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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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SMR. 758 - 20

SPRING COLLEGE IN CONDENSED MATTER ON QUANTUM PHASES

(3 May - 10 June 1994)

MAGNETOTRANSPORT MEASUREMENTS IN QUANTUM HALL EFFECT

LECTURES 1 and 4

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MAGNETOTRANSPORT MEASUREMENTS OF QHE

I. General introduction
   1. Experimental (sample structures and fabrication)
   2. Basic IQHE and FQHE characteristics: Localization and Correlation
   3. Energy gap in FQHE

II. Ground state at low fillings
   1. Quest for low fillings & Wigner crystal (WC)
   2. The case for a pinned WC near $\nu=1/5$ in GaAs/AlGaAs 2 DEG
   3. Other systems: 2DHG, bilayer systems, etc.
   4. Role of disorder: single-particle localization vs pinned WC

III. Correlated states in bilayer electron systems
   1. Quest for even-denominator FQHE
   2. Magnetic-field-induced collapse of tunneling gap
   3. Observation of $\nu=1/2$ & $3/2$ FQHE states
   4. 1-component to 2-component transitions
   5. Many-body QHE at $\nu=1$

IV. Half-filled Landau levels
   1. Composite Fermions & the existence of a fermi surface at $\nu=1/2$
   2. FQHE as IQHE of composite Fermions

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MAGNETOTRANSPORT MEASUREMENTS IN QUANTUM HALL EFFECT

LECTURE 1

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Magnetotransport measurements of QHE

(mostly FRACTIONAL QHE)

Part I: General introduction

1. Experimental details (samples)
2. Basic IQHE and FQHE characteristics:
   Localization & Correlation
3. Energy gap in FQHE

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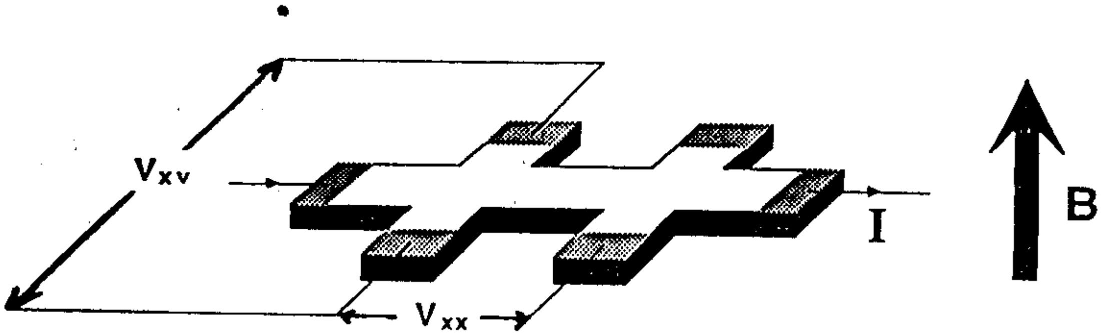
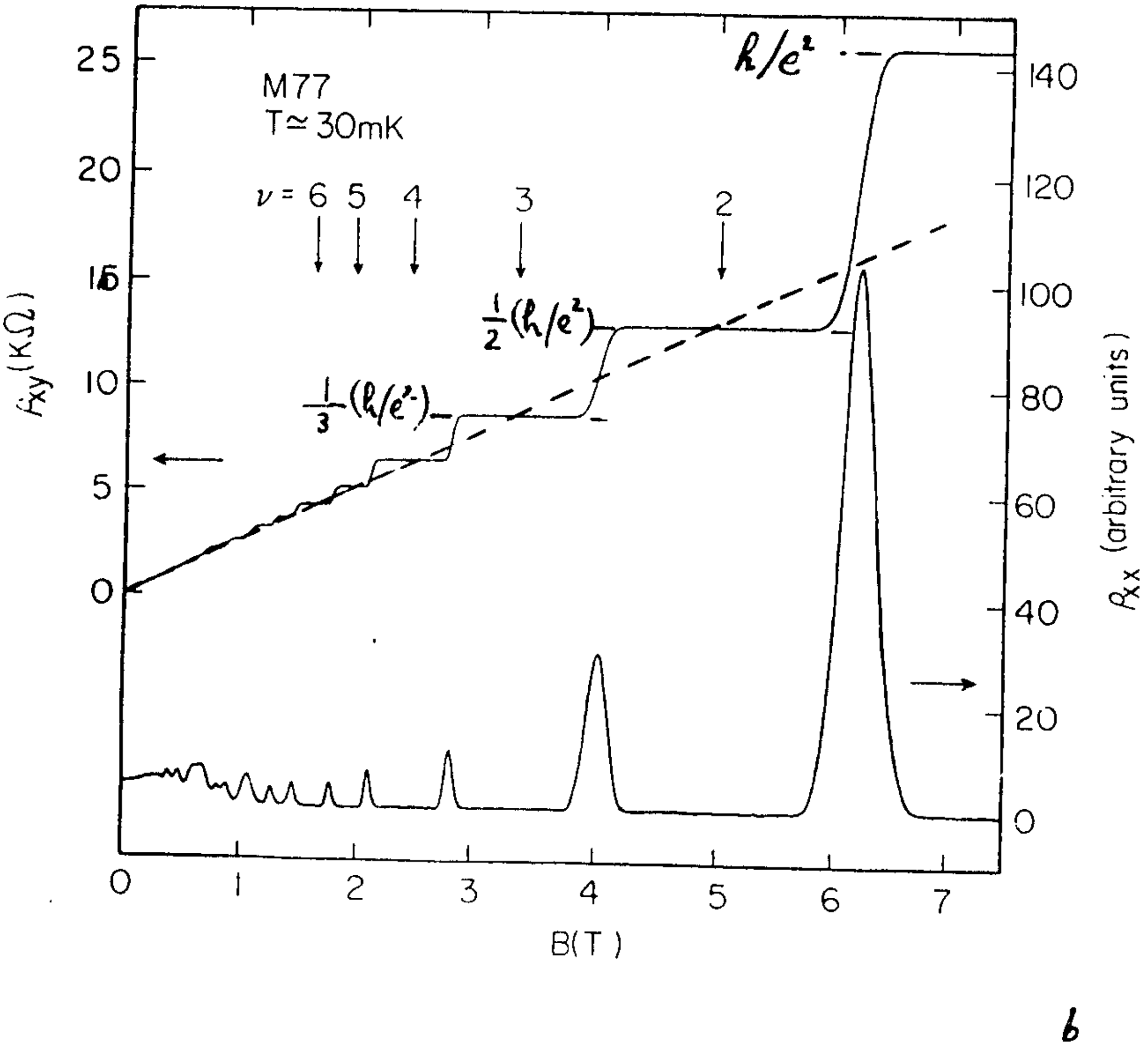
May 1994

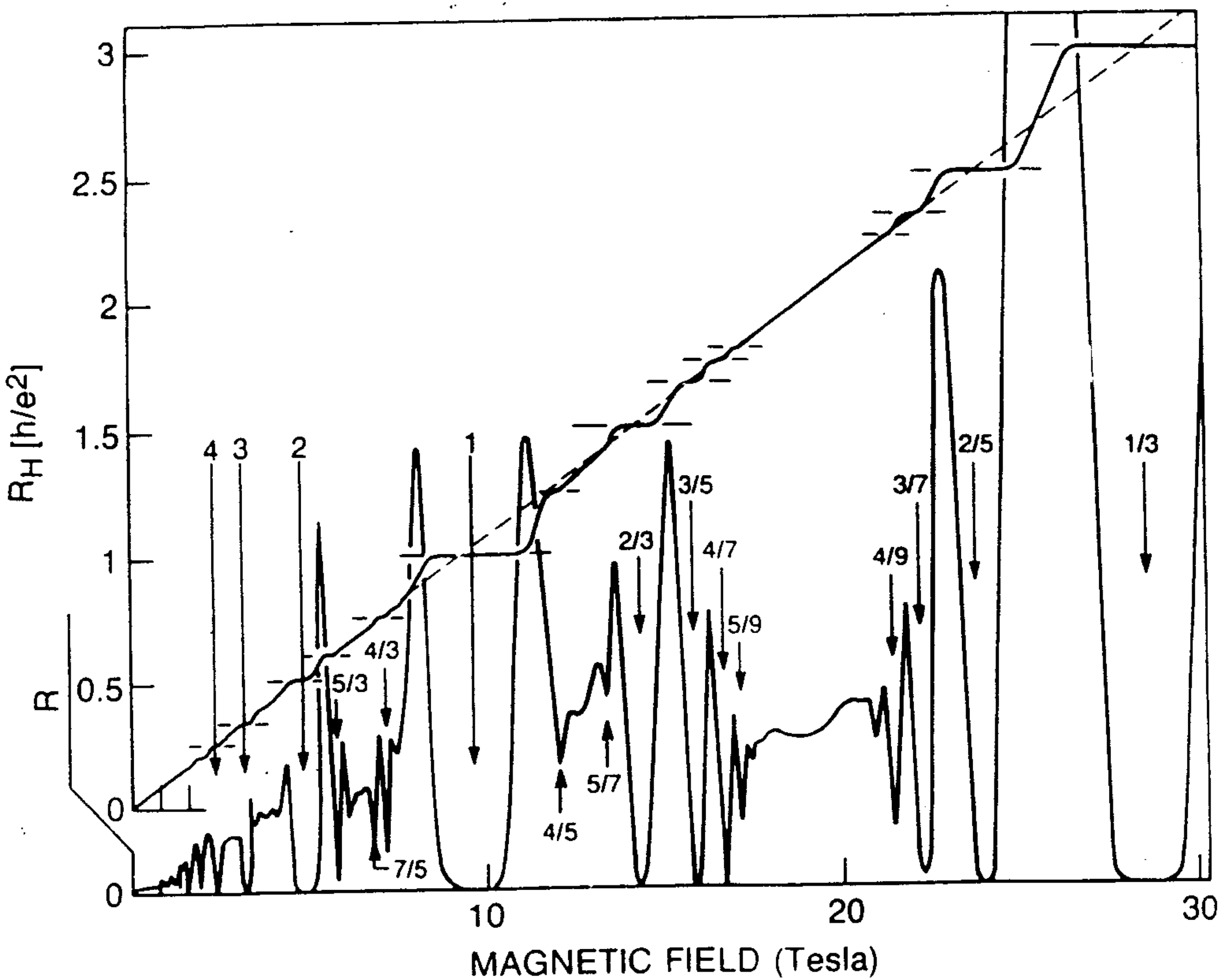
IQHE (Integer Quantum Hall Effect)





Sajoto (1989b)

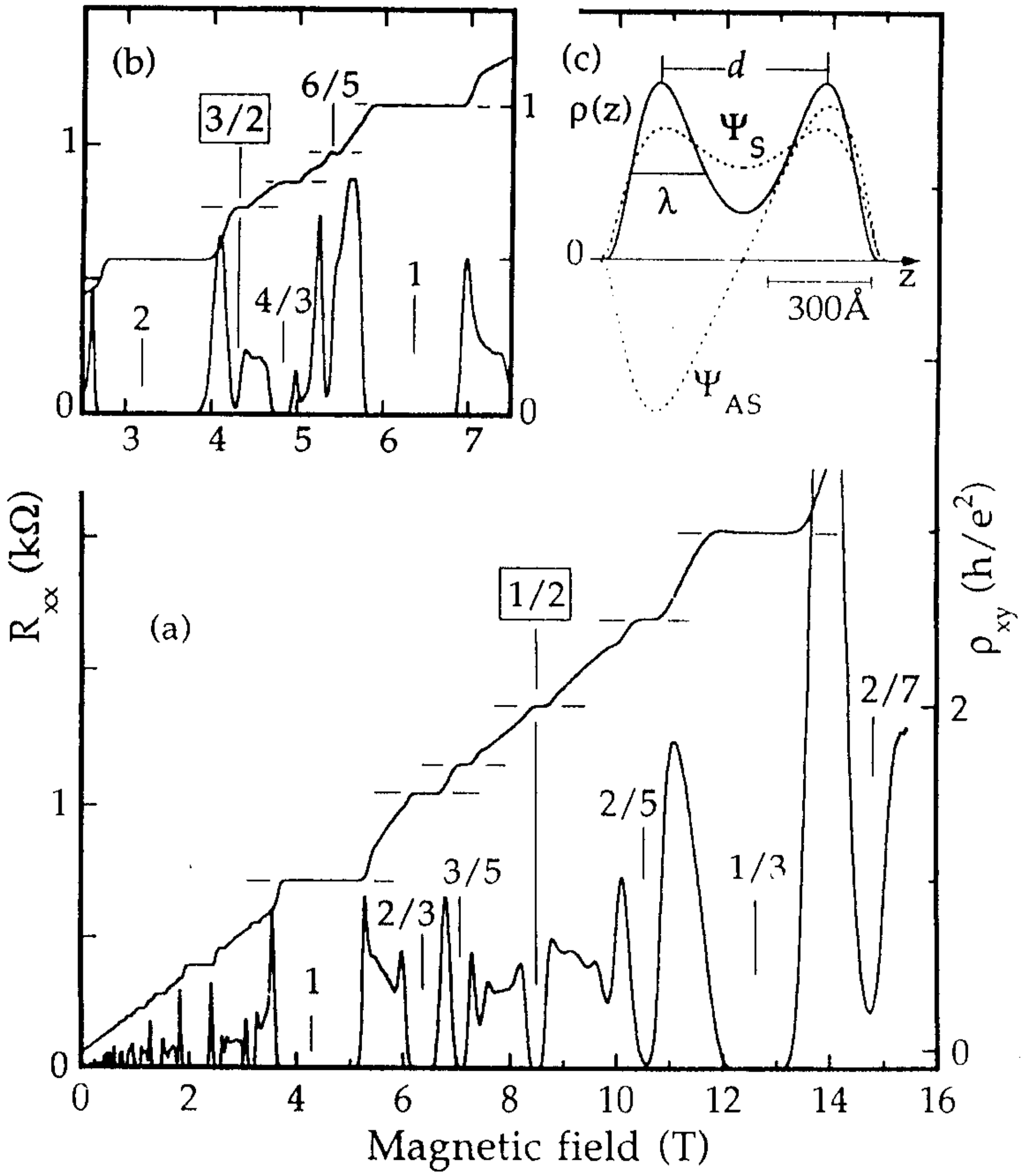
FQHE (Fractional Quantum Hall Effect)



Willett (~1983)

Even "even-denominator" FQHE!

Part III



Suen (1994)

Outline

1. Experimental
    - a. sample structure (GaAs/AlGaAs)
    - b. molecular beam epitaxy
    - c. sample geometry (contacts, gating, etc.)
    - d. mobility & disorder

2. IQHE
    - a. 2D electron system in a magnetic field
    - b. localization & scaling
    - c. edge states

    b. and c. ← topics of current interest

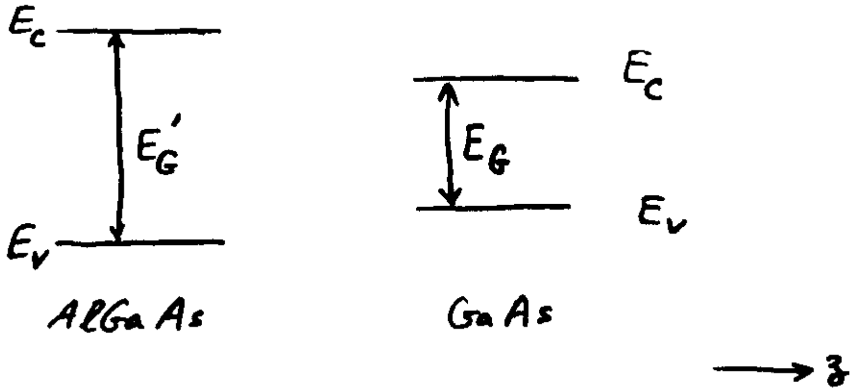
3. FQHE
    - a. one-page introduction to the FQHE
    - b. energy gap in FQHE: experiment vs theory
    - c. Hierarchy: "old" and "new" pictures
    - d. experimental evidence for "composite fermions"

    c. and d.: supposed to be lecture IV

Selectively-doped Structures

2DEG @ GaAs/AlGaAs Heterojunction Interface

①



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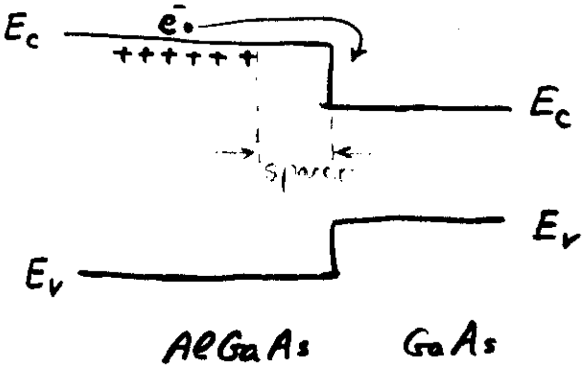
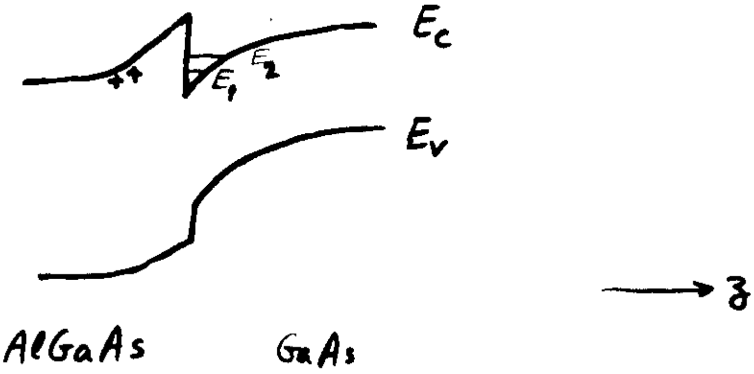


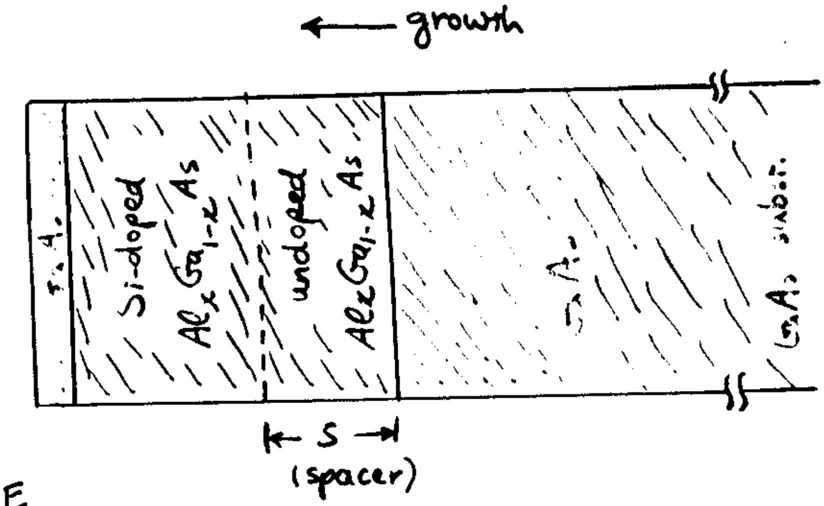
③



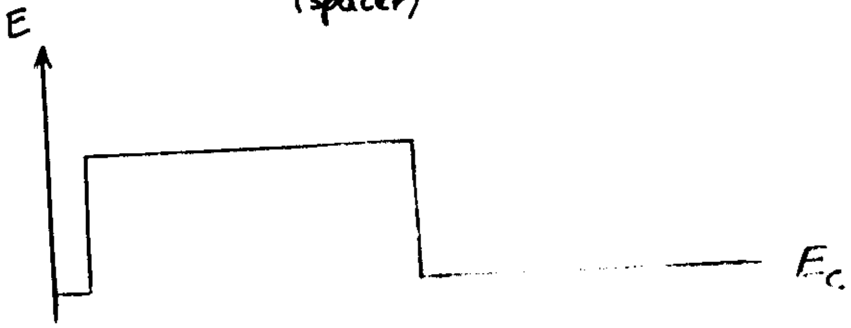
(Stormer, 1978)

Doping & Band Diagram

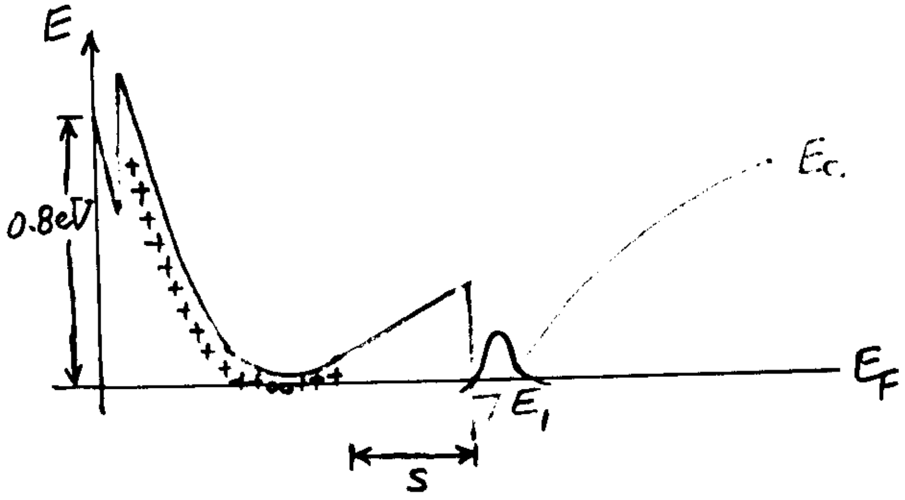
material



band diagram
(before charge
transfer)



band diagram
(after charge
transfer)
(see, e.g., Stern
1983)

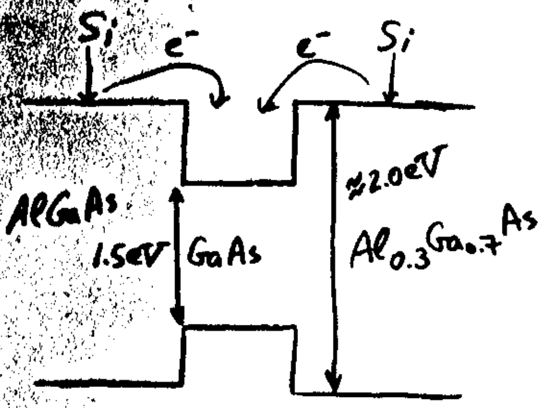


E_F pinned @ $\simeq 0.8$ eV below GaAs E_c at surface because of surface states.

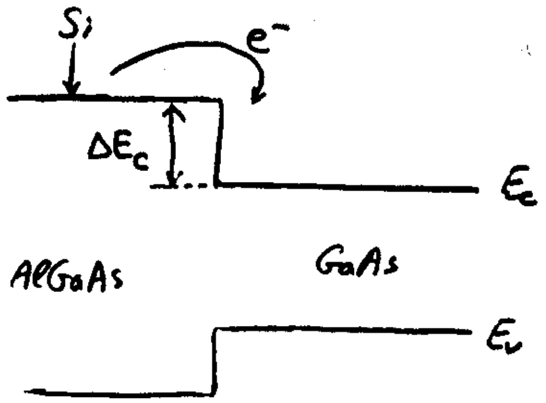
More on Modulation (selective) doping

- Before charge transfer:

Quantum Well

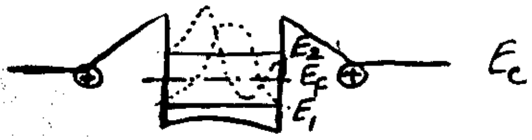


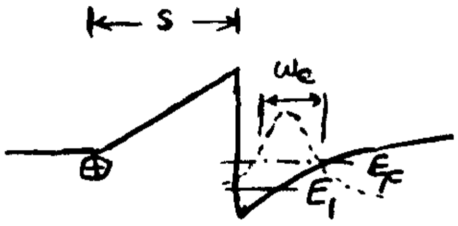
Single heterojunction



Conduction band offset for $GaAs/Al_xGa_{1-x}As$: $\Delta E_c \simeq 900x$ (meV)
(small x)

- After charge transfer:



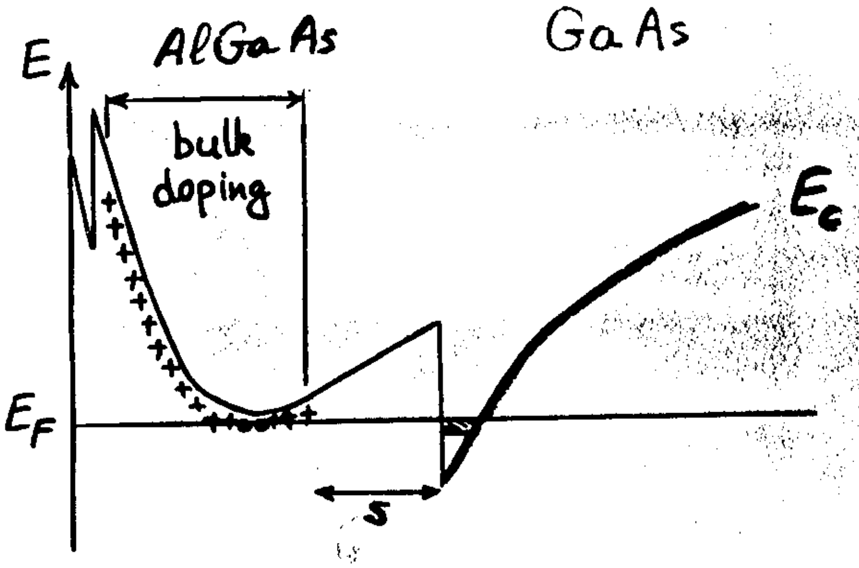


- spacer thickness $s \sim 100\text{Å} - 10{,}000\text{Å}$
- electron layer thickness (in GaAs/AlGaAs heterojunctions) $w_e \sim 100\text{Å}$

State-of-the-art modulation-doped heterostructures

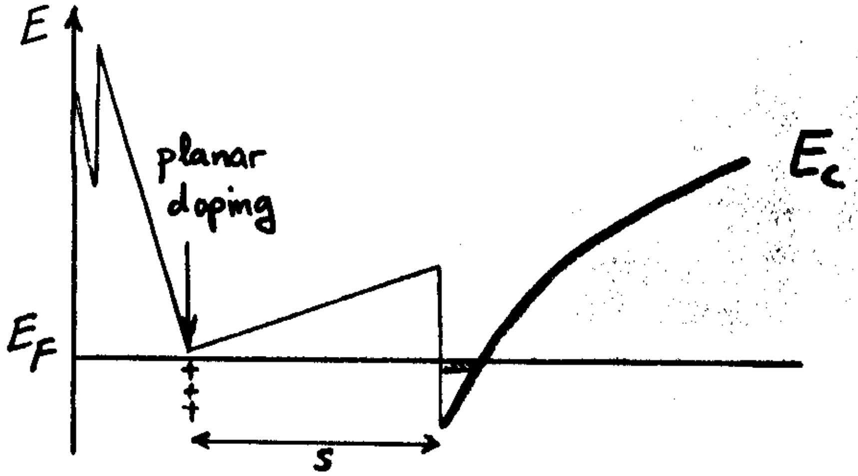
- "conventional"

for simple calculation see Stern, 1983



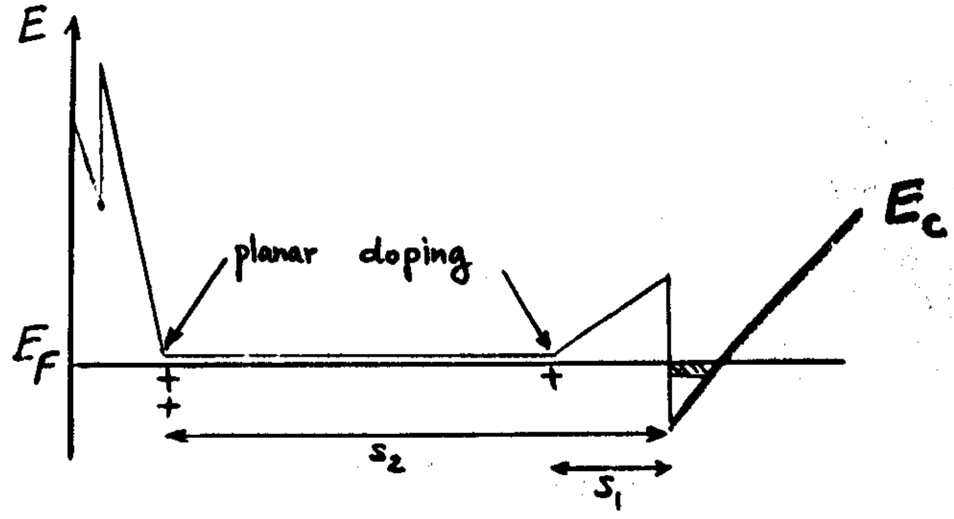
- single δ

Shayegan, 1998 a

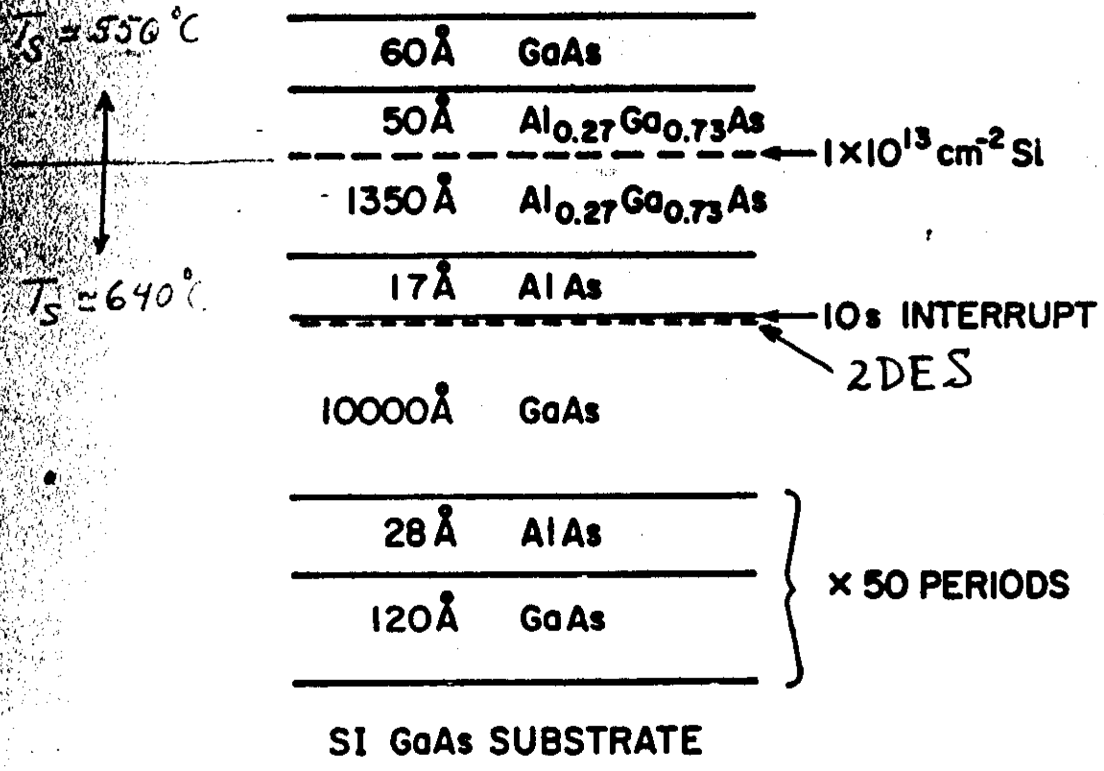


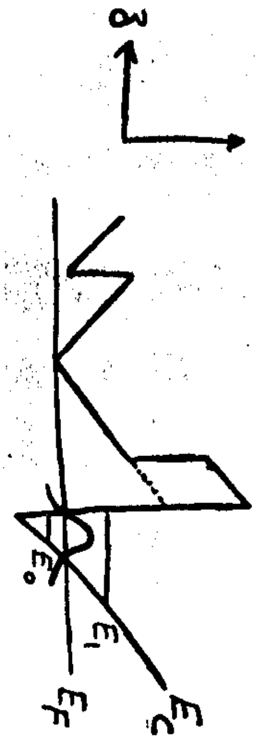
- double δ

Etienne 1987
Shayegan 1988 d



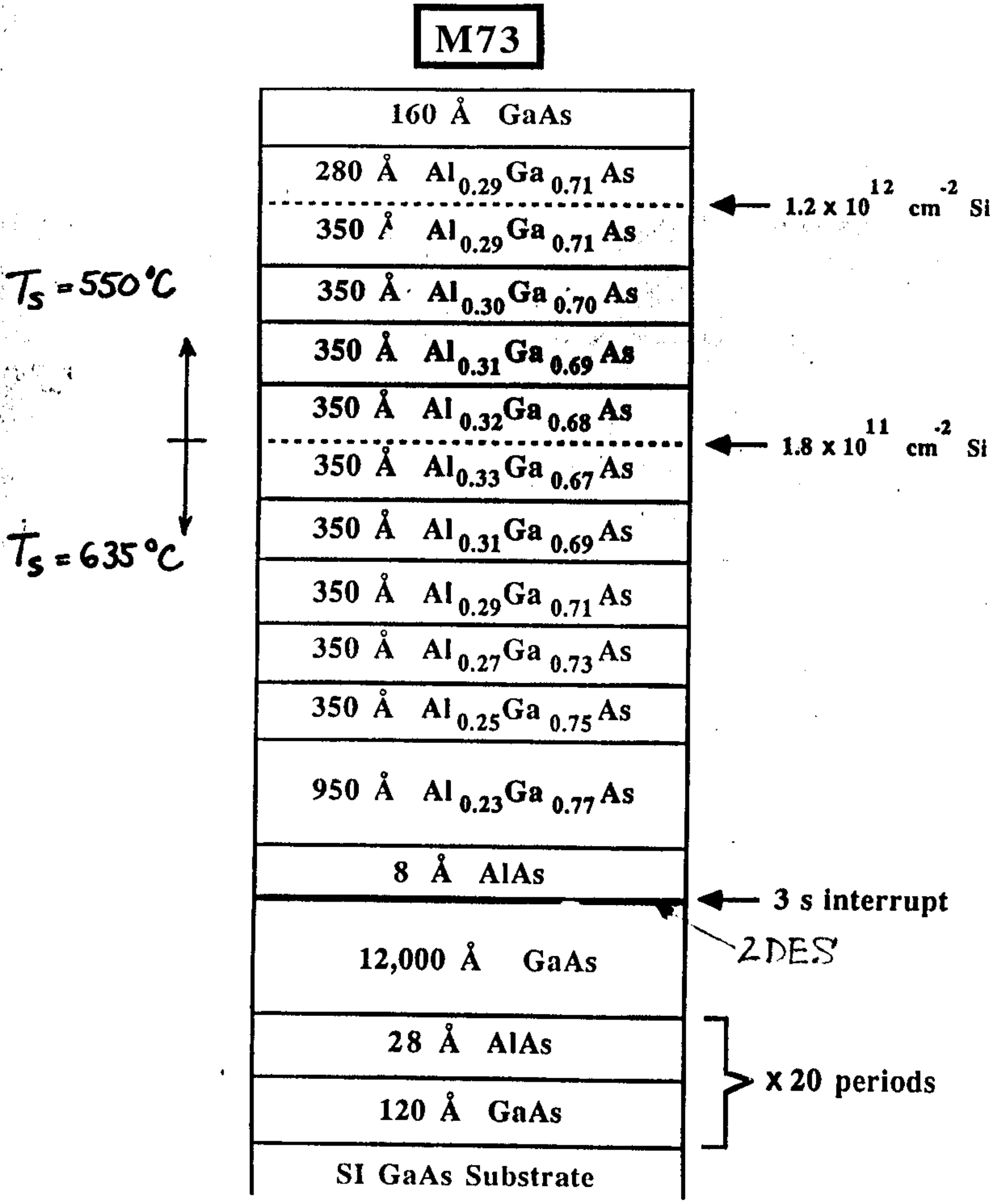
Structure for Sample M42





- ultrathick spacer
- atomic plane doping
- AlAs barrier
- lower T_S after doping

