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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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SMR. 758 - 22

SPRING COLLEGE IN CONDENSED MATTER
ON QUANTUM PHASES
(3 May - 10 June 1994)

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MAGNETOTRANSPORT MEASUREMENTS IN QUANTUM HALL EFFECT

LECTURES 2 and 3

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MAGNETOTRANSPORT MEASUREMENTS IN QUANTUM HALL EFFECT

LECTURE 2

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Part II. Ground State @ Low Fillings:

The Case for a Wigner Crystal

M. Shayegan

Trieste, May 1994

Electron System

1. Correlation Effects

Free electron gas

$E_K \sim E_F \sim n^{2/3} \sim 1/r^2$

$E_C \sim e^2/r \sim 1/r$

r = average e-e spacing

Large r => $E_C/E_K >> 1$

=> gas → solid
(Wigner Crystal)

=> "Localization"

- Magnetic field:
  (extreme quantum limit)

$$E_F = \frac{2\pi^4\hbar^4}{m^* e^2} \cdot \frac{n^2}{B^2}$$

2. Disorder

Real electron system

-impurities,
-inhomogeneities,
-defects,
-etc.

=>Potential fluctuations

=> Localization

In uniformly doped semiconductors (InSb):

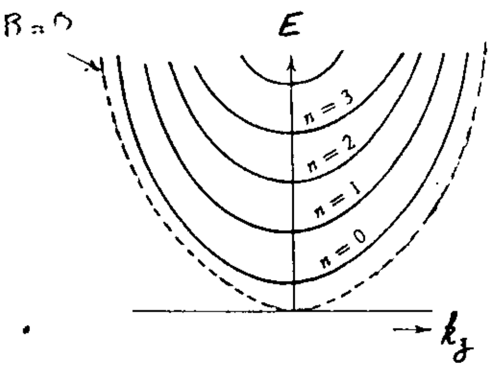
- shallow donors
  (hydrogenic model)
- electron-impurity
  interaction $\sim e^2/r$

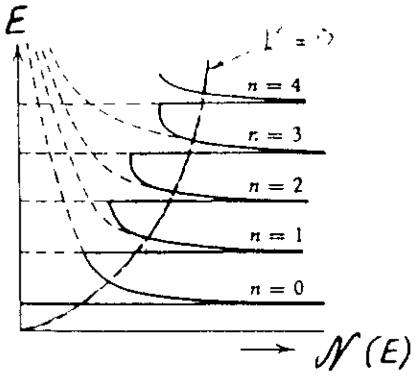
=> electrons are bound
on donors
(freeze-out)

Free electrons in a magnetic field (3D)

- $E = (n+1/2)\hbar\omega_c + \frac{\hbar^2}{2m} k_z^2 \qquad (n = 0,1,2,\ldots)$

$\omega_c = eB/m$ cyclotron frequency





- *Extreme Quantum Limit*

$$E_F = \frac{2\pi^4\hbar^4}{me^2} \cdot \frac{n^2}{B^2}$$

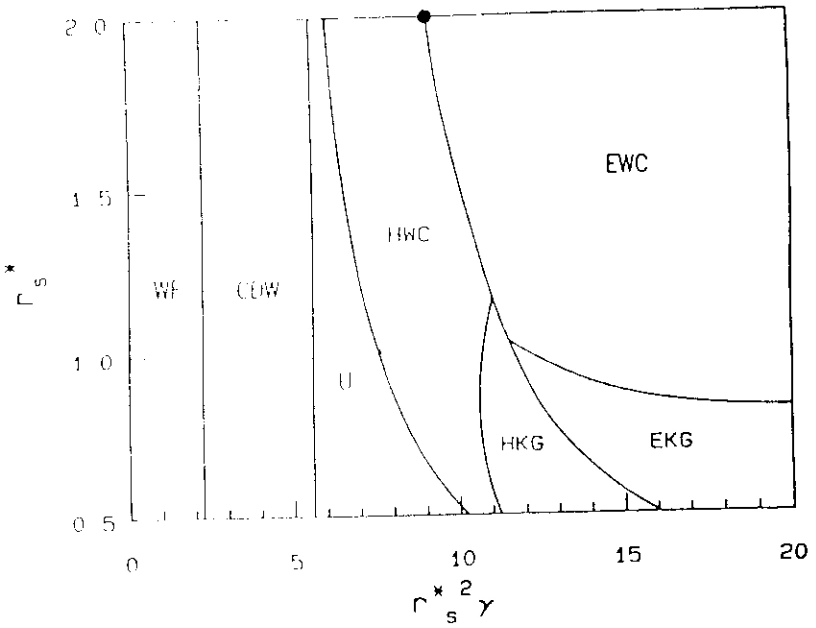
=> *E_F decreases with B*

- Metal: $B \sim 10^9$T needed

  Semiconductor: B ~ few T

Strong-Magnetic-Field States of the Pure Electron Plasma (3D)

MacDonald & Bryant (PRL, 1987)



$$\frac{4\pi}{3}(a_B^* r_s^*)^3 = \frac{V}{N} = \frac{1}{n} \qquad \gamma = \frac{\hbar\omega_c}{2R_y^*}$$

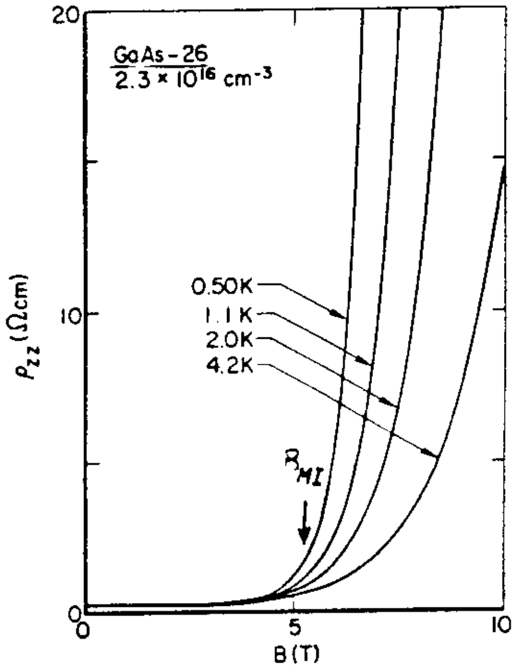
For GaAs with $\underline{r_s^* = 2.0}$: $\underline{n = 3.0\mathrm{x}10^{16}\mathrm{cm}^{-3}}$

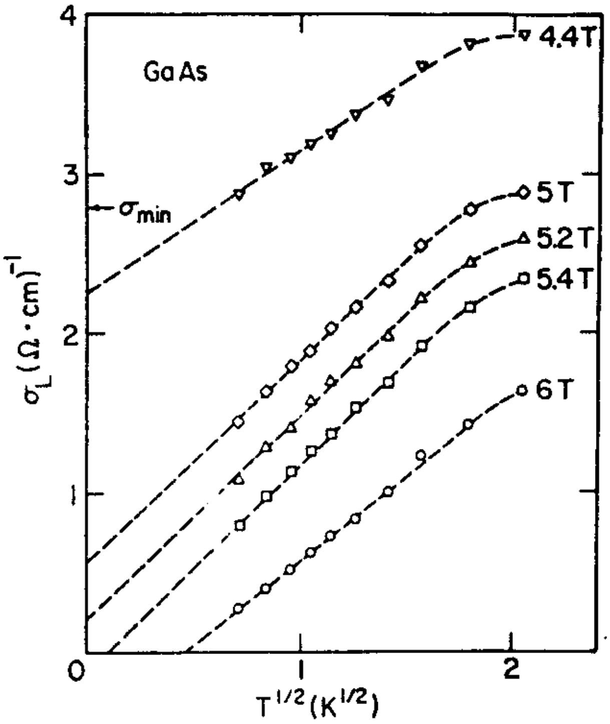
- => $\gamma = 2.3$ E_F (@14 T)= 0.42 meV

$B = 14$ T $E_C = \frac{e^2}{\epsilon r_s^* a_B^*} = 5.5$ meV

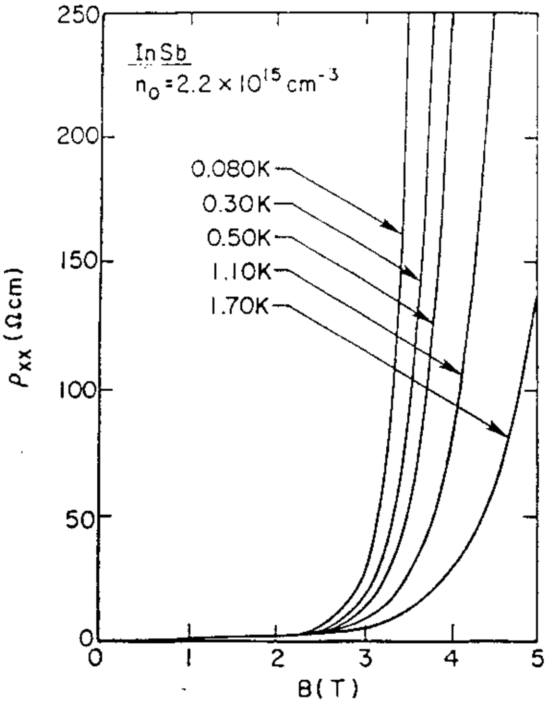
=> $E_C/E_K \simeq 40$

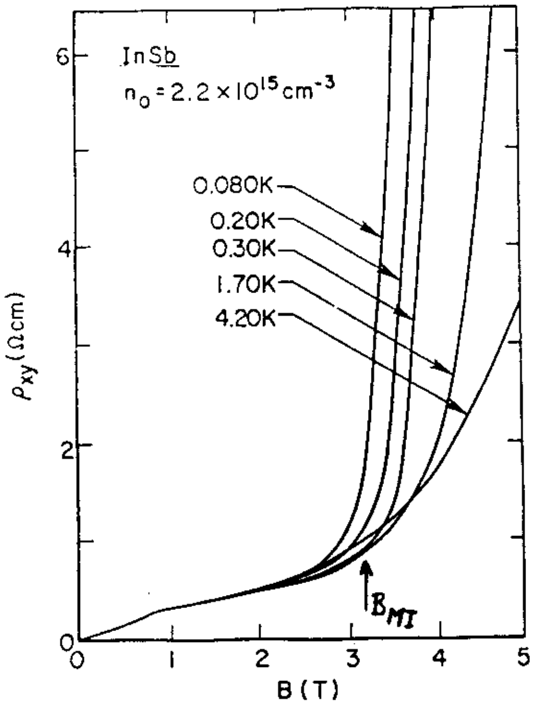
$$\sigma(T) = \sigma(0) + \underset{\text{Coulomb}}{A}\,T^{1/2} + \underset{\text{Weak Localization}}{B}\,T$$





Shayegan (~1986)





Shayegan, 1988b

2D Wigner Crystal $(B=0)$

(review: Platzman, 1987)

$$\langle P.E.\rangle = (\pi n_s)^{1/2}\,\frac{e^2}{\epsilon}$$

$$\langle K.E.\rangle = \begin{cases} k_B T & T\to\infty \\ E_F = \dfrac{\pi\hbar^2 n_s}{m^*} & T\to 0 \end{cases}$$

- Classical limit $(k_B T \gg E_F)$

  - $\Gamma = \dfrac{\langle P.E.\rangle}{\langle K.E.\rangle} = \dfrac{(\pi n_s)^{1/2}\,\frac{e^2}{\epsilon}}{k_B T}$

  - gas $\longrightarrow$ solid as $\Gamma\nearrow$

    $(n_s\uparrow$ or $T\downarrow)$

  - e^-'s on Liquid He:

    solid for $\Gamma \gtrsim 130$

    both theory & experiment

- Quantum limit $(k_B T \ll E_F)$

  - $\Gamma = \dfrac{(\pi n_s)^{1/2}\,\frac{e^2}{\epsilon}}{\frac{\pi\hbar^2 n_s}{m^*}} = \dfrac{\langle r\rangle}{a_B^*} = r_s$

  - gas $\longrightarrow$ solid as $r_s\nearrow$

    $(n_s\downarrow)$

  - theory:

    solid for $r_s \gtrsim 33$

  - no experiments

    (Pudalov, 1993 ?)

- Physical parameters of 2D systems

		GaAs/AlGaAs	
	2DES on He	2DES	2DHS
$n_s\,(\mathrm{cm}^{-2})$	$10^5 - 10^9$	$10^{10} - 10^{12}$	$10^{10} - 10^{12}$
m^*/m_e	1.0	0.067	$\simeq 0.38$
$E_F\,(\mathrm{meV})$	$2.4\times10^{-7} - 2.4\times10^{-3}$	0.36 - 36	0.064 - 6.4
$\epsilon\ (\epsilon_0)$	1.0	13	13
$a_B^*\,(\mathring{A})$	0.53	103	18
r_s	$3.4\times10^5 - 3.4\times10^3$	5.5 - 0.55	31 - 3.1

2D Wigner Crystal $(B > 0)$

$\epsilon_N = \hbar\omega_c\left(N + \frac{1}{2}\right)$ L.L.'s

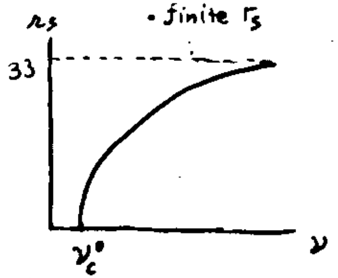
$\ell_B = (\hbar/eB)^{1/2}$ (in the plane $\perp \vec{B}$, e^- in the lowest L.L. confined to length $\propto \ell_B$)

$\therefore$ Kintetic energy "quenched"

- Important parameters:
  - Filling factor
    $$\nu = \frac{2\ell_B^2}{a^2} = \frac{E_F}{\hbar\omega_c} \qquad \left(\pi a^2 = \frac{1}{n_s}\right)$$
  - L.L. mixing parameter
    $$\lambda = \frac{e^2/a}{\hbar\omega_c} \equiv r_s\,\nu$$
  - For finite T
    $$\Gamma_\infty = \frac{e^2/a}{k_B T}$$
- Theory:
  - for $r_s \to 0$

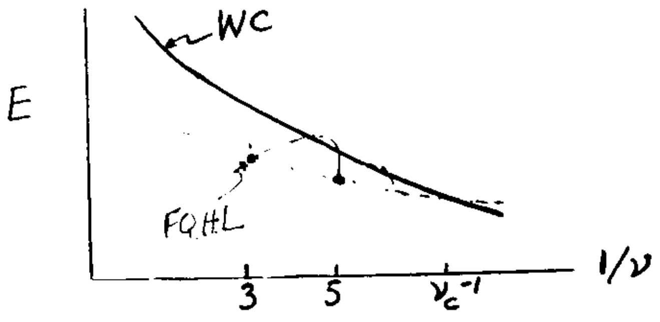
$$\nu < \underbrace{\nu_c^\circ}_{\sim \frac{1}{6} \text{ to } \frac{1}{11}} \Leftrightarrow \text{W.C.}$$

Lozovik, 1975
Lesvesque, 1984
Lam & Girvin, 1984



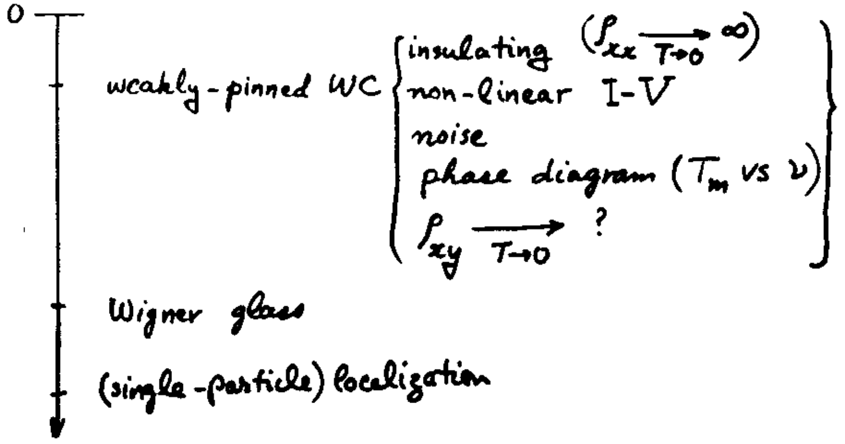
Chui, 1991
Zhu, 1993 Price, 1993

- Wigner Crystal in 2D systems at low ν?



$$\frac{1}{10} \lesssim \nu_c \lesssim \frac{1}{6}$$

- Reentrant ?
- Effect of Disorder ?



weakly-pinned WC
- insulating $\left(\rho_{xx} \xrightarrow[T\to 0]{} \infty\right)$
- non-linear I-V
- noise
- phase diagram (T_m vs ν)
- $\rho_{xy} \xrightarrow[T\to 0]{}$?

Wigner glass

(single-particle) localization

Outline

I. 2D electron system at GaAs/AlGaAs

  A. Historical

  B. Insulating phase near $\nu=1/5$

  - Non-linear I-V
  - Noise
  - Melting Phase diagram ?

II. Other systems

  A. Comparison with CDW in 1D ($NbSe_3$)

  B. 2D electrons on He

  C. 2D holes at GaAs/AlGaAs

  D. 2D electrons at Si/SiO_2

III. Summary

Quest for Low ν

2DES in GaAs

1983/84: "feature" near $\nu = 1/5$ (*Mendez, Chang*)

1988: Minimum in ρ_{xx} at $\nu = 1/5$
(*Willett, Nicholas, Goldman, Mallet, Glattli, Shayegan*)
"inflection point" near 1/7 (*Goldman*)

1989: Non-linear I-V near 1/5 (*Willett*)

1990: Non-linear I-V, phase diagram (*Goldman*)
Vanishing ρ_{xx} at $\nu = 1/5$ (*Jiang*)

1991: More non-linear I-V (*Williams, Jiang, Li*)
Noise (*Li*)
High frequency & SAW (*Willett, Paalanen*)

1992: "Normal" $\rho_{xy} = B/ne$ near $\nu = 1/5$ (*Sajoto, Goldman*)

Other Systems

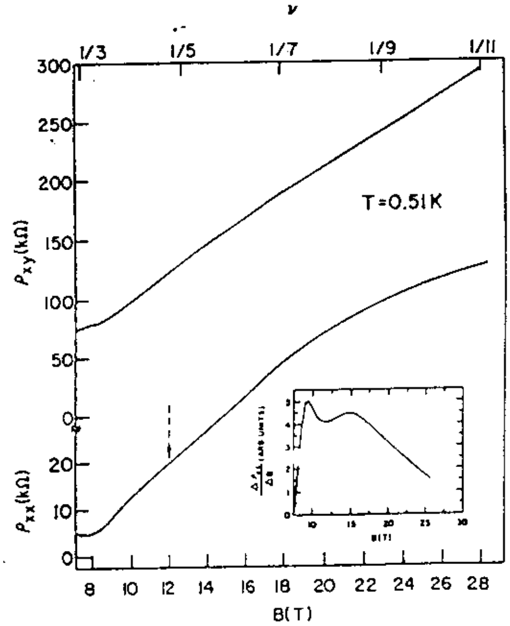
1989: Electrons on He (*Jiang*)

1990/91: 2D electrons at Si/SiO_2 (*D'Iorio, Pudalov*)

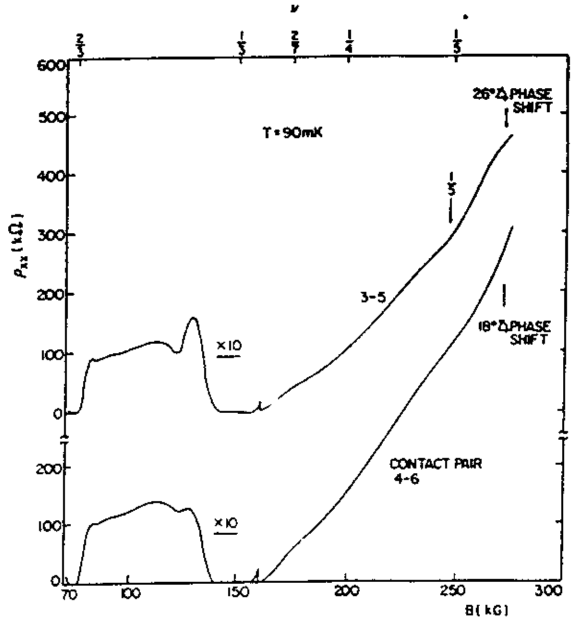
1991/92: 2D holes at GaAs/AlGaAs (*Santos*)

1992: Electrons in wide GaAs wells (*Suen*)

Early "features" near $\nu = 1/5$



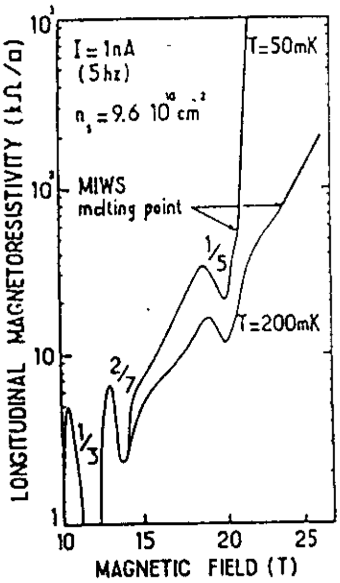
Mendez (1983)

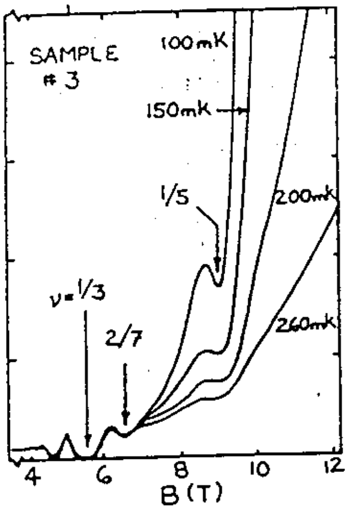


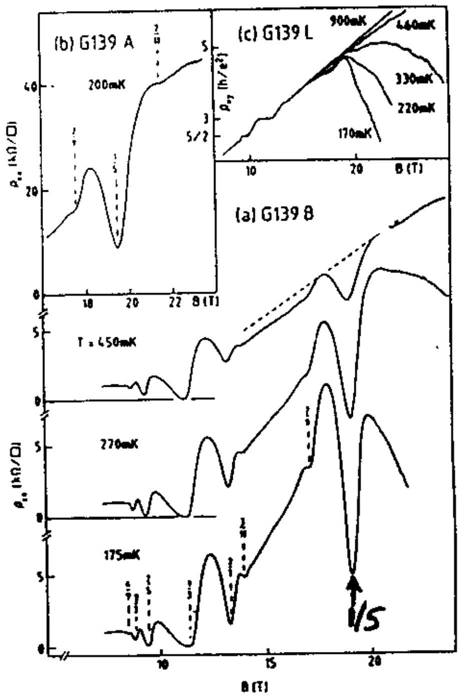
Chang (1984)

Würzburg, August 1988

$\nu = \frac{1}{5}$ minimum emerging!





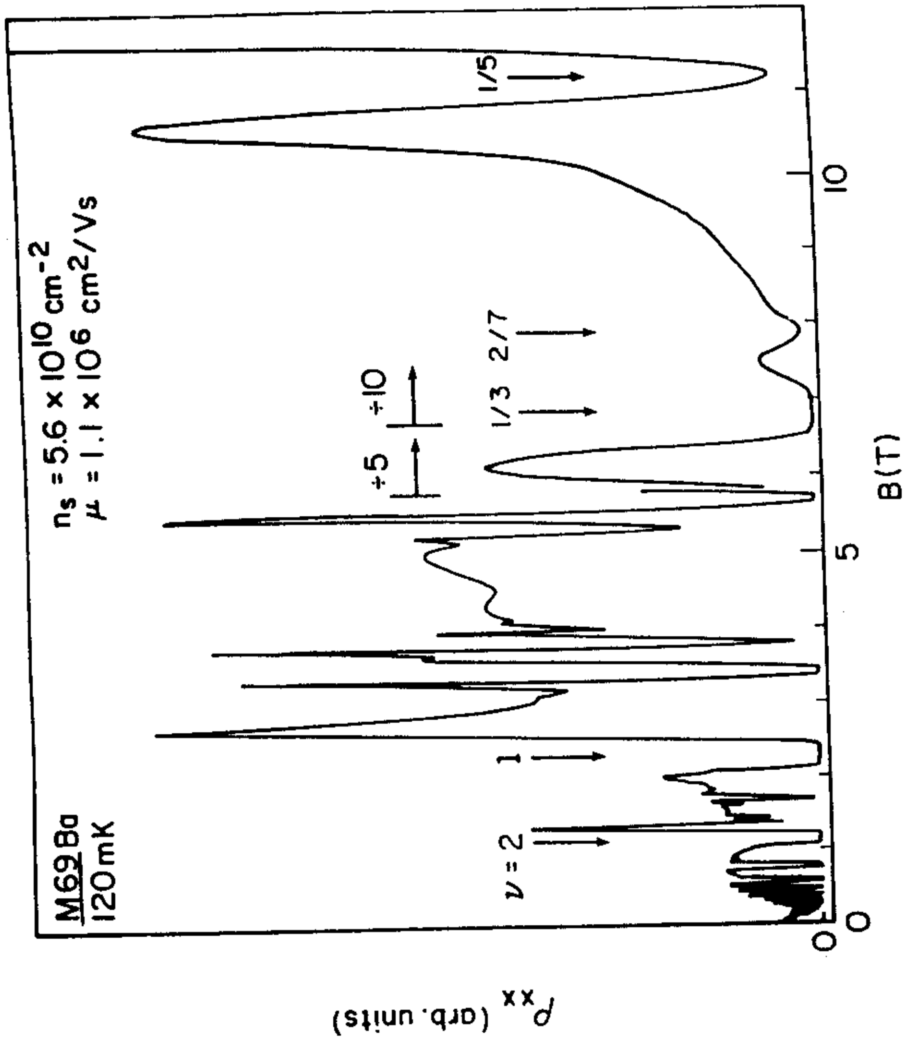


Glattli et al.

Willett et al.

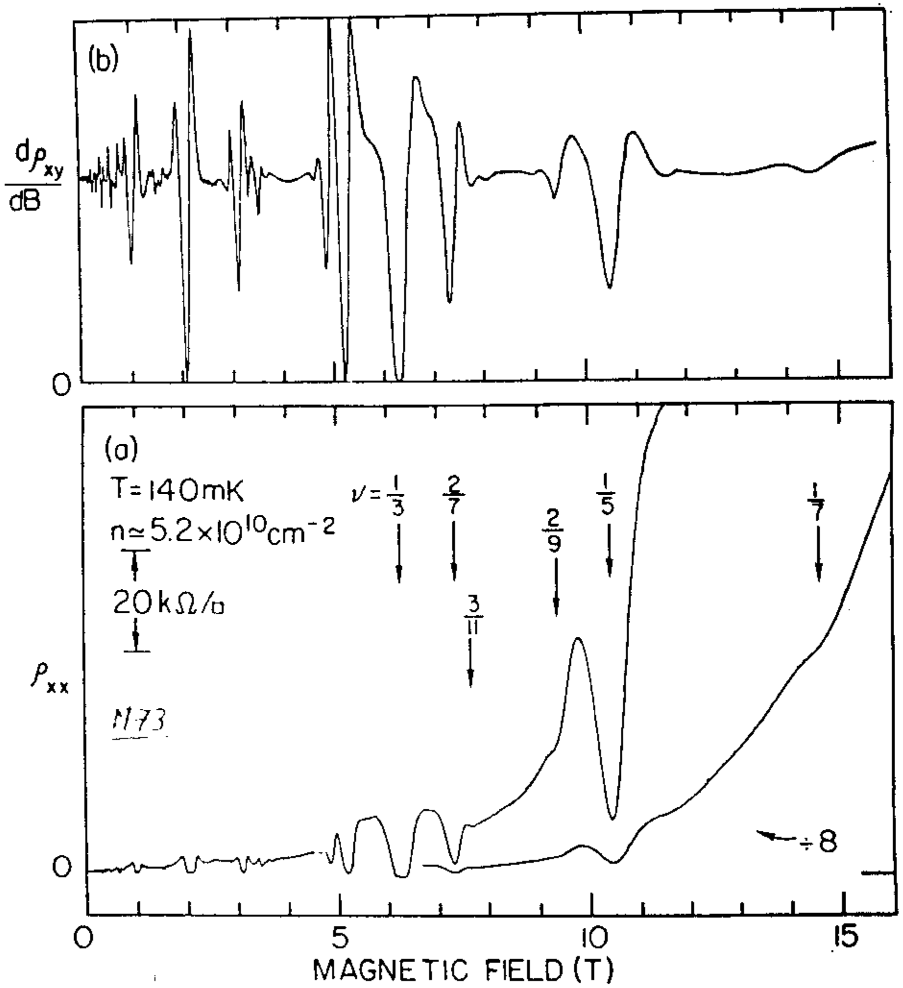
Nicholas et al.

