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SPRING COLLEGE IN CONDENSED MATTER
ON QUANTUM PHASES
(3 May - 10 June 1994)

QUANTUM PHASE TRANSITIONS AND QUENCHED DISORDER

PART III

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QUANTUM PHASE-TRANSITIONS AND QUENCHED DISORDER

OUTLINE

1. EXPERIMENTAL MOTIVATION

   $La_{2-x}Sr_xCuO_4$

   Si:P

2. SIMPLEST QUANTUM MODELS WITH INTERACTIONS AND DISORDER

   QUANTUM ROTORS

   TRANSVERSE-FIELD ISING

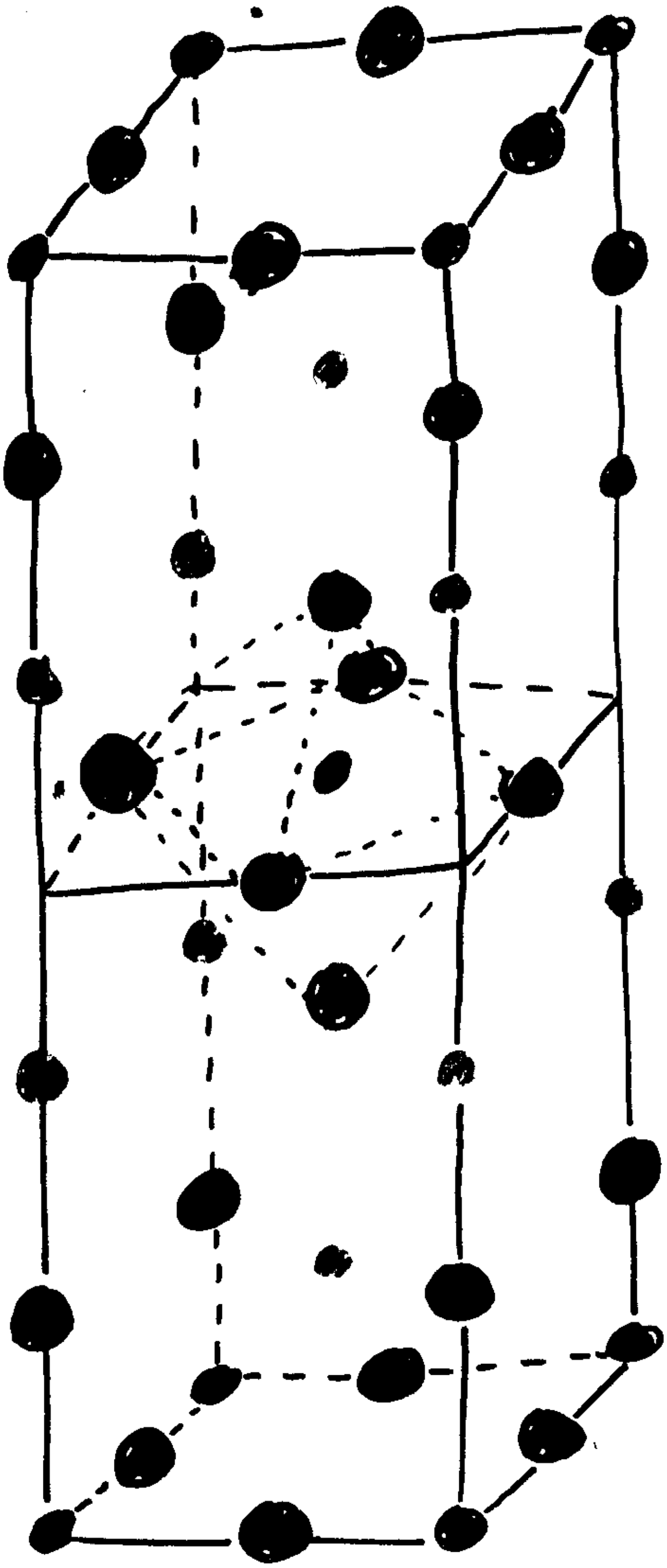
   POSSIBLE PHASES

3. $\nu \geq 2/d$

4. GRIFFITHS SINGULARITIES

5. FIELD THEORY FOR MAGNETIC ORDER — SPIN FLUID TRANSITION

6. FIELD THEORY FOR SPIN-GLASS — SPIN-FLUID TRANSITION [N. READ J. YE]



La

O

Cu

STRUCTURE OF La_2CuO_4

$La \rightarrow [Xe]5d^1 6s^2$

$Cu \rightarrow [Ar]3d^{10} 4s^1$

$O \rightarrow [He]2s^2 2p^4$

$$\mathcal{H} = \sum_{ij} J \vec{S}_i \cdot \vec{S}_j$$

tively large magnetic correlation length, which greatly facilitates the neutron experiments, makes the transition regime $0.015 \leq x \leq 0.05$ an ideal testing ground for the role of magnetic fluctuations in the normal state of the copper-oxide superconductors. Based on a specific microscopic model, Aharony *et al.*[9] predicted that the spin system in this concentration regime would exhibit canonical spin-glass behavior. Recent experiments[10] have supported this prediction. Hence, this concentration regime