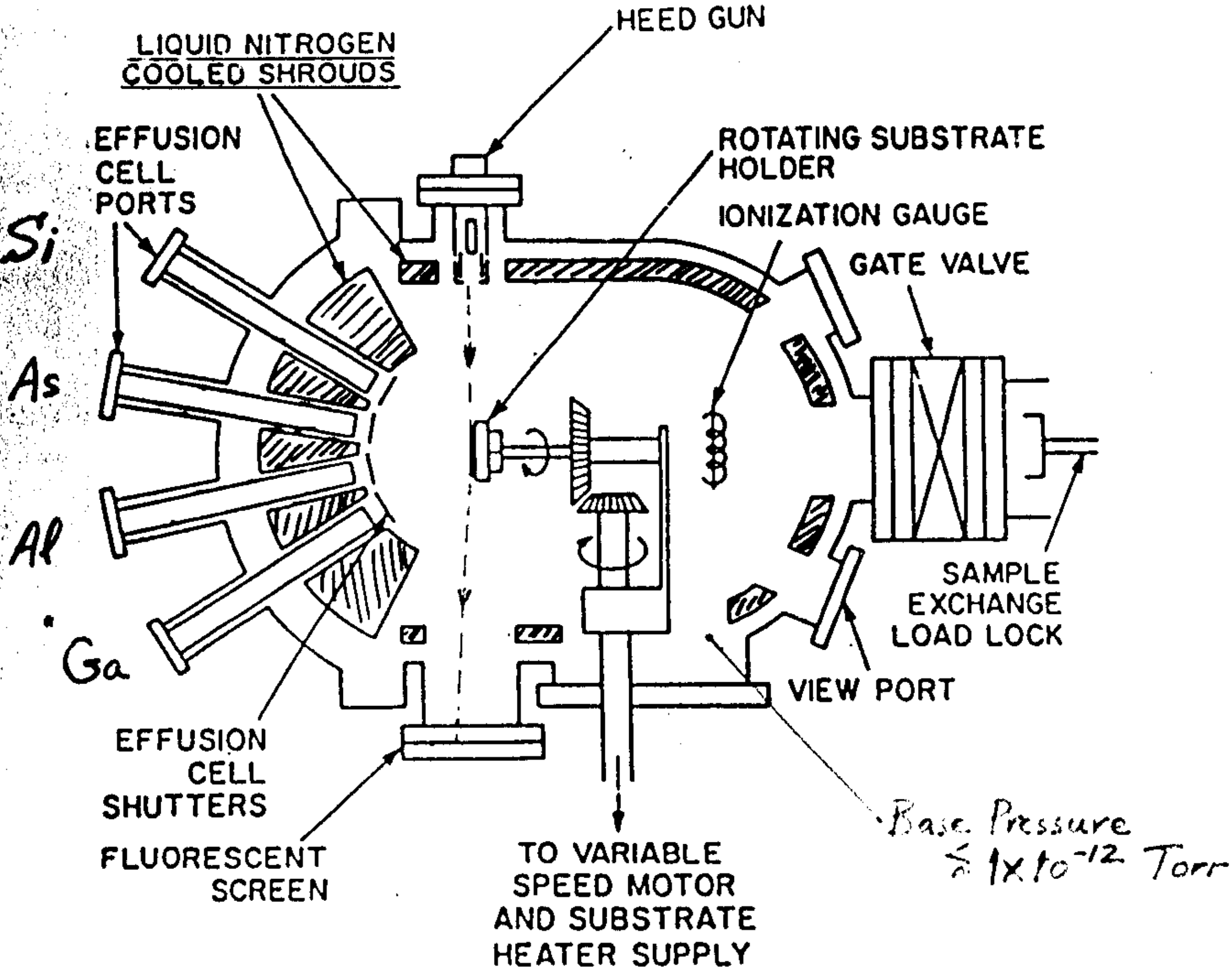
(Shayegan, 1988a)



- ultrathick graded spacer
- double-δ doping

(Shayegan, 1988d)

Molecular Beam Epitaxy (MBE)



Base Pressure $\lesssim 1 \times 10^{-12}$ Torr

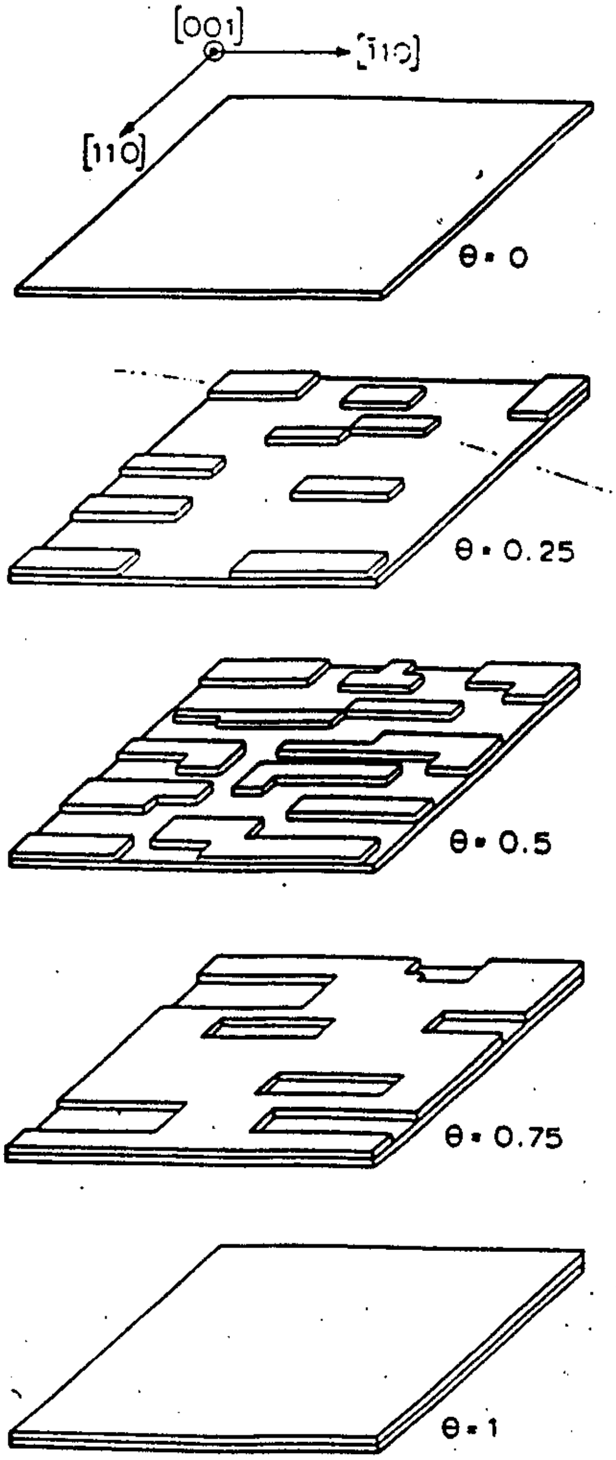
MBE provides accurate control of:

- thickness
- composition
- doping

=> structure

Monitoring of layer-by-layer growth via RHEED oscillations

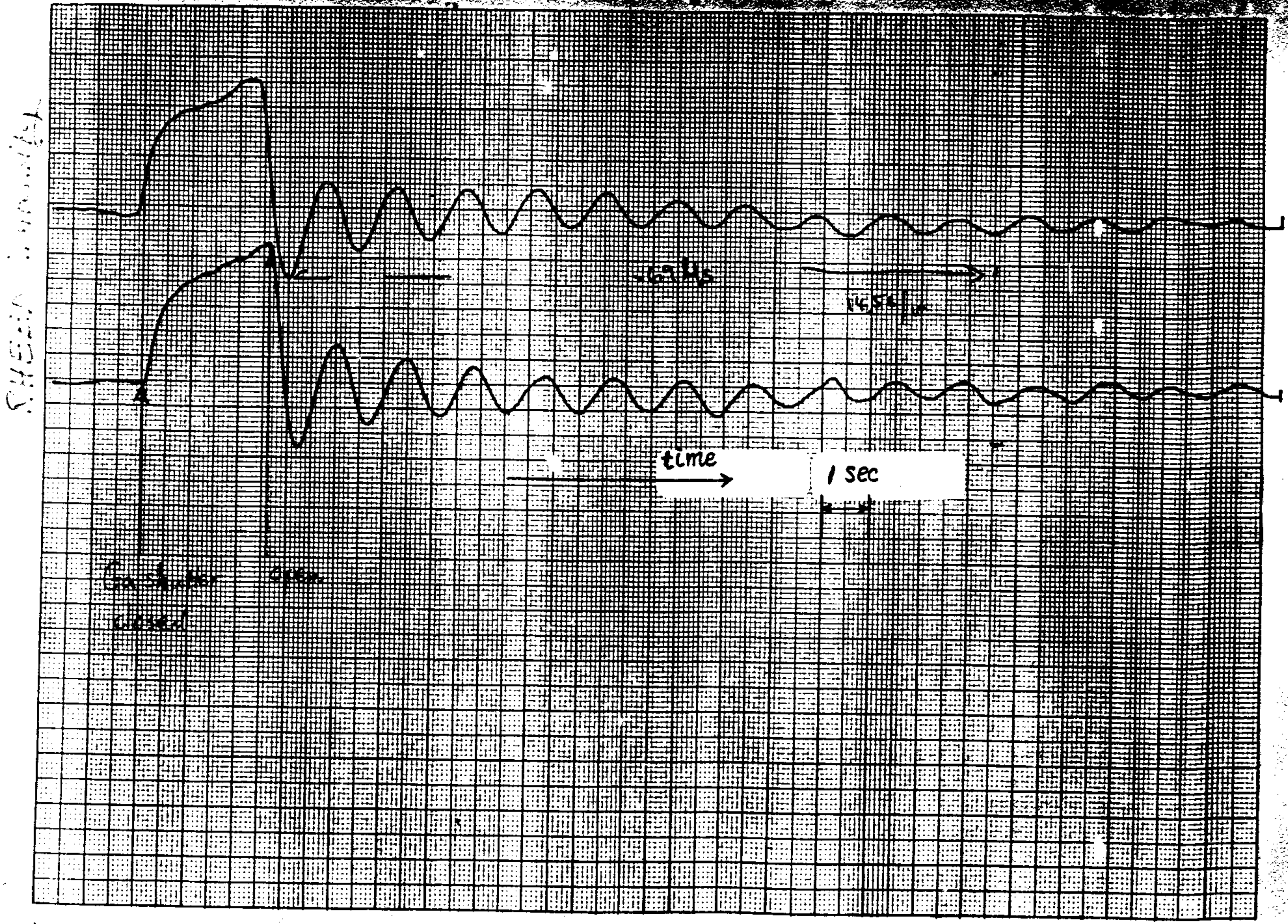
reflection high energy electron diffraction



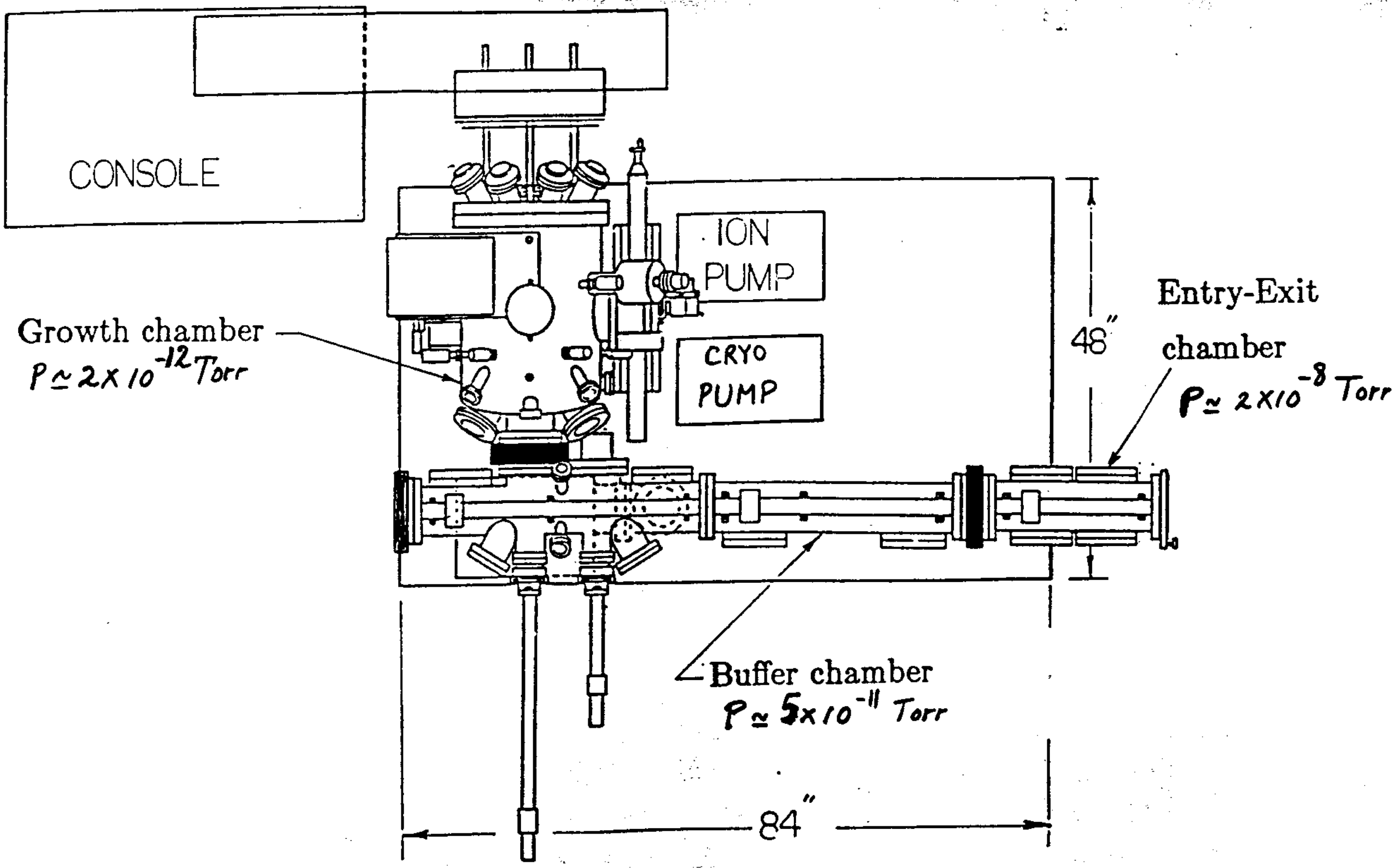
θ Is fractional layer coverage

Fig. 6. Real space representation of the formation of a single complete layer

RHEED oscillations

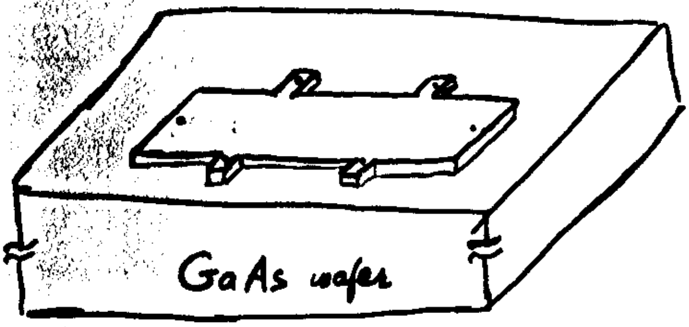


Varian Gen II (Modular)

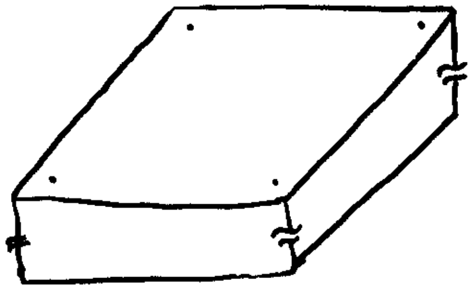


• Sample preparation: Contacts & Gates

① Define geometry:

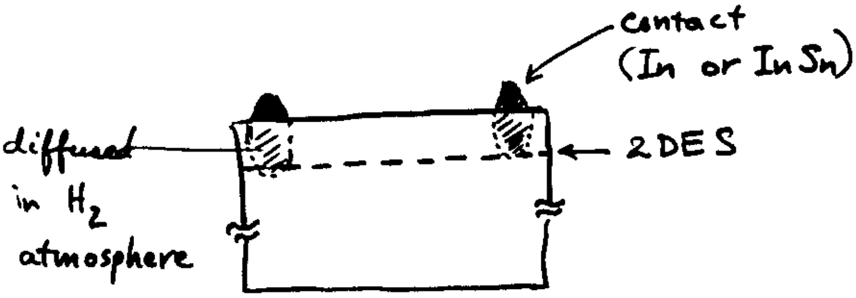


Hall bar
(requires etching)



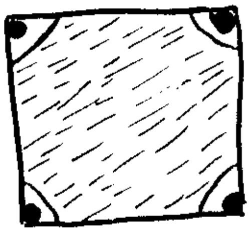
van der Pauw

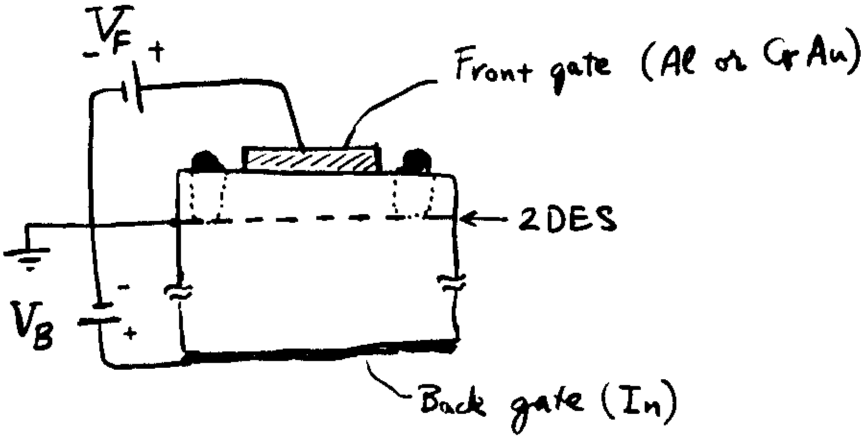
② Put contacts:

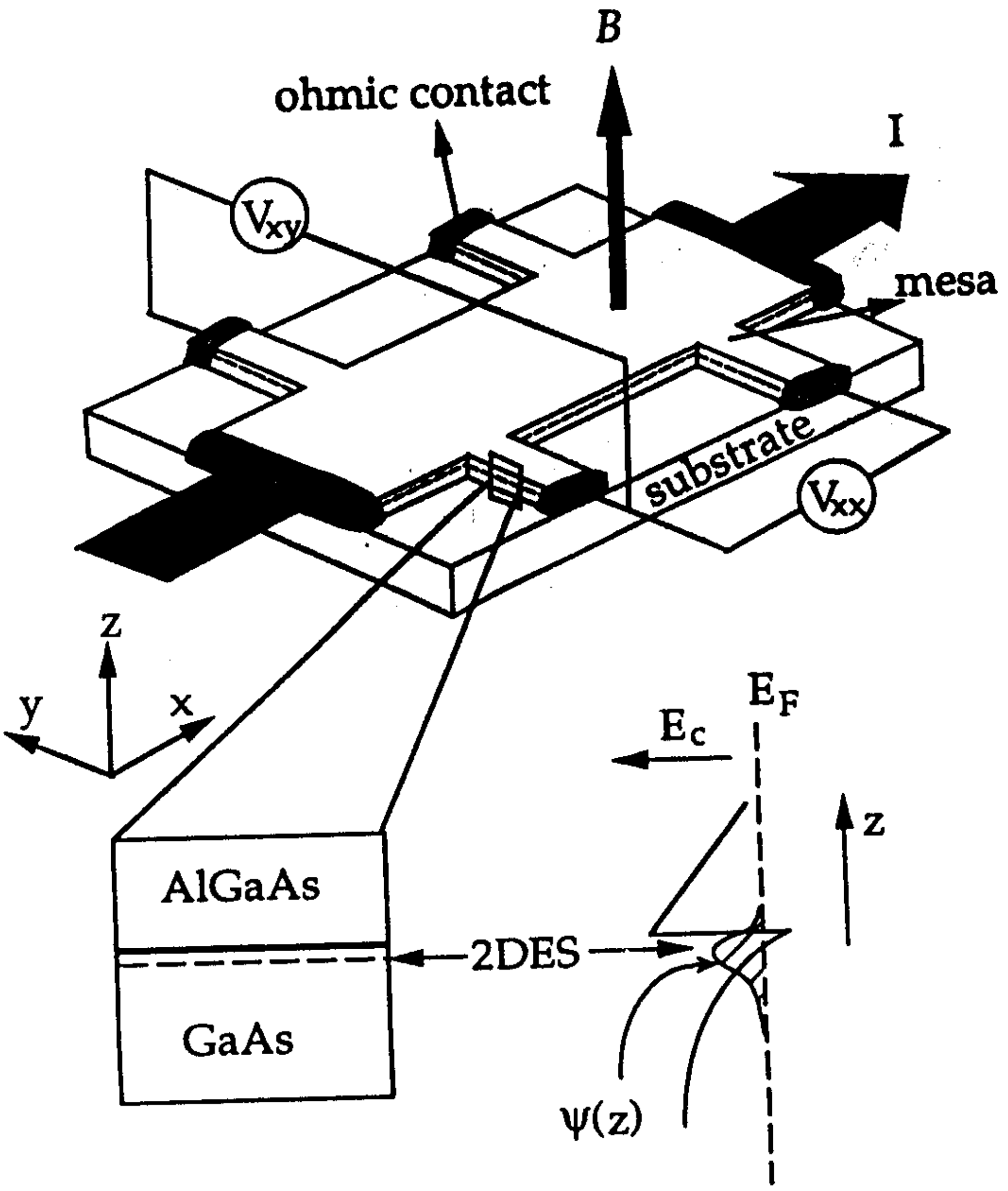


③ Deposit gate:
(if needed)

top view:

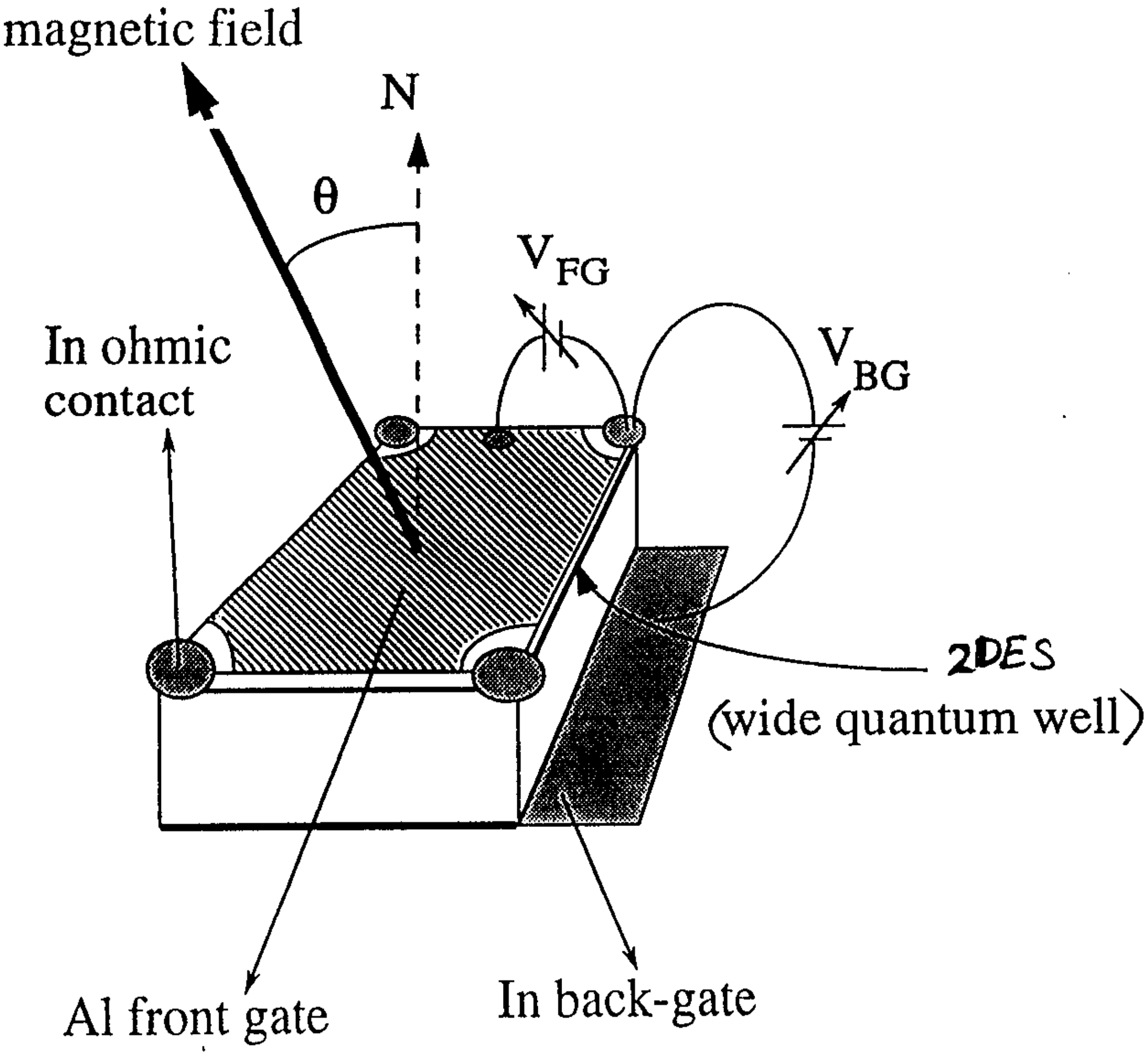






Suen, PhD thesis, 1993

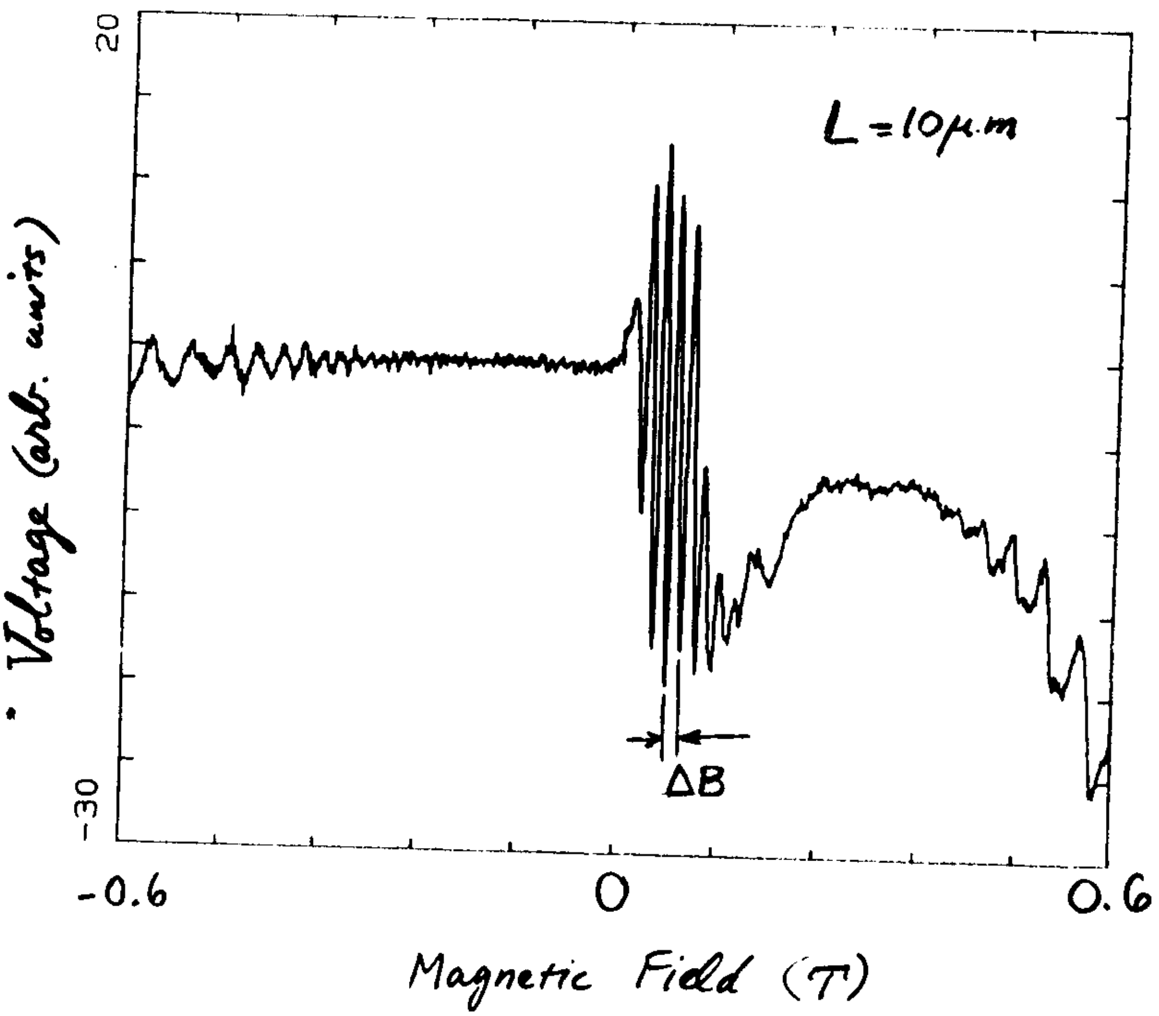
Figure 1.1: Schematic diagram of a magnetotransport measurement performed on a 2D system in a typical "Hall bar" geometry. Also shown schematically are the conduction band edge, E_c, and the electron distribution, $\psi(z)$.

