INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

SMR. 758 - 30

SPRING COLLEGE IN CONDENSED MATTER
ON QUANTUM PHASES
(3 May - 10 June 1994)

==

METAL-INSULATOR TRANSITION

"Experimental Highlights"

Mikko PAALANEN
Department of Physics
University of Jyväskylä
Jyväskylä, Finland

==

These are preliminary lecture notes, intended only for distribution to participants.

==

METAL-INSULATOR TRANSITION

"Experimental highlights"

Prof Mikko Paalanen
Univ. of Jyväskylä, FINLAND

OUTLINE

I INTRODUCTION
- I.1. Crystalline materials
- I.2. Transport measurements in B-field
- I.3. Classical microscopic picture

II DISORDERED MATERIALS
- II.1. Amorphous metal-semiconductor alloys
- II.2. Doped semiconductors
- II.3. Mott's criteria for MIT
- II.4. Conductivity in disordered materials; interesting questions
- II.5. Answers: Minihistory of benchmark theories
  - II.5.1. First scaling ideas for noninteracting electrons
  - II.5.2. Including e-e interactions

III HIGHLIGHTS OF TRANSPORT EXPERIMENTS
- III 1. Weak localization
  - III 1.1. T-dependence
  - III 1.2. B-dependence
  - III 1.3. Sharvin & Sharvin experiment
  - III 1.4. Bergmann's visualization of single electron loc.
- III 2. Strong localization
  - III 2.1. Uniaxial stress tuning of Si:P through MIT
  - III 2.2 Continuous tuning of AlGaAs by photo-irradiation
  - III 2.3. $Nb_x Si_{1-x}$
  - III 2.4 Few comments about Hall coefficient

IV HIGHLIGHTS OF THERMODYNAMIC MEASUREMENTS
- IV 1 In Fermi-Liquid
- IV 2 Near MIT
  - IV 2.1. Specific heat near MIT
  - IV 2.2 Susceptibility measurements
- IV 3 Insulating phase of Si:P

IV 4 Local Moment Formation in Disordered Metals

IV 5 Two Fluid Model

V Concluding Remarks

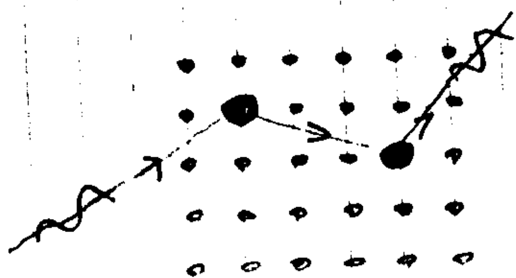
I INTRODUCTION

I 1. Crystalline materials:

Electron motion in crystalline materials was solved by early QM on 1930's.

- Bloch waves
- Band structure
- Fermi liquid theory 1950's

Electrical conductivity: ($j = \sigma E$)



Scattering of Bloch waves from impurities
ℓ = mean free path

Boltzmann equation (Magnetic field $B = 0$)

$$\sigma = \frac{ne^2\tau}{m} = \frac{e^2}{\hbar}\left(\frac{n}{k_F^2}\right)(k_F \ell)$$

τ = scattering time
$\ell = v_F \tau = \hbar k_F \tau / m$
v_F = Fermi velocity $= \hbar k_F / m$
k_F = Fermi momentum
n = electron density

$[\sigma] = \begin{cases} \frac{1}{\Omega m} & (3D) \\ \frac{1}{\Omega} & 2D \\ \frac{m}{\Omega} & (1D) \end{cases}$, $[n] = \begin{cases} \frac{1}{m^3} & \text{in 3-} \\ \frac{1}{m^2} & \text{in 2-} \\ \frac{1}{m} & \text{in 1-} \end{cases}$ dimensional samples

More in Ashcroft - Mermin text book

I 2 Transport measurements in B-field :

In Fig. 1 we see the measuring geometry for longitudinal resistivity ρ_{xx} and Hall resistance ρ_{xy}

- V_L measures the voltage drop along the current path
- V_H measures the voltage drop ⊥ current path
- I is total current
- magnetic field. B is usually ⊥ current path
- A is the area of 3D sample
  W is the width of 2D and 3D sample
  L is the distance between the V_L contacts

$$\rho_{xx} = \begin{cases} (V_L/I)(A/L) & 3D \\ (V_L/I)(W/L) & 2D \\ V_L/I\,(1/L) & 1D \end{cases}$$

$$\rho_{xy} = \begin{cases} (V_H/I)(A/W) & 3D \\ (V_H/I) & 2D \quad !! \\ \text{— ?} & 1D \end{cases}$$

Experimental notes :