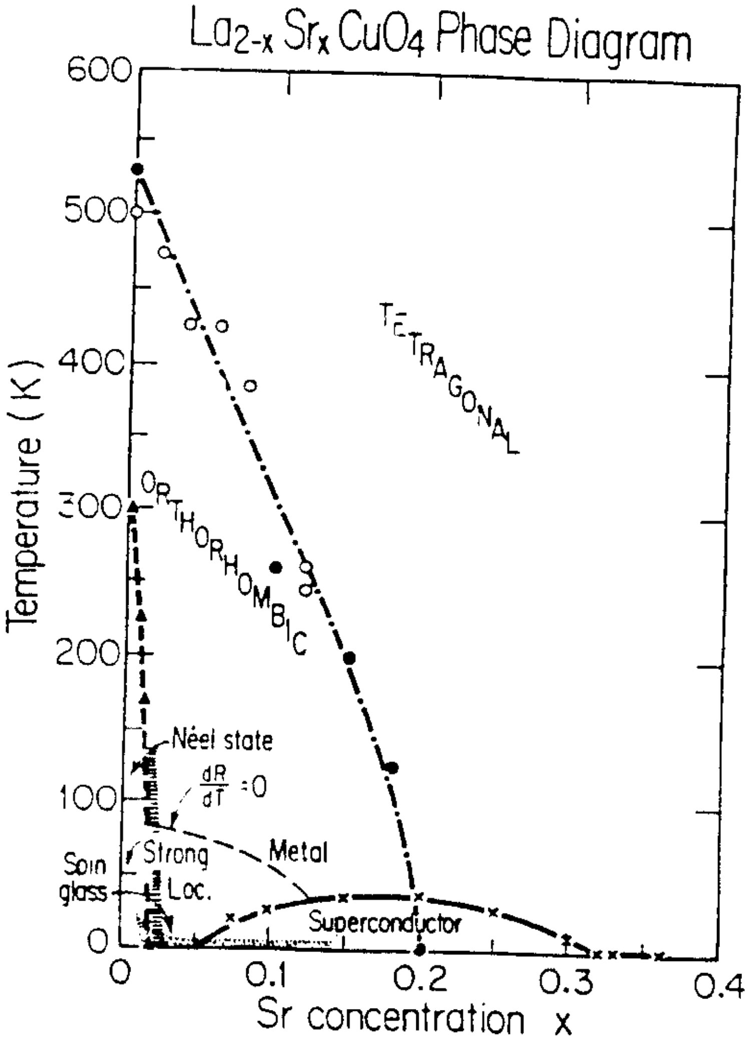


FIG. 1. Phase diagram for $La_{2-x}Sr_xCuO_4$ summarizing structural, magnetic, and transport properties. The narrow dashed line ($dR/dT=0$) separates the region of metallic linear resistance from that of logarithmically increasing resistance. The conductance in the Néel state is strongly localized. The nature and the extent of the "spin-glass" phase require further investigation.

free La_2CuO
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mobile holes
correlations.
placement o
those of elec
pounds; furt
capable of e
effects of hol
that a temper
into the syste
the inverse m
of the inverse
and the inve
$\kappa(x,T)=\kappa(x,$
magnetic cor
inconsistent
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simple scaling
of $La_{1.96}S$
variable ω/T
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universal cur
able effect ove
$(0.75 \leq \omega \leq 45$
scaling functi
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below ~20 K
become appare
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spectrum. A
the spin distor
ready quite l
speculative at t

At the end
scattering of

Doped Cuprates:

Neutron scattering studies local dynamic spin correlations. From the neutron scattering cross-section we can deduce the dynamic susceptibility $\chi_L(\omega)$

$$\chi''_L(\omega) = \sum_n \frac{e^{-E_n/k_BT}}{Z} |\langle m|S_+(i)|n\rangle|^2 [\delta(\hbar\omega - E_m + E_n) \\ -\delta(\hbar\omega - E_n + E_m)]$$

$\hbar\omega\chi''_L(\omega)$ is the energy dissipation rate of a *local* field $h_i e^{i\omega t}$ on site i

The imaginary part of the dynamic susceptibility was found to obey a scaling form (Keimer *et. al.*, Varma *et. al.*)

$$\chi''_L(\omega) = I(\omega)F\left(\frac{\hbar\omega}{k_BT}\right)$$

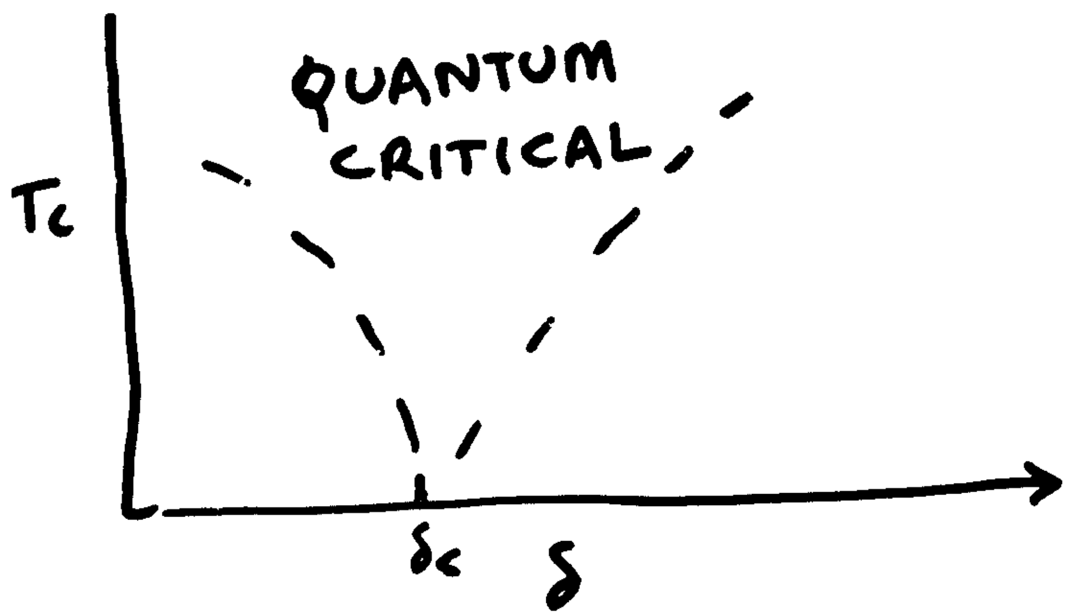
Our theory predicts that

$$I(\omega) = \mathrm{sgn}(\omega)|\omega|^\mu$$

this fits experiments with $\mu \approx -0.4$.

$\mu = \frac{\eta}{z}$. Negative value

→ disorder is important



IN QUANTUM-CRITICAL REGION

$$\chi(q,\omega) = \frac{C_2}{T^{(2-\eta)/z}} \Phi\left(\frac{c_1 q}{T^{1/z}}, \frac{\hbar\omega}{k_B T}\right)$$

$c_1, c_2 \to$ NON-UNIVERSAL CONSTANTS

$$\chi_L''(\omega) = \int \frac{d^d q}{(2\pi)^d} \chi''(q,\omega)$$

$$= \mathrm{sgn}(\omega)\, |\omega|^{(d-2+\eta)/z} F\left(\frac{\hbar\omega}{k_B T}\right)$$

