INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

H4.SMR/709-6

Second Workshop on Non-Linear Dynamics
and Earthquake Prediction

22 November - 10 December 1993

Numerical Simulation of Block Structure Dynamics

A. Gabrielov, A. Soloviev
Russian Academy of Sciences
International Institute of Earthquake Prediction
Theory and Mathematical Geophysics
Moscow 113556
Russian Federation

NUMERICAL SIMULATION OF BLOCK STRUCTURE DYNAMICS

A.Gabrielov, A.Soloviev

INTRODUCTION

Mathematical models of lithosphere dynamics are tools for the study of the earthquake preparation process. These models are also useful in earthquake prediction studies. An adequate model should indicate the physical basis of premonitory patterns determined empirically before large events. Note that the available data often do not constrain the statistical significance of the premonitory patterns. The model can be used also to suggest new premonitory patterns that might exist in real catalogs.

The basic principles of a model investigated here are developed in Gabrielov et al. (1990). The model produces an artificial catalog of earthquakes.

Although there is no adequate theory of the seismo-tectonic process, various properties of the lithosphere, such as spatial heterogeneity, hierarchical block structure, different types of non-linear rheology, gravitational and thermodynamic processes, physico-chemical and phase transitions, fluid migration and stress corrosion, are probably relevant to the properties of earthquake sequences. The qualitative stability of these properties in different seismic regions suggests that the lithosphere can be modeled as a large dissipative system that does not essentially depend on the particular details of the specific processes active in a geological system.

The model exploits the hierarchical block structure of the lithosphere (Alekseevskaya et al., 1977). Blocks of the lithosphere are separated by comparatively thin, weak, less consolidated fault

zones, such as lineaments and tectonic faults. In the seismotectonic process major deformation and most earthquakes occur in such fault zones.

In the model, a seismically active region is represented as a system of absolutely rigid blocks divided by infinitely thin plane faults. Relative displacement of all blocks is supposed to be infinitely small relative to their geometric size. Blocks interact between themselves and with the underlying medium. The system of blocks moves as a consequence of prescribe motion of boundary blocks and the underlying medium.

As the blocks are rigid, all deformation takes place in the fault zones and the block bottoms separating the blocks and the underlying medium. The relative displacements of the blocks take place along the fault planes. This is justified by the fact that for the lithosphere the affective elastic modules of the fault zones are essentially smaller than those within the blocks.

The blocks are in viscous-elastic interaction with the underlying medium. The corresponding stresses depend on the value of relative displacement. This dependence is assumed to be linear elastic. The motion of the medium underlying different blocks may be different.

The motion of the blocks of the structure is defined so that the system is in quasistatic equilibrium state.

The interaction of the blocks along the fault planes is viscous-elastic ("normal state") while the stress is below a certain strength level. After such a level is exceeded for some part of a fault plane an elastic stress-drop ("a failure") occurs. It can cause a failure for other parts of fault planes. Each sequence of failures is considered as an earthquake.

After an earthquake the corresponding parts of the fault planes are in creep state. In this state the interaction along the fault plane is viscous-elastic but the values of constants are different of those for normal state. Creep state lasts until the stress falls below some other level. Then normal state returns.

As a result of the numerical simulation a synthetic earthquake catalog is produced.

BLOCK STRUCTURE GEOMETRY

A layer with a depth (thickness) H between two horizontal planes is considered. A block structure is a part of this layer limited and divided into blocks by planes intersecting the layer. Parts of these planes which are inside the block structure or adjoin to it are called "faults".

Block structure geometry is defined by intersection lines of faults (they will also be called faults below) with the upper plane and by angles of dip for the fault planes.

It is considered that three or more faults cannot have a common point on the upper plane. A common point of two faults is called "vertex". There are three types of vertices:

C (corner vertex) - a vertex which is an end point of a fault and at the same time an initial point of another one;

E (end vertex) - a vertex which is an initial (or end) point of a fault and belongs to another one but isn't its initial (or end) point;

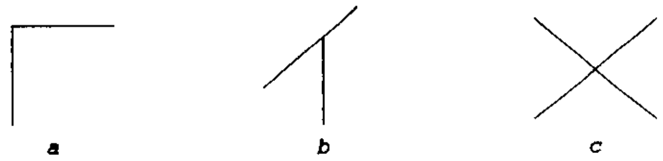

FIGURE 1. Types of vertices: a - corner (C); b - end (E); c - intersection (I).

I (intersection) - a point of intersection of two faults which is not an initial (or end) for each of them.

The examples of these types of vertices are shown in Figure 1.

Corner vertices are defined by indication of their coordinates on the upper plane. Coordinates of other vertices are calculated by using information about their positions in faults.

A fault is defined by indication of its consecutive vertices. For each vertex its type is indicated. For a corner vertex its number is indicated. For an end vertex or an intersection the number of the another fault to which this vertex also belongs is indicated. If the end vertex is not an initial or end point for the fault defined its relative position in the fault (the ratio of the distance between the initial point of the fault and the vertex to the fault length) is indicated.

The angle of dip for the fault plane is measured on the left of the fault. The fault direction is the direction from its initial point to its end point.

The structure is separated by the faults into blocks. A common part of any block with the upper plane is a polygon.

"Boundary blocks" are defined in the structure. A boundary

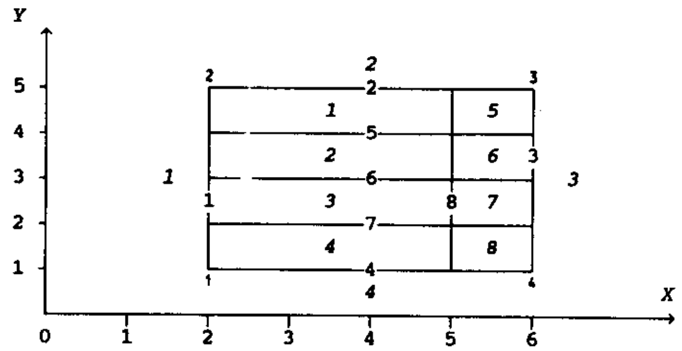

FIGURE 2. The vertices and the faults on the upper plane.

block is a continuous part of the structure boundary between two vertices. It is defined by indication of its initial and end vertices. The direction is selected to have the structure on the right.

