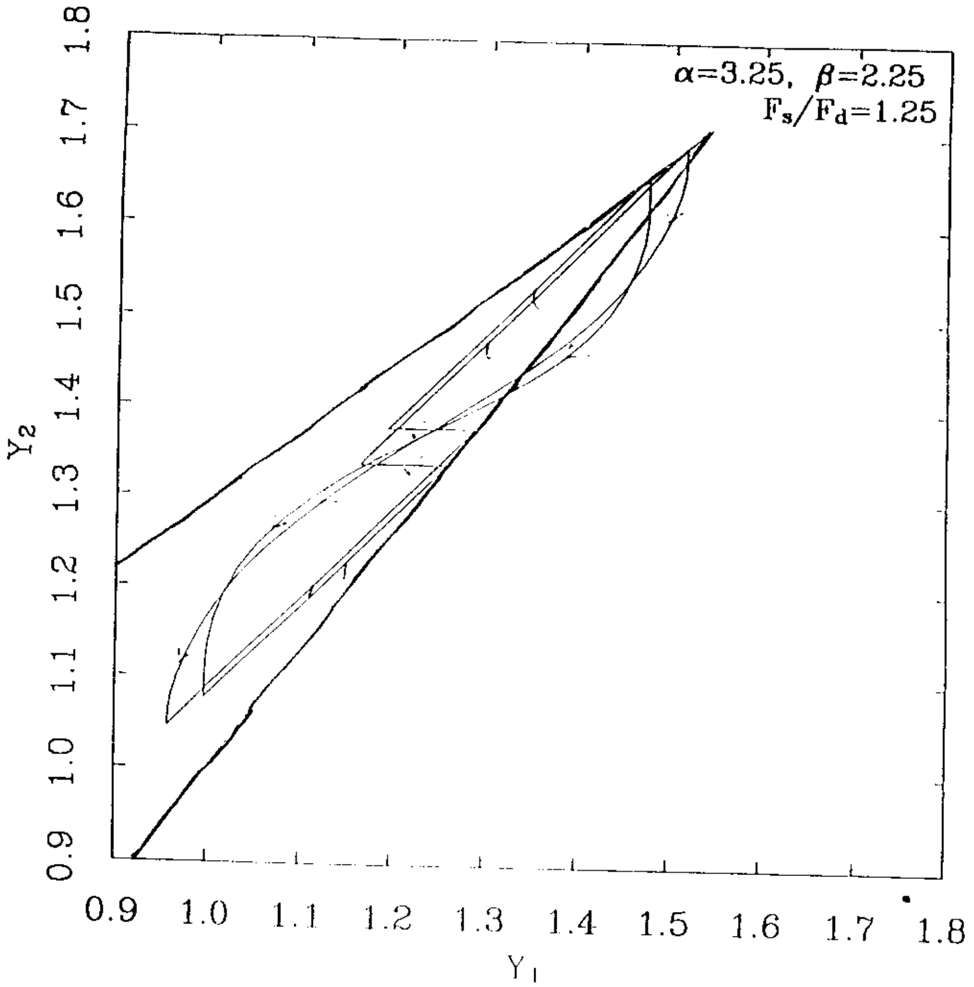
Parameters:

