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Workshop on Fluid Mechanics

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Core mantle coupling

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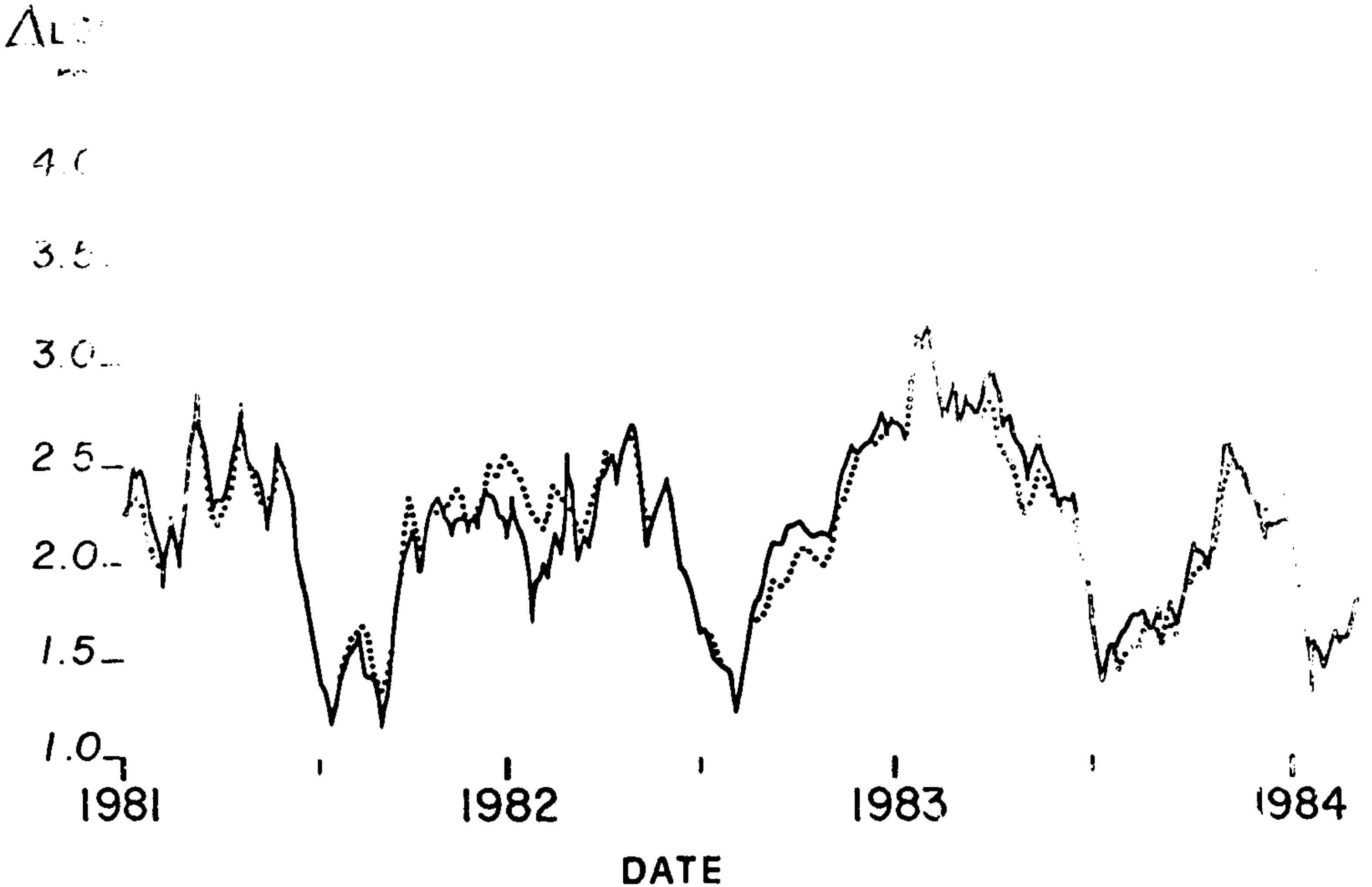
These are preliminary lecture notes, intended only for distribution to participants

Core - mantle interaction.

A. Observations.

- Decade variations of the length of the day.
- Decade variations of the mean pole position (Markowitz wobble)



4.0
3.5
3.0
2.5
2.0
1.5
1.0
1981
1982
1983
1984
DATE

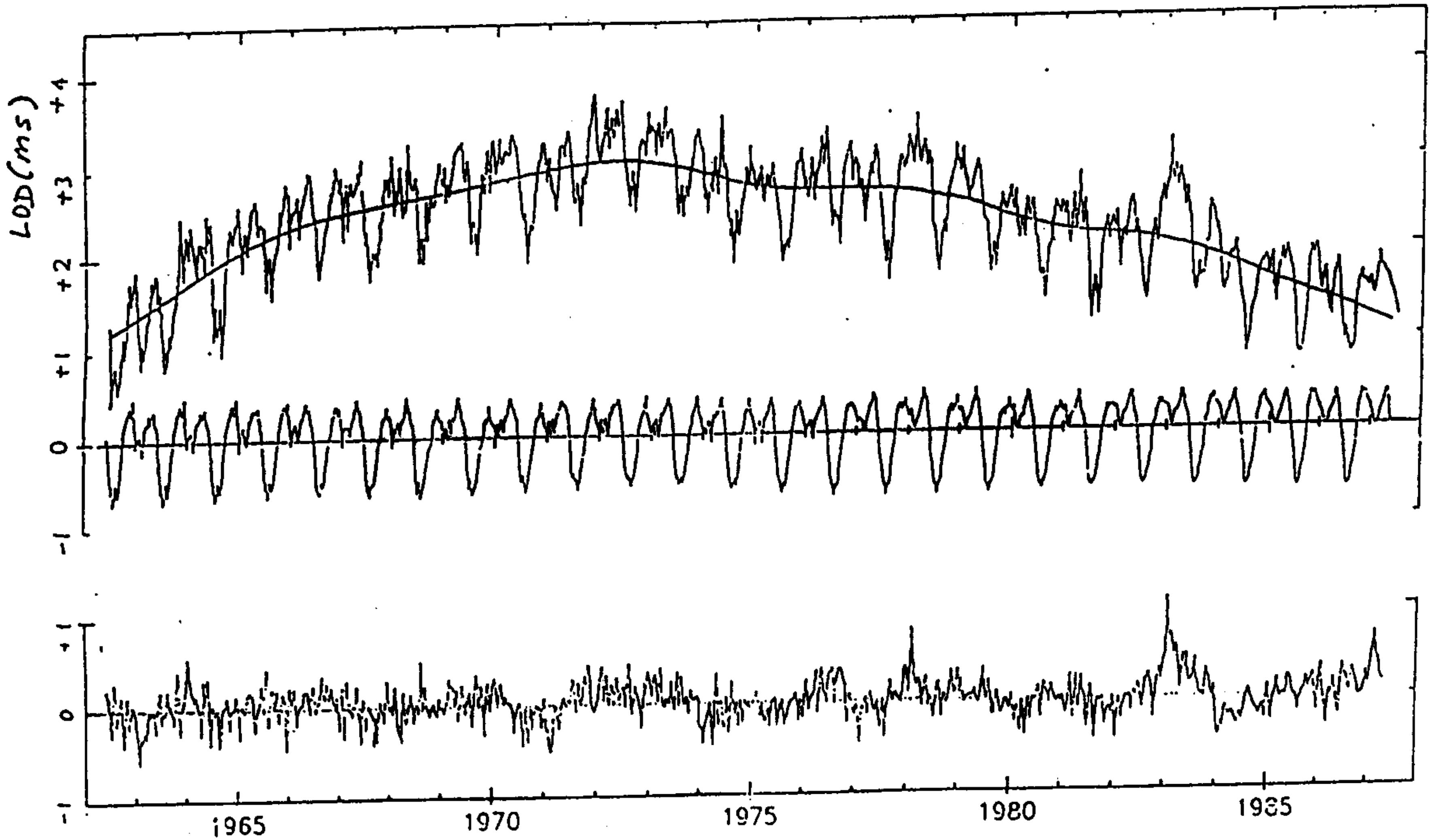
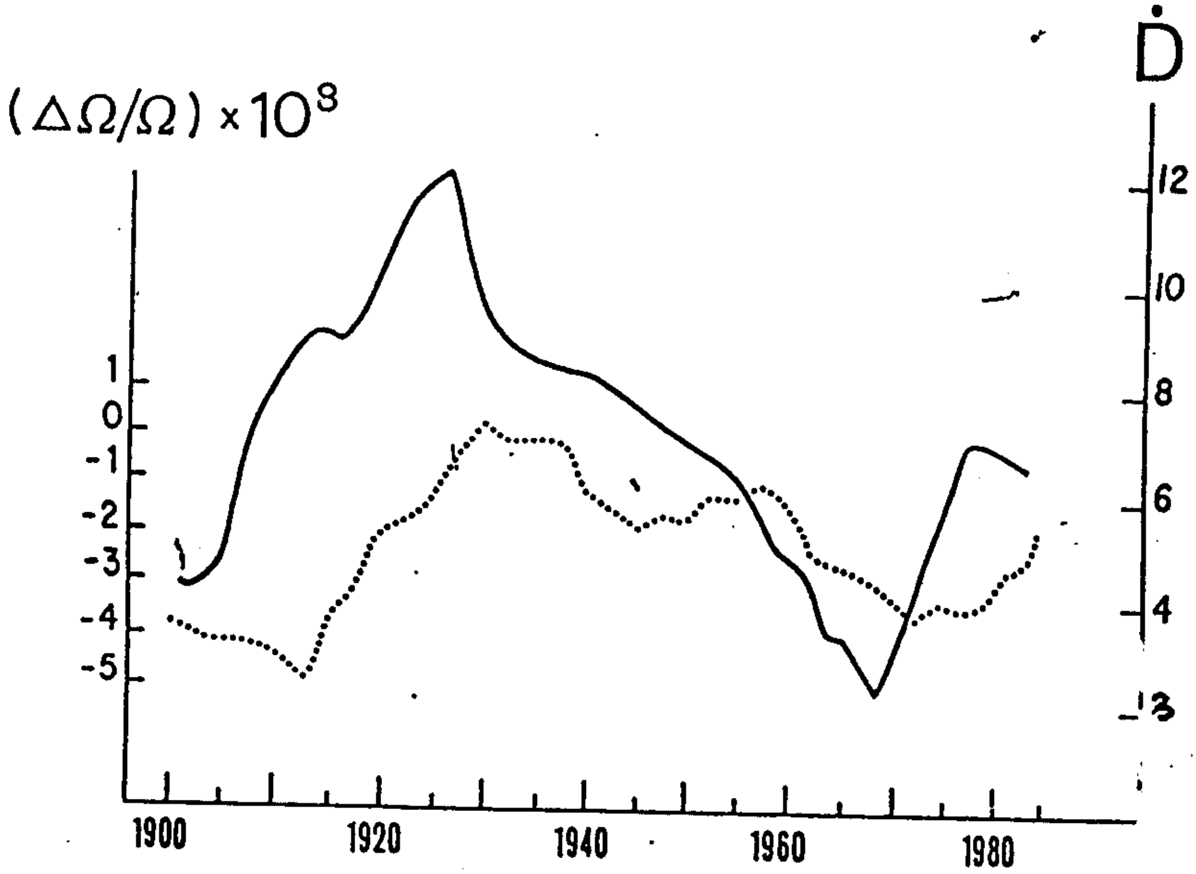
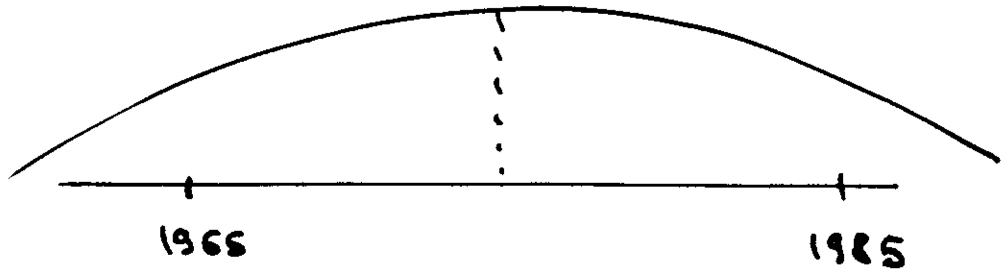


