INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

SMR. 758 - 18

SPRING COLLEGE IN CONDENSED MATTER
ON QUANTUM PHASES
(3 May - 10 June 1994)

THE QUANTUM HALL EFFECT: THEORY

Part II

Allan H. MACDONALD
Department of Physics
Indiana University
Swain Hall - West 117
Bloomington, IN 47405, U.S.A.

Digression: Correlation Function Moments

- 2nd-Quantized Expression

$$n^{(2)}(\vec r_1,\vec r_2) = \sum_{\substack{m_1,m_2\\ m_1',m_2'}} \bar\varphi_{m_1'}(\vec r_1)\varphi_{m_1}(\vec r_1)\bar\varphi_{m_2'}(\vec r_2)\varphi_{m_2}(\vec r_2)\langle c^\dagger_{m_1'}c^\dagger_{m_2'}c_{m_2}c_{m_1}\rangle_0$$

uniform system

$$g(r) \equiv n^{-2} n^{(2)}(\vec 0,\vec r) = \nu^{-2}\sum_{m\neq 0}\left(\frac{r^2}{2}\right)^m \frac{1}{m!}\, e^{-r^2/2}\,\langle \hat n_m \hat n_0\rangle_0$$

↑ pair distribution ftn

- Zeroth Moment $x = r^2/2$

$$h(r) \equiv g(r) - 1 = \nu^{-2}\sum_{m=0}^{\infty} \underbrace{\frac{x^m}{m!}e^{-x}}\Big((1-\delta_{m,0})\langle\hat n_m\hat n_0\rangle - \langle\hat n_m\rangle\langle\hat n_0\rangle\Big)$$

↑ pair correlation ftn

$n\int d^2\vec r \to \nu$

$n\int d^2\vec r\, x \to \nu(m+1)$

$$n\int d^2\vec r\; h(r) = \nu^{-1}\Big[\langle\hat N\hat n_0\rangle - \langle\hat n_0\hat n_0\rangle - \langle\hat N\rangle\langle\hat n_0\rangle\Big] = -1$$

- First Moment:

$\hat M = \sum_m m\,\hat n_m$

$$n\int d^2\vec r\;\frac{r^2}{2}h(r) = -1 + \nu^{-1}\Big[\langle\hat M\hat n_0\rangle - \langle\hat M\rangle\langle\hat n_0\rangle\Big]$$

$$= -1$$

Digression: Structure Factor & Projected Structure Factors

- Static Structure Factor & Pair Correlation Ftn.

$$S(k) \equiv \left\langle \frac{1}{N}\sum_{i\neq j} e^{i\vec k\cdot(\vec r_i - \vec r_j)}\right\rangle_0 \equiv \frac{1}{N}\langle \rho_{-\vec k}\,\rho_{\vec k}\rangle_0$$

$$\downarrow\quad n^{(2)}(\vec r,\vec r\,') = \sum_{i\neq j}\langle\delta(\vec r-\vec r_i)\,\delta(\vec r\,'-\vec r_j)\rangle_0$$

$$S(k) = \underset{\substack{\uparrow\\ i=j\\ \text{terms}}}{1} + \frac{1}{N}\int d\vec r\int d\vec r\,'\, e^{i\vec k\cdot(\vec r-\vec r\,')}\, n^{(2)}(\vec r,\vec r\,')$$

$$= 1 + \underset{\substack{\uparrow\\ g(r)=h(r)+1}}{N\delta_{\vec k,0}} + \underbrace{n\int d\vec r\, e^{i\vec k\cdot\vec r}\, h(r)}_{\equiv\, h(k)}$$

- Moments & Long Wavelength h(k) behavior

$$h(k) = n\int d\vec r\, h(r) + \frac{k^2}{2}\left(-n\int dr\,\frac{r^2}{2}h(r)\right) + \mathcal{O}\,k^4$$

$$= -1 + \frac{k^2}{2} + \cdots$$

- Relationship of $s(k)$ & $\bar{s}(k)$

$$\rho_{\vec{k}} \equiv \sum_i e^{-i\vec{k}\cdot\vec{r}_i}$$

$$\bar{\rho}_{\vec{k}} \equiv \sum_i {}_i\langle 0| e^{-i\vec{k}\cdot\vec{r}} |0\rangle_i = \sum_i B_i(k)$$

$$B_i(k) = e^{-ik b_i/\sqrt{2}} \, e^{-ik b_i^\dagger/\sqrt{2}}$$

Note $B_i(k_1) B_i(k_2) = e^{k_1 \bar{k}_2/2} B_i(k_1+k_2)$

$$\bar{s}(k) \equiv \frac{1}{N}\langle \bar{\rho}_{-\vec{k}} \bar{\rho}_{\vec{k}} \rangle_0 = \frac{1}{N}\sum_{i\neq j} \langle e^{i\vec{k}\cdot\vec{r}_i} e^{-i\vec{k}\cdot\vec{r}_j}\rangle_0 + \frac{1}{N}\sum_i \langle \cancel{B_i(-k) B_i(k)} \rangle$$

$$\cancel{B_i(-k) B_i(k)} \to e^{-|k|^2/2}$$

$$= s(k) - 1 + e^{-|k|^2/2} = h(k) + e^{-|k|^2/2}$$

- Long Wavelength $\bar{s}(k)$ behavior

$$\bar{s}(k) = \left(-1 + \frac{|k|^2}{2} + \Theta(|k|^4)\right) + \left(1 - \frac{|k|^2}{2} + \Theta(|k|^4)\right)$$

$$\Theta(|k|^4) = \sum_m |\langle \Psi_m | e^{-i\vec{k}\cdot\vec{r}} |\Psi_0\rangle|^2$$

no dipole coupling within lowest Landau level !! $\left(\sigma(\omega) \propto \delta(\omega - \omega_c)\right)$
no dirt

MAGNETOROTONS - LOWEST LANDAU LEVEL SMA

- Trial Wavefn. for Collective Excitation $|\Psi_{\vec{k}}\rangle \propto \bar{\rho}_{\vec{k}} |\Psi_0\rangle$

$$E(\vec{k}) = \frac{\langle \Psi_{\vec{k}} | H | \Psi_{\vec{k}} \rangle}{\langle \Psi_{\vec{k}} | \Psi_{\vec{k}} \rangle} = \frac{\langle \Psi_0 | \bar{\rho}_{-\vec{k}} [H, \bar{\rho}_{\vec{k}}] | \Psi_0 \rangle}{\langle \Psi_0 | \bar{\rho}_{-\vec{k}} \bar{\rho}_{\vec{k}} | \Psi_0 \rangle} \equiv \frac{\bar{f}(\vec{k})}{\bar{s}(\vec{k})}$$

- Evaluating Projected f-sum Rule

$$\bar{f}(k) \equiv \frac{1}{N} \sum_m |\langle \Psi_m | \bar{\rho}_k | \Psi_0 \rangle|^2 (E_m - E_0) = \bar{f}(-k)$$

$$\Downarrow$$

$$\bar{f}(k) = \frac{1}{N} \langle \Psi_0 | [\bar{\rho}_{-k}, [H, \bar{\rho}_k]] | \Psi_0 \rangle$$

but

$$H = \frac{1}{2} \int \frac{d^2q}{(2\pi)^2} V(q) \bar{\rho}_{-q} \bar{\rho}_q + \text{constant}$$

$\Downarrow$ ← after a little patient work

$$\bar{f}(k) = \int \frac{d^2q}{(2\pi)^2} V(q) \left(1 - \cos(\hat{z}\cdot(\vec{q}\times\vec{k}))\right)\left(\tilde{\bar{s}}(k+q) - \tilde{\bar{s}}(q)\right) e^{-|k|^2/2}$$

$$\tilde{\bar{s}}(q) \equiv e^{|q|^2/2} \bar{s}(q)$$

SMA MAGNETOROTONS

Coulomb Model

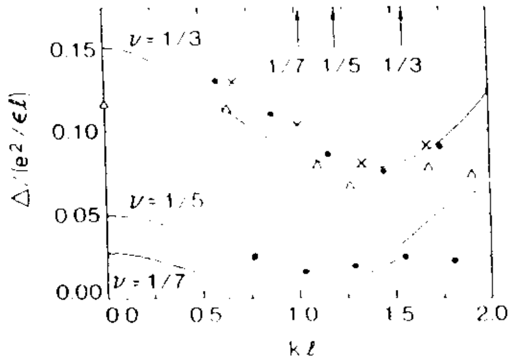


Figure 9.1 Comparison of SMA prediction of collective mode energy for $\nu = 1/3$, $1/5$, and $1/7$ (solid lines) with small-system numerical results. Crosses indicate ($N=7$, $\nu = 1/3$) spherical system, triangles ($N = 6$, $\nu = 1/3$) hexagonal unit cell (Haldane and Rezayi 1985a). Solid dots are from ($N = 9$, $\nu = 1/3$) and ($N = 7$, $\nu = 1/5$) spherical system calculations of Fano *et al.* (1986). Arrows at the top indicate the magnitude of the reciprocal lattice vector of the Wigner crystal at the corresponding density.

Girvin, MacDonald, Platzman
PRB 33, 2481 (1986)

Exact Diagonalization

344 F. D. M. Haldane

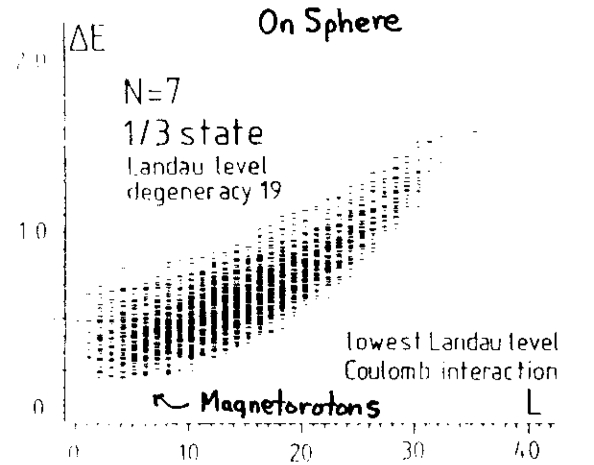


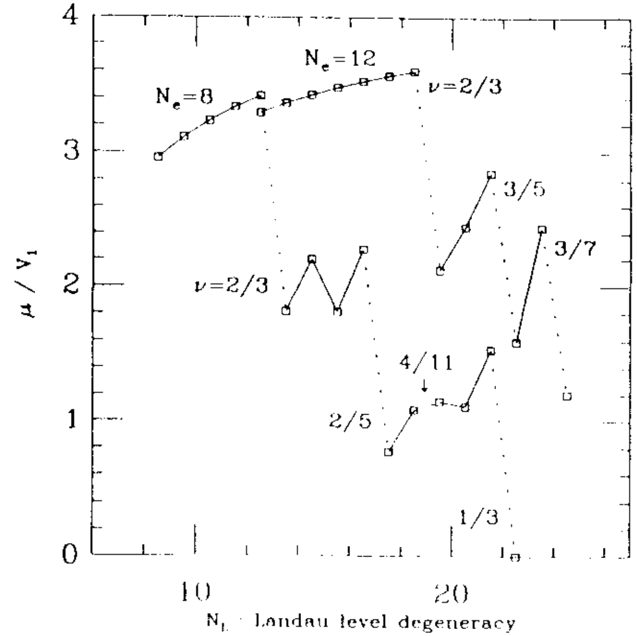
Figure 8.11 Complete excitation spectrum for the seven-particle system in spherical geometry corresponding to $\nu=1/3$, with the lowest-Landau-level Coulomb interaction.