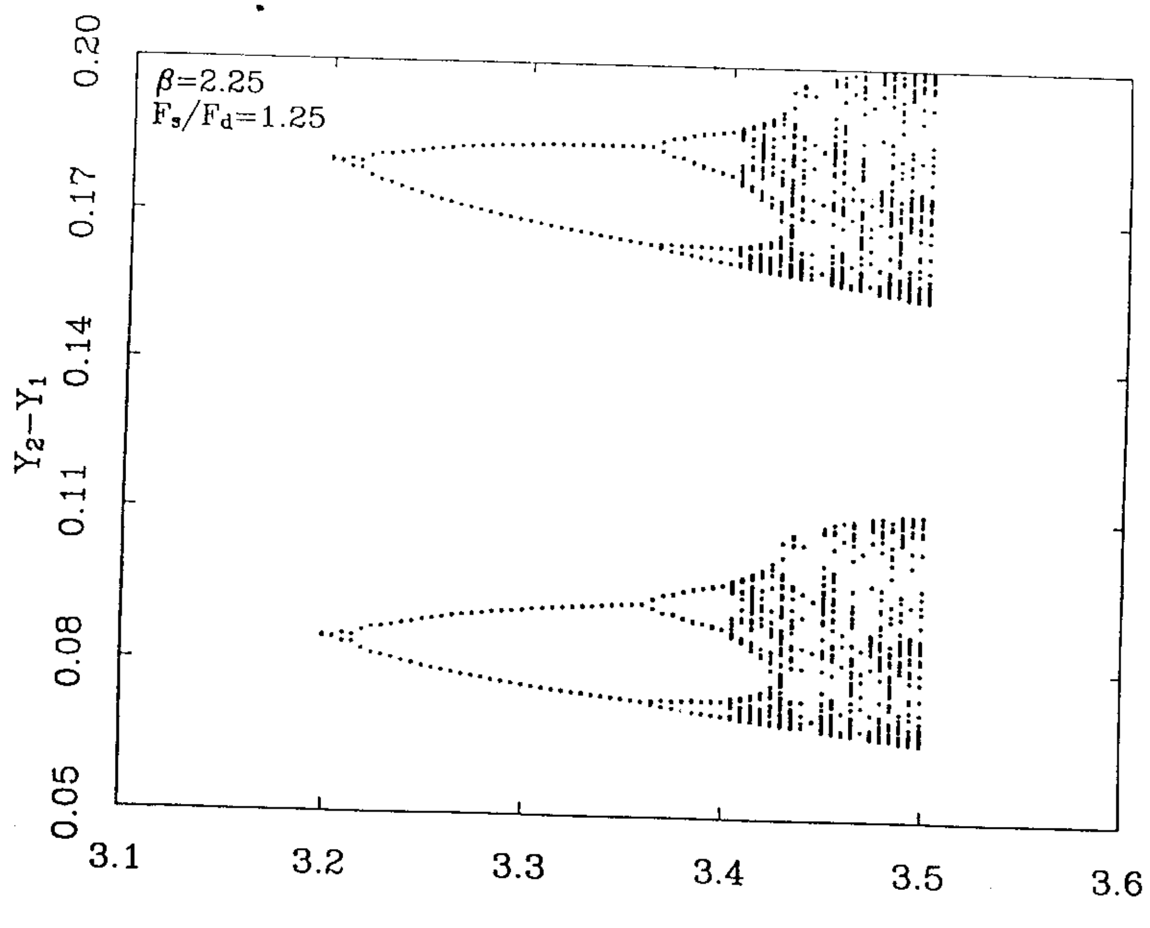
α ~ coupling ratio

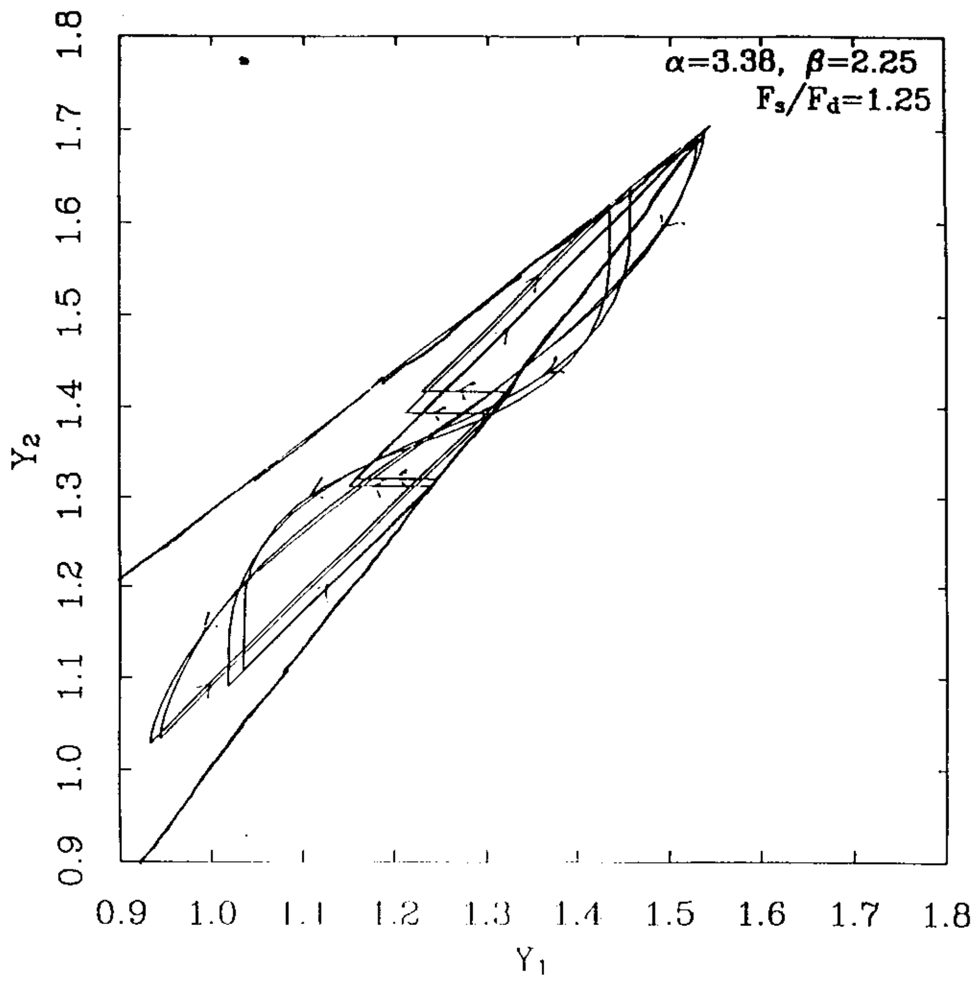
β ~ spatial heterogeneity

ϕ ~ ratio of static/dynamic friction



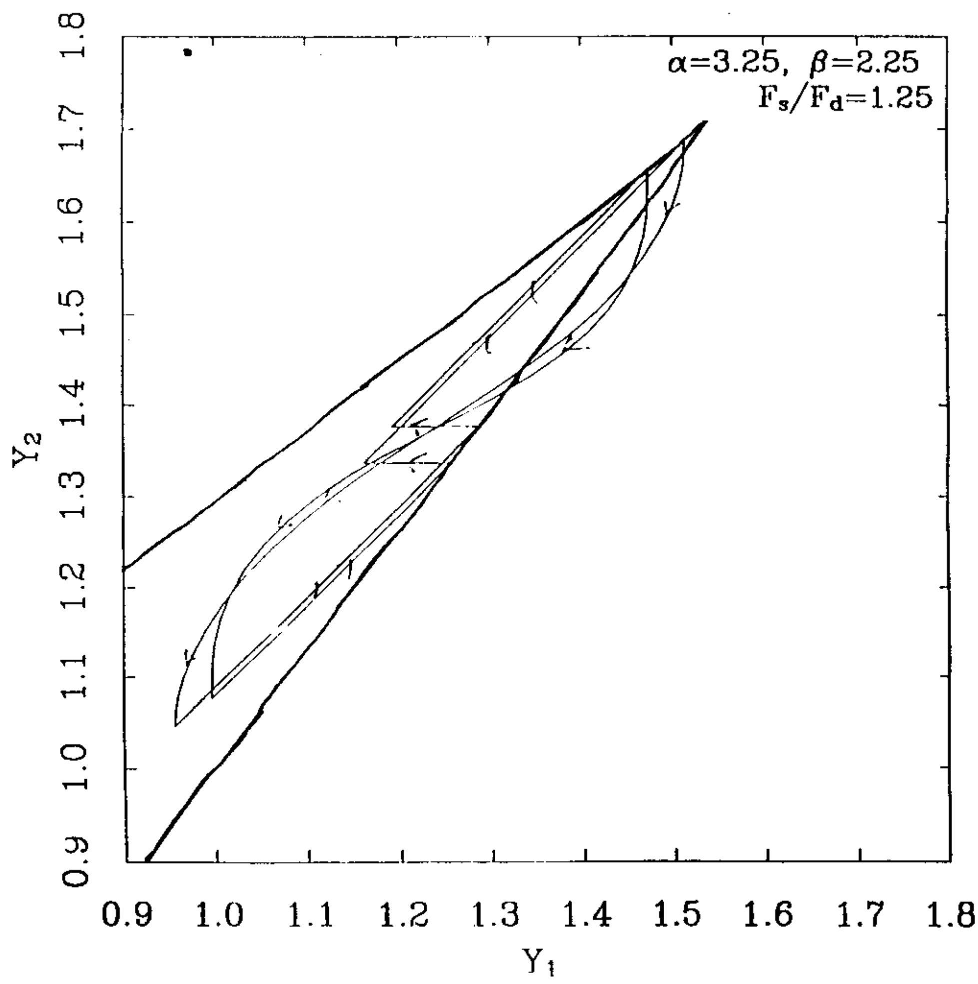

α=3.25, β=2.25
Fs/Fd=1.25
Y2
Y1
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8