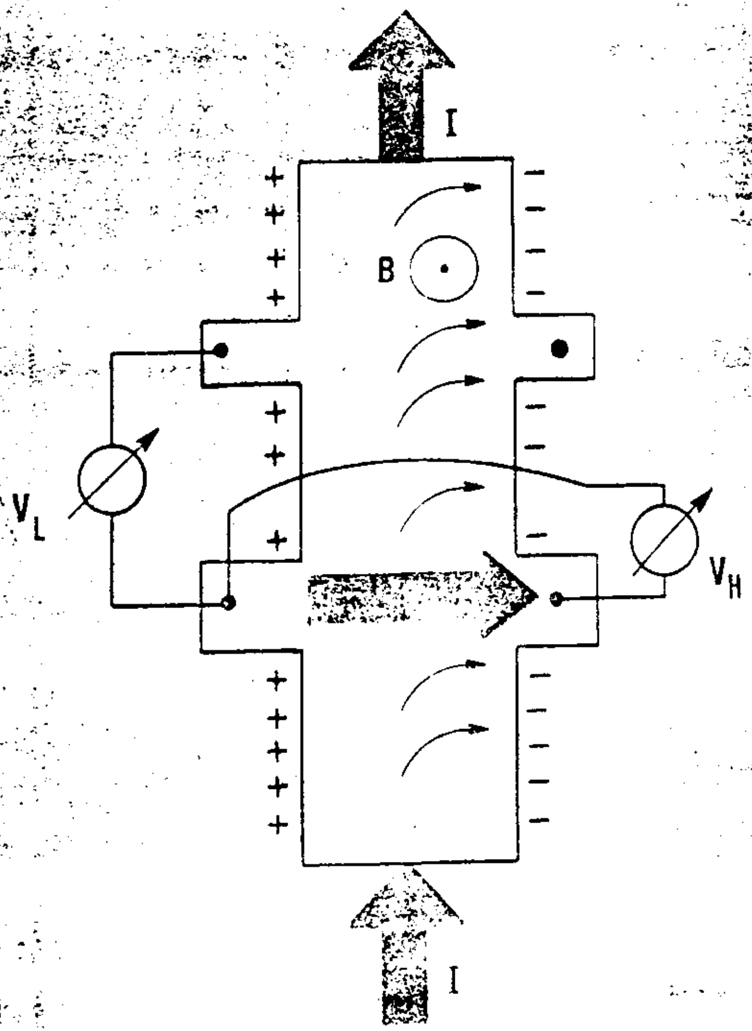
- In addition of V_L, V_H & I, we have to measure accurately also sample dimensions!
  The accuracy is usually few %
- Interesting exception is ρ_{xy} in 2D samples. No geometrical information is needed: Leads to accurate determination of Quantum Hall Effect!

Ex 1 How do we measure resistance (ρ_{xx}) and Hall resistance (ρ_{xy})

$$\rho_{xx} = \frac{V_L}{I} \times \text{geom. factor}$$

$$\rho_{xy} = \frac{V_H}{I} = \frac{B}{ne} \quad (2\text{-dim})$$

For 2-dimensional samples we get the following formulas



$= nev$

HALL RESISTANCE

MAGNETO RESISTANCE

- Generally $\rho_{xy} \ll \rho_{xx}$ ($V_H \ll V_L$) and ρ_{xy} measurements are more difficult and less accurate. Alignment of V_H contacts $\perp$ current path often impossible

- Solution $\rho_{xy}(B) = \dfrac{\rho_{xy}(B) - \rho_{xy}(-B)}{2}$

  (Homework: Why is this helping?)

- Resistivity is a tensor in magnetic field ($\vec{E} = \overleftrightarrow{\rho} \cdot \vec{j}$)

  For example in 2D (isotropic case):

$$\overleftrightarrow{\rho} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ -\rho_{xy} & \rho_{xx} \end{pmatrix} = \overleftrightarrow{\sigma}^{-1}$$

Hence $\rho_{xx} = \dfrac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2} \neq \dfrac{1}{\sigma_{xx}}$

(Homework: a) Why $\rho_{xx} = (V_L/I)(W/L)$ and $\sigma_{xx} \neq (I/V_L)/(L/W)$ in Fig 1?

b) Create a measuring geometry for σ_{xx})

I 3. Classical microscopic picture: (Boltzmann)