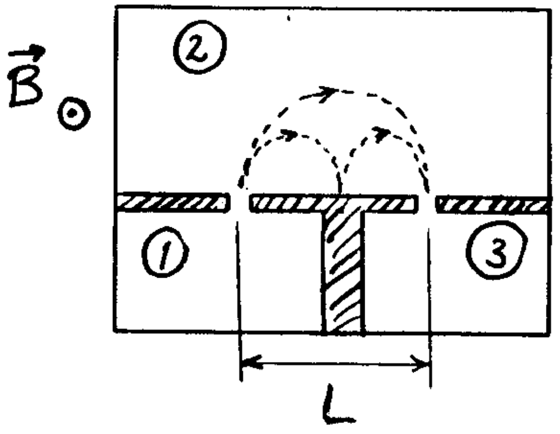


van der Pauw Geometry

Suen, Ph.D. Thesis 1993

Magnetic Focusing





$$k_F = \sqrt{2\pi n_s}$$

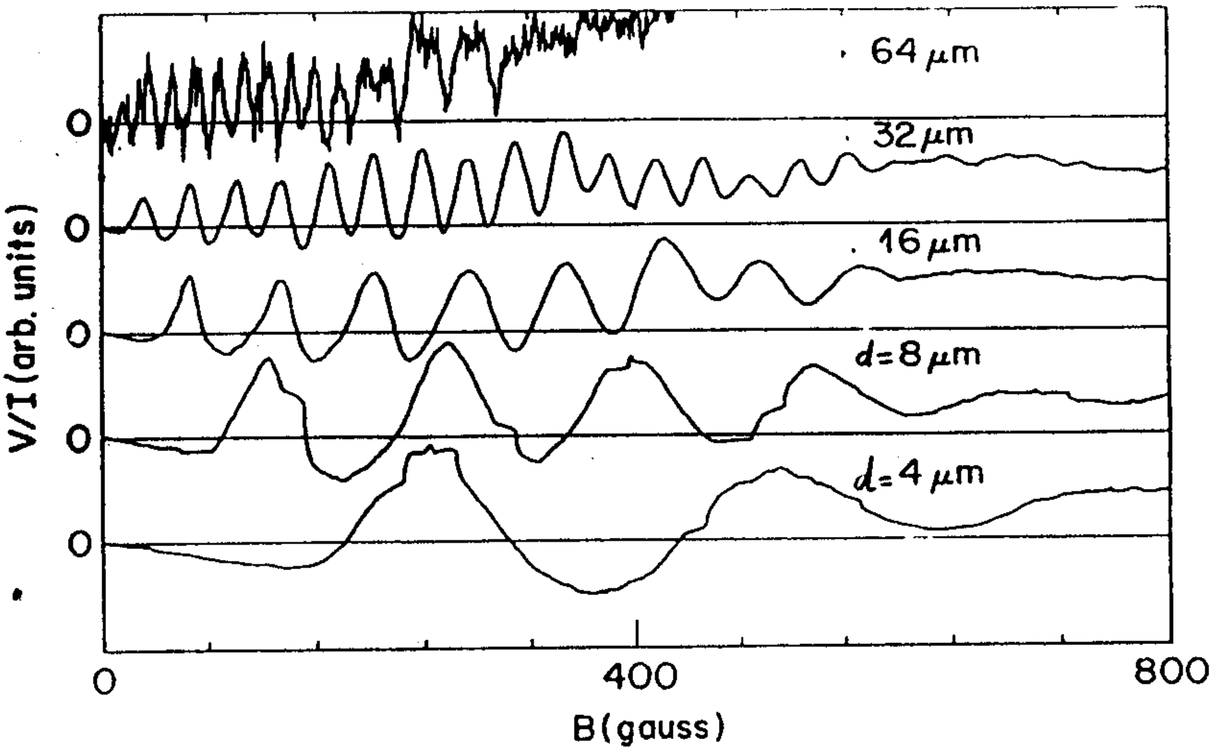
$$L_{cycl.} = \frac{2\hbar k_F}{eB}$$

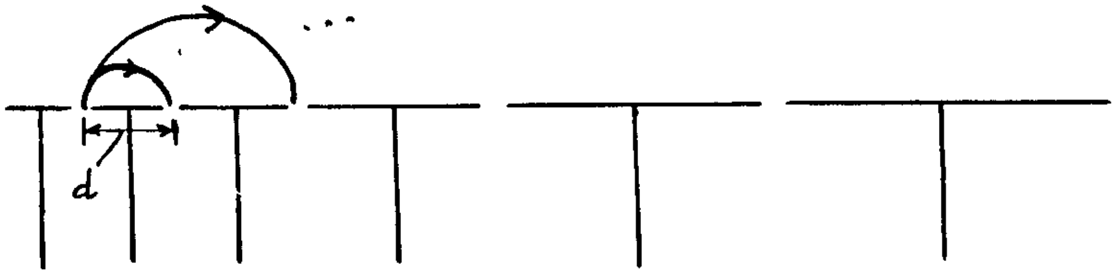
$$\Delta B = \left(\frac{2\hbar}{e}\right)\left(\frac{k_F}{L}\right)$$

Heremans et al.

- Ballistic transport over distances $\lesssim \underline{\underline{100\,\mu m}}$ $(= \frac{\pi}{2}\, 64\,\mu m)$!

Spector (1990)





- Mobility exceeding $\sim 10^7 \, cm^2/Vs$

(Pfeiffer, 1989)

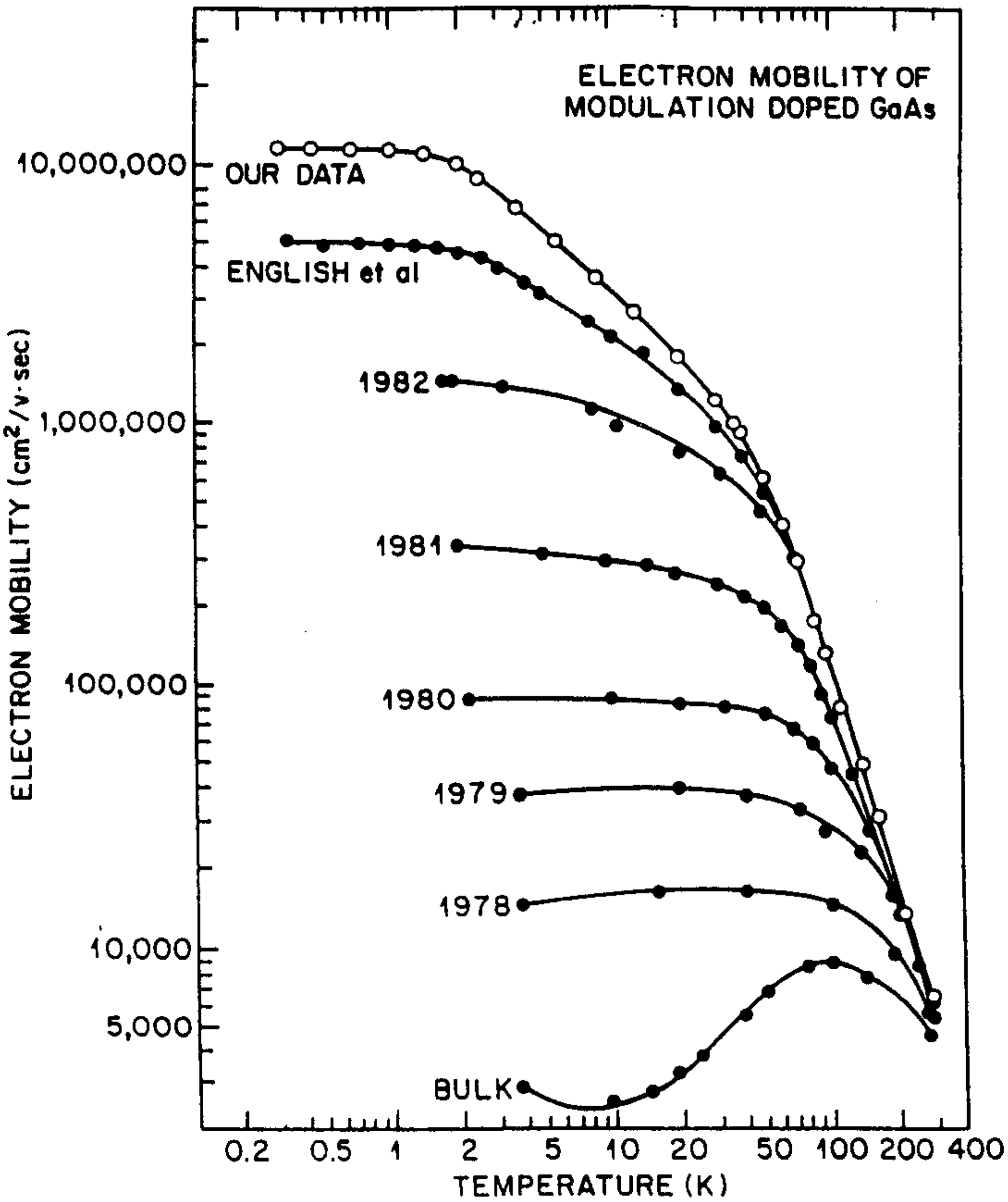
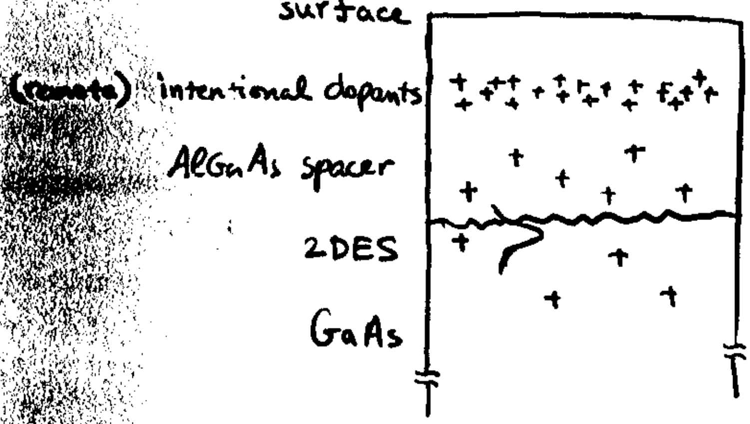


Figure 1

- Mobility vs density (low T)



# of ionized impurities		mobility
N_{rm}	$\Rightarrow$	$\mu_{rm} \propto 1/N_{rm}$
N_{sp}	$\Rightarrow$	$\mu_{sp} \propto 1/N_{sp}$
← interface roughness	$\Rightarrow$	μ_{ir}
N_{bk}	$\Rightarrow$	$\mu_{bk} \propto 1/N_{bk}$

Matthiessen's Rule: $\frac{1}{\mu_{tot.}} = \sum_i \frac{1}{\mu_i} = \frac{1}{\mu_{rm}} + \frac{1}{\mu_{sp}} + \frac{1}{\mu_{bk}} + \frac{1}{\mu_{ir}} + \cdots$

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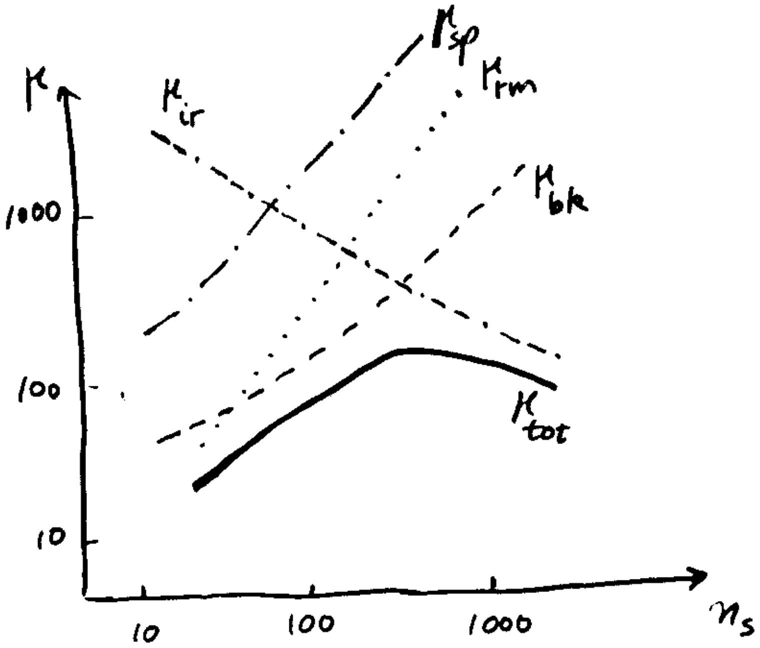
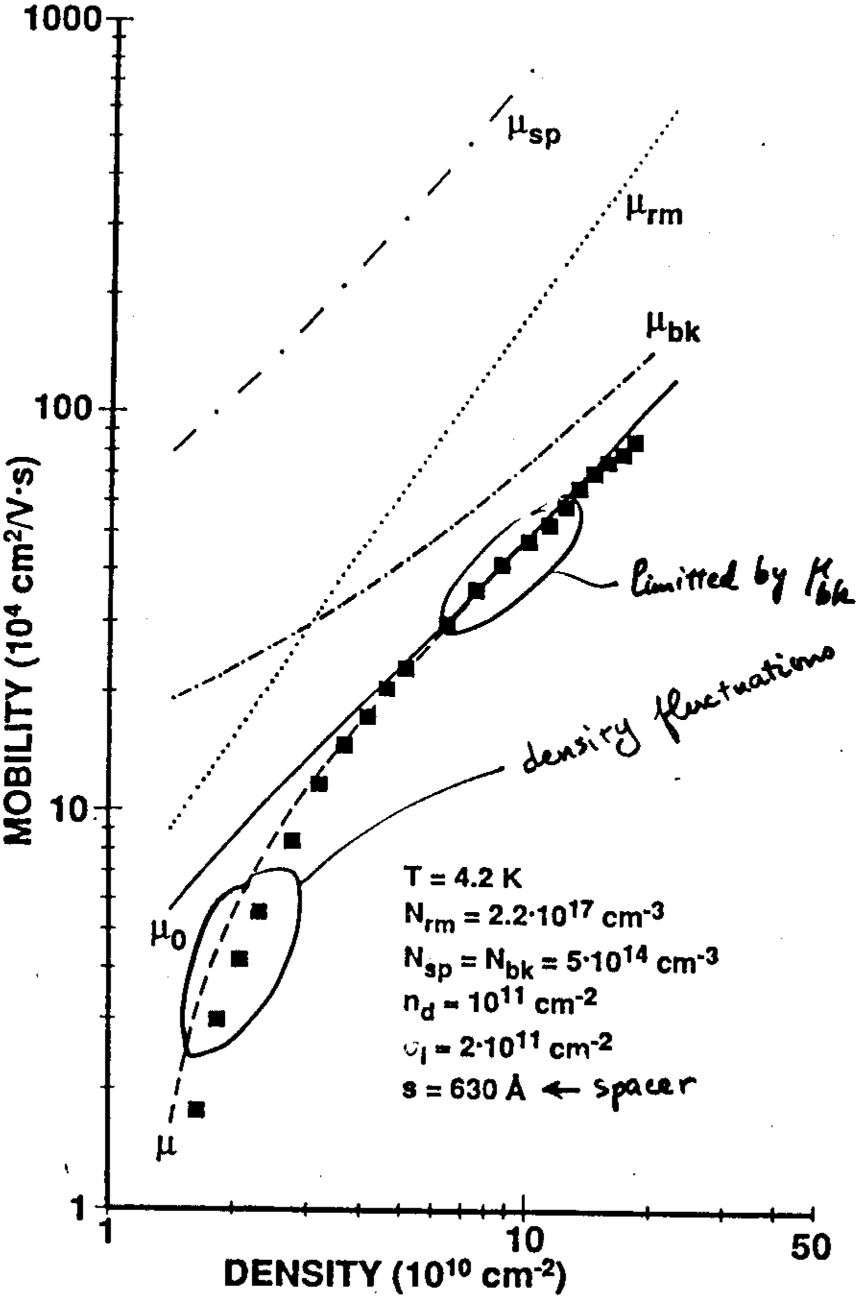


Stern 1983
Jiang 1988



Jiang et al., 1988

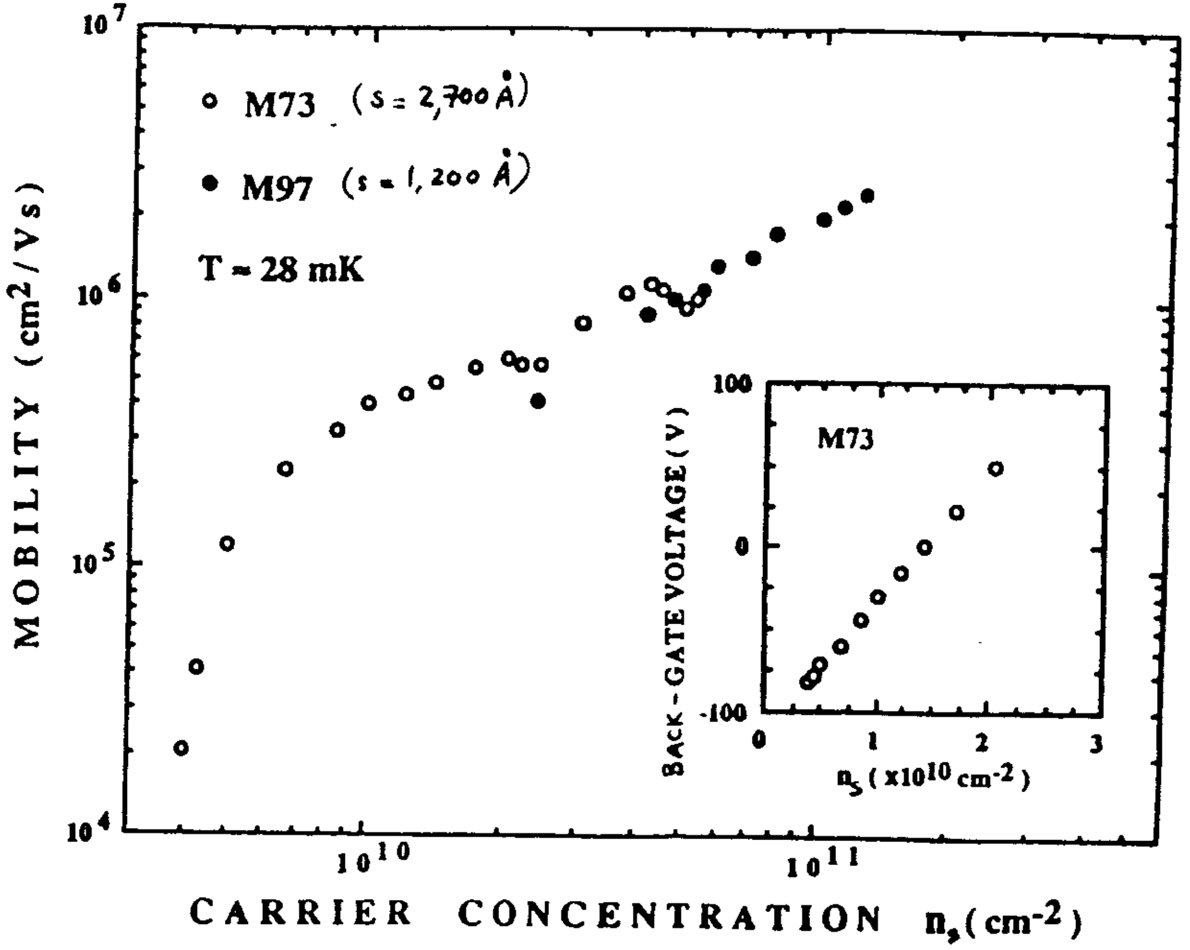


Figure 2

$N_{bk} \lesssim 10^{14}\,\mathrm{cm}^{-3}$

- Key factor in getti
highest μ samples

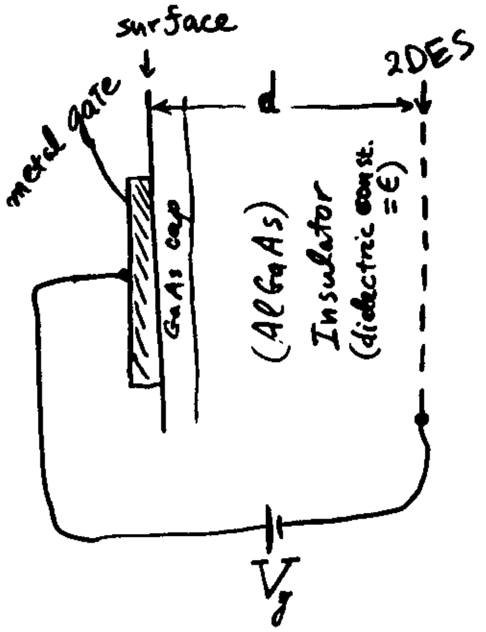
REDUCE N_{bk}

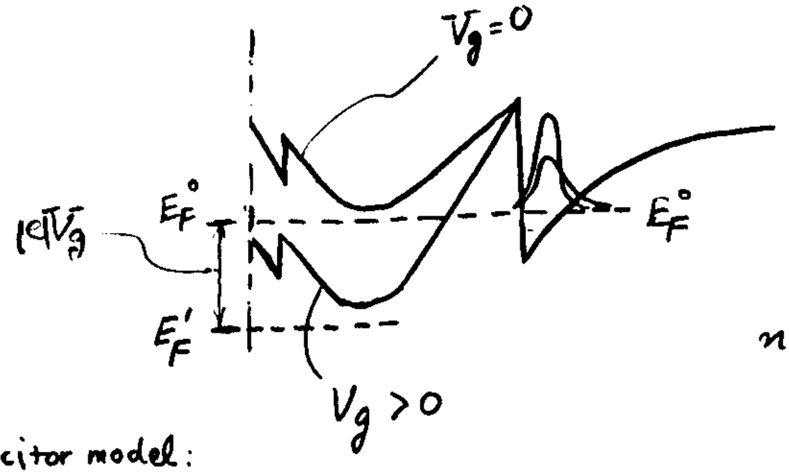
i.e. CLEAN MBE!

(Sajoto, 1990)

• Controlling n_s

① Capacitively:

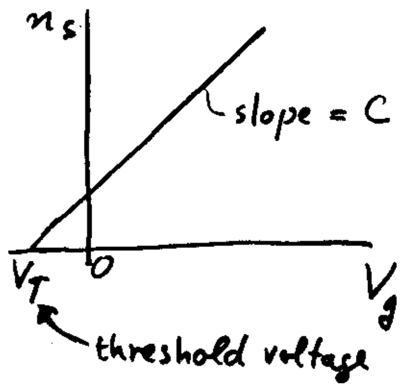




simple capacitor model:

$$dn_s = \underbrace{\left(\frac{\epsilon}{d}\right)}_{\text{capacitance per unit area}} dV_g$$

($\epsilon \approx 13\epsilon_0$ for GaAs & AlGaAs)



② By shinning light

- persistent photoconduc.
  positive ($n_s \uparrow$ with light)
  & negative ($n_s \downarrow$ " ")
- not reversible
- often reproducible !

- Typical cryostat

Pumped ^{3}He system

$T \gtrsim 0.3\,K$

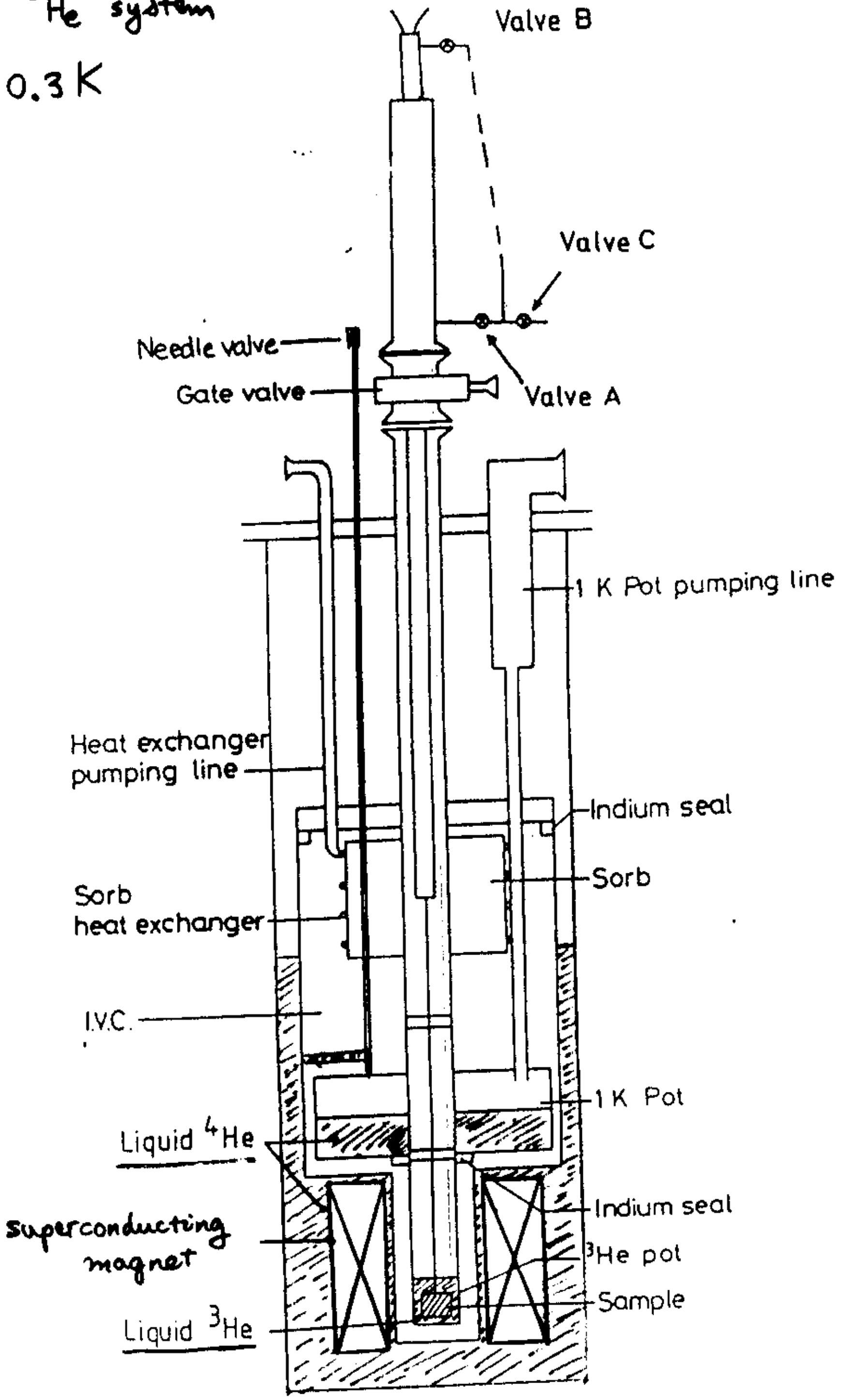


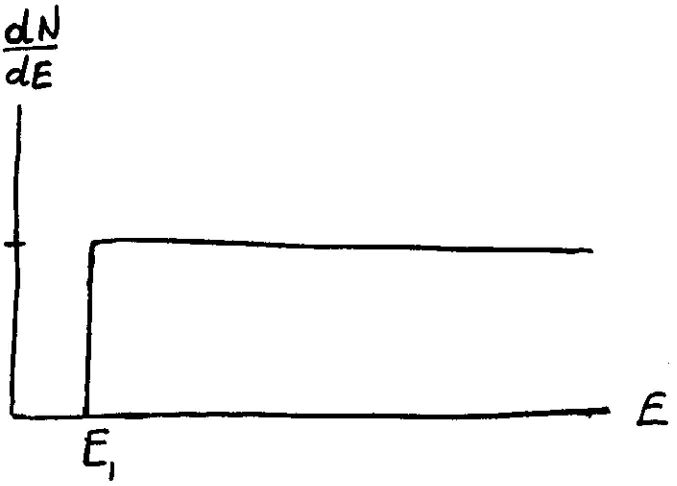
Figure 1. Schematic representation of the top loading sorption pumped helium -3 insert.

2. IQHE

- 2DES in a perpendicular magnetic field

B=0

$2.8\times10^{13}/\text{cm}^2\cdot\text{eV} = \frac{m^*}{\pi\hbar^2}$

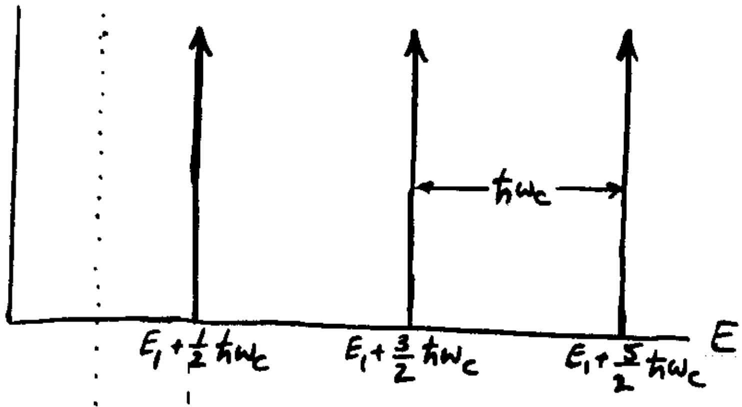


$B>0,\ g^*=0$

L.L. deg $= \frac{2e}{h}B = 4.8\times10^{13}/\text{cm}^2\cdot\text{T}$

$\hbar\omega_c = \hbar\frac{eB}{m^*} \simeq 20\,\text{K/T}$

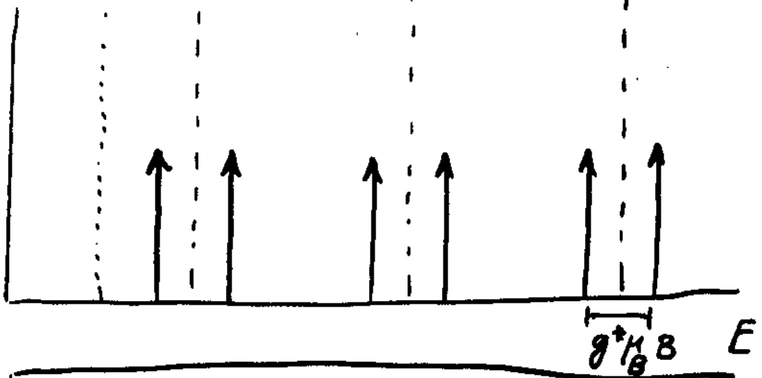
m^*: $0.067\,m_0$ for 2DES in GaAs



$B>0,\ g^*>0$

$g^*\mu_B B \simeq 0.3\ \text{K/T}$

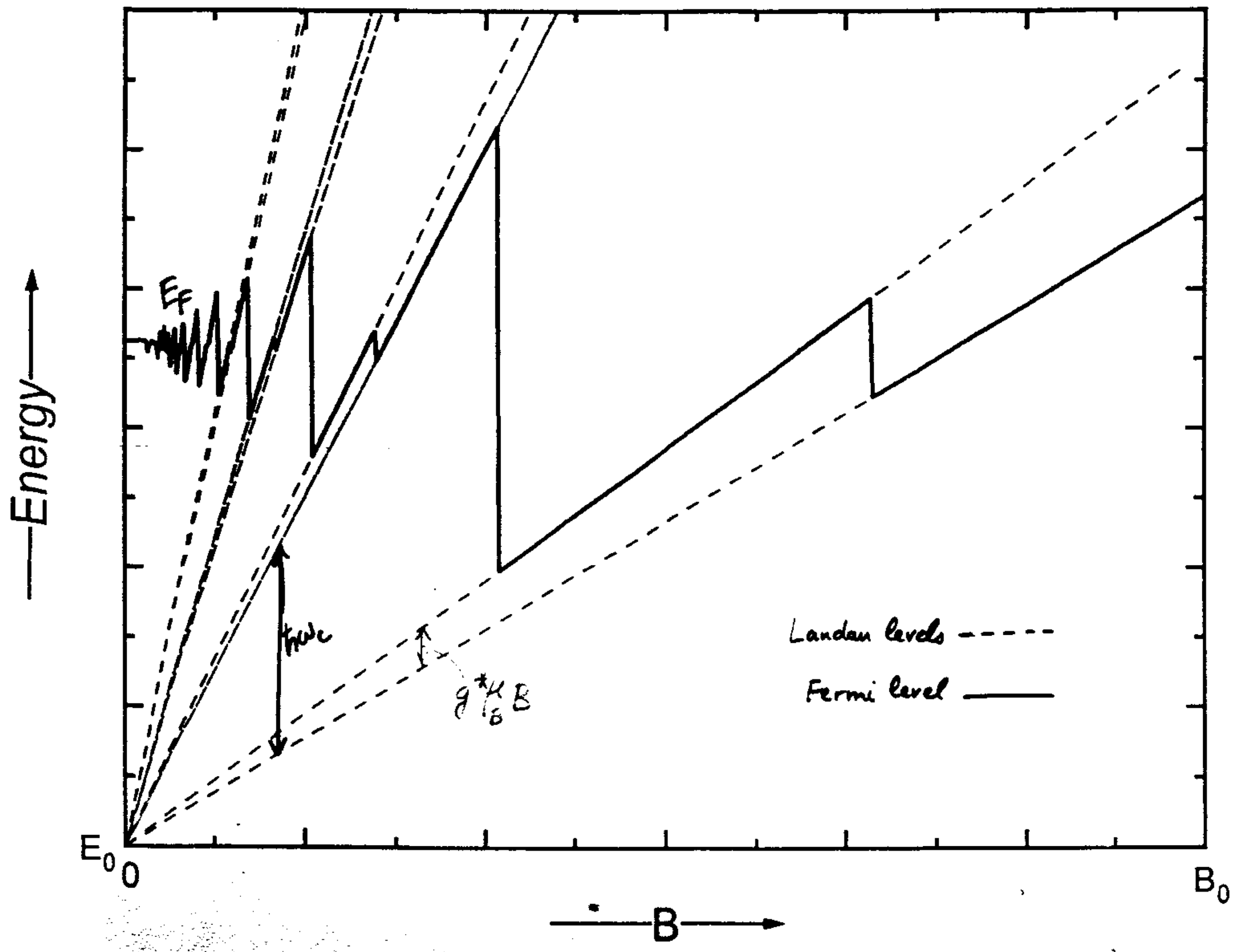
g^*: 0.44 for bulk GaAs (can be much larger for partially-filled Landau-levels)