β=2.25
Fs/Fd=1.25
Y2−Y1
0.05
0.08
0.11
0.14
0.17
0.20
3.1
3.2
3.3
3.4
3.5
3.6

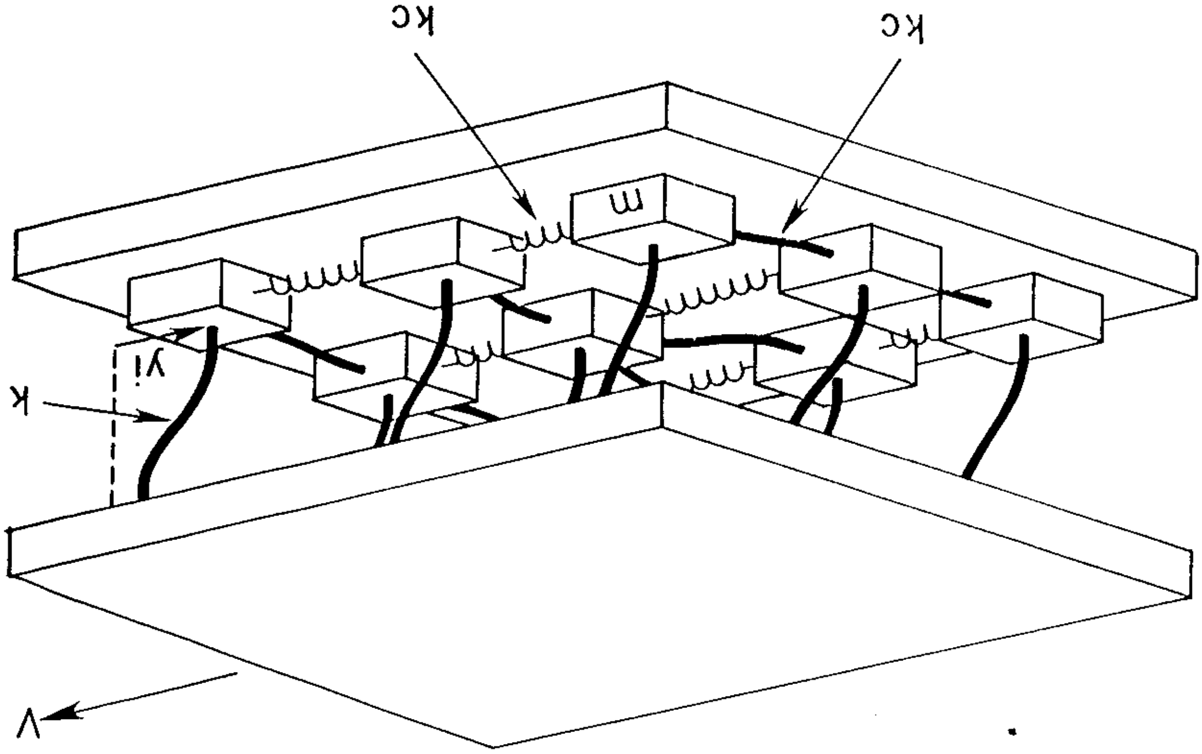


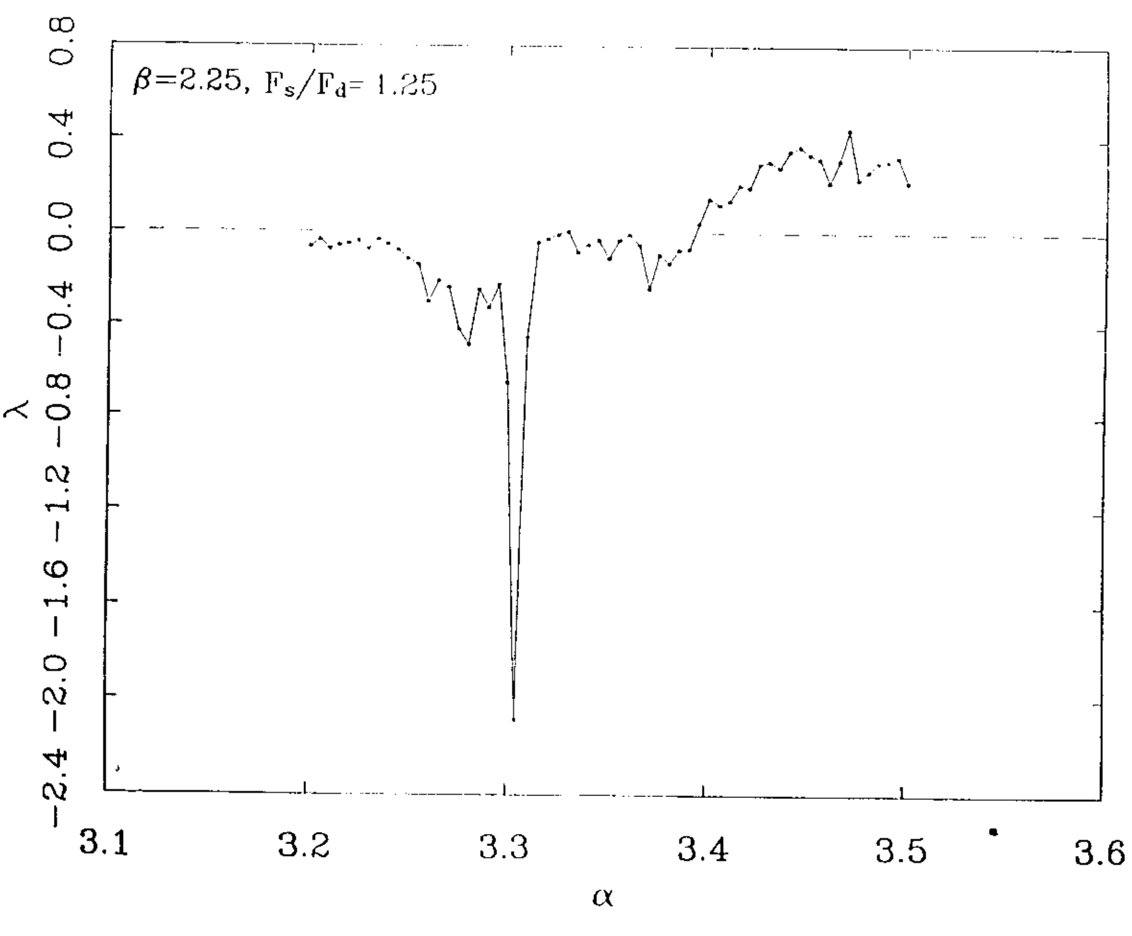
α=3.38, β=2.25
Fs/Fd=1.25
Y2
Y1
0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8