Int'l MBE Conf. • (Sapporo, Aug. 1988)

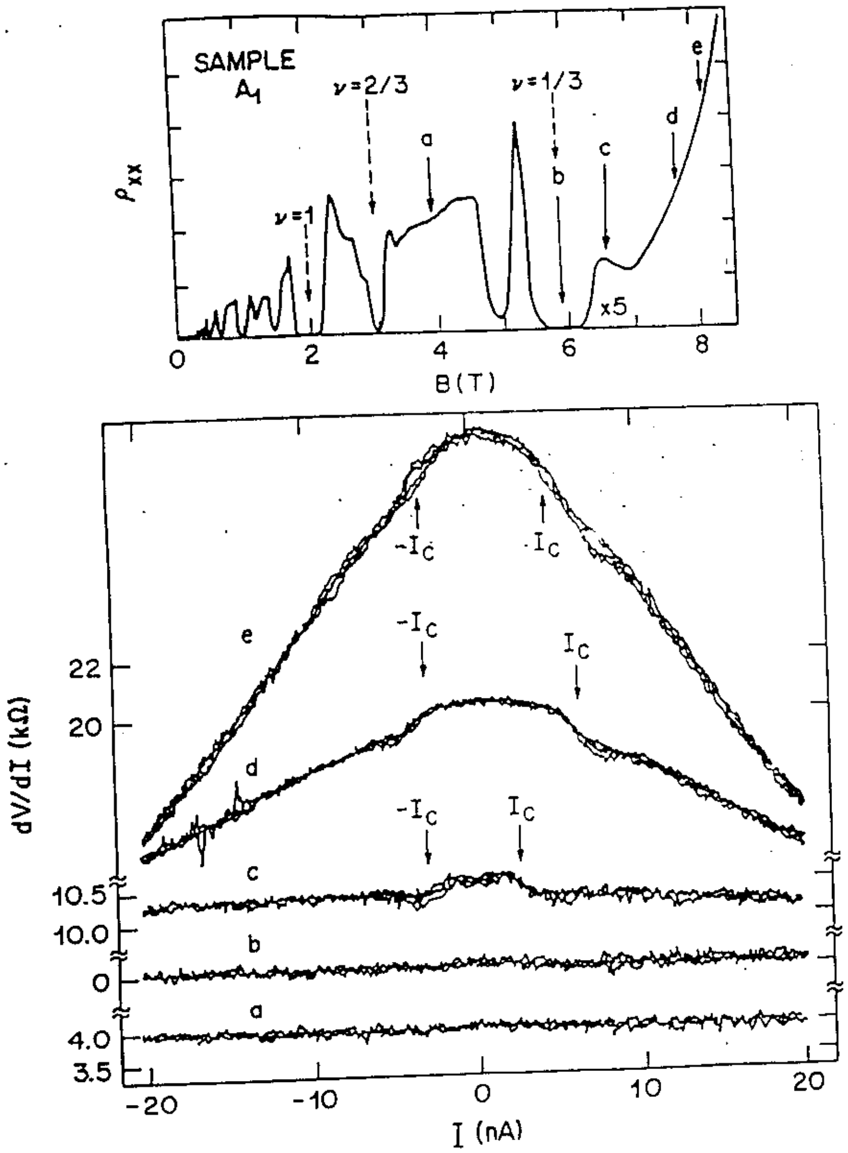


Shayegan et al. [J. Cryst. Growth, vol. 95, 1989]

Even a FQHE feature near $\nu = \frac{1}{7}$!

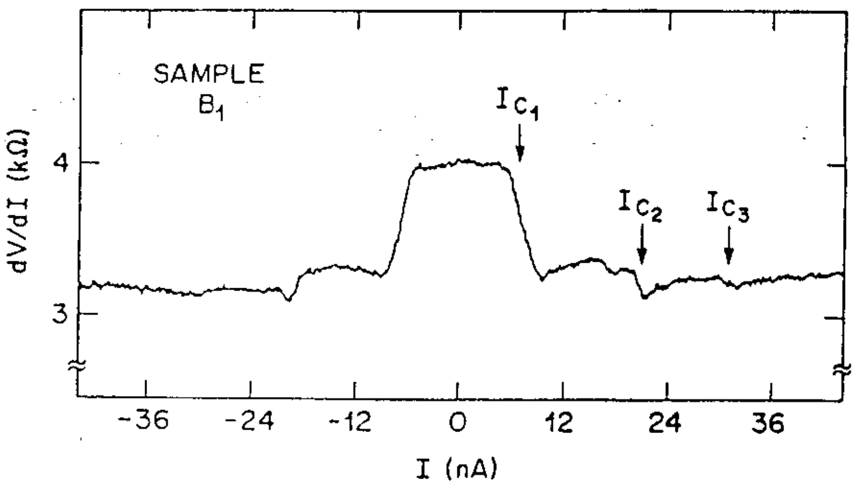


Goldman, Shayegan, & Tsui (Phys. Rev. Lett. Aug. 15, 1988)



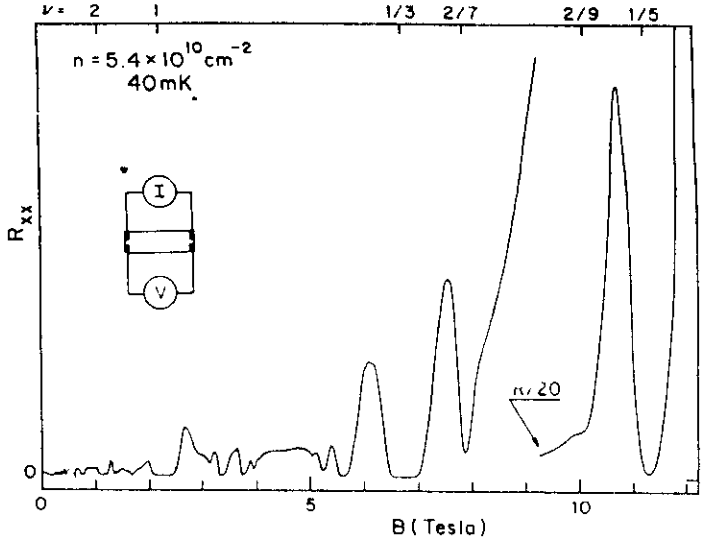
Willett et al. (1989)

- Non-linear I-V
- No clear T-dependence

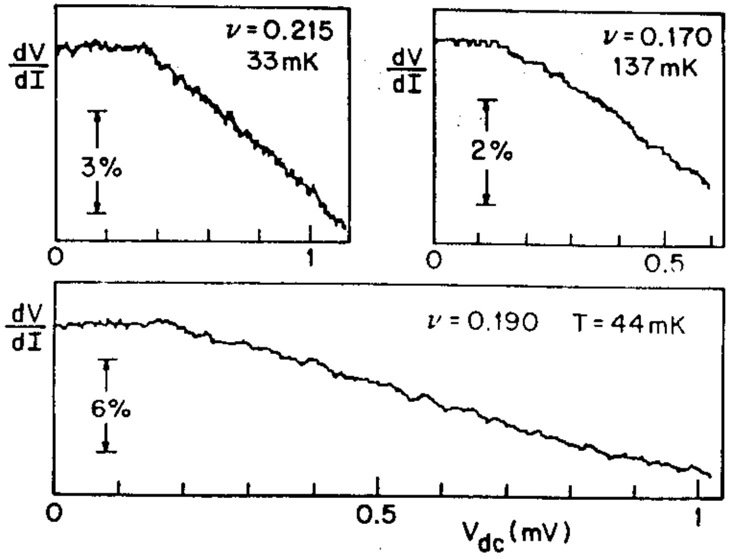


Willett et al. (1989)

- Observation of a Pinned Quantum Wigner Crystal

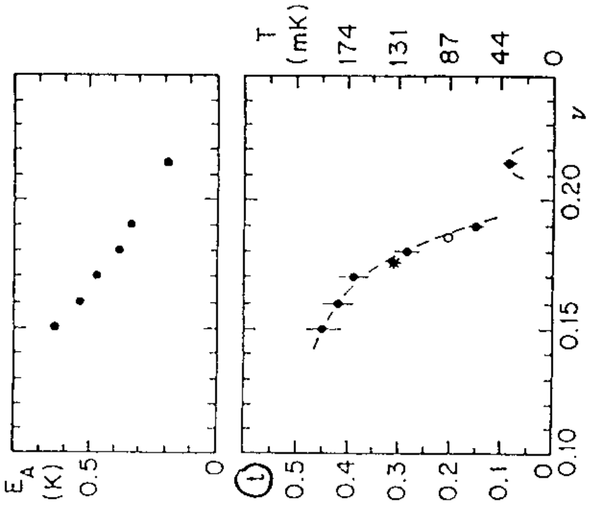


- Non-linear I-V



Goldman et al. (1990)

W.C. melting phase diagram ?

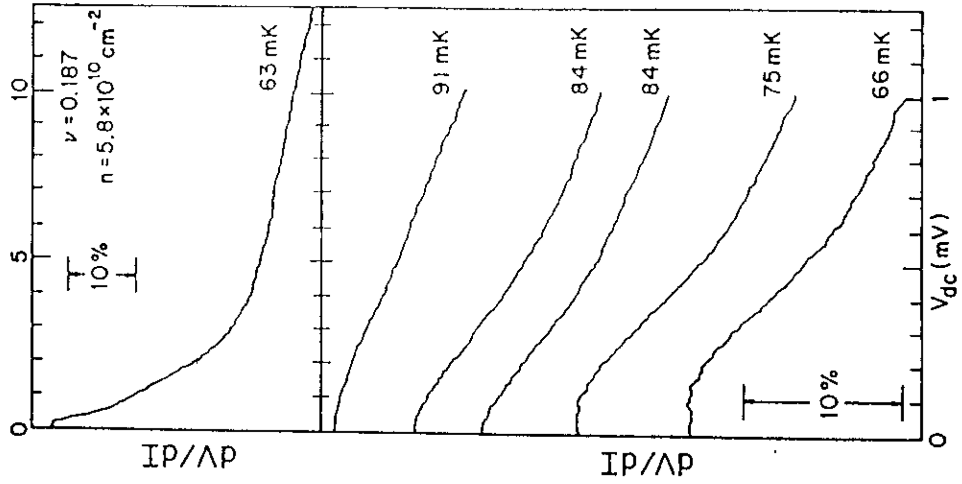


τ = reduced temperature = $\frac{T}{T_{mc}}$ where: $T_{mc} = \frac{e^2}{\epsilon a \Gamma_m}$

$\Gamma_m = 127$

$\begin{cases} a = (\pi n_s)^{-1/2} \\ \epsilon = 13\epsilon_0 \end{cases}$

Goldman et al. (1990)



- Reentrance around $\nu = \frac{1}{5}$ FQH liquid

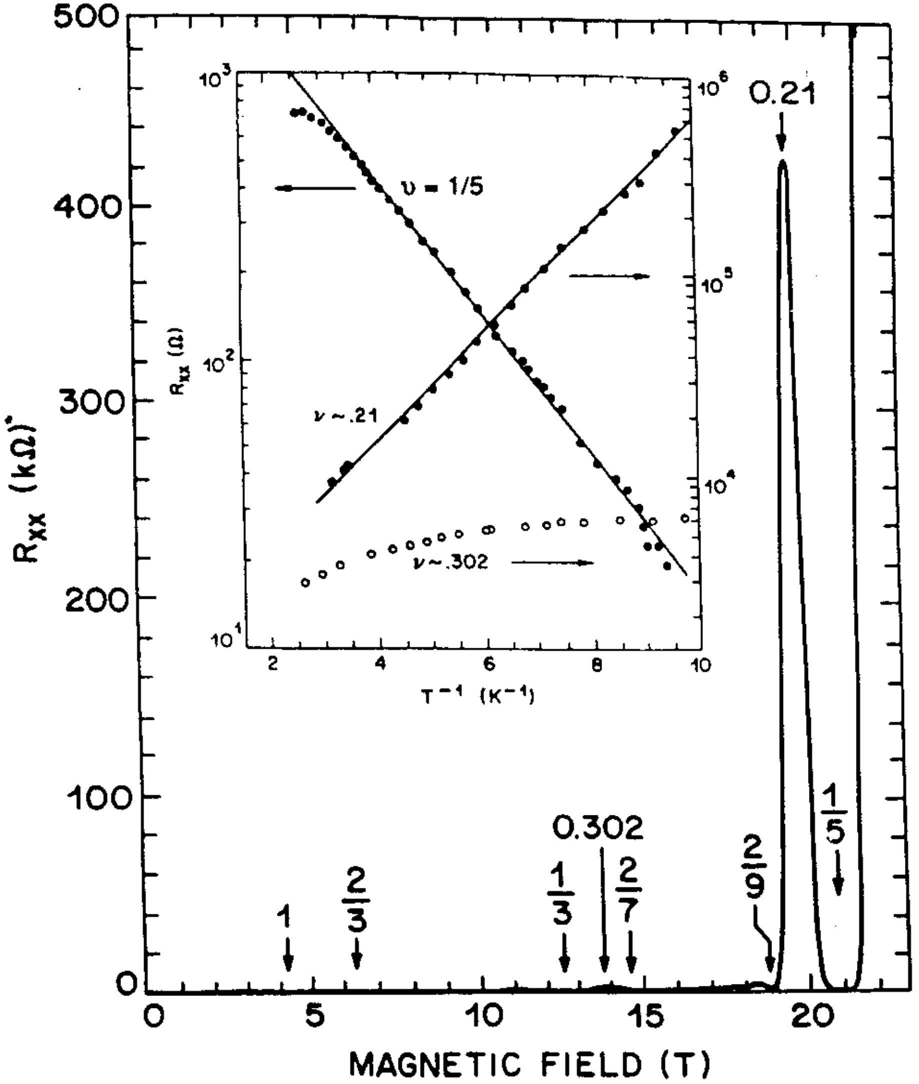
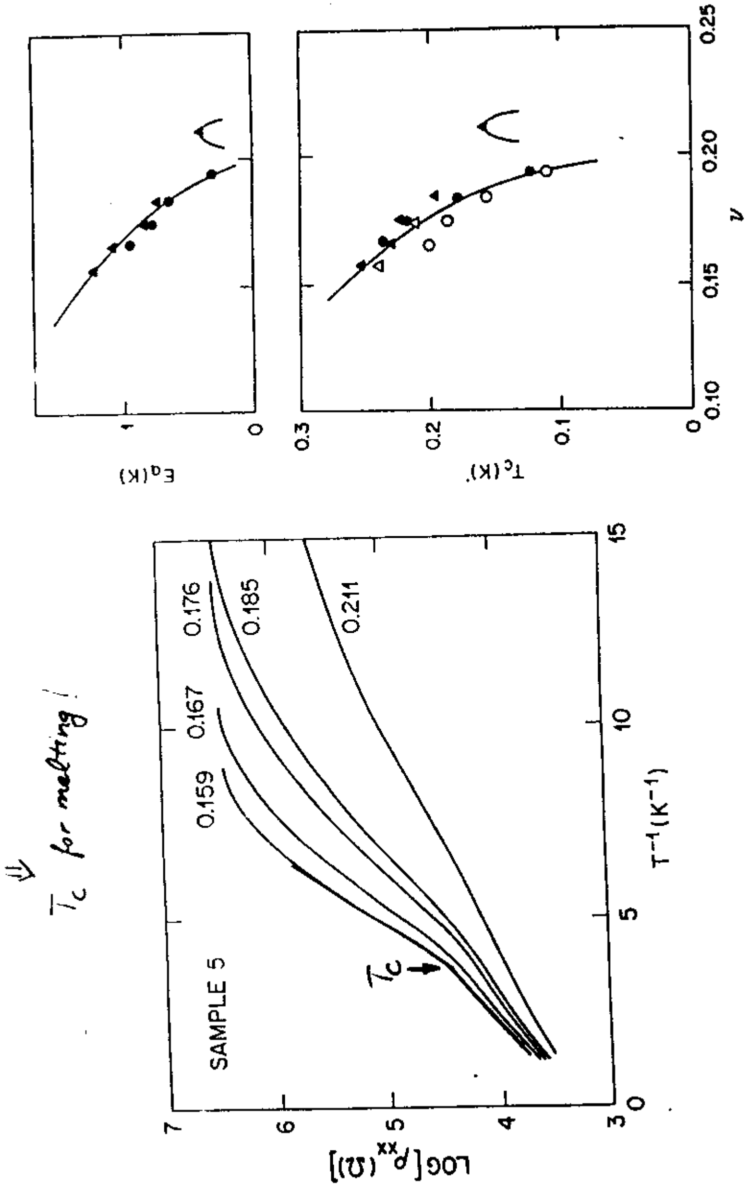


Fig. 6 Jiang et al. (1990)



"Kink" in ρ_{xx} vs T

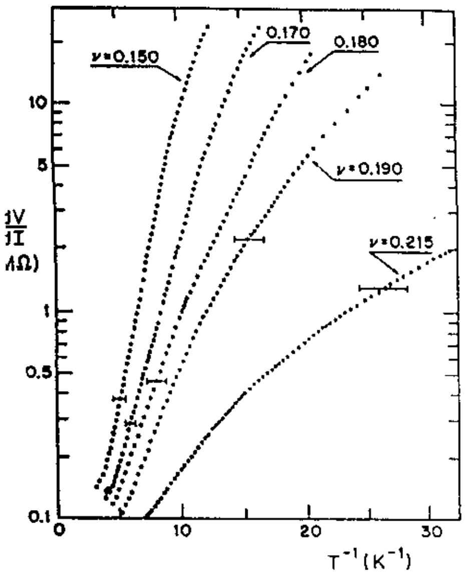
⇓

T_c for melting!

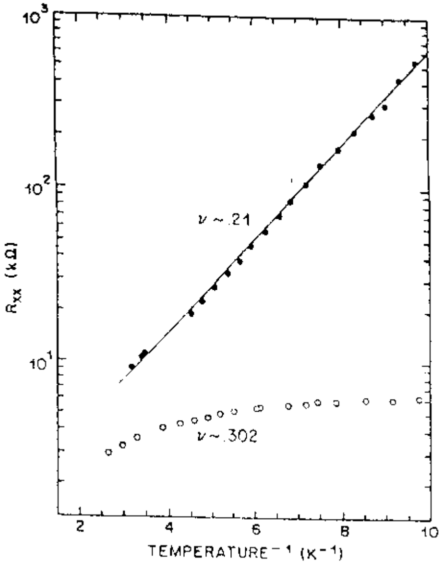
Paalanen et al. (1992)

PRB 45, 13,784

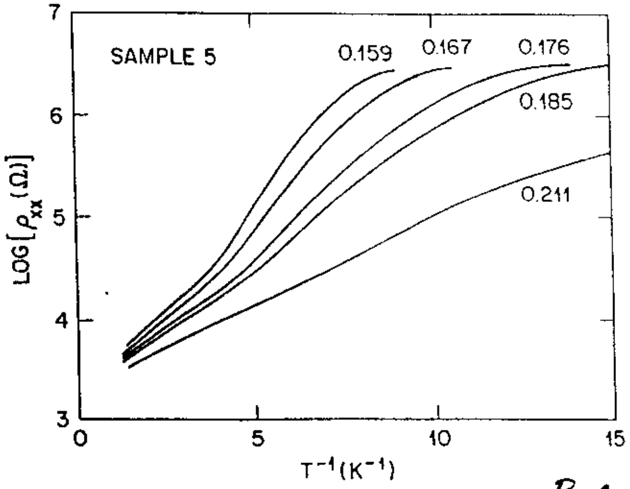
R_{xx} vs $\frac{1}{T}$ near $\nu = \frac{1}{5}$: A comparison!



Goldman, 1990



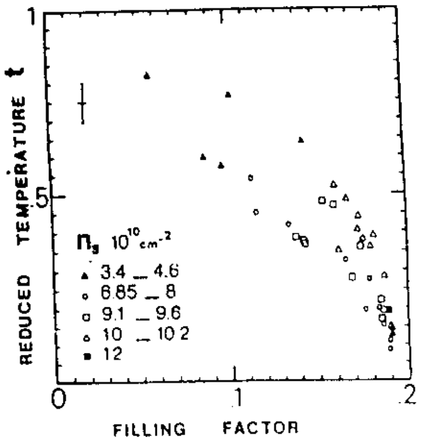
Jiang, 1990



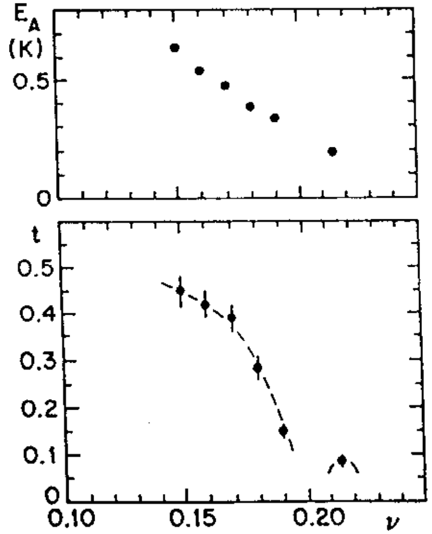
Paalanen, 1992

- 2D WC melting phase diagrams'

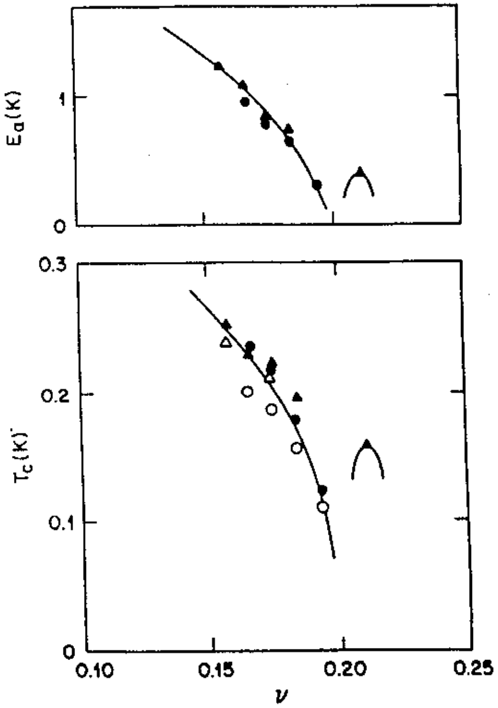
t vs ν : A comparison



Williams, 1991



Goldman, 1990



Paalanen, 1992

Transport and noise in the insulating phase around the $\frac{1}{5}$ FQH liquid

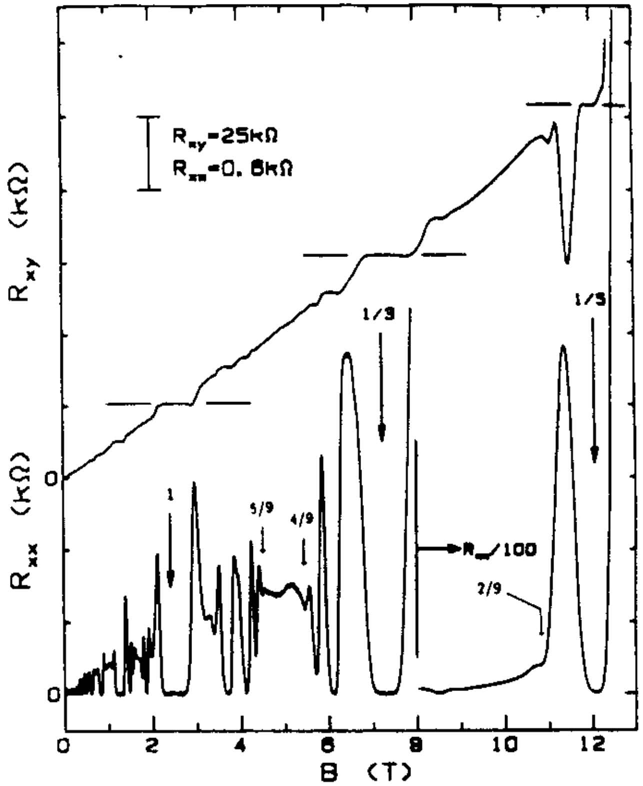
L. W. Engel, Y. P. Li, T. Sajoto, D. C. Tsui, and M. Shayegan
Dept of Electrical Engineering, Princeton University

Li, 1991
Engel, Surf. Sci. 263, 44 (1992)
Sajoto, Ph.D. Thesis, 1992
Li, Ph. D. Thesis, 1994

Systematic Studies (B,T dependence):

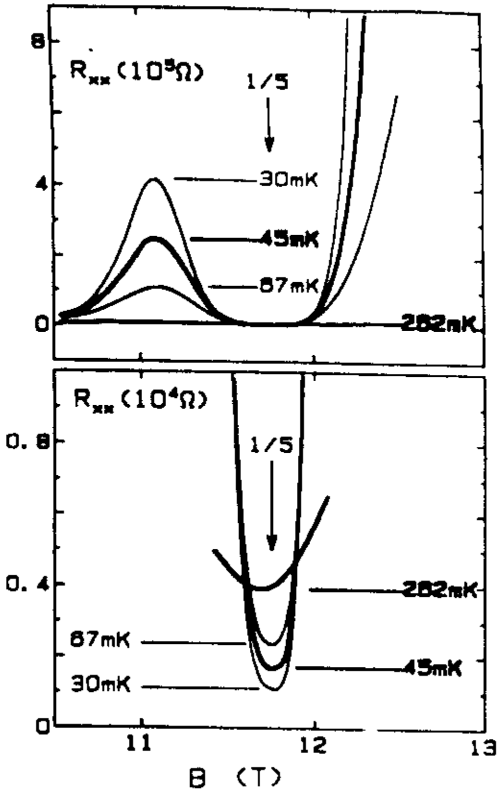
- Re-entrant insulating phase (IP).
- I-V curves: dV/dI drops off at threshold V_T.
- Current noise: also has V threshold.

Transport:



- Sample, 1.5×5mm^2, unpatterned. $n \approx 5.75 \times 10^{10}\text{cm}^{-2}$.
- Well-developed $\frac{1}{5}$ FQHE, with quantized R_{xy} plateau.
- High R_{xx} regions either side.