Hard-Core Model on Sphere

Some fractions more robust than others

$\nu_n = \frac{n}{2n+1}$

$\nu_n' = \frac{n+1}{2n+1}$



C. Gros & AHM, PRB $\underline{42}$, 10811 (1990)

$\nu \neq 1/m$ What we really know

(Quite a bit but not quite Enough)

$(\psi[z] = P[z] \prod_k \exp(-|z_k|^2/4))$

- Bosonization (AHM + D.B. Murray, PRB $\underline{32}$, 2707 (1985))

$\boxed{\nu = 1}$ $P[z] = \begin{vmatrix} z_1^0 & \cdots & z_N^0 \\ \vdots & & \vdots \\ z_1^{N-1} & & z_N^{N-1} \end{vmatrix} = \prod_{i<j} (z_i - z_j) \equiv P_V[z]$

← Vandermonde determinant

$\boxed{\nu > 1}$ $P[z] = Q[z]\, P_V[z]$

$\hat{O}_B[z] = P_V[z]\, \hat{O}_F\, P_V[z]$

$\boxed{\nu_F^{-1} \equiv \nu_B^{-1} + 1}$

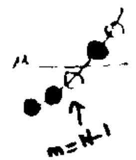
$N_\Phi^B = N_\Phi - N$

$M^F = M + \frac{N(N-1)}{2}$

→ examples

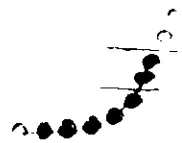
$M^B = 1$ $Q[z] = \sum_i z_i$ $P[z] = \begin{vmatrix} z_1^0 & \cdots & z_N^0 \\ \vdots & & \vdots \\ z_1^{N-2} & \cdots & z_N^{N-2} \\ z_1^N & & z_N^N \end{vmatrix}$

edge phonon



$M^B = N$ $Q[z] = \prod_i z_i$ $P[z] = \begin{vmatrix} z_1^1 & \cdots & z_N^1 \\ \vdots & & \vdots \\ z_1^N & & z_N^N \end{vmatrix}$

quasi hole at origin



- **Particle-hole symmetry** $\boxed{\nu' = 1-\nu}$

$$c_m^{+} \longrightarrow c_m \quad \Rightarrow \quad H \longrightarrow \frac{N^2}{N_\phi}\,\epsilon(\nu=1) + H$$

[illegible]

$$\frac{1}{3} \longrightarrow \frac{2}{3}$$

$$N \rightarrow N_h = N_\phi - N$$

$$\Rightarrow \left(\nu = 1 - \frac{1}{m} \text{ understood} \right)$$

e.g. ν near 1

$$N_h = N_\phi - N \quad \longleftarrow \quad \boxed{\begin{array}{l} N_h \text{ particles},\ N+N_h \text{ states} \\ N_h \text{ bose particles},\ N \text{ states} \end{array}}$$

(or also bosonization $\equiv$

- **Correlation Factor** – Relative Angular Momentum Boost

$$\Psi'[z] = \prod_{i<j} (z_i - z_j)^2\, \Psi[z] \qquad (V_1 \gg V_3 \cdots)$$

$$\nu^{-1\,\prime} = 2 + \nu^{-1} \qquad 2/3 \rightarrow \frac{2}{7}$$

$$\boxed{2 \pm \nu^{-1}}\ ? \qquad \text{not } \frac{2}{5}$$

Relationship to flux attachment?

e.g. ν near $1/3$

N_h fermion quasiparticles, $N + N_h$ states

N_h boson quasiparticles, N states

Hierarchy Pictures

(Classical Hierarchies)

- First Level of Hierarchy

N_{qp} ← charge $1/m$ gas with N_{qp} particles

N_ϕ = [illegible] # of flux quanta seen by qp's

[illegible]

$$\nu^{-1} = m \pm \frac{1}{2k}$$

$$\begin{array}{ccccc} 1 & & 1/3 & & 1/5 \\ 2/3 & 2/5 & 2/7 & 2/9 & 2/11 \end{array}$$

$\vdots$

all odd denominator fractions

Ms. Experimenter

Neoclassical Hierachies

- Chern-Simons Landau-Ginsburg
  - Bosonization + Charge-Vortex Duality in Boson Superfluid
  - $\Rightarrow$ all fractions with odd denominator

(Hall [illegible] / [illegible])

- Composite Fermions (Jain) (Read) (Halperin, Lee, Read)
  - Expanded Hilbert Space + Correlation Factors

$$\nu^{-1} = \pm \frac{1}{n} + 2 \quad \Rightarrow \quad \nu = \frac{n}{2n \pm 1}$$

  - Fermion $\rightarrow$ Fermion Flux Attachment

$$\Psi_{2/5}[z] = \overline{\prod_{i<j} (z_i - z_j)^2 \, \Psi_{\nu=2}[z]}$$

Compressible States & Composite Fermions

- CS mean-field theory

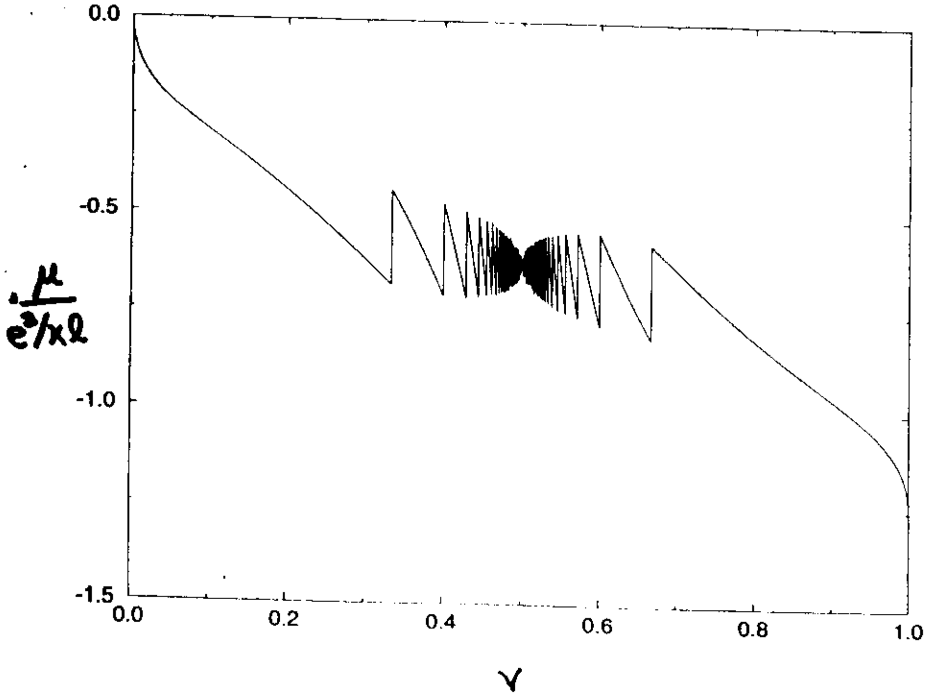
$$\nu^{-1} \Rightarrow \frac{N_\phi}{N} \rightarrow \nu^{-1} - 2 \quad \Rightarrow \nu \simeq 1/2 \text{ like weak field}$$

ok because of interactions

Consequences Predicted	Seen?
$\nu = \frac{n}{2n \pm 1}$ dominant	✓
$\Delta \propto \hbar\omega_c \propto n^{-1}$ ($\Rightarrow \Delta\mu$ constant !!)	maybe
Semiclassical magnetotransport. eff. cyclotron radius $\overline{R} \propto n$	Seems to be amazing !!

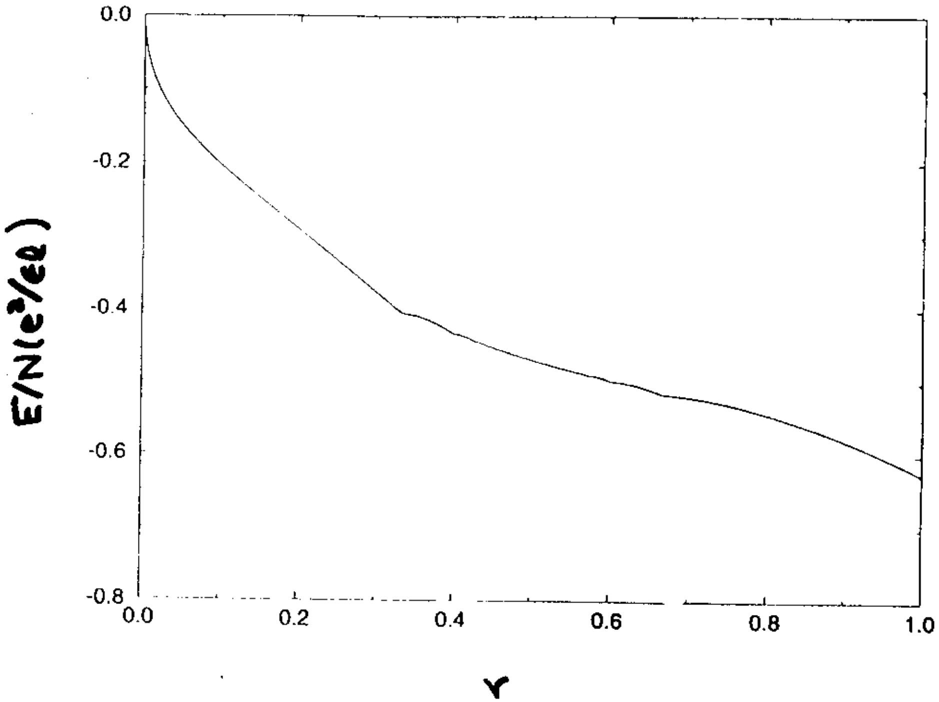
- Problems
  - higher Landau levels unphysical
  - dependence on unphysical band mass
  - theory difficult