The following is an example of description of the structure having the upper plane faults represented in Figure 2.

This structure has 4 corner vertices with the coordinates: (2, 1), (2, 5), (6, 5), (6, 1). The are 8 faults. Their description is given in Table 1.

With the layer depth $H = 1$, the faults and the vertices have on the lower plane the position shown in Figure 3.

The structure has 8 blocks.

For the structure 4 boundary blocks can be defined. In this case the boundary blocks are connected with the pairs of adjacent boundary vertices. The boundary block can be consolidated. For example two boundary blocks could be considered for the structure: the boundary block between the corner vertices with the numbers 1 and 3 and the boundary block between the corner vertices with the numbers 3 and 1.

TABLE 1. The description of the faults.

Type of vertex	Number of vertex or fault	Relative position of vertex
Fault 1, Angle = 45°		
C	1	
E	7	0.25
E	6	0.5
E	5	0.75
C	2	
Fault 2, Angle = 45°		
C	2	
E	8	0.75
C	3	
Fault 3, Angle = 135°		
C	3	
E	5	0.25
E	6	0.5
E	7	0.75
C	4	
Fault 4, Angle = 135°		
C	4	
E	8	0.25
C	1	

Type of vertex	Number of vertex or fault	Relative position of vertex
Fault 5, Angle = 45°		
E	1	
I	8	
E	3	
Fault 6, Angle = 45°		
E	1	
I	8	
E	3	
Fault 7, Angle = 45°		
E	1	
I	8	
E	3	
Fault 8, Angle = 45°		
E	4	
I	7	
I	6	
I	5	
E	2	

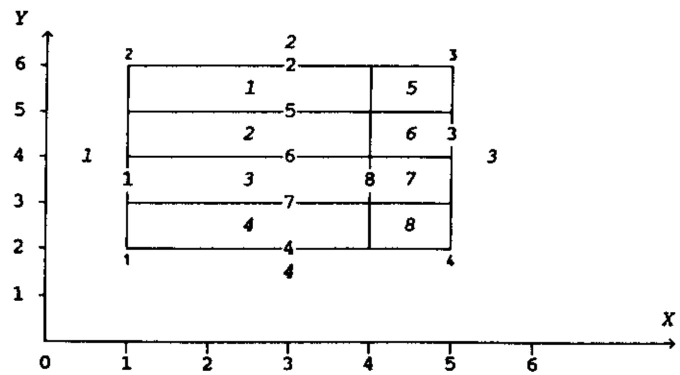

FIGURE 3. The vertices and the faults on the lower plane.

MOVEMENT OF BLOCKS

The blocks are assumed to be rigid and all their relative displacements take place along the corresponding fault plane.

The movements of the the boundaries of the block structure (boundary blocks) and the medium underlying the blocks is assumed to be the outer action on the structure. The rates of these movements are considered to be horizontal and known.

At each moment of time the displacements of the blocks are defined so that the structure is in a quasistatic equilibrium.

All displacements are supposed to be infinitely small relative to the geometrical sizes of the blocks.

INTERACTIONS

Interaction between a block and the underlying medium. The elastic force which is due to relative displacement of the block and the underlying medium in some point of the block bottom is supposed to be proportional to the difference between the total relative displacement vector and the vector of slippage (inelastic displacement) in this point.

The density of the elastic force $\mathbf{f}^u = (f_x^u, f_y^u)$ acting at the point with coordinates (X,Y) at some moment t is defined by the formulas

$$f_x^u = K_u(x - x_u - (Y - Y_c)(\varphi - \varphi_u) - x_a),$$
$$f_y^u = K_u(y - y_u + (X - X_c)(\varphi - \varphi_u) - y_a). \quad (1)$$

Here X_c, Y_c are the coordinates of the geometrical center of the block bottom; (x_u, y_u) and φ_u are the shear vector and the angle of the rotation around the geometrical center of the block bottom of the underlying medium at the moment t; (x,y) and φ are the shear vector of the block and the angle of its rotation around the geometrical center of its bottom at the moment t; (x_a, y_a) is the inelastic displacement vector at the point at the moment t.

The evolution of the inelastic displacement at the point is described by the equations

$$\frac{dx_a}{dt} = V_u f_x^u, \quad \frac{dy_a}{dt} = V_u f_y^u. \quad (2)$$

The values of the coefficients K_u and V_u in formulas (1) and equations (2) can be different for different blocks.

Interaction between blocks along the fault plane. At the moment t at some point of the fault plane separating the blocks with the numbers i and j (the block with the number i is on the left and the block with the number j is on the right of the fault) the components Δx, Δy of the relative displacement of the blocks are defined by the formulas

$$\Delta x = x_i - x_j - (Y - Y_c^i)\varphi_i + (Y - Y_c^j)\varphi_j,$$
$$\Delta y = y_i - y_j + (X - X_c^i)\varphi_i - (X - X_c^j)\varphi_j. \quad (3)$$

Here X_c^i, Y_c^i, X_c^j, Y_c^j are the coordinates of the geometrical centers of the block bottoms; (x_i, y_i), (x_j, y_j) are the shear vectors of the blocks at the moment t; φ_i, φ_j are the angles of the block rotations around the geometrical centers of their bottoms at the moment t.

Accordingly to the assumption that the block relative displacements take place only along the fault plane the displacements along the fault plane are connected with the horizontal relative displacement by the formulas

$$\Delta_t = e_x \Delta x + e_y \Delta y,$$
$$\Delta_l = \frac{\Delta_n}{\cos\alpha}, \quad \text{where} \quad \Delta_n = e_x \Delta y - e_y \Delta x . \qquad (4)$$

Here Δ_t, Δ_l are the displacements along the fault plane parallel (Δ_t) and normal (Δ_l) to the fault line on the upper plane; (e_x, e_y) is the unit vector having direction of the fault line on the upper plane; α is the anjle of dip for the fault plane; Δ_n is the horizontal displacement normal to the fault line on the upper plane.

The density of the elastic force $f = (f_t, f_l)$ acting along the fault plane at the point is defined by the formulas

$$f_t = K(\Delta_t - \delta_t),$$
$$f_l = K(\Delta_l - \delta_l). \qquad (5)$$

Here δ_t, δ_l are the inelastic displacements along the fault plane at the point at the moment t parallel (δ_t) and normal (δ_l) to the fault line on the upper plane.