Figure 1: écart à 86400s de la longueur du jour, corrigé de l'effet des marées zonales; décomposition en une tendance et des termes saisonniers et irréguliers (Feissel et Gavoret, 1987).



(ΔΩ/Ω) x 10^8
Ḋ
1
0
-1
-2
-3
-4
-5
12
10
8
6
4
3
1900
1920
1940
1960
1980

Order of magnitude of the axial torque



$$T = T(1973) - 10^{-2}(t-1973)^2$$

T ms

t years

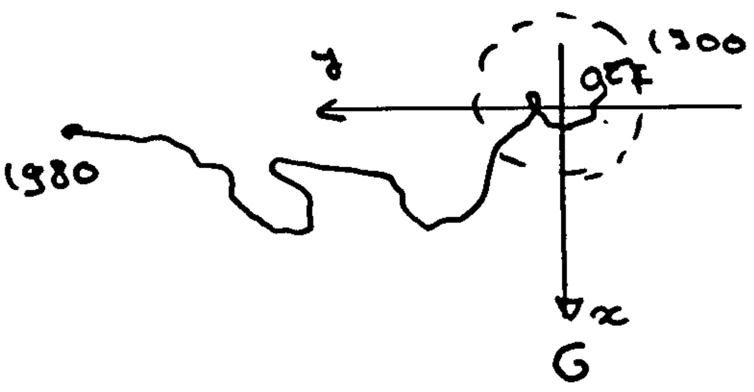
$$\Gamma_{ax} = C^m \frac{d\Omega}{dt} = -C^m \frac{2\pi}{T^2}\frac{dT}{dt}$$

$$\sim 4\,10^{16}(t-1972)\ \mathrm{N.m}$$

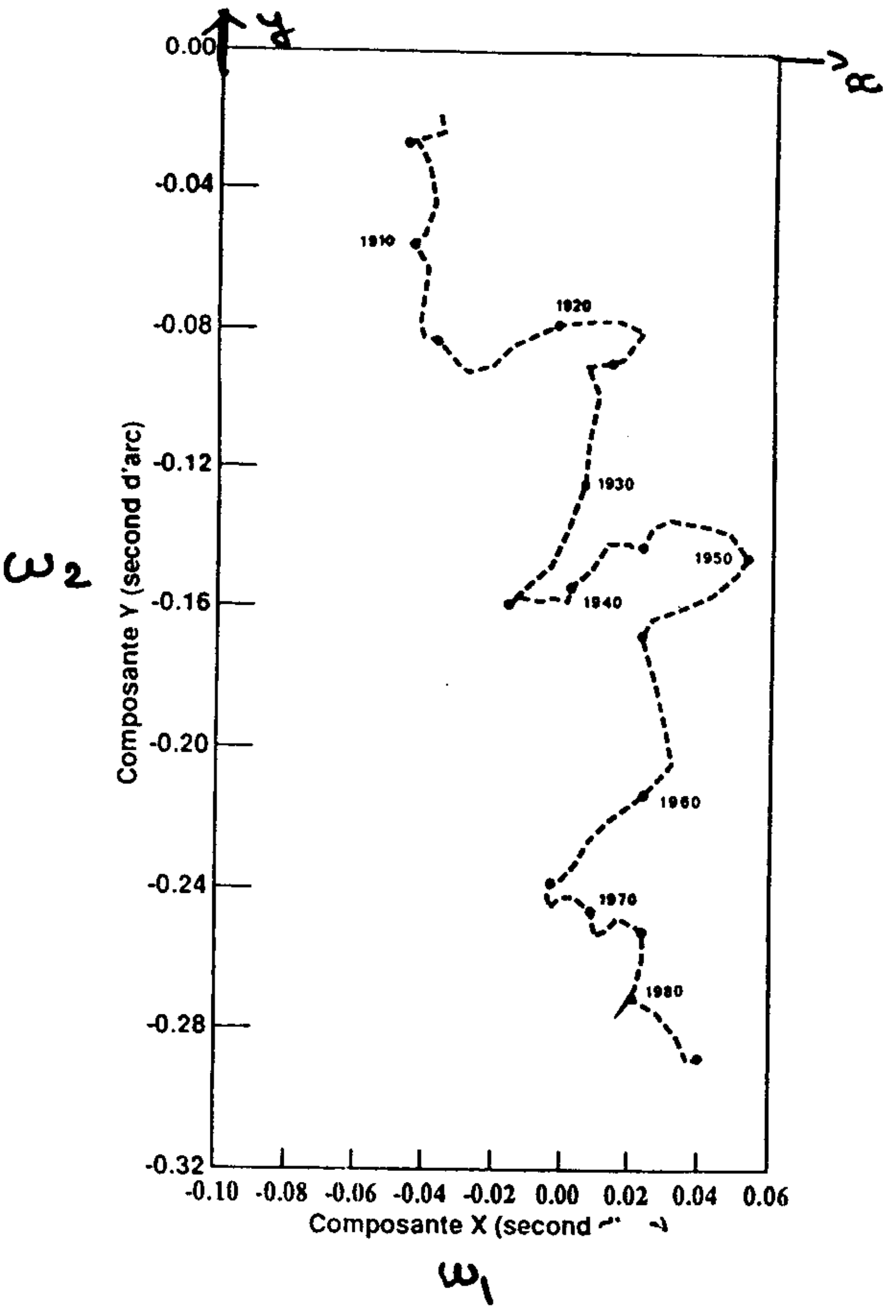
Γ is increasing of $4\,10^{16}$ N.m per year since 1972

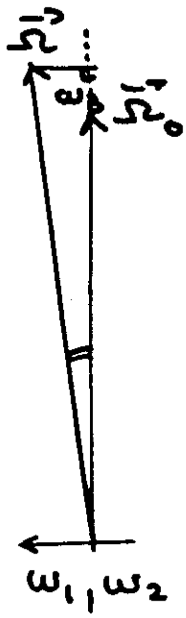
$\underline{\Gamma_{1960} \sim 10^{17}\ \mathrm{N.m}}$