q integral $\to$ on shell $\to$ convergent

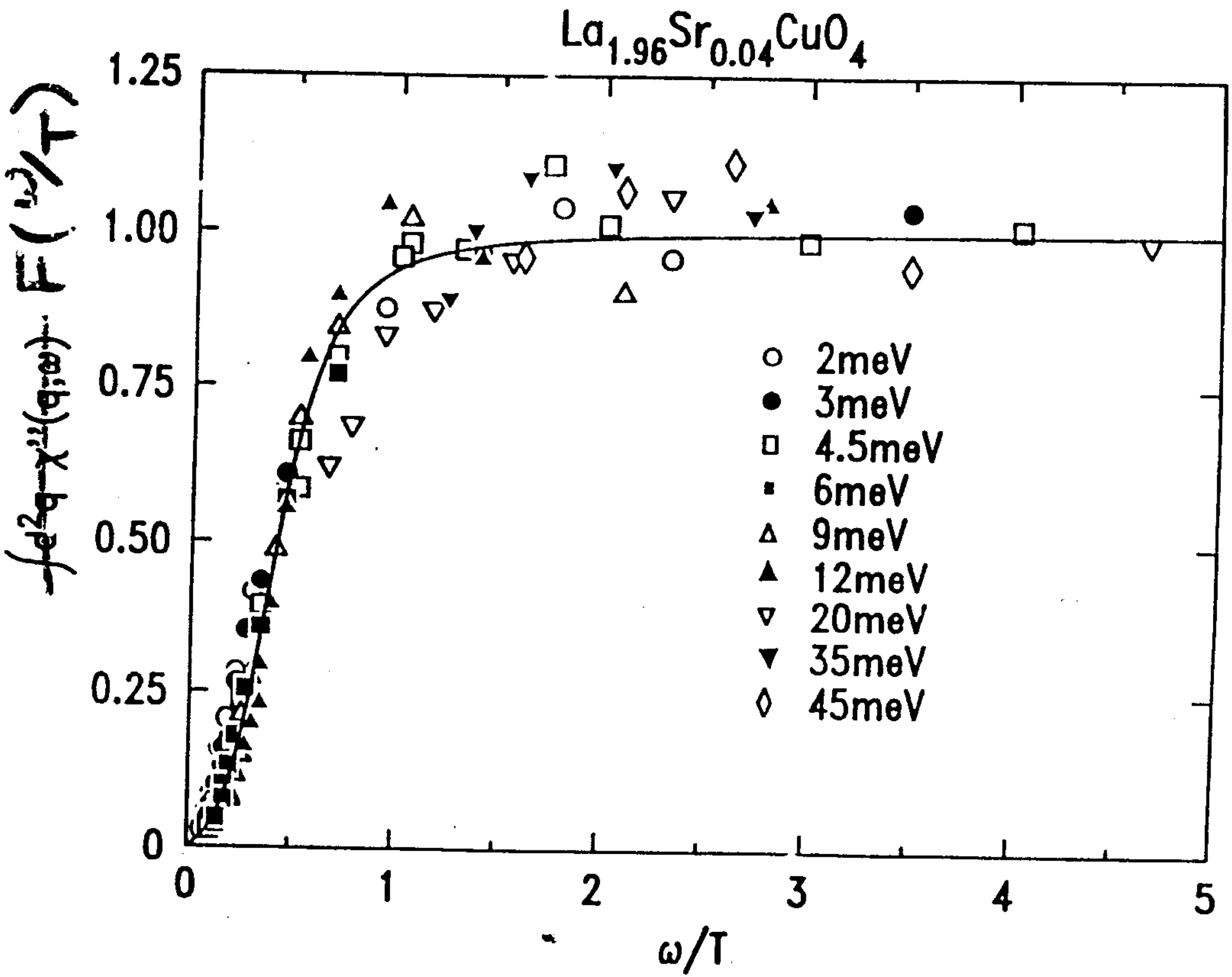

$La_{1.96}Sr_{0.04}CuO_4$
$F(\omega/T)$
1.25
1.00
0.75
0.50
0.25
0
0
1
2
3
4
5
ω/T
2meV
3meV
4.5meV
6meV
9meV
12meV
20meV
35meV
45meV

Fig. 18

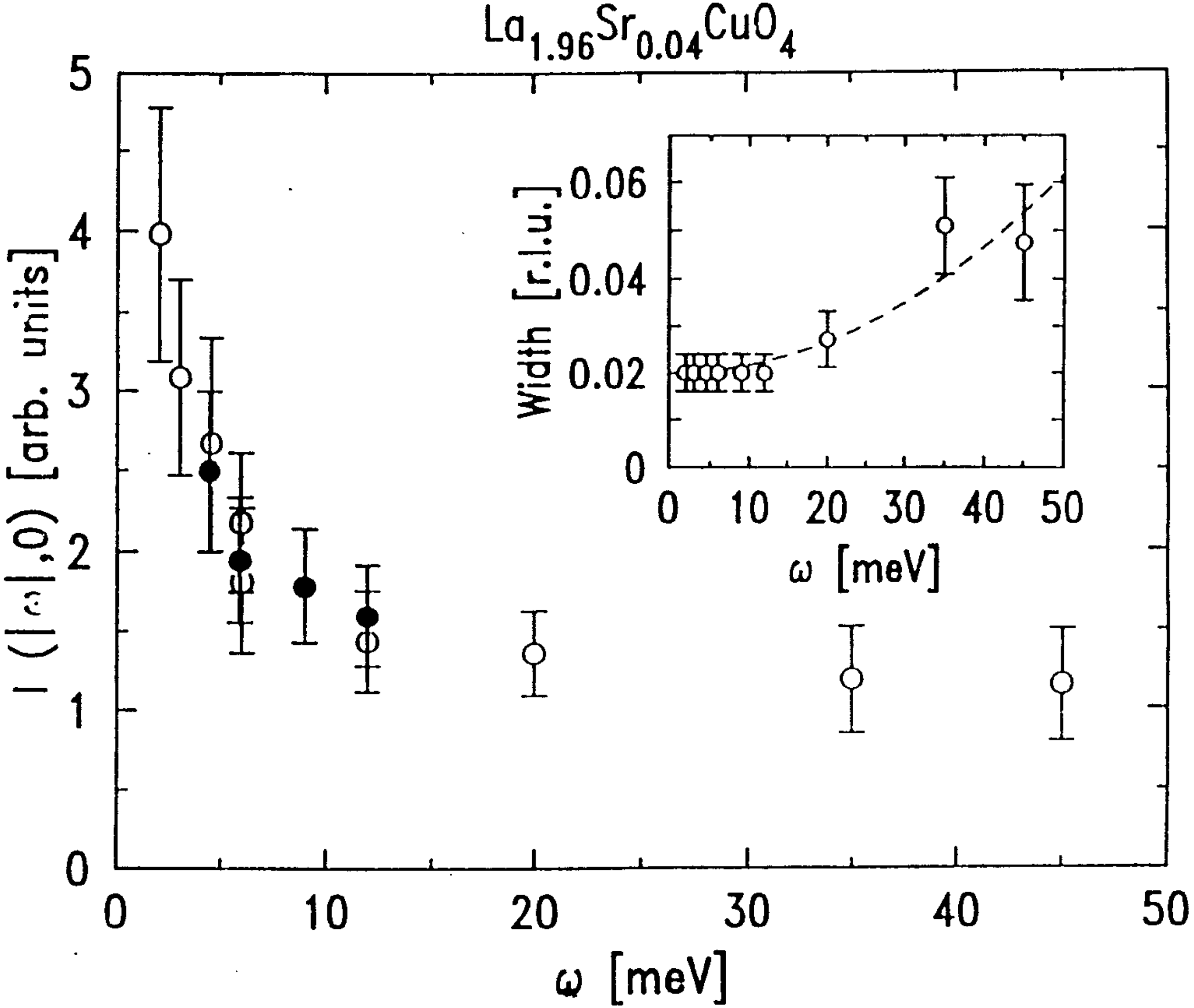


Fig. 16

2. SIMPLEST QUANTUM MODELS WITH INTERACTIONS + DISORDER

QUANTUM ROTORS

$$\mathcal{H} = g\sum_i \vec{L}_i^2 - \sum_{\langle ij\rangle} J_{ij}\, \vec{n}_i \cdot \vec{n}_j$$