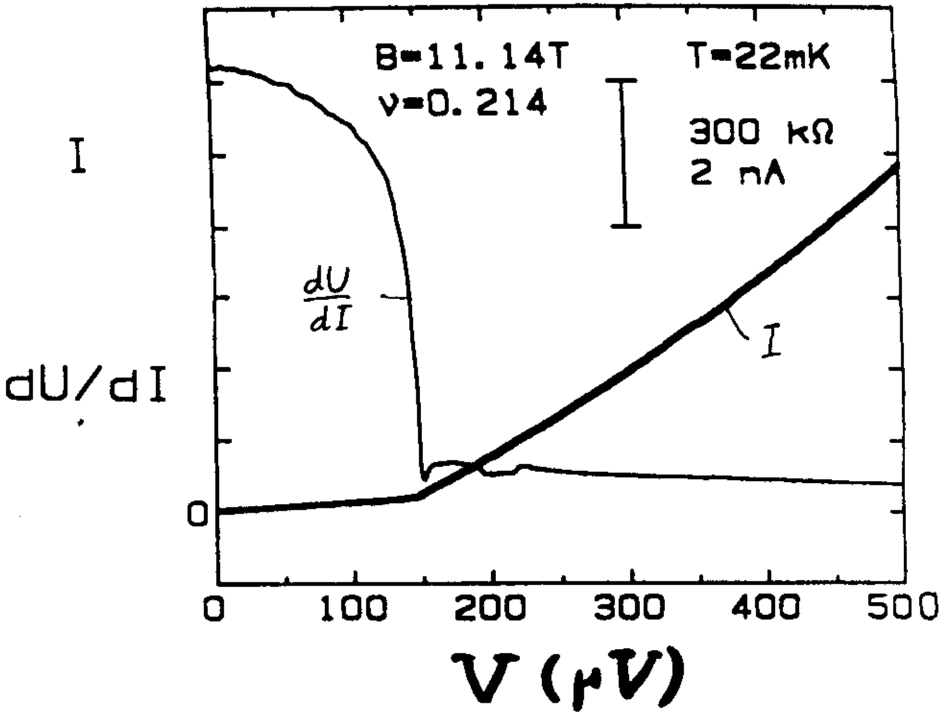
Re-entrance of the insulating phase (IP)



- 1/5 FQH liquid, decreasing R_{xx} with decreasing T to lowest T.
- IP, increasing R_{xx} as T falls.

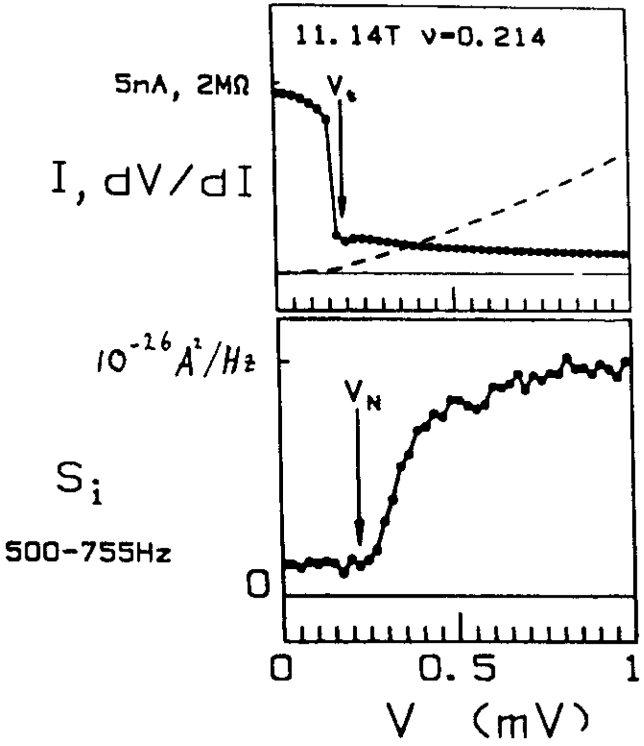
Re-entrance ⟹ correlations important in IP, at least on low-B side of $\frac{1}{5}$ FQH liquid.

Current-Voltage thresholds in IP regions



- 100-300μV thresholds in IP regions.
- Sharp steps in this sample.
- Curve shown is at R_{xx} peak between $\frac{1}{5}$ and $\frac{2}{9}$ FQHE's and was measured with separate current and voltage contacts.

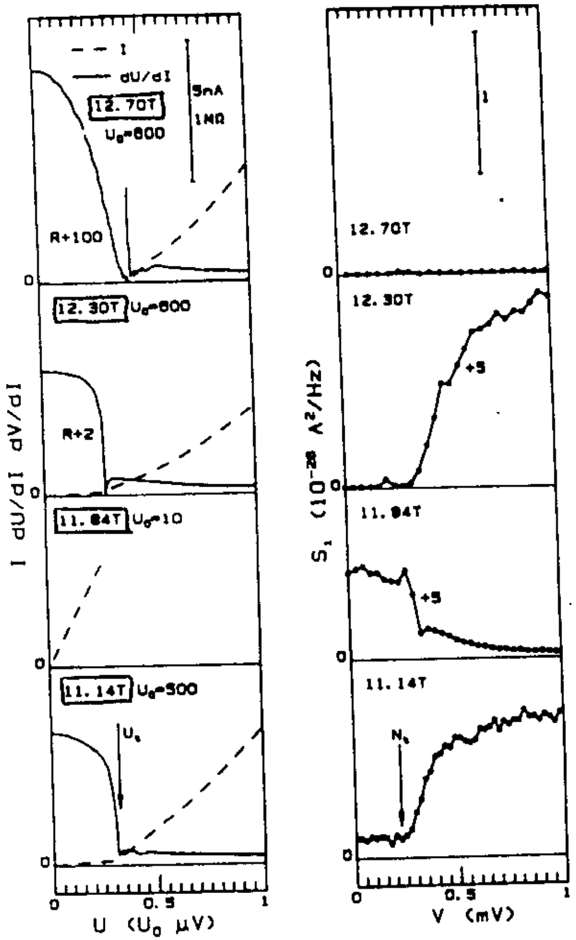
I-V & Noise Measurements



- S_i is current noise power, measured two-terminal with transimpedance amplifier.

Li et al. (1991)

I-V and noise data: 4 regions of ν



Noiseless IP

IP, $\nu < \frac{1}{5}$

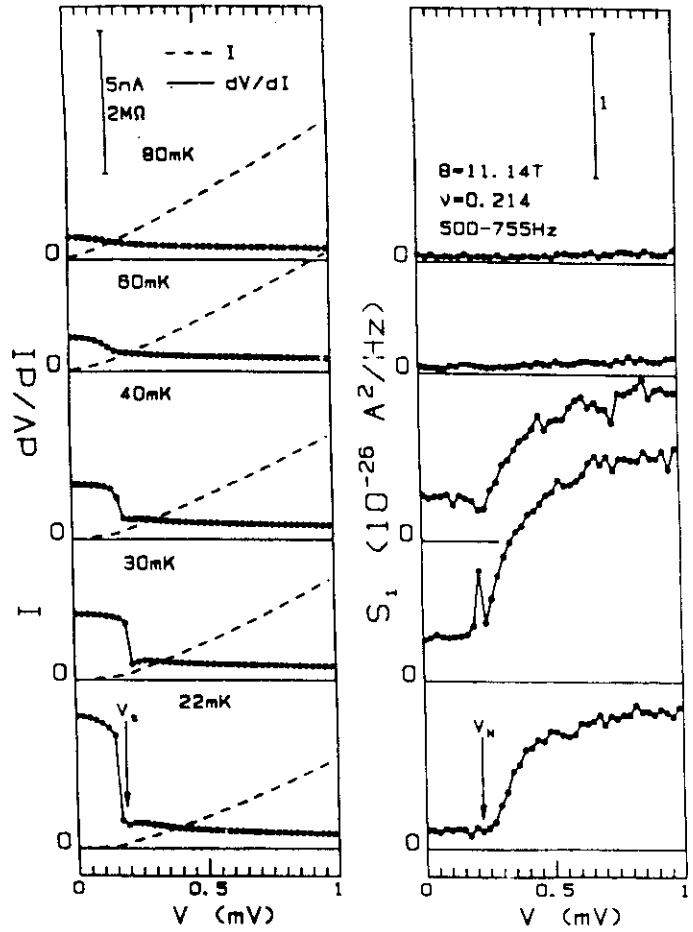
$\frac{1}{5}$ FQH Liquid

IP, $\nu > \frac{1}{5}$

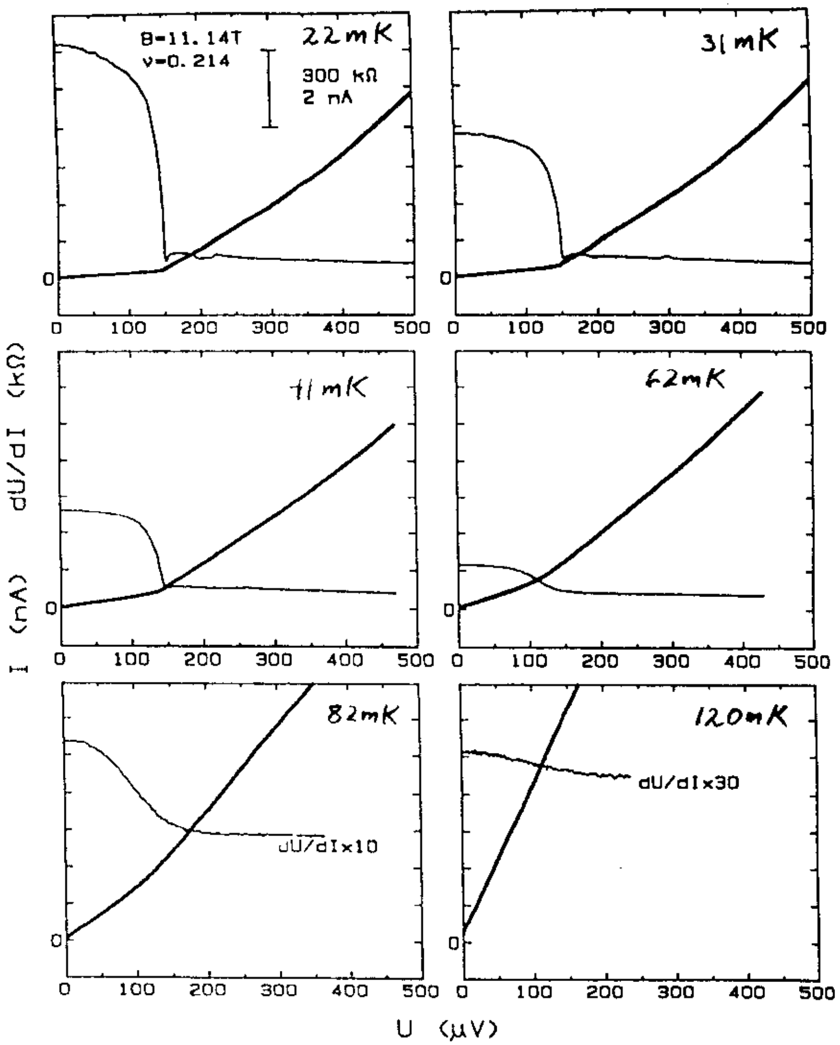
In the IP...

- S_i averaged over 550-775 Hz. $T \sim 22$mK.
- I-V: Ohmic in $\frac{1}{5}$ FQH liquid. High R_{xx}, $B \geq 12.7$T.
- Noise: Noise disappears in high R_{xx} region, $B \geq 12.7$T.

I-V and S_i-V T-Dependence

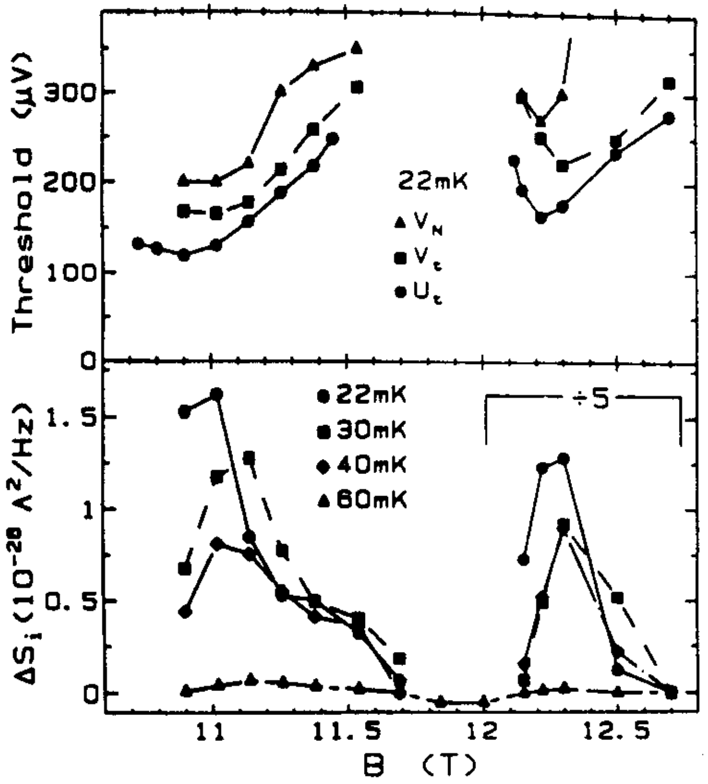


- Zero-bias dV/dI decreases with T.
- When threshold observed, $V_T \sim$ constant with T.
- Relative change in dV/dI at threshold *gradually* disappears by T ~120mK.
- ΔS_i disappears at lower T of ~ 60mK.



Li, Sajoto, et al. (1991)

Threshold and Noise: Survey Results



($\Delta S_i = S_i(V = 1\text{mv}) - S_i(V = 0)$ averaged 550 to 775 Hz.)

- Threshold voltages and noise power vary oppositely.
- As $\frac{1}{5}$ FQH liquid is approached, thresholds *increase*, noise *decreases*.
- same as "noiseless" IP approached, as B goes from 12.3 to 12.7 T.

(?) Interpretation for B<12.7T: ~~Weakly~~ pinned WC model, domain size L_0

- Pinning energy ~ Work done in threshold field E_T, to slide WC by one lattice constant a. Use theoretical shear modulus: $L_0^2 \sim 0.02e/\epsilon E_T$.
- Our $E_T \lesssim 1.2$mV/cm, $\Longrightarrow$ $L_0 \lesssim 4\mu$m ~ 100a.
- Sliding WC $\Longrightarrow$ S_i thresholds $V_N \gtrsim V_T$.
  Each domain ~ independent noise source,
  $L_0 \ll$sample size $\Longrightarrow$ $S_i \sim L_0^2$,
- Decreasing L_0 thus explains *both* increasing threshold voltages *and* decreasing ΔS_i seen as $\frac{1}{5}$ FQH liquid approached, and as B goes from 12.3 to 12.7T.
- The more insulating phase for $B \geq 12.7$T, where $S_i \to 0$, is *not* described by weak pinning model —better described as a Wigner glass, since $L_0 \longrightarrow a$.

(?) Later measurements by Li (Ph.D. thesis) of

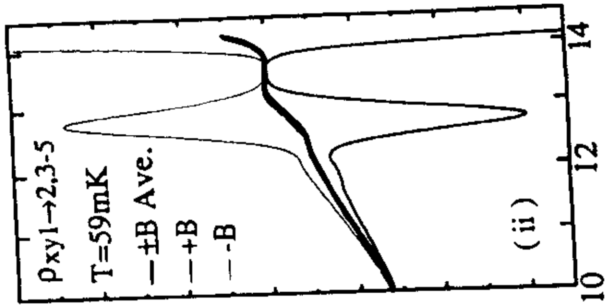
$\left\{ \begin{matrix} \sigma(\omega) \\ \epsilon(\omega) \end{matrix} \right\} \Rightarrow$ "strong" pinning by residual impurities near 2DES more appropriate!

Also, see: Zhu, Littlewood, & Millis, PRL <u>72</u>, 2255 (199
& submitted to PRB ?
for latest theoretical work!

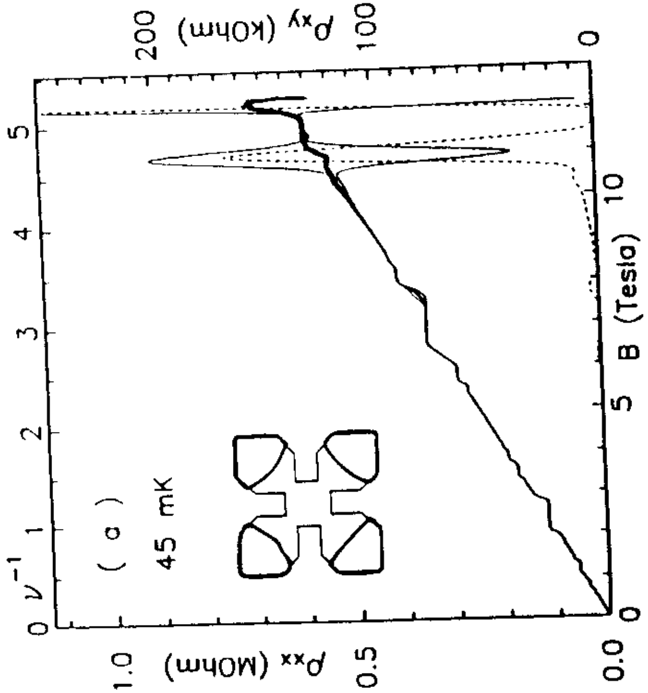
Summary

- IP on either side of region with $\frac{1}{5}$ FQH liquid ground state.
- Differential resistance falls, noise jumps at threshold $E_T \sim$ 1.2mv/cm.
- Opposite V_T and S_i variation with ν: as $\frac{1}{5}$ FQHL approached, V_T increases while S_i decreases.
- Interpret I-V and noise data using weakly pinned WC model, $L_0 \lesssim 4\mu$m.
- Transition to "Wigner glass" at $B \geq 12.7$T, $\nu \leq 0.187$.
- Gradual T-dependence of I-V's. Different temperatures for disappearance of S_i-V and I-V thresholds. *Cannot draw a reasonable B, T phase diagram.*

Hall coefficient · stays normal! (at least down to ≲ 40 mK)



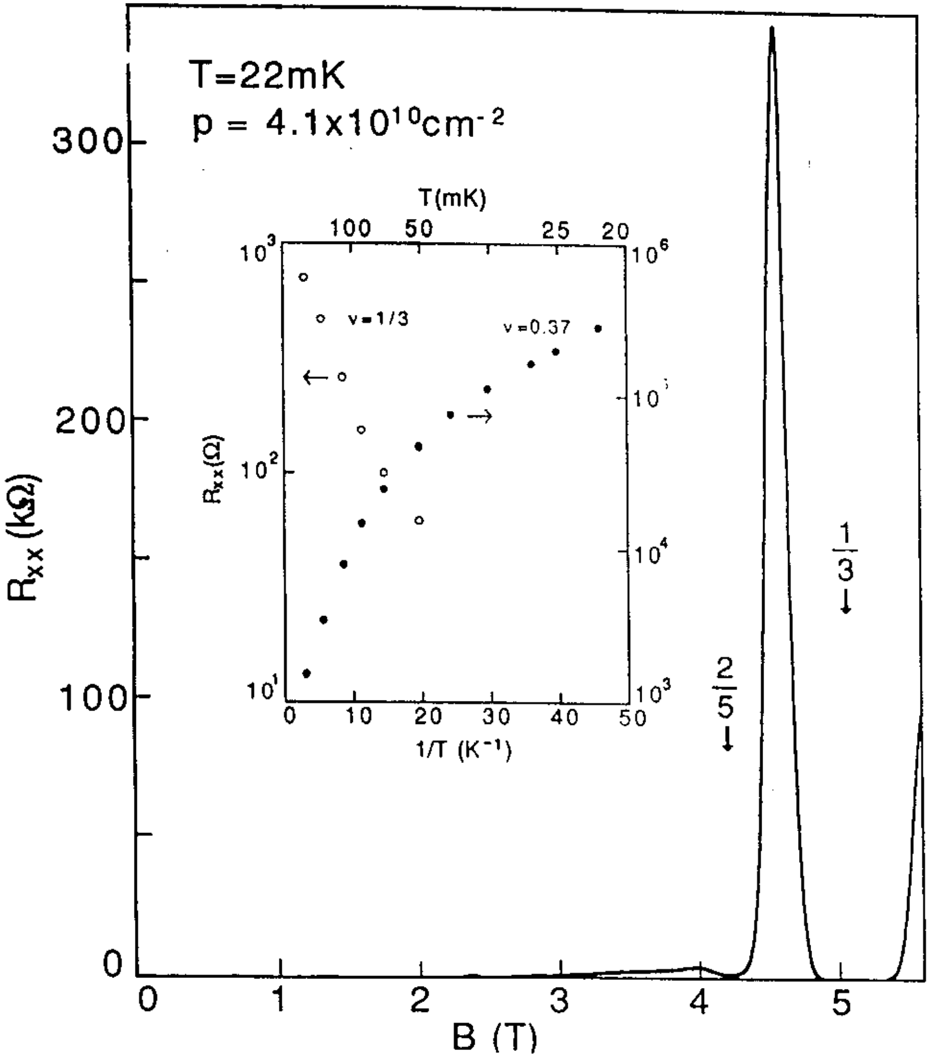
Sajoto et al. (1993)



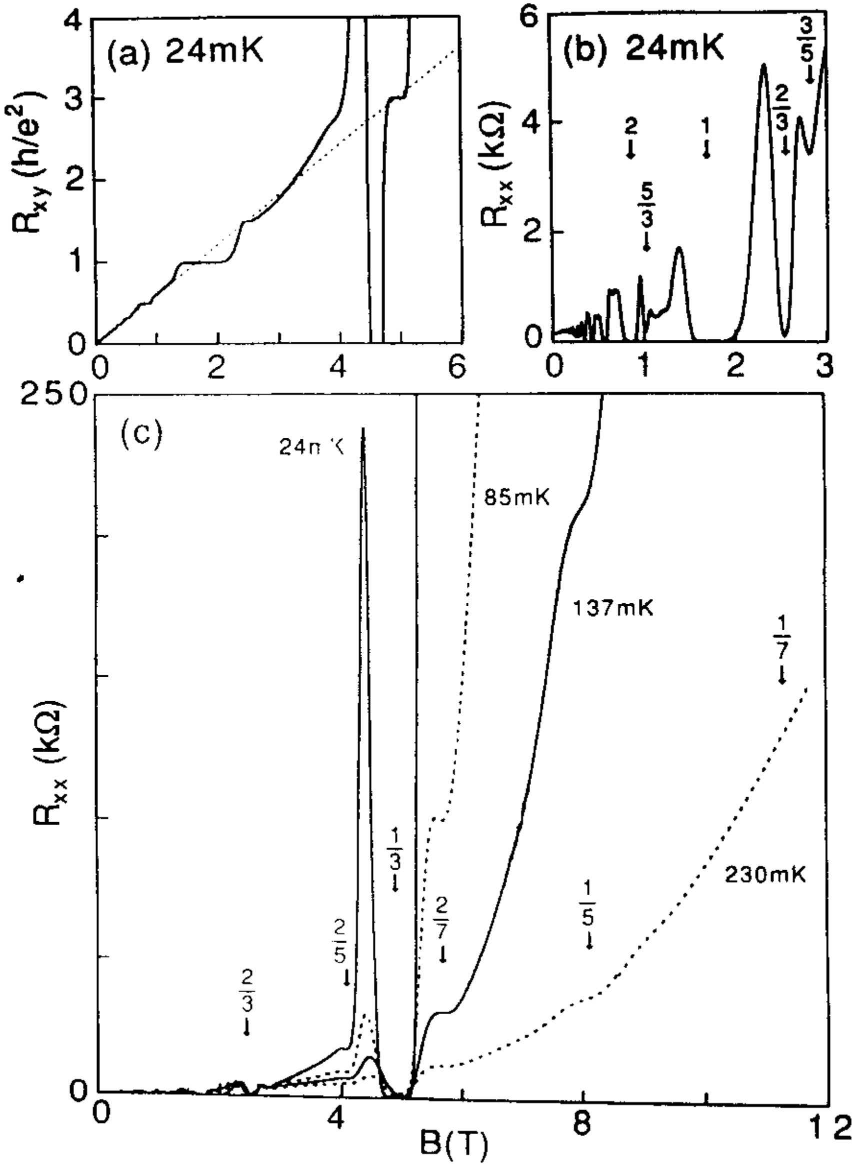
Goldman et al. (1993)

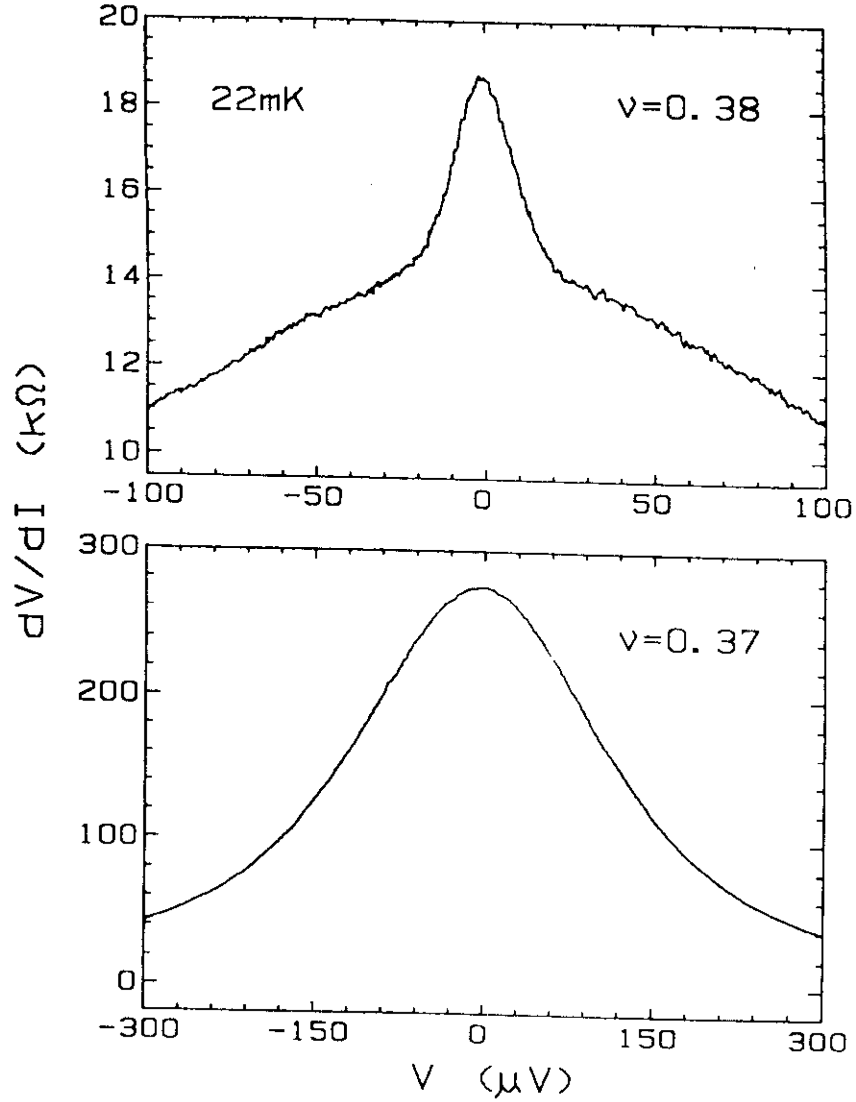
2D H.H. System :

Reentrant insulating phase near $\nu = 1/3$!!