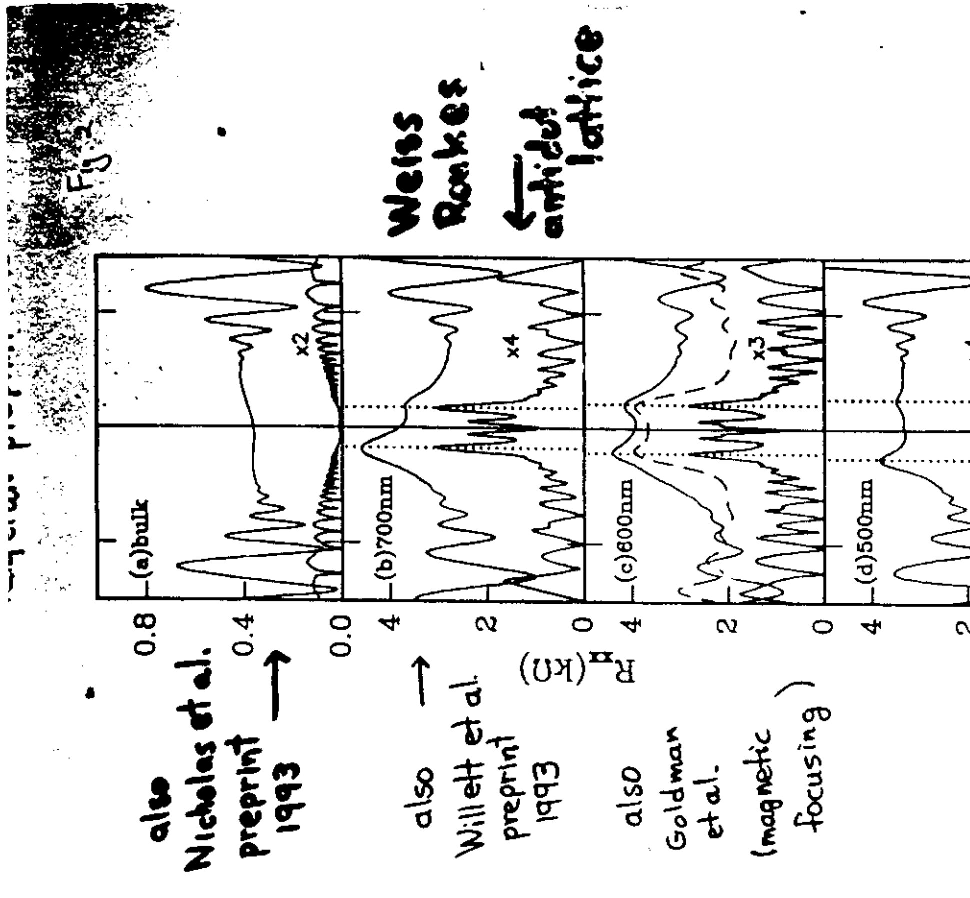
Composite Fermion Chemical Potential



AHM unpublished

Composite Fermion 'Model' Ground State Energy





On the Interaction of Electrons in Metals

E. Wigner, *Princeton University*

(Received October 15, 1934)

The energy of interaction between free electrons in an electron gas is considered. The interaction energy of electrons with parallel spin is known to be that of the space charges plus the exchange integrals, and these terms modify the shape of the wave functions but slightly. The interaction of the electrons with antiparallel spin, contains, in addition to the interaction of uniformly distributed space charges, another term. This term is due to the fact that the electrons repell each other and try to keep as far apart as possible. The total energy of the system will be decreased through the corresponding modification of the wave function. In the present paper it is attempted to calculate this "correlation energy" by an approximation method which is, essentially, a development of the energy by means of the Rayleigh-Schrödinger perturbation theory in a power series of e^2.

1.

The attempt has been made in previous work[1] to give a more general expression for the wave function of free electrons in metals than that provided by Hartree's method of the self-consistent field[2,3] or Fock's equations. The form of the wave function assumed in Fock's equations for a system of $2n$ electrons, occupying n doubly-degenerate states is

$$\frac{1}{n!}\begin{vmatrix}\psi_1(x_1)&\cdots&\psi_1(x_n)\\ \vdots&&\vdots\\ \psi_n(x_1)&\cdots&\psi_n(x_n)\end{vmatrix}\begin{vmatrix}\psi_1(y_1)&\cdots&\psi_1(y_n)\\ \vdots&&\vdots\\ \psi_n(y_1)&\cdots&\psi_n(y_n)\end{vmatrix}, \quad (1)$$

where x stands for three Cartesian coordinates of electrons with upward spin, and y for those of electrons with downward spin. The ψ_ν are the solutions of a Schrödinger equation in which the potential of the charge distribution of the other electrons enters as well as the potential arising from the ions.

In a metal the charge distribution of all electrons is practically unaltered by removing one so that the second quantity may be replaced by the former and the potential for a given electron at the point u is given by adding to the Coulomb field of the ions the fields of all electrons with parallel and with antiparallel spin. The former distribution may be obtained by inserting u for x_n in (1) and integrating over all coordinates except x_1 and u, while the latter is obtained by a similar operation with the exception that the integration should be carried out over all coordinates except y_1 and u.

Actually, it had been shown in[1,4] that the wave functions ψ_ν of the free electrons in a Na-lattice are very nearly plane waves $e^{2\pi i \nu \cdot x/L}$ where L is the cube edge of the crystal and ν stands for a set of three integers, $\nu \cdot x$ denotes the scalar product of ν and x. Hence the charge distribution of the electrons with opposite spin is practically uniform, that of the electrons with parallel spin uniform with a "hole" around u.[5]

In no wave function of the type (1) is there a statistical correlation between the positions of electrons with antiparallel spin. The purpose of the aforementioned generalization of (1) is to allow for such correlations. This will lead to an improvement of the wave function and, therefore, to a lowering of the energy value. This energy gain will be called "correlation energy."

2.

The new form of the wave function, assumed in[1] was

$$\frac{1}{n!}\begin{vmatrix}\psi_1(y_1\cdots y_n;x_1)&\cdots&\psi_1(y_1\cdots y_n;x_n)\\ \vdots&&\vdots\\ \psi_n(y_1\cdots y_n;x_1)&\cdots&\psi_n(y_1\cdots y_n;x_n)\end{vmatrix}\begin{vmatrix}\psi_1(y_1)&\cdots&\psi_1(y_n)\\ \vdots&&\vdots\\ \psi_n(y_1)&\cdots&\psi_n(y_n)\end{vmatrix}, \quad (2)$$

[1] E. Wigner and F. Seitz, Phys. Rev. 46, 509 (1934).
[2] D. R. Hartree, Proc. Camb. Phil. Soc. 24, 89 (1928).
[3] J. C. Slater, Phys. Rev. 35, 210, 1930; V. Fock, Zeits. f. Physik 61, 126 (1930).
[4] J. C. Slater, Phys. Rev. 45, 794 (1934); A. Sommerfeld and H. Bethe, Geiger-Scheel's Handbuch der Physik, Vol. 24, 2nd part, 2nd edition, p. 406.
[5] E. Wigner and F. Seitz, Phys. Rev. 43, 804 (1933).

WC E.P. WIGNER, Phys. Rev. 46, 1002 (1934)

ENERGY SCALES

$$\frac{T}{N} \equiv t \sim \frac{\hbar^2 k_F^2}{2m} \quad \propto n^{2/3}$$

$$\frac{U}{N} \equiv u \sim e^2 n^{1/3}$$

$$\langle \psi_0 | T | \psi_0 \rangle = \left(\frac{\hbar^2}{2ma_0^2} \right) \frac{2.21}{r_s^2}$$

$$\langle \psi_0 | U | \psi_0 \rangle = \left(\frac{e^2}{2a_0} \right) \frac{-.916}{r_s}$$

$$n^{-1} = \frac{4\pi r_s^3 a_0^3}{3}$$

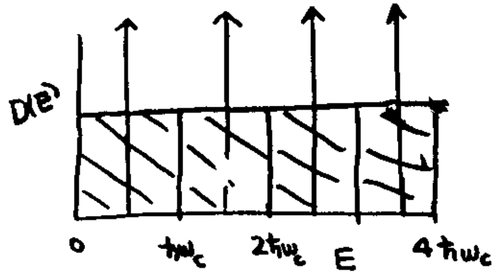
$$\langle \psi_{WC} | U | \psi_{WC} \rangle = \left(\frac{e^2}{2a_0} \right) \frac{-1.79}{r_s} \qquad \text{BCC LATTICE}$$

MFIC "MAGNETIC FIELD INDUCED Crystallization"

$$\frac{\hbar^2 (k_z^2 + k_\perp^2)}{2m} \rightarrow \frac{\hbar^2 k_z^2}{2m} + \hbar\omega_c (n + \tfrac{1}{2})$$

$$\frac{N_L}{A} = \frac{1}{(2\pi \ell^2)} \qquad \ell^2 = \frac{\hbar c}{eB} = \left(\frac{25.56\,\text{nm}}{\sqrt{B[\text{Tesla}]}} \right)^2$$

$$n \rightarrow \frac{k_F \nu}{\pi (2\pi \ell^2)}$$



$$\Rightarrow k_F \propto \ell^2 \propto \frac{1}{B}$$

$$\Rightarrow \langle T_z \rangle \propto \frac{1}{B^2}$$

$$\langle \psi_0 | T_z | \psi_0 \rangle = \frac{2\gamma \delta^2}{6} \propto \frac{1}{B^2} \qquad \delta \equiv k_F \ell = \frac{3\pi}{2 r_s^3 \gamma^{1.5} \nu}$$

$$\gamma = \frac{\hbar\omega_c}{2Ry^*}$$

IRRELEVANT IN STRONG FIELDS

lock-in techniques were used to measure longitudinal resistance (R_{xx}) and Hall resistance (R_{xy}) of the square

zero-r
in sam
quantu
With
a separ
ticle a
(Fig. 2
calcula
and ε

JIANG, WILLETT et. al, PRL $\underline{65}$, 633 (1990)