$\vec{n}_i$ unit N-component vector on a d-dimensional lattice

$\vec{L}_i$ generator of O(N) rotations

$N \geq 2$

TRANSVERSE-FIELD ISING

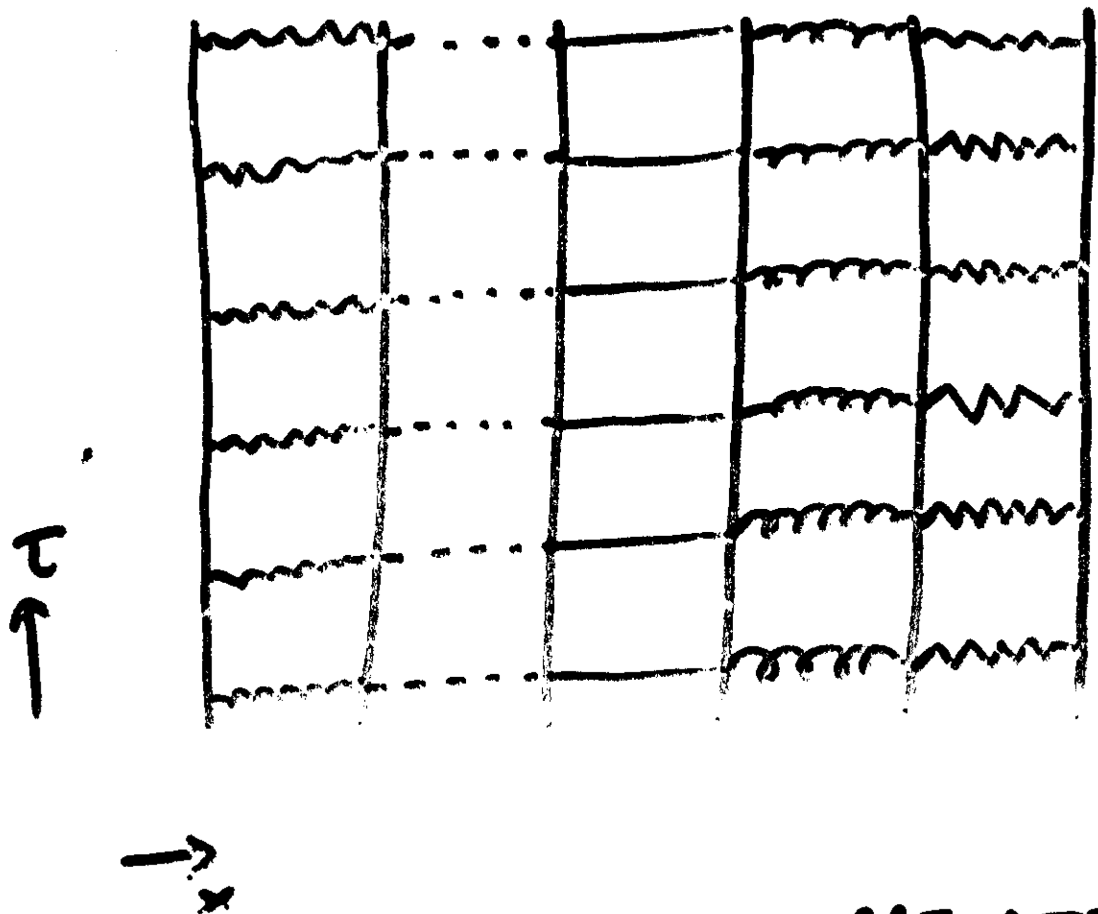
$$\mathcal{H} = g\sum_i \sigma_i^x - \sum_{\langle ij\rangle} J_{ij}\, \sigma_i^z \sigma_j^z$$

Natural extrapolation of rotors to $N=1$

$J_{ij} \rightarrow$ near-neighbor interaction; random

$$\overline{J_{ij}} = J_0 \qquad \overline{\left(J_{ij} - J_0\right)^2} = \Delta^2$$

MODELS MAP ONTO CLASSICAL O(N) VECTOR SPINS MODELS IN $d+1$ DIMENSIONS $[N \geq 1]$



DISORDER IS CORRELATED ALONG TIME DIRECTION.

NOTE HOWEVER : $z \neq 1$

WHEN $\Delta \neq 0$

$g \rightarrow$ QUANTUM PARAMETER

$J_0 \rightarrow$ "CLASSICAL" INTERACTION STRENGTH

$\Delta \rightarrow$ RANDOMNESS PARAMETER

POSSIBLE PHASES

(I) SPIN-FLUID $\boxed{g \gg J_0, \Delta}$

$$\langle \vec{n}_i \rangle = 0$$

$$\lim_{\tau \to \infty} \langle \vec{n}_i(0) \cdot \vec{n}_i(\tau) \rangle = 0$$

(II) MAGNETIC ORDER $\boxed{J_0 \gg g, \Delta}$

$$\langle \vec{n}_i \rangle \neq 0$$

$$\overline{\langle \vec{n}_i \rangle} = \vec{N}_0 \neq 0$$

$$\lim_{\tau \to \infty} \overline{\langle \vec{n}_i(0) \cdot \vec{n}_i(\tau) \rangle} = \vec{N}_0^2 \neq 0$$

