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Functions on Earthquake Flow

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FUNCTIONS ON EARTHQUAKE FLOW

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EARTHQUAKE FLOW

The subject of the lecture is a description of the earthquake flow. This description was particularly used in the intermediate-term earthquake prediction studies. These studies are based on seismicity data - earthquake catalogs. In the catalogs an earthquake is described by the origin time, hypocenter coordinates (latitude, longitude and depth), and energy or magnitude. The earthquake flow is a sequence of earthquakes. The intermediate-term earthquake prediction studies show that the sequence of earthquakes contains information about a coming strong earthquake. However, there are the following difficulties.

(i) The existing theory is not sufficiently enough for data analysis. Seismology has no adequate model for occurrence of earthquakes. That is why the description of an earthquake flow will be based on empirical data. Space, time and energy will be considered independently.

(ii) The earthquake flow includes a large chaotic component. Therefore, symptoms of the incipient strong earthquake can be different from time to time and even more so in different regions. This situation calls for integral descriptions. The earthquake flow is described in an area which is defined by regionalization of the territory or regular scanning by square or circles. The earthquake sequence is estimated in a sliding time window and within certain magnitude ranges.

The scheme of definition is outlined in Figure 1. It shows an earthquake sequence in an area. The vertical axis indicates the magnitude of an earthquake. The horizontal axis is a time axis.

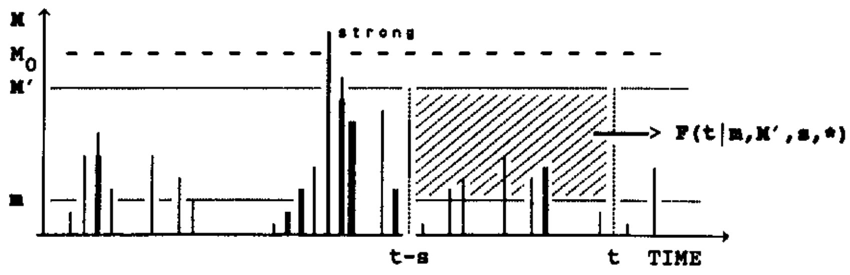


FIGURE 1

The vertical dashed line indicates a sliding moment of time. At each moment we look back in time and count functions which represent some traits of the earthquake sequence.

What traits will be represented? The list of these traits is the following:

- LEVEL OF SEISMIC ACTIVITY
- VARIATION OF SEISMIC ACTIVITY IN TIME
- CLUSTERING IN SPACE AND TIME
- LONG-RANGE INTERACTION
- SPATIAL CONCENTRATION

Why were these traits selected?

-- The intensity of an earthquake flow was chosen because abnormal activation and quiescence (separately or in combination) were reported before the occurrence of many strong earthquakes; for example - different kinds of seismic gap.

-- The next three traits represent the phenomenon common for many non-linear systems. When instability is coming the fluctuation increases as well as the response to excitation. The response may be expressed in fluctuation of activity, in clustering and in increase of distance within which the earthquakes are interdependent.

-- The last one, concentration, is suggested by laboratory experiments with rocks: when the density of microstructure exceeds some rather universal threshold, the failure of the whole sample occurs (Zhurkov et al., 1978).

Certainly, this list of traits is not complete.

FUNCTIONS

These traits are represented by several not independent functions. One trait can be represented by different functions, or the function with different parameters (different time-window or different range of magnitude). Aftershocks are eliminated, so that earthquake sequence would not be dominated by relatively strong earthquakes. The number of aftershocks is used as one of the parameters of main shocks.