Level degeneracy $= d = B/\Phi_0 \quad (\Phi_0 = h/e)$

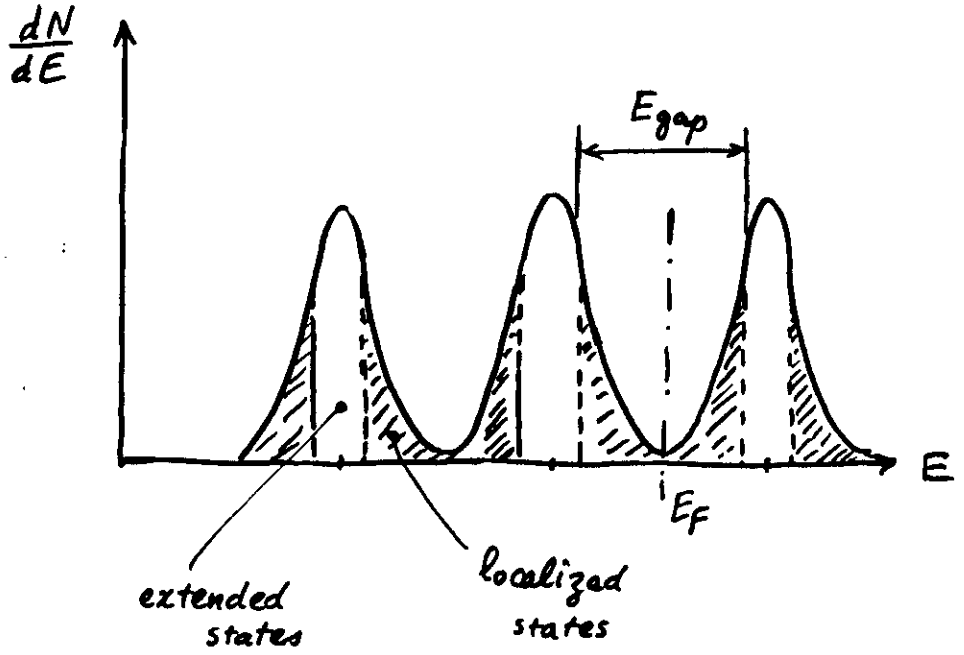
Filling factor $= \nu = n_s/d = nh/eB$

Fan Diagram with Spin Splitting for One Subband



frs.org/frs.p1/25 apr 94

- Effect of <u>disorder</u> (and finite T): Broaden the levels



- E_F in localized states => $\rho_{xx} \to 0$, $\rho_{xy} \to \frac{h/e^2}{i}$ => QHE

- E_F in extended states => finite ρ_{xx}

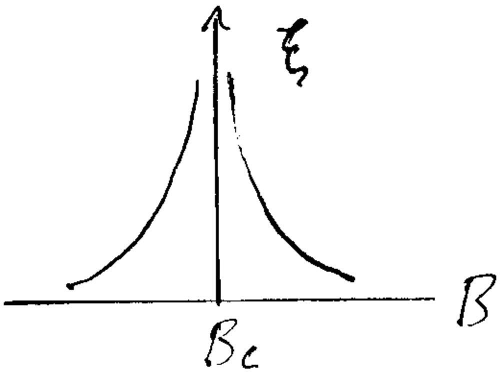
$$\boxed{\left.\begin{array}{l}\cdot\ \text{Gap in DOS}\\ \cdot\ \text{Localized states}\end{array}\right\} \Rightarrow \text{IQHE}}$$

Finite length scaling

(will be covered by R. Bhatt)

CRITICAL $B = B_c$ when E_F on a Landau level

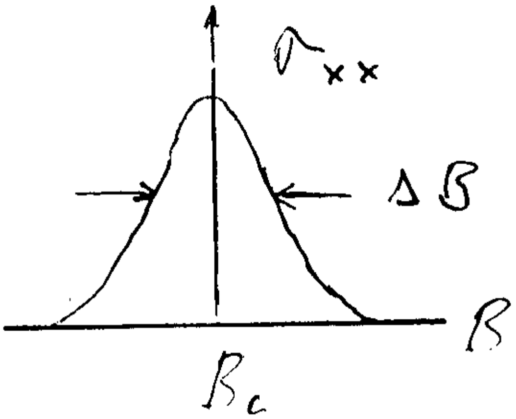
$T = 0$



ξ
- localization length,
- diverges as $B \rightarrow B_C$:
  $\xi \sim |B - B_c|^{-x}$

L
- Inelastic length
- $\Longrightarrow$ effective sample size.
- $L = L(T) \sim T^{-p/2}$

SCALING: $\xi/L \Longrightarrow \sigma_{xx}, \sigma_{xy}$



As $B \rightarrow B_c$
- ξ approaching L limits σ_{xx}
- $\xi \sim L \Longrightarrow \Delta B$,

Peak Widths:
- $\Delta B^{-x} \sim L \quad \Delta B \sim L^{-1/x}$
- $\Delta B \sim T^{p/2x} = T^{\kappa}$
- $\kappa = p/2x$: observed by Wei *et al.*

dc peakwidth ΔB vs T H. P. Wei *et al.*, PRL (1988)

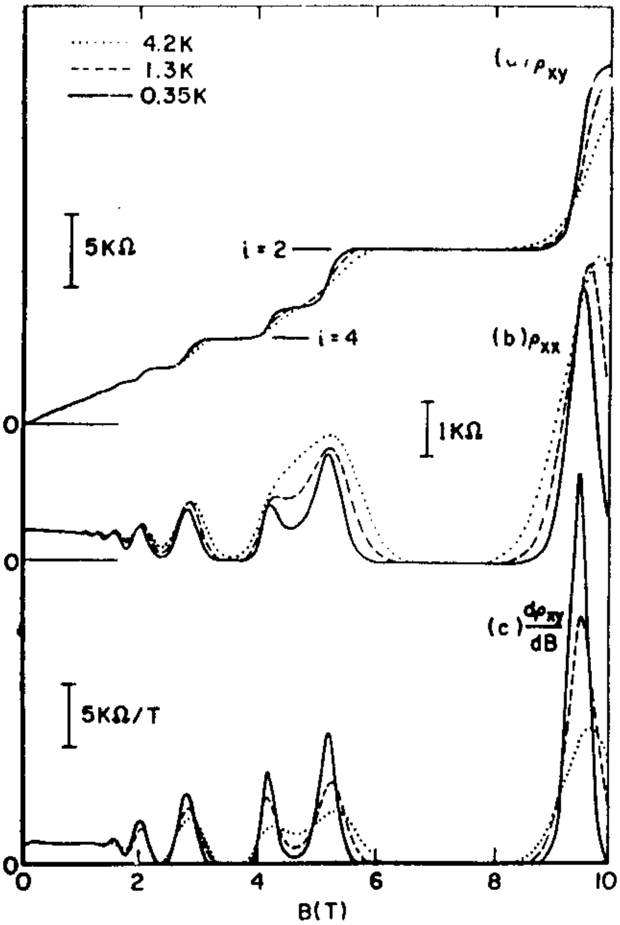


FIG. 1. Quantum transport coefficients (a) ρ_{xy} and (b) ρ_{xx} as functions of B at three temperatures, $T=4.2$, 1.3, and 0.35 K. (c) The corresponding $d\rho_{xy}/dB$. The sample is an $In_{0.53}Ga_{0.47}As/InP$ heterostructure with a two-dimensional electron density $n_{2D}=3.3\times10^{11}$ cm^{-2} and mobility $\mu=34000$ cm^2/V s at $T=0.8$ K.

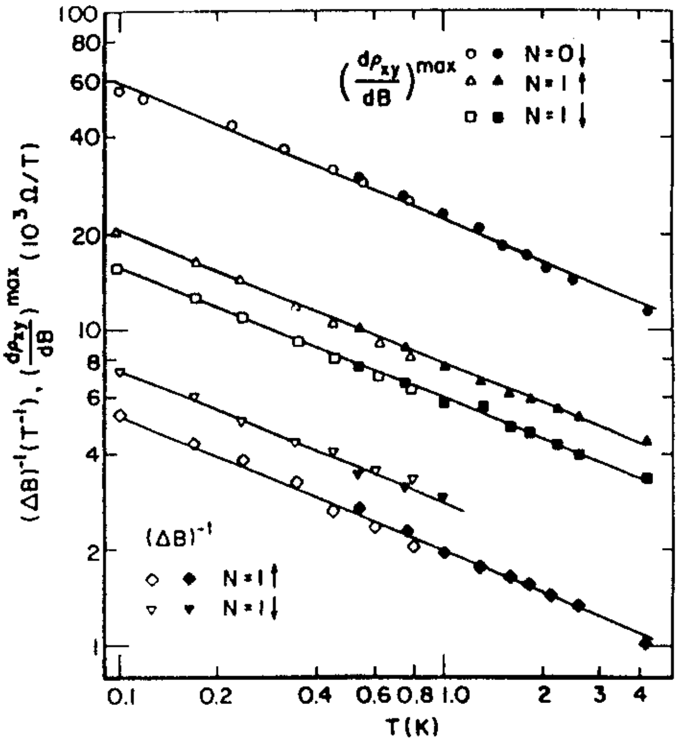
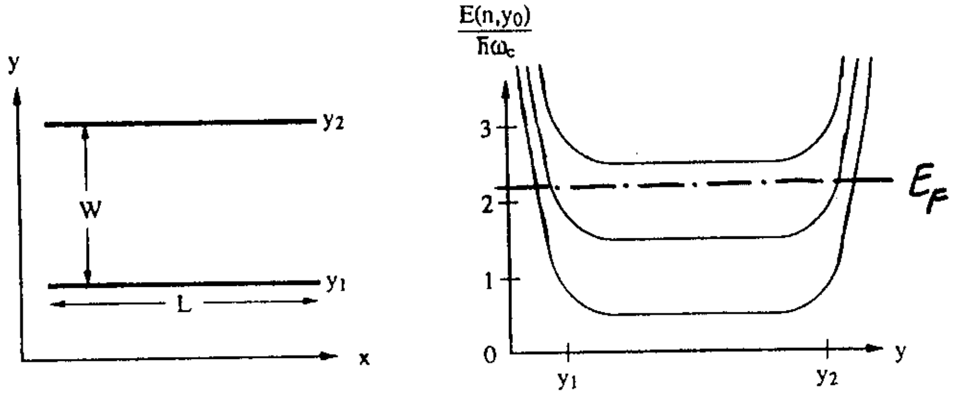


FIG. 2. The upper portion shows the T dependence $(d\rho_{xy}/dB)^{max}$ for Landau levels $N=0\downarrow$, $1\uparrow$, and $1\downarrow$; the low portion shows the T dependence of $1/\Delta B$ for the $N=1\uparrow$ and 1 Landau levels. The open symbols are data taken in a dilutio refrigerator, whereas the filled symbols are data taken in a ^{3}H system. The slope of the straight lines gives $(d\rho_{xy}/dB)^{m}$ $\sim T^{-\kappa}$ and $\Delta B\sim T^{\kappa}$ with $\kappa=0.42\pm0.04$. The typical unce tainty in T is ~0.02 K at 0.4 K.

- Width vs T: $\Delta B \propto T^{\kappa}$, $\kappa = 0.42 \pm 0.04$
- Power Law interpreted with finite length scaling

- Edge States

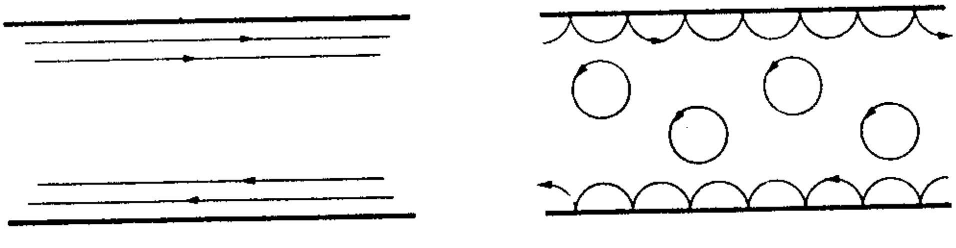
(Halperin, 1982)
Streda, 1987



(a)

(b)

FIG. 4.1. (a) An ideal non-interacting 2DEG with infinitely hard reflecting walls at $y = y_1, y_2$, and periodic boundary conditions in the x-direction. (b) The energy branch spectrum of (a) in a strong magnetic field. When the guiding center y_0 is within l_B of the hard walls at y_1, y_2, the 'Landau levels' are perturbed to higher energies.



(a) semi-classical

(b) classical

FIG. 4.2. Current-carrying edge states. (a) The semiclassical picture. An edge state forms at each intersection of E_F with an energy branch or Landau level, and current is carried in opposite directions for opposite sides of the 2DEG. (b) The classical analogue of (a), in which current-carrying edge states correspond to skipping orbit trajectories. Non-skipping orbits carry no current.

From J. Simmons
Ph. D. thesis
1990

- Experimental evidence for "edge states" in QHE

Haug, 1988

Washburn, 1988

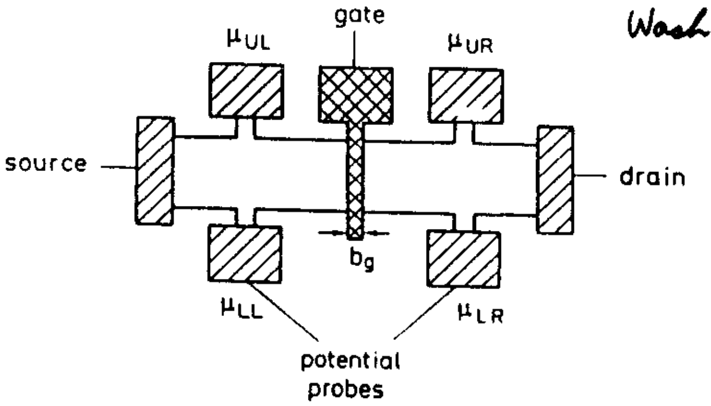


FIG. 1. Schematical view of the device.

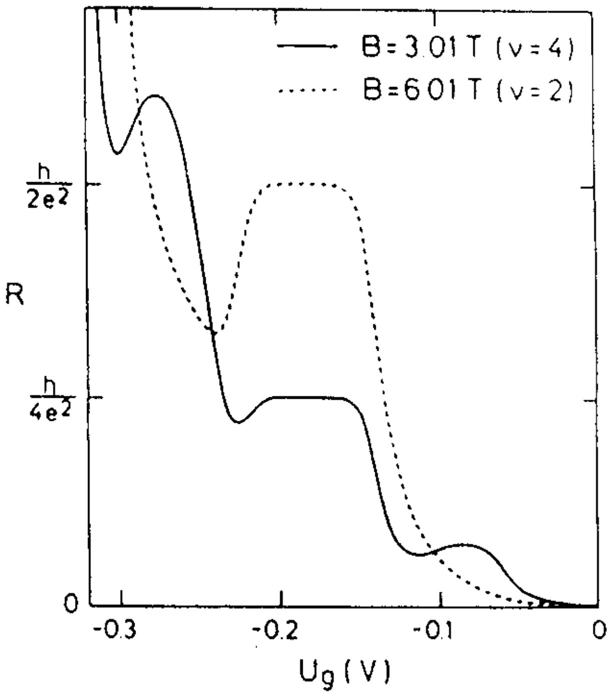


FIG. 2. The resistance measured across a gate region with $b_g = 10$ μm vs the gate voltage U_g at a temperature of $T = 0.55$ K for magnetic fields corresponding to the filling factors $\nu = 4$ and $\nu = 2$.

- Explanation based on "edge states"

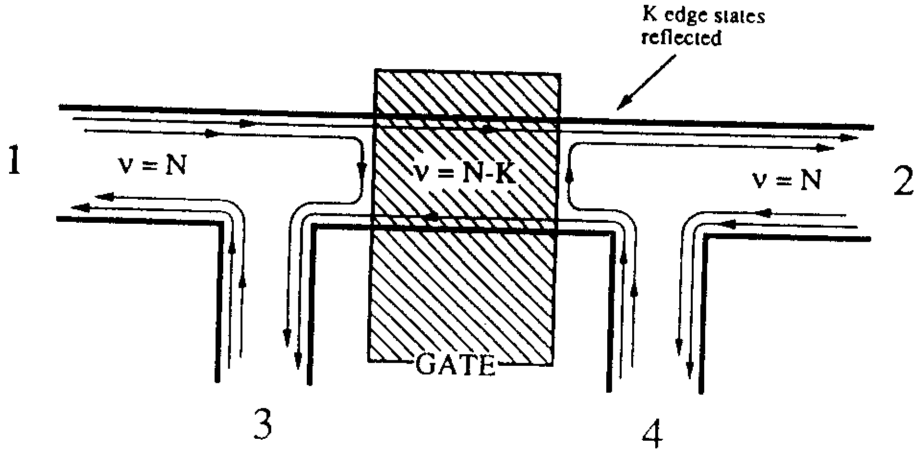


FIG. 4.5. A four terminal device with a gate across the center. If the gate voltage and B are varied so that the filling factor away from the gate is N, and under the gate is $N-K$, with N, K both integers, the longitudinal resistance $R_{1,2;3,4}$ will be quantized to $h/e^2[1/(N-K)-1/N]$.

$$R = \frac{h}{e^2}\left(\frac{1}{\nu_{gate}} - \frac{1}{\nu}\right)$$

From J. Simmons PhD thesis

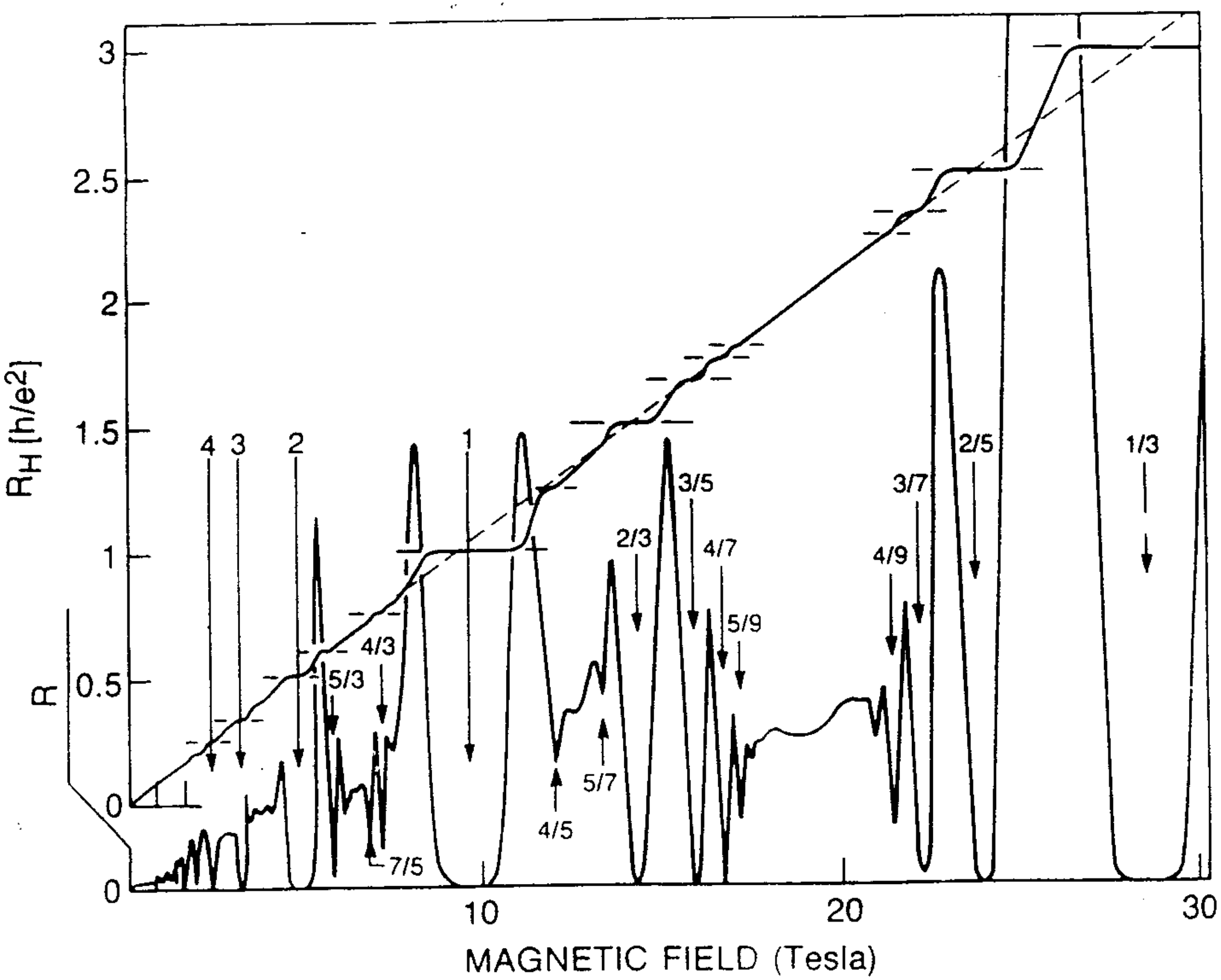
Refs.

Büttiker, 1986

Streda, 1987

Haug, 1988

FQHE (Fractional Quantum Hall Effect)



Willett

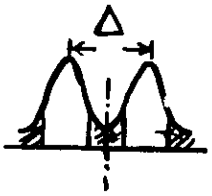
3. FQHE

- At filling factors $\nu = 1/m$ (m = odd) the stable ground state is an **incompressible quantum fluid** (=> no dissipation at T=0)

Lecture #

- A many-electron state (Laughlin's ψ) $\Psi_m = \prod_{j<k}^{N} (z_j - z_k)^m \exp\left(-\frac{1}{4}\sum_{\ell}^{N} |z_\ell|^2/\ell_0^2\right)$

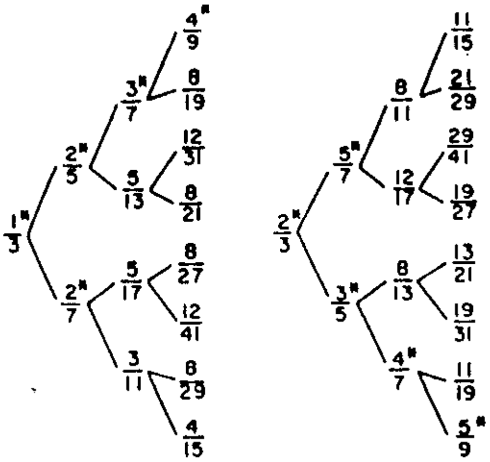
1 → • Excitations separated from ground state by finite energy gap (Δ)



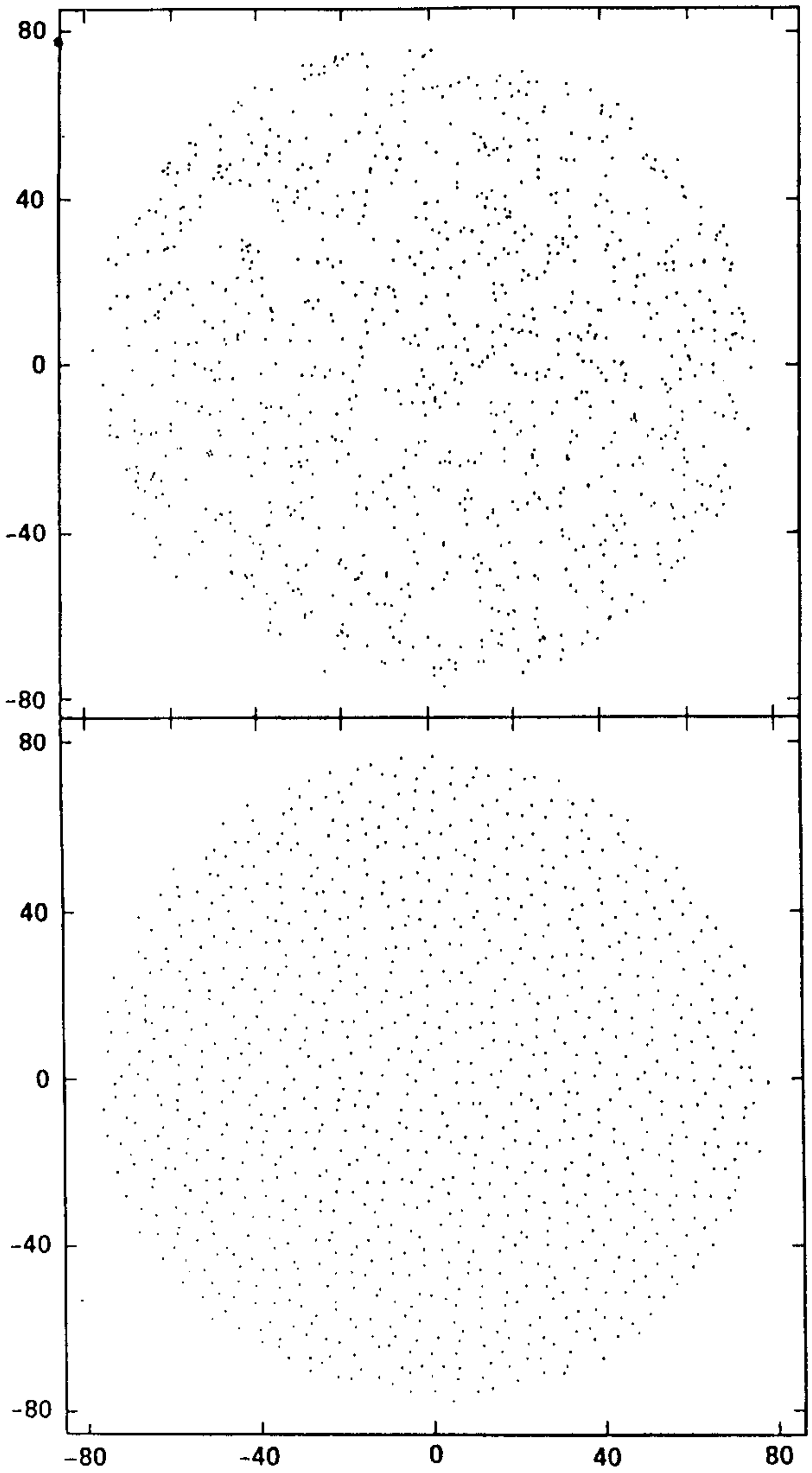
3) 4 → • Excitations are quasiparticles with fractional charge. The quasiparticles can interact, and under appropriate conditions, form new ground states · · ·

=> "Old" Hierarchical model (Haldane, Halperin, Laughlin)

$$\nu = \cfrac{1}{p + \cfrac{\alpha_1}{p_1 + \cfrac{\alpha_2}{p_2 + \cdots}}}$$



2 → • For $\nu < \nu_W$, the ground state is expected to be a solid (Wigner crystal) rather than a fluid. ν_W **theoretically** predicted to be ~ 1/5 to 1/11



2DES
random gas

$\nu = \frac{1}{3}$ FQH
Liquid state

Laughlin in: Prange & Girvin, "The QHE"

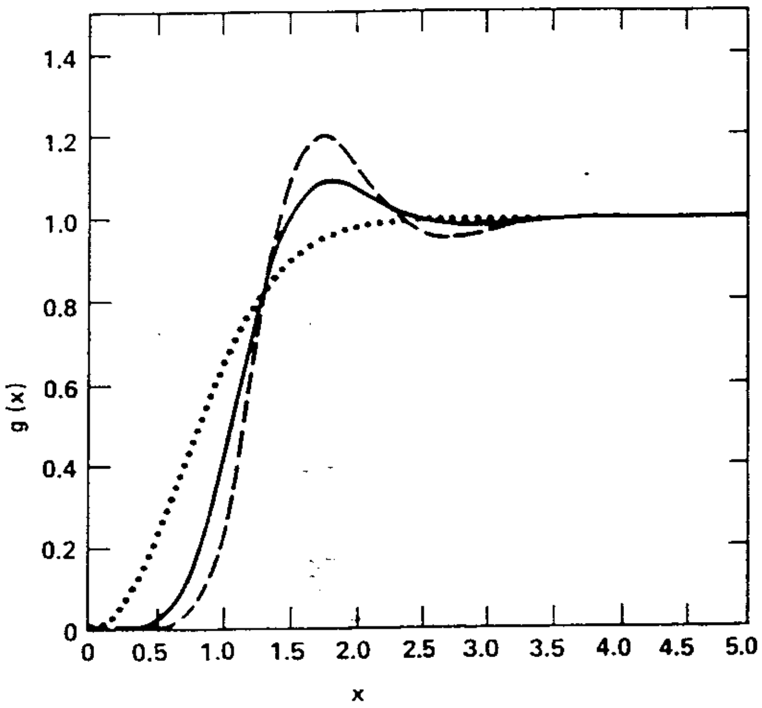


Figure 7.4 Radial distribution function for Ψ_m, as defined in Eq. 3.8, for $m=1$ (dotted), $m=3$ (solid), and $m=5$ (dashed) plotted against the reduced variable $x=|z_1-z_2|/(2m)^{\frac{1}{2}}$.

$$\Psi_W(z_1,\ldots,z_n) = \sum_\sigma sgn(\sigma)$$

$$\times\ \varphi_{j_1k_1}[z_{\sigma(1)}]\ \cdots\ \varphi_{j_nk_n}[z_{\sigma(n)}]\ , \qquad (7.3.10)$$

where the orbitals $\varphi_{jk}[z]$ are Gaussians

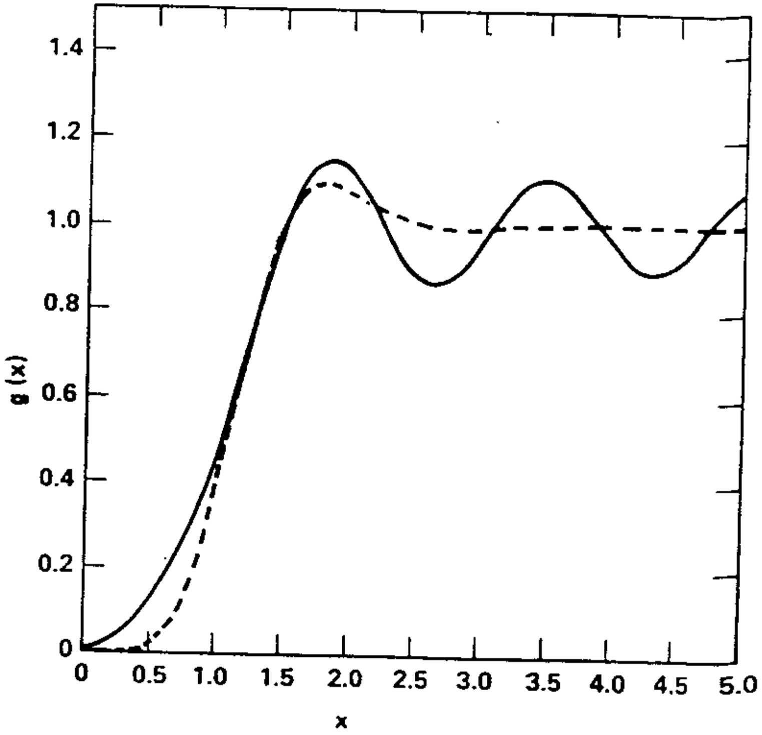


Figure 7.5 Comparisons of radial distribution function of Ψ_m (dashed) with that of Hartree-Fock Wigner crystal (solid) at the same density, for $m=3$, plotted against reduced variable $x=|z_1-z_2|/(2m)^{\frac{1}{2}}$.

centered at hexagonal lattice sites $z_{jk}^{(o)}$:

$$z_{jk}^{(o)} = \sqrt{\frac{4\pi m}{\sqrt{3}}}\left[j + \left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)k\right] . \qquad (7.3.12)$$

One sees that the two are almost identical

FQHE

- Laughlin w.f.

$$\Psi_m = \prod_{j<k}^{N} (z_j - z_k)^m \exp\left(-\frac{1}{4}\sum_{\ell}^{N} \frac{|z_\ell|^2}{\ell_B^2}\right)$$

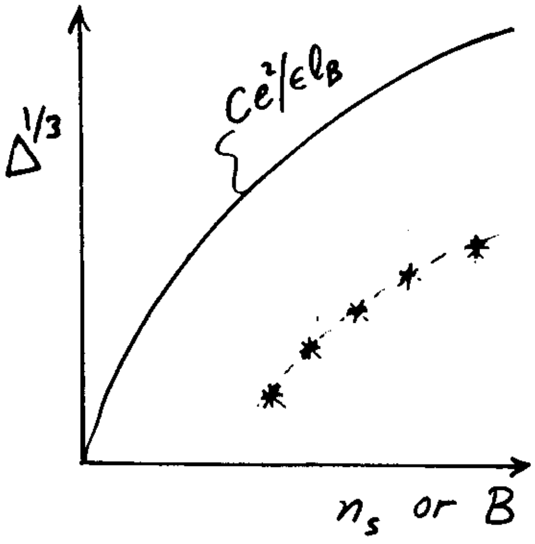
$$z \triangleq x - iy$$

$\nu = \frac{1}{m}$ $m =$ odd integer $\left[\nu = \frac{n}{d} = \frac{nh}{eB}\right]$