C014



α=3.25, β=2.25
Fs/Fd=1.25
Y2
Y1
0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8


kc
kc
m
yi
k
V


β=2.25, Fs/Fd= 1.25
λ
α
0.8
0.4
0.0
−0.4
−0.8
−1.2
−1.6
−2.0
−2.4
3.1
3.2
3.3
3.4
3.5
3.6

N block model

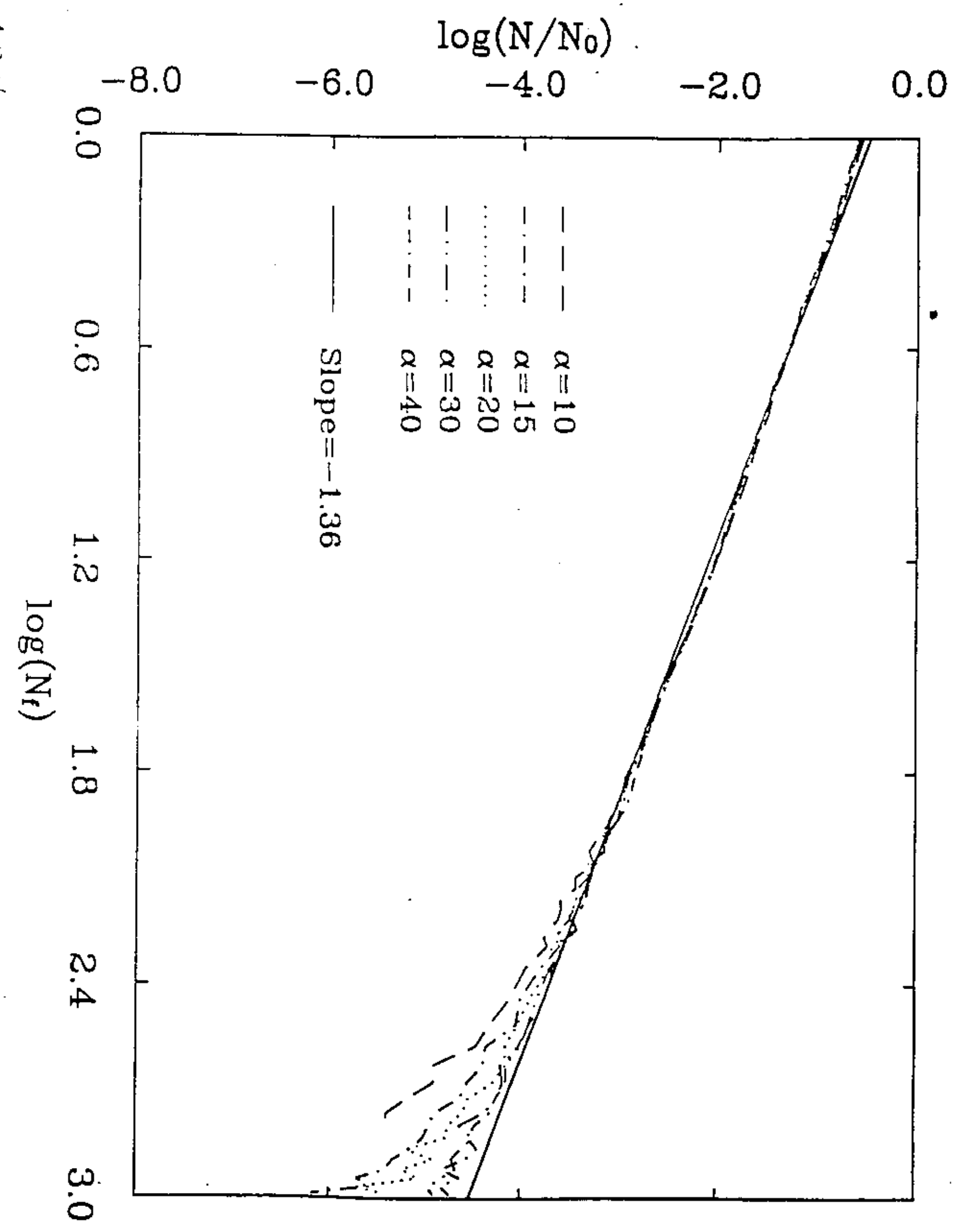
1. Displacement is increased until a block slips.

2. The block slides until its velocity is zero (analytical solution).

3. The four adjoining blocks (except for edge blocks) are checked to see if the slip criteria is violated.

4a. If the slip criteria *is not* violated the step is terminated.

4b. If the slip criteria *is* violated on one or more of the blocks, one block is allowed to slide until its velocity is zero. Further blocks are allowed to slide sequentially until all blocks are stable and the step is terminated.