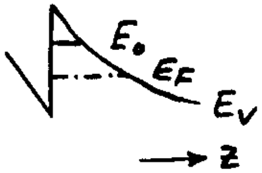


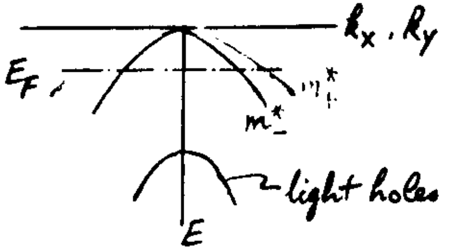
Santos et al. (1992)



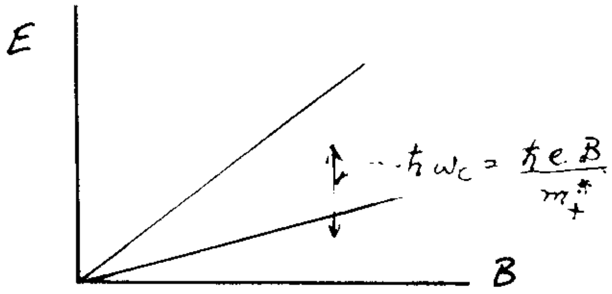


- Effective mass of 2D holes in:





$m_+^* \simeq \overset{0.37}{\cancel{0.6}}\ m_o \sim \overset{5}{\cancel{10}}\ m^*_{\rm electrons}$



→ $\hbar\omega_c << $ Coulomb energy

$\frac{e^2(\pi n)^{1/2}}{4\pi\epsilon_o\epsilon}$	$\frac{e^2}{4\pi\epsilon_o\epsilon l}$	$^{1/3}\Delta$	$\hbar\omega_c = \frac{\hbar eB}{m_+^*}$
50K	100K	10K	20K

@B=5T

2D *holes* at $\nu=1/3$ ↔ 2D *electrons* at $\nu=1/5$

Why !?

$\boxed{m^*_{hole} \approx 5\, m^*_{electron}}$

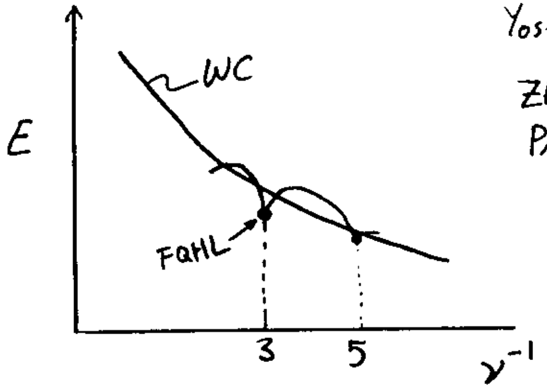
At $B \approx 5T$:

$e^2/(4\pi\varepsilon\ell)$	${}_{1/3}\Delta$	$\hbar\omega_c = \hbar eB/m^*$
100K	10K	20K

Landau level mixing

↓

Both ${}_{1/3}\Delta$ and $E_{WC}-E_{FQHL}$ are lowered.



Yoshioka (1984, '86)

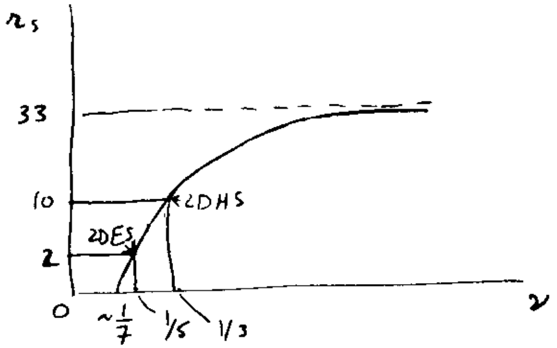
Zhu et al. (1993)

Price et al. (1993)

- consider "phase" diagram in ν vs. r_s plane

$$r_s \triangleq \frac{\langle \text{particle separation} \rangle}{\text{Bohr radius}}$$

- $r_s \gtrsim 33$ → WC for even $B=0$
- $r_s=0$ → WC for $\nu \lesssim 1/7$



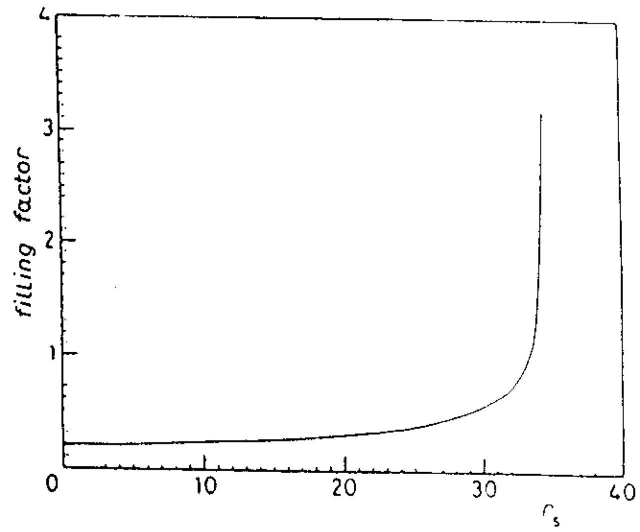
Experimental observations so far:

2DES: $\begin{cases} r_s \sim 2 \\ \nu_c \simeq 1/5 \end{cases}$ ←● (for $n_s = 1\times10^{11}\,\mathrm{cm}^{-2}$)

2DHS: $\begin{cases} r_s \sim 10 \\ \nu_c \simeq 1/3 \end{cases}$ ← very dilute system!

S.T. Chui and K. Esfarjani

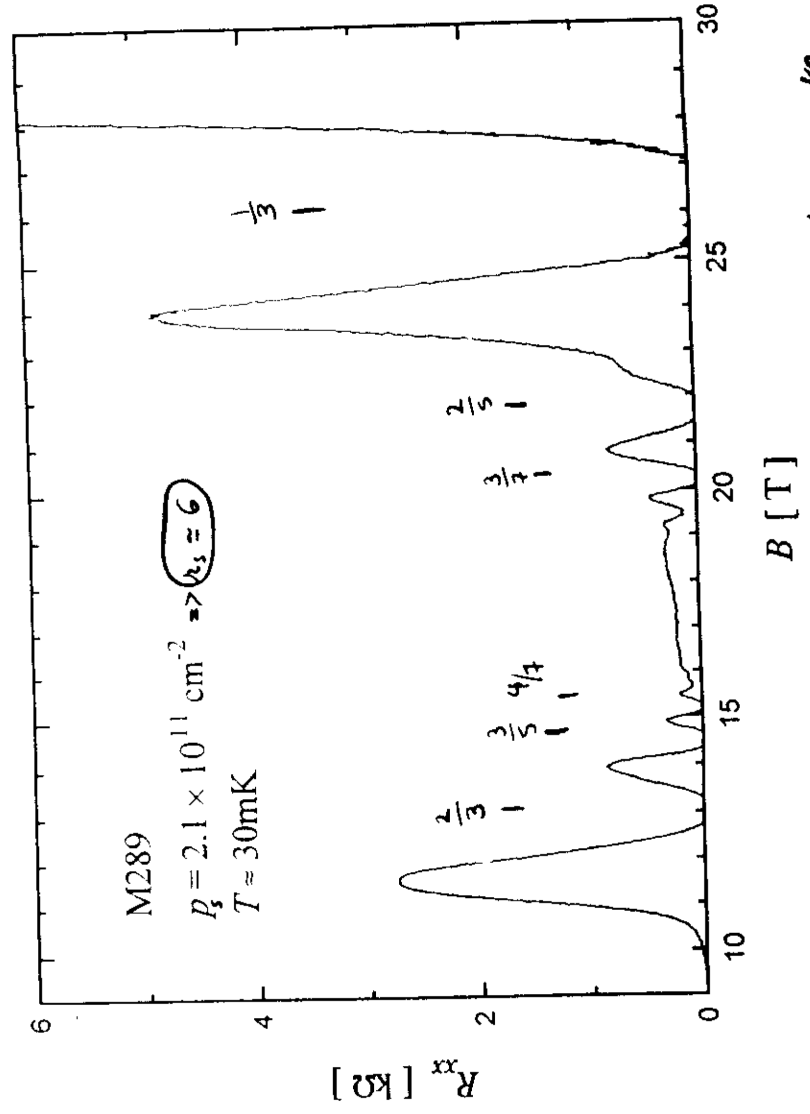
Europhysics Letters (1991)



$$r_s = \frac{\text{<distance between charge carriers>}}{\text{effective Bohr radius}}$$

• Does the IP near $\nu = \frac{1}{3}$ disappear as we go to higher p_s in 2DHS !?
⇓
smaller r_s

YES!



Manoharan, Shayegan, Klepper (1994)
unpublished.

- Other experiments:

1. RF measurements
   - Andrei 1988 (?)
   - Glattli 1990
   - Williams 1991
   - Paalanen 1992

→ – Li, Ph. D. thesis, 1994:

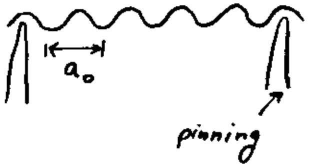
- $\left\{\begin{matrix}\sigma(\omega)\\ \epsilon(\omega)\end{matrix}\right\}$ large dielectric const. => polarizability => WC

- Washboard frequency (ac + dc):

apply sufficiently large dc $\vec{E}$ to depin W.C.

=> expect an ac current @ frequency:

$$f_{WB} = \frac{\overset{\text{ave. drift velocity}}{v_d}}{\underset{\text{W.C. lattice const.}}{a_0}} = \frac{J}{n_s e a_0}$$



=> f_{WB} depends linearly on J with n_s & a_0 known

Li's results qualitatively consistent with f_{WB}.

2. Magnetooptics
   - photoluminescence
   - cyclotron resonance

   } see: Surf. Sci. 305 (1994) (Proc. EP2DS11 Conf.)

3. Thermopower
   - diverging S_{xx} in IP => Energy gap! Bayot, 1994

Summary

- Although experiments so far vary in details and don't prove W

  they are VERY SUGGESTIVE!

  - reentrance around well-developed FQH liquid states (=> correlations important)
  - non-linear I-V
  - noise
  - large $\epsilon(\omega)$

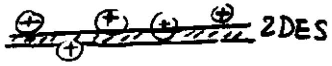
    ⋮
  - 2DHS with large r_s => IP around $\nu = \frac{1}{3}$ rather than $\frac{1}{5}$

    ⋮

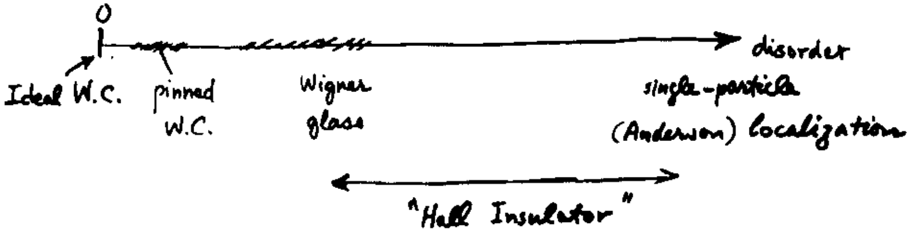
- But what about experiments in "dirtier" 2DES?

  - GaAs/AlGaAs (Jiang, 1992, 1993)
  - Si/SiO$_2$ (D'Iorio, Pudalov 1990-1993)

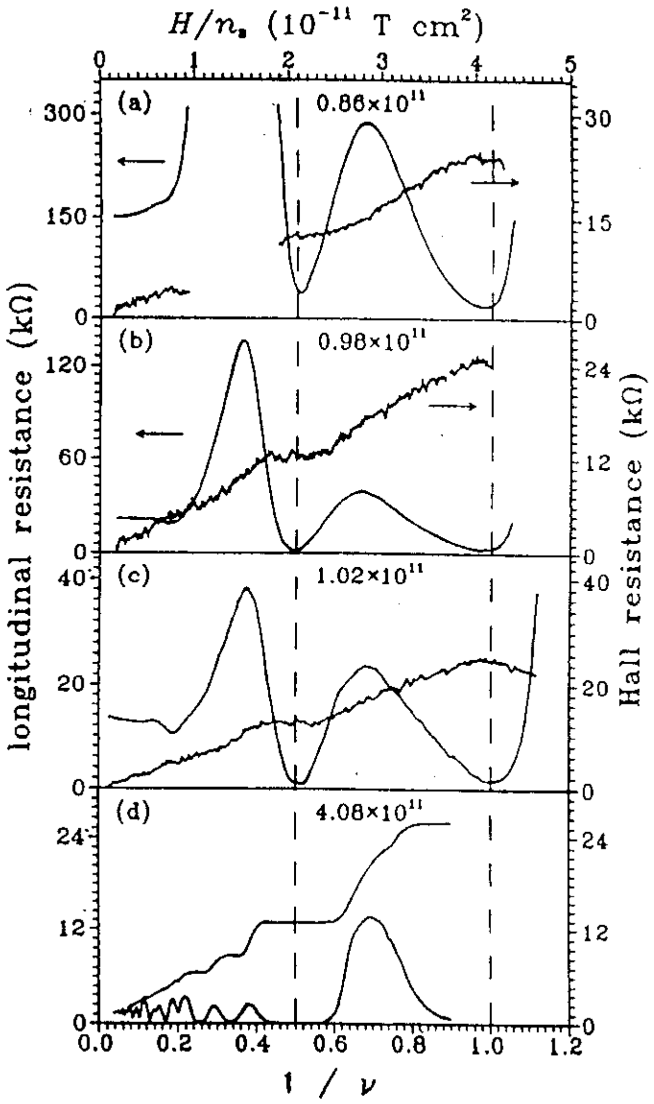
  which show reentrant IP behavior around IQH states and other similar features, e.g., $\rho_{xx} \to \infty$; $\rho_{xy} \to$ normal



- Can the "Hall Insulator" picture explain it all?



2D electrons at Si/SiO$_2$



D'Iorio et al. (1990); Pudalov et al. (1991)

- Two important points

(1) Is $\left\{\begin{matrix}\rho_{xx}\to\infty\\ \rho_{xy}\to \text{normal}\end{matrix}\right\}$ evidence for Hall Insulator & against W.C. ?

NO !

a. limit $\left\{\begin{matrix}\omega=0\\ T\to 0\end{matrix}\right\}$ more physical, not $\left\{\begin{matrix}T=0\\ \omega\to 0\end{matrix}\right\}$ as in HI

b. Pinned W.C. (at $T=0$) and de-pinned W.C. also have finite & normal ρ_{xy}

[Chui, Solid State Commun. <u>86</u>, 605 (1993)
Zhu, Littlewood, Millis, PRL <u>72</u>, 2255 (1994)
& preprint]

(2) Global phase diagram does <u>not</u> allow for the transitions like:

FQHL → ~~IR~~ → FQHL

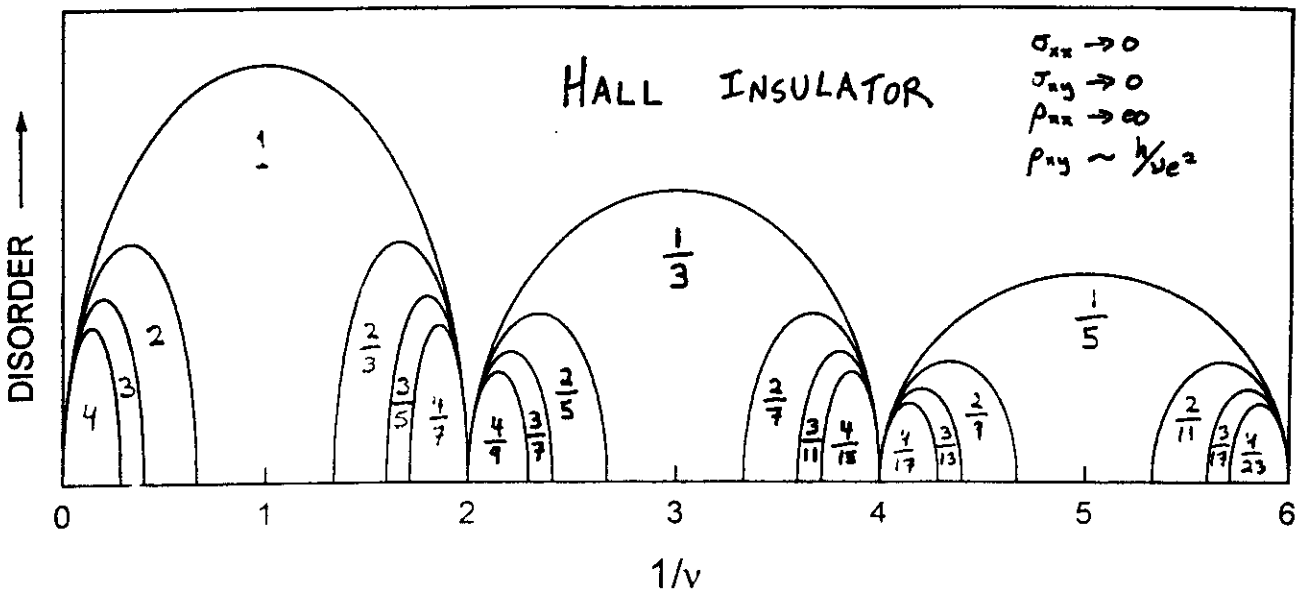
$\frac{2}{5} \longrightarrow \frac{1}{3}$

or $\frac{2}{9} \longrightarrow \frac{1}{5}$

But these are experimentally observed !

Global Phase Diagram

(Kivelson, Lee, and Zhang; 1992)

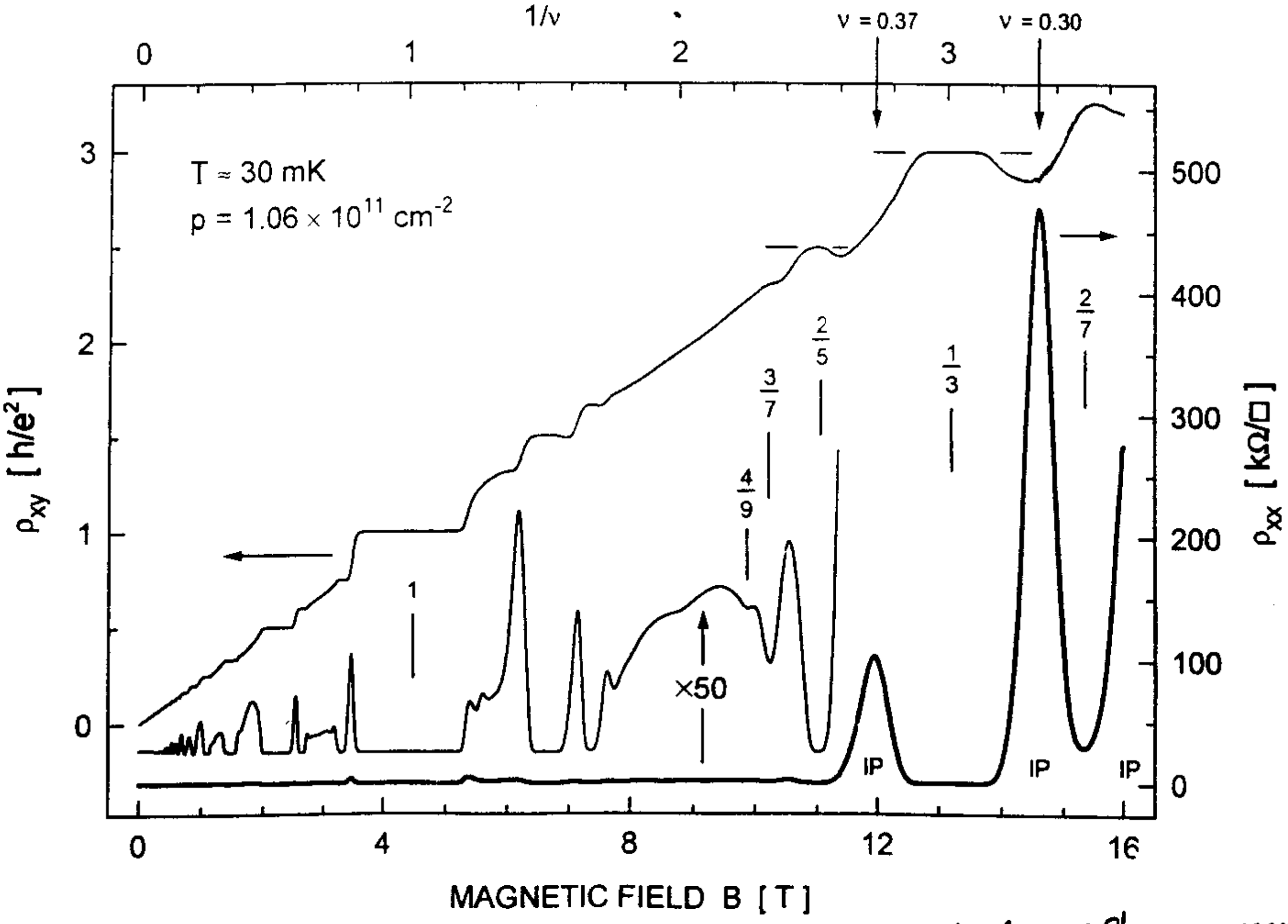


"CORRESPONDING STATES":

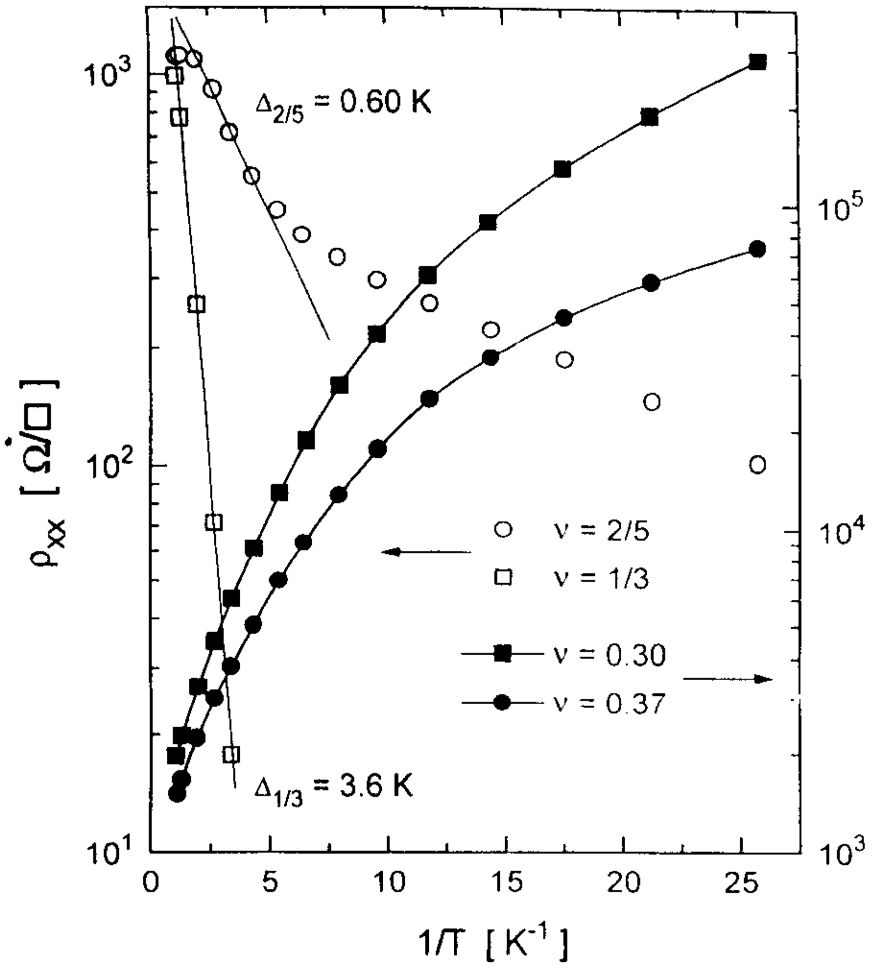
$\nu \longleftrightarrow \nu + 1$ LL promotion

$\nu \longleftrightarrow 1 - \nu$ P-H symmetry

$\frac{1}{\nu} \longleftrightarrow \frac{1}{\nu} + 2$ Flux attachment



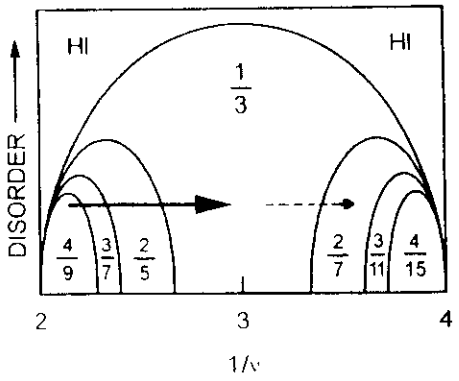
Manoharan & Shayegan, 1994



SUMMARY

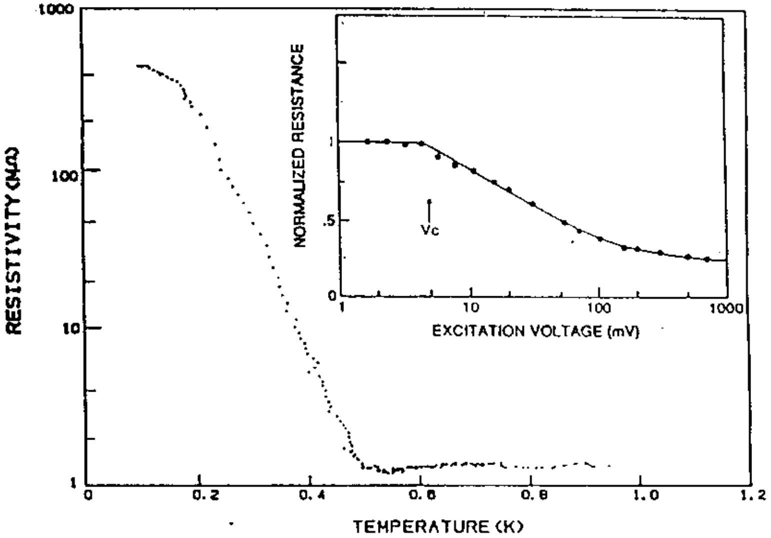
- EXPERIMENTAL OBSERVATIONS
  - Direct transition from high-order ($\nu = 2/5$) FQH liquid to an IP
  - Finite and ~classical ρ_{xy} in IP regions

- INCONSISTENT WITH HALL INSULATOR AND GLOBAL PHASE DIAGRAM TOPOLOGY:



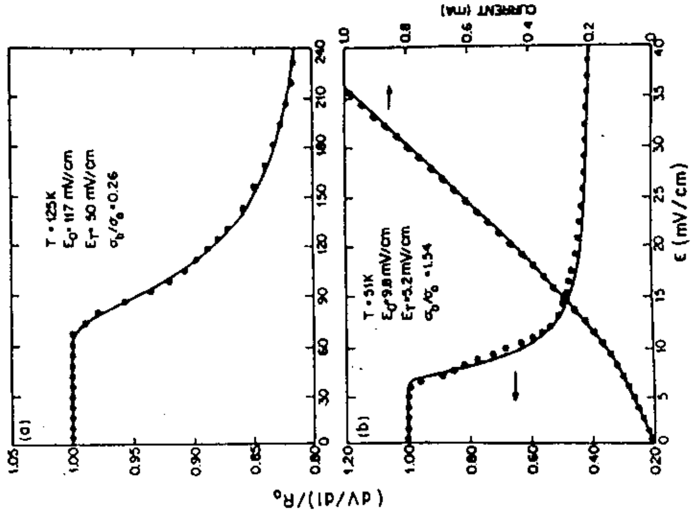
- CONSISTENT WITH PINNED WIGNER CRYSTAL
  - IP around $\nu = 1/5$ for 2DES
    LL MIXING → $\nu = 1/3$ for 2DHS
  - Higher density 2DHS shows direct $\nu = 2/7$ FQHL ⟷ IP