$\ell_B = (\hbar/eB)^{1/2}$ magnetic length

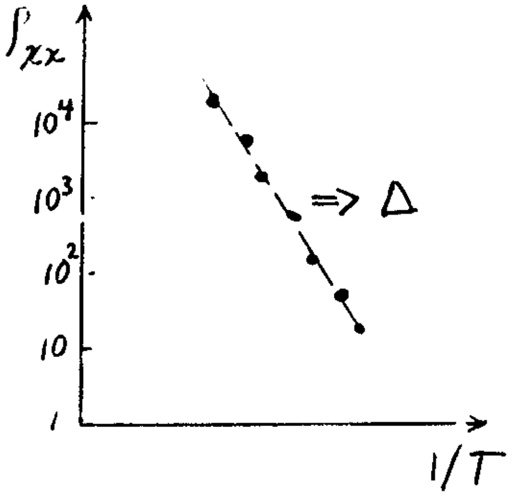
- Gap $\triangleq$ $\Delta \simeq Ce^2/\epsilon\ell_B \propto \sqrt{B} \propto \sqrt{n_s}$

$C \simeq 0.1$



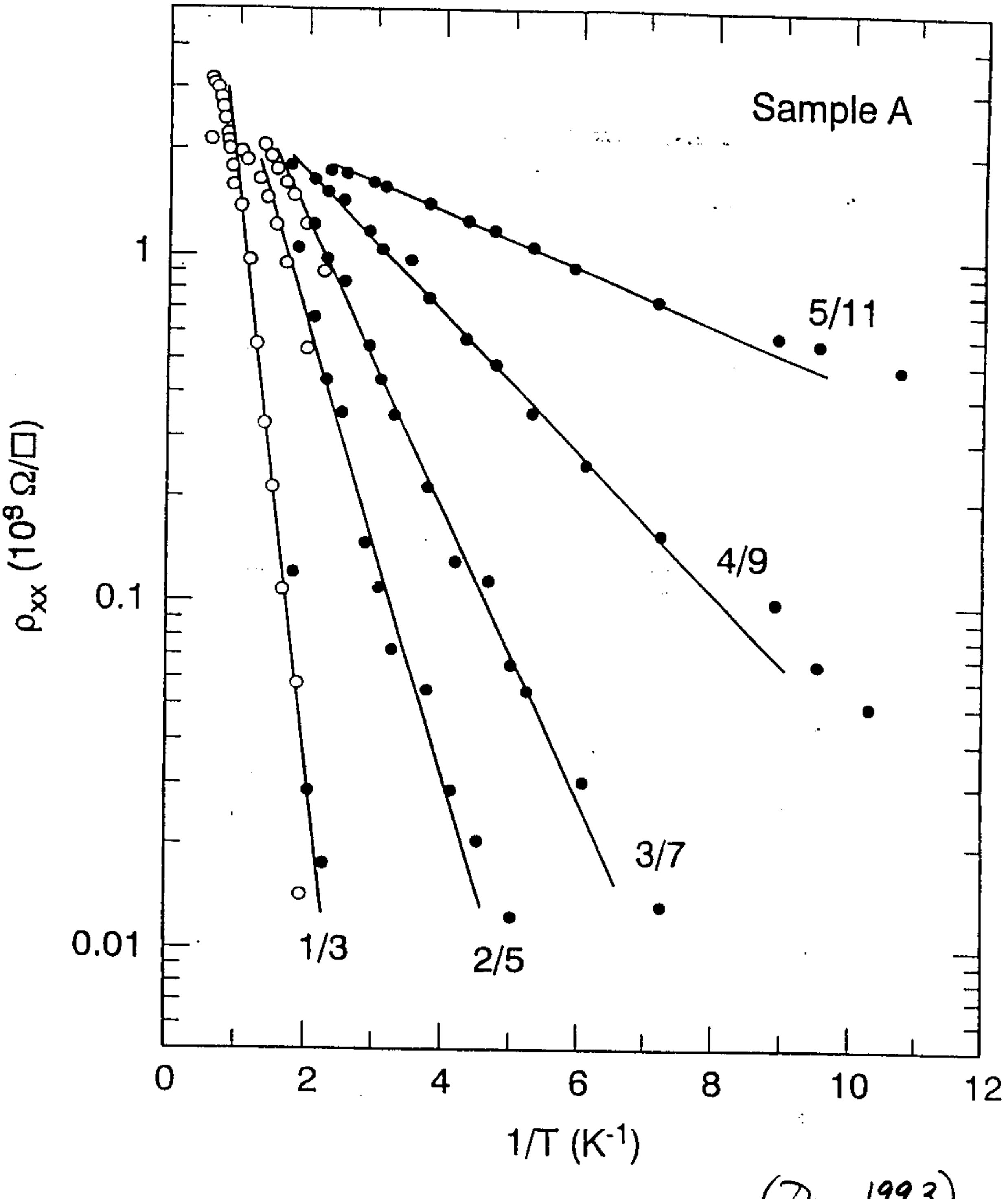
$*$ Experimental gap:

$$\rho_{xx} \sim \exp(-\Delta/2kT)$$



- Measurements of FQHE gaps

$$\rho_{xx} \propto \exp(-\Delta/2k_B T)$$

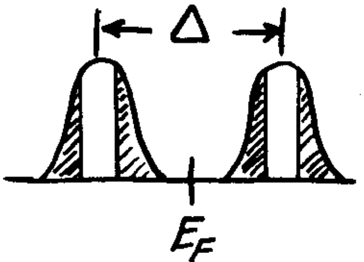


(Du, 1993)

Fig. 2

C&C 2M22686.1

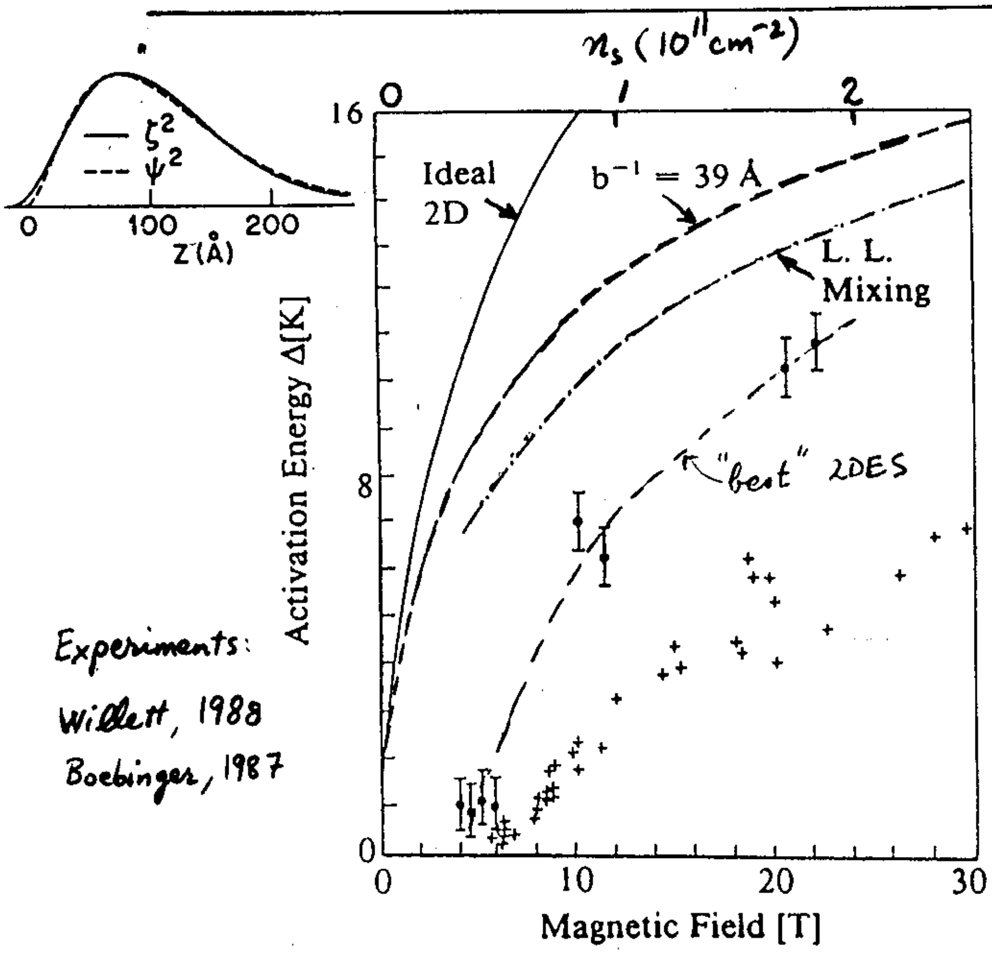
Energy Gap (Δ) for FQHE (@ $\nu = 1/3$)



Ideal 2DES: $\Delta \simeq 0.1 \frac{e^2}{\epsilon \ell_B} \propto B^{1/2} \propto n_s^{1/2} \propto 1/r$

$\ell_B = (\hbar / eB)^{1/2}$

Real 2DES: $\Delta < 0.1 \frac{e^2}{\epsilon \ell_B}$



Experiments:
Willett, 1988
Boebinger, 1987

Real 2DES

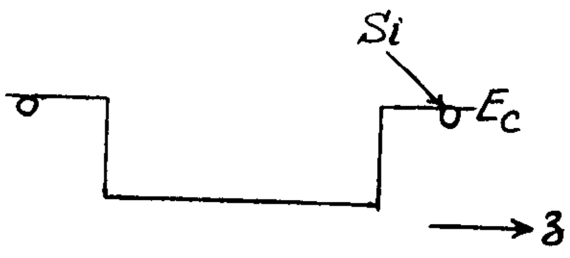
- Finite layer thickness (Zhang, 1986)
- Landau level mixing (Yoshioka, 1986)
- Disorder (Platzman, 1989
  MacDonald, 1985
  Gold, 1986)

→ • Experimental study of dependence of Δ on layer thickness

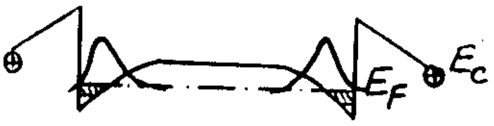
Clean 2D systems with additional degree of freedom

Quasi-2DES in Wide Parabolic Wells

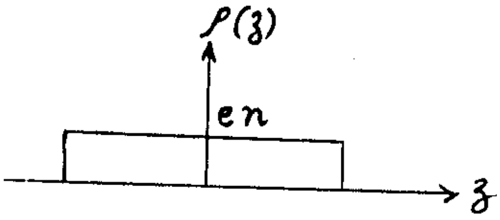
potential (empty)



potential (with e⁻)



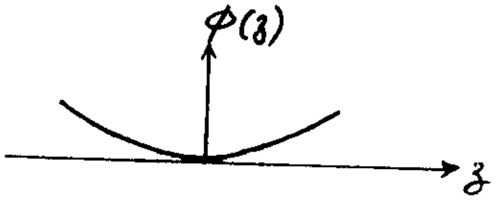
"fictitious" jelium



$$\frac{d^2\phi}{dz^2} \propto n$$

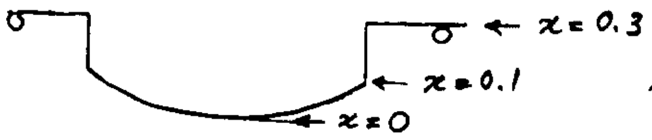
$$\Downarrow$$

"fi-titious" potential



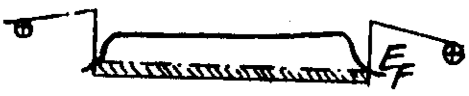
$$\phi = \frac{e^2}{2\epsilon_s} n z^2$$

potential (empty)



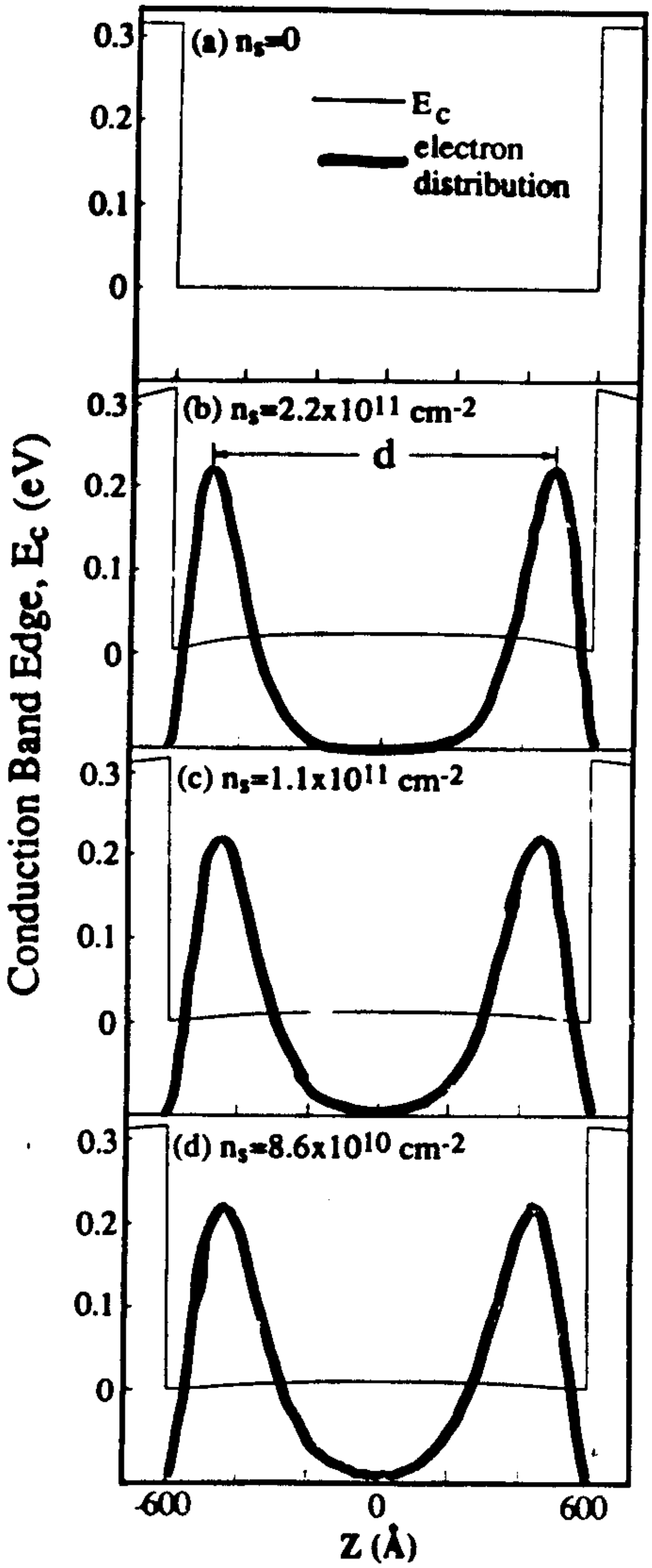
$Al_x Ga_{1-x} As$

potential (with e⁻)

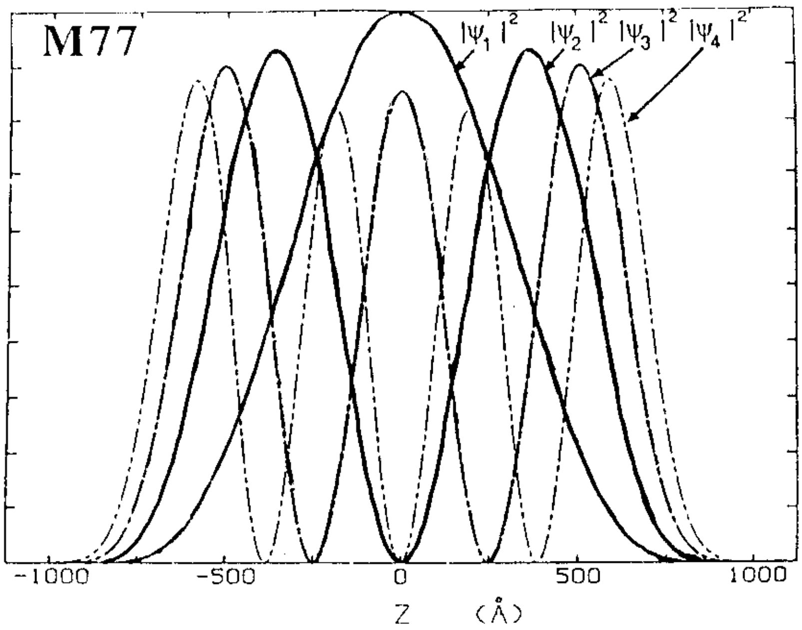


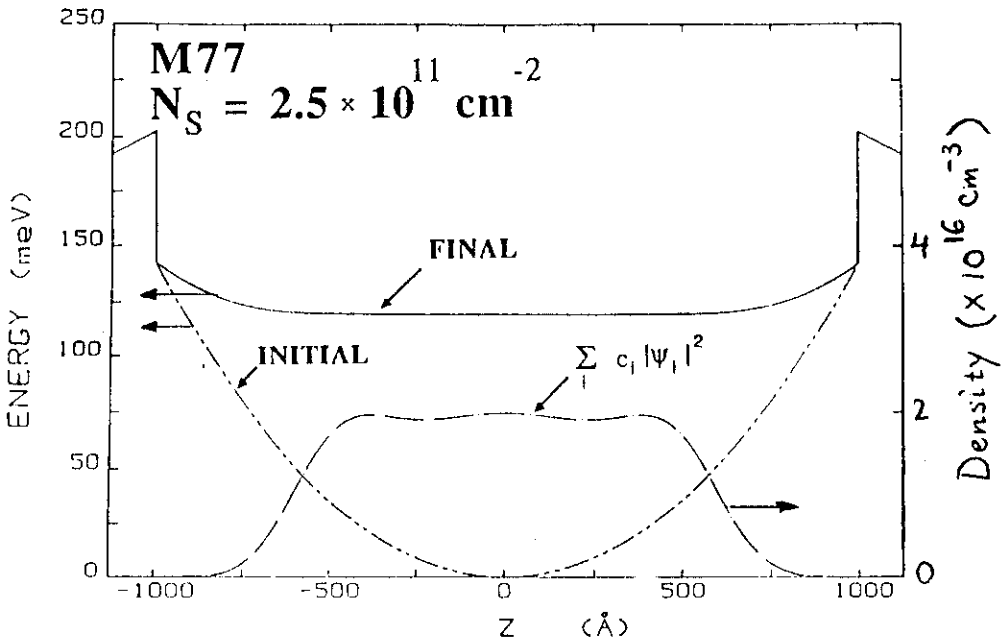
Ref. Shayegan 1988c
1990

- Double-layer electron system (Lecture III)

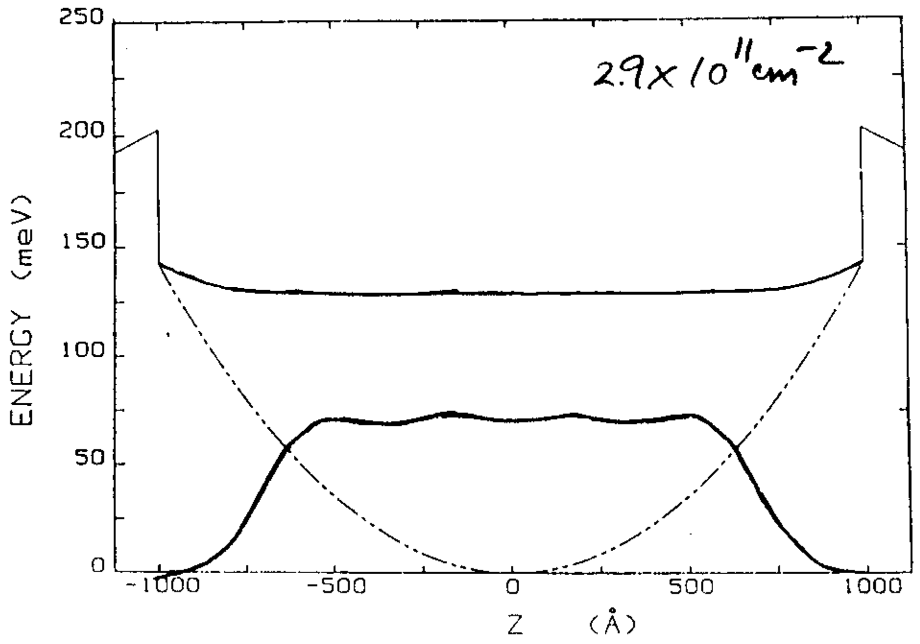


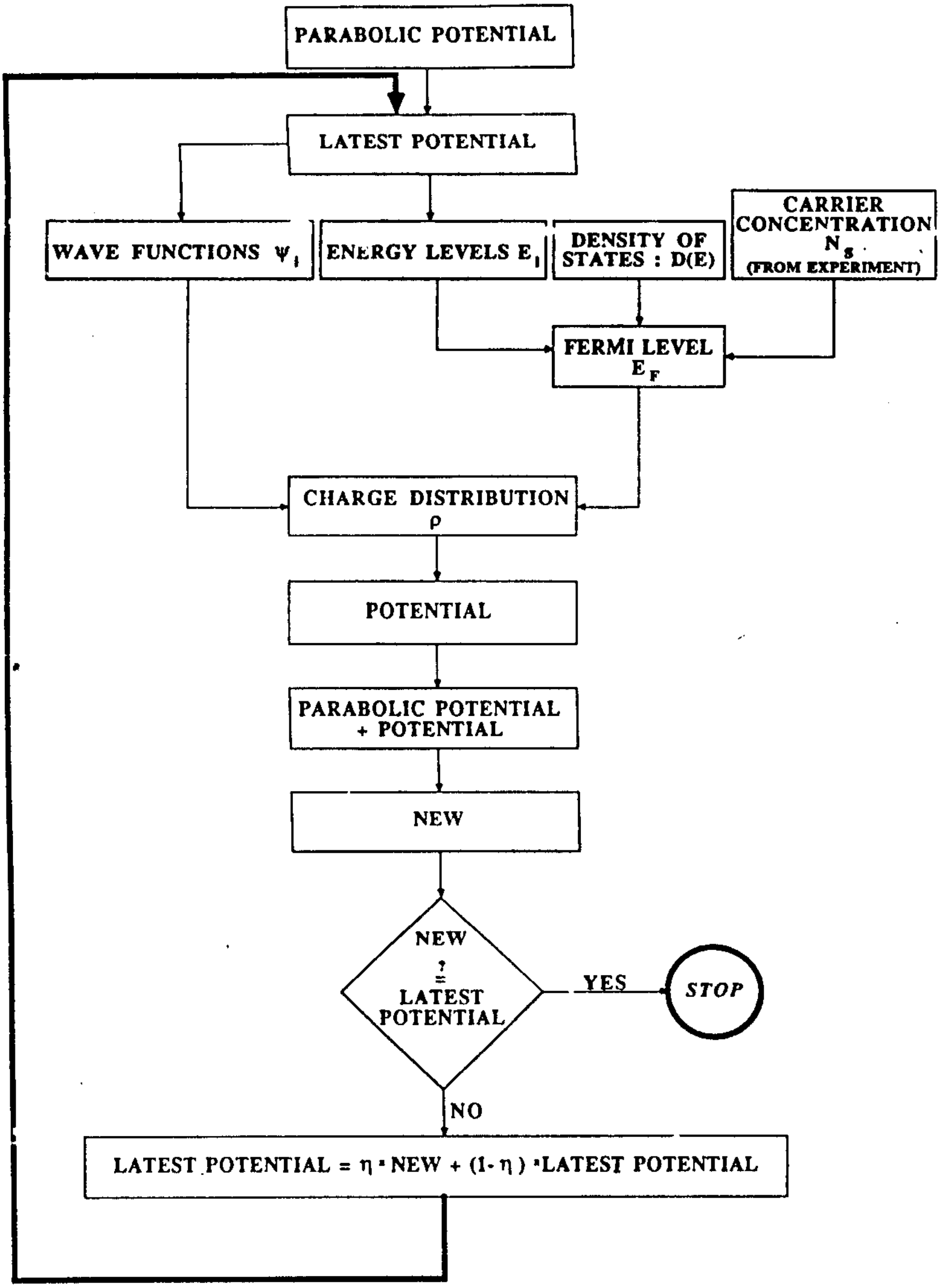
Quasi - 2D system in wide parabolic well





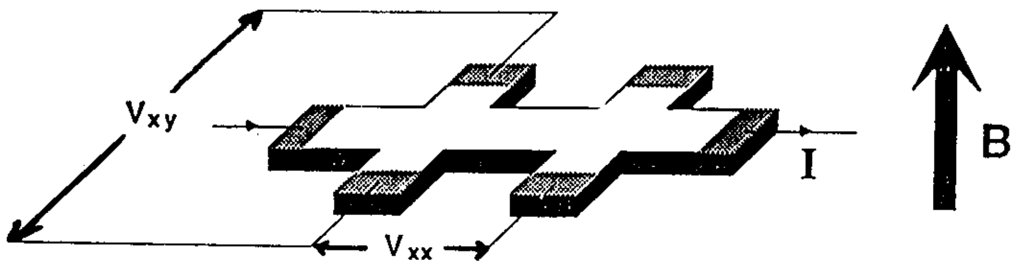
(Sajoto, 1989)

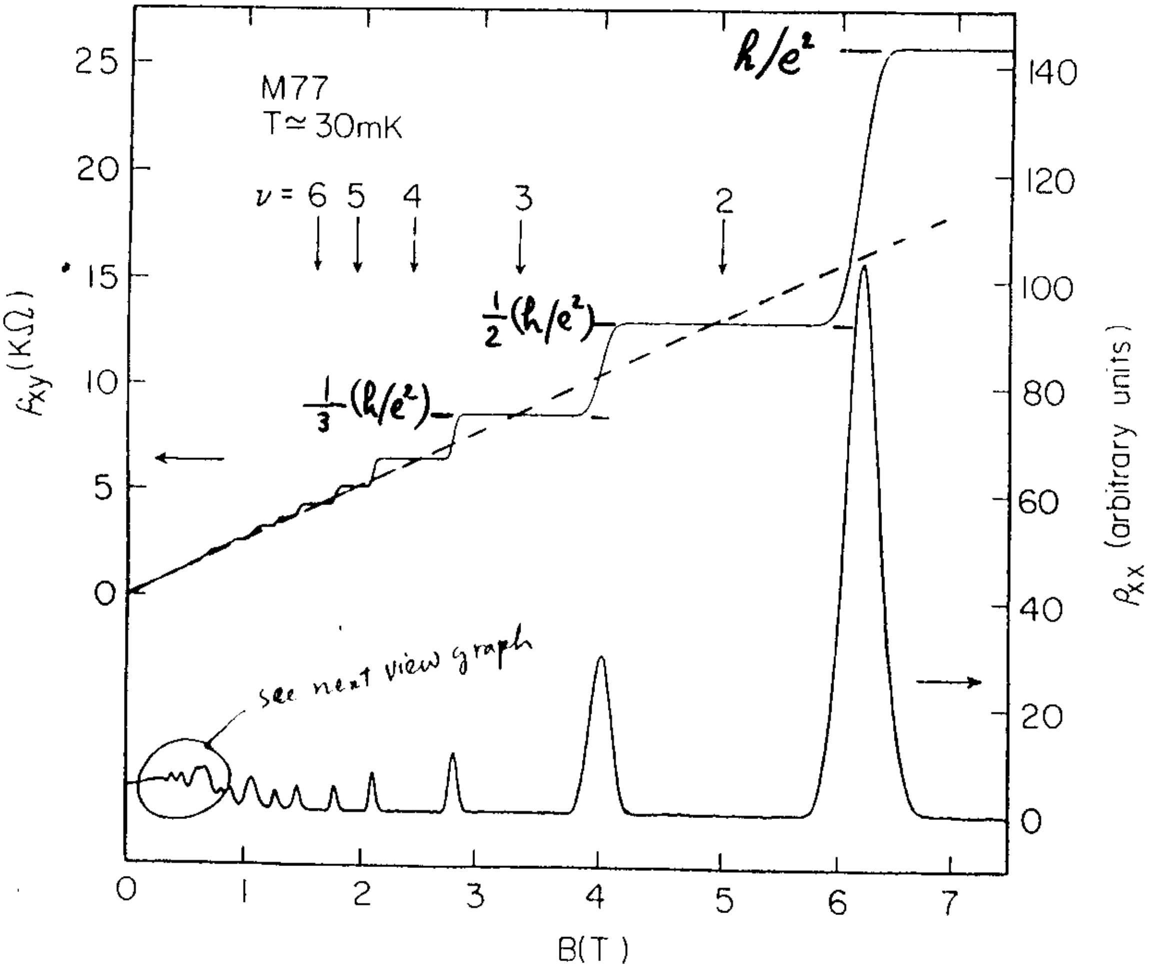

2.9x10^11 cm^-2
250
200
150
100
50
0
ENERGY (meV)
-1000
-500
0
500
1000
Z (Å)


PARABOLIC POTENTIAL
LATEST POTENTIAL
WAVE FUNCTIONS ψ_i
ENERGY LEVELS E_i
DENSITY OF STATES : D(E)
CARRIER CONCENTRATION N_s (FROM EXPERIMENT)
FERMI LEVEL E_F
CHARGE DISTRIBUTION ρ
POTENTIAL
PARABOLIC POTENTIAL + POTENTIAL
NEW
NEW ≟ LATEST POTENTIAL
YES
STOP
NO
LATEST POTENTIAL = η · NEW + (1- η) · LATEST POTENTIAL

J. Jo (1988)

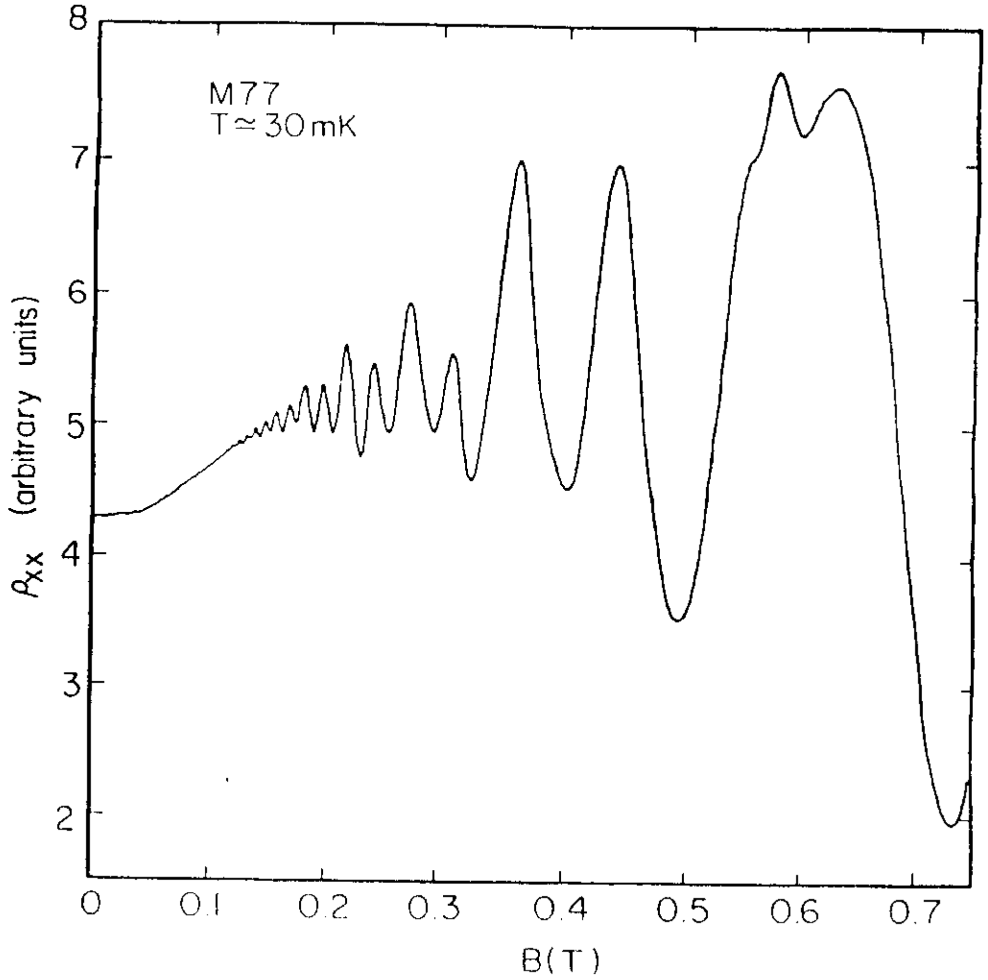
IQHE (Integer Quantum Hall Effect)