II) SPIN GLASS $(\Delta \gg J_0, g)$

$$\langle \vec{n}_i \rangle \neq 0$$

$$\overline{\langle \vec{n}_i \rangle} = 0$$

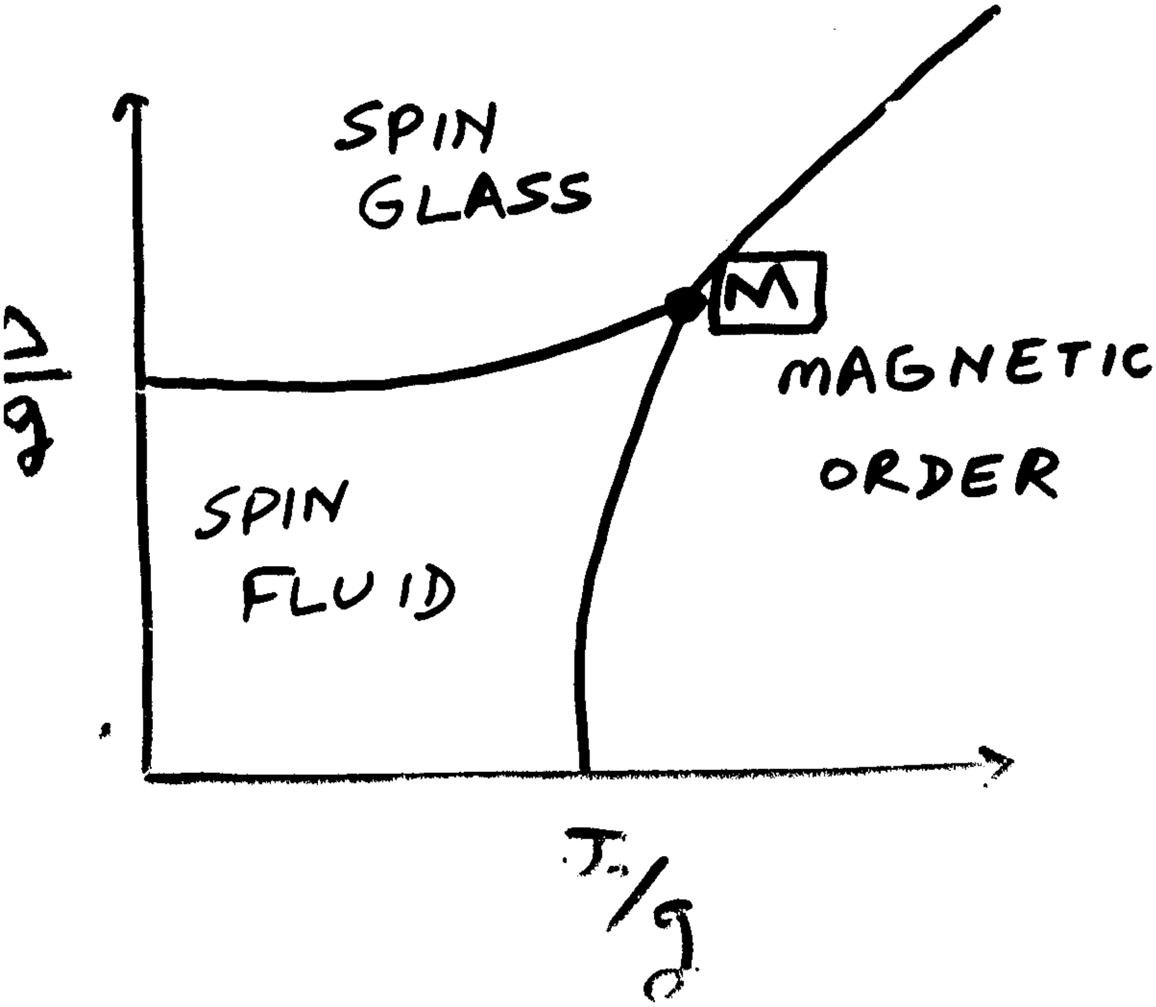
$$\lim_{\tau \to \infty} \langle \vec{n}_i(0) \cdot \vec{n}_i(\tau) \rangle \neq 0$$

$$\lim_{\tau \to \infty} \overline{\langle \vec{n}_i(0) \cdot \vec{n}_i(\tau) \rangle} = q_{EA}$$

$$\overline{\langle \vec{n}_i \rangle^2} = q_{EA}$$

$q_{EA} \to$ Edwards Anderson order parameter

PHASE-DIAGRAM (T=0)

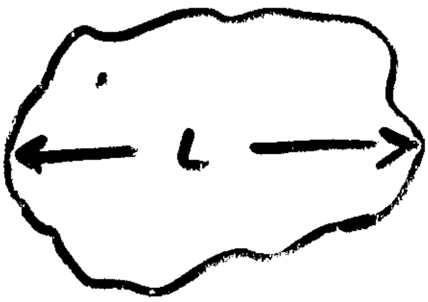


EXPTS IN CUPRATES
NOT TOO FAR FROM $\boxed{M}$

3. $\nu \geq 2/d$

Consider effect of small Δ near the SPIN-FLUID $\leftrightarrow$ MAGNETIC ORDER transition at $J_0 = J_c$ [take $g = 1$]

There are fluctuations in the average value of J_{ij} in a region of size L



$$\overline{J_{ij}}^{\,L} \sim J_0\left(1 + \frac{c}{L^{d/2}}\right)$$

$c \sim$ random variable of order unity

AT WHAT SCALE L_R DOES $\overline{J_{ij}}^{\,L_R} - J_0$ become of order $J_0 - J_c$?

$$\boxed{L_R \sim (J_0 - J_c)^{-2/d}}$$

FOR RANDOMNESS TO BE <u>UNIMPORTANT</u> WE WANT

$$L_R \ll \xi$$

or $$(J_0 - J_c)^{-2/d} \ll (J_0 - J_c)^{-\nu}$$

or

$$\boxed{\nu \geqslant 2/d}$$

A) "Disorder is <u>relevant</u> at a pure fixed point if $\nu_{pure} < 2/d$

[HARRIS]

B) Even at the random fixed point we must have

$$\nu \geqslant 2/d$$

[CHAYES ET. AL.]

4. Griffiths Singularities in the Spin Fluid phase

Consider first the spin-fluid phase in non-random system ($\Delta = 0$).

Magnon excitation has a gap m.

For frequencies $\omega < 3m$, the magnon cannot decay, and spectral peak is a delta function

$$\chi''(k, \omega) = \frac{A}{2\sqrt{k^2+m^2}} \, \delta\left(\omega - \sqrt{k^2+m^2}\right)$$

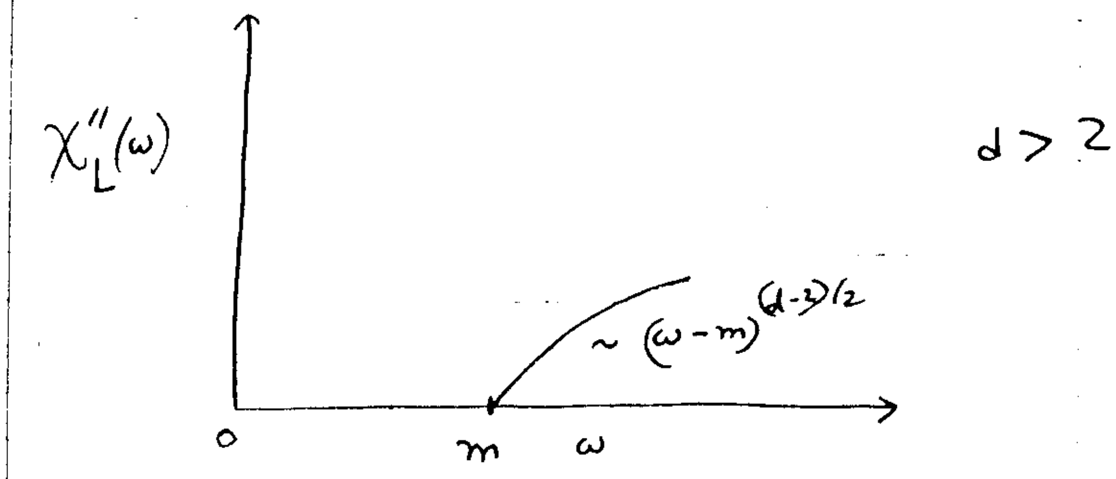
$[\omega > 0]$.