2D Electrons on He

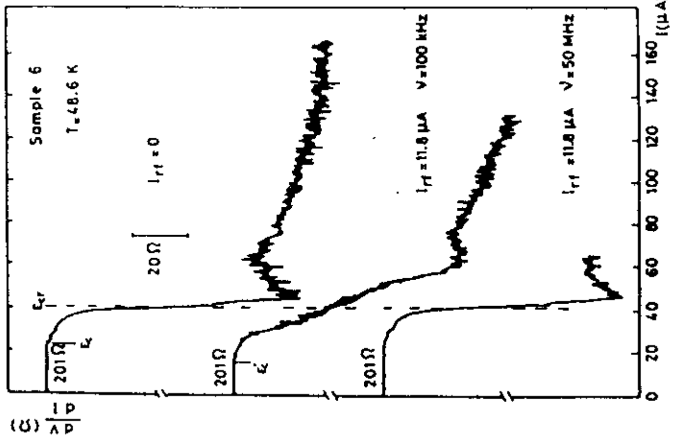


Jiang & Dahm (1989)

CDW in 1D Systems ($NbSe_3$)



Grüner (1988)



Monceau et al. (1982)

SPRING COLLEGE IN CONDENSED MATTER
ON QUANTUM PHASES
(3 May - 10 June 1994)

MAGNETOTRANSPORT MEASUREMENTS IN QUANTUM HALL EFFECT

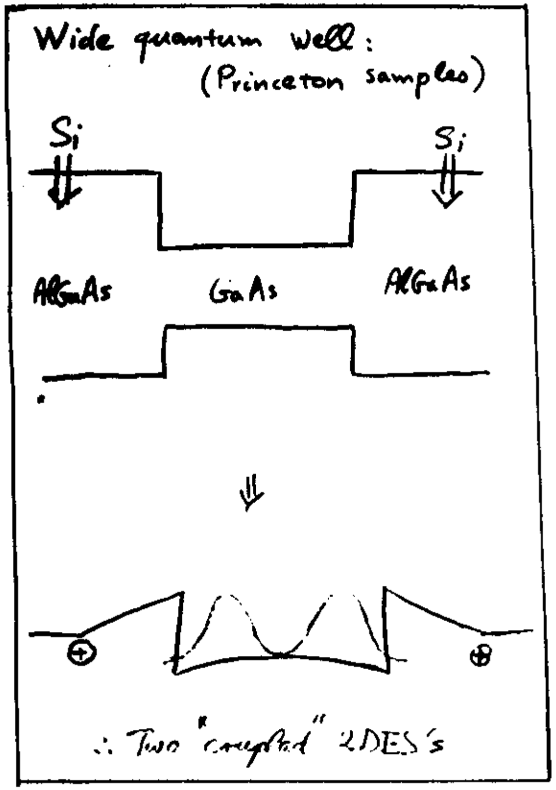
LECTURE 3

Lecture III, Trieste, May 1994 -M. Shayegan

CORRELATED QHE in BILAYER ELECTRON SYSTEMS

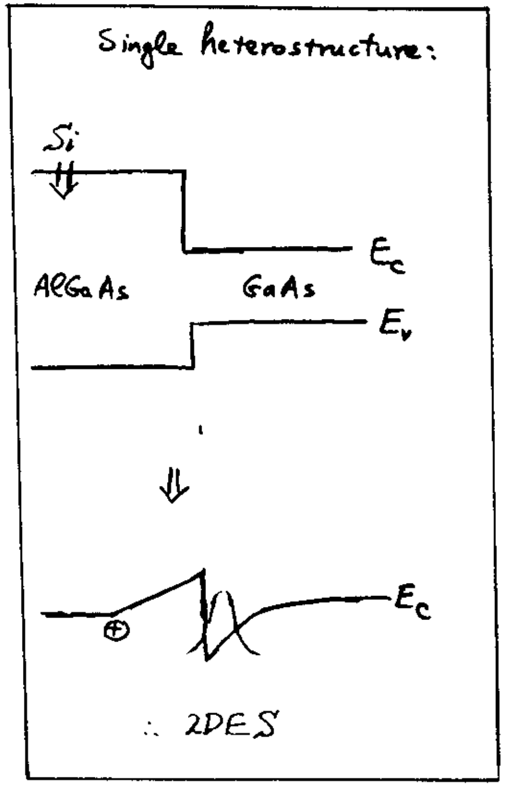
SUEN, MANOHARAN, LAY, YING, SANTOS, SHAYEGAN

Wide quantum well:
(Princeton samples)



∴ Two "coupled" 2DES's

Single heterostructure:



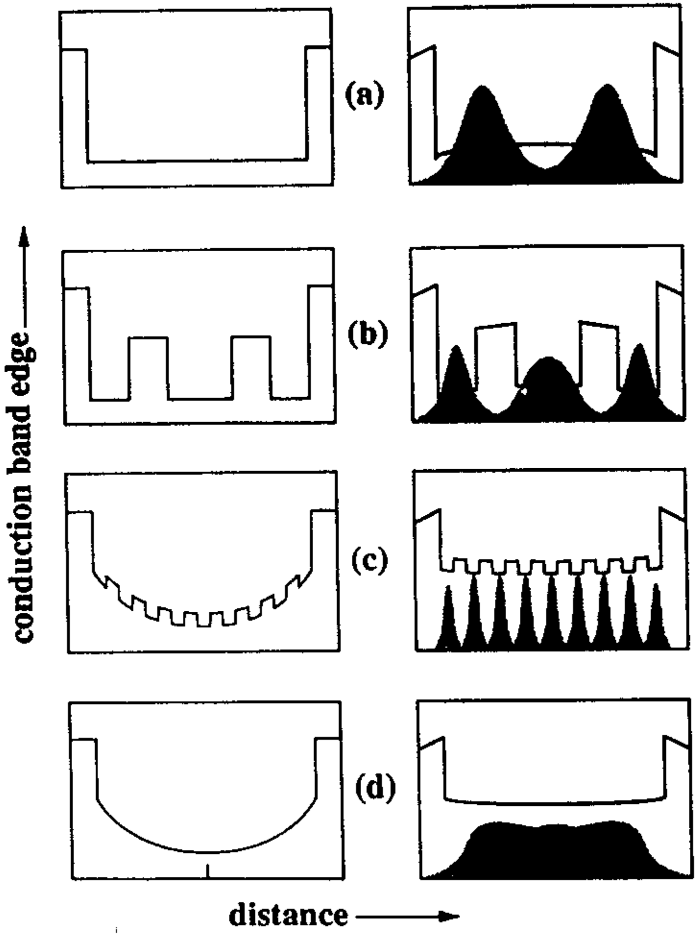
∴ 2DES

← Also in double-quantum-wells (DQW's)
(Bell Labs' samples)

Boebinger, Eisenstein, Murphy,
Pfeiffer, & West

More to come!!

"Quasi-2D" Systems with an Additional (Spatial) Degree of Freedom



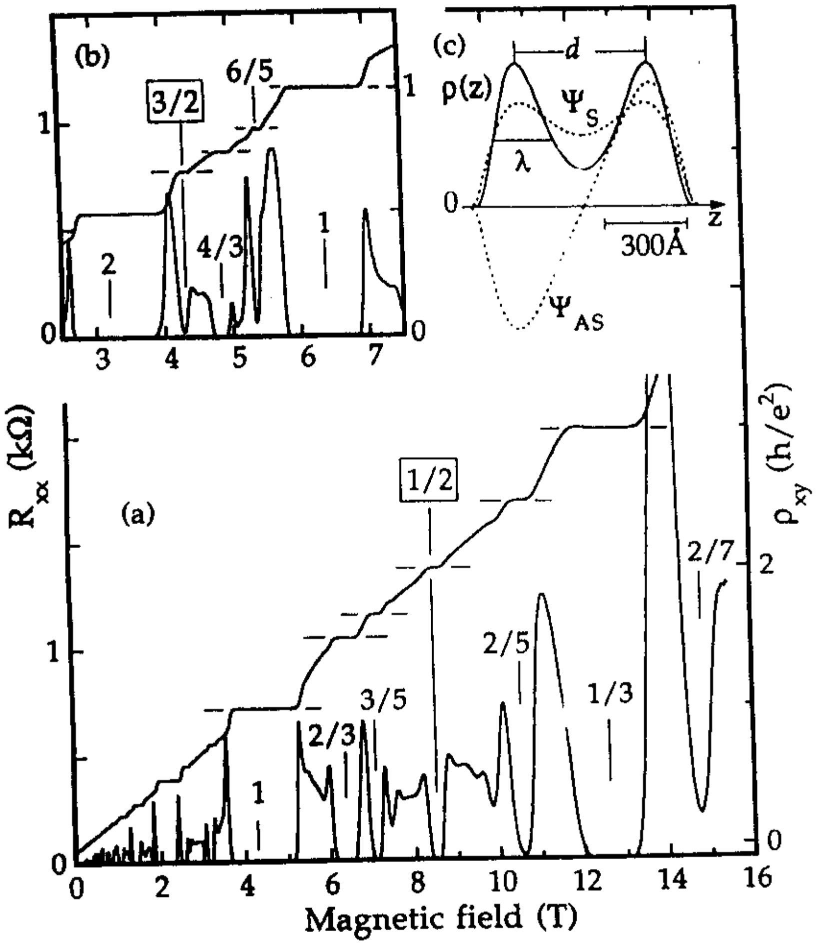
Suen,... (1991)

Jo,... (1992)

Jo,... (1990)

Shayegan... (198
Sundaram... (19

Experimental Surprises!



FQHE @ $\nu_{tot.} = \frac{1}{2}$!

(i.e., $\nu_{\text{each layer}} = \frac{1}{4}$)

IQHE @ $\nu_{tot.} = 1$!

(i.e., $\nu_{\text{each layer}} = \frac{1}{2}$)

Suen, 1994b

Simple picture doesn't work!

Two limits:

- Two isolated 2DES's in parallel

⇓

IQHE

@ $\nu_{\text{each layer}} = 1, 2, 3, \cdots$

$\Rightarrow \nu_{tot.} = 2, 4, 6, \cdots$

FQHE

@ $\nu_{\text{each layer}} = \frac{1}{3}, \frac{2}{5}, \cdots \frac{3}{5}, \frac{2}{3}$

$\Rightarrow \nu_{tot.} = \frac{2}{3}, \frac{4}{5}, \cdots \frac{6}{5}, \frac{4}{3}$

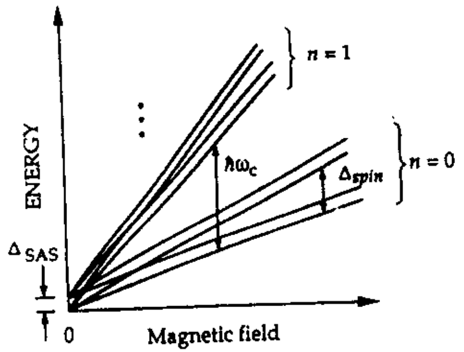
$\Delta_{SAS} < \Delta_{spin} \ll \hbar\omega_c$

- One (wide) 2DES

⇓

IQHE

@ $\nu_{tot.} = 1, 2, 3, \cdots$

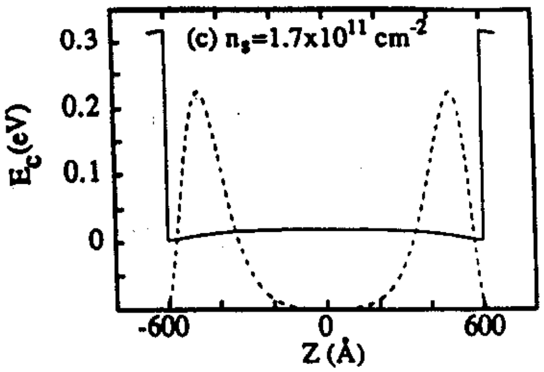


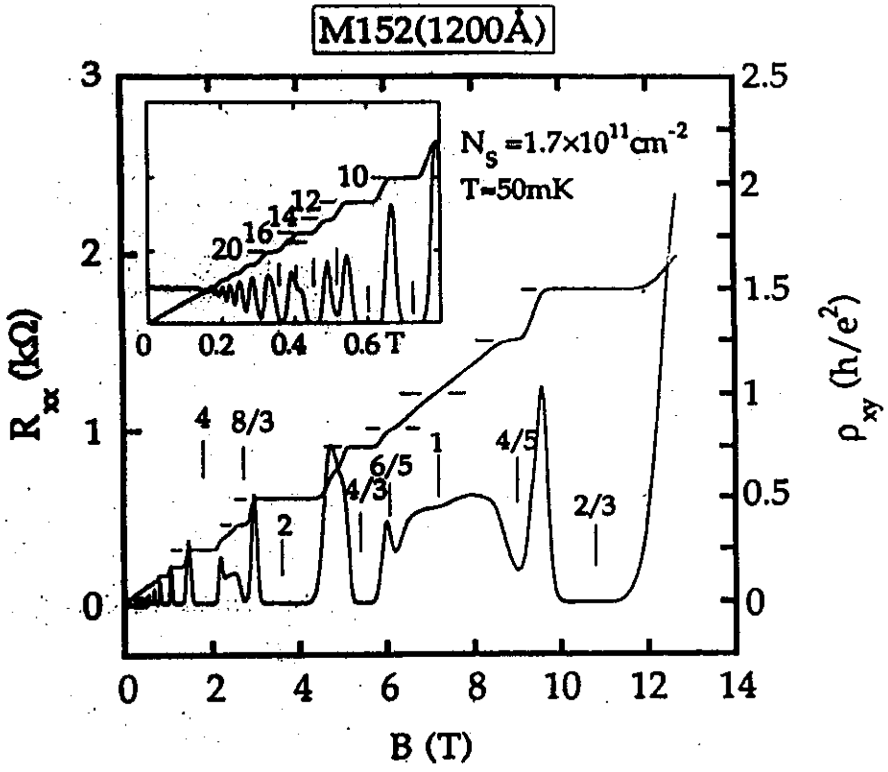
FQHE

@ $\nu_{tot.} = \frac{1}{3}, \frac{2}{5}, \cdots \frac{3}{5}, \frac{2}{3}$

$\nu = \frac{1}{2}$?

$\nu = 1$ "✓"





HISTORICAL (EXPERIMENTS)

1. Collapse of odd-ν IQHE in BLES's

→ Boebinger, 1990, 1992

Suen 1991, 1992

2. Observation of $\nu = \frac{1}{2}$ FQHE in BLES's

→ Suen, 1992a,b; 1994 a, b ~ May 23 issue of PRL!
(has $\nu = 3/2$ FQHE also)

→ Eisenstein, 1992

3. Many-body $\nu = 1$

Eisenstein 1992 (a hint!)

Murphy 1994 (commensurate to incommensurate transition with in-plane B)

→ Lay 1994 (finite-T QHE → metal transition!)
↖ preprint

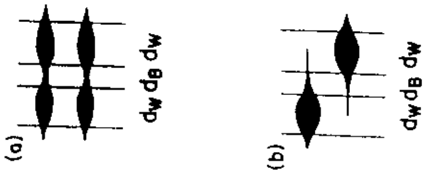
4. Reentrant insulating phases around $\nu = \frac{1}{3}$ and even $\frac{1}{2}$ FQHL's!

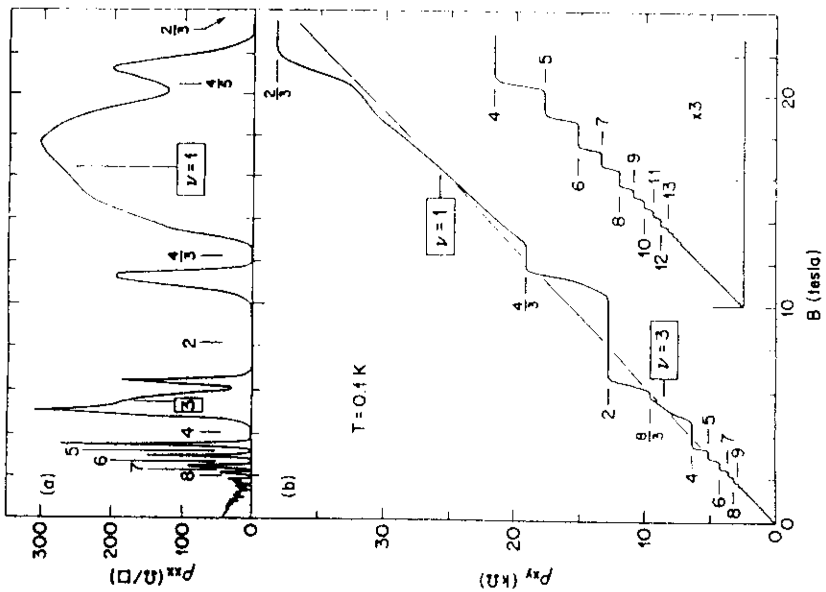
Evidence for Bilayer Wigner Crystal!?

if time? --> Suen, 1992b; 1994a; unpublished.

- Collapse of odd-ν IQHE at high B (Boebinger 1990)

DQW with finite Δ_{SAS}

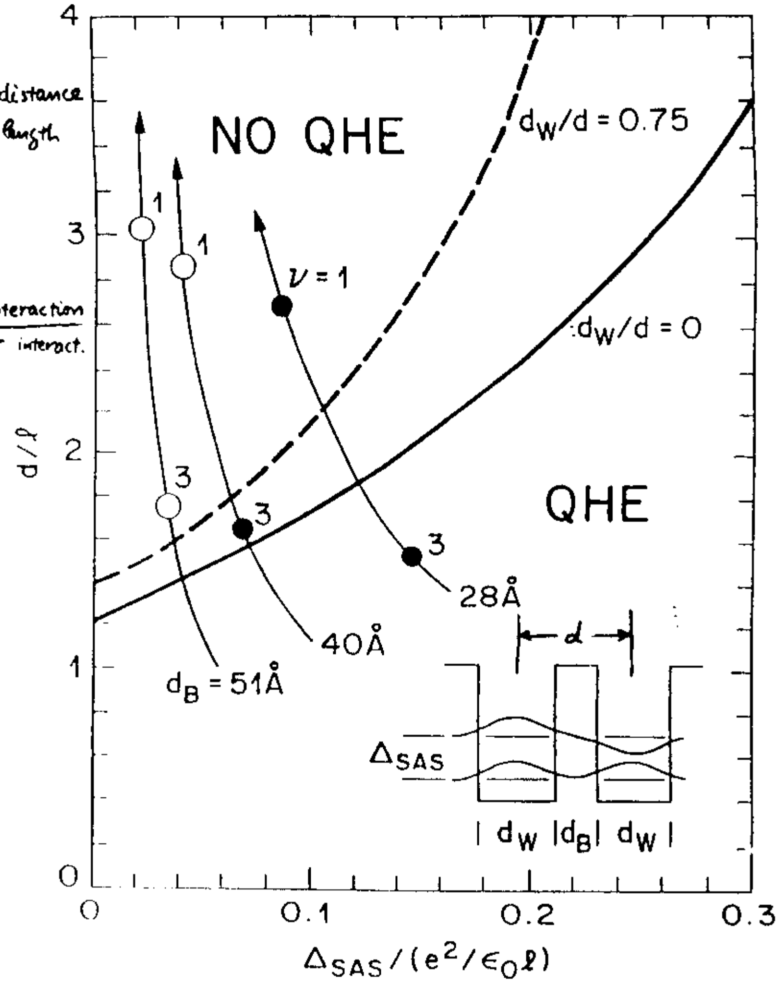




Explanation

d = interlayer distance
ℓ = magnetic length

$$\frac{d}{\ell} = \frac{e^2/\ell}{e^2/d} = \frac{\text{in-plane interaction}}{\text{interlayer interact.}}$$

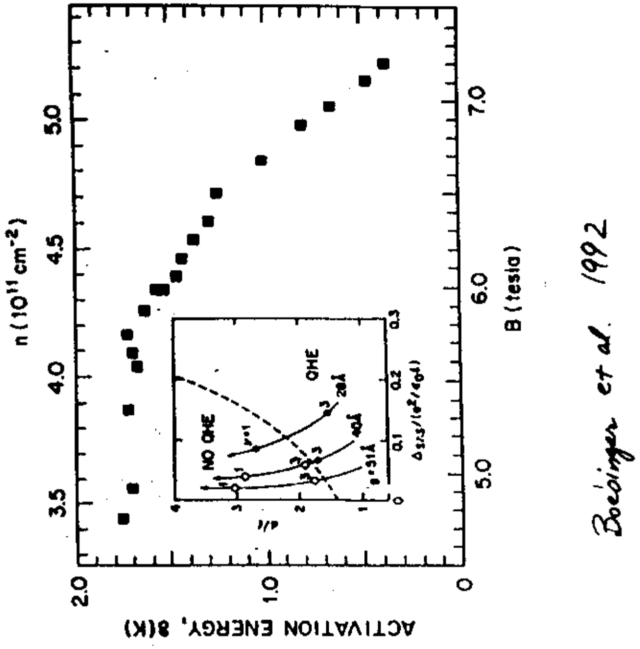


MacDonald, Platzman, Boebinger (1990) FIG ①

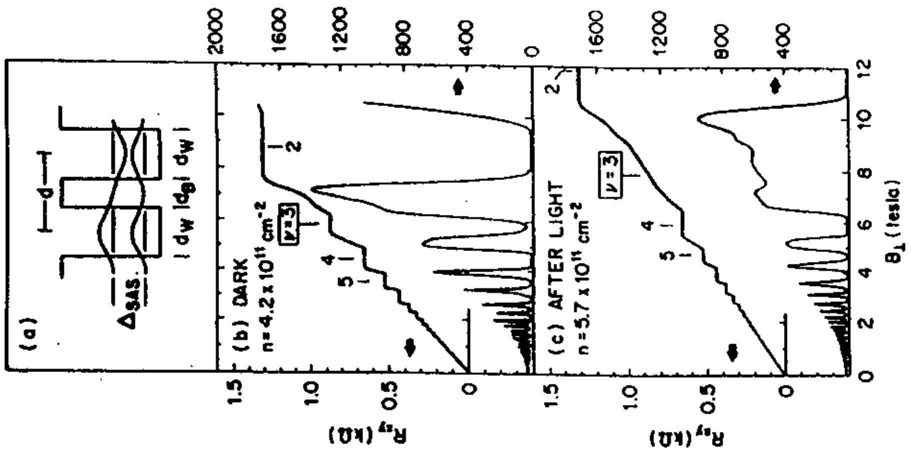
Also, see Brey, 1990

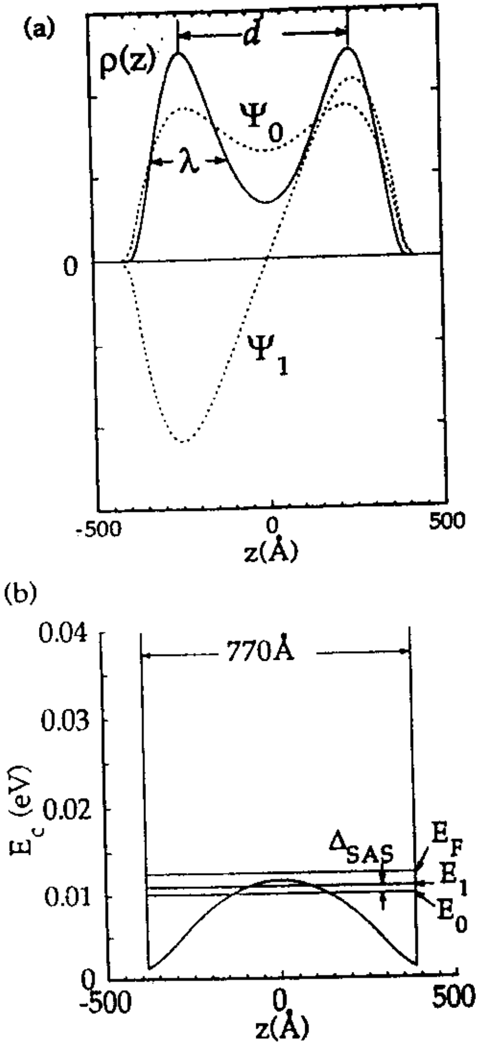
How to (slowly) kill $\nu = 3$ QHE !

raise density in DQW



Boebinger et al. 1992

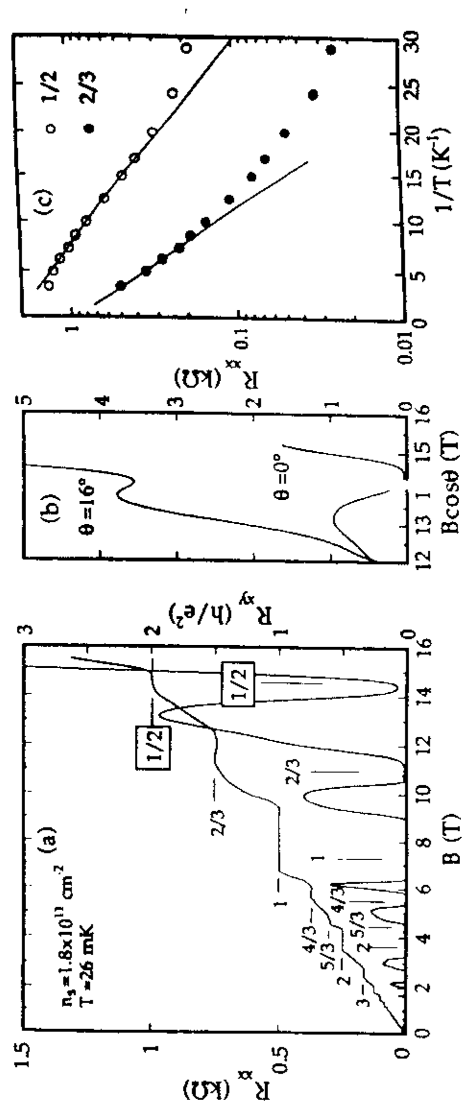




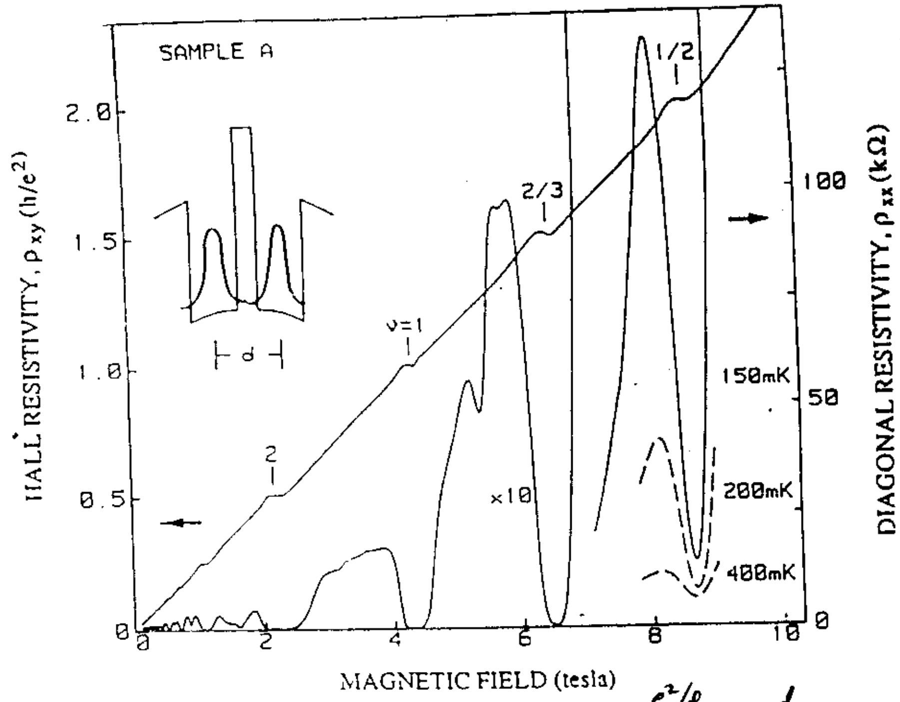
Suen, PhD thesis
1993

Figure 2.5: (a) The calculated wavefunctions of two occupied states, Ψ_0 and Ψ_1, the electron density distribution function $\rho(z)$, and (b) the conduction band edge E_c, for a 770 Å-wide well with $N_s \simeq 1 \times 10^{11}$ cm^{-2}.