EXPECT REENTRANT BEHAVIOR

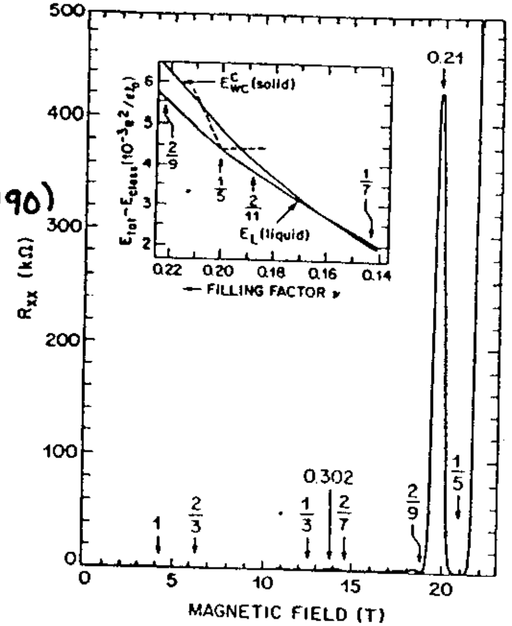
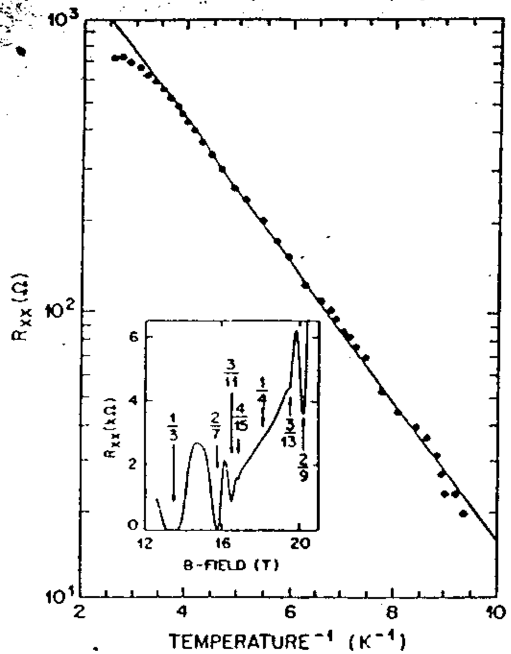


FIG. 1. Diagonal resistance R_{xx} vs magnetic field at $T\approx 90$ mK. Data are taken on a square sample so that $\rho_{xx}=\alpha R_{xx}$ with $\alpha\sim 1$. At $\nu=\frac{1}{5}$, $\rho_{xx}\rightarrow 0$ indicating that the $\nu=\frac{1}{5}$ quantum liquid forms the ground state. The resistivity ρ_{xx} in the sharp spike at $\nu\sim 0.21$ and for all $\nu\lesssim\frac{1}{5}$ is rising exponentially on lowering the temperature. All FQHE features at lower magnetic field are well developed but practically invisible on this scale. Inset: Result of a calculation for the total energy per flux quantum of the solid (E^c_{WC}) and interpolated $1/m$ quantum liquids (E_L) as a function of filling factor (Ref. 4). A classical energy ($E_{class}=-0.782133\nu^{-1/2}$) is subtracted for clarity. The dashed lines represent the cusp in the total energy (Ref. 13) of the liquid at $\nu=\frac{1}{5}$. Its extrapolation intersects the solid at $\nu\sim 0.21$ and 0.19 suggesting *two* phase transitions from quantum liquid to solid around $\nu=\frac{1}{5}$.

FIG. 2
activated
with an e
R_{xx} vs ma
density th
mized to
FQHE sta



ACTIVATED DISSIPATION

FIG. 2. Temperature dependence of R_{xx} at $\nu=\frac{1}{5}$. R_{xx} is activated $[R_{xx}\propto\exp(-\Delta^5/2T)]$ over 2 orders of magnitude with an energy gap of $\Delta^5=1.1$ K. Inset: Diagonal resistance R_{xx} vs magnetic field B at $T\sim 250$ mK and at a slightly higher density than in Fig. 1. The temperature for this trace is optimized to show the features in R_{xx} associated with various FQHE states.

JIANG et al. PRL $\underline{65}$, 633 (1990)
also
GLATTLI et al. Surf. Sci &
to be published
GOLDMAN et al. PREPRINT PRL $\underline{65}$, 2189 (1990)
PLAUT, KUKUSHKIN et al. ..
Clark et al. ... PRL JETP Lett. $\underline{52}$, 925 (1990)

...features present in its vicinity. The inset to ... shows several new structures associated with the ... higher-order states between $\nu = \frac{1}{5}$ and $\frac{1}{3}$. The ... minimum is unprecedentedly strong and appears ... very close to forming a zero-resistance state.

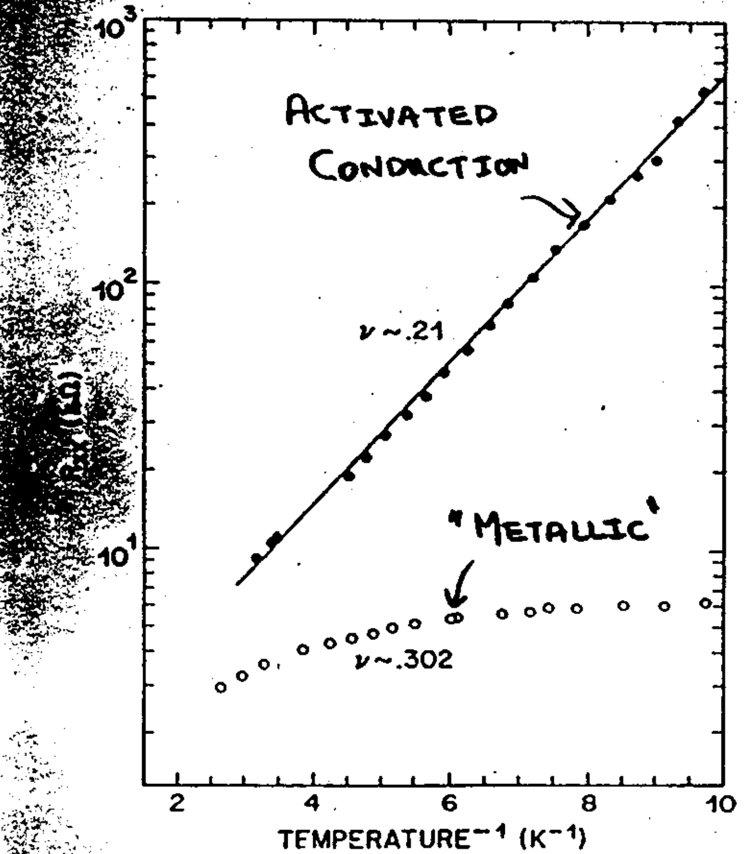


FIG. 3. The upper trace shows the temperature dependence of R_{xx} at $\nu = 0.210$. R_{xx} is strictly exponential $[R_{xx} \propto \exp(E_g/T)]$ with $E_g = 0.63$ K. R_{xx} of all other maxima at $\nu \gtrsim \frac{1}{5}$ lack this exponential dependence and saturate as $T \to 0$. This is shown in the lower trace for the second largest peak at

HF THEORY FOR 2D WC

$$V_{HF}(\vec{r}) = \int d\vec{r}\,' \, U(\vec{r}-\vec{r}\,') \, n(\vec{r}\,')$$

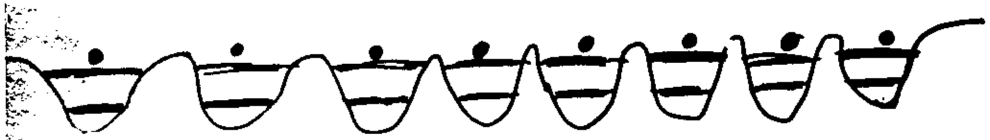
$$U(\vec{q}) = \frac{2\pi e^2}{q} - 2\pi e^2 \ell \sqrt{\frac{\pi}{2}} \, e^{q^2\ell^2/4} \, I_0(q^2\ell^2/4)$$

$$\equiv V_c(q) - I(q)$$

Analyticity

AHM & SMG PRB <u>38</u>, 6295 (1988)

- EXCHANGE ≡ SIC



MAGNETOPHONON'S (HARMONIC APPROX.)

$$-\omega^2 \vec{u}(\vec{k}) = \underbrace{-\overleftrightarrow{D}(\vec{k})\cdot\vec{u}(\vec{k})}_{\text{Coulomb Force}} \quad \underbrace{-i\omega\omega_c \hat{z}\times\vec{u}(\vec{k})}_{\text{Lorentz Force}}$$

$$(D_{11}-\omega^2)(D_{22}-\omega^2) - (D_{12}-i\omega\omega_c)(D_{12}+i\omega\omega_c) = 0$$

$$\omega^4 - \omega^2(D_{11}+D_{22}+\omega_c^2) + D_{11}D_{22}-D_{12}^2 = 0$$

Strong Field Limit

$$\omega_- = \frac{\sqrt{\det D}}{\omega_c} = \frac{\omega_0^2}{\omega_c}\sqrt{\det \tilde{D}}$$

INDEPENDENT OF m !