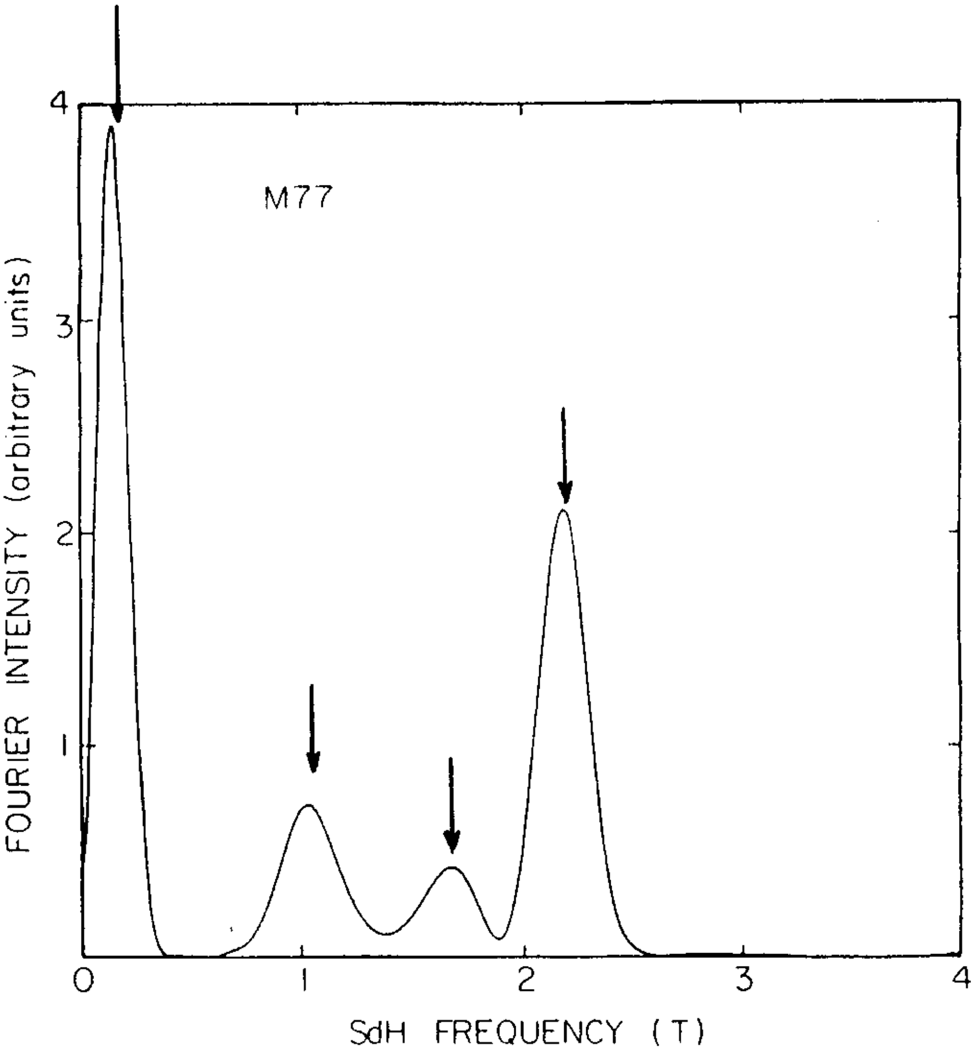


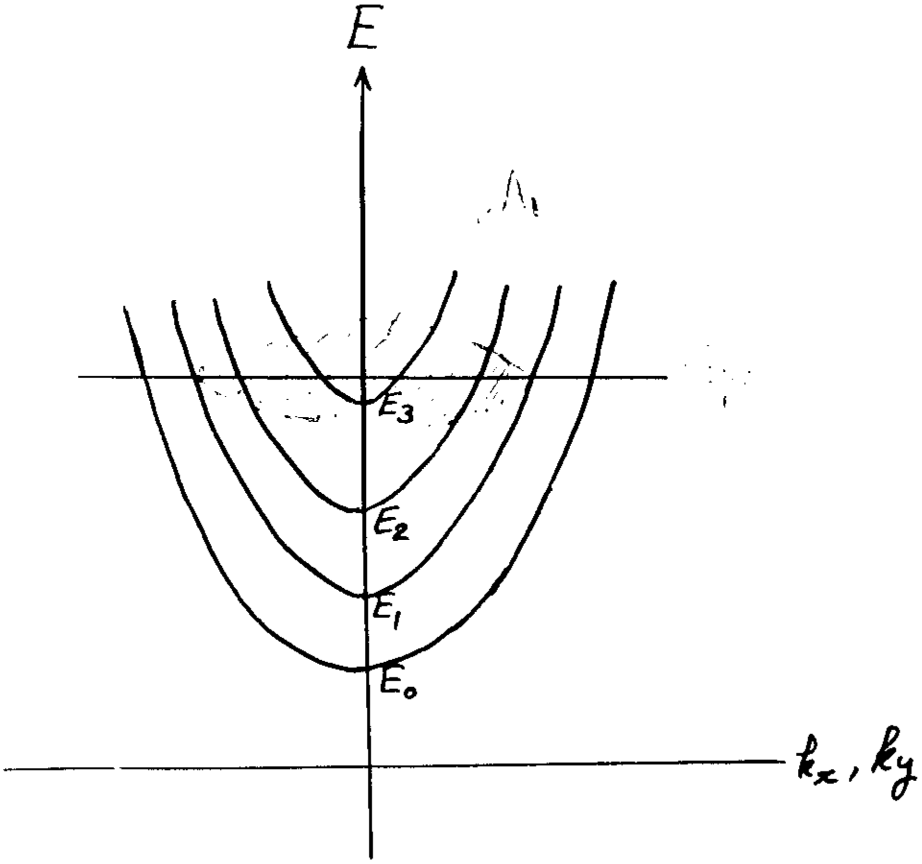
Sajoto (1989)

Expansion of Low-B data
∴ multisubband occupation



- Fourier transform of last viewgraph's (ρ_{xx} vs $\underline{\underline{\frac{1}{B}}}$) data



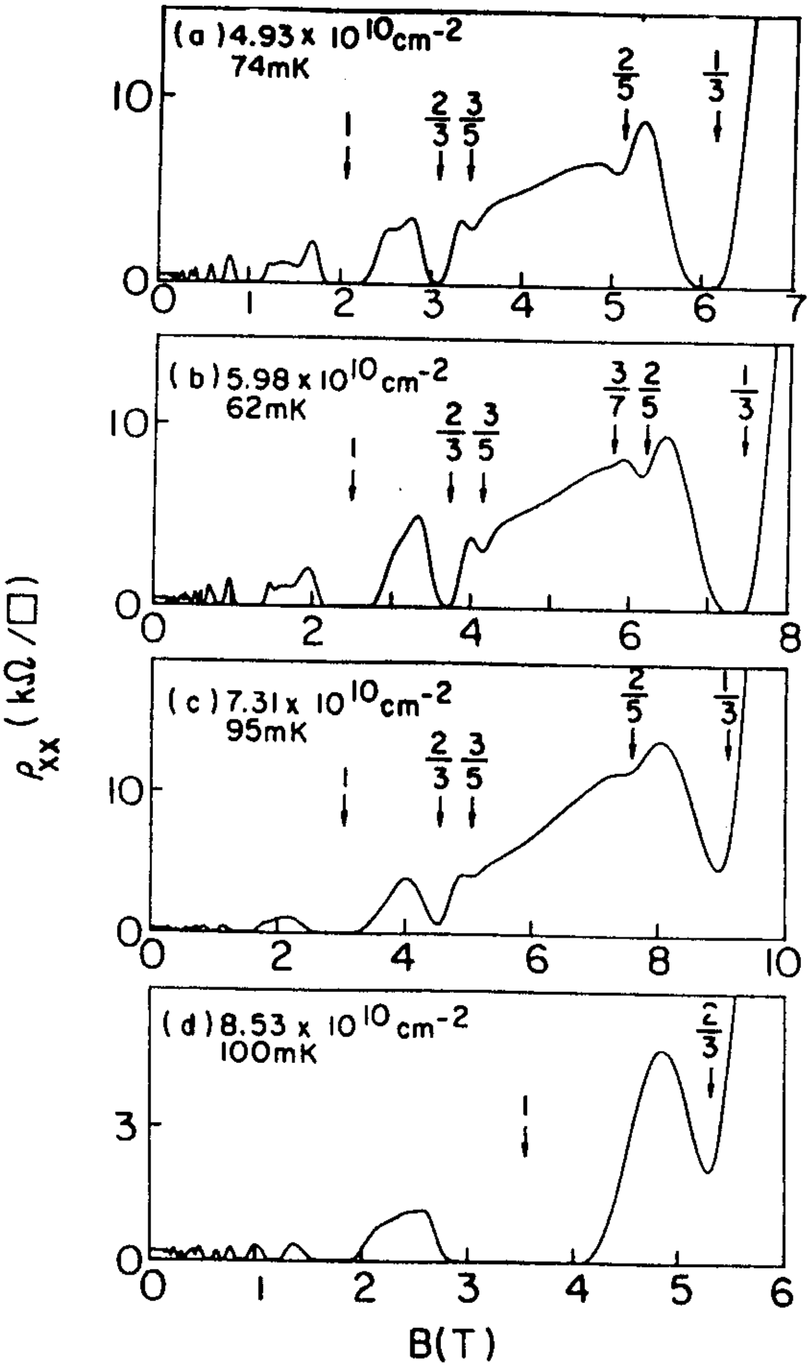


Shubnikov-de Haas Frequency ∝ Fermi Surface X-section
∝ Subband Density

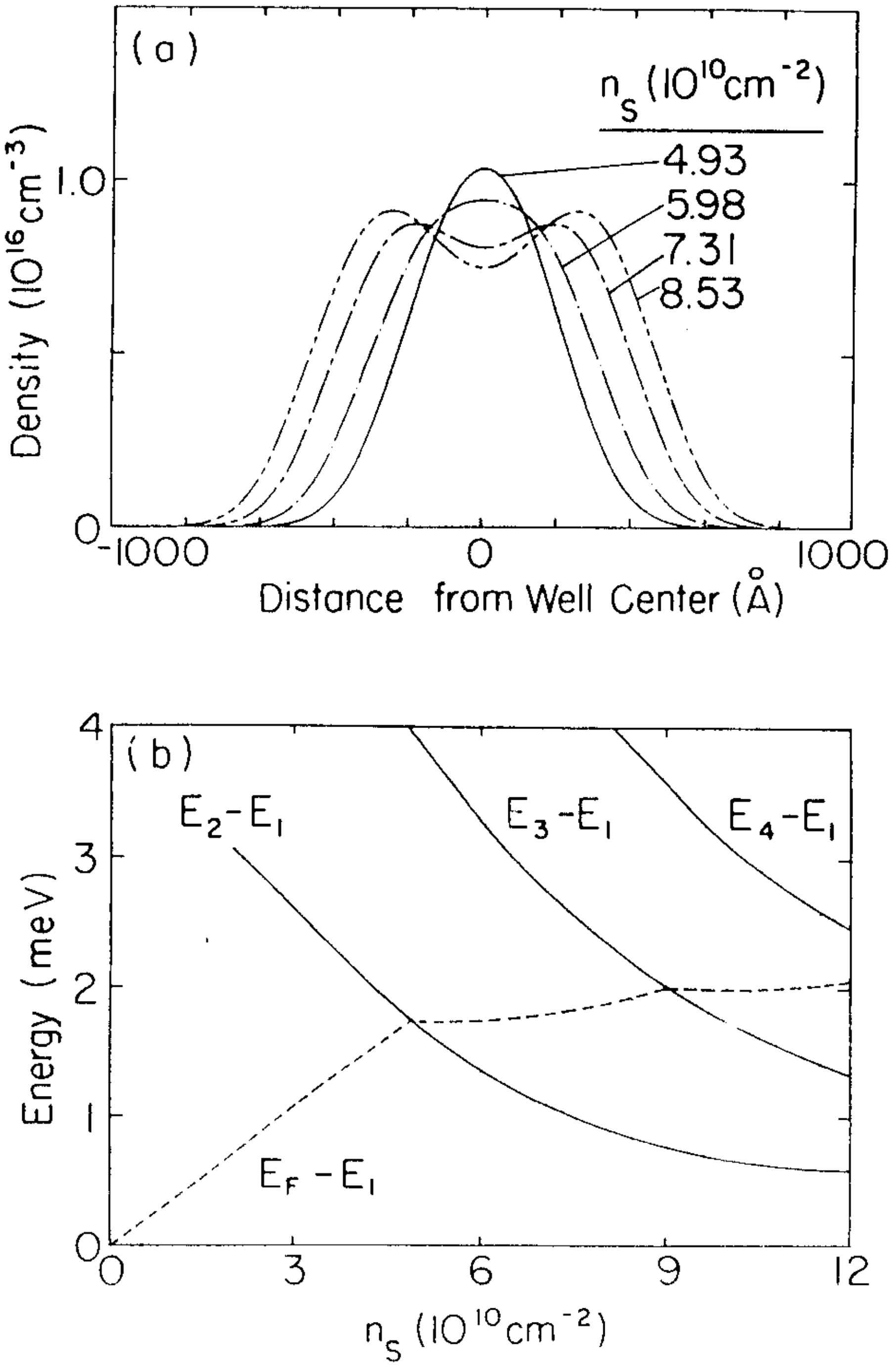
$$\underbrace{0.48 \times f_{SdH}}_{\text{Tesla}} = \underbrace{n_{\text{sub.}}}_{\times 10^{11}\ \text{cm}^{-2}}$$

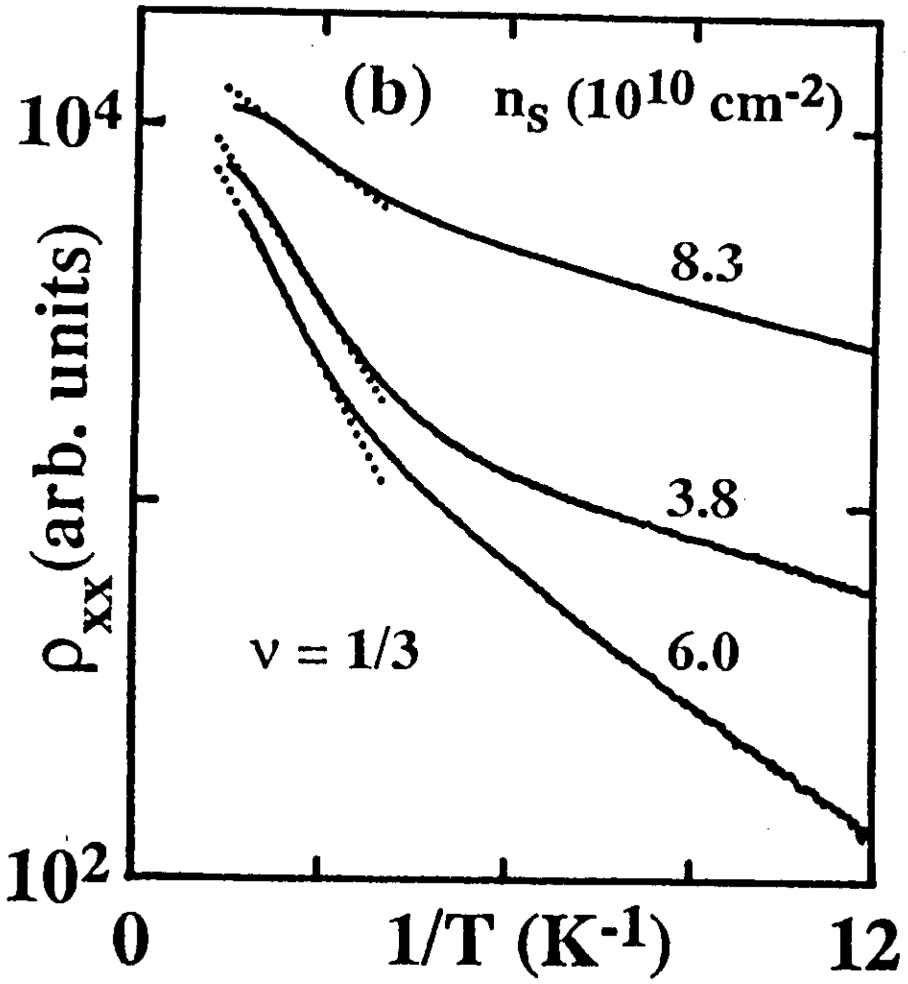
- FQHE in wide parabolic wells

(charge density profile & energy structure shown on next V.G.)

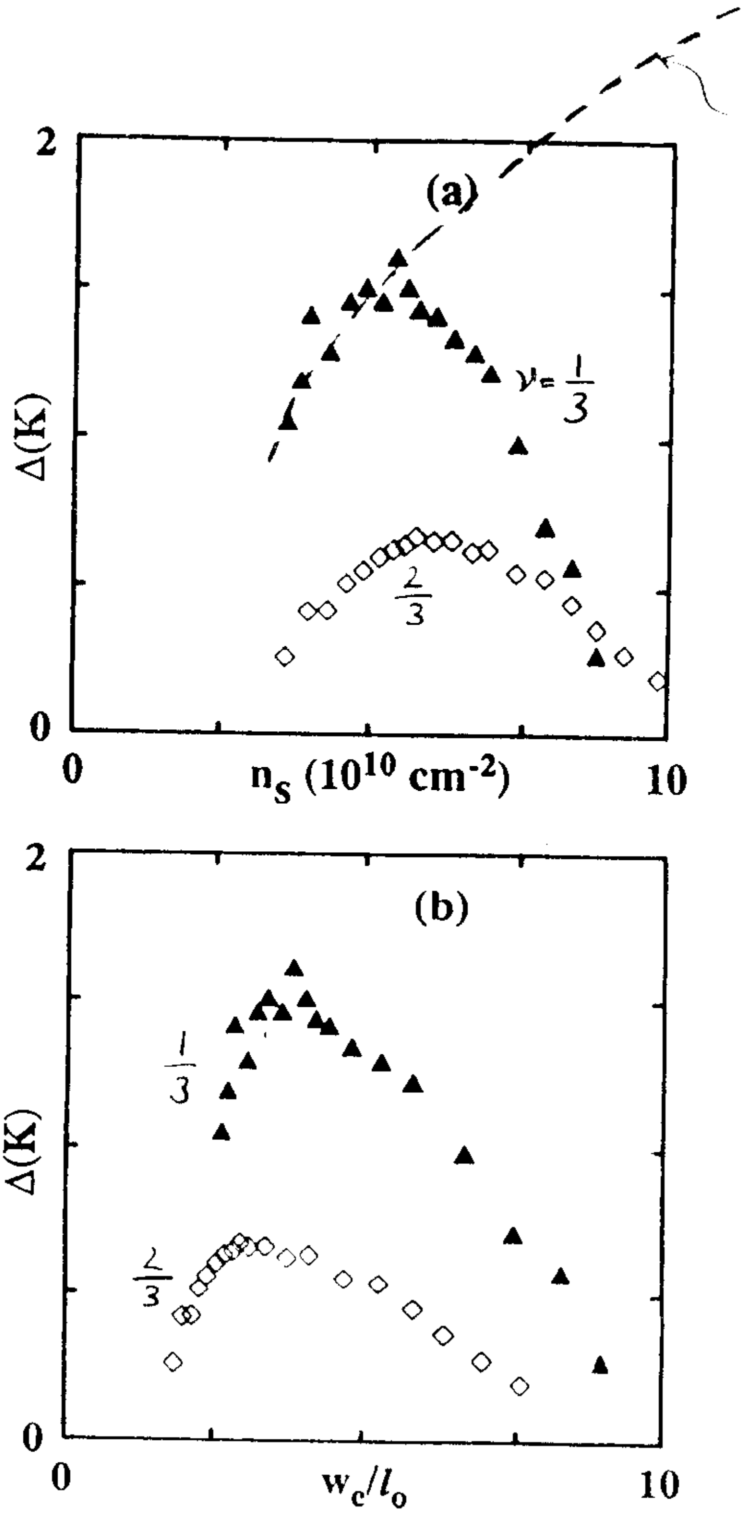


Shayegan 1990


(a)
n_s (10^{10}cm^{-2})
4.93
5.98
7.31
8.53
Density (10^{16}cm^{-3})
1.0
0
-1000
0
1000
Distance from Well Center (Å)
(b)
E_2-E_1
E_3-E_1
E_4-E_1
E_F-E_1
Energy (meV)
4
3
2
1
0
0
3
6
9
12
n_s (10^{10}cm^{-2})

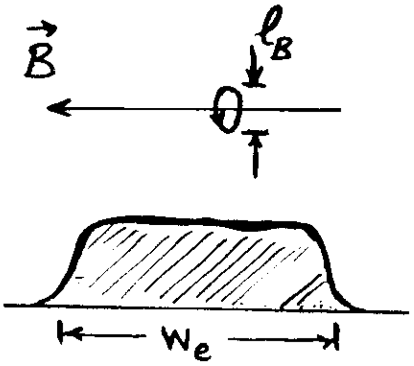

(b)
n_s (10^{10} cm^{-2})
10^4
10^2
ρ_{xx}(arb. units)
8.3
3.8
6.0
$\nu = 1/3$
0
1/T (K^{-1})
12

Collapse of the FQHE in "thick" electron systems



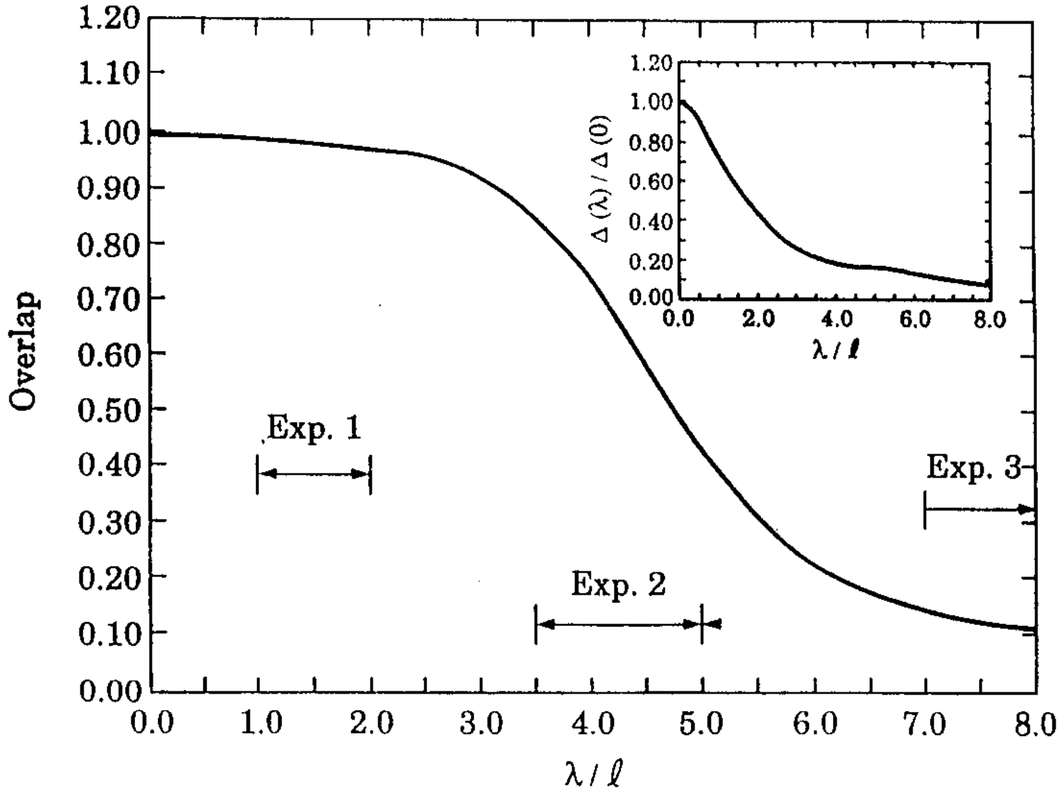
Standard "thin" 2DES:

$$\Delta \sim \frac{e^2}{\epsilon l_B} \sim \sqrt{B} \sim \sqrt{n_s}$$



Destruction of Fractional Quantum Hall Effect
In Thick Systems

Song He, F.C. Zhang,* X.C. Xie, and S. Das Sarma (1990)
Department of Physics
University of Maryland
College Park, Maryland 20742



RAPID COMMUNICA

SICAL REVIEW B VOLUME 47, NUMBER 15 15 APRIL 1993-

M. Shayegan

Collapse of the fractional quantum Hall effect in a parabolic quantum well

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Department of Theoretical Physics, University of Oulu, SF-90570 Oulu, Finland

(Received 13 April 1992; revised manuscript received 24 November 1992)

We study, using finite-system calculations, the behavior of the quasiparticle-quasihole energy gap and the collective excitation energy gap of a parabolically confined quasi-two-dimensional electron system subjected to a perpendicular magnetic field. We find the collapse of the energy gap as the areal density of the electron system is increased. Our results, which are in an excellent qualitative agreement with recent experiments, indicate that the collapse of the fractional quantum Hall effect in a parabolic quantum well is due to the electric-subband mixing. The effect of the finite layer thickness is also discussed.

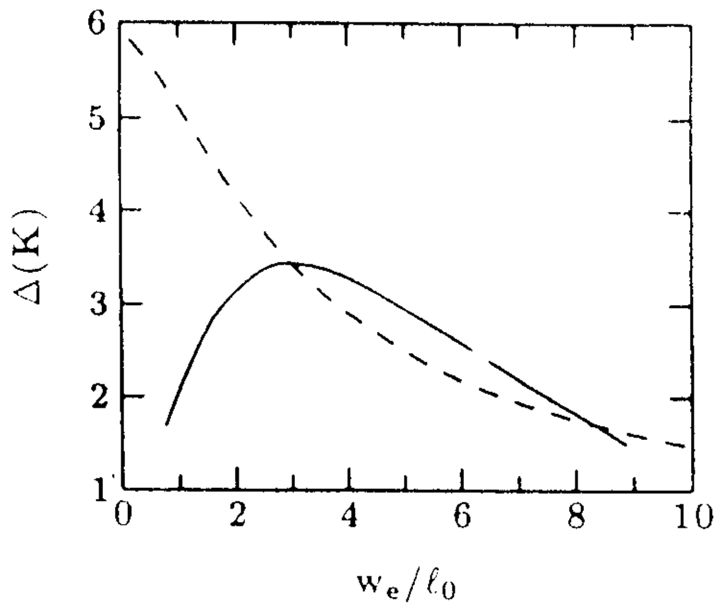


FIG. 2. The quasiparticle-quasihcle energy gap (Δ) at $\frac{1}{3}$ filling as a function of the dimensionless parameter w_e/ℓ_0. The dashed curve is the result calculated at $B = 5$ T without subband mixing, and the solid curve is for fixed confinement potential $\hbar\omega_z = 4$ meV (subband mixing included).

SPRING COLLEGE IN CONDENSED MATTER
ON QUANTUM PHASES
(3 May - 10 June 1994)

MAGNETOTRANSPORT MEASUREMENTS IN QUANTUM HALL EFFECT

LECTURE 4

IV. COMPOSITE FERMIONS & FQHE

General refs. (theory)

1. Jain, 1992
2. Halperin, Lee, Read, 1993

Mansour Shayegan

May 1994

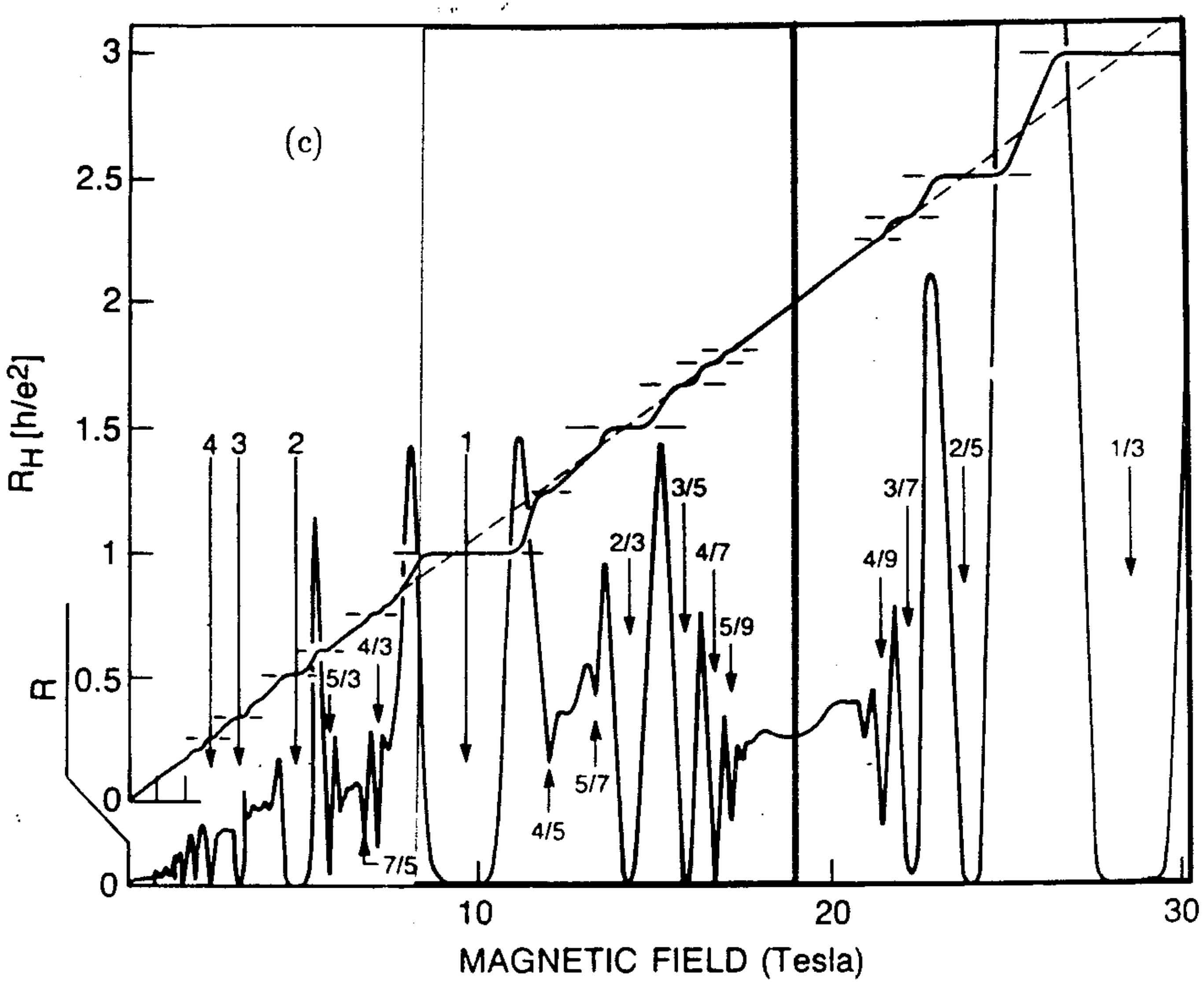

(c)
R_H [h/e²]
R
0
0.5
1
1.5
2
2.5
3
0
10
20
30
MAGNETIC FIELD (Tesla)
4
3
2
5/3
4/3
7/5
1
4/5
5/7
2/3
3/5
4/7
5/9
4/9
3/7
2/5
1/3

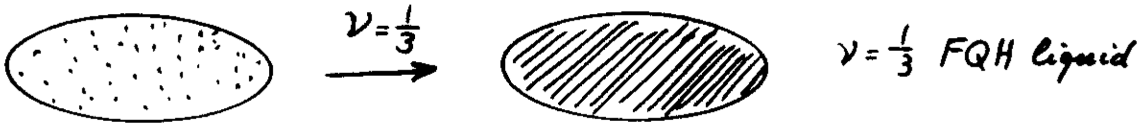
Fig. 1(c)

- "Old" or "Standard" hierarchy scheme :

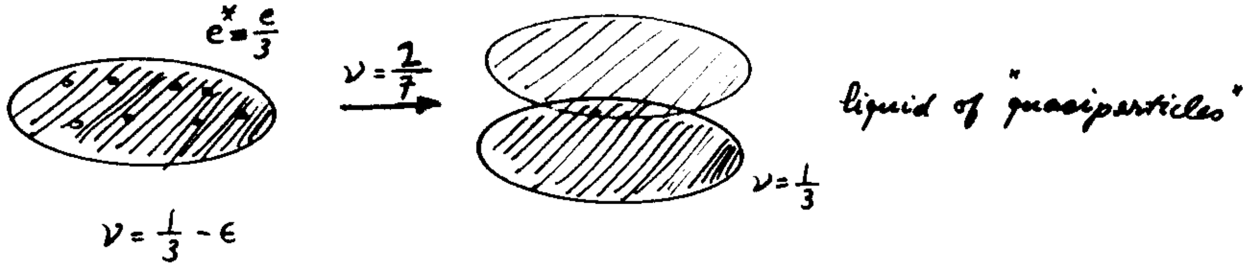
(Haldane, Halperin, Laughlin)

— Anyonic - quasiparticle condensation picture

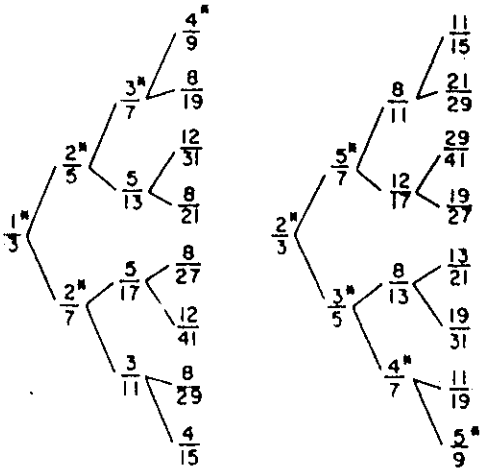
— "Primive" (fundamental) states : $\nu = 1/3, 1/5, 1/7$, etc.