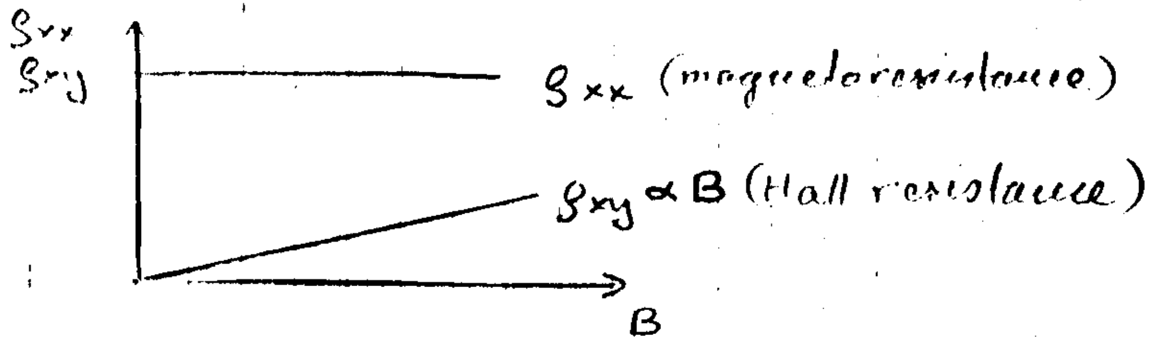
I 3.1. B-dependence:

$$\rho_{xx} = \frac{m}{ne^2\tau}$$

$$\rho_{xy} = \frac{B}{ne}$$

This is valid only at low magnetic fields $\omega_c\tau = \frac{eB}{m}\tau \ll 1$ (no Landau level formation!)

ω_c = cyclotron frequency

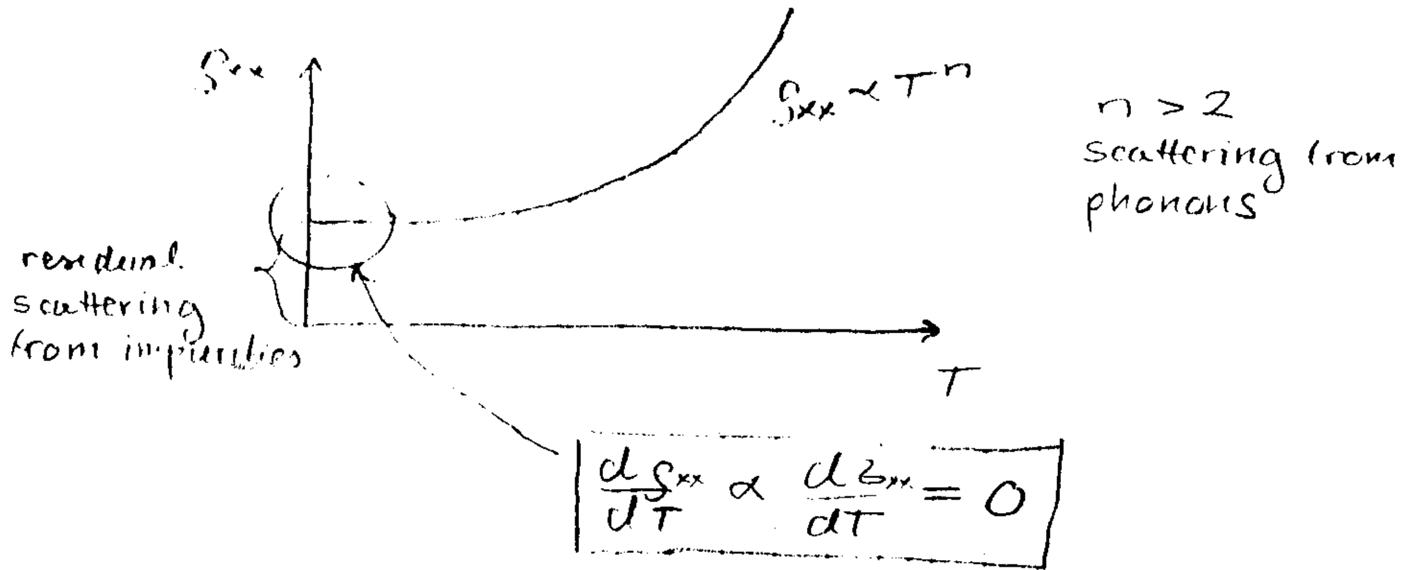


Notice: - In classical theory ρ_{xx} is independent of B and measures the scattering rate $\frac{1}{\tau}$ ($\omega_c\tau \ll 1$)

- $\rho_{xy} \propto B$ and the slope determines electron density n and the sign of current carrier charge.

(Homework: If you have both positive and negative current carriers present at the same time, how does ρ_{xy} and ρ_{xx} eqs change?)