"MAGNETOPHONONS"

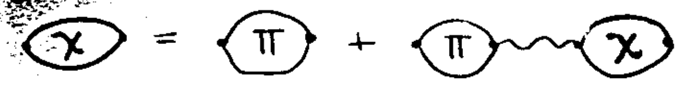
$$\omega_+^2 = \omega_c^2 + \mathrm{tr}\, D$$

"MAGNETOPLASMONS"

$$\Rightarrow \omega_+ = \omega_c + \frac{1}{2}\frac{\mathrm{tr}\, D}{\omega_c} = \omega_c + \frac{\omega_0^2}{2\omega_c}\mathrm{tr}(\tilde{D})$$

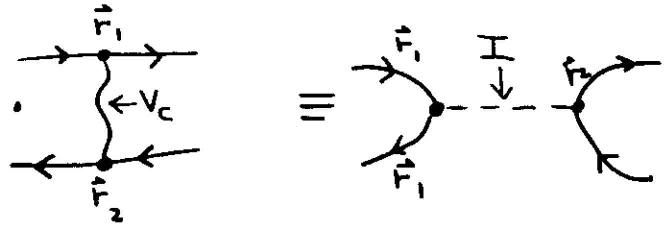
$$\omega_0^2 = \frac{8e^2}{ma_0^3} \qquad \frac{\omega_0^2}{\omega_c} = \left(\frac{e^2}{\hbar\epsilon\ell}\right)\left(\frac{\sqrt{3}r}{\pi}\right)^{1.5}$$

TIME-DEPENDENT HFA



BUT

ANALYTICITY



$\Rightarrow$

$\Rightarrow$

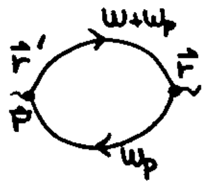
RPA-phonons

Brenig
Werthamer } solid He

LIFE ON EASY STREET

I. "No" KINETIC ENERGY

$$H_{HF} = \frac{1}{A}\sum_{\vec G} U(\vec G)\rho_{HF}(\vec G)\,\overline{e^{i\vec G\cdot\vec r}}$$



$$\chi^0(\vec r,\vec r\,':\omega_n) = \frac{1}{\beta\hbar}\sum_{\omega_p} G(\vec r,\vec r\,';\omega_n+\omega_p)\,G(\vec r\,',\vec r:\omega_p)$$

$$= \sum_{\alpha,\beta}\frac{n_F(\epsilon_\alpha)-n_F(\epsilon_\beta)}{i\omega_n-(\epsilon_\alpha-\epsilon_\beta)}\;\varphi^*_\alpha(\vec r)\varphi_\alpha(\vec r\,')\varphi_\beta(\vec r)\varphi^*_\beta(\vec r\,')$$

Eq. of Motion

$$[\text{illegible}]_{\vec G,\vec G'}(\vec k:i\omega_n) = \int_0^{\beta\hbar} d\tau\, e^{-i\omega_n\tau}\Big\{-\big\langle T_\tau\big[e^{H_{HF}\tau}\bar\rho_{\vec k+\vec G}\,e^{-H_{HF}\tau}\,\bar\rho_{-\vec k-\vec G'}\big]\big\rangle\Big\}$$

$$\left[\bar\rho(\vec k_1),\bar\rho(\vec k_2)\right] = 2i\exp\left(\frac{\vec k_1\cdot\vec k_2}{2}\right)\sin\left(\frac{(\vec k_1\times\vec k_2)\cdot\hat z}{2}\right)\bar\rho(\vec k_1+\vec k_2)$$

$$\bar\rho(\vec k) = \sum_i \overline{e^{i\vec k\cdot\vec r_i}}$$

$$H_{HF} = \frac{1}{A}\sum_{\vec G} W(\vec G)\,\bar\rho(\vec G,\tau) \qquad W(\vec G) = U(\vec G)\rho_{HF}(\vec G)$$

!! Eq. of Motion Closes !!

$$\sum_{\vec G''}\left(\omega\delta_{\vec G,\vec G''} - A_{\vec G,\vec G''}(\vec k)\right)\chi^0_{\vec G'',\vec G'}(\vec k,\omega) = -B_{\vec G,\vec G'}(\vec k)$$

$$\left(\omega I - A(\vec k) + \underbrace{B(\vec k)\,U(\vec k)}_{\uparrow}\right)\chi(\vec k:\omega) = -B(\vec k)$$

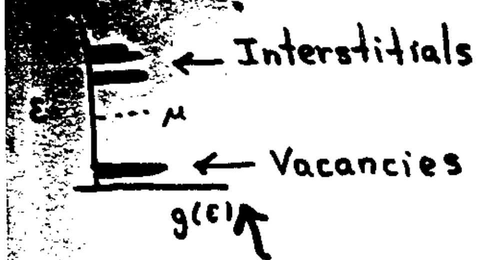
Local Field Corrections

$$U(\vec k) = \delta_{\vec G,\vec G'}\, U(\vec k+\vec G)$$

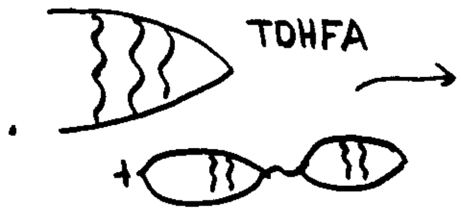
$$= \delta_{\vec G,\vec G'}\left(\frac{2\pi e^2}{|\vec k+\vec G|} - I(|\vec k+\vec G|)\right)$$

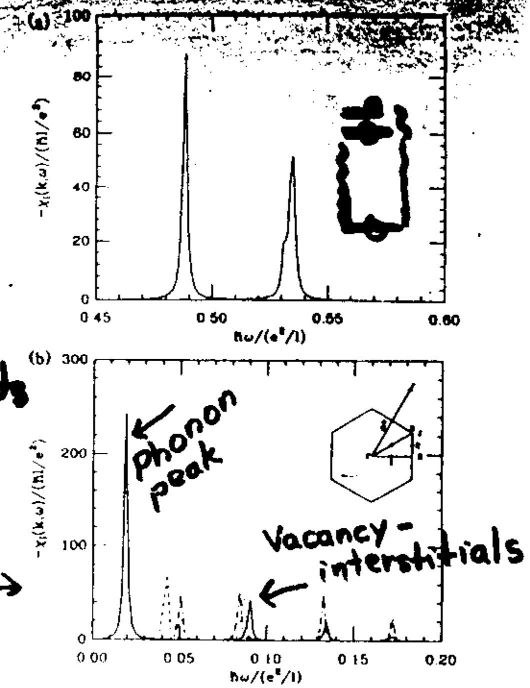
- RECOVERS HARMONIC LIMIT
- EXCHANGE TREATED EXACTLY
- "SELF-CONSISTENT PHONON" TYPE EFFECTS + OTHER ANHARMONICITY WITHOUT EXPANSION IN DISPLACEMENTS

TDHFA for Wigner Crystal ($\nu = 1/4$)



SC Hofstader bands





$$\omega \sim \frac{v}{r} \sim \frac{c\,\overset{m\omega_0^2 r}{\vec F}}{eB}\cdot\frac{1}{r}$$

$$\sim \frac{\omega_0^2}{\omega_c}$$

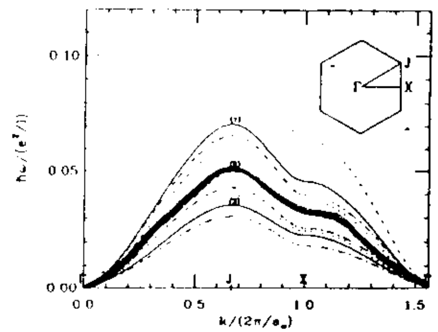


FIG. 2. Dispersion relation of the magnetophonon along the edges of the irreducible Brillouin zone (see inset) for filling factors (1) $\nu = $ [illegible], (2) $\nu = $ [illegible], (3) $\nu = $ [illegible], and for the TDHFA (solid line), the harmonic approximation (dash-dotted line), and the form-factor approximation (dashed line). At a given filling factor, the TDHFA result lies between the other results.

René Côté + AHM
PRB 44, 8759 (1991)

QUESTIONS IN THE THEORY OF THE WIGNER X-TAL STATE

— ≡ Editorial Remarks

- PHASE TRANSITION

  1st order or continuous? — 1st order with no disorder

  Quantitative $\frac{k_B T_c}{e^2/\epsilon l}(\nu)$ ⟵ Classical limit ?

- CONDUCTIVITY

  $\sigma(\omega)$ — Low Freq. ? Pinning Peak. Continuous transition with disorder ?

  dc non-linear — Why is depinning signature so weak ?

  $\sigma_H(\omega)$ — Hall insulator ?

  dc activation energy — ??

DOUBLE & MULTIPLE-LAYER SYSTEMS

(or non spin-polarized systems)

- CHARGE GAPS AT 'NEW' FILLING FACTORS

  - Generalized Laughlin Wave functions (Halperin)
  - Complex Fractionally (Multilayer ≡ Irrational Charge) charged excitations
  - Crossovers with strength of interlayer tunneling

- NOVEL BROKEN SYMMETRIES & RELATED PHASE TRANSITIONS

LAUGHLIN THEORY (SINGLE LAYER)

$$\longrightarrow \psi_m(z) = \frac{z^m}{\sqrt{2\pi \ell^2 2^m m!}} e^{-|z|^2/4\ell^2} \qquad \ell^2 = \frac{\hbar c}{eB}$$

$$\Psi_1 = \begin{vmatrix} z_1^0 & \cdots & z_N^0 \\ \vdots & & \vdots \\ z_1^{N-1} & \cdots & z_N^{N-1} \end{vmatrix} \prod_k e^{-|z_k|^2/4\ell^2}$$

$\nu = 1$ droplet

$$= \prod_{i<j} (z_i - z_j) \prod_k e^{-|z_k|^2/4\ell^2}$$

$$\Psi_m = \prod_{i<j} (z_i - z_j)^m \prod_k e^{-|z_k|^2/4\ell^2}$$

$\nu = 1/m$ droplet

$$\mathcal{H}_m^{(0)} = \sum_{\langle ij \rangle} \sum_{\ell < m} V_\ell P_\ell^{ij}$$

Haldane pseudopotentials

PLASMA ANALOGY

$$|\psi_m|^2 \equiv \exp(-\mathcal{U})$$

$$\Rightarrow \mathcal{U} = \sum_{\langle ij \rangle} m \left(-2 \ln |z_i - z_j|\right) + \sum_k \frac{x_k^2 + y_k^2}{2\ell^2} \Rightarrow n_B = -\frac{1}{2\pi \ell^2}$$

$$\Rightarrow n = \frac{1}{m} \cdot \frac{1}{2\pi \ell^2} \qquad \left[\nu = \frac{1}{m} \right]$$

LAUGHLIN THEORY
- DOUBLE LAYER -

$[i] = i + \frac{N}{2}$

$$\Psi_{m,m,n} = \prod_{i<j} (z_i - z_j)^m (z_{[i]} - z_{[j]})^m \prod_{k,l} (z_k - z_{[l]})^n \otimes \prod_l e^{-|z_l|^2/4l^2}$$

$$\left.\begin{aligned} m \cdot n_L + n \cdot n_R &= (2\pi l^2)^{-1} \\ n \cdot n_R + m \cdot n_L &= (2\pi l^2)^{-1} \end{aligned}\right\} \text{Plasma Analogy}$$

$$\Rightarrow n_L = n_R = (2\pi l^2)^{-1} \left(\frac{1}{m+n} \right)$$

Charge neutrality condition

$$\text{i.e. } \nu_L = \nu_R = \frac{1}{m+n}$$

Table I

Generalized Laughlin states for two component systems. S is the total spin quantum and * denotes a state which is not an eigenstate of $\vec{S}^2_T$.

m	m′	n	$\nu_\uparrow$	$\nu_\downarrow$	ν	S
1	1	0	1	1	2	0
1	1	1	1/2	1/2	1	N/2
1	3	0	1	1/3	4/3	N/4
1	5	0	1	1/5	6/5	N/3
3	3	0	1/3	1/3	2/3	*
3	3	1	1/4	1/4	1/2	*
3	3	2	1/5	1/5	2/5	0
3	3	3	1/6	1/6	1/3	N/2
3	5	0	1/3	1/5	8/15	*
3	5	1	2/7	1/7	3/7	*
3	5	2	3/11	[redacted]/11	[redacted]/11	N/4
5	5	0	1/5	1/5	2/5	*
5	5	1	1/6	1/6	1/3	*
5	5	2	1/7	1/7	2/7	*
5	5	3	1/8	1/8	1/4	*
5	5	4	1/9	1/9	2/9	0
5	5	5	1/10	1/10	1/5	N/2

MAXIMALLY POLARIZED

Table II

Quantum numbers and filling factors for some generalized Laughlin states in three layer systems.