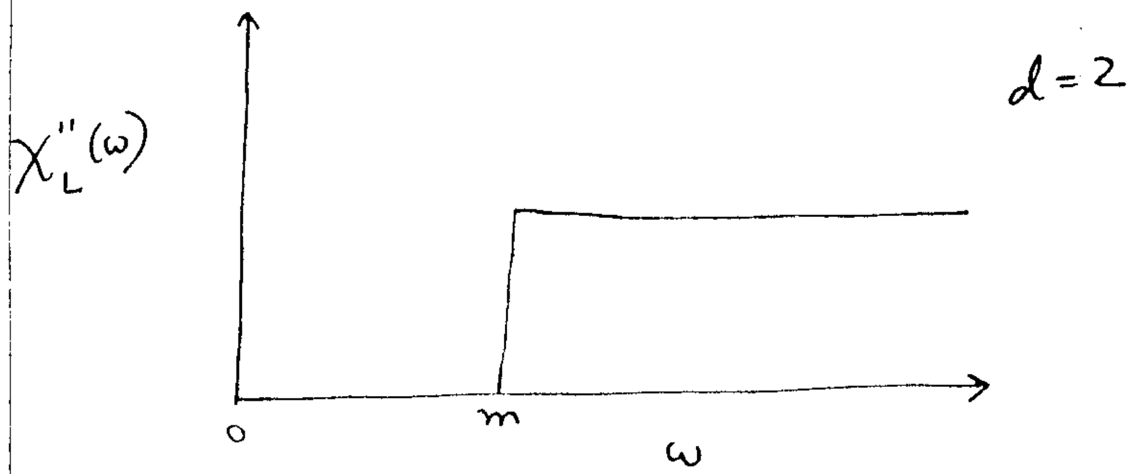
So the local susceptibility

$$\chi''_L(\omega) = \int \frac{d^d k}{(2\pi)^d} \, \chi''(k, \omega)$$

$$= \frac{m^{d/2} S_d}{\pi^d \, 2^{d/2+2}} \, \frac{1}{\omega} \, (\omega - m)^{(d-2)/2} \, \theta(\omega - m)$$

$$0 \leq \omega \leq 3m$$





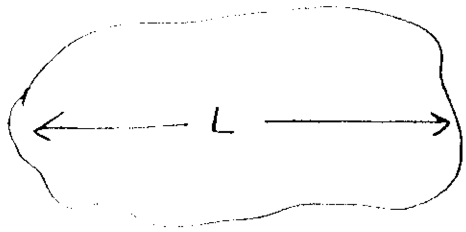
"Griffiths effects" → gap in $\overline{\chi''_L(\omega)}$
is wiped out for any non-zero randomness $(\Delta > 0)$.

$\chi''_L(\omega) > 0$ for all $\omega > 0$.

(The above is true if the disorder probability distribution is unbounded. Bounded disorder requires finite Δ)

RARE

Consider a fluctuation in a spatial region of size L, in which all the bonds are roughly equal and of a magnitude $J > J_c$



This region is locally in the magnetically-ordered phase.

Probability of fluctuation $\sim e^{-c_1 L^d}$

Contribution of this region to

$$\overline{\left\langle \vec{n}_i(0) \cdot n_i(\tau) \right\rangle} = \int_0^\infty \frac{d\Omega}{\pi} \chi_L''(\Omega)\, e^{-\Omega|\tau|}$$

For τ large enough the region behaves like a one-dimensional, classical, N-component chain with exchange coupling $\sim L^d$

$$\left[\beta H_{1d} = -K \sum_{\tau} \vec{n}_{\tau} \cdot \vec{n}_{\tau+1} \right.$$
$$\left. K \sim L^d \right]$$

It is known that the correlation length of such a chain has the behavior

$$\xi_\tau \sim \begin{cases} e^K & \text{for } N=1 \\ K & \text{for } N \geq 2 \end{cases} \quad \text{as } K \to \infty$$

So contribution of such a region to

$$\langle \vec{n}_i(0) \cdot \vec{n}_i(\tau) \rangle \sim \exp\left(-\tau/\xi_\tau\right)$$

with ξ_τ given above.

Now we sum over all values of L.

Consider the cases $N=1$ and $N \geq 2$ separately

(A) $N=1$

$$\overline{\left\langle \vec{n}_i(0)\cdot\vec{n}_i(\tau)\right\rangle} \geqslant c_4\int_0^\infty dL\, \exp(-c_1L^d) \otimes \exp\left(-c_2\tau e^{-c_3L^d}\right)$$

for some constants c_{1-4}.

As $\tau\to\infty$, integral can be evaluated by a saddle point at $L=L^*$

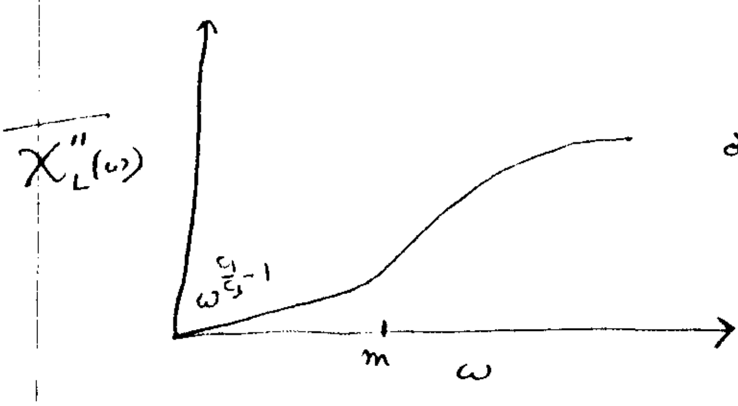
$$L^* \sim (\log\tau)^{1/d}$$

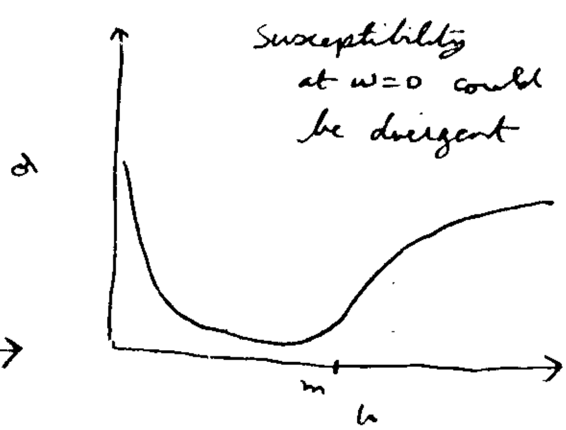
and

$$\overline{\left\langle \vec{n}_i(0)\cdot\vec{n}_i(\tau)\right\rangle} \geqslant \frac{\tilde{C}}{\tau^{c_1/c_3}}$$

Power-law with continuously varying exponents

So $\overline{\chi''_L(\omega)} \geqslant \bar{\bar{C}}\,\omega^{c_1/c_3-1}$





(B) $N \geq 2$

$$\overline{\langle \vec{n}_i(0) \cdot \vec{n}_i(\tau) \rangle} \geq c_3 \int_0^\infty dL \, \exp(-c_1 L^d) \exp(-c_2 \tau L^{-d})$$