The evolution of the inelastic displacement at the point is described by the equations

$$\frac{d\delta_t}{dt} = Vf_t, \quad \frac{d\delta_l}{dt} = Vf_l . \qquad (6)$$

The values of the coefficients K and V in formulas (5) and equations (6) can be different for different faults.

Besides the elastic force the reaction force which is normal to the fault plane acts also. But this force does not perform any work because all relative movements are tangent to the fault plane. The density of elastic energy at the point is equal to

$$e = (f_t(\Delta_t - \delta_t) + f_l(\Delta_l - \delta_l))/2 . \qquad (7)$$

From formulas (4) and (7) the elastic force density horizontal component normal to the fault line on the upper plane can be obtained. This component is equal to

$$f_n = \frac{\partial e}{\partial \Delta_n} = \frac{f_l}{\cos\alpha} . \qquad (8)$$

Formula (8) confirms that the reaction force is normal to the fault plane (see Figure 4). The density of the reaction force is equal to

$$p_0 = f_l \,\mathrm{tg}\alpha . \qquad (9)$$

Formulas (4), (5) and (8) lead to the following formulas for the horizontal components of the vector (f_x, f_y) of the elastic force density at the point.

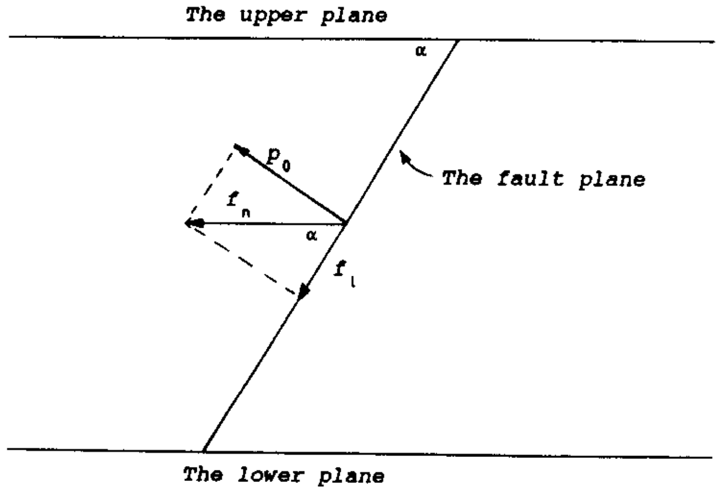

FIGURE 4. The forces in the plane orthogonal to the line of the fault intersection with the upper plane.

$$f_x = K\left(\left(e_x^2 + \frac{e_y^2}{\cos^2\alpha}\right)\Delta x + e_x e_y\left(1 - \frac{1}{\cos^2\alpha}\right)\Delta y - \delta_t e_x + \frac{\delta_l e_y}{\cos\alpha}\right),$$
$$f_y = K\left(\left(e_y^2 + \frac{e_x^2}{\cos^2\alpha}\right)\Delta y + e_x e_y\left(1 - \frac{1}{\cos^2\alpha}\right)\Delta x - \delta_t e_y - \frac{\delta_l e_x}{\cos\alpha}\right). \qquad (10)$$

The formulas given above are also valid for the boundary faults. In this case one of the blocks separated by the fault is the boundary block. The movement of these blocks is described by their shears and rotations around the coordinate origin. Therefore the coordinates of the geometrical center of the block bottom in formulas (3) are equal to zeros for the boundary block. For example if the block with the number j is the boundary block then $X_c^j = Y_c^j = 0$ in formulas (3).

EQUILIBRIUM EQUATIONS

The components of the shear vectors of the blocks and the angles of their rotations around the geometrical centers of the bottoms are defined from the condition that the total force and the total moment of forces acting on each block are equal to zero. This is the condition of the quasi-static equilibrium of the system and at the same time the condition of the energy minimum.

In accordance with formulas (1) and (10) the system of equations which describes the equilibrium has the following form

$$\mathbf{Az} = \mathbf{b}\ , \qquad (11)$$

where the components of the unknown vector $\mathbf{z} = (z_1, z_2, \ldots, z_{3n})$ are the components of the shear vectors of the blocks and the angles of their rotations around the geometrical centers of the

bottoms (n is the number of the blocks), i.e. $z_{3m+1} = X_m$, $z_{3m+2} = Y_m$, $z_{3m+3} = \varphi_m$ (m is the number of the block, $m = 0, 1, \ldots, n-1$).

For each block the moment of forces is calculated relative to the geometrical center of its bottom.

The elements of the matrix A $(3n\times 3n)$ can be calculated by the formulas

$$a_{3m+1,3m+1} = S_u^m K_u^m c_m + \sum_{p=1}^{r_m} S_p^m K^{mp} C_1^{mp},$$

$$a_{3m+1,3m+2} = a_{3m+2,3m+1} = \sum_{p=1}^{r_m} S_p^m K^{mp} C_2^{mp},$$

$$a_{3m+1,3m+3} = a_{3m+3,3m+1} = \sum_{p=1}^{r_m} S_p^m K^{mp}(C_2^{mp}(X_c^{mp} - X_c^m) - C_1^{mp}(Y_c^{mp} - Y_c^m)),$$

$$a_{3m+2,3m+2} = S_u^m K_u^m c_m + \sum_{p=1}^{r_m} S_p^m K^{mp} C_3^{mp},$$

$$a_{3m+2,3m+3} = a_{3m+3,3m+2} = \sum_{p=1}^{r_m} S_p^m K^{mp}(C_3^{mp}(X_c^{mp} - X_c^m) - C_2^{mp}(Y_c^{mp} - Y_c^m)),$$

$$a_{3m+3,3m+3} = K_u^m c_m\left(\int_S (X^2 + Y^2)\,dS - S_u^m\left((X_c^m)^2 + (Y_c^m)^2\right)\right) +$$
$$+ \sum_{p=1}^{r_m} K^{mp}\left(\int_S (C_3^{mp}X^2 + C_1^{mp}Y^2 - 2C_2^{mp}XY)\,dS - S_p^m\left(C_3^{mp}\left(2X_c^{mp}X_c^m - (X_c^m)^2\right) + C_1^{mp}\left(2Y_c^{mp}Y_c^m - (Y_c^m)^2\right) + 2C_2^{mp}(X_c^m Y_c^m - X_c^{mp}Y_c^m - Y_c^{mp}X_c^m)\right)\right).$$