M_{11}	M_{22}	M_{33}	M_{12}	M_{23}	ν_1	ν_2	ν_3	ν
1	1	1	0	0	1	1	1	3
1	1	1	1	0	1/2	1/2	1	2
3	3	3	0	0	1/3	1/3	1/3	1
3	3	3	1	0	1/4	1/4	1/3	[redacted] 5/6
3	3	3	1	1	2/7	1/7	2/7	5/7
5	3	3	1	0	1/7	2/7	1/3	16/21
5	3	3	1	1	6/37	7/37	10/37	23/37
3	5	3	1	1	12/39	3/39	12/39	27/39
5	5	5	1	1	4/23	[redacted]/23	4/23	11/23
5	5	5	2	2	3/17	1/17	3/17	7/17

AHM, Surface Science, 1990

Spin States

$$\Psi_{m,m,n} = \mathcal{A}\left[\alpha_1 \cdots \alpha_{N/2}\, \beta_{1+\frac{N}{2}} \cdots \beta_N\, \psi_{m,m,n}\right]$$

$$\vec{S}^2 \Psi_{m,m,n} = \frac{1}{2}\left[S_- S_+ + S_+ S_-\right] \Psi_{m,m,n}$$

$$= \frac{N}{2} + \frac{1}{2}\sum_{i \neq j} \left(S_{-i} S_{+j} + S_{+i} S_{-j}\right) \Psi_{m,m,n}$$

$$= \mathcal{A}\left[\alpha_1 \cdots \alpha_{\frac{N}{2}}\, \beta_{1+\frac{N}{2}} \cdots \beta_N\, \tilde{\psi}_{m,m,n}\right]$$

$$\tilde{\psi}_{m,m,n} = \frac{N}{2}\, \psi_{m,m,n} + \sum_{i,j} \left(-e(i,[j])\right) \psi_{m,m,n}$$

e.g.

$$\tilde{\psi}_{m,m,m} = \left[\frac{N}{2} + \left(\frac{N}{2}\right)^2\right] \psi_{m,m,m} \Rightarrow S = \frac{N}{2}$$

$$S_+ \psi_{1,1,0} \equiv 0$$

$$\Rightarrow \sum_j e(i,[j])\, \psi_{m,m,m-1} = \psi_{m,m,m-1} \Rightarrow S = 0$$

Quasiparticle Charges

Double-Layer

$$|\psi'_\alpha|^2 = \left(\prod_i z_{i,\alpha}\right) |\psi|^2$$

$$\Rightarrow U' = \sum_{i,\gamma} \frac{|z_{i,\gamma}|^2}{2\ell^2} - 2m \sum_{i<j,\ \gamma} \ln|z_{i,\gamma} - z_{j,\gamma}|$$

$$- 2n \sum_{i,j} \ln|z_{i,\alpha} - z_{j,\beta}|$$

$$+ \ldots$$

$$\Rightarrow \quad m\,\delta q_\alpha + n\,\delta q_\beta = -1$$

$$n\,\delta q_\alpha + m\,\delta q_\beta = 0$$

$$\Rightarrow \delta q_\alpha = \frac{-m}{m^2 - n^2} \qquad \delta q_\beta = \frac{n}{m^2 - n^2}$$

$$\delta q_\alpha + \delta q_\beta = \frac{1}{m+n}$$

MULTI-LAYER SYSTEMS

Qiu, Joynt & AHM
PRB 40, 11943 (1989)

$$\sum_j{}' A_{i,j}\,\delta q_j = -\delta_{i,0}$$

$$\Rightarrow \quad \delta q_j = -\int_{-\pi}^{\pi}\frac{dk}{2\pi}\,A(k)\,e^{-ikj}$$

$$A(k) = \sum_{i-j=-\infty}^{\infty} A_{i-j}\,e^{ik(i-j)}$$

e.g. $|1,3,1\rangle$ $\quad A(k) = 3 + 2\cos k$

$$\Rightarrow \quad \delta q_j = \int_{-\pi}^{\pi}\frac{dk}{2\pi}\,\frac{e^{-ikj}}{3+2\cos k}$$

Irrational!

$$= \frac{\left[(\sqrt{5}-3)/2\right]^{|j|}}{\sqrt{5}}$$

SPONTANEOUS INTERLAYER COHERENCE

IN TWO-LAYER QUANTUM HALL SYSTEMS

PRL 72, 732 (1994)
+ live TeX file

Ψ

Steve Girvin
Kun Yang (→ Princeton)
Lian Zheng (→ Kentucky)
Kyungsun Moon
(Hiroshima →) Hiro Mori
AHM

Shou-Cheng Zhang (Stanford)
René Côté (Sherbrooke)
Daijiro Yoshioka (Tokyo)
Herb Fertig (Kentucky)
Luis Brey (Madrid)

MULTI- LAYER- QUASIPARTICLES

$$\Psi = \prod_{j,i,k\geq k'} \left(z_{i,k} - z_{j,k'}\right)^{m_{k'-k}} \prod_{\ell} \exp\left(-|z_\ell|^2/4\ell^2\right)$$

$$U = -2 \sum_{\substack{i,j \\ k\geq k'}} m_{k'-k} \ln|z_{i,k} - z_{j,k'}| + \sum_{\ell} \frac{|z_\ell|^2}{2\ell^2}$$

$$\Rightarrow \rho \sum_{k'\geq k} m_{k'-k} = \frac{1}{(4\pi\ell^2)}$$

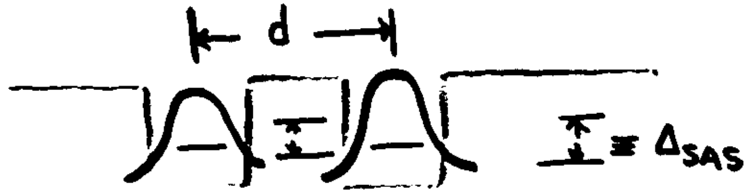
$$\Psi_k' = \prod_i z_{i,k}\, \Psi$$

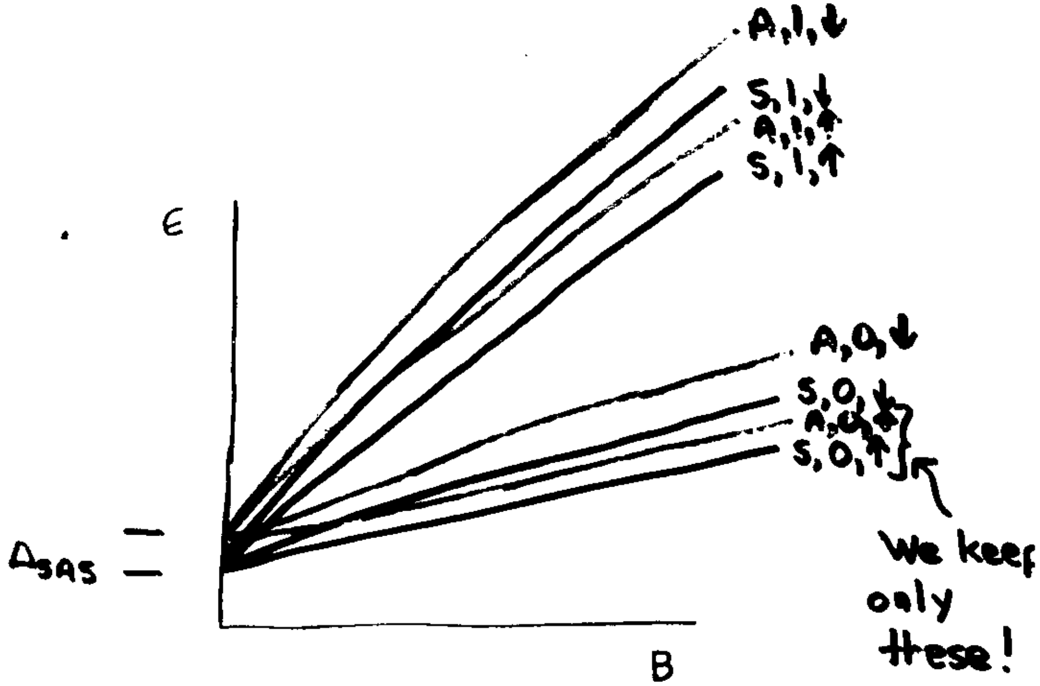
$$U' = U - 2 \sum_{i,k} \ln|z_{i,k}|$$

$$\sum_{k'}{}' \delta n_{k'} m_{k'-k} = -\delta_{k,0}$$

Solve by Fourier X-form

Double-Layer Systems





Strong Fields $\equiv \hbar\omega_c,\ g\mu_B B \gg \Delta_{SAS}$

Related Work

Spins
- Halperin, Helv. Phys. Acta. 56, 75 (1983)
- Haldane, .. in Steve's book
- Kallin, Halperin, PRB 31, 3635 (1985)
- Rasolt + AHM, PRB 34, 5530 (1986)

Layers
- Rezayi + Haldane, BAPS 32, 892 (1987)
- Chakraborty + Pietiläinen, PRL 59, 2784 (1987)
- Yoshioka, AHM, Girvin, PRB 39, 1932 (1989)

[illegible]
- Fertig, PRB 40, 1087 (1989)
- Brey, PRL 65, 903 (1990)
- AHM, Platzman, Boebinger, PRL 65, 775 (1990)

Broken Sym.
- Wen + Zee, PRL 69, 1811 (1992)
- Ezawa + Iwazaki, Int. J. Mod. Phys. B 19, 3205 (1992)
- He, Das Sarma, Xie, PRB 47, 4394 (1993)
- Chen + Quinn, PRB 45, 11 054 (1992)
- Coté, Brey, AHM, PRB 46, 10239 (1992)

Parallel Fields
- Murphy, Eisenstein, Boebinger, ... PRL 72, 728 (1994)
- Sondhi, Karlhede, Kivelson, Rezayi PRB 47, 16419 (1993)
- Yang + many others

WHAT?

$$\lim_{\vec{R}\to\infty} \left\langle \left(\hat{\Psi}_L^{+}(\vec{r}+\vec{R})\, \hat{\Psi}_R(\vec{r}+\vec{R}) \right)^{+} \hat{\Psi}_L^{+}(\vec{r})\, \hat{\Psi}_R(\vec{r}) \right\rangle_0 \neq 0$$

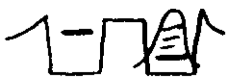
or $\quad \left\langle \hat{\Psi}_L^{+}(\vec{r})\, \hat{\Psi}_R(\vec{r}) \right\rangle_0 \neq 0$

- even for no hopping

WHY ?