Now saddle point is at

$$L^* \sim \tau^{1/2d}$$

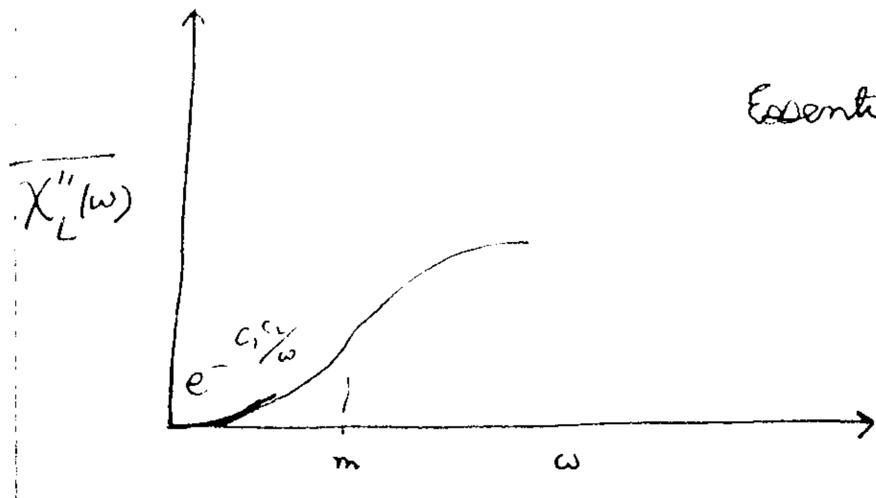
$$\overline{\langle \vec{n}_i(0) \cdot \vec{n}_i(\tau) \rangle} \geq \tilde{C} \exp\left(-2\sqrt{c_1 c_2 \tau}\right)$$

Stretched exponential

So, ~~χ''_L~~

Performing inverse Laplace transform

$$\overline{\chi''_L(\omega)} \geq \tilde{\tilde{C}} \exp\left(-\frac{c_1 c_2}{\omega}\right)$$



Essential singularity at $\omega = 0$.

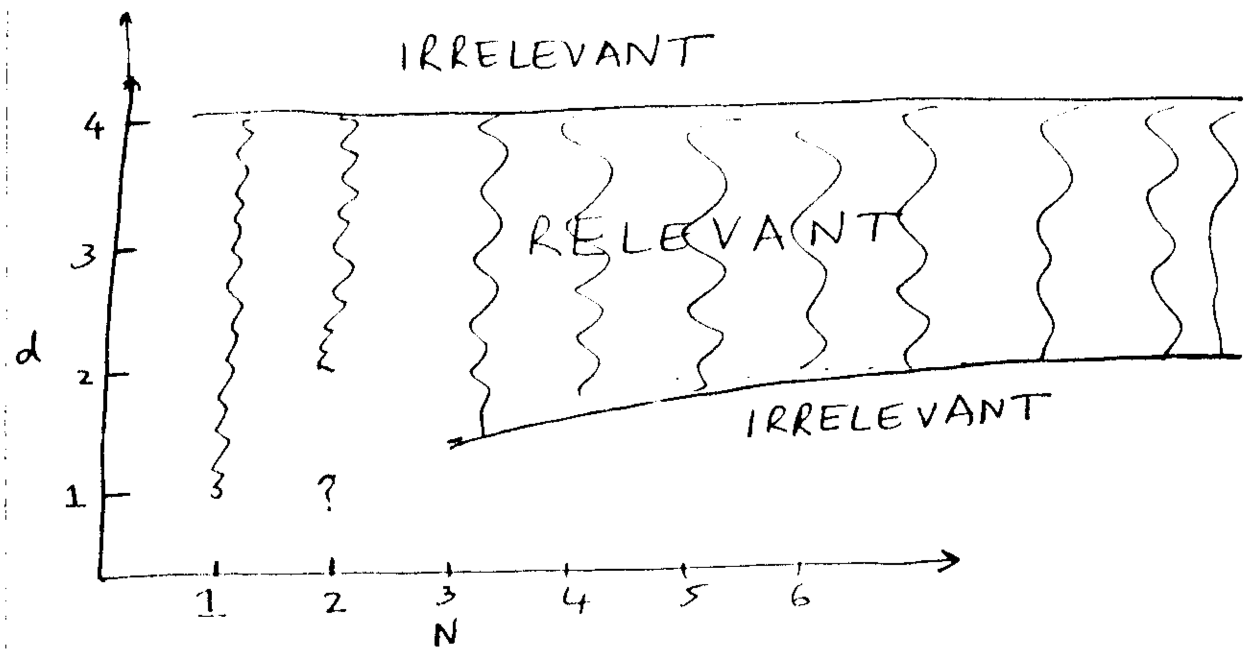
5. Field Theory for magnetic order — spin fluid transition

When is a small Δ relevant about transition in clean system?

Compare ν_{pure} with $2/d$.

$$\nu_{pure} = \begin{cases} 1/2 & \text{for } d \geq 3 \\ > 1/2 & \text{for } d < 3 \end{cases}$$

and $\sim \frac{1}{d-1}$ as $d \to 1$ for $N \geq 3$



Continuum field theory

Use $\vec{\psi}(x,\tau)$ which is the average $\vec{n}$ value over some ~~co~~ coarse-grained region. No length constraint on $\vec{\psi}(x,\tau)$.

$$Z = \int \mathcal{D}\psi \exp\left(-\int d\tau\, d^d x\, \mathcal{L}[\psi]\right)$$

$$\mathcal{L}[\psi] = \frac{1}{2}\left[\left(\frac{\partial \vec{\psi}}{\partial \tau}\right)^2 + (\nabla\vec{\psi})^2 + r\,\vec{\psi}^2 + v(x)\,\vec{\psi}^2\right] + \frac{u}{8}\left(\vec{\psi}^2\right)^2$$

$v(x)$ is a random variable

$$\langle v(x)\rangle = 0 \qquad \langle v(x)\,v(x')\rangle = \Delta^2\,\delta(x-x')$$

Introduce n replicas and average over disorder $\quad \psi_{\alpha a}(x,\tau) \quad$ $\alpha \to$ vector index $1,\ldots N$

$a \to$ replica index $1,\ldots n$

$$\overline{Z^n} = \int \mathcal{D}\psi_{\alpha a} \exp\Big(-\int d^d x\, d\tau \Big(\frac{1}{2}\left(\frac{\partial \psi_{\alpha a}}{\partial \tau}\right)^2 + \frac{1}{2}(\nabla\psi_{\alpha a})^2 + \frac{r}{2}\psi_{\alpha a}^2 + \frac{u}{8}\left(\psi_{\alpha a}^2\right)^2\Big) + \frac{\Delta^2}{8}\int d^d x\, d\tau\, d\tau' \Big\{ \psi_{\alpha a}^2(x,\tau)\,\psi_{\beta b}^2(x\,\tau')\Big\}\Big) \qquad (\star)$$