An even-denominator $\nu = 1/2$ FQH state!



Suen et al. (1992a)



$$\frac{e^2/\ell_B}{e^2/d} = \frac{d}{\ell_B}$$

Sample	Barrier width (Å)	Density (10^{11} cm^{-2})	Mobility (10^6 cm^2/V-s)	$\Delta_{2/3}$ (K)	d/ℓ_B at $\nu = 1/2$	Strength of $\nu = 1/2$
A	31	1.04	0.5	1.7	2.4	strongest
B	31	1.29	1.5	2.3	2.7	strong
C	31	1.52	1.0	1.7	2.9	weak
D	99	1.31	0.5	2.8	3.6	absent

Eisenstein et al. (1992)

Fig 3

- *Explanation for $\nu = 1/2$ FQHE?*

1. *"Two-component" Laughlin trial wavefunction:*

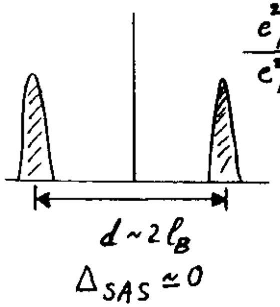
   (Halperin, 1983)

$$\Psi_{m m' n}(u_i, w_i) =$$

$$\prod_{i<j}(u_i - u_j)^m \prod_i \exp\left(-\frac{1}{4}|u_i|^2\right) \prod_{i<j}(w_i - w_j)^{m'} \prod_i \overset{\wedge}{\underset{\exp}{}}\left(-\frac{1}{4}|w_i|^2\right) \prod_{i,j}(u_i - w_j)^n$$

$\Rightarrow$ *The state $(m\,m'\,n) = (331)$ has filling $\nu = 1/2$ & can be a FQH ground state.*

$\left\{\begin{array}{l}\text{Rezaayi, Haldane (1987)} \\ \text{Yoshioka et al. (1989)} \\ \text{He et al. (1991, 93)}\end{array}\right.$



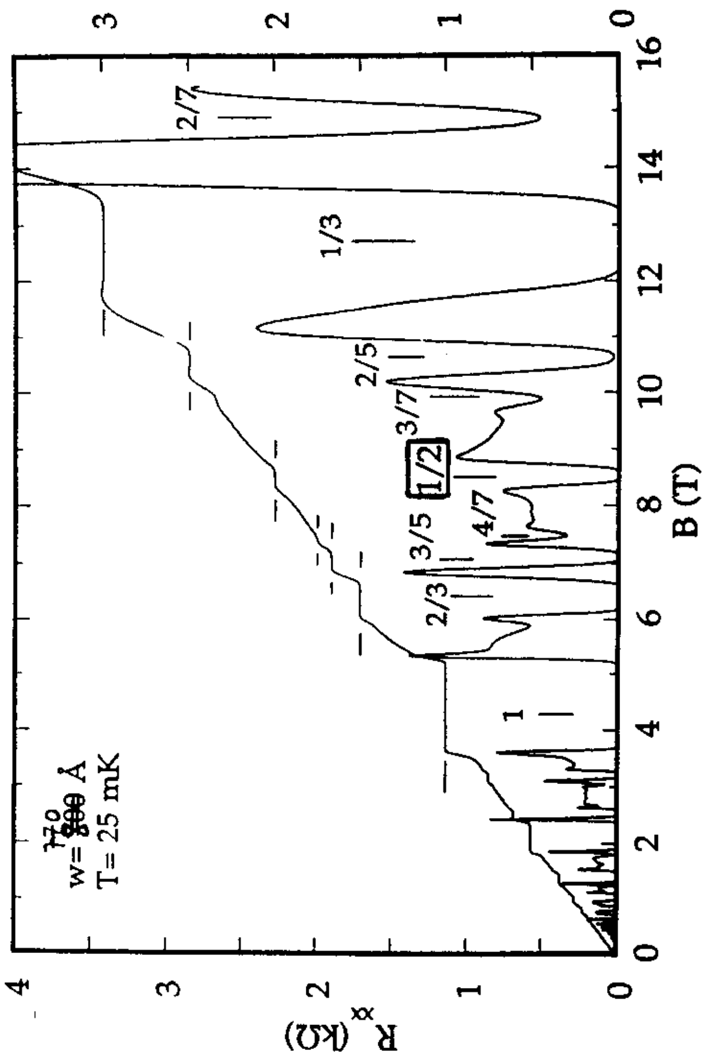
$$\frac{e^2/\ell_B}{e^2/d} = \frac{d}{\ell_B}$$

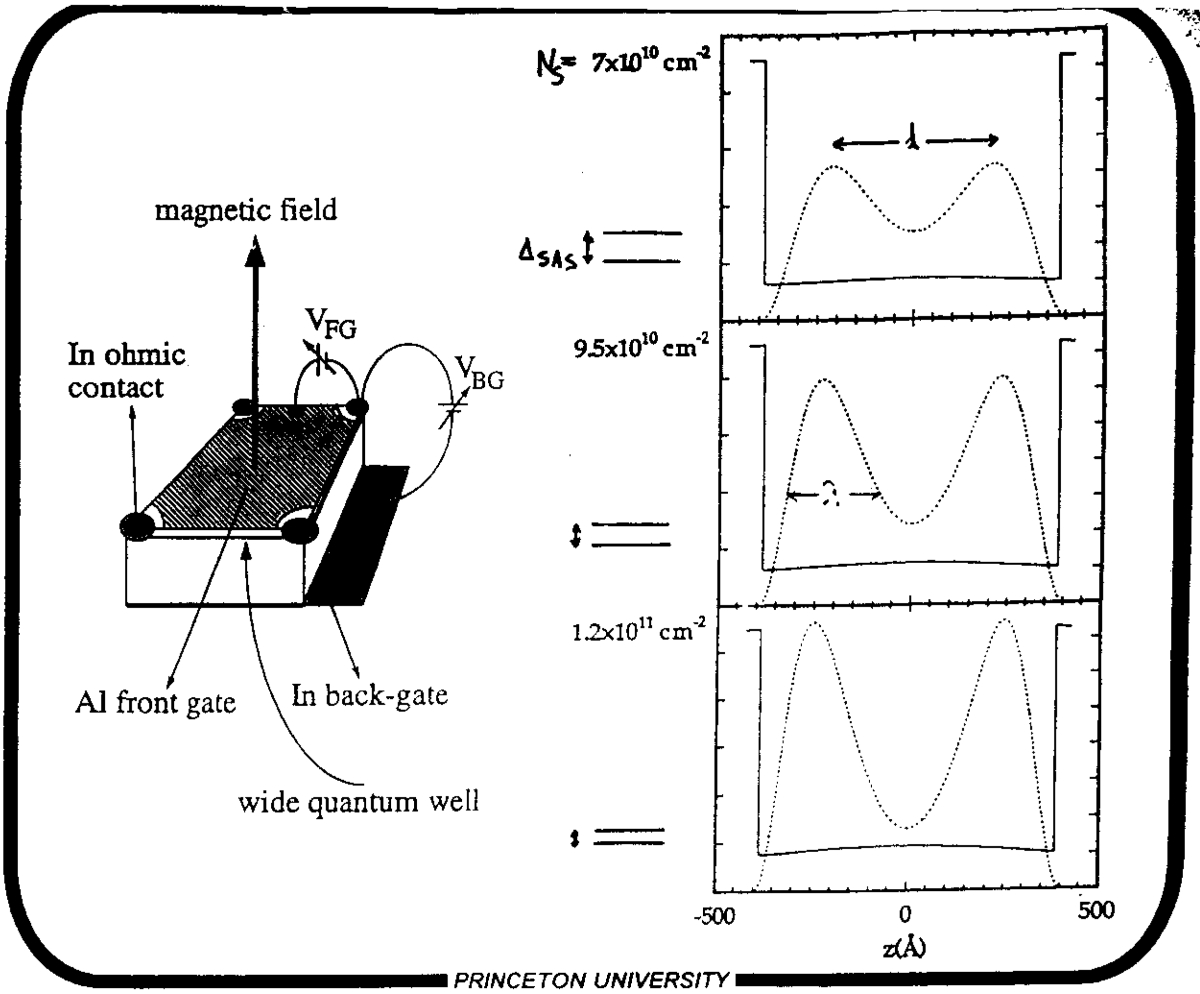
2. *"Paired Hall State"*

   (Greiter, Wen, Wilczek, 1991, 1992):

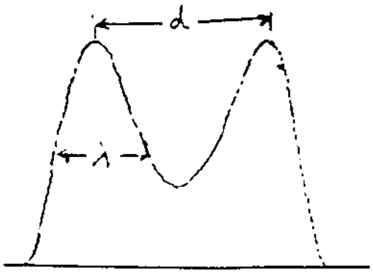
   - *Quasi-2D system with reasonably large thickness ($w_e/\ell_B \gtrsim 3$) can show $\nu = 1/2$ FQH state*
   - *effective reduction of short-range Coulomb repulsion $\Rightarrow$ "pairing"!*

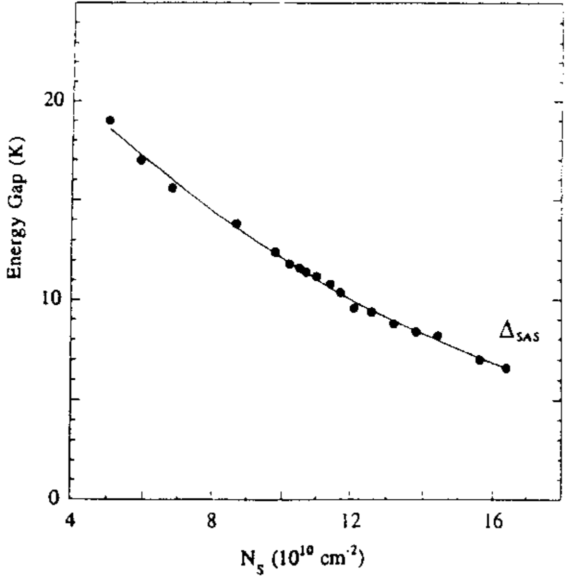
Origin of $\nu = \frac{1}{2}$ FQH state: One-component or two-component?
(in wide single QW's)

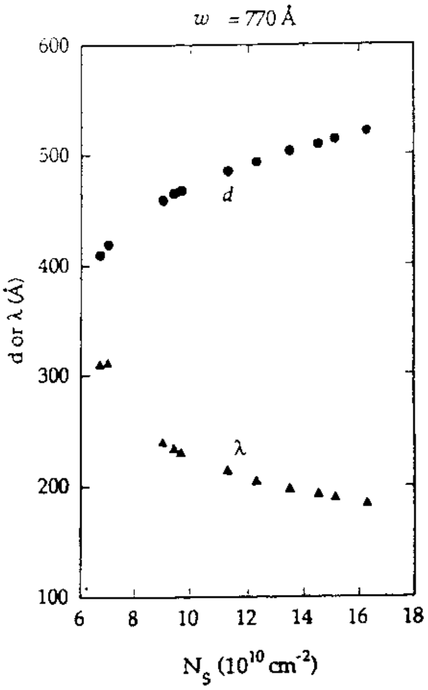


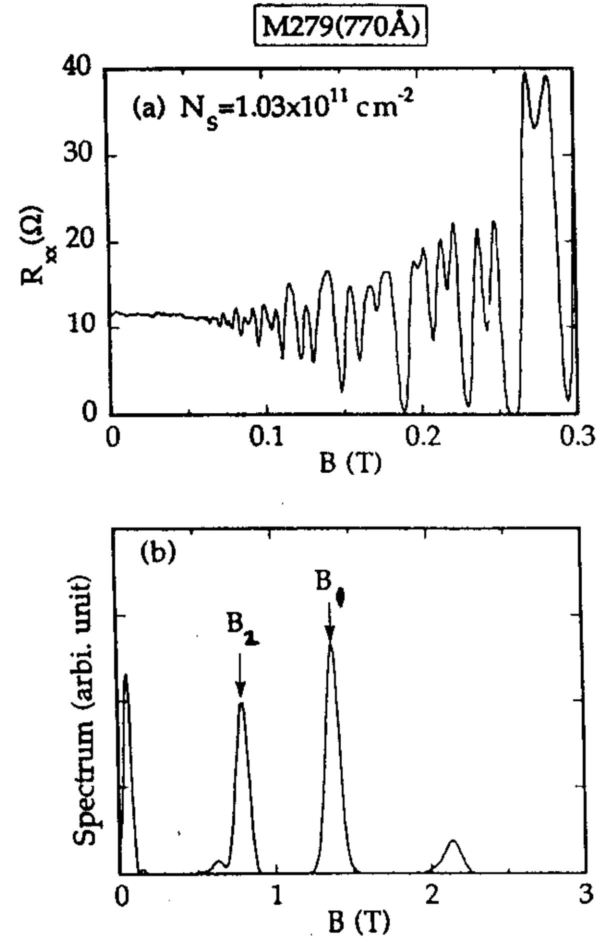


- Dependence of Δ_{SAS}, d, λ on N_s









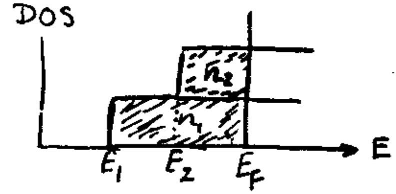
FFT of R_{xx} vs $\frac{1}{B}$

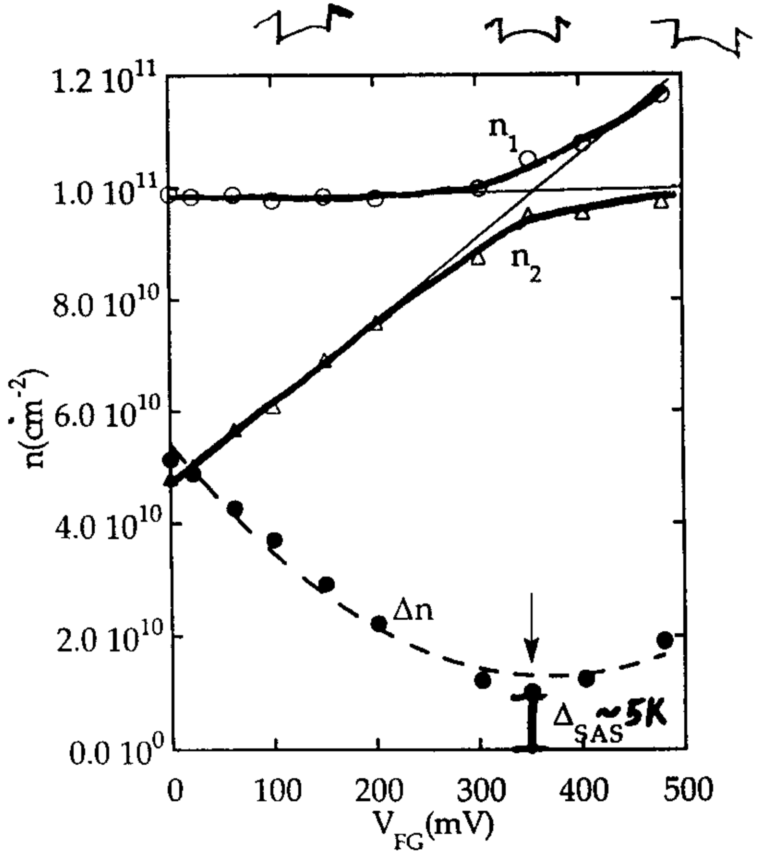
$n_i = 4.83\times10^{10}\ \mathrm{cm}^{-2} \times B_i$ (Tesla)

$\Rightarrow B_1 - B_2$ directly gives Δ_{SAS}

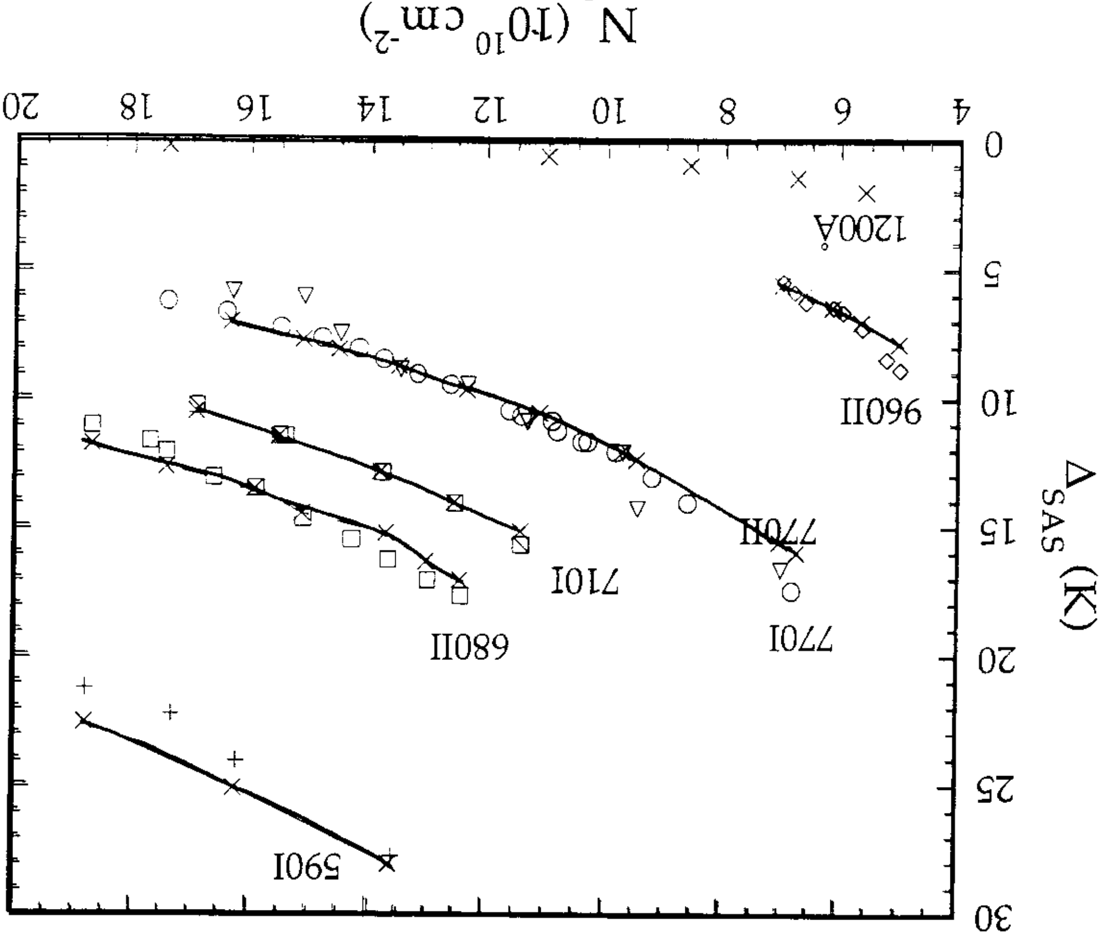
Suen, PhD thesis, 1993

Figure 2.3: (a) The low-magnetic-field Shubnikov-de Haas oscillations, and (b) the FFT spectrum of the oscillatory R_{xx} vs $1/B$ data.

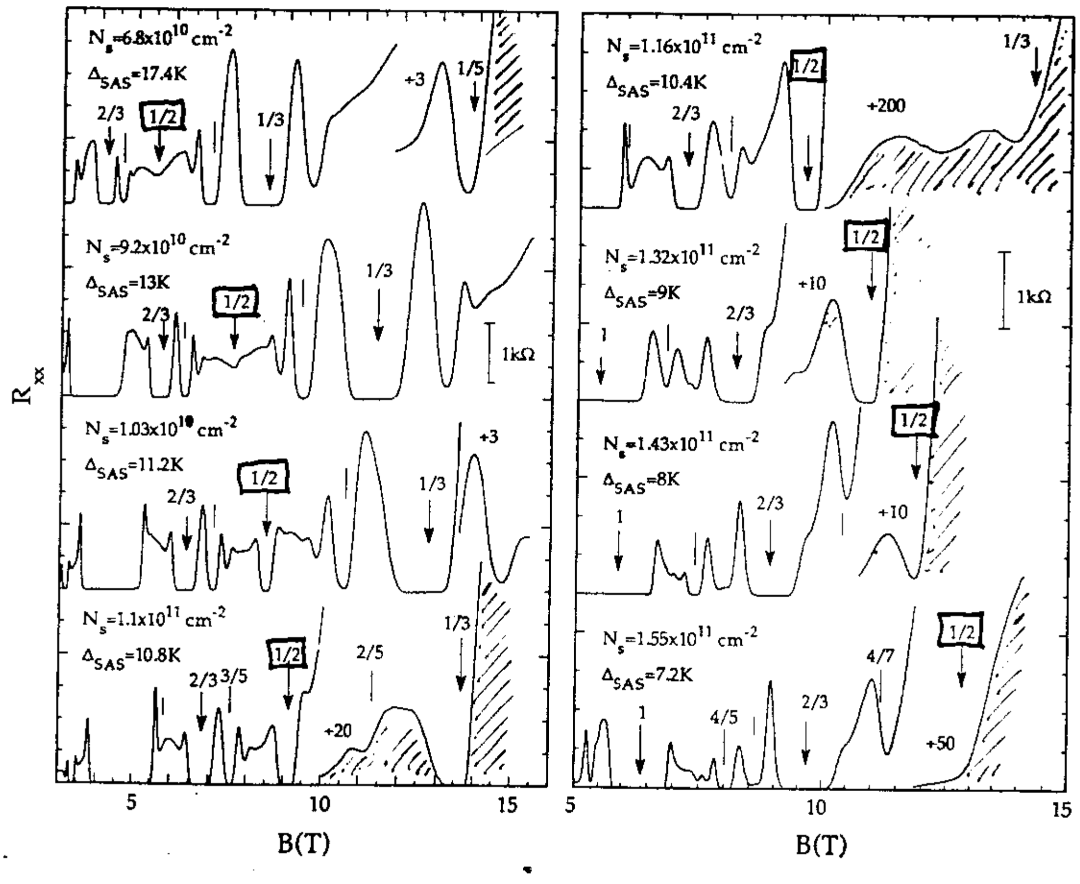




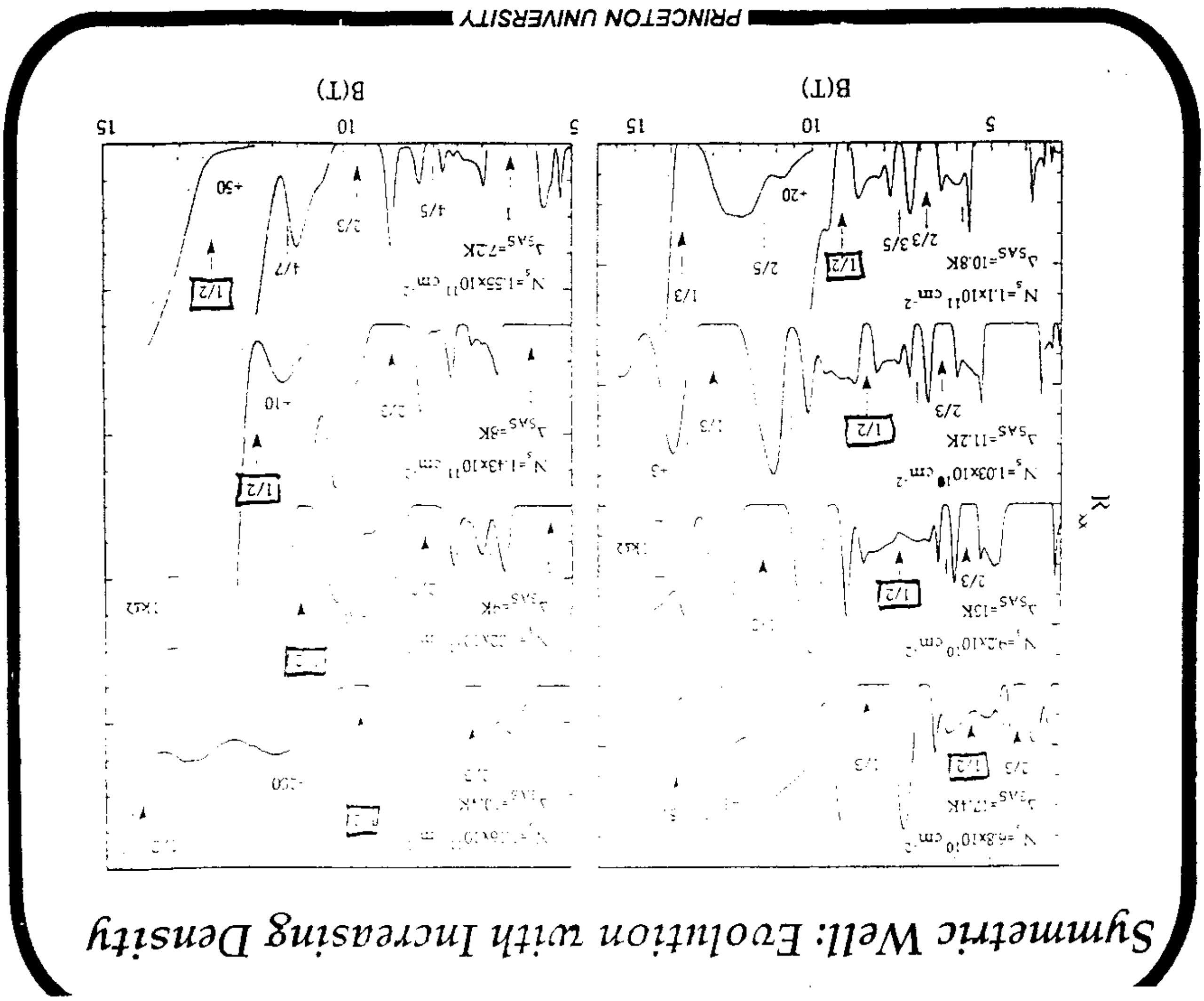
Pages 75 and 76 of Professor SHAYEGAN's LECTURE NOTES
SMR. 758/22