- Good Interlayer Correlations
- No Kinetic Energy Cost

- STUPID STATE



$$|\Psi_{HF}\rangle = \prod_X c^+_{R,X} |0\rangle$$

$$\varepsilon_{RHF} = E_0 + \frac{\hbar\omega_c}{2} + \frac{\pi e^2 d}{\ell^2} - I_A$$

$$\varepsilon_{LAF} = E_0 + \frac{\hbar\omega_c}{2} - \frac{\pi e^2 d}{\ell^2}$$

- SMARTER STATE

$$|\Psi_{HF}\rangle = \prod_X \frac{1}{\sqrt{2}} \left(c^+_{LX} + e^{i\varphi} c^+_{RX} \right)$$

$$\varepsilon_{+HF} = E_0 + \frac{\hbar\omega_c}{2} - \left(\frac{I_A + I_E}{2}\right)$$

$$\varepsilon_{-HF} = E_0 + \frac{\hbar\omega_c}{2} - \left(\frac{I_A - I_E}{2}\right)$$

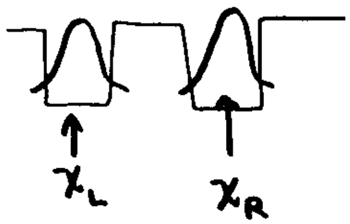
$$g_{RR}(r) = g_{LR}(r)$$

$$g_{RR}(r=0) = g_{LR}(r=0) = 0$$

Spin & 'Iso-spin'

$$\Psi(z) = \alpha\, \chi_L(z) + \beta\, \chi_R(z)$$

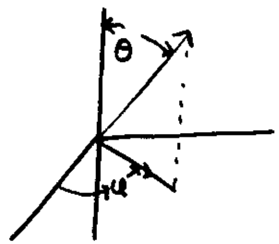
$$\longrightarrow \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$



- $\sigma_z |L\rangle = |L\rangle$ $\qquad \langle\sigma_z\rangle \neq 0 \Rightarrow$ Polarization
- $\sigma_z |R\rangle = -|R\rangle$ $\qquad \langle\vec{\sigma}_\perp\rangle \neq 0 \Rightarrow$ Coherence
- $\sigma_x |S\rangle = |S\rangle$
- $\sigma_x |A\rangle = -|A\rangle$

General Isospinor

$$\begin{pmatrix} \cos\theta/2 \\ \sin\theta/2\, e^{i\varphi} \end{pmatrix}$$



Corresponding Operators

Bias Potential $\longrightarrow$ Zeeman Field $V\hat{z}$

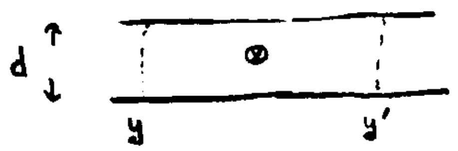
$$\begin{pmatrix} V/2 & 0 \\ 0 & -V/2 \end{pmatrix} \longrightarrow \frac{V}{2}\sigma_z$$

Hopping $\longrightarrow$ Zeeman Field $-\Delta_{SAS}\hat{x}$

$$\begin{pmatrix} 0 & -t \\ -t & 0 \end{pmatrix} \qquad -t\,\sigma_x$$

Hopping & Parallel Field $\longrightarrow$ Rotating Zeeman Field

$$\begin{pmatrix} 0 & -te^{iQy} \\ -te^{-iQy} & 0 \end{pmatrix} \qquad -t\,\vec{\sigma}\cdot(\hat{x}\cos Qy + \hat{y}\sin Qy)$$



$$\Phi = B_{\parallel} d(y'-y) \equiv \frac{\Phi_0}{2\pi} Q(y'-y)$$

Spin-Texture Properties

$$|\hat{m}(\vec{r})\rangle \equiv e^{-iO}|\Psi_0\rangle$$

- Charge $\equiv$ Topological Charge

$$\delta\rho(\vec{r}) = -\frac{1}{8\pi}\varepsilon_{\mu\nu}\vec{m}(\vec{r})\cdot(\partial_\mu\vec{m}(\vec{r})\times\partial_\nu\vec{m}(\vec{r}))$$

Sondhi et al. PRB **47**, 16419 (1993)

Fradkin, <u>Field Theories in Condensed Matter</u>

e.g. Skyrmion has charge 1

- Energy Functional

$$E[\vec{m}] \equiv \langle m(\vec{r})|\mathcal{H}|m(\vec{r})\rangle$$

$$= \int d\vec{r}\left[\beta m_z^2(\vec{r}) + \frac{\rho_A}{2}|\vec{\nabla}m_z|^2 + \frac{\rho_E}{2}|\vec{\nabla}m_\perp|^2\right] + \cdots$$

= anisotropic $\Rightarrow$ KT transition

- Equation of Motion

$$\left.\begin{aligned} \frac{dm_y(\vec{q})}{dt} &= \frac{4\pi}{\nu}\frac{\partial E[\vec{m}]}{\partial m_z(-\vec{q})} \\ \frac{dm_z(\vec{q})}{dt} &= -\frac{4\pi}{\nu}\frac{\partial E[\vec{m}]}{\partial m_y(-\vec{q})} \end{aligned}\right\} \text{Precession}$$

Hartree-Fock Energy Functional Parameters

$$\rho_E \sim \int d\vec{k}\; V_E(\vec{k})\, e^{-|\vec{k}|^2/2}\, k^2$$

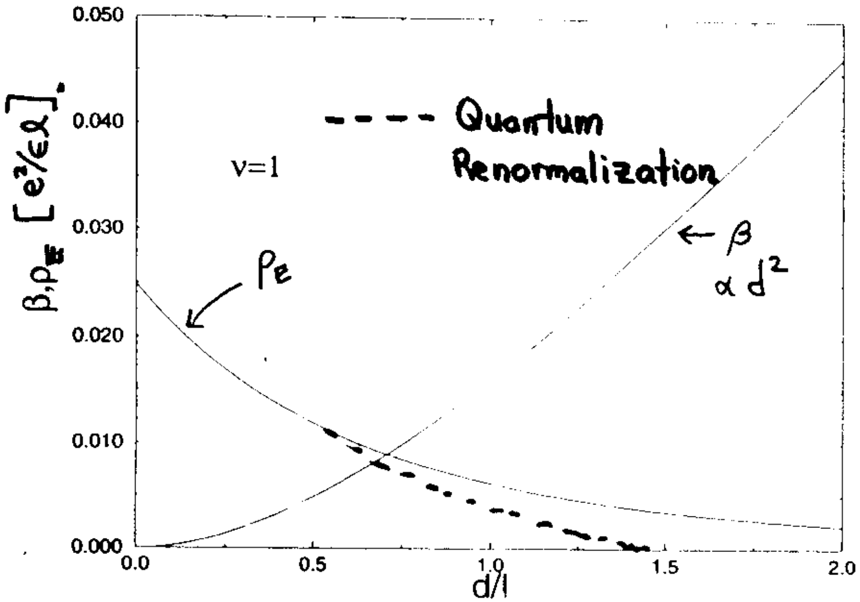
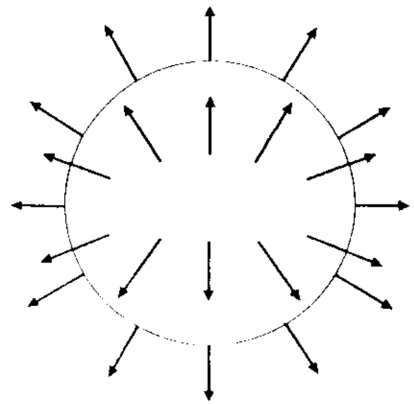


FIG. 4.



Skyrmion on Sphere

$$\Delta E = \frac{e^2}{\epsilon \ell} \cdot \frac{1}{2} \sqrt{\frac{\pi}{2}}$$

Charge Gap > 1993

Sondhi, Karlheds, Kivelson, Rezayi
PRB <u>47</u>, 16419 (1993)