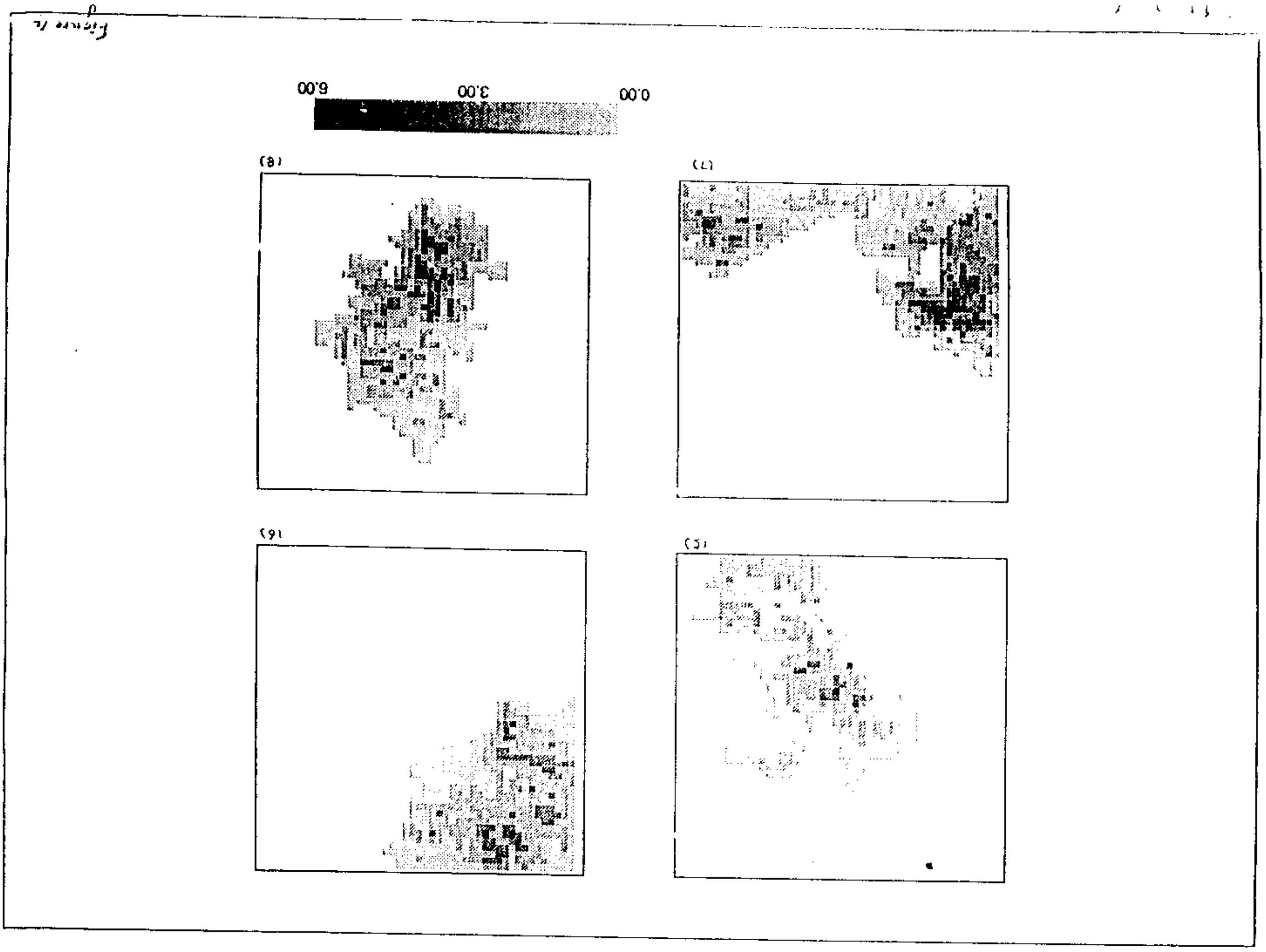
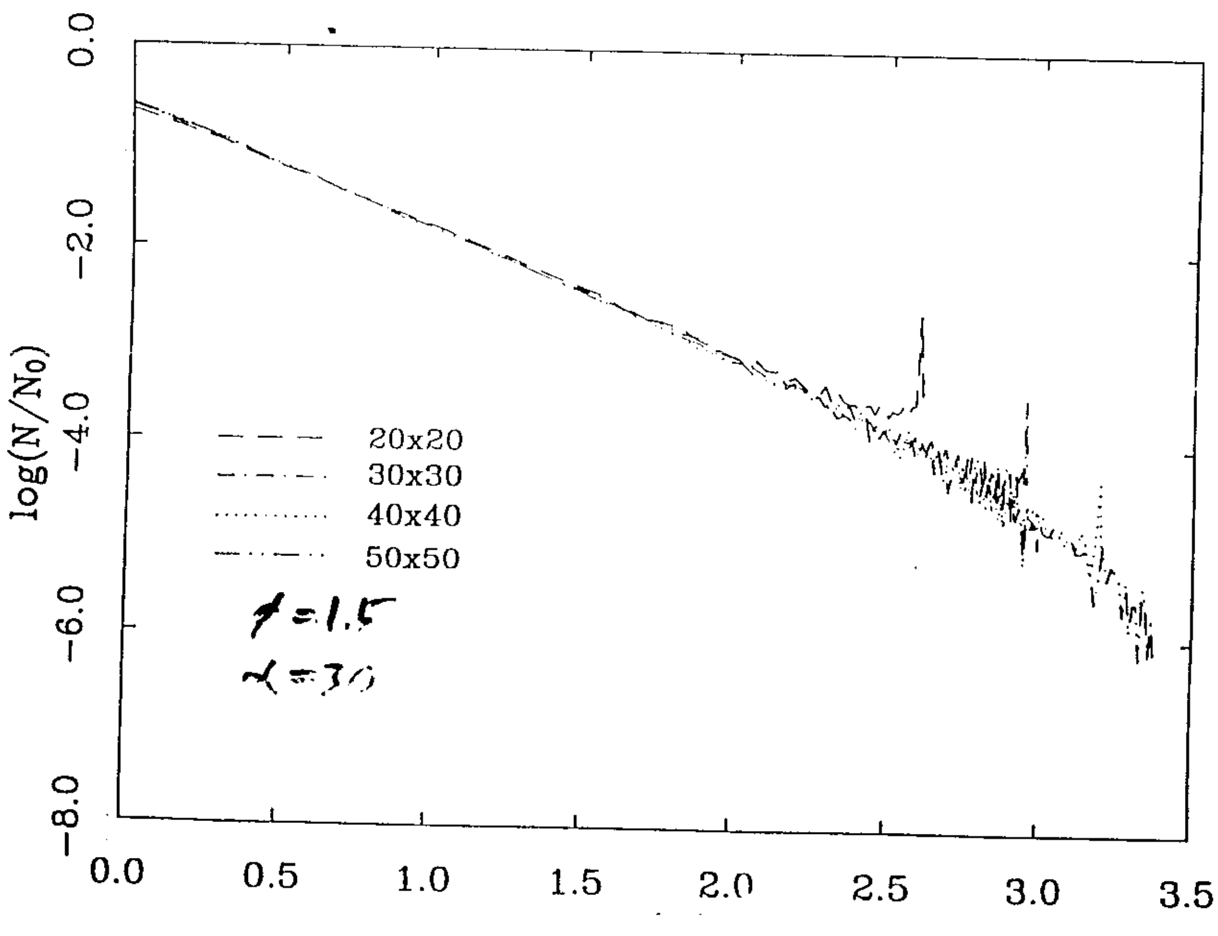
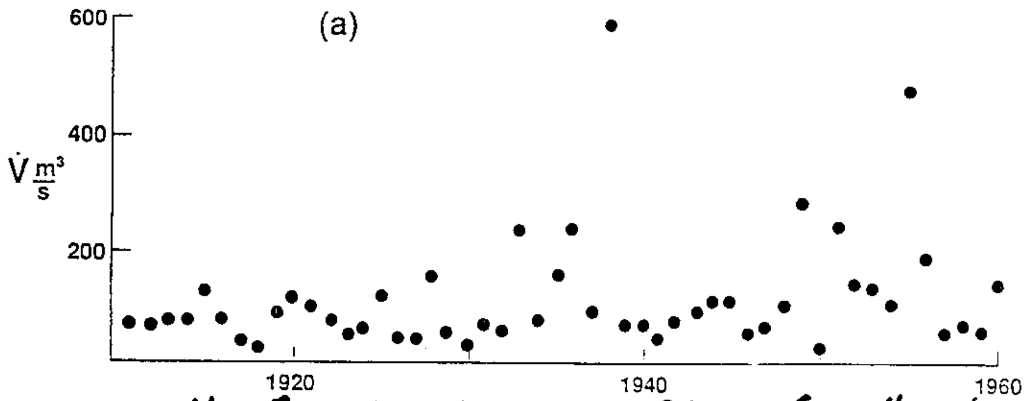
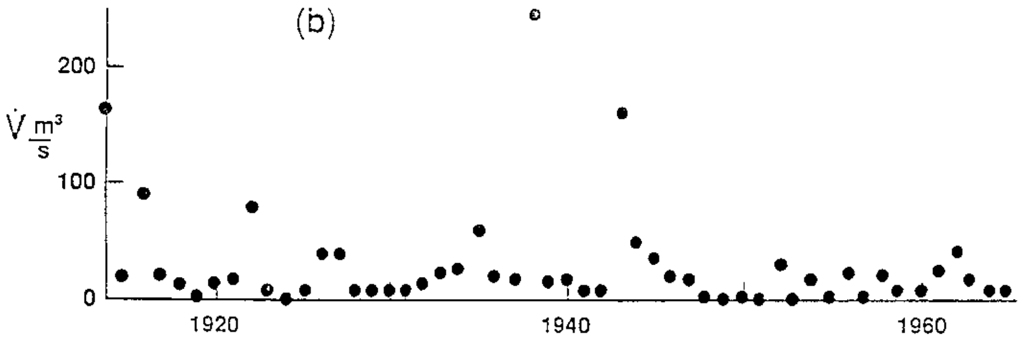