Here S_u^m, X_c^m, Y_c^m are the square and coordinates of the geometrical center of the bottom of the block with the number m; K_u^m is the coefficient K_u in formula (1) for the block with the number m; r_m is the number of vertices of the block with the number m; S_p^m, X_c^{mp}, Y_c^{mp}

are the square and co-ordinate of the geometrical center of the fault segment between the block vertices with the numbers p and $p+1$ (if $p < r_m$) or r_m and 1 (if $p = r_m$); K^{mp} is the coefficient K in formulas (5) and (10) for the fault to which the segment belongs. Here and below a fault segment means a part of the fault plane limited by the upper and lower planes and lines which connect positions on the upper and lower planes of two sequential vertices of the fault.

The coefficients c_m, C_1^{mp}, C_2^{mp}, C_3^{mp} are calculated by the formulas

$$c_m = \min_{1 \le p \le r_m} \cos^2\alpha_{mp}, \quad C_1^{mp} = (e_x^{mp})^2 c_m + \frac{(e_y^{mp})^2 c_m}{\cos^2\alpha_{mp}},$$

$$C_2^{mp} = e_x^{mp} e_y^{mp}\left(c_m + \frac{c_m}{\cos^2\alpha_{mp}}\right), \quad C_3^{mp} = (e_y^{mp})^2 c_m + \frac{(e_x^{mp})^2 c_m}{\cos^2\alpha_{mp}},$$

where α_{mp}, e_x^{mp}, e_y^{mp} are the values of α, e_x, and e_y for the fault to which the segment belongs.

Let $m \ne k$. If the blocks with the numbers m and k have no common segments, the elements $a_{3m+i,3k+j}$ ($i, j = 1, 2, 3$) of the matrix A are equal to zero. Otherwise

$$a_{3m+1,3k+1} = -\sum_p{}' S_p^m K^{mp} C_1^{mp},$$

$$a_{3m+1,3k+2} = a_{3m+2,3k+1} = -\sum_p{}' S_p^m K^{mp} C_2^{mp},$$

$$a_{3m+1,3k+3} = \sum_p{}' S_p^m K^{mp} \left(C_1^{mp}(Y_c^{mp} - Y_c^k) - C_2^{mp}(X_c^{mp} - X_c^k)\right),$$

$$a_{3m+2,3k+2} = -\sum_p{}' S_p^m K^{mp} C_3^{mp},$$

$$a_{3m+2,3k+3} = \sum_p{}' S_p^m K^{mp} \left(C_2^{mp}(Y_c^{mp} - Y_c^k) - C_3^{mp}(X_c^{mp} - X_c^k)\right),$$

$$a_{3m+3,3k+1} = \sum_p{}' S_p^m K^{mp} \left(C_1^{mp}(Y_c^{mp} - Y_c^m) - C_2^{mp}(X_c^{mp} - X_c^m)\right),$$

$$a_{3m+3,3k+2} = \sum_p{}' S_p^m K^{mp} \left(C_2^{mp}(Y_c^{mp} - Y_c^m) - C_3^{mp}(X_c^{mp} - X_c^m)\right),$$

$$a_{3m+3,3k+3} = \sum_p{}' K^{mp} \Bigg(\int_S (2C_2^{mp}XY - C_3^{mp}X^2 - C_1^{mp}Y^2)\,dS +$$

$$+ S_p^m \bigg(C_3^{mp}\left[X_c^{mp}(X_c^m + X_c^k) - X_c^m X_c^k\right] + C_1^{mp}\left[Y_c^{mp}(Y_c^m + Y_c^k) - Y_c^m Y_c^k\right] +$$

$$+ C_2^{mp}\left[X_c^m Y_c^k + X_c^k Y_c^m - X_c^{mp}(Y_c^m + Y_c^k) - Y_c^{mp}(X_c^m + X_c^k)\right]\bigg)\Bigg).$$

In these formulas summarizing is made only for the common segments of the blocks with the numbers m and k.

The components of the vector $\mathbf{b} = (b_1, b_2, \ldots, b_{3n})$ are calculated by the formulas

$$b_{3m+1} = c_m\left(K_u^m\left(S_u^m x_u^m + \int_{S_u^m} x_s\,dS\right) + \sum_{p=1}^{r_m} K^{mp} \int_{S_p^m}\left(\delta_t e_x^{mp} - \frac{\delta_l e_y^{mp}}{\cos\alpha_{mp}}\right)dS\right) + d_{3m+1},$$

$$b_{3m+2} = c_m\left(K_u^m\left(S_u^m y_u^m + \int_{S_u^m} y_s\,dS\right) + \sum_{p=1}^{r_m} K^{mp} \int_{S_p^m}\left(\delta_t e_y^{mp} + \frac{\delta_l e_x^{mp}}{\cos\alpha_{mp}}\right)dS\right) + d_{3m+2},$$

$$b_{3m+3} = c_m\Bigg(K_u^m\bigg(\int_{S_u^m}\left[y_s(X - X_c^m) - x_s(Y - Y_c^m) + \varphi_u^m(X^2 + Y^2)\right]dS -$$

$$- S_u^m \varphi_u^m\left((X_c^m)^2 + (Y_c^m)^2\right)\bigg) + \sum_{p=1}^{r_m} K^{mp} \int_{S_p^m}\bigg(\delta_t\left[e_y^{mp}(X - X_c^m) - e_x^{mp}(Y - Y_c^m)\right] +$$

$$+ \delta_l \frac{e_x^{mp}(X - X_c^m) + e_y^{mp}(Y - Y_c^m)}{\cos\alpha_{mp}}\bigg)dS\Bigg) + d_{3m+3}.$$

Here x_u^m, y_u^m are the components of the shear vector of the medium underlying the block with the number m; φ_u^m is the angle of the underlying medium rotation around the geometrical center of the block bottom.

If the block with the number m has no common segments with the boundary blocks, the items d_{3m+i} ($i = 1, 2, 3$) are equal to zero.