The following functions represent *the level of seismic activity*:

-- The number of earthquakes with magnitude greater than or equal to some threshold (m) calculated for the time interval $(t - s,\ t)$:

$$N(t|m,s)\ .$$

-- The weighted number of earthquakes with magnitudes M_i $m \le M_i \le M'$ calculated for the time interval $(t - s,\ t)$:

$$\Sigma(t|m,M',s,\alpha,\beta) = \sum_i 10^{\beta(M_i - \alpha)}\ .$$

This function can have different sense. If β is about $B/3$ where B is the coefficient in the relation $\lg E = A + BM$ between the energy E and the magnitude M of an earthquake, then $\Sigma(t|m,M',s,\alpha,\beta)$ is

proportional to the linear size of the earthquake sources. If β is about $2B/3$, then the function is proportional to the total area of the sources, and if β is about B, it is proportional to the energy released.

--The next function (Figure 2) is the ratio of the number of earthquakes with magnitudes $m_1 \le M < m_2$ to the number of earthquakes with $M \ge m_1$ $(m_1 \le m_2)$ calculated for the time interval $(t - s,\ t)$:

$$G(t|m_1,m_2,s) = 1 - \frac{N(t|m_2,s)}{N(t|m_1,s)}\ .$$

-- The next function represents the deficiency of activity i.e. quiescence:

$$q(t|m,s) = \sum_i{}_{+}[sa(m) - N(t_i|m,s)]\ .$$

Here a(m) is an average annual number of earthquakes. $\sum_+$ denotes the sum of the positive terms only, i.e. the sliding time intervals $(t_i - s,\ t_i)$, for which the number of earthquakes is less than the average, are only considered (Figure 3). The horizontal dot line in the figure indicates the average number $\overline{N} = sa(m)$ of earthquakes with $M \ge m$ for s years.

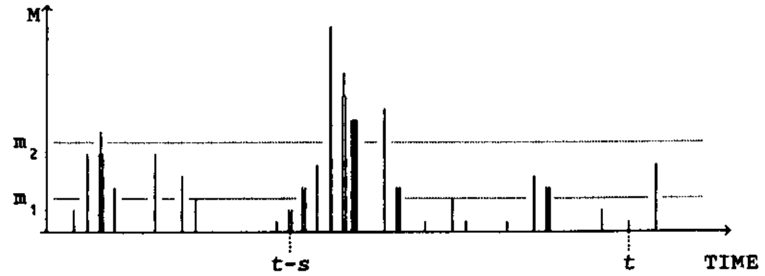


FIGURE 2

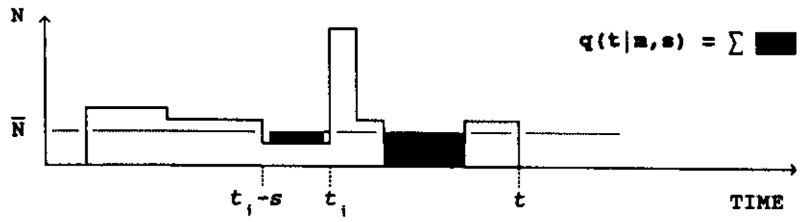


FIGURE 3

The following functions represent *variation of seismic activity*.

-- Variation of the number of earthquakes (Figure 4). That is a sum of differences between the numbers of earthquakes for two successive time intervals $(t_i - s,\ t_i)$ and $(t_{i+1} - s,\ t_{i+1})$ where the moments t_i belong to the time interval $(t - u,\ t)$:

$$V(t|m,s,u) = Var\ N(t|m,s) = \sum_i |N(t_{i+1}|m,s) - N(t_i|m,s)| \ .$$

-- The next function is "drop-and-increase" in activity. It is the same function as the previous one, but variation of $N(t)$ is counted between t and the previous time of the local maximum of $N(t)$:

$$Q(t|m,s) = Var\ N(t|m,s) = \sum_i |N(t_{i+1}|m,s) - N(t_i|m,s)| \ .$$

Here the moments t_i belong to the time interval $(t_{max},\ t)$ where t_{max} is the last moment of the local maximum of $N(t)$ before t.