0.00
3.00
6.00

fig. 3


log(N/N0)
0.0
-2.0
-4.0
-6.0
-8.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
20x20
30x30
40x40
50x50
f = 1.5
α = 30



Middle Branch, Westfield River, Goss Heights



Arroyo Seco, Pasadena, CA

Flood statistics

$\dot{V}_m$ = peak discharge during a flood $\dot{N} \sim \dot{V}_m^{-F/3}$

$\dot{N}(\dot{V}_m)$ = number of floods per year with peak discharge exceeding $\dot{V}_m$

$T(\dot{V}_m)$ = recurrence time interval for a flood with peak discharge $\dot{V}_m$

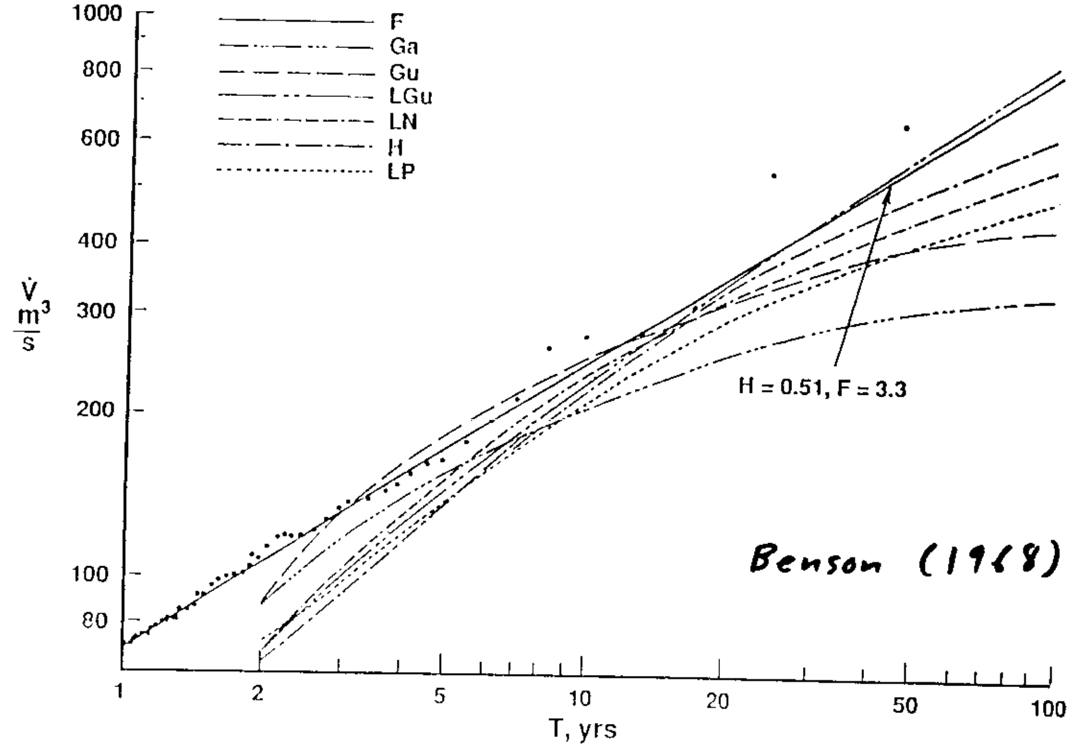
$$\dot{N}(\dot{V}_m) = \frac{1}{T(\dot{V}_m)}$$