Otherwise

$$d_{3m+1} = \sum_p{}' S_p^m K^{mp} (C_1^{mp}(x_{mp} - \varphi_{mp} Y_c^{mp}) + C_2^{mp}(y_{mp} + \varphi_{mp} X_c^{mp})),$$

$$d_{3m+2} = \sum_p{}' S_p^m K^{mp} (C_2^{mp}(x_{mp} - \varphi_{mp} Y_c^{mp}) + C_3^{mp}(y_{mp} + \varphi_{mp} X_c^{mp})),$$

$$d_{3m+3} = \sum_p{}' K^{mp} \Big(S_p^m \big[x_{mp}(C_2^{mp}(X_c^{mp} - X_c^m) - C_1^{mp}(Y_c^{mp} - Y_c^m)) +$$

$$+ y_{mp}(C_3^{mp}(X_c^{mp} - X_c^m) - C_2^{mp}(Y_c^{mp} - Y_c^m)) + \varphi_{mp}(X_c^{mp}(C_2^{mp} Y_c^m - C_3^{mp} X_c^m) +$$

$$+ Y_c^{mp}(C_2^{mp} X_c^m - C_1^{mp} Y_c^m)) \big] + \varphi_{mp} \int_{S_p^m} (C_3^{mp} X^2 + C_1^{mp} Y^2 - 2C_2^{mp} XY)\, dS \Big) .$$

In these formulas summarizing is made only for the common segments of the block with the number m and the boundary blocks; x_{mp}, y_{mp} are the components of the shear vector of the corresponding boundary block; φ_{mp} is the angle of the boundary block rotation around the coordinate origin.

DISCRETIZATION

Time discretization is performed by introducing a time step Δt. The block structure state is considered for the discrete moments of time $t_i = t_0 + i\Delta t$ ($i = 1, 2, \ldots$), where t_0 is the initial moment. Transition from the state at t_i to the state at t_{i+1} is made as follows. First, the new values of the inelastic displacements x_a, y_a, δ_t, δ_l are calculated from equations (2) and (6). Next the shear vectors and the rotation angles at t_{i+1} for the boundary blocks and the medium underlying the blocks are calculated. Then the components of the vector **b** in the system of equations (11) have been calculated and this system is used to define the shear vectors and the angles of rotation for the blocks. As the elements of A in (11) do not

depend on time, the matrix A and the associated inverse matrix may be calculated just ones at the beginning of the calculation.

Space discretization is defined by the parameter ε. Discretization is made for the surfaces of the fault segments and the block bottoms.

Discretization of a fault segment is performed as follows. Note that any fault segment is a trapezium. Let a and b be the bases of the trapezium. The trapezium height h is given by the formula $h = H/\sin\alpha$, where H is the depth of the layer, α is the dip angle of the fault plane. Let

n_1 = ENTIRE(h/ε) + 1, n_2 = ENTIRE(max(a,b)/ε) + 1.

The trapezium is divided into $n_1 n_2$ small trapeziums by two groups of lines inside it: n_1-1 lines parallel to the trapezium bases with the distance between them equals to h/n_1 and n_2-1 lines connecting the points spaced at intervals of a/n_2 and b/n_2, respectively, on the bases (see Figure 5). The small trapeziums obtained will be called cells. The coordinates X, Y of the center of the mean line of the cell are assigned to all its points. The inelastic displacements δ_t, δ_l are supposed to be the same for all points of the cell.

The block bottom is a polygon. Before discretization it is divided into trapeziums (triangles) by lines passing through its vertices and parallel to Y axis (see Figure 6). The discretization of these trapeziums (triangles) is performed in the same way as in the case of fault segments. The small trapeziums (triangles) are also called cells. For all points of the cell the coordinates X, Y and the inelastic displacements x_a, y_a are supposed to be the same.

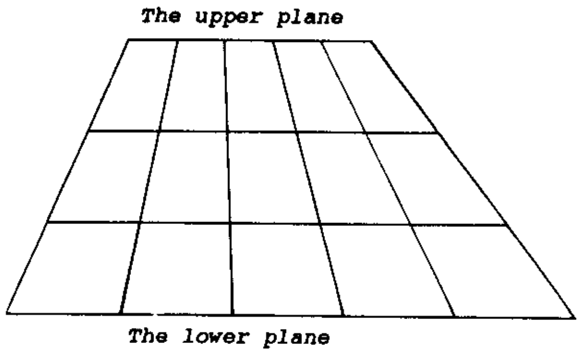

FIGURE 5. Discretization of the fault segment ($n_1 = 3$, $n_2 = 5$).

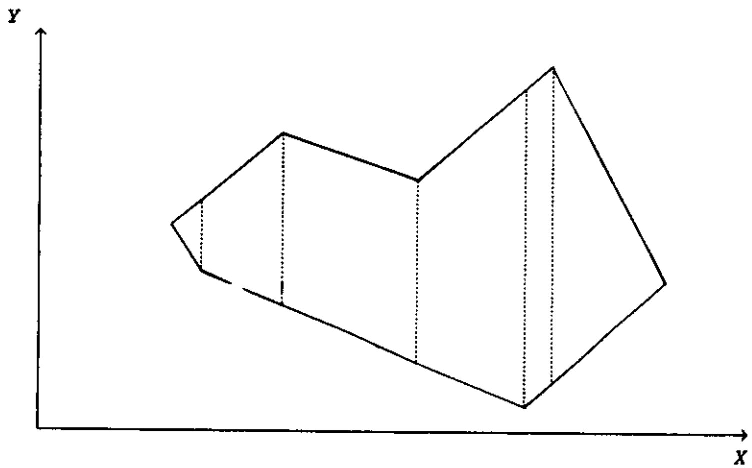

FIGURE 6. Division of the block bottom into trapeziums and triangles.

Denote

$$\kappa = \frac{\|(f_t, f_l)\|}{P - p_0} \tag{12}$$

where (f_t, f_l) is the vector of the density of the elastic force defined by formulas (5), P is the difference between lithostatic and hydrostatic pressure which has the same value for all faults, p_0 is the density of the reaction force which is defined by formula (9).

For each fault the values of the following three levels are indicated

$B > H_f \geq H_s$.