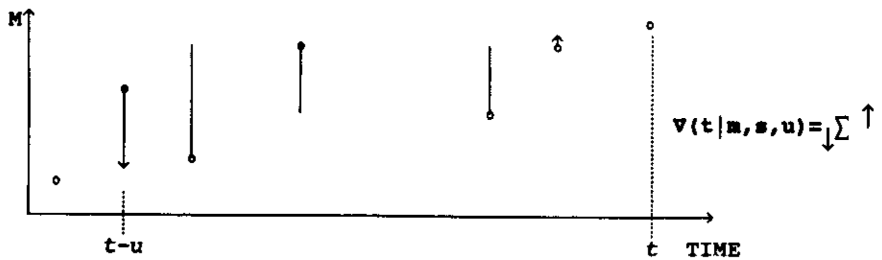


FIGURE 4

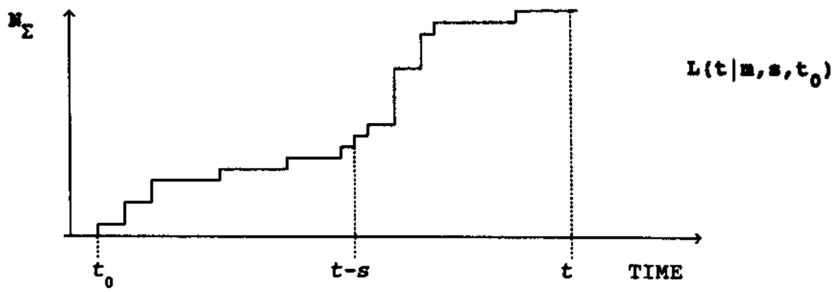


FIGURE 5

-- The next function (Figure 5) is deviation from long-term trend of activity in the period from t_0 to t. Usually t_0 is the beginning of a catalog:

$$L(t|m,s) = N(t|m,t-t_0) - N(t-s|m,t-t_0-s)\ \frac{t - t_0}{t - t_0 - s} \ .$$

The second term is a linear extrapolation of $N(t)$ from $(t - s)$ to t.

-- The next function (Figure 6) represents the difference of activity, that is, the difference between the number of earthquakes for two successive time intervals $(t - s,\ t)$ and $(t - 2s,\ t - s)$:

$$K(t|m,s) = N(t|m,s) - N(t-s|m,s) \ .$$

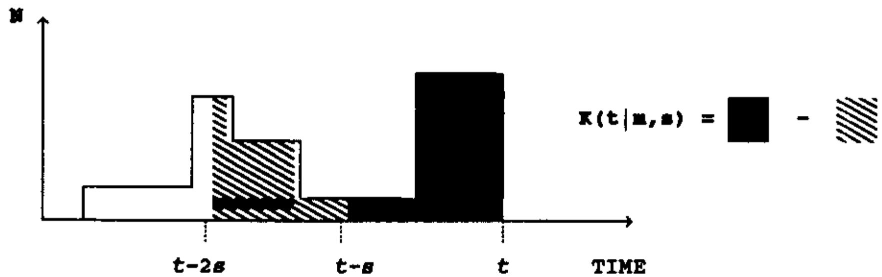


FIGURE 6

The next trait of the earthquake flow is *clustering of earthquakes in space and time*. Only one function represents this trait. For each main shock the number $B(e,M_a)$ of aftershocks with magnitude $M \geq M_a$ in the first e days after the main shock is calculated. The measure of clustering is the maximal $B(e,M_a)$ for main shocks with magnitude $m \leq M \leq M'$ from the time interval $(t - s,\ t)$:

$$B_{max}(t|m,M',s,M_a,e) = \max B(e,M_a) \ .$$

Many other measures of clustering have been described in the literature. We have chosen the number of aftershocks because to our knowledge it is so far the only precursor for which statistical significance has been established (Molchan et al., 1988).

The long-range interaction of earthquakes can be characterized by two phenomena. One was suggested by Prozorov (1982). A strong earthquake is usually followed by some activation in the area where the next strong earthquake is expected. The earthquakes which represent the activation are called long-range aftershocks. Their maximal magnitude is one of the measures of long-range interaction $M_l(t|s,M_0,u)$. Here M_0 is the threshold magnitude for definition of strong earthquakes; the long-range aftershocks are identified for u years after a strong earthquake.