— "Daughter" states : $\nu = 2/5, 3/5, 3/7, 2/7$, etc.



$$\nu = \cfrac{1}{p + \cfrac{\alpha_1}{p_1 + \cfrac{\alpha_2}{p_2 + \cdots}}}$$



- Problems with the "old" hierarchy (Jain, 1992)
  - no easy way to write wavefunctions
  - density of quasiparticles very large
  - strength of the predicted fractions not clear
  - → [FQHE treated differently from IQHE while experimentally they are very similar]

- Jain's wavefunctions:

$$\Psi_{1/3} = \prod_{j<k} (z_j - z_k)^3 \exp\left(-\frac{1}{4}\sum_i |z_i|^2\right) \quad \leftarrow \text{Laughlin}$$

$$= \underbrace{\prod_{j<k} (z_j - z_k)^2}_{D} * \underbrace{\prod_{j<k} (z_j - z_k) \exp\left(-\frac{1}{4}\sum_i |z_i|^2\right)}_{1^{st} \text{ Landau level } \Phi_1}$$

Jain:

$$\therefore \quad \Psi_{1/3} = D * \Phi_1$$

$$\Psi_{2/5} = D * \Phi_2$$

$$\Psi_{3/7} = D * \Phi_3$$

$$\vdots$$

$$\Psi_{\nu'} = D * \Phi_\nu \qquad \nu' = \frac{\nu}{2\nu+1} \qquad \text{e.g. } \frac{2}{5} = \frac{2}{2\cdot 2+1}$$

Two other operators:

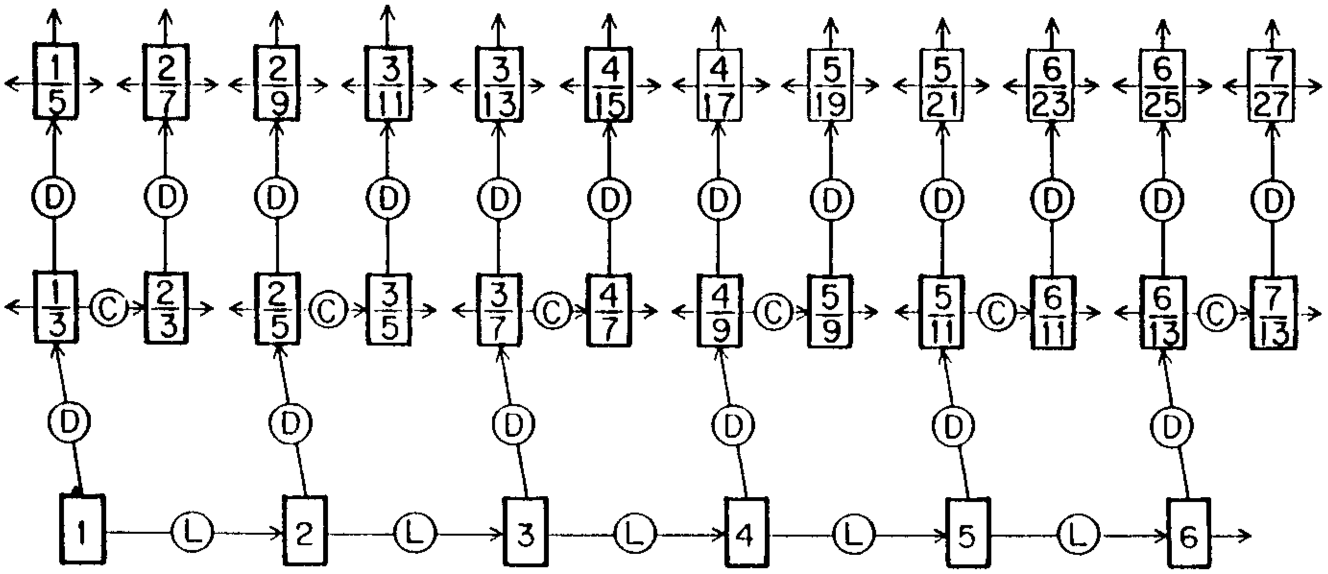
- particle-hole conjugation

$$\Psi_{\nu'} = C * \Psi_\nu \qquad \nu' = 1 - \nu \qquad \text{e.g. } \frac{2}{3} = 1 - \frac{1}{3}$$

- adds a filled LL to a state

$$\Psi_{\nu'} = L * \Psi_\nu \qquad \nu' = 1 + \nu \qquad \text{e.g. } \frac{4}{3} = 1 + \frac{1}{3}$$

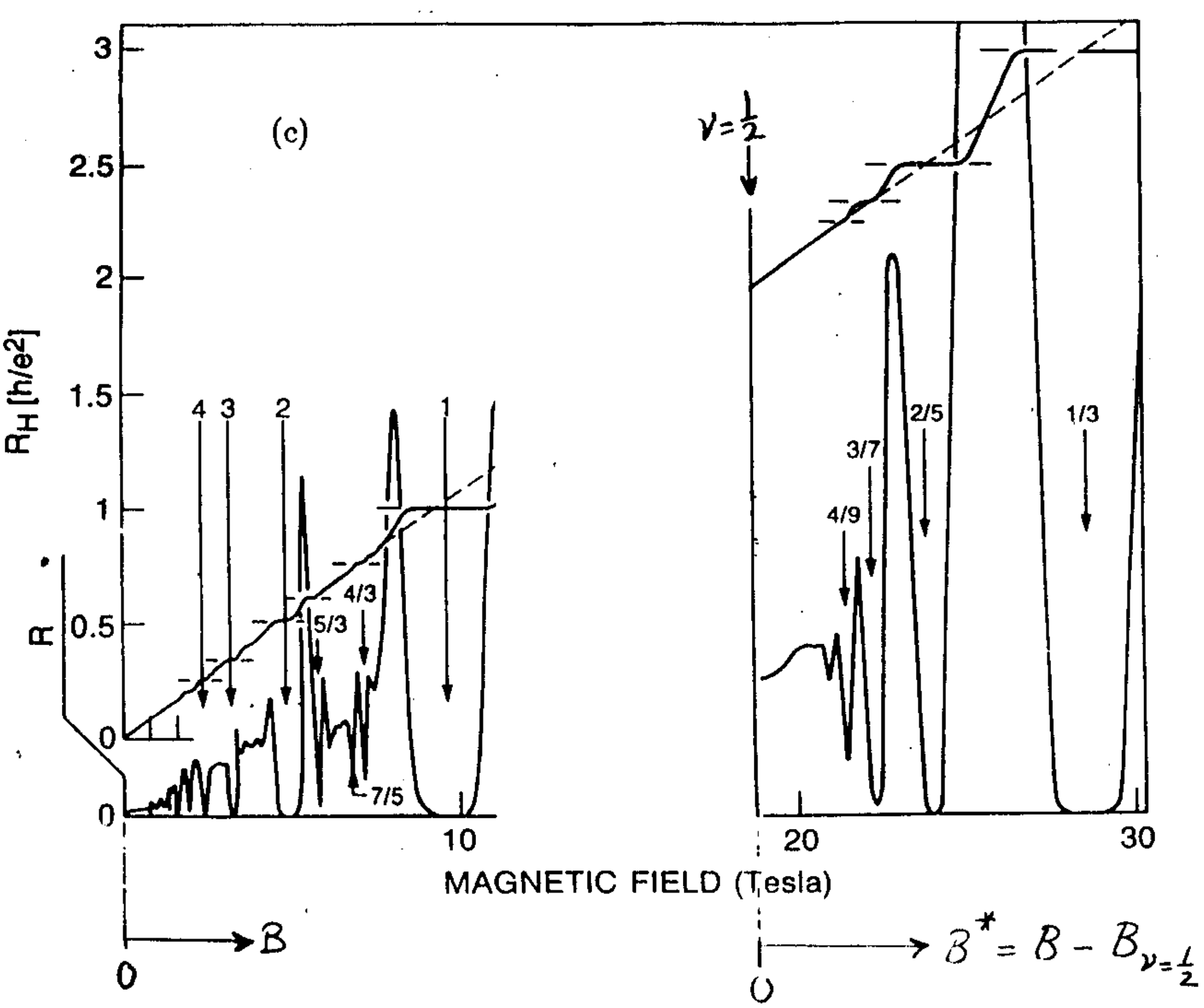
"New" Hierarchy
(Jain & Goldman, 1992)



D : $\nu' = \dfrac{\nu}{2\nu+1}$

C : $\nu' = 1-\nu$

L : $\nu' = 1+\nu$



Energy gap $= \hbar\omega_c = \dfrac{\hbar e B}{m_b^*}$

↑

$0.067 m_e$ for GaAs 2DES

Energy gap $= \dfrac{\hbar e B^*}{m_{CF}^*}$

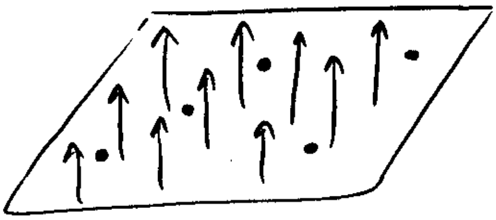
↑

?

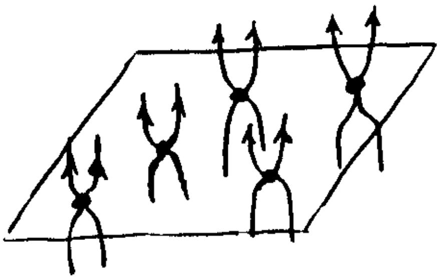
Composite Fermions

(Jain 1989, 1992; Halperin 1992)

$\nu = \frac{1}{2} \Rightarrow 2\Phi_0$ flux per e^-
$\uparrow$
h/e



Attach 2 flux quanta
to each e^-



$\Rightarrow$ "Composite Fermion" $\triangleq$ electron + 2 flux quanta
(CF)
sees no magnetic field (i.e., $B^* = 0$)

Two implications:

- IQHE of CFs $\rightarrow$ FQHE
- Sea of CFs at $\nu = 1/2 \rightarrow$ "Fermi surface"

Experiments:

A. Existence of a Fermi surface at $\nu = \frac{1}{2}$

1. Surface acoustic waves (SAW) (Willett, 1992, 1993
2. Geometrical resonances
   a. magnetoresistance of antidots (Kang, 1993)
   b. magnetic focusing (Goldman, 1994)
3. Thermopower (Ying, 1994)

B. FQHE as IQHE of CF's

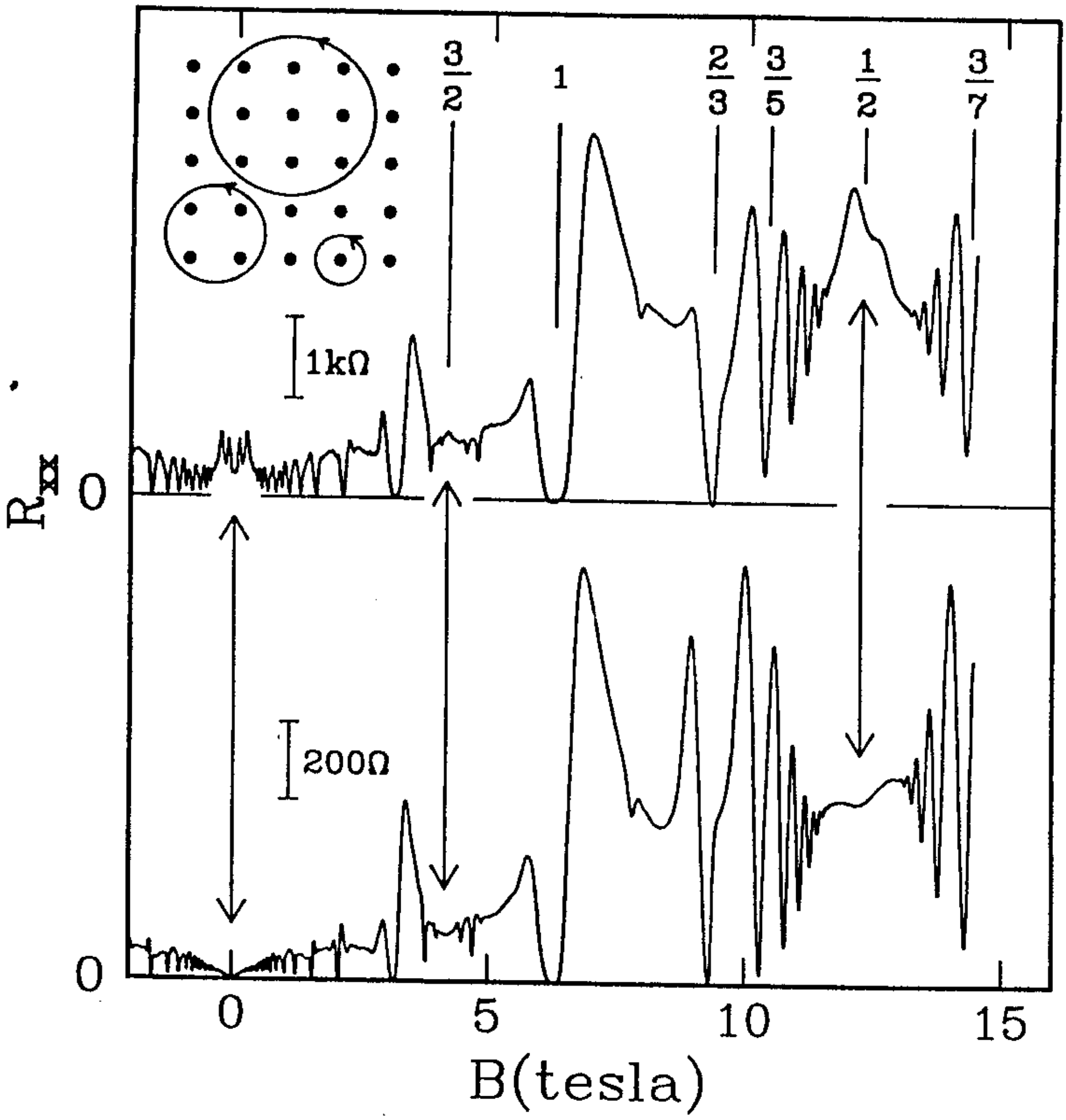
1. Energy gaps	(Du, 1993, 1994)	2DES
2. Effective masses	(Leadley, 1994)	
3. Scattering times	(Manoharan, 1994)	2DHS

- Geometrical Resonances

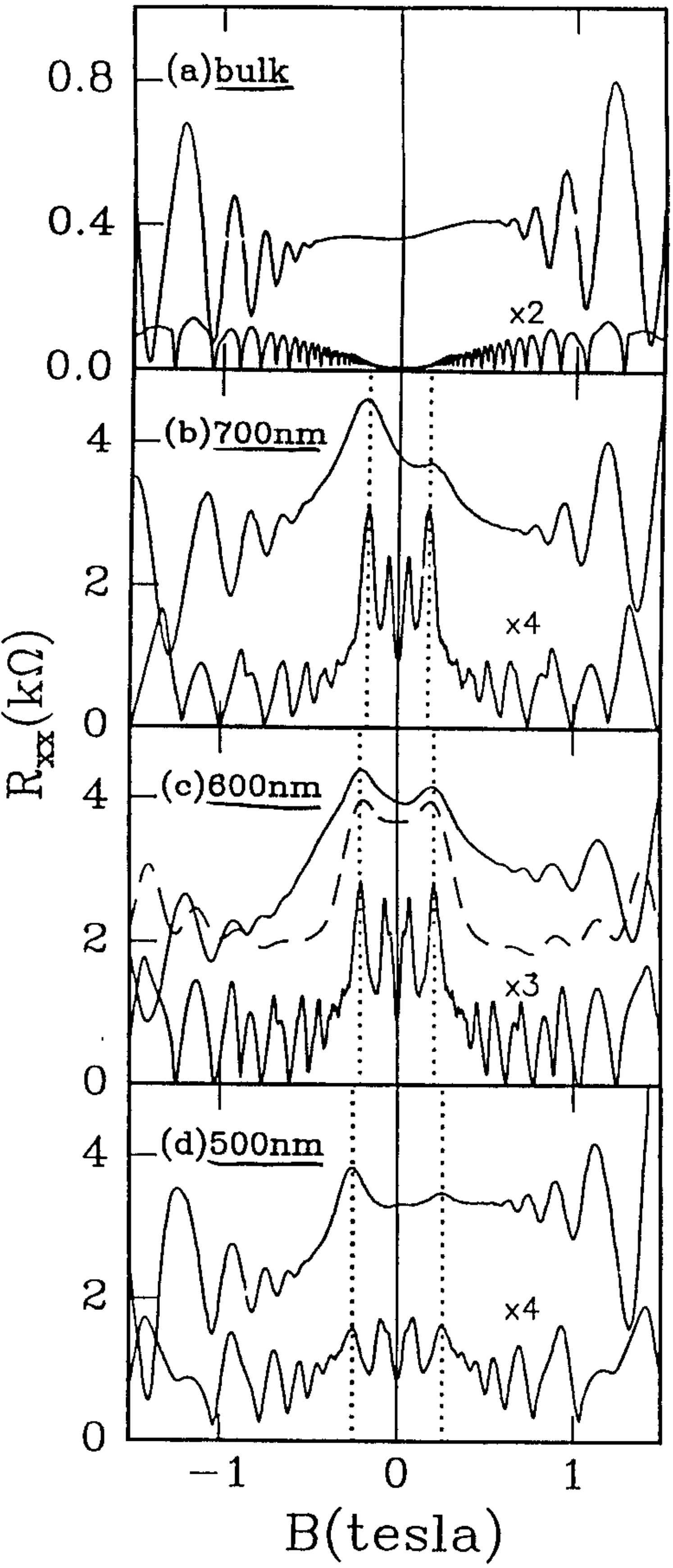
  - Magnetoresistance of "Anti-dot Lattices" (Kang, 1993)

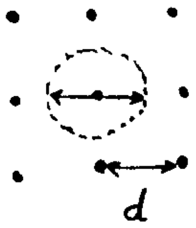
↑

Swiss cheese with periodic holes!



- lower curves:
  - zero of B is 0
- uper curves:
  - zero of B is $B_{\nu=1/2}$
  - also B-scale compressed by $\sqrt{2}$





Near $B = 0$

d = cyclotron diam. for e^- = $\frac{2\hbar k_F^0}{eB}$

$\Rightarrow B^0_{reson.} = \left(\frac{2\hbar}{e}\right)\cdot\left(\frac{k_F^0}{d}\right)$

$k_F^0 = (2\pi n_s)^{1/2}$

Near $B_{\nu=1/2}$

Expect same $B_{reson.}$ except that:

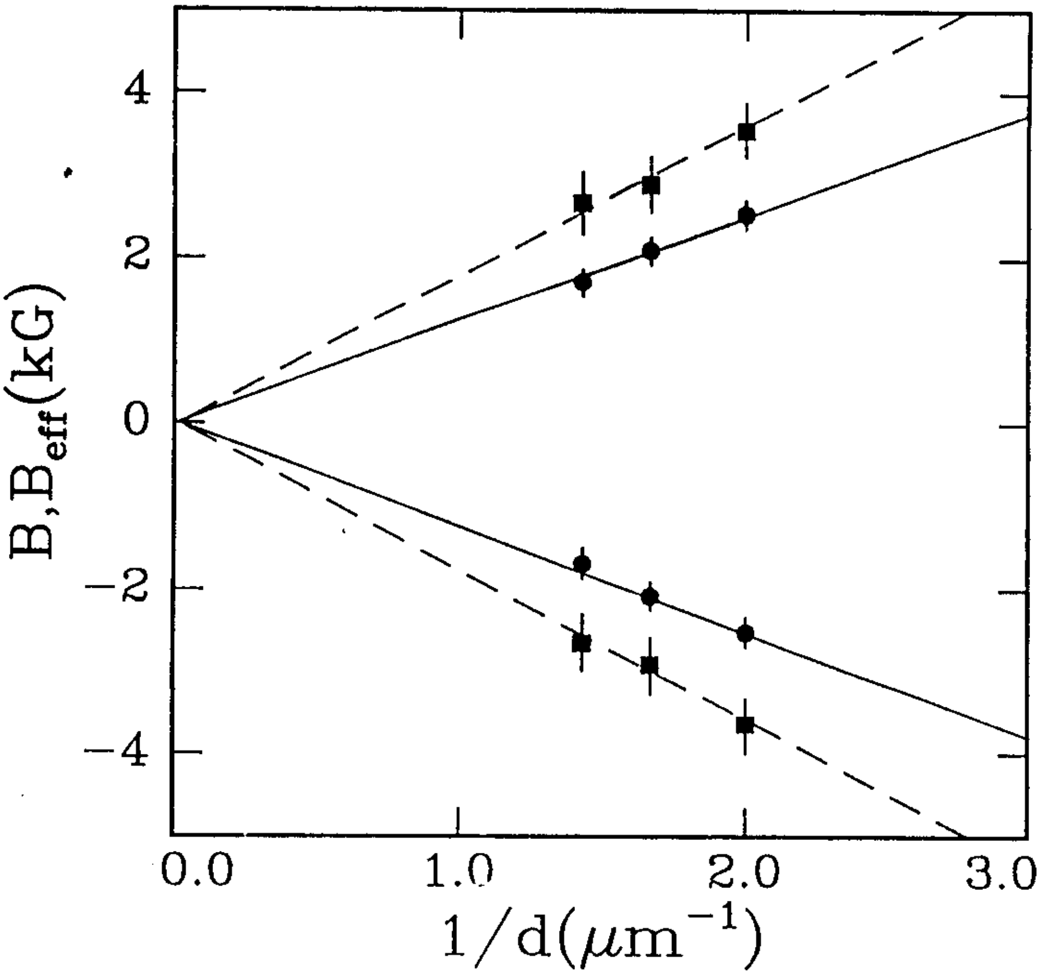
$k_F^{CF} = \sqrt{2}\ k_F^0$

(spin polarized)

- Dependence of $B_{reson.}$ on d :

Expect:

$$B_{reson.} = \left(\frac{2\hbar}{e}\right) \cdot \left(\frac{1}{d}\right) \cdot k_F \begin{array}{l} \nearrow k_F^{\circ} \\ \searrow k_F^{CF} = \sqrt{2}\, k_F^{\circ} \end{array}$$

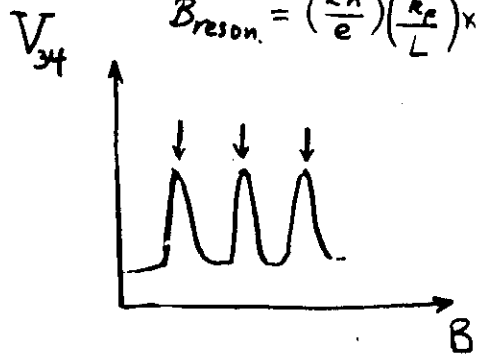


- Transverse Magnetic Focusing of CF's !?

(Goldman, 1994)

$I_{inj.}$: 2→1

$B_{reson.} = \left(\frac{2\hbar}{e}\right)\left(\frac{k_F}{L}\right) \times \text{integer}$



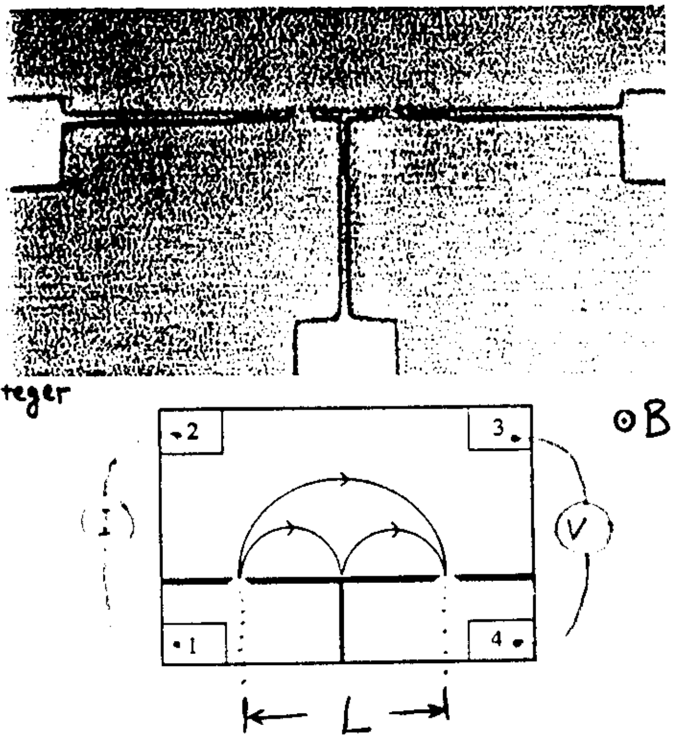


Fig. 1. Optical micrograph of the double-constriction area of Sample D; the width of the wide regions of the gates is 5 μm. Schematic geometry of a sample is shown below; the dark regions represent the gates deposited in the etched trenches, and the numbered rectangles represent four ohmic contacts.

- Magnetic Focusing near B=0

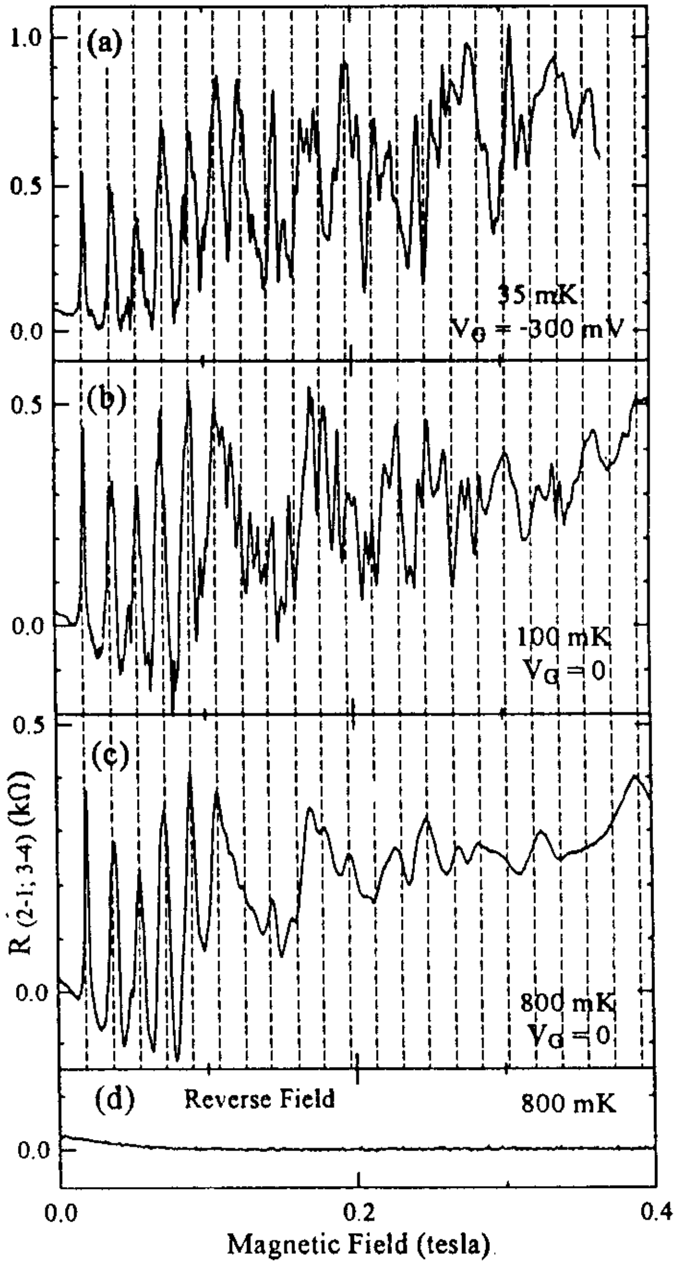


Fig. 2. Magnetoresistance of Sample E. (a) - (c) show the magnetic focusing spectra, (d) gives the resistance in the opposite field direction. The effect of lowering the temperature is seen in (b) and (c); the effect of a negative gate voltage is seen in (a) and (b). The dashed vertical lines are equidistant and spaced by ΔB = 17.8 mT.

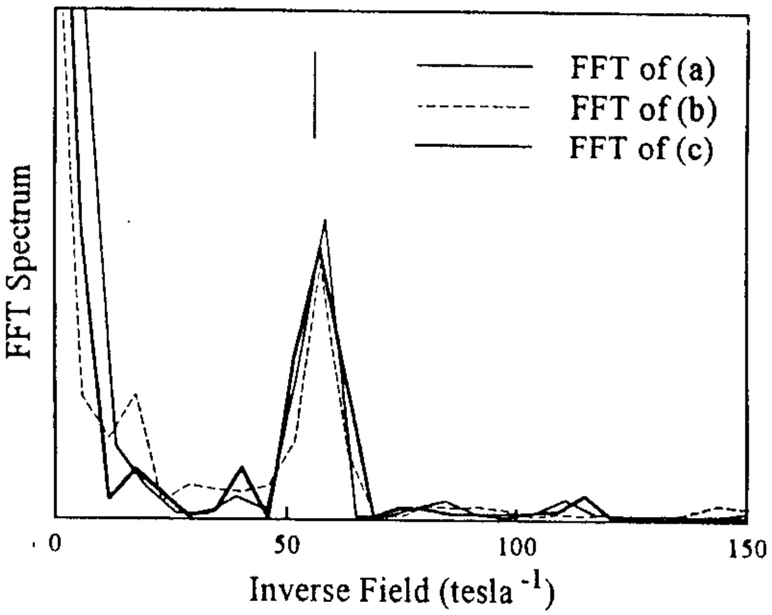
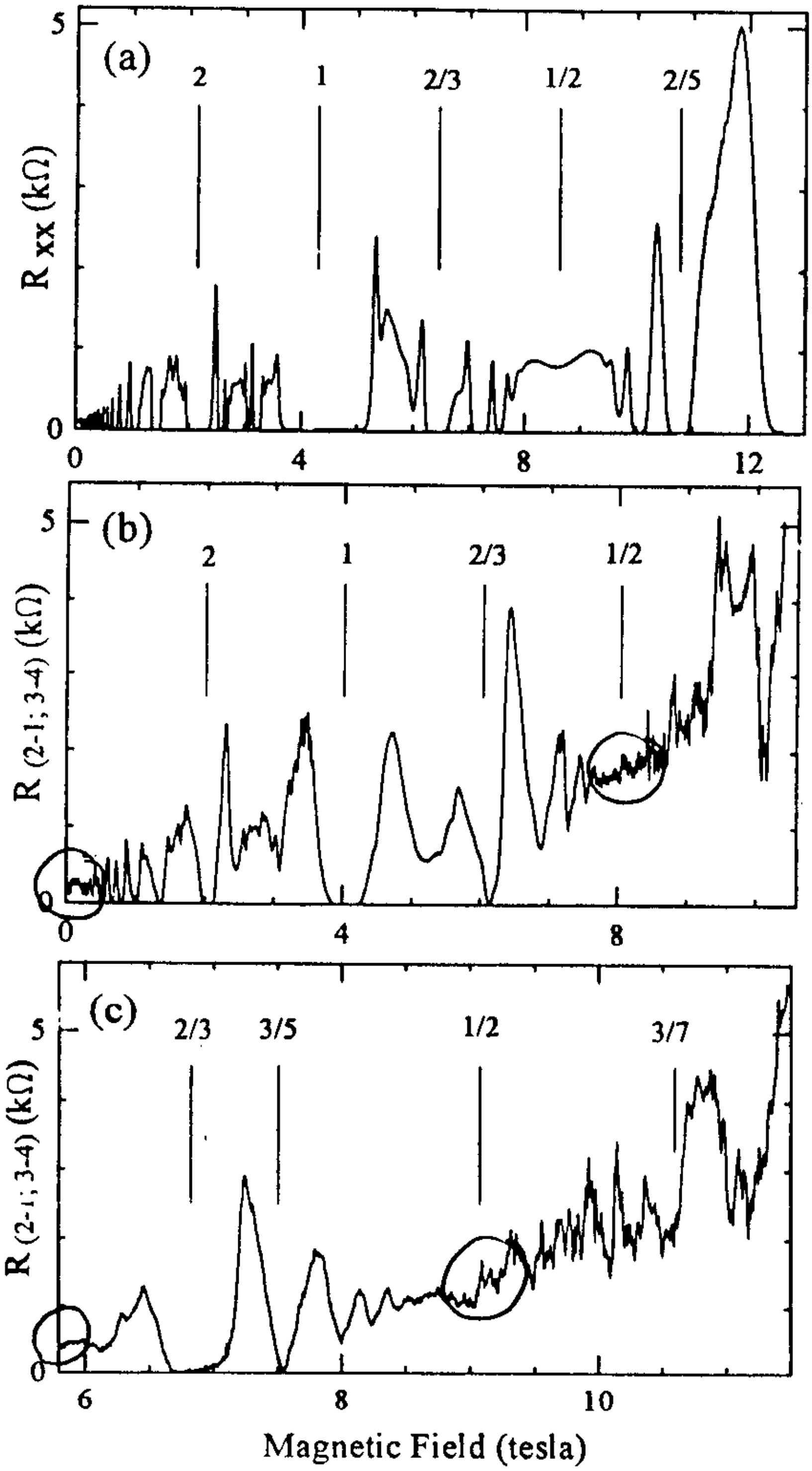
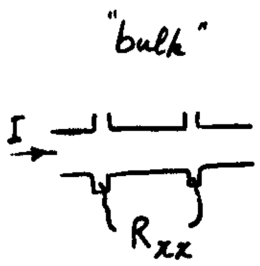


Fig. 3. Fourier transform spectra of the data of Fig. 2. The vertical dash is located at 56.2 mT^{-1}, which corresponds to ΔB = 17.8 mT.