I32 Temperature dependence: (classical picture)



$\rho_{xx} \propto \frac{1}{\tau}$ scattering rate

- scattering rates (phonon-, impurities-) are additive (Matthiessen's rule)

In disordered materials we find interesting deviations from classical picture in the low T and low B behavior of ρ_{xx} and ρ_{xy}

- interference effects of electron waves?
- e-e interaction effects?

Beyond Boltzmann transport and Fermi-liquid theory

Weak localization — perturbation calculations

Strong localization — scaling theories

II DISORDERED MATERIALS:

There are more disordered than crystalline materials (liquids, gases, glasses, amorphous)

- no crystalline local order
  - are there Bloch waves ?
  - no underlying symmetries
    → 'isotropic'!

II 1. Metal-semiconductor amorphous alloys:

- quenched condensed
- $Nb_x Si_{1-x}$ ($x_c = 11.5\%$)
- $Au_x Si_{1-x}$ ($x_c = 14\%$)
- $Au_x Ge_{1-x}$
- a $Mo_x Ge_{1-x}$ ($x_c = 10.4\%$)

x small, no connecting metallic paths ⇒ no electrical conductivity

$x > x_c$ (critical consent), metallic behavior
However, large disorder dominates

‖ We can tune disorder by changing x.

II 2. Doped semiconductors :

- Crystalline semiconductor host material provides a high ε (dielectric constant) "vacuum" for donors (n-type) or acceptors (p-type). These are placed randomly either substitutionally or interstitially in the host material.

- Si : P — P phosphorous doped silicon, P is donor, electron type

- Si : B — Boron doped silicon, B is acceptor (hole type)

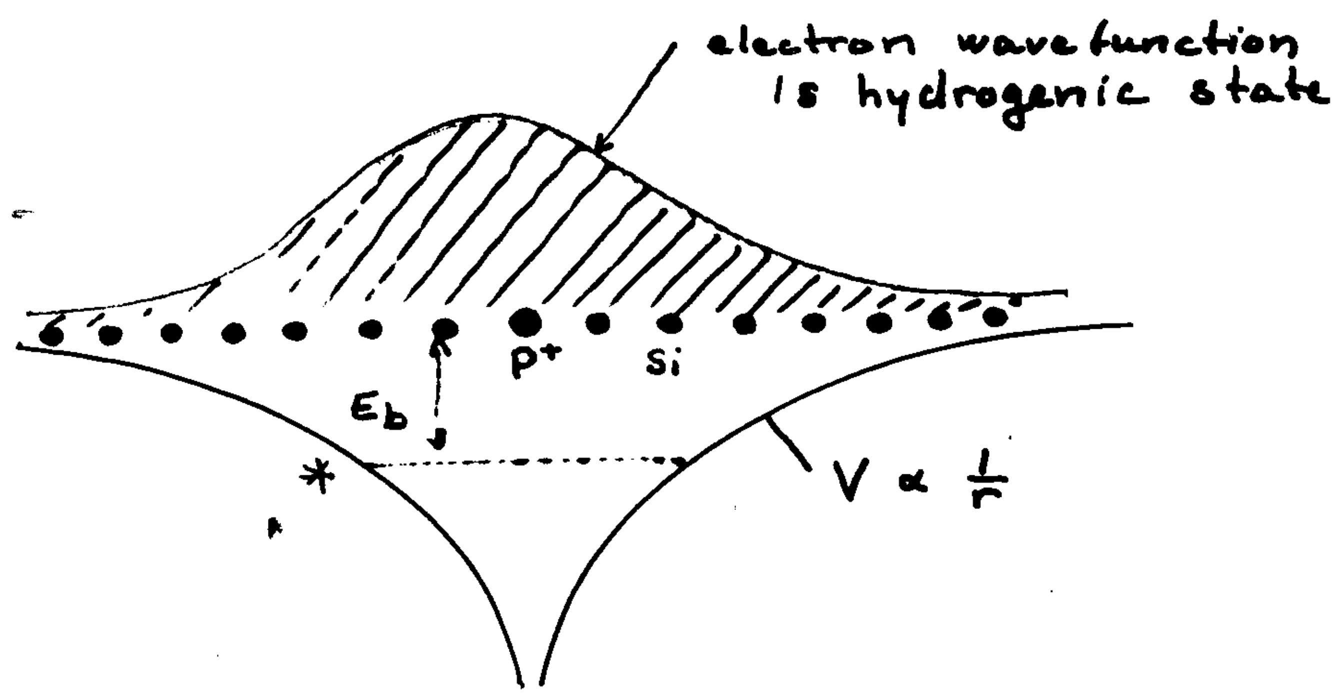
etc

In Fig 2a P^+ ion and electron form a hydrogenic type bound state with large effective Bohr radius of 16 Å. (See page 10.)