Since 1840 $\underline{\Gamma < 10^{18}\ \mathrm{N.m}}$



Polhodie 1903 - 1985





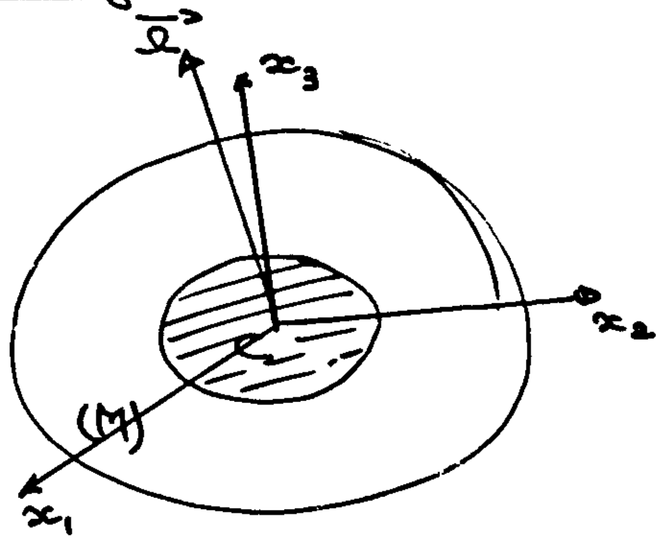
$\left\{\begin{array}{l}\omega_1, \omega_2. \\ \sim 10^{-6}\Omega_0\end{array}\right.$

$\omega_3 < \sim 10^{-7}\Omega_0$

ω_1, ω_2 measured in arcsec.

1 arcsec $\sim 5\,10^{-6}$ rad.

3. Torque budget



We will first suppose the mantle rigid.

Ox_1, x_2, x_3 : principal axes of the mantle

$$\bar{\bar{C}} = \begin{vmatrix} A^m & 0 & 0 \\ 0 & A^m & 0 \\ 0 & 0 & C^m \end{vmatrix} + c_{ij}$$

$\vec{\Omega}$: mantle rotation $\quad \Omega \sim 7.\ 10^{-5}$

— No motion in the core $\quad \vec{\Omega} = \vec{\Omega}_0 = \Omega_0 \hat{z}$
$= \Omega_0 \hat{x_3}$

— Motions in the core:

$$\vec{\Omega} = \vec{\Omega}_0 + \vec{\omega} \quad \begin{cases} \omega_1 \\ \omega_2 \\ \Omega_0 + \omega_3 \end{cases} \qquad \vec{\omega}(t)$$

$$\omega \ll \Omega_0 \quad , \quad \omega \sim 10^{-6}\, \Omega_0$$

1) Navier - Stokes equation.

$$\rho \frac{d\vec{u}}{dt} = \vec{F} - 2\rho \vec{\Omega} \wedge \vec{u} - \rho \dot{\vec{\omega}} \wedge \vec{r} - \rho \left(\vec{\Omega} \wedge (\vec{\Omega} \wedge \vec{r})\right) \qquad (1)$$

In the absence of motion ($\vec{u} = 0$, $\vec{\omega} = 0$) we get the hydrostatic solution in the core:

$$\vec{\nabla} P_h = \rho_h \vec{\nabla} U_h - \rho_h \left(\vec{\Omega}_0 \wedge (\vec{\Omega}_0 \wedge \vec{r})\right) \qquad (2)$$

$$= \rho_h \vec{\nabla} (U_h + \psi_0) \qquad \psi_0 = \frac{\Omega^2}{2} r^2 \sin^2\theta$$

Linearizing (1) - (2), Boussinesq approximation:

$$\rho \frac{d\vec{u}}{dt} = \vec{F}_{el} + \vec{F}_v + \rho_h \vec{\nabla}(U_f + \psi_f) + \rho_f \vec{\nabla}(U_h + \psi_0)$$
$$- \vec{\nabla} P_f - 2\rho_h (\vec{\Omega}_0 \wedge \vec{u}) - \rho_h \dot{\vec{\omega}} \wedge \vec{r} \qquad (3)$$

$$\rho_f \sim 10^{-9} \rho_h \qquad \begin{cases} \nabla^2 U_f = -4\pi G \rho_f \\ \vec{\nabla}\psi_f = \vec{\Omega} \wedge (\vec{\Omega} \wedge \vec{r}) - \vec{\Omega}_0 \wedge (\vec{\Omega}_0 \wedge \vec{r}) \end{cases}$$

Slow motions.

$$T \gg 1 \text{ day}$$

$$\vec{\nabla} P_f + 2\rho \vec{\Omega}_0 \wedge \vec{u} = F_{el,v} + \rho_h \vec{\nabla}(U_f + \psi_f) + \rho_f \vec{\nabla}(U_h + \psi_h)$$

At the core surface.

$$F_{el} \sim 0 \qquad F_v \sim 0 \qquad \text{(main stream)}$$

$$\rho_h \vec{\nabla}(U_g + \psi_g) + \rho_g \vec{\nabla}(U_h + \psi_h)$$

$$= \vec{\nabla}(\rho_h (U_g + \psi_g)) + \underbrace{\rho_g \vec{\nabla}(U_h + \psi_h) - (U_g + \psi_g)\vec{\nabla}\rho_h}$$

$\sim$ radial
horizontal part of
the order of $\rho_g (\hat{n} \cdot \hat{r})$
$= O(\varepsilon^2)$

It comes

$$\nabla P_g + 2\rho\, \Omega_0 \wedge U - \nabla \rho_h (U_g + \psi_g) = A\, \hat{e}_r$$

$$\boxed{\nabla (P_g - \rho_h (U_g + \psi_g)) + 2\rho(\vec{\Omega} \wedge \vec{u}) = A\, \hat{e}_r} \qquad (5)$$

Comparing with the former equation (3)

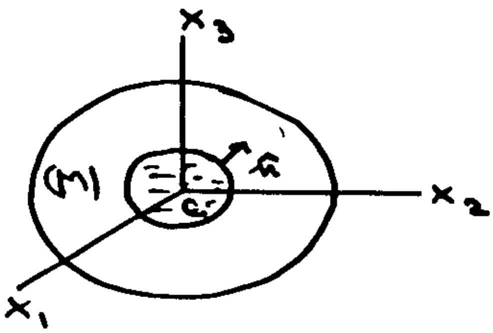
$$\vec{\nabla} P_{geo} + 2\rho\, \vec{\Omega} \wedge \vec{u} = -\rho_1 g \hat{n}$$

$\hat{n} = -\hat{e}_r$
in this approximation

It comes that the pressure determined at the core surface, P_{geo}, when inverting S.V. data, is not P_g but:

$$\boxed{P_{geo} = P_g - \rho(U_g + \psi_g)} \qquad (6)$$

2) Expression of the acting torques.



$$\vec{\Gamma}_M = \vec{\Gamma}_p + \vec{\Gamma}_g = \iint_{CMB} P_f (\vec{r} \wedge \vec{n})\, dS + \iiint_{(M)} \vec{r} \wedge \rho_R \vec{\nabla} U_f \, dV$$

$\Rightarrow$ (from (6)):

$$\vec{\Gamma}_M = \vec{\Gamma}_{geo} + \iiint_{Terre} \vec{r} \wedge \rho_R \vec{\nabla} U_f \, dV + \iiint_{core} \vec{r} \wedge \rho_R \vec{\nabla} \varphi_f \, dV$$

$$+ \iiint_{core} (U_f + \varphi_f)\, \vec{r} \wedge \vec{\nabla} \rho_R \, dV$$

Action and reaction principle $\longrightarrow$

$$\iiint_{Earth} \vec{r} \wedge (\rho_R \vec{\nabla} U_f + \rho_f \vec{\nabla} U_R)\, dV = 0$$

$\rho_f = 0$ in (M)

$\longrightarrow$

$$\Gamma_M = \Gamma_{geo} + \iiint_{core} \vec{r} \wedge \left((U_F + \varphi_F) \vec{\nabla} \rho_R - \rho_f \vec{\nabla} U_R \right) dV$$

$$+ \iiint_{core} \vec{r} \wedge \rho_R \vec{\nabla} \varphi_f \, dV$$

$$\vec{\Gamma}_M = \vec{\Gamma}_{geo} + \iiint_{core} \rho_R \vec{r} \wedge \nabla \psi_f \, dV + \Gamma_{rest} \quad (7)$$

3) Angular momentum equation.

$$\vec{H}^m = \bar{\bar{C}} \, \vec{\Omega} \qquad \bar{\bar{C}} = \begin{vmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & C \end{vmatrix}$$

$$\left(\frac{d\vec{H}^m}{dt}\right)_r + \vec{\Omega} \wedge \vec{H}^m = \vec{\Gamma}_M$$

$$\bar{\bar{C}} \, \dot{\vec{\omega}} + (C^m - A^m) \vec{\Omega} \wedge \vec{\omega} = \vec{\Gamma}_M$$

But, in (7):

$$\iiint_{core} (\vec{r} \wedge \rho_R \vec{\nabla} \psi_f) \, dV = (C^c - A^c) \vec{\Omega} \wedge \vec{\omega}$$