Initial conditions for numerical simulation of block structure dynamics are supposed to satisfy the inequality $\kappa < B$ for all cells of the fault segments. If at some moment t_i the value of κ in any cell of a fault segment reaches the level B, failure ("earthquake") occurs. Failure means slippage during which the inelastic displacements δ_t, δ_l in the cell change abruptly to reduce the value of κ to the level H_f.

The new values of the inelastic displacements are calculated by the formulas

$$\delta_t^* = \delta_t + \gamma f_t, \qquad \delta_l^* = \delta_l + \gamma f_l \tag{13}$$

where δ_t, δ_l, f_t, f_l are the values of the inelastic displacements and the components of the elastic force density vector just before the failure. The value of the coefficient γ is defined by the formula

$$\gamma = \left(1 - \frac{PH_f}{\sqrt{f_t^2 + f_l^2} + H_f f_l \operatorname{tg}\alpha}\right)\frac{1}{K}\ . \tag{14}$$

It follows from formulas (5), (9), (12)-(14) that just after the failure the value of κ equals to the value of the level H_f.

After calculation of new values of inelastic displacements for the failed cells the new values of the components of the vector **b** are calculated and from the system of equations (11) the shear vectors and the angles of rotation for the blocks are found. If for some cell of the fault segments $\kappa \geq B$, the procedure given above is repeated for this cell (or cells). Otherwise the earthquake has ended, and the state of the block structure at the moment t_{i+1} is calculated in the ordinary manner.

The cells in which the failures occurred are considered to be in the creep state. It means that for these cells the parameter V_s ($V_s \geq V$) is used instead of V in equations (6) which describe the evolution of the inelastic displacement. The values of V_s can be different for different faults. A cell is in the creep state while $\kappa > H_s$ for it. When $\kappa \leq H_s$ the cell returns to the ordinary state and henceforth for this cell the parameter V is used in (6).

The parameters of an earthquake are defined as follows: time is t_i; coordinates and depth are weighted sums of coordinates and depths of the cells in which failures occurred (the weights of the cells are their squares divided by the sum of squares of these cells); magnitude is calculated as

$$M = D\lg S + E, \qquad (15)$$

where D and E are constants; S is the sum of the squares of the cells (in km^2) in which failures occurred during the earthquake.

HIERARCHY OF FAULTS

Fault features can be taken into consideration through the values of the constants K, V, V_s and the levels B, H_f, H_s.

The hierarchy of faults is controlled by the hierarchy of structures separated by them. Larger faults separate larger structures. Note that accordingly to the fault definition the lager fault does not mean the longer fault.

It seems natural that the same value of elastic displacement leads to a smaller elastic force for the larger fault than for a smaller one. Thus the value of K has to be smaller for a larger fault.

Larger faults separating larger structures are usually the more strongly fractured and less consolidated zones than smaller faults, and the same force can lead to larger slippage (inelastic displacement) for a larger fault than for a smaller one. Thus the values of V and V_s have to be larger for larger faults than for smaller ones.

The more strongly fracturing of the larger faults can be a cause that earthquakes occur in the larger faults for smaller values of the parameter κ than in the smaller ones. This can be reflected in smaller values of the levels B, H_f, H_s for the larger faults than for the smaller ones.

The qualitative arguments given above can be used as some indications for selecting the values of constants K, V, V_s and levels B, H_f, H_s if the fault hierarchy is known.

THE BLOCK STRUCTURE BASED ON THE SCHEME OF MORPHOSTRUCTURAL ZONING OF THE WESTERN ALPS

The scheme of morphostructural zoning of the Western Alps constructed by A.Gorshkov and E.Ranzman (Cisternas el al., 1985) was used as the basis for the definition of the block structure (see Figure 7).

The point with the geographic coordinates 43°N and 5°E was selected as the coordirates origin. *X* axis is the north-directed meridian coming through the coordinate origin. *Y* axis is the east-directed parallel coming through the coordinate origin. The scheme of morphostructural zoning of the Western Alps is interpreted as the scheme of the fault lines on the upper plane.

79 vertices are defined on the scheme. 17 vertices are corner vertices. They have the numbers from 1 to 17. Their coordinates (in km) are: (355, 495), (163, 402), (5, 190), (5, 60), (255, 90), (315, 142), (318, 200), (190, 240), (285, 316), (334, 310), (160, 360), (160, 340), (120, 250), (75, 205), (60, 150), (205, 150), (213, 280).

50 vertices with the numbers from 18 to 67 are end vertices. They have the following relative positions in the corresponding faults: 0.13, 0.53, 0.72, 0.15, 0.53, 0.68, 0.88, 0.26, 0.54, 0.63, 0.82, 0.16, 0.49, 0.68, 0.95, 0.43, 0.22, 0.53, 0.10, 0.75, 0.23, 0.64, 0.77, 0.50, 0.37, 0.56, 0.35, 0.45, 0.63, 0.20, 0.94, 0.25, 0.23, 0.84, 0.17, 0.33, 0.22, 0.73, 0.21, 0.55, 0.16, 0.26, 0.62, 0.70, 0.82, 0.88, 0.50, 0.36, 0.69, 0.43.

12 intersections have the numbers from 68 to 79.