In Fig 2b the consentration of random P-dopants has increased and the electrons can hop from P^+ site to another. Metallic type behavior starts. P^+ ions form very disordered scattering system.

Si : P

Hydrogen atom model



Binding energy $E_b = \frac{e^4 m_e}{2(4\pi\varepsilon\varepsilon_0\hbar)^2} = 45\,meV$

"Bohr radius" $a_0^* = \frac{4\pi\varepsilon\varepsilon_0\hbar^2}{m_e e^2} \simeq 16\,Å$!

$$\begin{cases} \varepsilon_{Si} = 11.7 \\ m_e \approx .3\, m_0 \end{cases}$$

* In reality, the groundstate in Si is not pure s-state, but a mixture of the s-state and anisotropic orbital states. Consequently a uniaxial stress will increase the portion of orbital states and also increase the average Bohr radius.

Si : P

Hydrogenic electron state

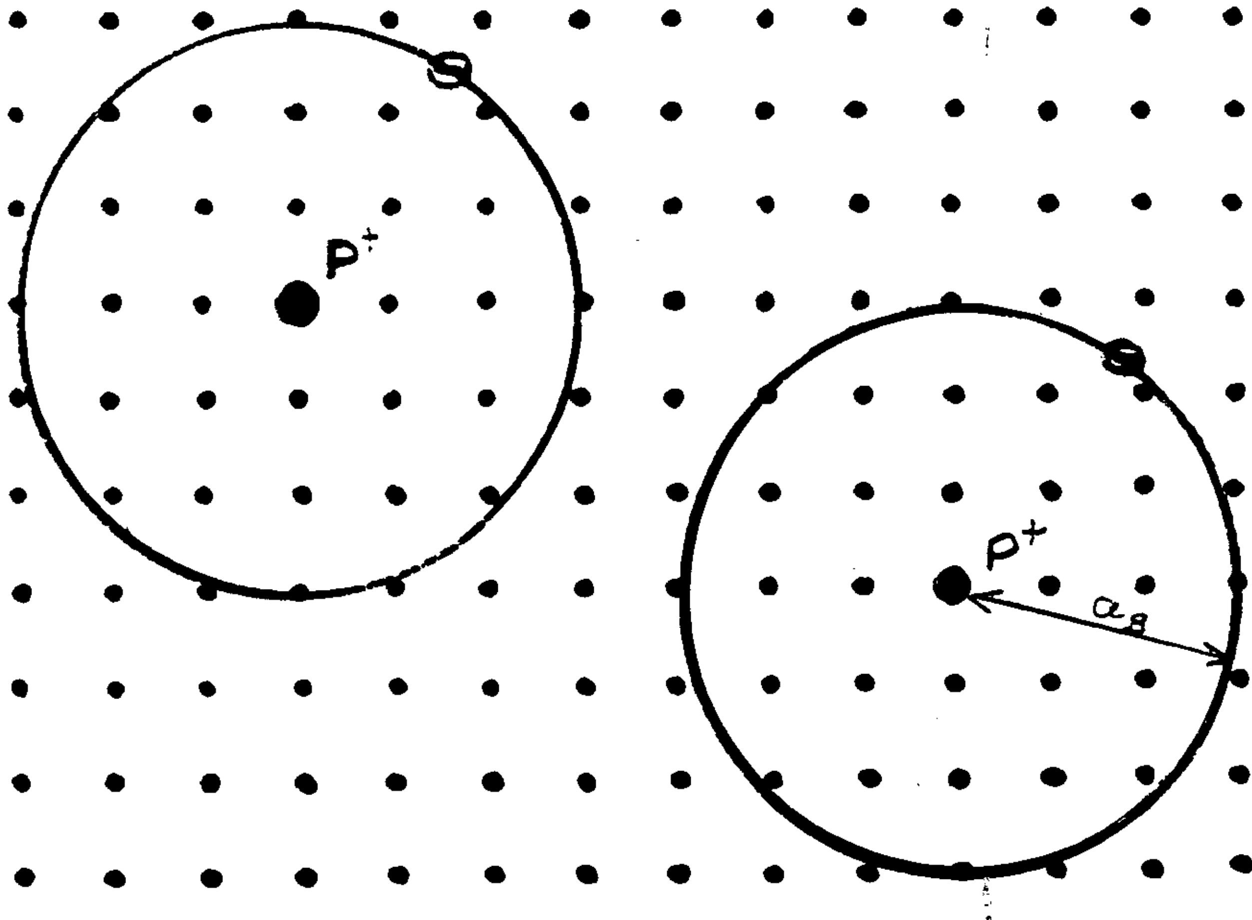


Fig 2a Randomly placed P-atoms in Si-host. P-atom has 5 p-electrons compared to 4 p-electrons of Si-host. Extra electron and P^+-ion form "a hydrogen-atom" with effective Bohr radius $a_B^* = 16$ Å !