RG transformation

$$x' = x/s$$

$$\tau' = \tau/s^{z}$$

$$\psi' = \psi \, s^{(d+z-2+\eta)/2}$$

At tree level

$$r' = r\, s^{2}$$

$$u' = u\, s^{4-d-z-2\eta}$$

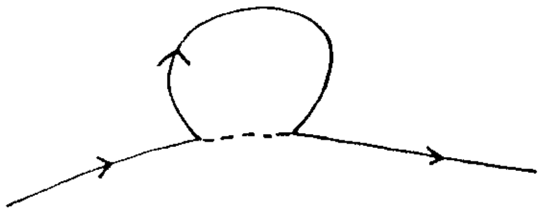
$$\Delta^{2\,\prime} = \Delta^{2}\, s^{4-d-2\eta}$$

and $\eta = 0$, $z = 1$.

So above $d=4$, u and Δ are both irrelevant. Exponents are gaussian

One-loop RG equations below $d=4$

$\epsilon = 4-d$ is small

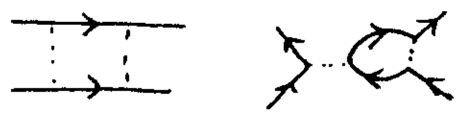


$$\frac{dr}{dl} = 2r - c_1 \Delta^2$$

$c_1, c_2 > 0$

$$\frac{du}{dl} = (-1+\epsilon)\, u$$

$$\frac{d\Delta^2}{dl} = \epsilon \Delta^2 + c_2 \Delta^4$$



Runaway flow to strong Δ^2

NO STABLE FIXED POINT!

[Interactions are necessary to stabilize localized band tails]

Cardy and Boyanovsky found a stable fixed point by letting disorder be correlated in ϵ_τ dimensions – then u and Δ^2 become relevant together and competition stabilizes system.

However this method almost certainly fails before $\epsilon_\tau \sim 1$.

6. Field Theory for spin-glass to spin-fluid transition

[N. READ J. YE]

Decouple quartic term in (★) by introducing Hubbard Stratonovich ~~field~~ field

$$Q^{ab}_{\alpha\beta}(x\,\tau\,\tau') \sim \psi^{a\alpha}(x\,\tau)\,\psi^{b\beta}(x\,\tau')$$

Then

$$\overline{Z^n} = \int \mathcal{D}\psi_{\alpha a}\,\mathcal{D}Q^{ab}_{\alpha\beta}(x\,\tau\,\tau')\ \exp\Big(-\int d^dx\,d\tau\Big(\frac{1}{2}\Big(\frac{\partial\psi_{\alpha a}}{\partial\tau}\Big)^2 + \frac{1}{2}(\nabla\psi_{\alpha a})^2 + \frac{r}{2}\psi^2_{\alpha a} + \frac{u}{8}(\psi^2_{\alpha a})^2\Big)$$

$$-\int d^dx\,d\tau\,d\tau'\Big(\frac{\Delta^2}{2}\big(Q^{ab}_{\alpha\beta}(x\,\tau\,\tau')\big)^2 - \frac{\Delta^2}{2}Q^{ab}_{\alpha\beta}(x\,\tau\,\tau')\,\psi_{\alpha a}(x\,\tau)\,\psi_{\beta b}(x\tau')\Big)\Big)$$

The order-parameter for the spin glass transition is contained in Q in 2 places.

(1) In spin-glass phase

$$\lim_{|\tau-\tau'|\to\infty} \overline{\langle Q^{aa}_{\alpha\beta}(x,\tau,\tau')\rangle} = \delta_{\alpha\beta}\, q_{EA} \neq 0$$

while $q_{EA} = 0$ in spin-fluid phase

(2) $$\overline{\left\langle Q^{ab}_{\alpha\beta}(x\,\tau\,\tau')\right\rangle} = \tilde{q}_{EA}\,\delta_{\alpha\beta} \text{ for } a \neq b$$

$$\downarrow a\neq b \quad \overline{\left\langle \psi_\alpha(x,\tau)\right\rangle\left\langle \psi_\beta(x\,\tau')\right\rangle}$$

Showing $q_{EA} = \tilde{q}_{EA}$ is not obvious
→ however equality is satisfied in our explicit solution.

To obtain a field theory we must integrate out $\psi_{\alpha a}$ field and obtain an effective action for Q

$$Z^n = \int \mathcal{D}Q^{ab}_{\alpha\beta}(x\,\tau\,\tau')\,\exp\left[-S_{eff}[Q]\right]$$

After performing a gradient expansion in S_{eff}, renormalizing the theory a little to generate all terms allowed by symmetry we get the following

$$S_{eff}[\varphi] = \frac{1}{W}\int d^dx\, d\tau \left[+\frac{d}{d\tau_1}\frac{d}{d\tau_2} + r \right] \varphi^{aa}_{\alpha\alpha}(x\tau_1\tau_2)\Big|_{\tau_1=\tau_2=\tau}$$

$$+ \frac{1}{2}\int d^dx\, d\tau\, d\tau' \left[\left(\nabla \varphi^{ab}_{\alpha\beta}(x\tau\tau')\right)^2 + t\left(\varphi^{ab}_{\alpha\beta}(x\tau\tau')\right)^2 \right]$$

$$-\frac{W}{6}\int d^dx\, d\tau_1 d\tau_2 d\tau_3\ \varphi^{ab}_{\alpha\beta}(x\tau_1\tau_2)\,\varphi^{bc}_{\beta\gamma}(x\tau_2\tau_3)\,\varphi^{ca}_{\gamma\alpha}(x\tau_3\tau_1)$$

$$+\frac{u}{2}\int d^dx\, d\tau\ \varphi^{aa}_{\alpha\beta}(x\tau\tau)\,\varphi^{aa}_{\alpha\beta}(x\tau\tau)$$

$$+\frac{v}{2}\int d^dx\, d\tau\ \varphi^{aa}_{\alpha\alpha}(x\tau\tau)\,\varphi^{aa}_{\beta\beta}(x\tau\tau)$$

$$-\frac{\lambda}{2}\int d^dx\, d\tau_1 d\tau_2\ \varphi^{aa}_{\alpha\alpha}(x\tau_1\tau_1)\,\varphi^{bb}_{\beta\beta}(x\tau_2\tau_2)$$

Key term is the first linear one.

We can shift $\varphi^{ab}_{\alpha\beta}(x\tau_1\tau_2) \to \varphi^{ab}_{\alpha\beta}(x\tau_1\tau_2) + v\,\delta^{ab}\delta^{\alpha\beta}\delta(\tau_1-\tau_2)$

~~to~~ ~~...~~ and choose v such that $t=0$.

Further analysis of S_{eff} in an upcoming paper.