Symmetric Well: Evolution with Increasing Density

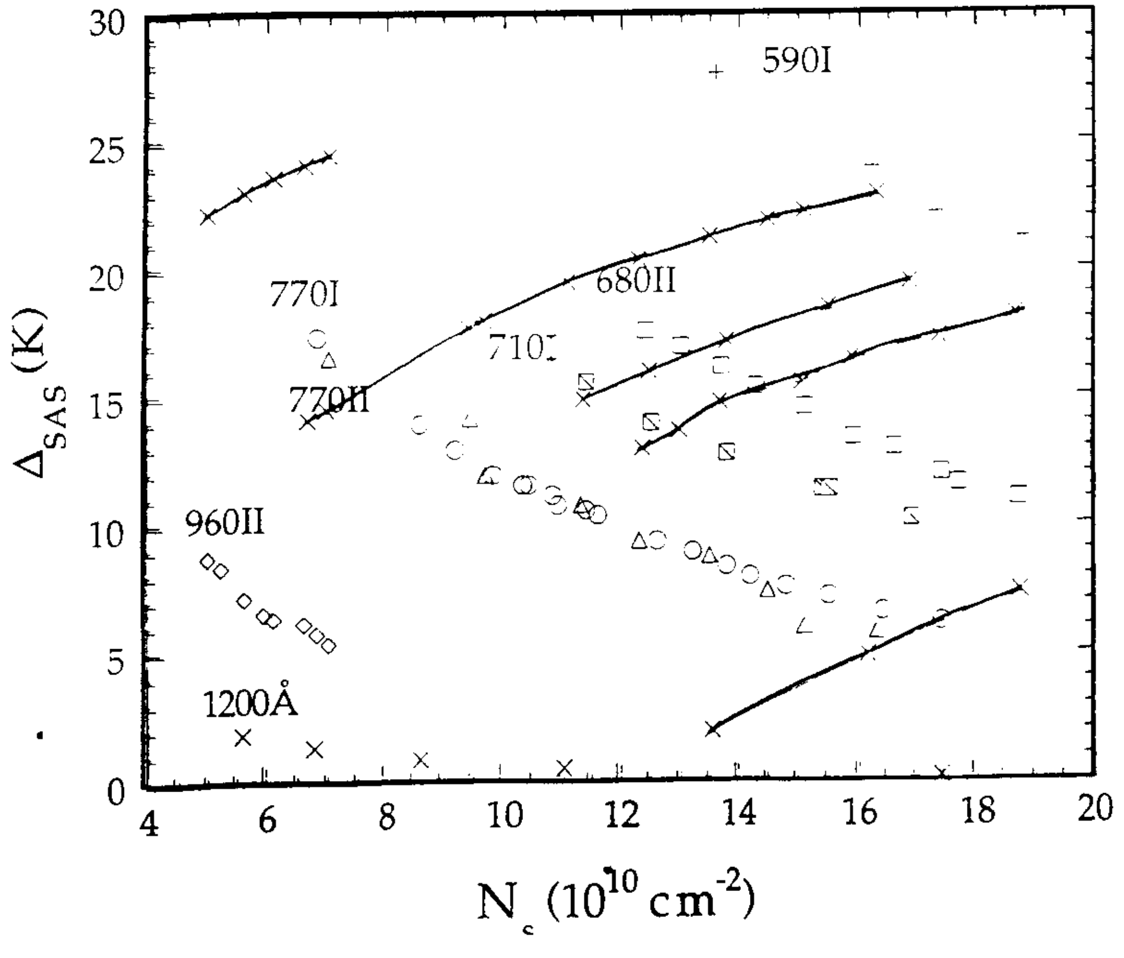


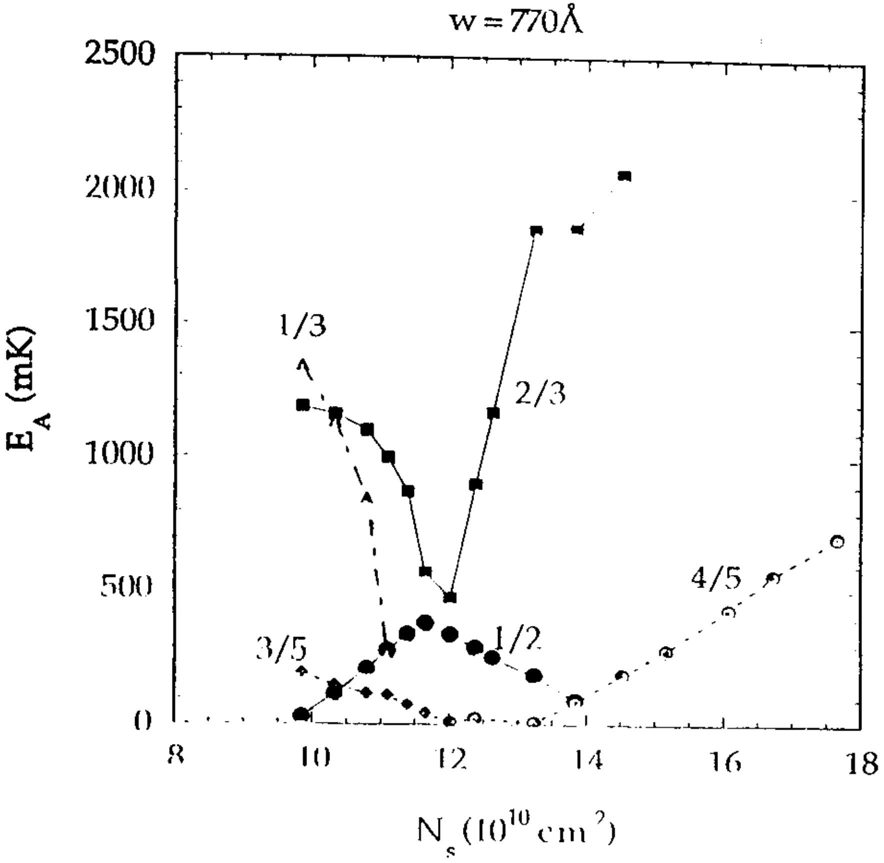
PRINCETON UNIVERSITY



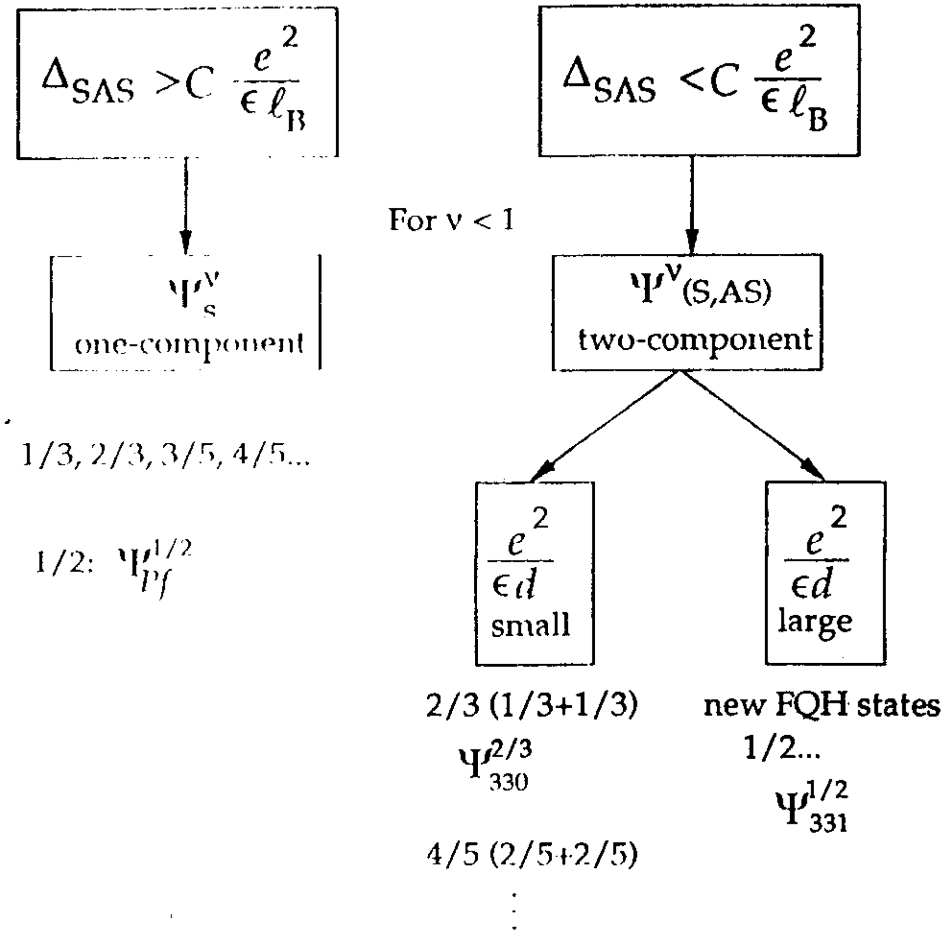
Symmetric Well: Evolution with Increasing Density

PRINCETON UNIVERSITY





What happens if Δ_{SAS} finite ?



$\Delta_{SAS} > C \frac{e^2}{\epsilon \ell_B}$

Ψ^{ν}_{s}
one-component

1/3, 2/3, 3/5, 4/5...

1/2: $\Psi^{1/2}_{Pf}$

For ν < 1

$\Delta_{SAS} < C \frac{e^2}{\epsilon \ell_B}$

Ψ^{ν}(S,AS)
two-component

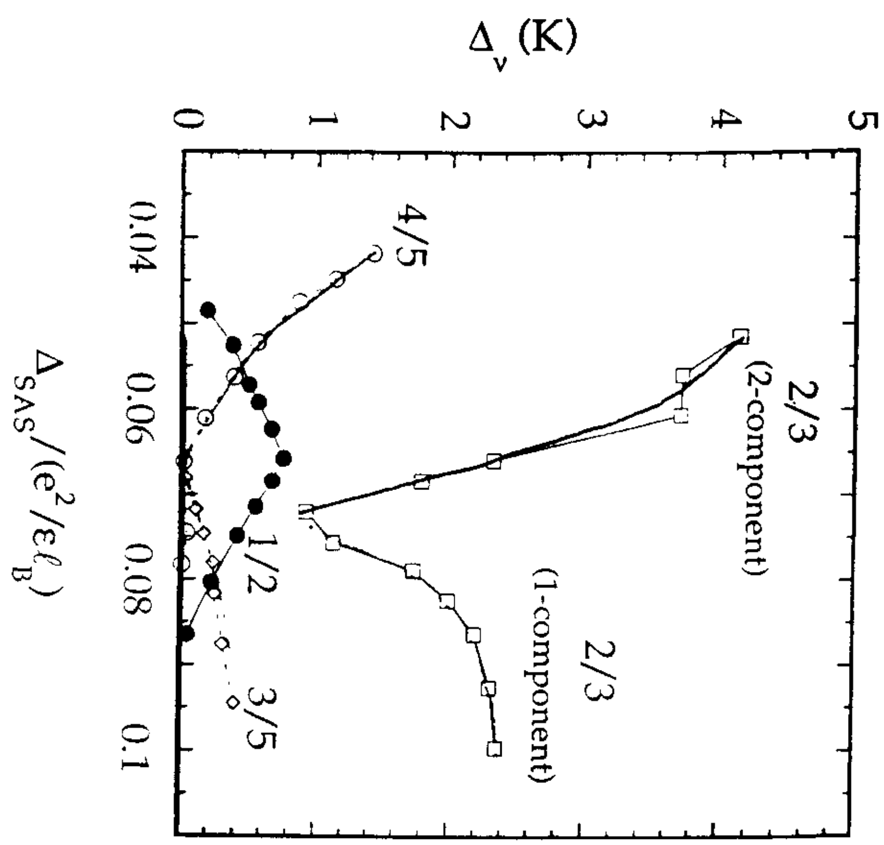
$\frac{e^2}{\epsilon d}$
small

2/3 (1/3+1/3)
$\Psi^{2/3}_{330}$

4/5 (2/5+2/5)
⋮

$\frac{e^2}{\epsilon d}$
large

new FQH states
1/2...
$\Psi^{1/2}_{331}$



UNBALANCING THE WELL (FIXED n_s)

ASYMMETRY CAUSES:

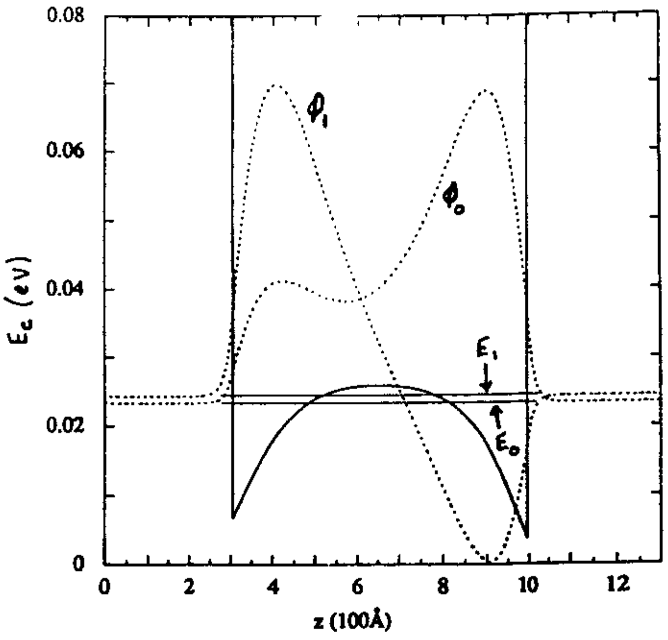
1) BOTH WAVEFUNCTIONS TO BECOME ASYMMETRIC

2) $E_1 - E_0 \equiv \Delta_{01} > \Delta_{SAS}$

ONE - COMPONENT STATES ↗

TWO - COMPONENT STATES ↘

SIMULATION



PRINCETON UNIVERSITY

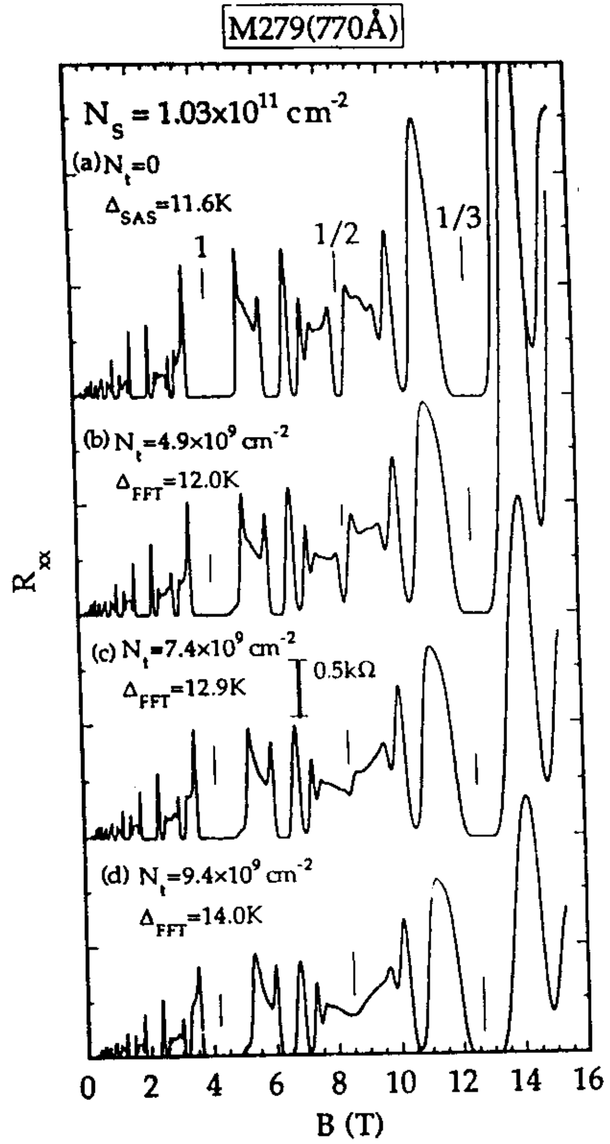
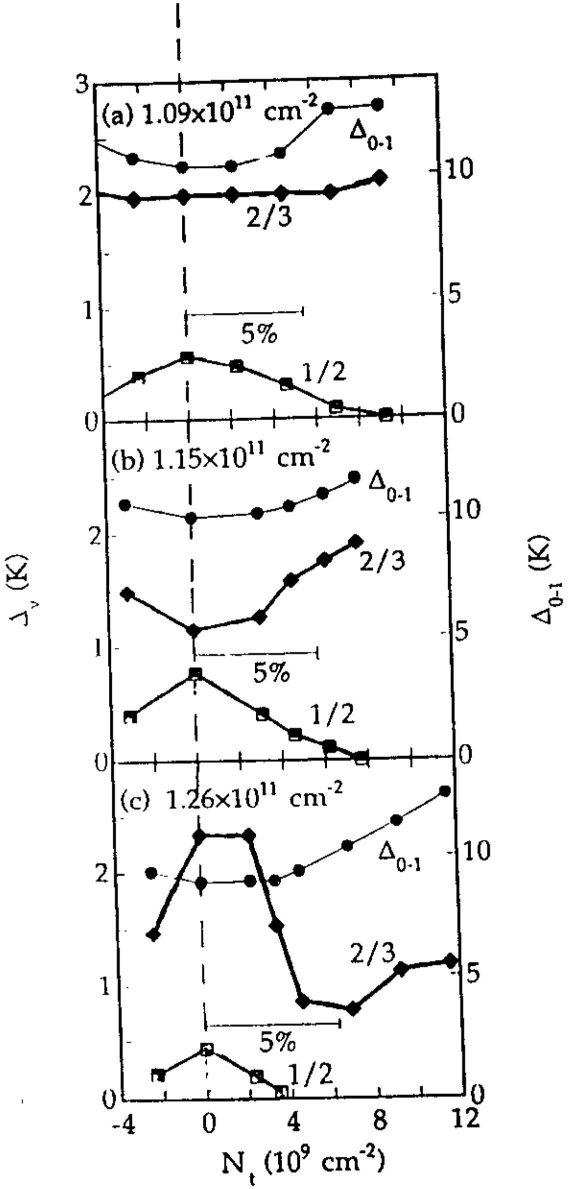
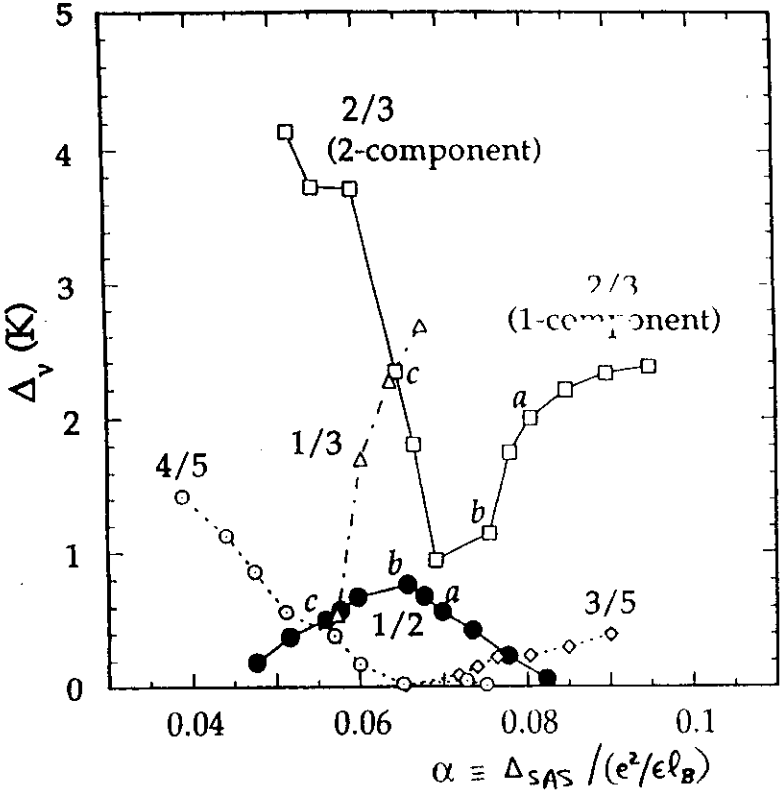


Figure 3.12: Low-temperature (~ 30 mK) R_{xx} data for different states along a constant-N_s ($= 1.03 \times 10^{11}$ cm^{-2}) line in the (V_{FG}, V_{BG}) parameter space for M279(770Å).

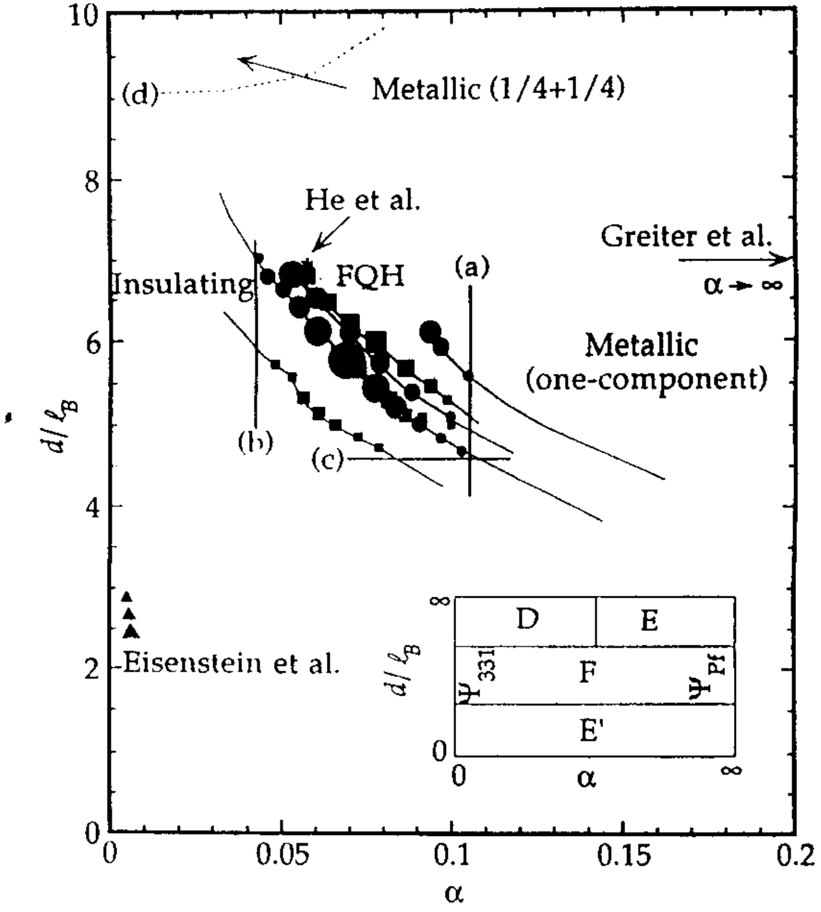


Suen, 1994b



Suen, 1994b

Summary of $\nu = 1/2$ FQH Data



Suen, 1994 b

FIG

SUMMARY

1) Observe $\nu = 1/2$ FQH state and one- to two-component transitions at the same energy scales.

2) Asymmetry effect at fixed density:

Δ_{0-1}, N_T ↑

1-component states ↑

2-component states ↓

(+ transition back to 1-component!)

$\frac{1}{2}$ state always ↓

↳ ∴ very sensitive to Δ_{0-1} and wavefunction symmetry

$$\Rightarrow \quad \boxed{\Psi_{331}^{1/2}}$$

$$\Rightarrow \quad \boxed{\Psi_{S}^{2/3} \xleftrightarrow{\text{DENSITY, ASYMMETRY}} \Psi_{330}^{2/3}}$$

{1} {2}

Finite-Temperature Phase Transition and Many-body Origin of $\upsilon = 1$ IQH state in Bilayer Electron Systems

T. S. Lay, Y. W. Suen, H. C. Manoharan,
X. Ying, M. B. Santos and M. Shayegan

Deparment of Electrical Engineering
Princeton University

APS Meeting, March 1994

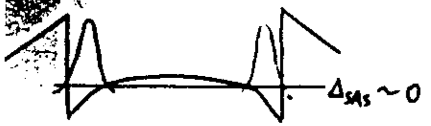
Also, Lay, 1994 (submitted)

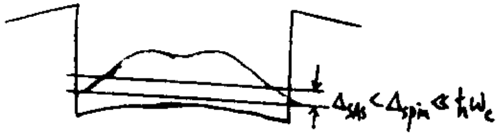
For exciting results & interpretation on $\nu = 1$ QHE in DQW's see:

Murphy, 1994

Yang, 1994

Two Limits





- Two isolated 2DES's in parallel (No tunneling)

$\Downarrow$

IQHE

at $\nu_{\text{each layer}} = 1, 2, 3 \dots$

$\Rightarrow \nu_{\text{total}} = 2, 4, 6 \dots$

FQHE

at $\nu_{\text{each layer}} = \frac{1}{3}, \frac{2}{5} \cdots \frac{3}{5}, \frac{2}{3}$

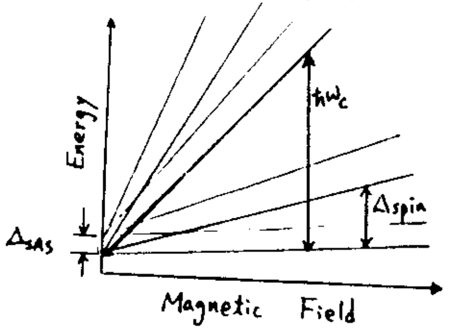
$\Rightarrow \nu_{\text{total}} = \frac{2}{3}, \frac{4}{5} \cdots \frac{6}{5}, \frac{4}{3}$

- One (Wide) 2DES

$\Downarrow$

IQHE

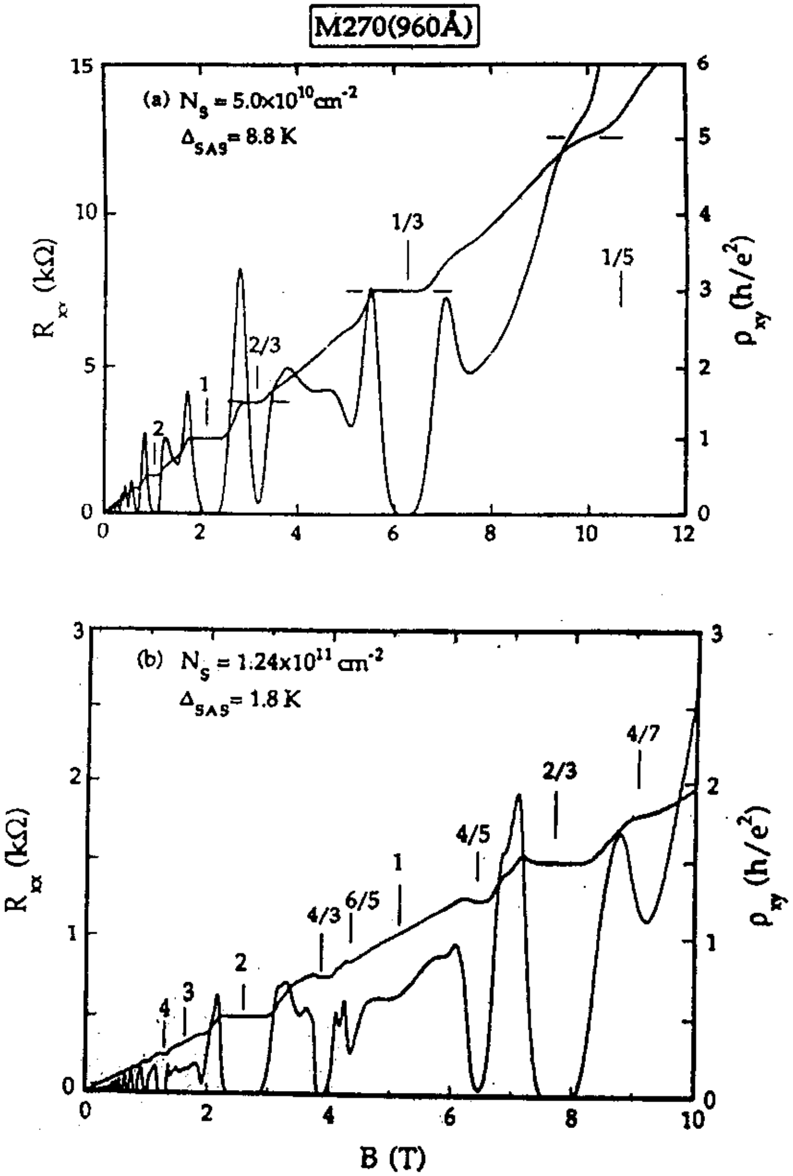
at $\nu_{\text{total}} = 1, 2, 3 \dots$



FQHE

at $\nu_{\text{total}} = \frac{1}{3}, \frac{2}{5} \cdots \frac{3}{5}, \frac{2}{3}$

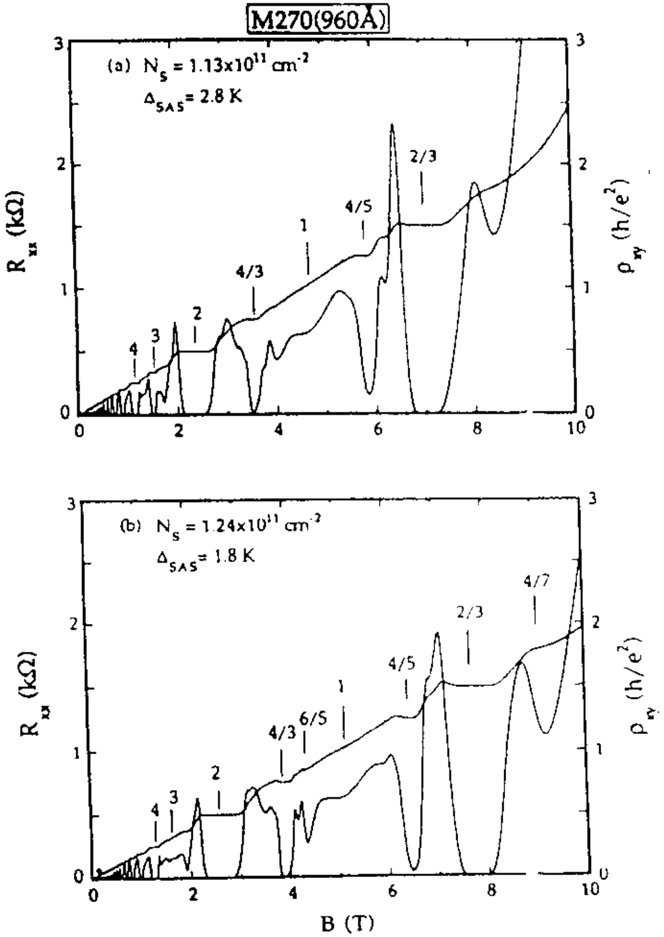
? evolution ?