Si : P

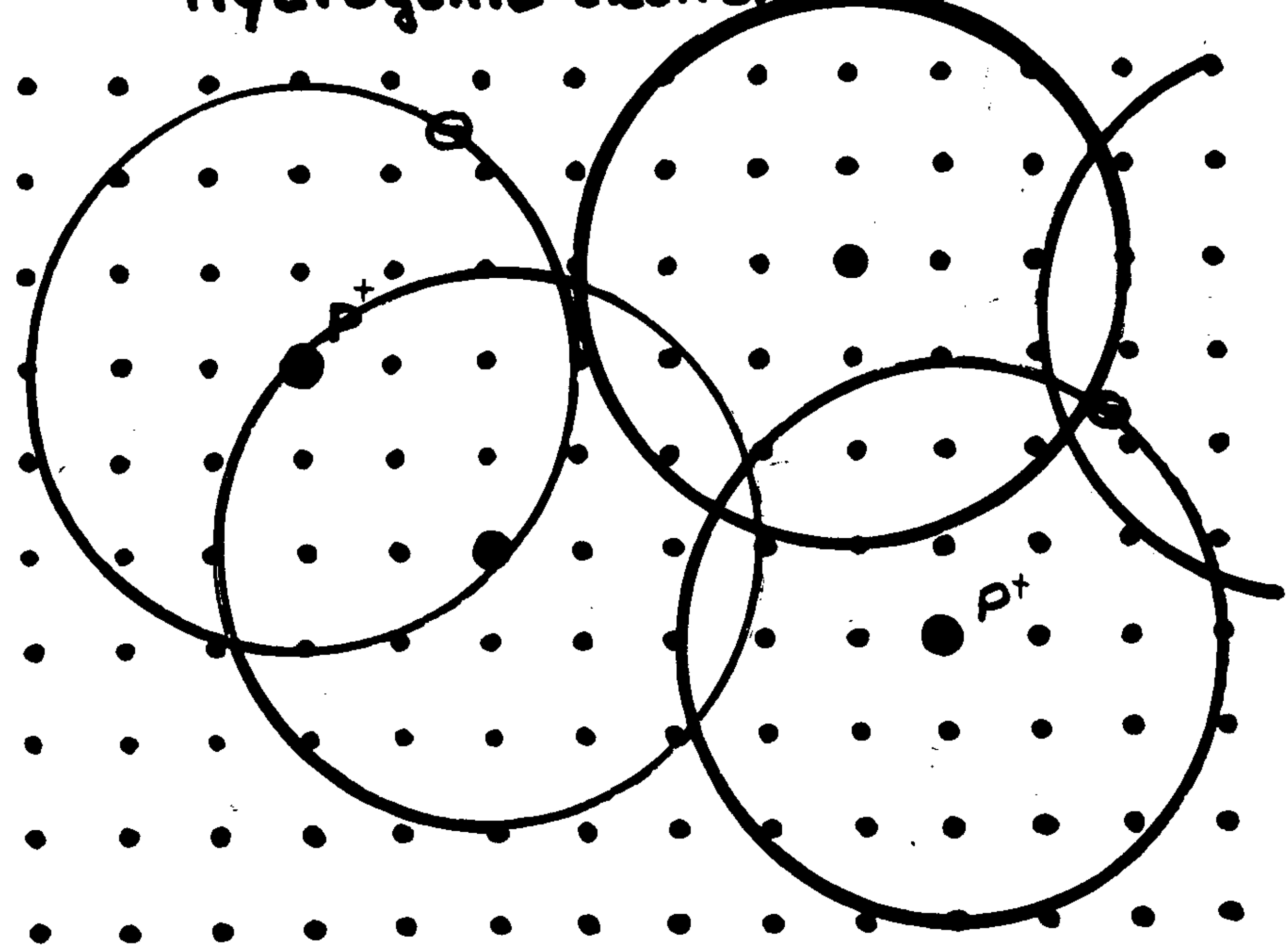


Fig 2b

Critical doping for metallic behavior

$n_c^{1/3} \times a_B^* \simeq 1/4$ Mott

$$\text{Si : P} \begin{cases} a_B^* \simeq 16\,\text{Å} \\ n_c \simeq 3.7\times10^{18}\ \text{cm}^{-3} \end{cases}$$

Notice: - Again we can tune the amount of disorder by changing the donor consentratio N_D.