Finally:

$$\boxed{\bar{\bar{C}}^m \dot{\vec{\omega}} - (C - A) \vec{\Omega}_0 \wedge \vec{\omega} = \Gamma_{geo} + \Gamma_{rest}} \quad (8)$$

C, A: whole Earth

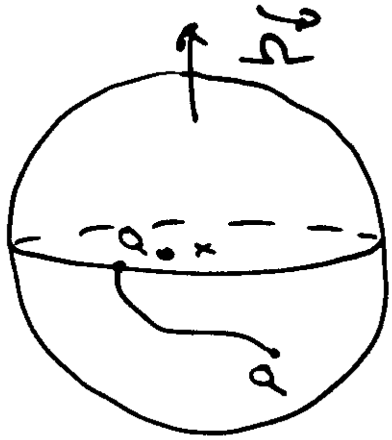
$$\Gamma_{rest} = \iiint_{core} \vec{r} \wedge [(U_f + \psi_f) \vec{\nabla} \rho_R - \rho_f \vec{\nabla} U_R] \, dV \quad (9)$$

$$O[f \times (\hat{n} - \hat{F})]$$

4) Computation of Γ_{geo}

Formula (9) of part I:

$$\vec{\nabla}_H p = -2\rho(\vec{\Omega} \wedge \vec{u})_H \qquad (10)$$



$\vec{u}$ known

p = cte on the equator : $(\vec{\Omega} \wedge \vec{u})_H = 0$

we can take $p(\varphi, \frac{\pi}{2}) = 0$

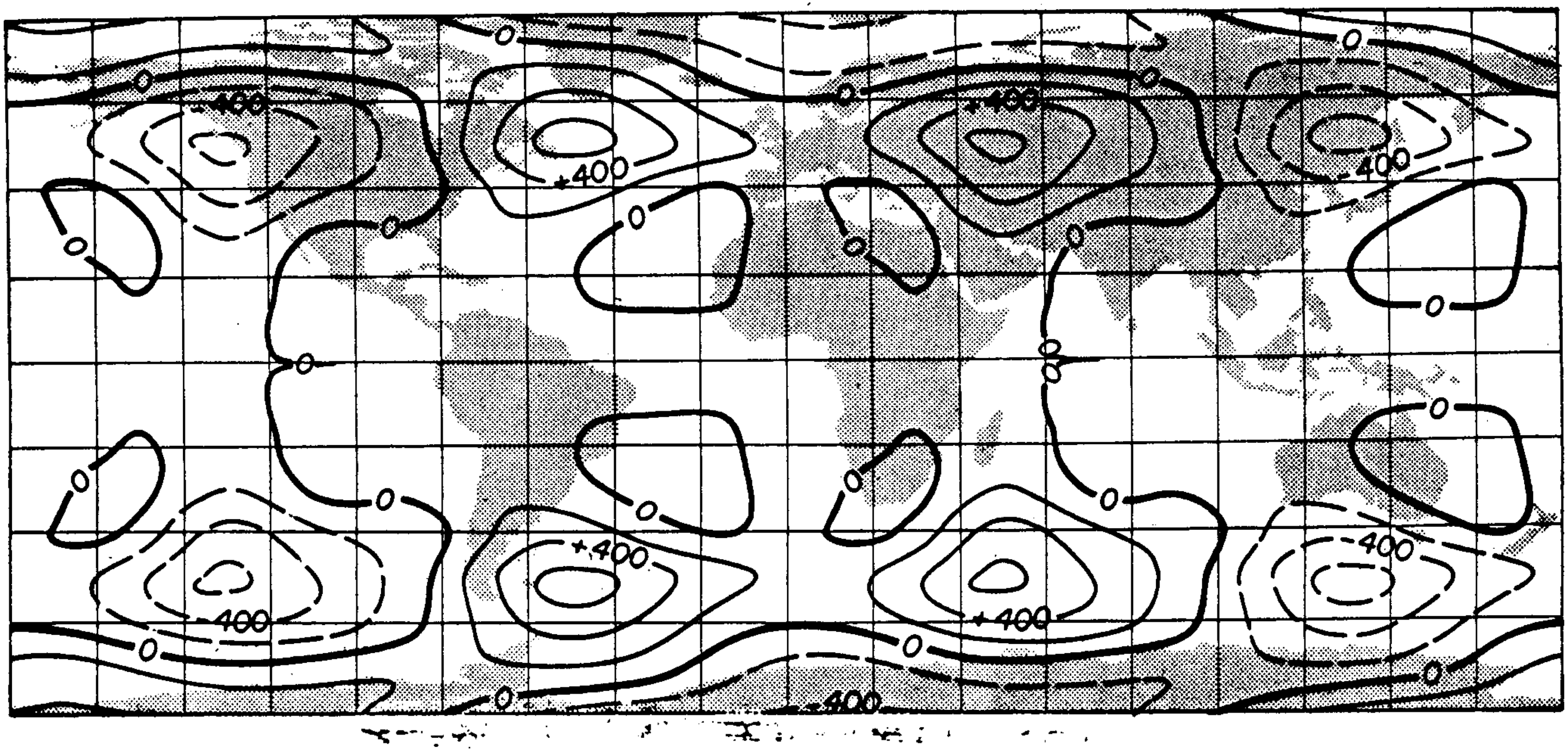
Then (10) gives

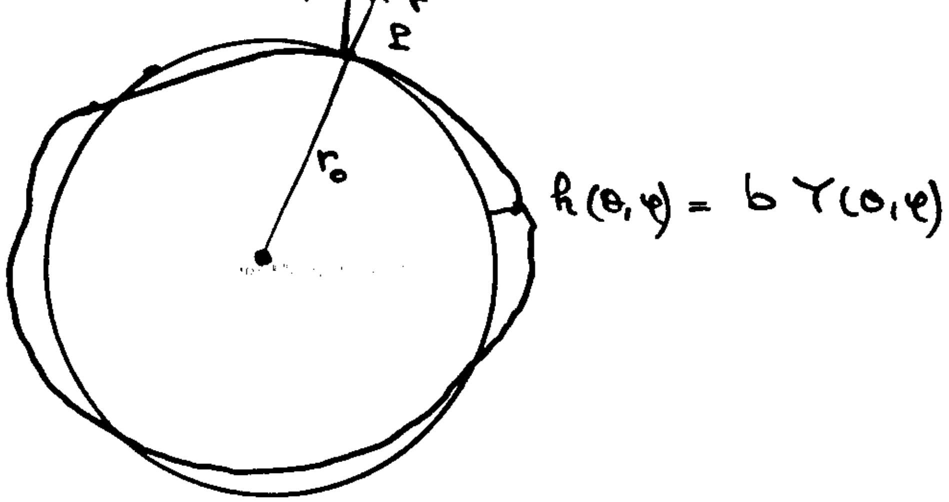
$$\boxed{p(a) = -\int_{a_0}^{a} 2\rho(\vec{\Omega} \wedge \vec{u})_H \cdot \vec{dl}} \qquad (11)$$

$$p = P_{geo} = \sum_{n,m} \left[p_n^m Y_n^{mc}(\theta, \varphi) + q_n^m Y_n^{ms}(\theta, \varphi) \right]$$

$$\Gamma_{geo} = \iint_{CMB} \vec{r} \wedge p \hat{n} \, dS$$

Contour interval 200 Pa





1) The CMB is axisymmetric elliptical (hydrostatic figure)

$$\hat{n} = \hat{r} - \alpha^c \sin 2\theta \, \hat{\theta}$$

α_c : geometrical ellipticity $\sim 1/400$

2) The CMB is bumpy

$$h(\theta,\varphi) = b\,Y(\theta,\varphi)$$

$$Y(\theta,\varphi) = \sum_{n,m} a_{nm} Y_n^{mc}(\theta,\varphi) + b_{nm} Y_n^{ms}(\theta,\varphi)$$

$$\hat{n} = \hat{r} - \frac{\partial Y}{\partial \theta}\hat{\theta} - \frac{1}{\sin\theta}\frac{\partial Y}{\partial \varphi}\hat{\varphi}$$

$\left\{ \begin{array}{l} p \quad \text{known from magnetic data} \\ Y \quad \text{from seismology ?} \end{array} \right.$

$\longrightarrow$ $\underline{\underline{\Gamma_{geo}}}$ $\qquad \vec{r} \wedge p\,\hat{n}\,dS = b\,p\left[\frac{\partial Y}{\sin\theta\,\partial\varphi}\hat{\theta} - \frac{\partial Y}{\partial\theta}\vec{\varphi}\right] dS$

$$\vec{\Gamma}_{geo} = \iint_{CMB} p \left(\frac{\partial h}{\sin\theta \, \partial\varphi} \hat{\theta} - \frac{\partial h}{\partial \theta} \hat{\varphi} \right) dS$$

Orthogonality relationships $\longrightarrow$ numerous interaction terms cancel

If the figure of the CMB is hydro-static, only the term

$$P = \Pi_2^{1c} + i\, \Pi_2^{1s}$$

is efficient

C. Comparison with observations.

C.1. Decade variations of the mean pole position.