$$\dot{V}_m(T) = C\,T^H \qquad D_a = 2 - H$$

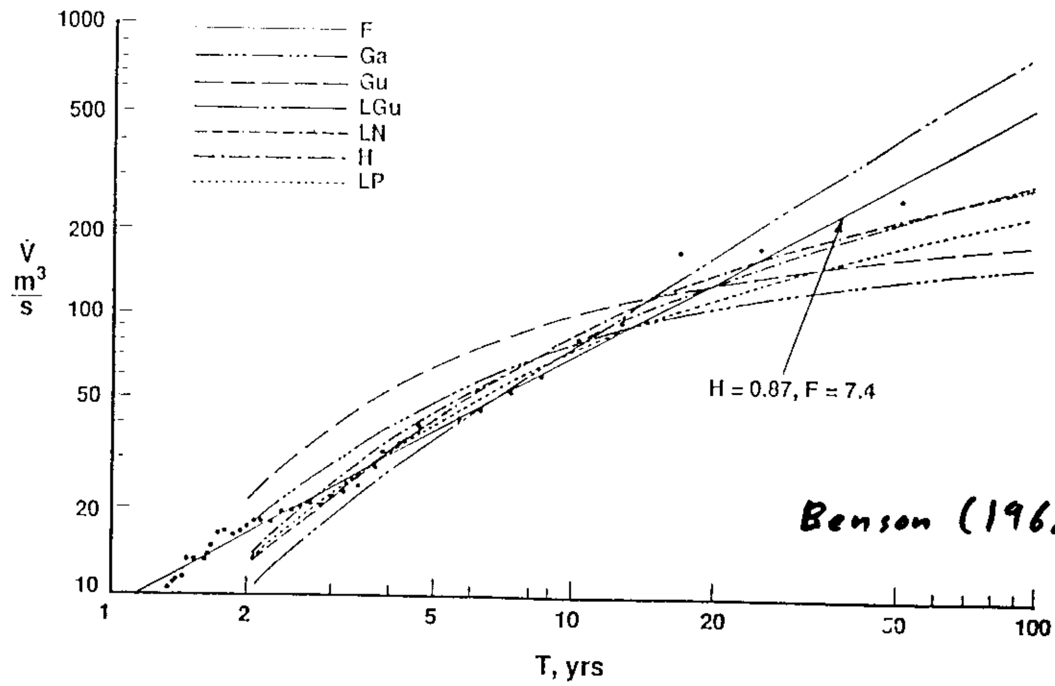
$$F = \frac{\dot{V}_m(10\text{ yr})}{\dot{V}_m(1\text{ yr})} = \frac{\dot{V}_m(100\text{ yr})}{\dot{V}_m(10\text{ yr})} = \frac{\dot{V}_m(1000\text{ yr})}{\dot{V}_m(100\text{ yr})} = 10^H$$