- If we have only donors present and 1 electron per p^+ donor (uncompensated semiconductor), we presumably have also strong e-e repulsion present. By adding acceptors (by compensating) we effectively build up empty p^+ sites and decrease e-e interaction. In principle, we can tune both disorder and e-e interactions!

II 3. Mott's criteria for MIT:

If we compare several semiconductors, both p- and n- type, we find universal onset for the metallic behavior (Fig 3.)

The critical density, n_c, for the Metal-Insulator-Transition (MIT) and the effective Bohr radius of the hydrogenic state are related (dimensional analysis gives the result!!)

$$n_c^{1/3} a_B^* \simeq 0.26 \pm 0.05$$

This is the famous Mott's relation originally based on e-e interaction effects and their screening properties rather than randomness.

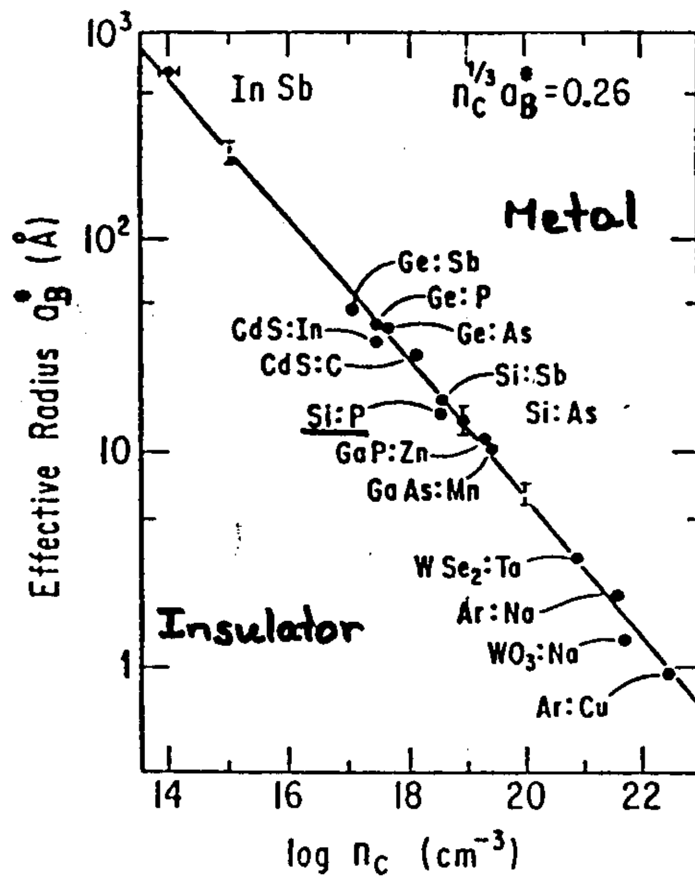


Fig.3 Mott's criterion for MI transition

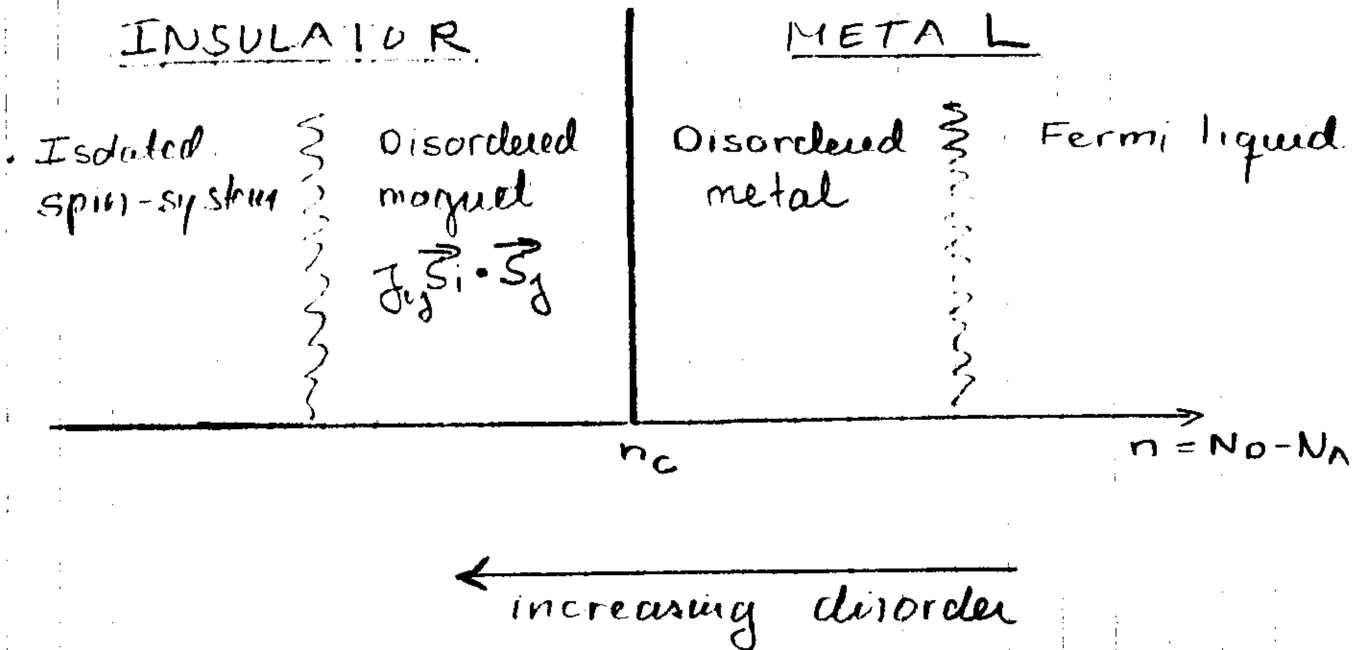
$$n_c^{1/3} a_B^* = 0.26 \pm 0.05$$

n_c critical density of donors

a_B^* effective Bohr radius

Doped semiconductors form "a standard system" for MIT studies
- disorder can be varied by N_D
- e-e interaction can be varied by N_A - acceptor density (the electron carrier density n, $n = N_D - N_A$)
- doped semiconductors are quite homogeneous, especially Si-based systems.

Example Si:P



Disordered metal: disorder, interference effects and interactions localize electrons

Disordered magnet: The wavefunctions of "hydrogen electrons" overlap $\Rightarrow$ antiferromagnetic interaction $J_{ij}\,\vec{S}_i \cdot \vec{S}_j$ (random Heisenberg antiferromagnet).

- $J_{ij} > 0$
- $J_{ij} \propto e^{-|r_i - r_j|/2a_B^*}$