Equatorial component of (8):

$$\boxed{A^m \frac{d\vec{\omega}_{eq}}{dt} - (C-A)\,\vec{\Omega}_o \wedge \vec{\omega}_{eq} = \vec{\Gamma}_{geo\,eq}} \quad (9)$$

$\vec{\Gamma}_{rest}$ being neglected

$$A^m \dot{\omega}_1 + \Omega_o (C-A)\,\omega_2 = \Gamma_{geo\,1}$$
$$A^m \dot{\omega}_2 - \Omega_o (C-A)\,\omega_1 = \Gamma_{geo\,2}$$

$$\boxed{A^m \dot{\omega} - i\,(C-A)\Omega_o\,\omega = \Gamma} \quad (10)$$

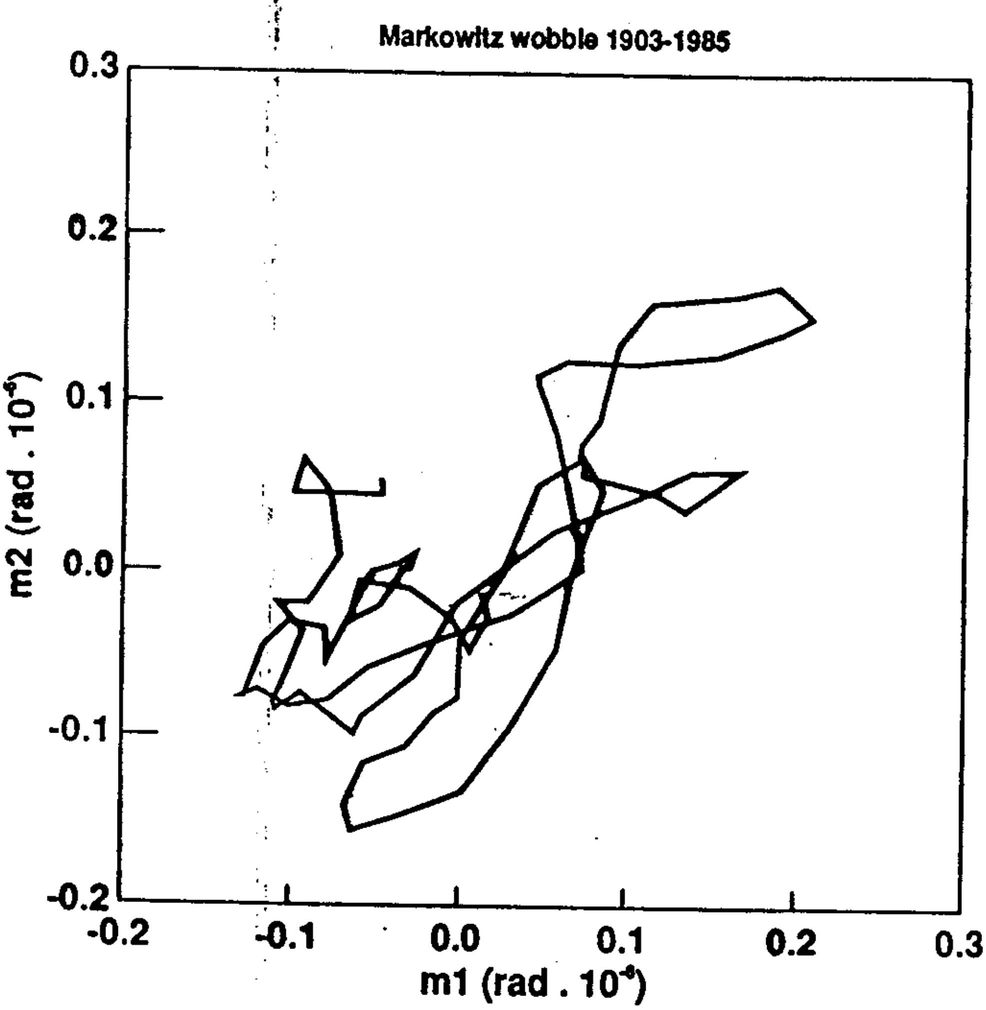
$$\omega = \omega_1 + i\,\omega_2$$

$$\Gamma = \Gamma_{geo\,1} + i\,\Gamma_{geo\,2}$$

$A^m \sim .33\ Ma^2 \sim 7.2\ 10^{37}$ kg m^2

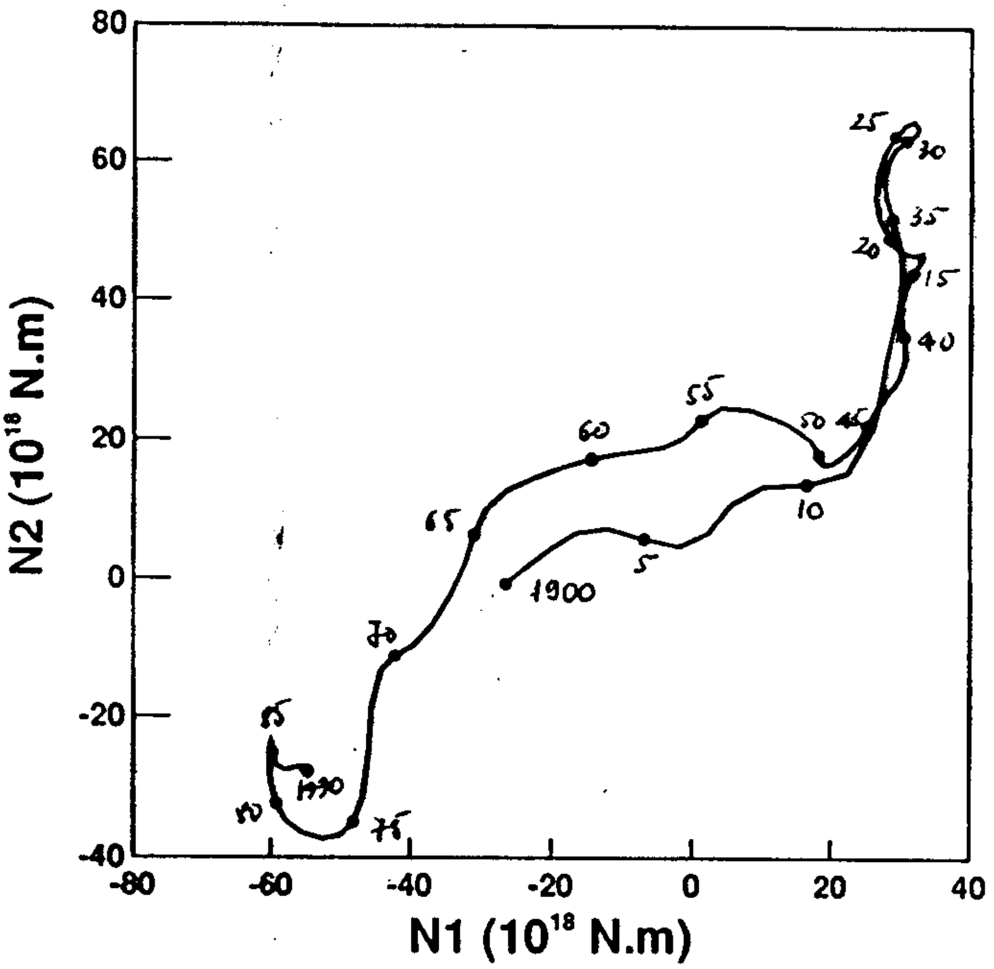
$(C-A) \sim 2.4\ 10^{35}$ kg m^2

$\Omega_o \sim 7.3\ 10^{-5}$



linear trend removed

Couple de Pression. (somme) 1900-1990



10^{-6} rad.

4

Axial rotation. Decade length of the day variatio

Component of (3) along the Ox_3 axis :

$$C^m \frac{d}{dt}\left(\Omega_0 + \omega_3\right) = \vec{k}\cdot\vec{\Gamma}_{geo} \qquad (13)$$

$$\vec{k}\cdot\vec{\Gamma}_{geo} = \iint_{CMB}\left(p\,\frac{\partial h}{\sin\theta\,\partial\varphi}\,\hat{\theta} - \frac{\partial h}{\partial\varphi}\,\hat{\varphi}\right) dS\cdot\vec{k}$$

$$= \iint_{CMB} p\,\frac{\partial h}{\sin\theta\,\partial\varphi}\;\vec{\theta}\cdot\vec{k}\; dS = -\iint p\,\frac{\partial h}{\partial\varphi}\, dS$$

$$\boxed{\Gamma_{geo\,3} = \iint_{CMB} h\,\frac{\partial p}{\partial\varphi}\, dS \qquad (14)}$$

$p = P_{geo}$

Remark. The pressure associated to the zonal (axisymmetric) toroidal part of $\vec{u}$ gives no contribution to the torque.

If $h \sim$ a few km (Morelli and Dziewonski)

$p \sim$ a few 10^2 Pa

$\Gamma_{geo\ axial}$ is 10 to 10^2 too large, i.e. larger than allowed by observations.

In fact $\Gamma_{geo.\,axial} \sim 10^{19}$ N m

One interpretation :

$$\int h \frac{\partial p}{\partial \varphi} \, dS \quad \ll \quad \langle h \rangle \langle p \rangle$$

As a consequence the flow is locked with respect to the mantle.