Another long-range interaction phenomena is the simultaneous activation of several regions belonging to the same fault zone or to the same morphostructure of a higher rank. Accordingly, the following two functions are counted:

$N_F(t|m,s)$ is the number of earthquakes for the whole fault zone;

$N_R(t|m,s)$ is the number of earthquakes for the morphostructure of a higher rank which includes the region under consideration.

The spatial concentration is represented by two functions.

The first one is roughly proportional to the average area of fracture in the source. The maximum of this area within u years is considered:

$$Smax(t|m,M',s,u,\alpha,\beta) = \max_{[t-u,t]} \frac{\Sigma(t|m,M',s,\alpha,\beta)}{N(t|m,s)-N(t|M',s)}$$

where $\beta = 2B/3$.

The second one is the ratio of average linear dimension of a fracture to average distance between fractures. The maximum of these values within u years is considered:

$$Zmax(t|m,M',s,u,\alpha,\beta) = \max_{[t-u,t]} \frac{\Sigma(t|m,M',s,\alpha,\beta)}{[N(t|m,s)-N(t|M',s)]^{2/3}}$$

where $\beta = B/3$.

The last function represents *simultaneous quiescence and activation in adjacent areas*. An area is in the quiescence state, if the number of earthquakes for s years is less than some threshold N_q and it is in activation state, if the number of earthquakes is greater than the other threshold N_a (Figure 7). The contrast of activity $T_{aq}(t|m,s,p)$ is measured by the time elapsed since the end

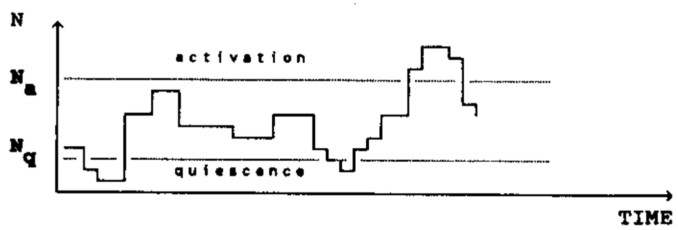


FIGURE 7

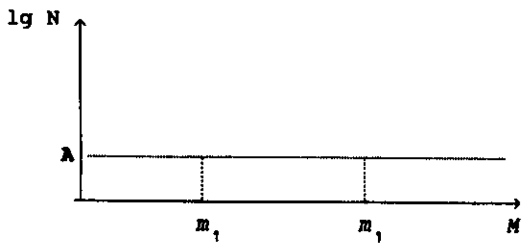
of the last time period of s years during which the area and its neighbour were in different states.

NORMALIZATION OF FUNCTIONS

The number of strong earthquakes, occurred in a single region is rather small. Thus the data analysis has to be accumulated for different regions. But there is different seismicity in different regions. That is why all functions are normalized so that their definitions to be applied uniformly to regions of different sizes and seismicity levels. Normalization is achieved by the choice of the magnitude range.

-- If in a definition of a function the earthquakes are considered with equal weights independent of their magnitudes, then the threshold for M is defined by the condition: the average annual number of earthquakes with magnitude greater than the threshold equals to the same constant for all regions. This normalization is illustrated in Figure 8.

-- If the weights depend on magnitude, the thresholds for M depend on the threshold magnitude M_0 for strong earthquakes (Figure 9).



$a(m) = A$

FIGURE 8

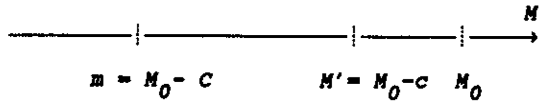


FIGURE 9

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The list of functions on an earthquake flow given above and the traits presented by them are neither complete nor universal. Other functions or other interpretations of the traits can be used to define the state of an earthquake flow in a region.

Note that the functions on an earthquake flow can also be used to analyse the features of synthetic catalogs obtained in models and to compare synthetic catalogs with real ones.

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