Suen, Ph.D. thesis
1993

$\alpha = 0.025$

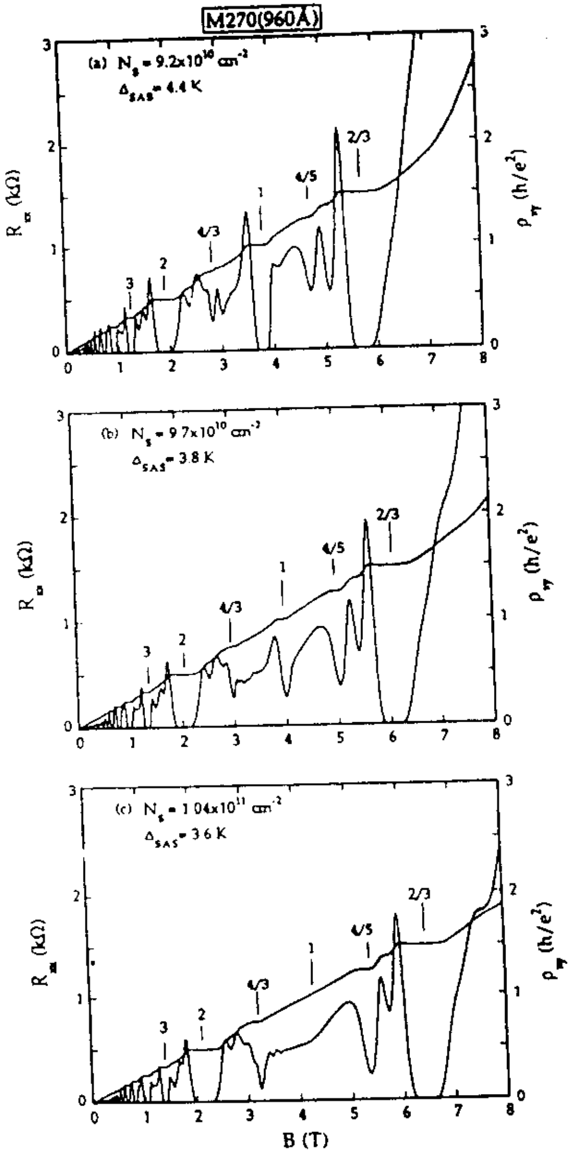
$\alpha = 0.016$



$\alpha = 0.044$

$\alpha = 0.037$

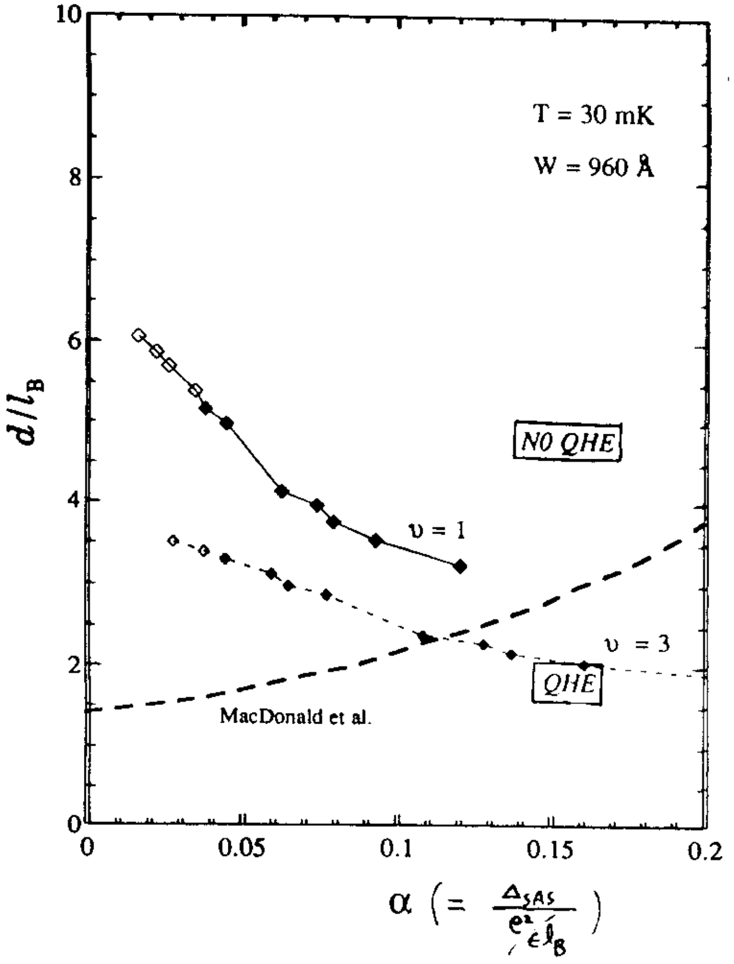
$\alpha = 0.034$

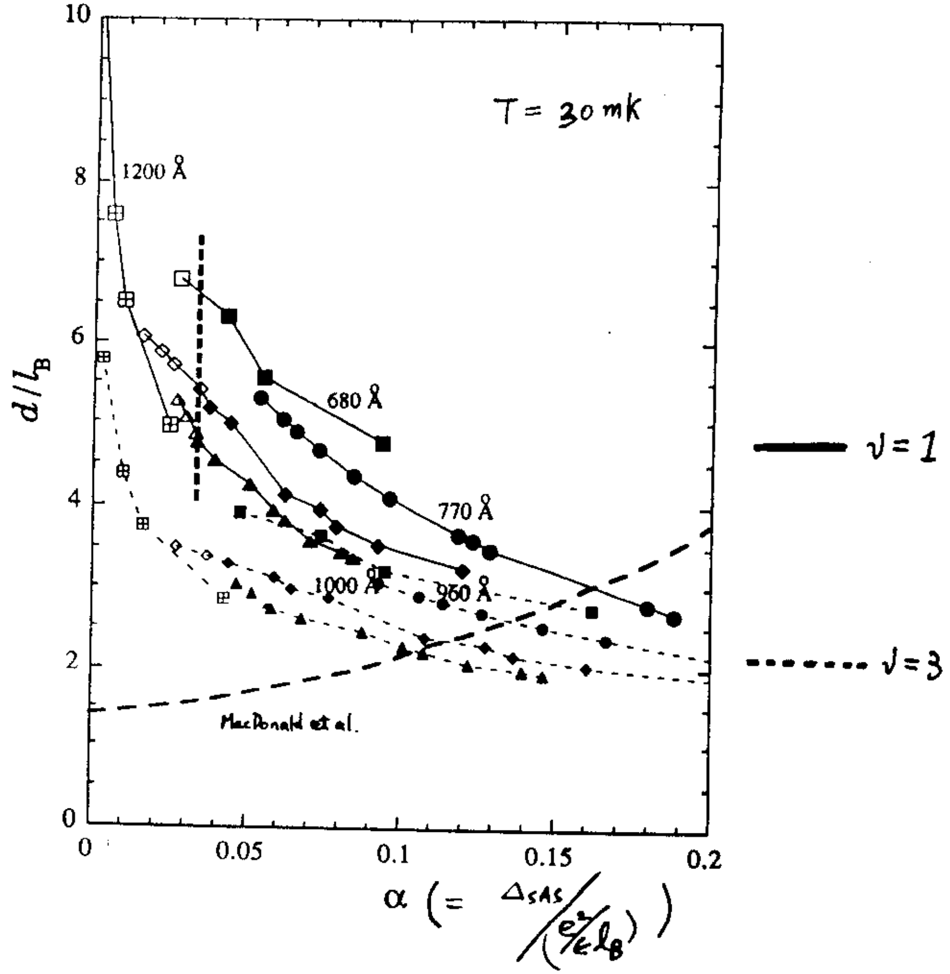


$$\alpha = \frac{\Delta_{SAS}}{e^2/\epsilon l_B} = \frac{\text{tunneling gap}}{\text{intralayer interaction}} \;; \qquad l_B = \sqrt{\hbar/eB}$$

(magnetic length)

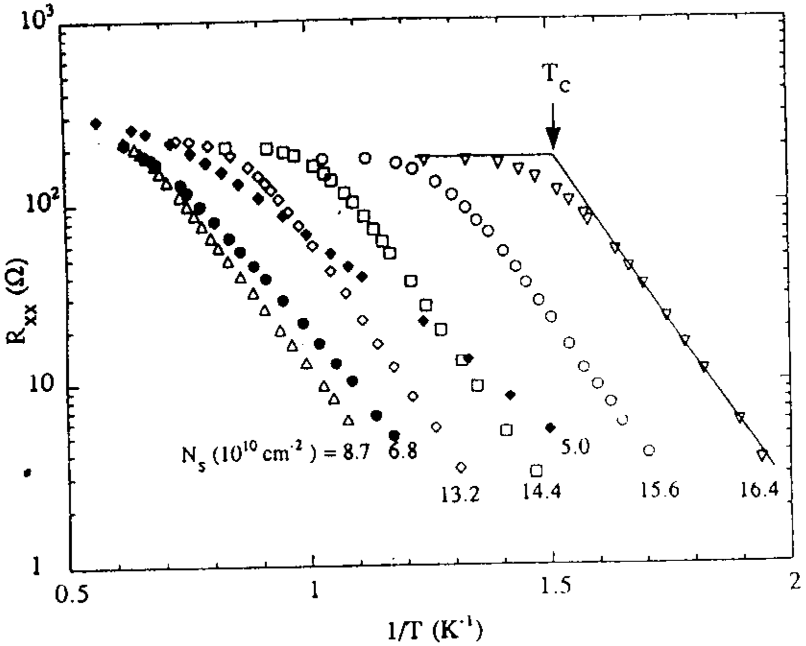
$$\frac{d}{l_B} = \frac{e^2/\epsilon l_B}{e^2/\epsilon d} = \frac{\text{intralayer interaction}}{\text{interlayer interaction}}$$





Observation :

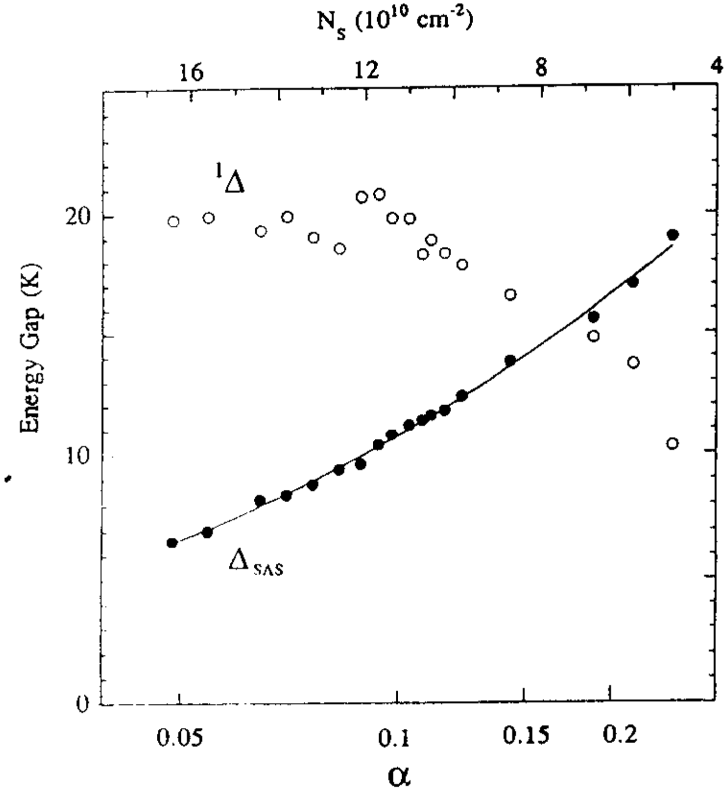
QHE in "forbidden" region



- For large N_s (small α):
  - --- "sharp" end to activated behavior above T_c.
  - --- $T_c \approx 1K \ll {}^1\Delta \approx 20K$.
  - --- T_c depends on N_s while ${}^1\Delta$ does not.

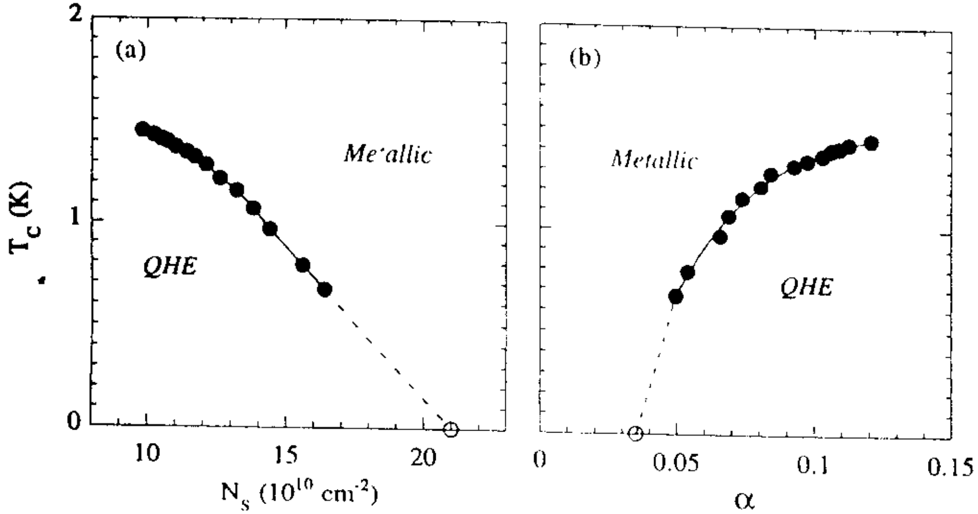
- For small N_s (large α):
  - --- No clear T_c.
  - --- ${}^1\Delta$ starts to drop.

W = 770 Å

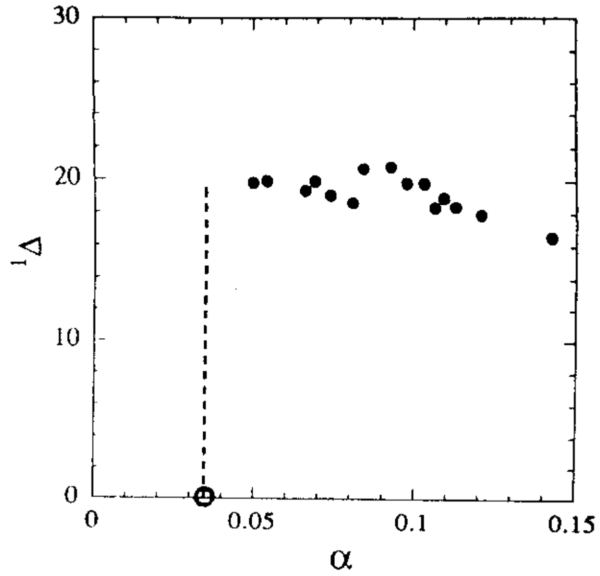


- Measured Δ_{SAS} is in excellent agreement with calculated result.

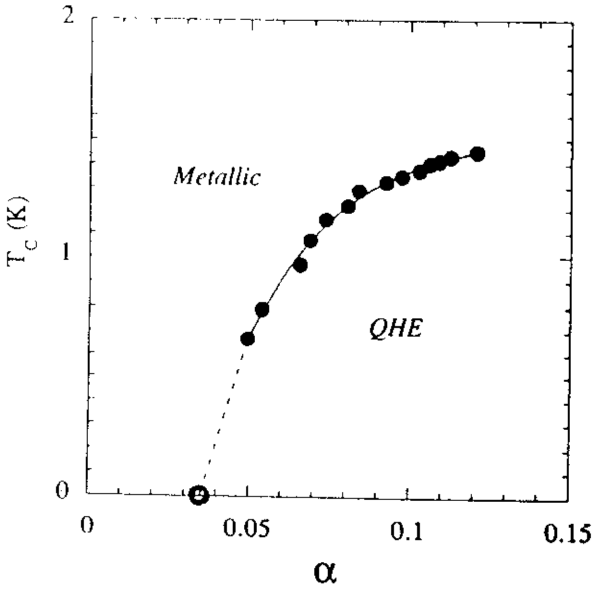
- For small α (large N_s):
  - --- ${}^1\Delta$ is independent of α (or N_s).
  - --- ${}^1\Delta$ is much larger than Δ_{SAS}.



(a)
(b)
T_c (K)
2
1
0
QHE
Me'allic
10
15
20
N_s (10^{10} cm^{-2})
Metallic
QHE
0
0.05
0.1
0.15
α



Δ
30
20
10
0
0
0.05
0.1
0.15
α



T_c (K)
2
1
0
Metallic
QHE
0
0.05
0.1
0.15
α

Summary

1. Many-body QHE at $\nu = 1$ in "forbidden" region:

- ${}^1\Delta \approx 3\,\Delta_{SAS}$
- ${}^1\Delta \gg T_C$
- two-component state?

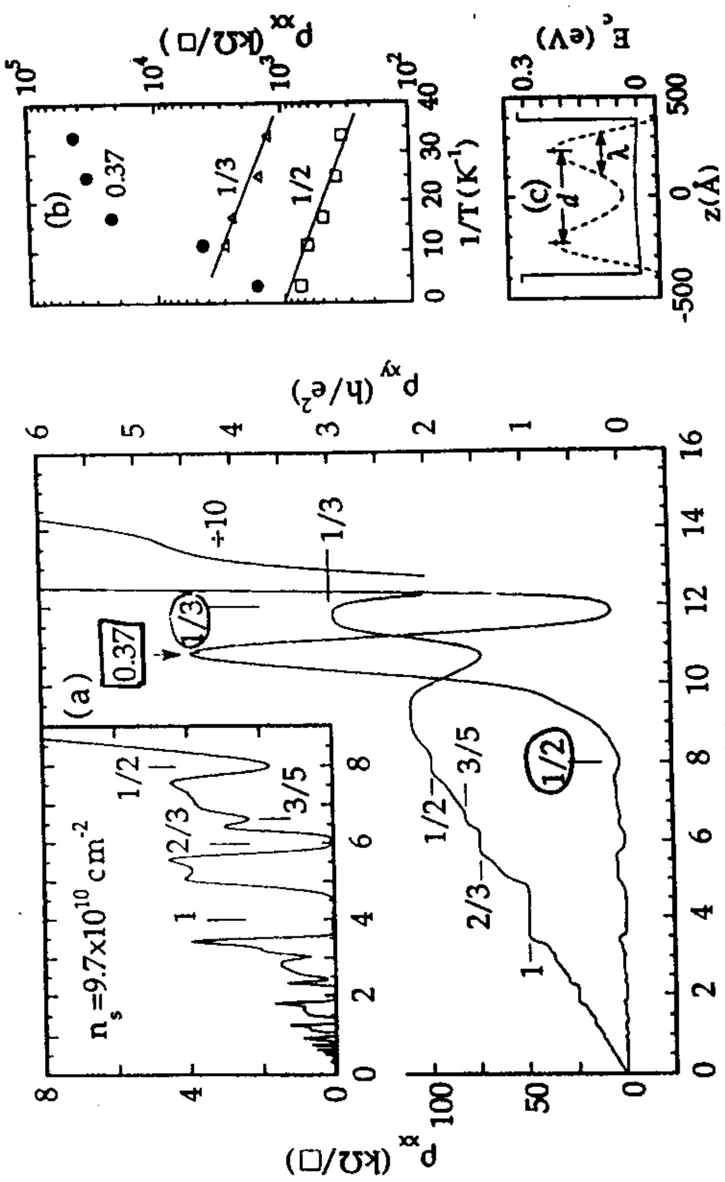
$$\Psi_{mm'n} = \prod_{i<j}(u_i-u_j)^m \prod_{i<j}(w_i-w_j)^{m'} \prod_{i,j}(u_i-w_j)^n \cdot \exp\left\{-\sum_i \frac{|u_i|^2}{4l_B^2} - \sum_i \frac{|w_i|^2}{4l_B^2}\right\}$$

$m = m' = n = 1 \Rightarrow \Psi_{111}$: QHE at $\nu = 1$

2. Finite-temperature QHE ⟶ metallic state transition:

- Kosterlitz-Thouless transition?
  - -- Theoretically proposed for bilayer system with $d > 0$ and $\Delta_{SAS} = 0$ (Wen and Zee; Yang et al.)
  - -- Estimated T_{KT} qualitatively consistent with our T_C
  - -- Problems: (i) $\Delta_{SAS} > 0$ expected to destroy KT transition
    (ii) not clear how R_{xx} vs T should behave near T_{KT}.
- Ψ_{111} (ground state) $\xrightarrow{T\uparrow}$ metallic state (larger entropy)?
  - --- The closer to the $\alpha = 0.035$ boundary, the lower the temperature to destroy Ψ_{111} state.

(related talks in session M2, Thursday)

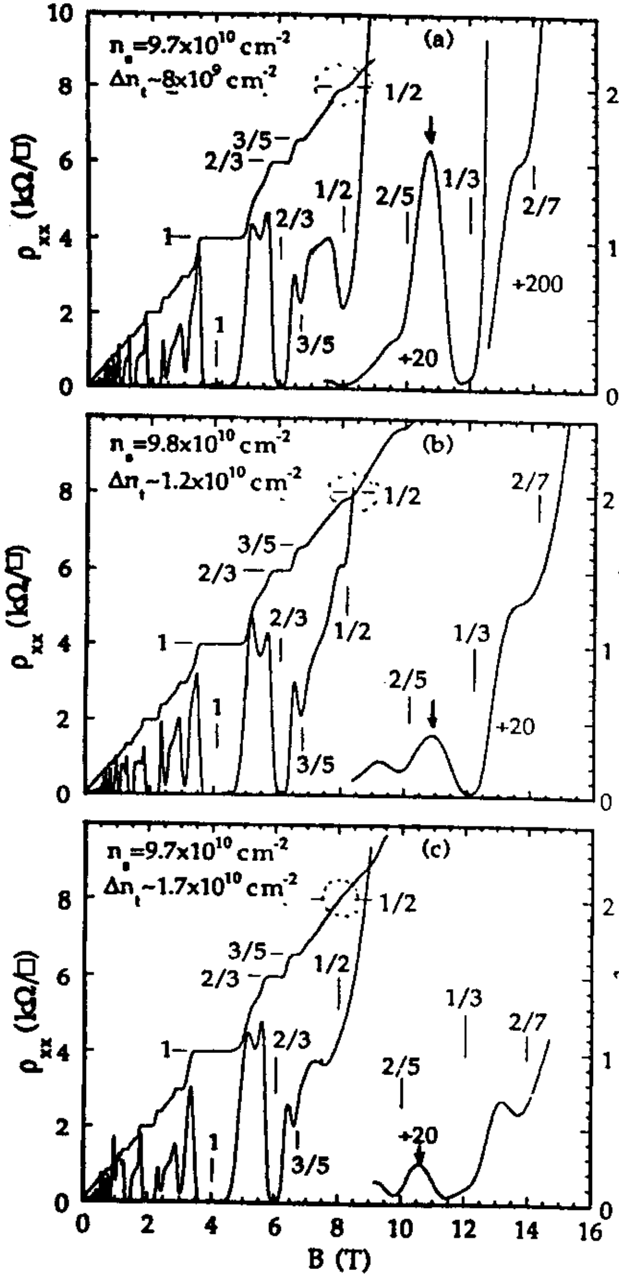


Insulating phase @ $\nu = 0.37$

FQH states @ $\nu = \frac{3}{5}, \frac{1}{2}, \frac{1}{3}$

Suen et al. 1992 b

Dependence of Asymmetry

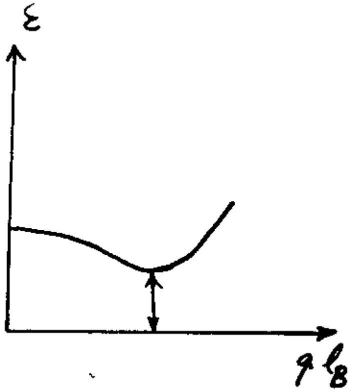


$n_L \sim 6.1 \times 10^{10}\,\mathrm{cm}^{-2}$

$n_R \sim 3.7 \times 10^{10}\,\mathrm{cm}^{-2}$

- IP for electrons in wide GaAs wells:
  - "reentrant" e.g., around $\nu = 1/3$ or $\nu = 1/2$ FQHL
  - nonlinear I-V
  - disappears in asymmetric systems
  - disappears as n_s is <u>lowered</u> !

- Explanation "?"
  - Oji, MacDonald, Girvin ('87)
    collapse of the energy gap at the magnetoroton minimum in a multilayer system
    => Wigner crystal ?



  - Disorder induced ? NO!