No hope to compute Γ_{axial}

but:

<u>Variations of the angular momentum of the core.</u>

For the time constants considered here – from a few years to a few tens of years –, the changes in the pressure field induce a time varying geostrophic flow organized in cylindrical annuli (proposition)

<u>Reason</u> (partial)

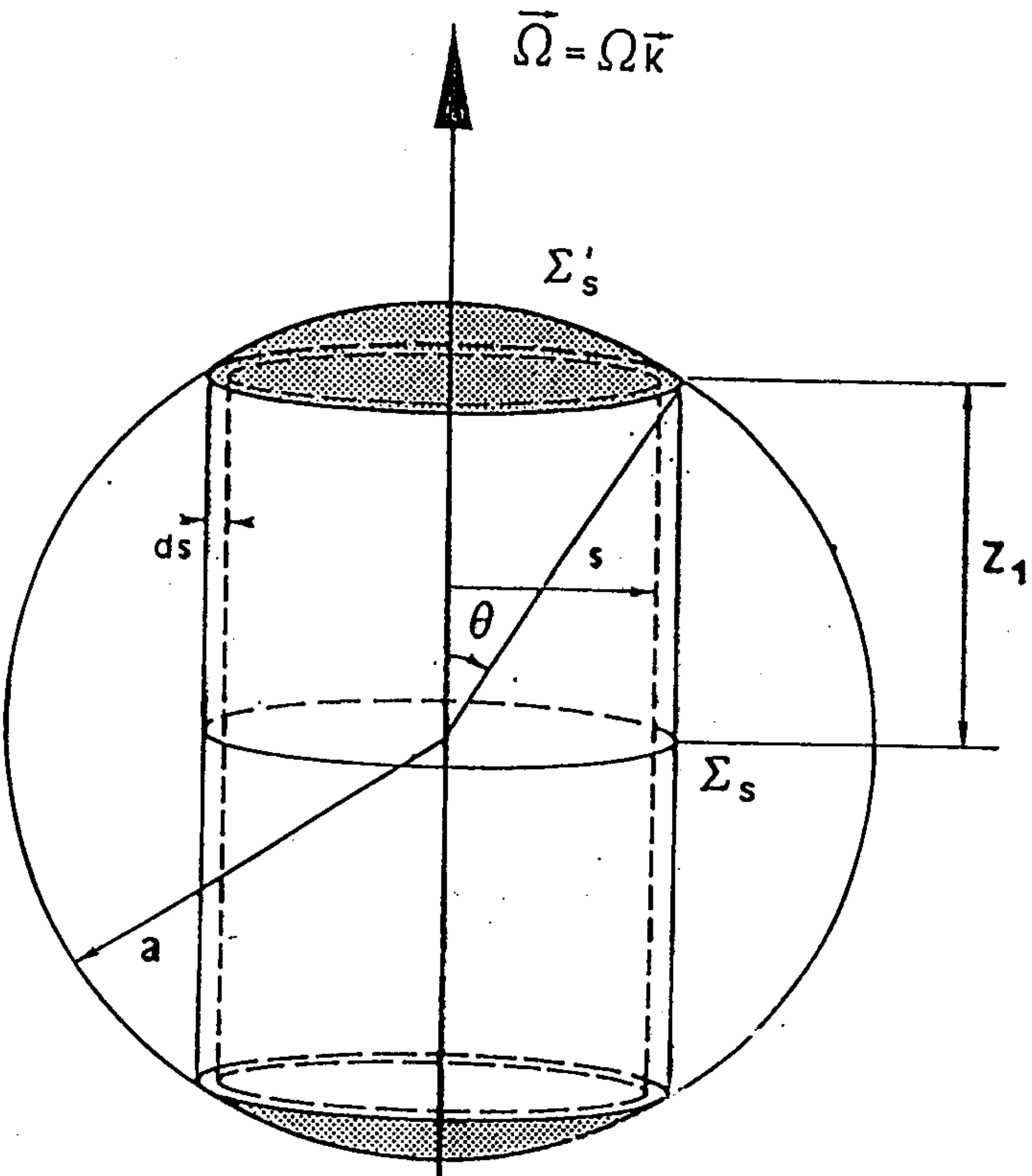
$$2\rho \vec{\Omega}_0 \wedge \vec{v} + \vec{\nabla} p = 0$$

$$\rightarrow \quad \vec{\nabla} \times (\vec{\Omega}_0 \wedge \vec{v}) = 0$$

$$\longrightarrow \quad \boxed{\frac{\partial \vec{v}}{\partial z} = 0} \qquad \text{Proudman-Taylor}$$

c.f dynamo lectures

Only $v(s)\,\hat{\varphi}$



angular rotation of $C(s)$: $\beta(s)$

linear velocity $s\beta(s)$

Angular momentum equation

$$C(s): \qquad I(s)\left(\frac{d\beta}{dt} + \frac{d\omega_3}{dt}\right) = b^2 \sin\theta \oint_{\theta,\, \pi-\theta} h \frac{\partial p}{\partial \varphi} d\varphi \; d\theta$$

$$I(s) = 4\pi \rho b^5 \cos^2\theta \sin^3\theta \; d\theta$$

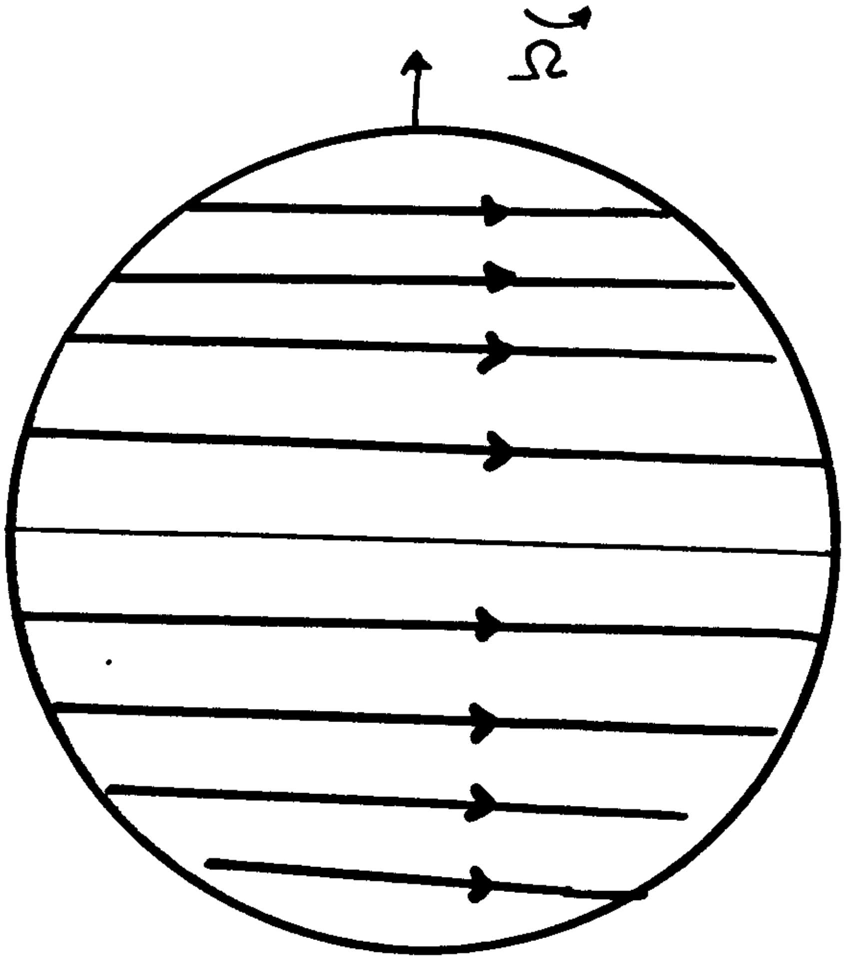
$$\boxed{\frac{d\beta}{dt} + \frac{d\omega_3}{dt} = \frac{1}{4\pi\rho b^3 \cos^2\theta \sin^2\theta} \oint_{\theta,\, \pi-\theta} h \frac{\partial p}{\partial \varphi} d\varphi}$$

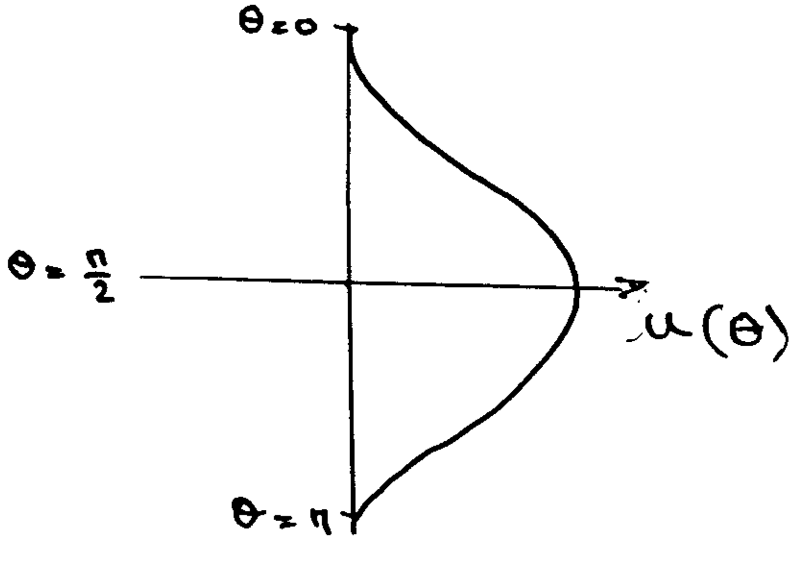
Same practical difficulty to compute the torque.