Total number of faults is 42. The lists of the numbers of the

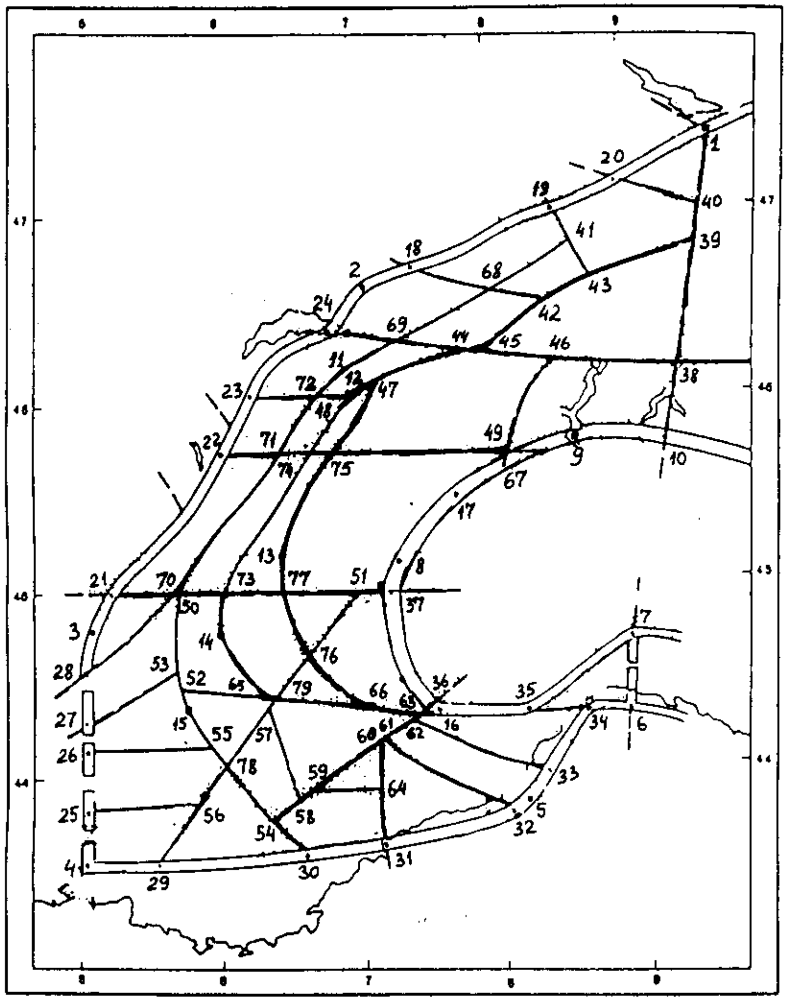

FIGURE 7. The block structure based on the scheme of morphostructural zoning of the Western Alps (the numbers of the vertices are indicated).

fault vertices in the order accordingly to the fault directions are given in Table 2. The dip angles of fault planes are also given in the table.

The number of the blocks is 38 and the number of the fault segments is 116.

The depth of the layer H = 20 km. The coefficients in formulas (1), (2), (5), and (6) have the same values for all blocks and faults:

$$K_u = K = 10^{-3}\text{Kbars/cm}, \quad V_u = V = 50 \text{ cm/Kbars}.$$

It is considered that the time t in equations (2) and (6) is a non-dimensional quantity.

Discretization is defined by the following values of the parameters ε and Δt:

$$\varepsilon = 5 \text{ км}, \quad \Delta t = 0.001.$$

The value of the difference between lithostatic and hydrostatic pressure P in formula (12) equals to 2 Kbars. The levels B, H_f, and H_s have the same values for all faults:

$$B = 0.1; \quad H_f = 0.085; \quad H_s = 0.07.$$

The constant V_s which defines the evolution of inelastic displacements during the creep state equals to $2\cdot10^6$ cm/Kbars for all faults. The earthquake magnitude is calculated with the following values of the constants in formula (15) (Utsu and Seki, 1954):

$$D = 0.98; \quad E = 3.92.$$

The square S in formula (15) is measured in square kilometers.

The movement of the boundary blocks is defined as follows. The boundary block which contains the vertices with the numbers 10, 9, 67, 17, 8, 37, 36, 16, 35, 7, and 6 and the segments between these

TABLE 2. The lists of the numbers of the fault vertices in the order accordingly to the fault directions and the angles of fault plane inclinations to the horizontal plane.

Fault	Vertices	Angle (in degrees)
1	65, 14	85
2	2, 18, 19, 20, 1	85
3	3, 21, 22, 23, 24, 2	85
4	4, 25, 26, 27, 28, 3	85
5	4, 29, 30, 31, 32, 5	85
6	5, 33, 34	85
7	6, 34, 35, 16	85
8	6, 7	85
9	7, 35	85
10	16, 36, 37, 8	60
11	8, 17	60
12	9, 10	60
13	10, 38, 39, 40, 1	85
14	40, 20	85
15	43, 41, 19	85
16	45, 42, 43, 39	85
17	42, 68, 18	85
18	24, 69, 44, 45, 46, 38	89
19	11, 69, 68, 41	85
20	28, 70, 71, 72, 11	85
21	12, 47, 44	85
22	14, 73, 74, 48, 12	75
23	23, 72, 48	85
24	67, 49, 46	85
25	22, 71, 74, 75, 49	85
26	13, 75, 47	60
27	66, 76, 77, 13	45
28	21, 70, 50, 73, 77, 51, 37	85
29	15, 52, 53, 50	85
30	27, 53	85
31	30, 54, 78, 55, 15	85
32	26, 55	85
33	25, 56	85
34	29, 56, 78, 57, 79, 76, 51	85
35	58, 57	85
36	54, 58, 59, 60, 61, 62, 63, 36	85
37	31, 64, 60	85
38	59, 64	85
39	32, 61	85
40	33, 62	85
41	52, 65, 79, 66, 63	85
42	17, 67, 9	60

vertices moves progressively with the components of the velocity -10 cm and 7.5 cm in one unit of time along X and Y axes respectively and rotates around the coordinate origin with the angle velocity $5 \cdot 10^{-6}$ radians in one unit of time. The other boundary blocks do not move.

The earthquake catalog obtained with zero initial conditions contains 26689 events in the time segment of 100 units. Magnitudes of these earthquakes are between 4.55 and 6.60. The graph of dependence of the accumulated number of earthquakes on the magnitude is represented in Figure 8.

The earthquakes occurred in the all cells of the following fault segments (the numbers of the vertices which limit the segment): (32, 5), (6, 34), (34, 35), (35, 16), (63, 36), (16, 36), (36, 37), (37, 8), (8, 17), (17, 67), (67, 49), (67, 9), (9, 10), (7, 35). The earthquakes occurred in some cells of the following fault segments: (45, 46), (6, 7), (33, 34), (31, 32), (32, 61), (60, 61). These cells adjoin to the vertices with the numbers: 45, 7, 34, 32, 61.

Irregularity in the earthquake distribution gives possibility to hope that the indication of the boundary block movement well approximating their real movement could cause the earthquake distribution being close to their real space distribution in the Western Alps. The values of constants P, K_u, V_u, K, V, and V_s and levels B, H_f, and H_s have to be close to reality and to reflect the hierarchy of the faults. The correction of the block structure geometry can also need.