$$\dot{N} = C'\,\dot{V}_m^{-1/H} \qquad D_s = \frac{2}{H}$$

... Branch, Westfield River ... Heights, MA 1911-1960

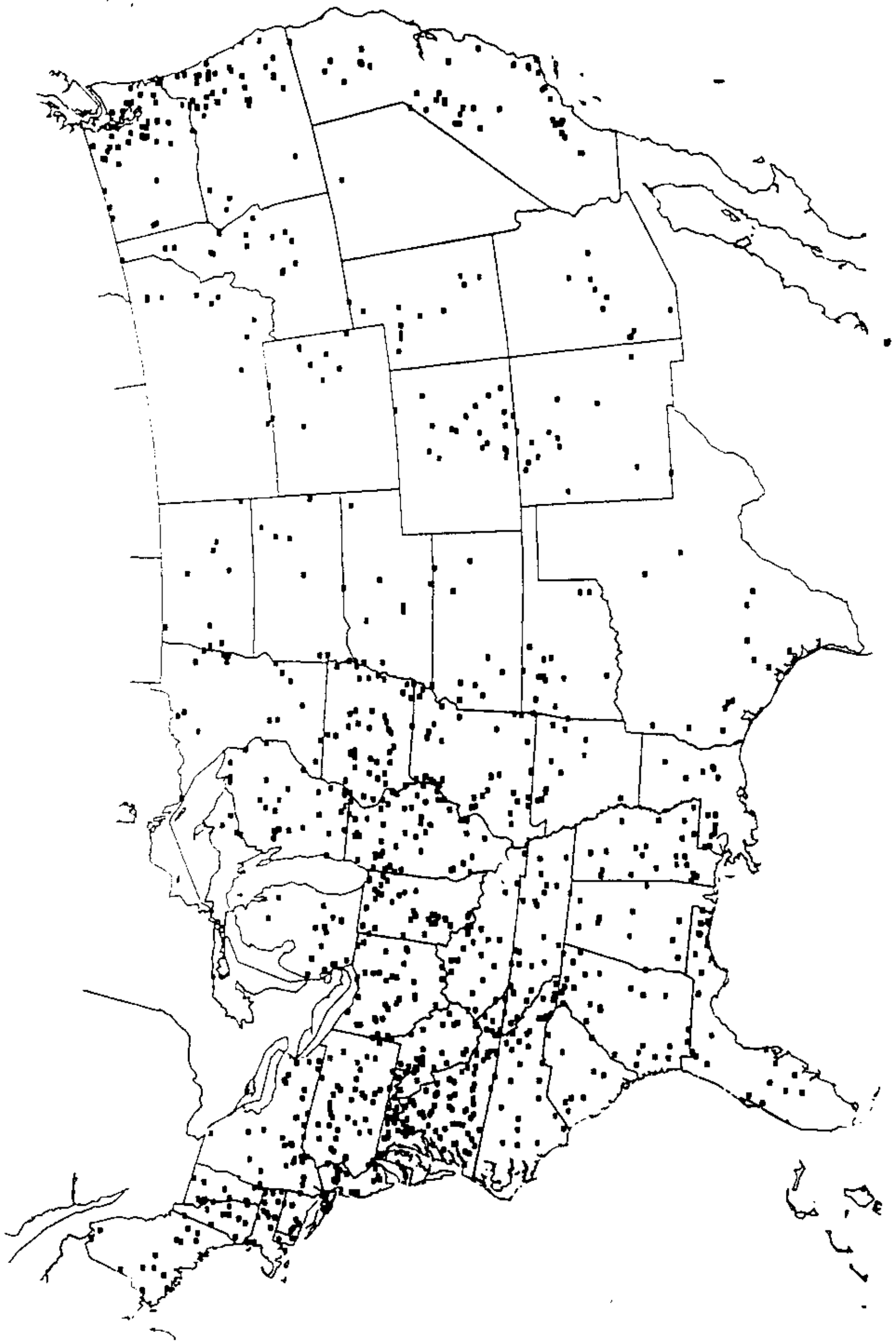


Arroyo Seco
Pasadena, CA 1914-1965



Benchmark Stations

Station	River (State)	H	D_a	F	H_1	D_s
-1805	Westfield (MA)	0.435	1.56	2.72	0.67	6.9
-2185	Oconee (GA)	0.540	1.46	3.47	0.72	5.6
-3310	Mississippi (MN)	0.470	1.53	2.95	0.72	6.4
-3440	Little Missouri (WY)	0.520	1.48	3.31	0.72	5.8
-8005	Elkhorn (NE)	0.540	1.46	3.47	0.67	5.6
-2165	Mora (NM)	0.630	1.37	4.27	0.73	4.8
-1500	Llano (TX)	0.719	1.28	5.24	0.70	4.2
0-3275	Humboldt (NV)	0.616	1.38	4.13	0.66	4.9
1-0980	Arroyo Seco (CA)	0.909	1.09	8.11	0.68	3.3
2-1570	Wenatchee (WA)	0.310	1.69	2.04	0.72	9.7



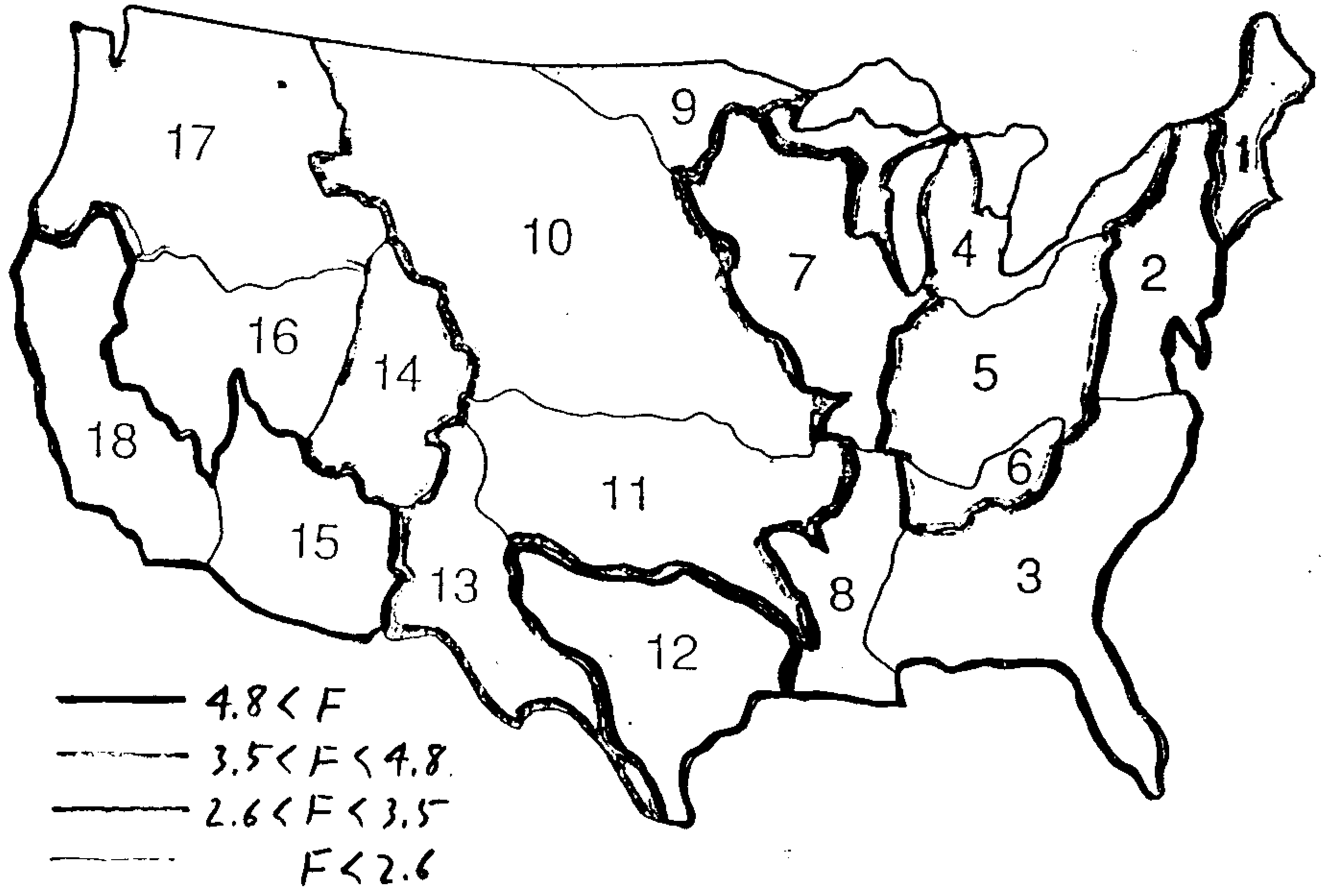


Figure 3: Hydrologic regions of the continental U.S.