But $\quad \vec{T}_n^{\,0}(\theta, t) = -b\, t_n^0(t)\, \vec{n} \wedge \vec{\nabla}_H P_n(\cos\theta)$

known from S.V data inversion. Let us assume that the toroidal zonal part of the computed tangentially geostrophic flow is the surface expression of the geostrophic flow $\beta(s)$. Then $\beta(s)$ is known. The geostrophic volume flow organized in cylindrical annuli is known.

$\vec{t}$





In fact the angular momentum (axial) of this flow can be shown to be:

$$H_c = C^c \left(\omega_3 + \frac{1}{6} \left(t_1^0 + \frac{12}{7} t_3^0 \right) \right)$$

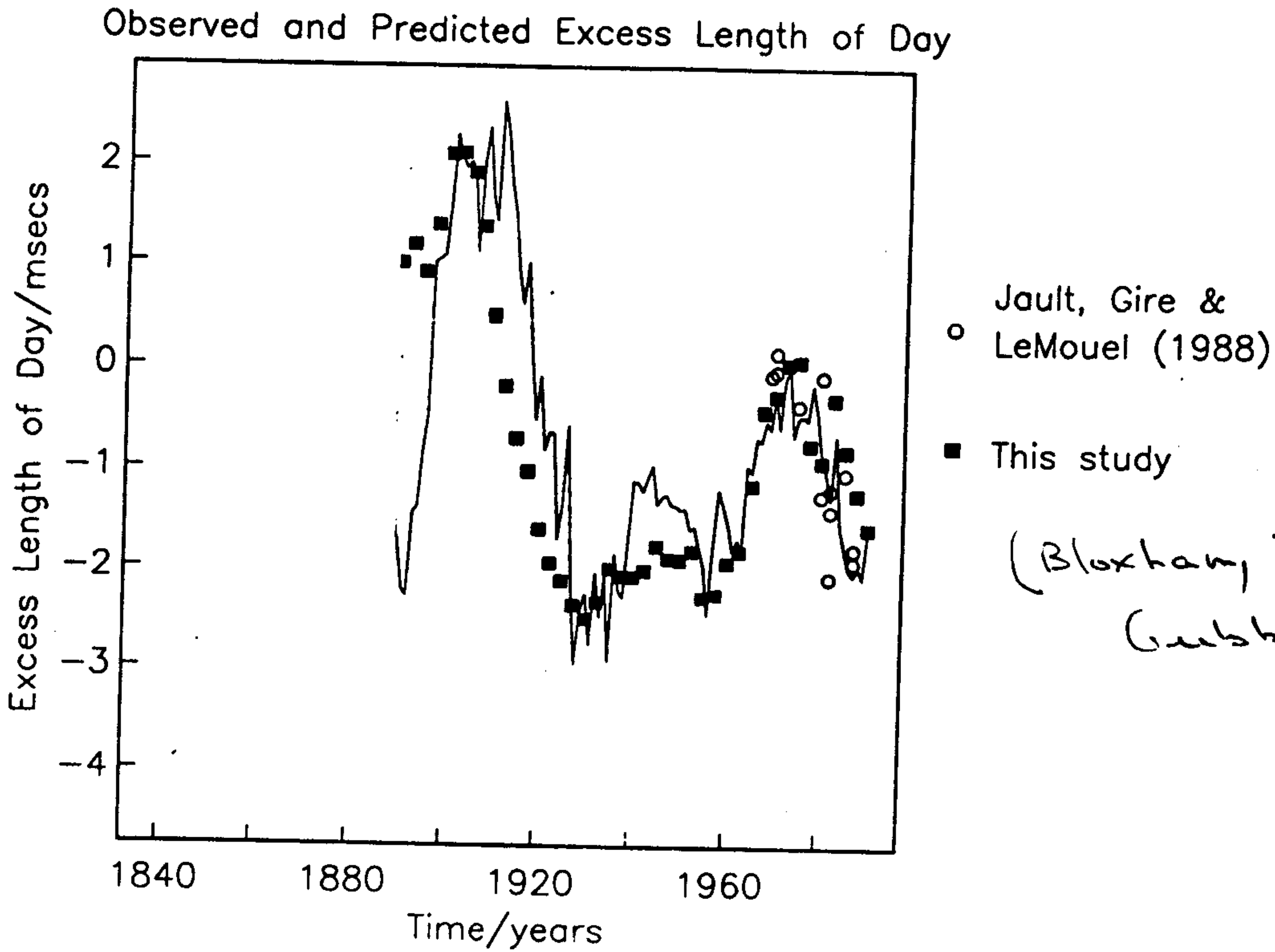
Rather:

$$\boxed{\frac{dH_c}{dt} = C^c \left[\frac{d\omega_3}{dt} + \frac{1}{6} \left(\frac{dt_1^0}{dt} + \frac{12}{7} \frac{dt_3^0}{dt} \right) \right]}$$

The whole Earth is isolated.

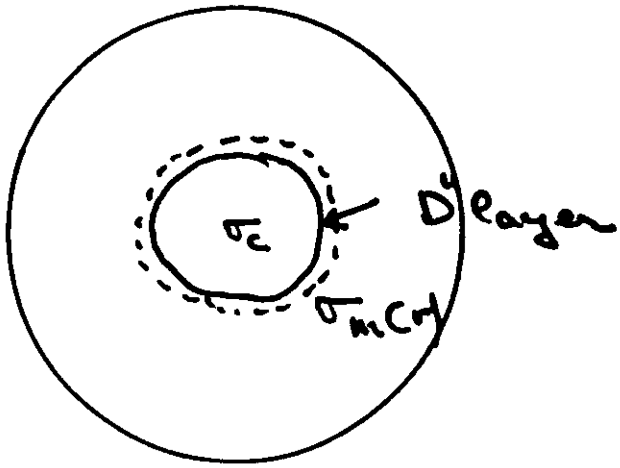
$$\frac{dH_m}{dt} + \frac{dH_c}{dt} = 0$$

$$\boxed{\underbrace{(C^c + C^m) \frac{d\omega_3}{dt}}_{\text{l.o.d data}} + \underbrace{C^c \frac{1}{6} \left(\frac{dt_1^0}{dt} + \frac{12}{7} \frac{dt_3^0}{dt} \right)}_{\text{magnetic data}}}$$



(Bloxham, Jackson, Gubbins)

D. Electromagnetic coupling.



$$\int \sigma_m(r)\, dr = \sigma_M L_M$$

1) Experimental results

$$\sigma_M L_M \sim 10^6\ \Omega + (\sigma L)_{D''}$$

2) Analysis of short events of secular variation

$$\tau_e \sim \mu \sigma_M L_M^2 \qquad < 1\text{ year}$$

$$\sigma_M L_M = \frac{\tau_e}{\mu L_M} \qquad \tau_e \text{ given}$$

$L_M \searrow,\ \sigma_M L_M \nearrow$

It is the quantity $\sigma_M L_M$ which is relevant in the computation of the electromagnetic torque.

$$\Gamma_{el} \sim A\, \sigma_M L_M \frac{t_i^0}{b} \qquad (15)$$

To get the observed accelerations, taken into account the estimated value of t_i^0, one needs

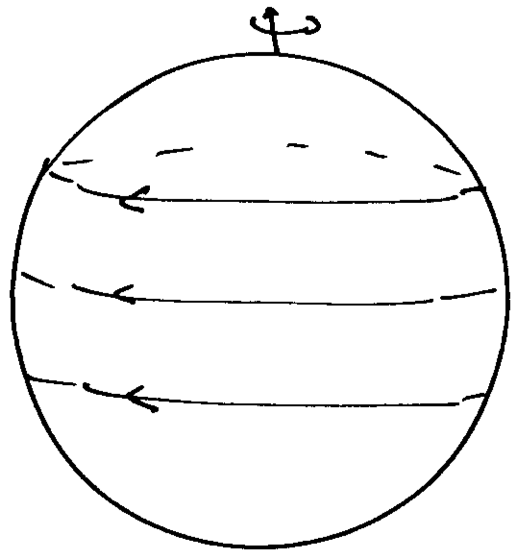
$$\sigma_M L_M \sim 3.10^8 \ S$$

D'' : <u>200 km × 1000 $(\Omega m)^{-1}$</u> ?

Contamination of oxydes by iron from the mantle ? (Poirier and le Mouël, 198)

Furthermore, for the last decades, the magnetic torque (15) seems to act the wrong way.

MHD interaction between a bumpy interface and a flow over it.