Finite-size exact diagonalization order parameter estimates

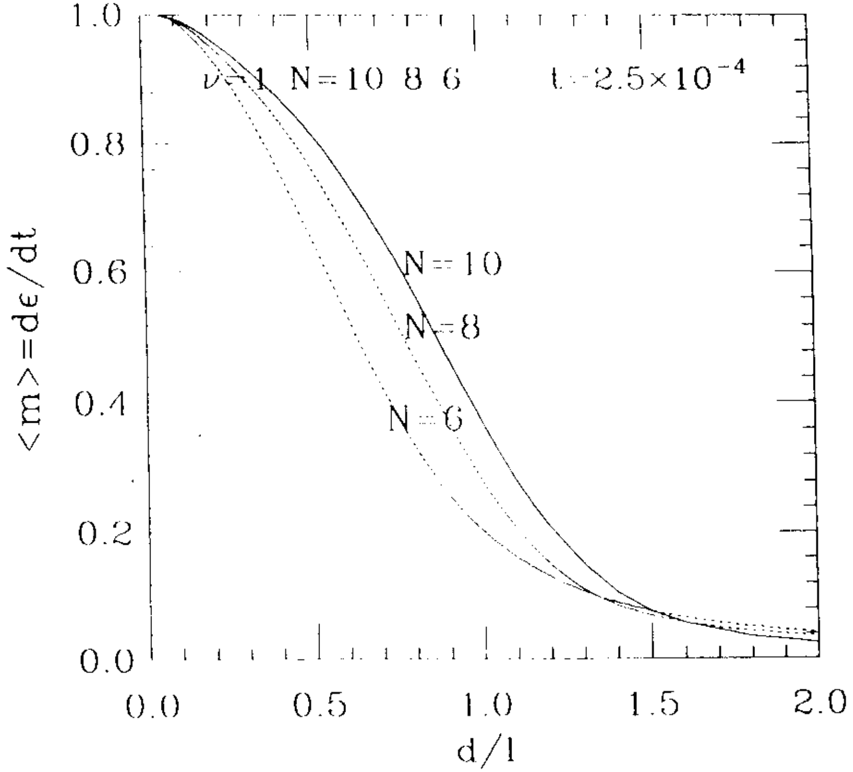
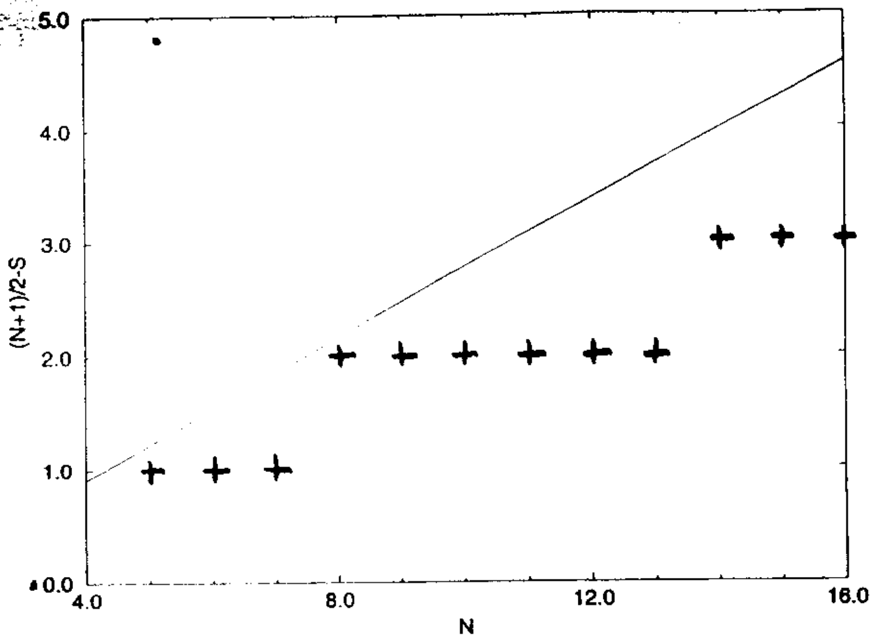


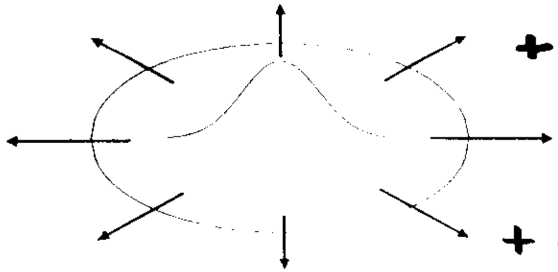
Figure 6 **Frustrated Skyrmion**



$\nu = 1^{\pm}$ Ground State Spin Quantum Number on Torus

FIG. 7.1

Meron Spin Textures



+ vorticity

+ polarization

$$\vec{m} \doteq \left(\sqrt{1-m_z^2}\cos\varphi, \sqrt{1-m_z^2}\sin\pm\varphi, \pm m_z\right)$$

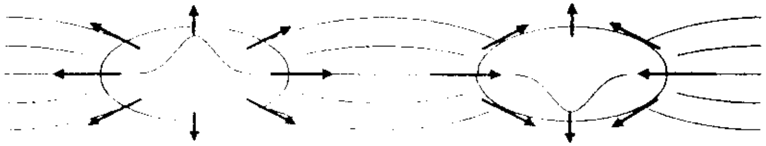
$$Q = -\frac{1}{2}\left[\text{vorticity} \otimes \text{polarization}\right]$$

$$= -\frac{1}{2}$$

FIG. 7.

Charged Spin Texture

$d/\ell \neq 0$



+ vorticity	− vorticity
+ polarization	− polarization
$Q = -\frac{1}{2}$	$Q = -\frac{1}{2}$

$$Q_{TOT} = -1$$

$$E = 2E_{core} + \rho_E \ln R + \frac{e^2}{4R}$$

'Josephson-Like' Effects

$$I \sim e\dot{M}_z \sim \frac{et}{\hbar} M_y$$

$\dot{M}_y \not\equiv 0 \Rightarrow$ no dc 'Josephson effect'

$$\frac{dm_y}{dt} \sim \frac{\partial E}{\partial m_z} \sim (\beta + t)\, m_z$$

$$\frac{dm_z}{dt} \sim -\frac{\partial E}{\partial m_y} \sim -t\, m_y$$

$$\Rightarrow \omega \sim \sqrt{t(\beta + t)} \ll \Delta$$

$\sim$ Wannier-Stark $\sim$ Bloch oscillations

'Meissner-Like' Effects

$$B_{\parallel} \neq 0$$

$$E \sim \int d^2\vec{r} \left[\underbrace{\frac{\rho_E}{2}}_{\substack{\text{Interlayer}\\ \text{Exchange}\\ \text{Energy}}} (\vec{\nabla}\varphi)^2 - \underbrace{t\,\xi}_{\substack{\text{Hopping Energy}\\ \text{in Parallel Field}}} \cos(\varphi - \overset{\propto B_{\parallel}}{Q} y) \right]$$

$$+ \frac{dA}{8\pi}(B_{\parallel} - H)^2 \leftarrow \text{Magnetic Energy}$$

Ground State

$$\varphi = Qy$$

$$\Rightarrow E \sim A\left[\frac{\rho_E}{2} Q^2 + \frac{d}{8\pi}(B_{\parallel} - H)^2\right]$$

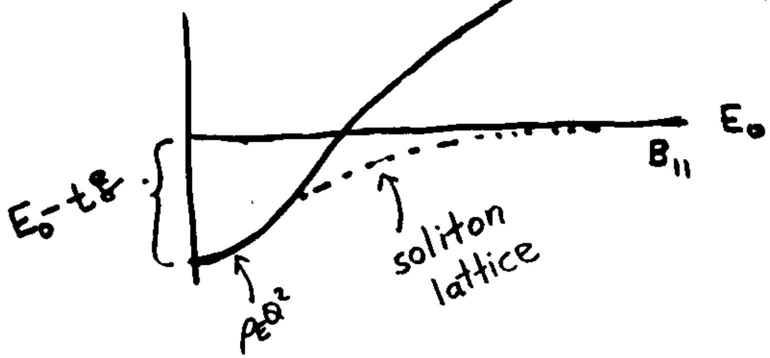
$$\Rightarrow (B_{\parallel} - H) \sim B_{\parallel}$$

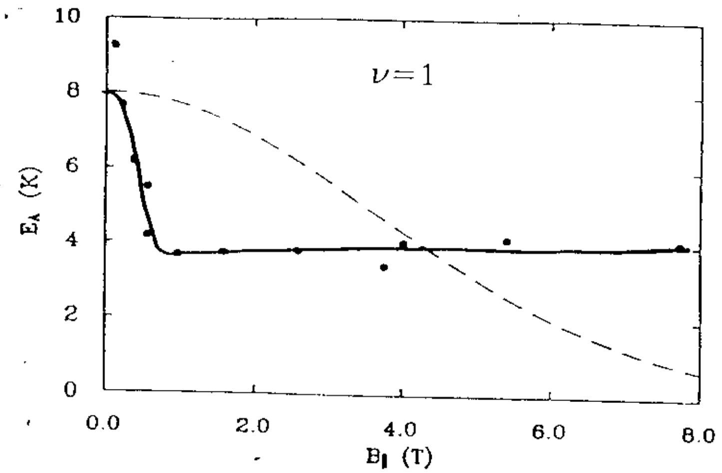
$\sim$ diamagnetic

Commensurate-Incommensurate Phase Transition

- Low $B_{\parallel}$ State $\qquad \varphi = Qy$ ← Satisfy hopping energy

High $B_{\parallel}$ State $\qquad \varphi =$ constant ← Satisfy exchange energy





3: Activation energy of the quantized state at $\nu=1$ versus in-plane magnetic field, $B_{\parallel}$. The hed line (normalized to the data at $B_{\parallel}=0$) is the calculated dependence of a single-particle neling gap at $\nu=1$. The relative independence of the activation energy over the range $1<B_{\parallel}<8$T trong evidence that the $\nu=1$ state of Fig. 2 does not arise from single-particle tunneling.

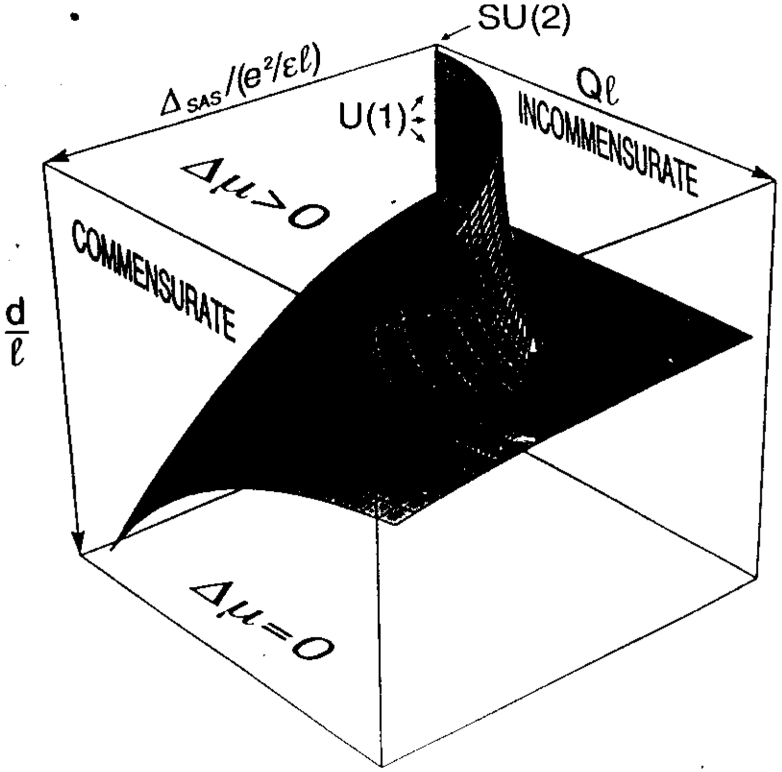
Murphy et al.

PRL

One Monday during January

PRL 72, 728 (1994)

SUMMARY



... more to come