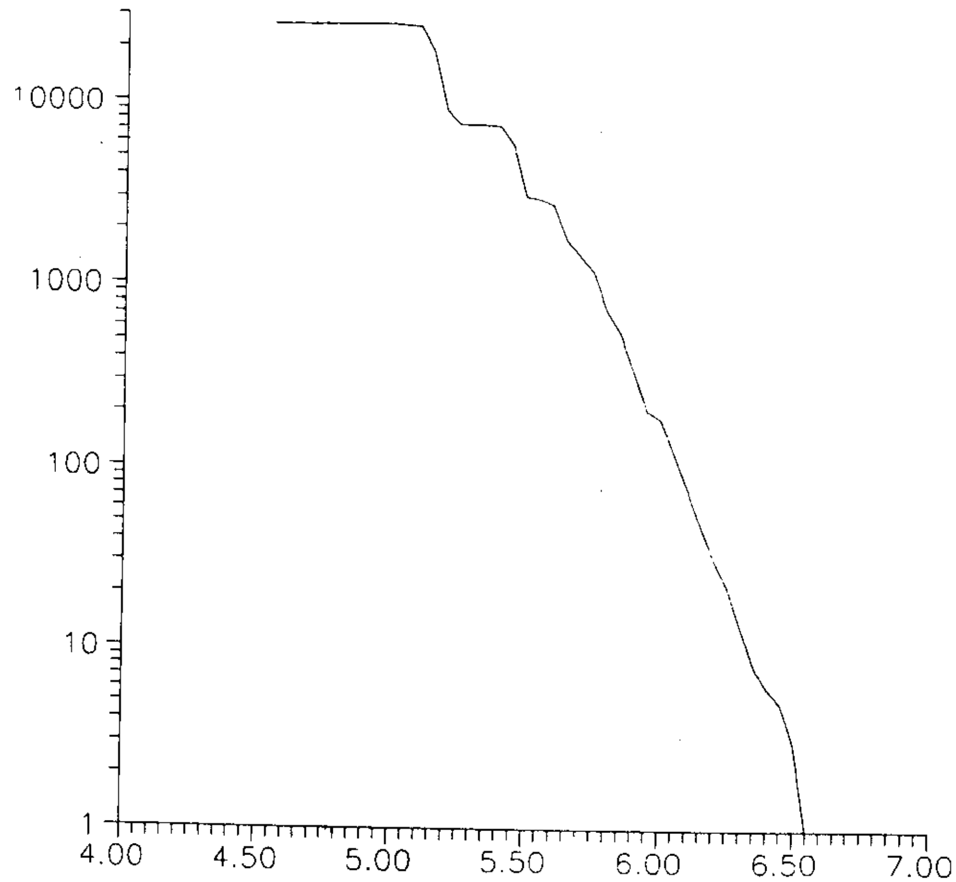

FIGURE 8. The graph of dependence of the accumulated number of earthquakes on the magnitude for the synthetic catalog.

APPLICATION OF M8 ALGORITHM TO THE SYNTHETIC CATALOG

The synthetic earthquake catalog obtained was used as the initial data for the intermediate-term earthquake prediction algorithm **M8** (Keilis-Borok and Kossobokov, 1990).

Before the algorithm application the magnitude transformation was made by the formula

$M' = 2M - 6.5$

where M is the catalog magnitude; M' is the magnitude used for the algorithm application. This transformation is explained by the fact that the inclination of the graph in Figure 8 is too steep. The values of the magnitude M' are from 2.70 to 6.66.

One unit of the non-dimensional time is interpreted as one year. The time of the initial moment is 0 h 0 m of January 1, 1900. In consequence the date of the first event in the catalog is December 29, 1901, th date of the last event is December 6, 1999.

Identification of main shocks and aftershocks was made by Gardner-Knopoff algorithm (Gardner and Knopoff, 1974). The catalog of the main shocks contains 2184 events. Statistical analysis of this catalog has given arguments to consider the earthquakes with $M' \geq 6,5$ as strong ones.

Adaptation of **M8** algorithm was made by using the initial time segment of the catalog (before July 1, 1915) and the standard set of values of its parameters in the circle with the center (45°N, 8°E) and radius 192 km. The data after 1915 were not used for the algorithm adaptation and were examined.

The results of the algorithm application are the following.

There is the only alarm period from July 1, 1913 to July 2,

1918. This period contains the strong earthquake with $M' = 6.66$ which occurred at August 10, 1915 and the two events with $M' = 6.06$ and $M' = 6.42$ which occurred less than one day before the strong one. Two other strong earthquakes occurred in 1904, i.e. during the initial part of the catalog when not all values of the functions used by the algorithm for the diagnosis of alarm periods are defined. After the end of the alarm period indicated there are not another alarm periods and there are not strong earthquakes, i.e. all earthquakes in the catalog after July 2, 1918 have $M' < 6.50$.

Thus the result of the application of **M8** algorithm to the synthetic catalog can be considered as satisfactory. But the lack of strong earthquakes in the catalog does not permit to estimate the reliability of the result obtained.

REFERENCES

Alekseevskaya,M.A., A.M.Gabrielov, A.D.Gvishiani, I.M.Gelfand, and E.Ya.Ranzman (1977) Formal morphostructural zoning of mountain territories, *J. Geophys*, 43: 227-233.

Cisternas,A., P.Godefroy, A.Gvishiani, A.I.Gorshkov, V.Kosobokov, M.Lambert, E.Ranzman, J.Sallantin, H.Saldano, A.Soloviev, and C.Weber (1985). A dual approach to recognition of earthquake prone areas in the western Alps, *Annales Geophysicae*, 3, 2: 249-270.

Gabrielov,A.M., T.A.Levshina, and I.M.Rotwain (1990). Block model of earthquake sequence, *Phys. Earth and Planet. Inter.*, 61: 18-28.

Gardner,J.K. and L.Knopoff (1974). Is the sequence of earthquakes in Southern California, with aftershocks removed, Poissonian?, *Bull. Seismol. Soc. Amer.*, 64: 1363-1367.

Keilis-Borok,V.I. and V.G.Kossobokov (1990). Premonitory activation of seismic flow: algorithm M8, *Phys. Earth and Planet. Inter.*, 61: 73-83.

Utsu,T. and A.Seki (1954). A relation between the area of aftershock region and the energy of main shock, *J. Seism. Soc. Japan*, 7: 233-240.