If $\vec{u}_0$ is zonal toroidal

$$g(s) = 0 \, , \qquad \Gamma_p \equiv 0$$

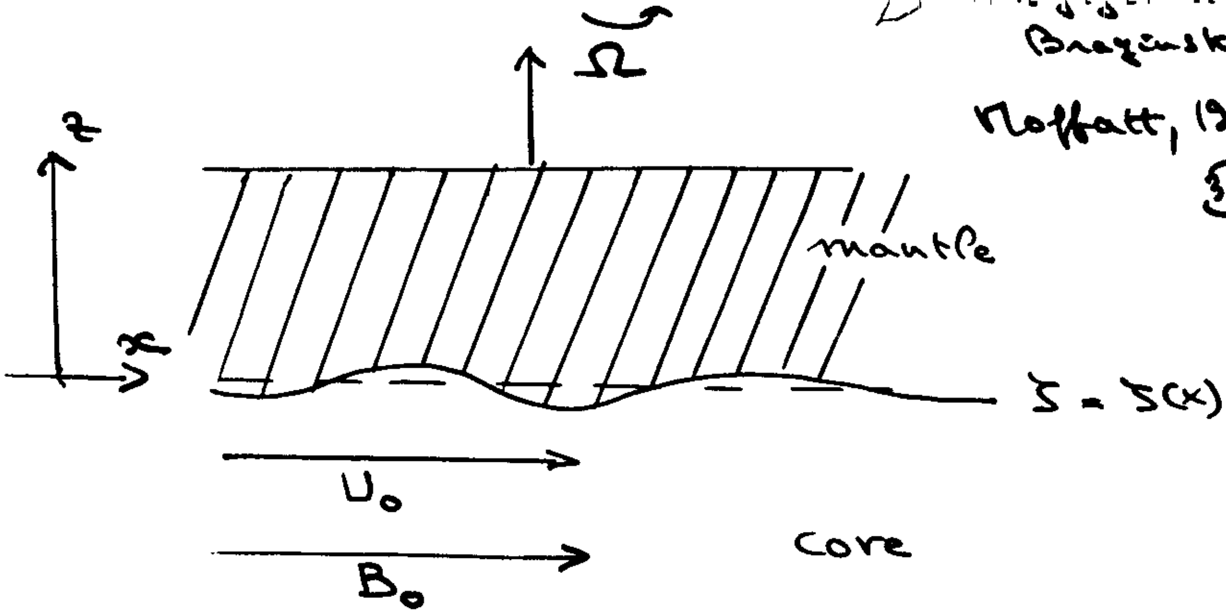
But considering the full MHD problem (Anufriyev and Braginsky, Moffatt, Moffatt and Dillon, El Tayeb and Hassan, Hassan and El Tayeb)

$$F_t \sim \underbrace{C_T}_{\text{drag coefficient}} \Omega \rho U_0 h \qquad \text{(Hide, 1977)}$$

Braginsky

Moffatt, 1977

33



$$\zeta(x) = \mathrm{Re}\,(\hat{\zeta}\, e^{ikx})$$

$$H_0 = (\mu_0 \rho_c)^{-1/2} B_0 \qquad \text{Alfven velocity}$$

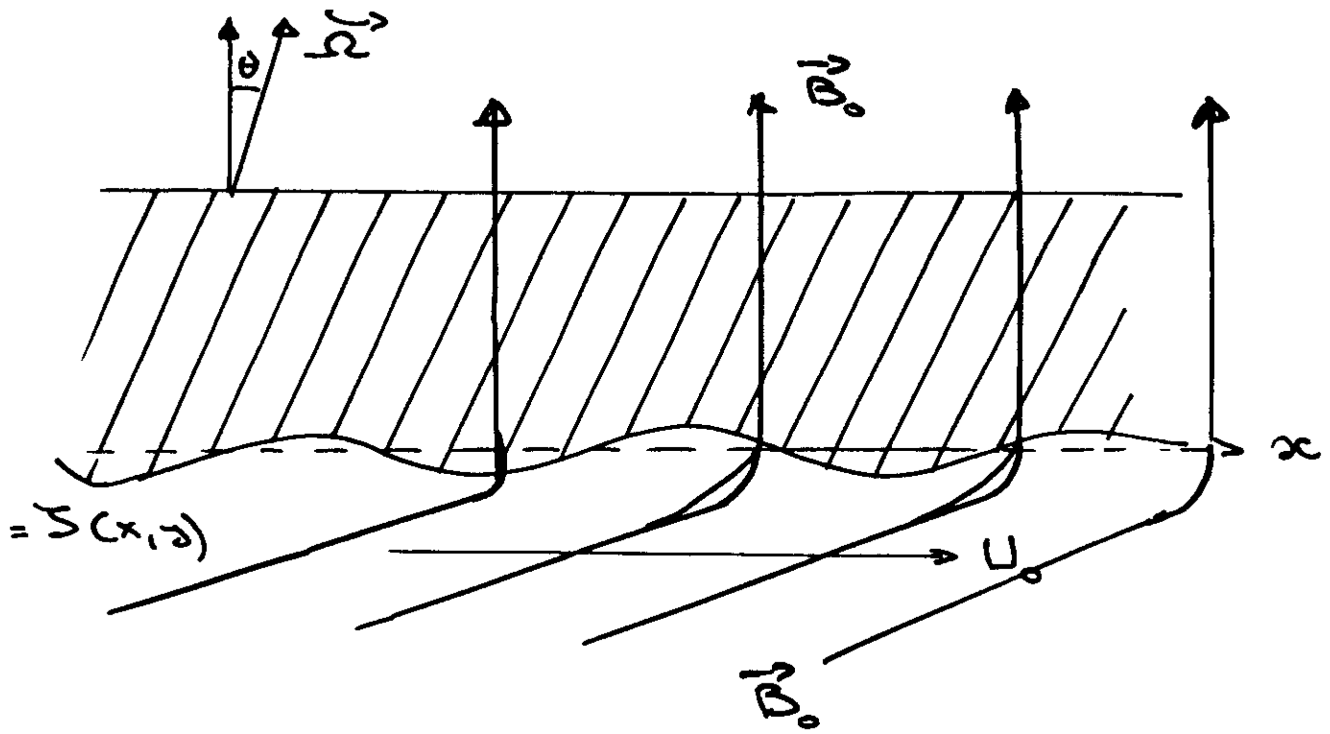
$\longrightarrow$

$$\boxed{F_t \sim \rho_c H_0^2 k^2 |\hat{\zeta}|^2}$$

can indeed reach 10^{-2} N. m^{2}

$(U_0 \sim 10^{-4}\,\mathrm{m\,s^{-1}})$

Hassan and El Tayeb, 1982



$$\longrightarrow F_t \sim |\zeta|^2 f(h_0)$$

$$h_0 = \frac{B_T}{B_P}$$

get $F_t \sim 10^{-2}\ N\,m^{-2}$

Difficulty to transpose to the spherical case:

$$\frac{d\sigma_r}{dt} = -(I_m + I_c)\frac{d\Omega}{dt} = \vec{k}\cdot\int \vec{r}\,\frac{\partial}{\partial t}(\rho\vec{u})\,dv$$

$$\frac{\partial \vec{u}}{\partial t} \equiv 0 \longrightarrow \frac{d\Omega}{dt} = 0 \text{ or } \Gamma_t = 0$$

Torsional oscillations. Braginsky, 1970

Lorentz force.

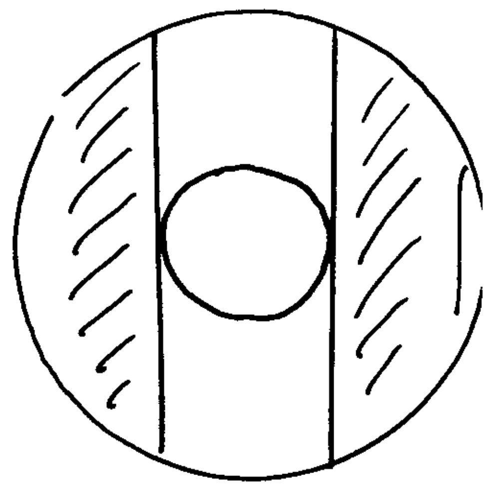
$$I(s)\,\frac{\partial \omega(s)}{\partial t} = \Gamma_p(s) + \Gamma_g(s) + \Gamma_L(s)$$

$$\Gamma_L(s) = \frac{2\pi}{\mu}\int_{-z_1}^{z_1} \frac{\partial}{\partial s}\left(\langle s^2 B_s B_\varphi \rangle\right) dz$$

with

$$\langle B_s B_\varphi \rangle = \frac{1}{2\pi}\int_0^{2\pi} B_s B_\varphi \, d\varphi$$

$$\frac{\partial B_\varphi}{\partial t} = s B_s \frac{\partial \omega}{\partial s}$$



→ string problem

elasticity $\sim \overline{B_s}^2$

<u>Period of oscillation</u>

$\sim \frac{1}{b}$ $\sim$ 60 years

for $b = \sqrt{\overline{B_s^2}} \sim 2$ Gauss

<u>But</u>: is there any evidence of this period in

l.o.d. data ?

magnetic data

Conclusion :

More studies of the CMB, of the D" layer : SEDI

More magnetic observations (INTERMAGNET) and data analysis