Magnetic Focusing:

L = 4.25 µm

L = 5.3 µm

Fig. 4. Magnetoresistance data for a "bulk" Hall bar sample at 35 mK (a) and for double-constriction Samples D [~50 mK, shown in (b)] and E [30 mK, shown in (c)]. Note the sudden onset of the focusing structure for $\nu < 1/2$.

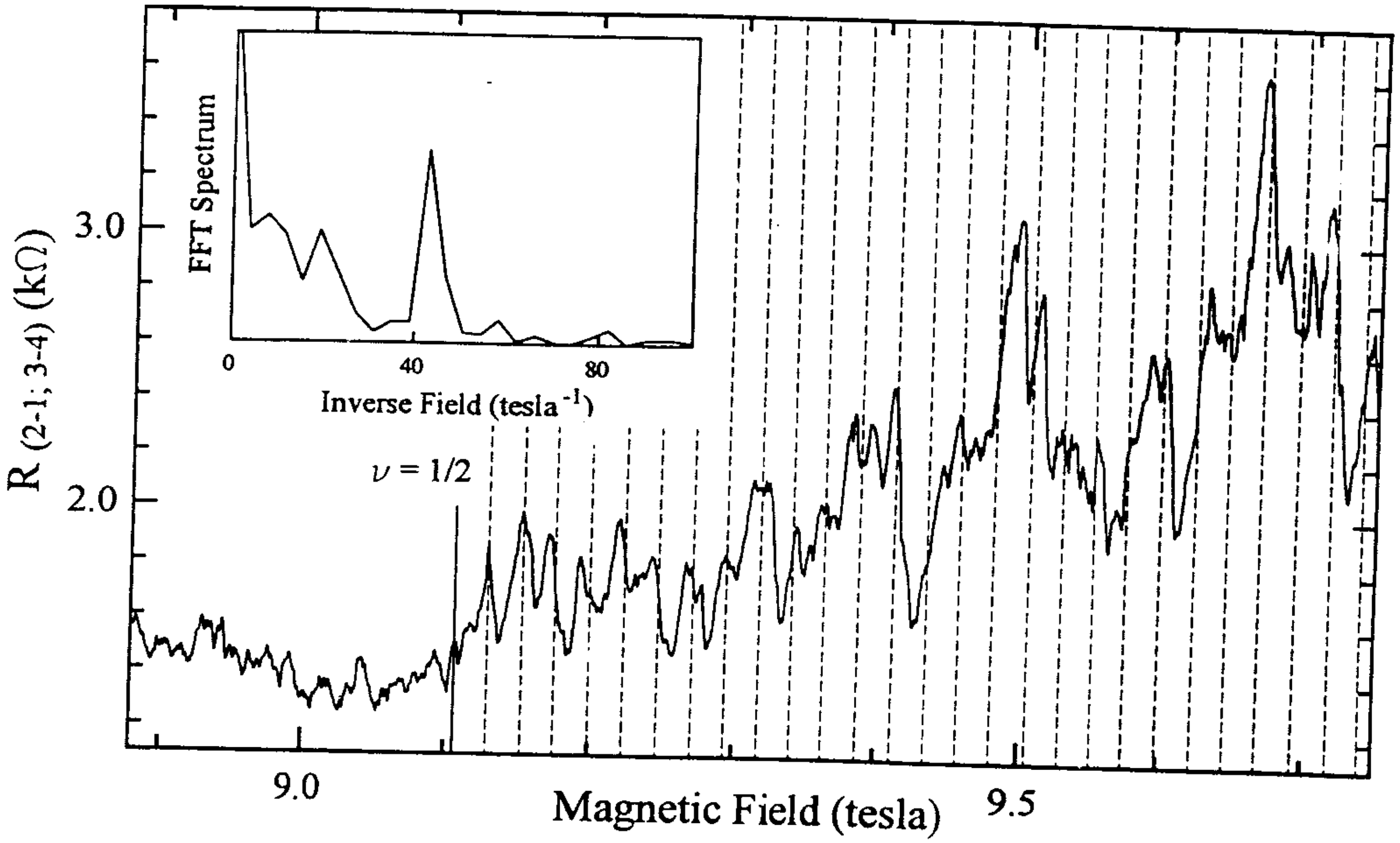


Fig. 6. Composite fermion magnetic focusing spectrum for Sample E at 25 mK. The vertical lines are equidistant, with spacing of 23.5 mT. The direction of *B* is the same as that in Fig. 2 (a) - (c), thus CF's are expected to be deflected to the right (as in Fig. 1) for $B > B(\nu = 1/2)$. The inset shows the fourier transform spectrum for $\nu < 1/2$.

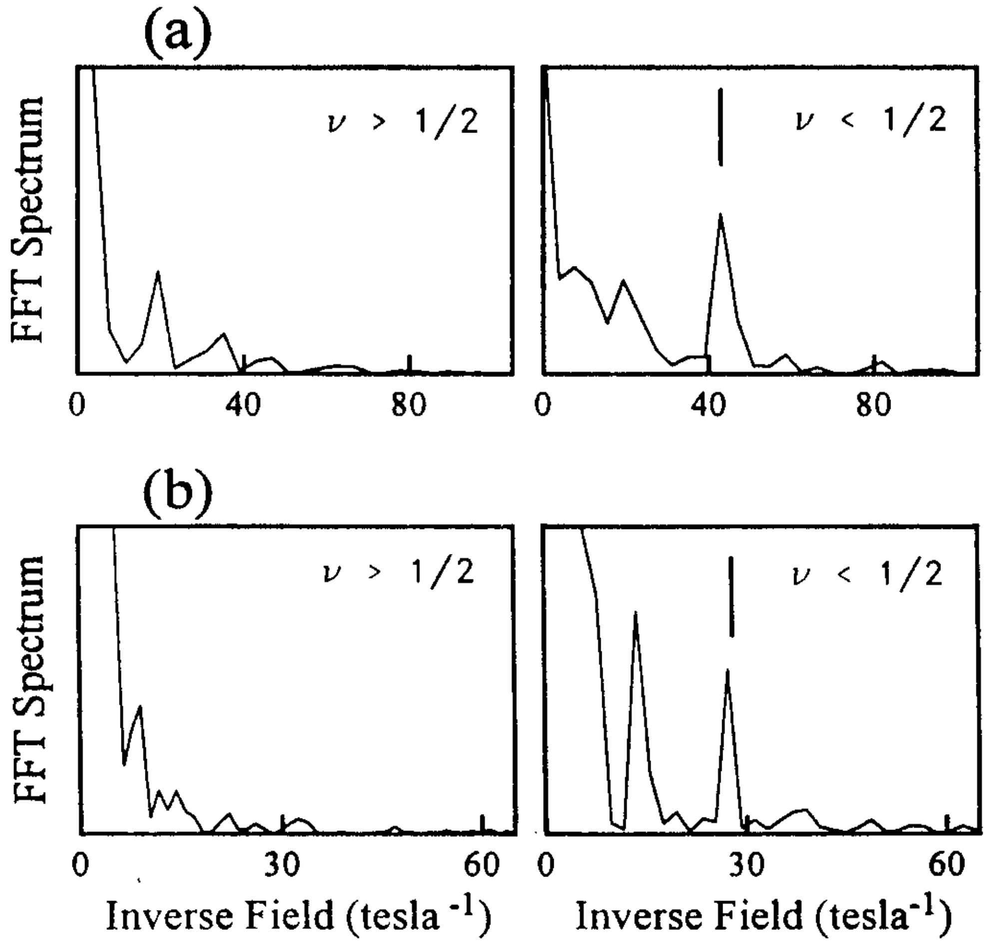


Fig. 9. Comparison of fourier transform spectra: (a) for the data of Fig. 6, Sample E, and (b) for the data of Fig. 7, Sample D. The vertical scales are the same for a given sample. The vertical dashes are located at $(\Delta B^{*})^{-1} = 42.6\ \mathrm{T}^{-1}$ in (a) and at $27.8\ \mathrm{T}^{-1}$ in (b).

Table I. Table of Samples. [18]

Sample	MBE material	L (μm)	n ($\times 10^{11}$ cm^{-2})	ΔB (mT)	$\Delta B^*(1/2)$ (mT)
A	97Ai	2.6±0.3	1.07	50±6	65±12
B	97Ah	3.1±0.3	1.09	30±5	50±10
C	97Cd	3.5±0.3	0.977	27.5±2	40±5
D	97Cc	4.25±0.3	0.973	25±2	36±2
E	124Ce	5.3±0.3	1.10	17.8±1	23.5±1.5

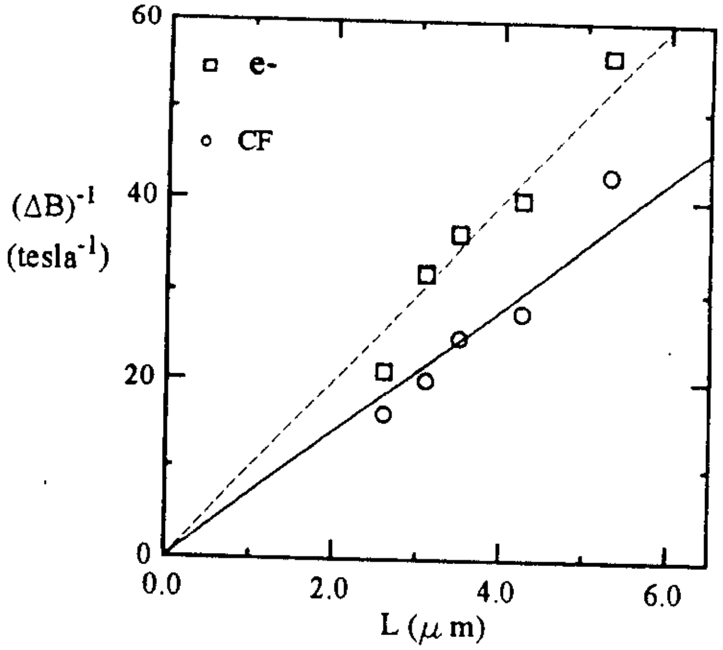
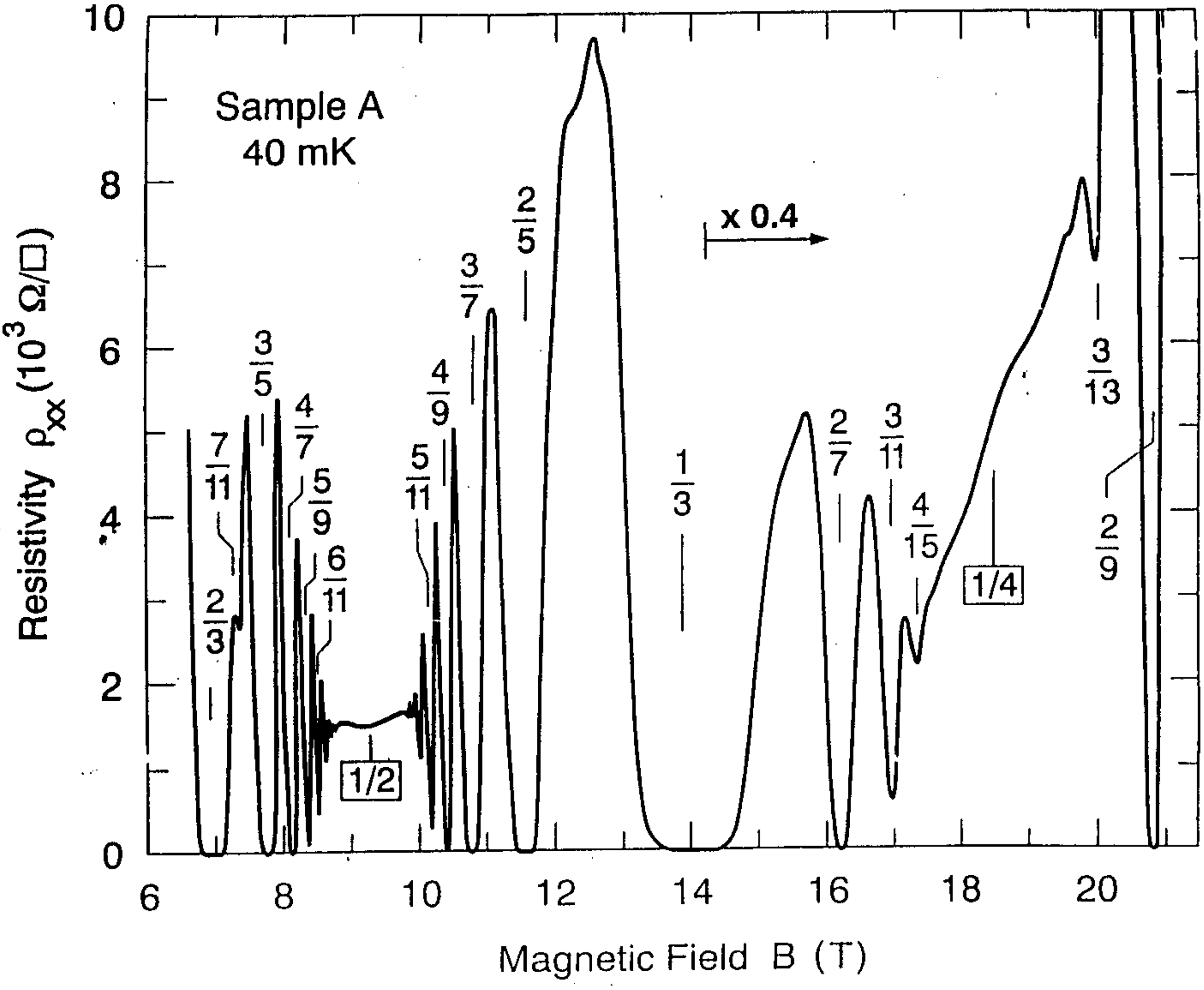


Fig. 10. Quasiperiods ΔB and ΔB^* of magnetic focusing of electrons (squares) and composite fermions (circles) for Samples A - E vs. constriction separation L. Expected periods are shown by the dashed line for electrons and the solid line for CF's; they involve no fitting parameters. The slope of the dashed line is $\sqrt{2}$ that of the solid line.

- Measurement of FQHE gap energies (Du, 1993)



$$\rho_{xx} \sim \exp\Big(-\underset{\substack{\uparrow \\ \text{Energy gap}}}{\Delta}/2k_BT\Big)$$

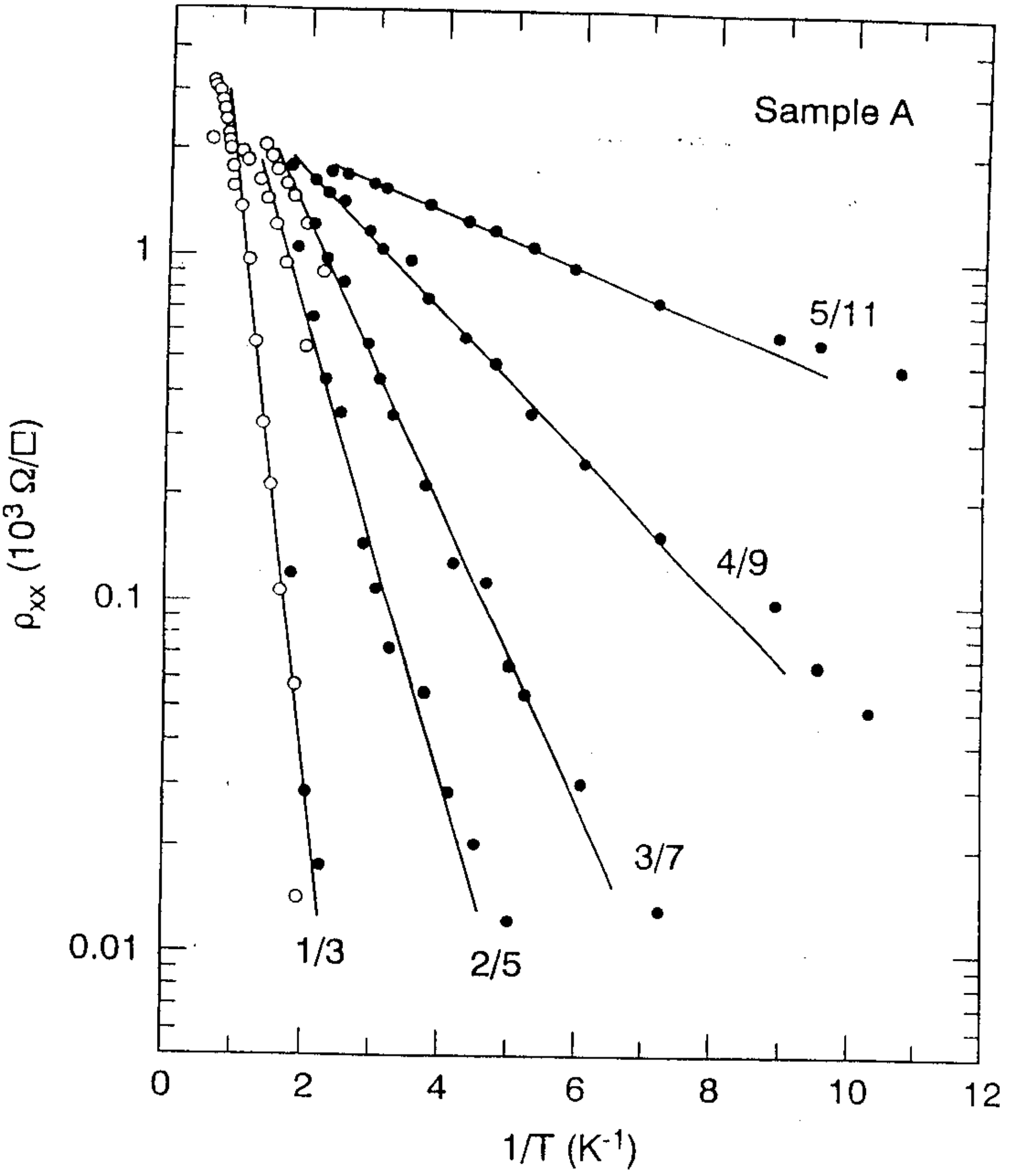
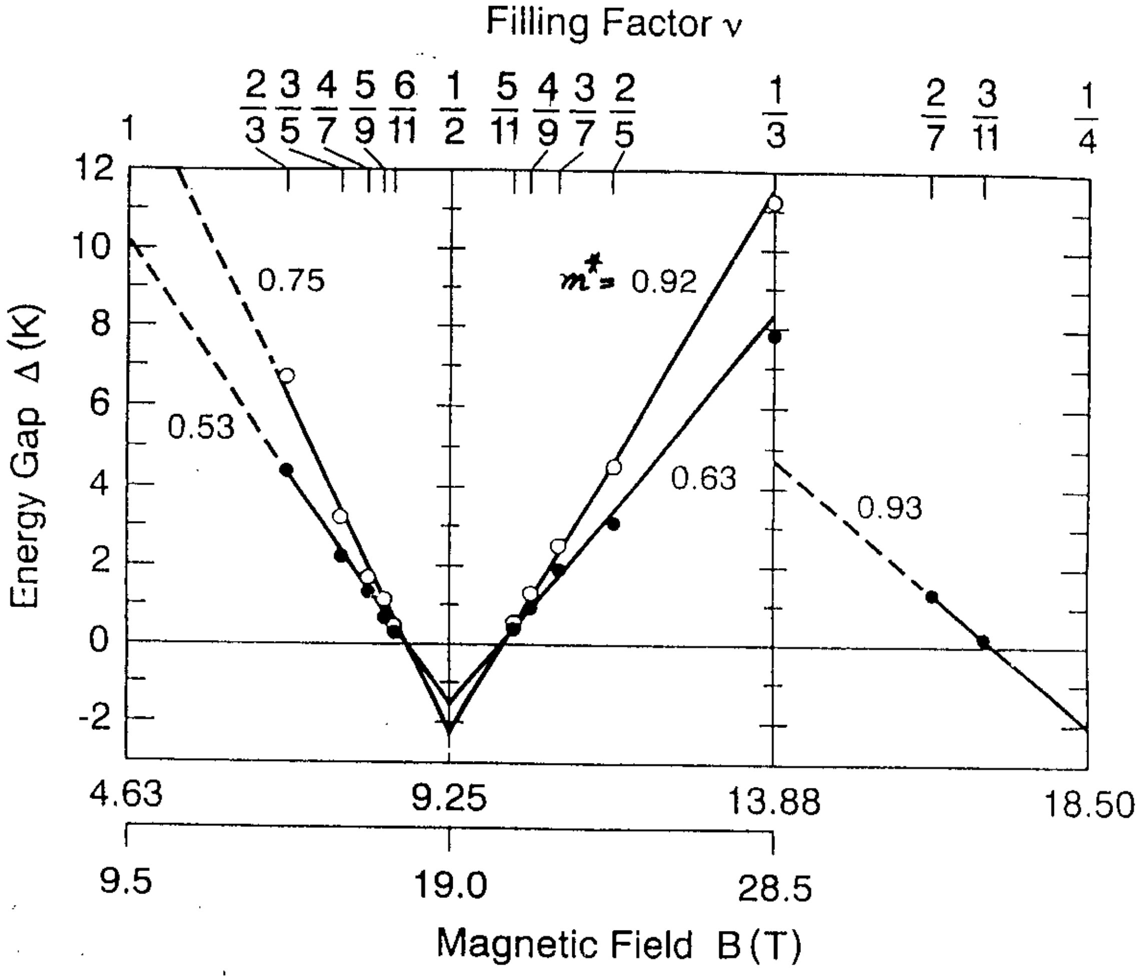
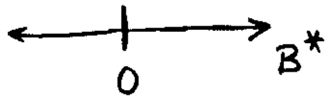


Fig. 2

C&D 2M22686.1





Energy gap = $\hbar\omega_c = \dfrac{\hbar e B^*}{m^*}$

assumes const. m^* ← ?

- Measurement of m^* from analysis of "Shubnikov-de Haas" data (Leadley, 1994)

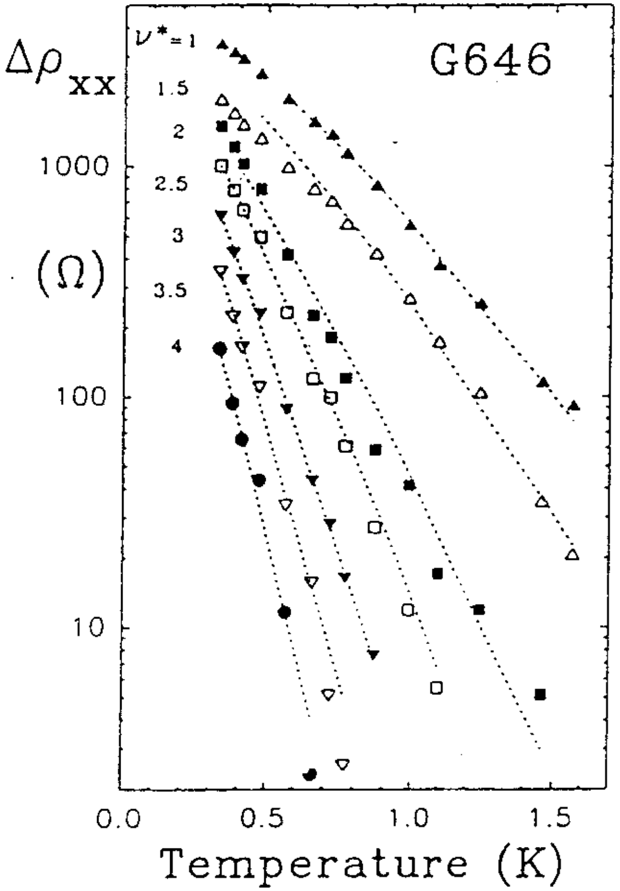


FIG. 2. Temperature dependence of the resistivity oscillations for sample G646 between $\nu = 1/2$ and $1/3$. Data are fitted by Eq. (2) for each minimum (filled symbol) and maximum (open). Values of ν^* are indicated in the figure.

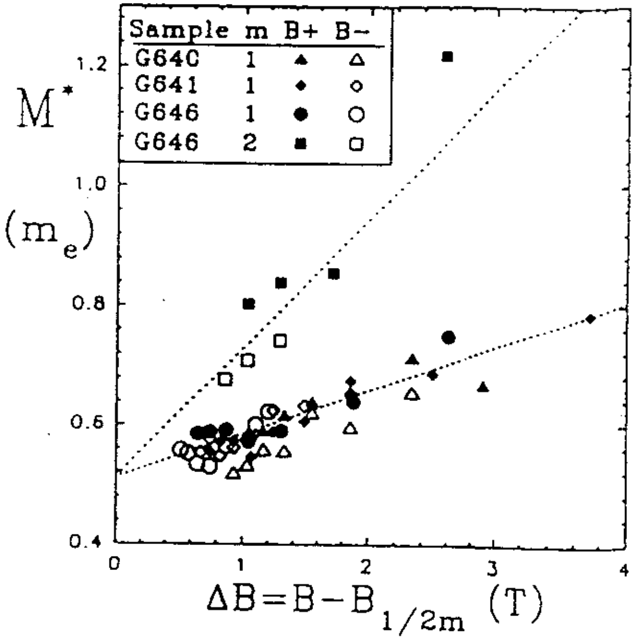


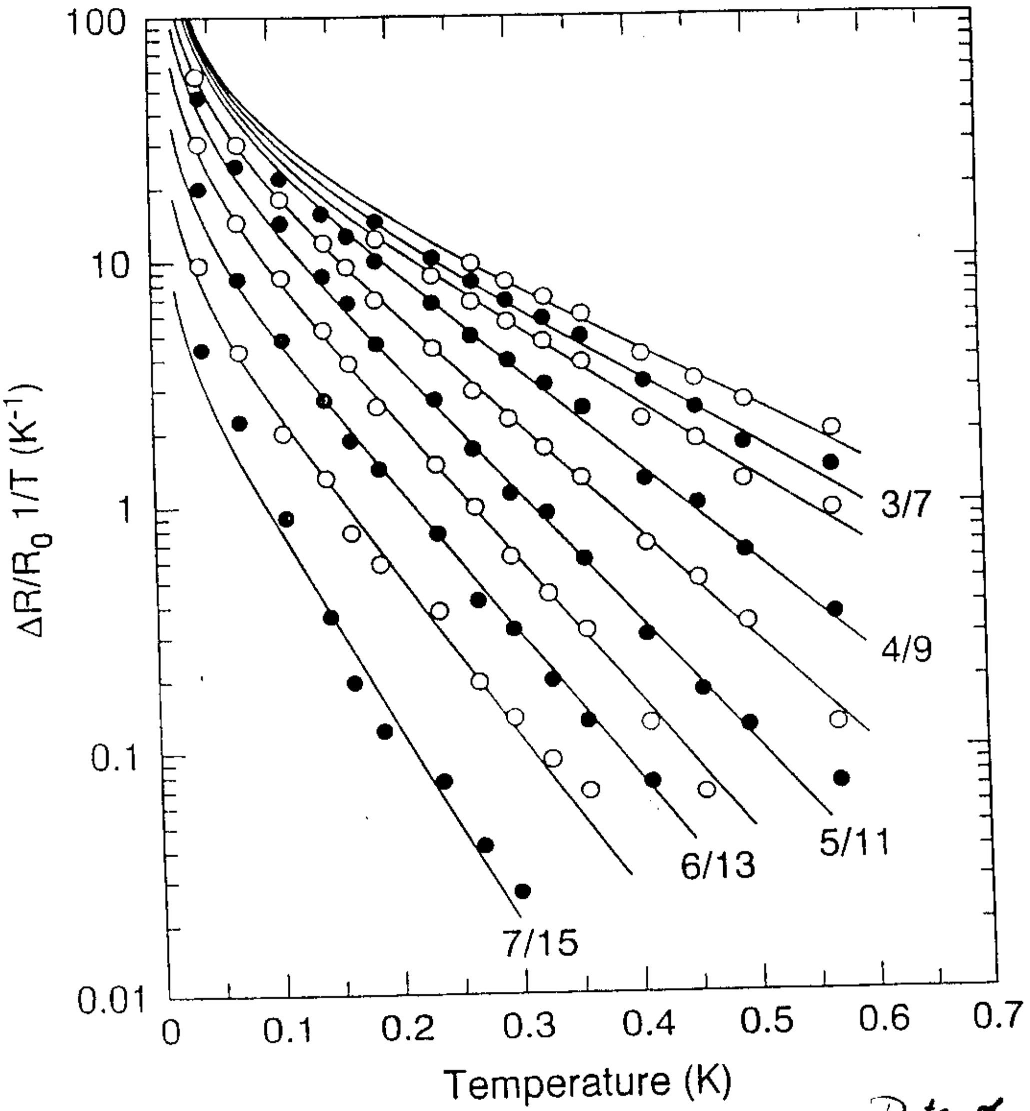
FIG. 3. Composite fermion effective mass M^* deduced fitting maxima and minima of ρ_{xx} by Eq. (2).

- m^* from analysis of "SdH" data

$$\frac{\Delta R_{xx}}{R_0} = 4 \exp\left(\frac{-\pi}{\omega_c \tau_q}\right) \frac{\xi}{\sinh \xi} \cos\left[\pi(2\nu - 1)\right]$$

$$\xi \equiv 2\pi^2 k_B T/\hbar\omega_c$$

$D(\xi) = \xi/\sinh \xi$ ← Dingle factor (gives damping of oscillations as $T\uparrow$)



Data of Du et al.

Fig. 2

● m* from SdH" analysis (Du, 1994)

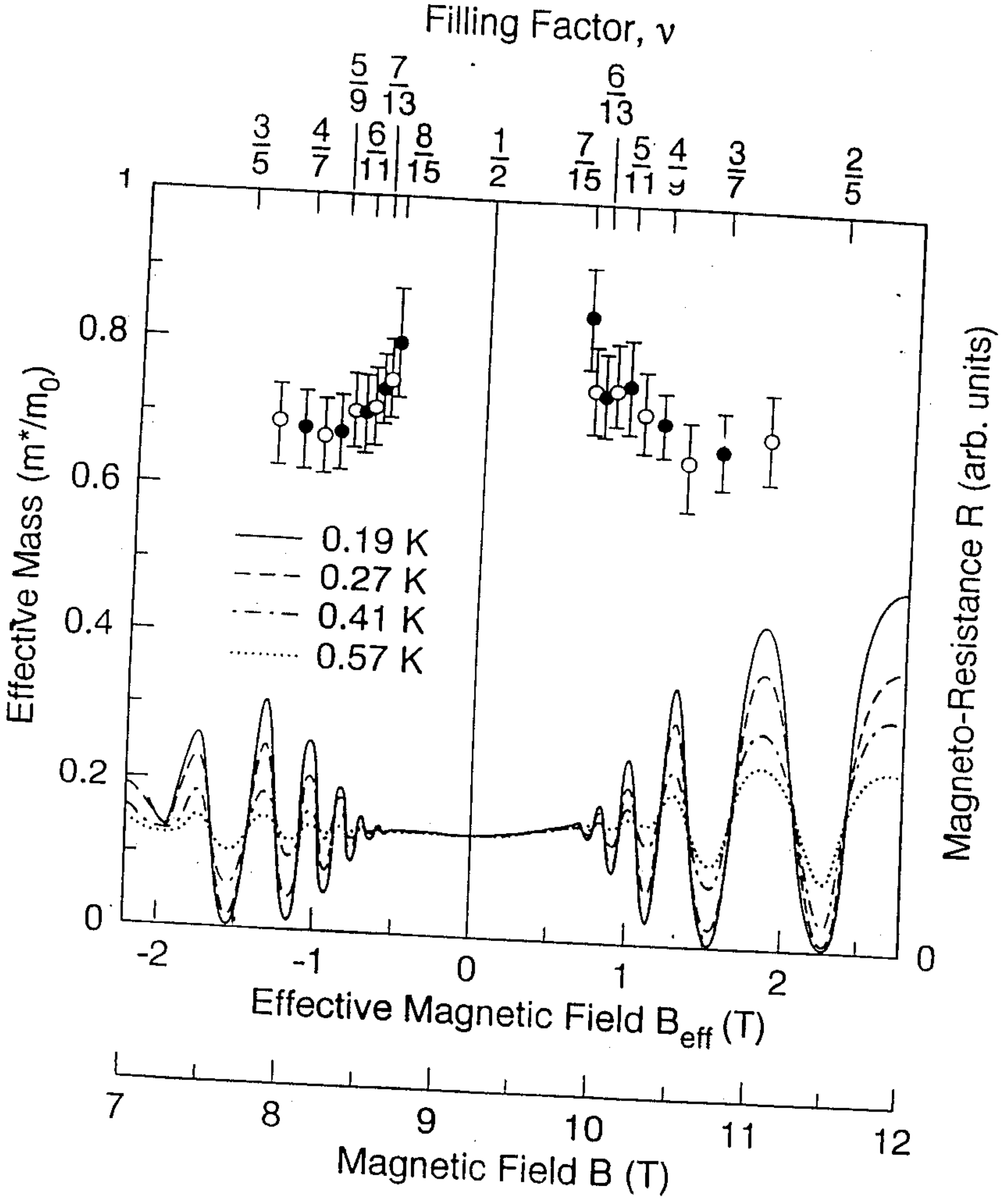


Fig. 1

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