II 4. Conductivity in disordered materials; Interesting questions:

- How do you calculate the conductivity, when Bloch waves disappear?

- How small can $k_F \ell$ be?
  - interference
  - multiple scattering

- Can we find universal laws at all in disordered world?

  T-dependence
  B-dependence
  $n - n_c$ dependence

- Is Fermi-liquid theory breaking down?

Valid in Fermi liquid case:

$$\begin{cases} C = \gamma T \\ \chi = \chi_{Pauli} \quad \text{(independent of } T) \\ T_1 T = \text{constant} \\ \tau_e \propto T^{-2} \quad \text{quasiparticle lifetime} \end{cases}$$

How is disorder affecting thermodynamic & spindynamic properties?

II 5 Answers (right and wrong) ; Minihistory of landmark theories :

Most of the theories and new ideas were based on rather extensive experimental work

1958 P. W. Anderson Phys. Rev 109, 1492

- Electron localizes by disorder. Localized wavefunction $|\psi(r)| \sim e^{-|r-r_0|/\xi}$

  ξ is localization length (depends on disorder)
  (Bloch wave $\psi(r) \propto e^{-i\vec{k}\vec{r}}$)

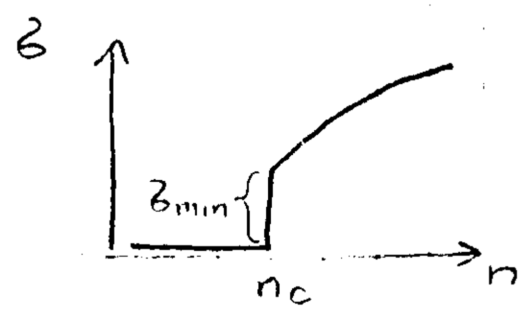
- impetus from Feher's ESR experiment

1960 Ioffe and Regel Prog. Semicond 4, 237
- MIT takes place at $k_F l \simeq 1$

$$\sigma = \frac{e^2}{\hbar}\left(\frac{n}{k_F^2}\right)(k_F l) \sim \frac{e^2}{\hbar}\frac{n}{k_F^2}$$

Notice in 2D $\sigma_{MIT} \sim \frac{e^2}{h} \sim (26k\Omega)^{-1}$

1973 Sir N. F. Mott
- bold prediction of minimum metallic conductivity σ_{min}



3D : $\sigma_{min} \approx \frac{e^2}{\hbar a_B^*}$

2D $\sigma_{min} \approx \frac{e^2}{h}$

- at $T=0$ conductivity change is discontinuous at MIT

- never seen experimentally

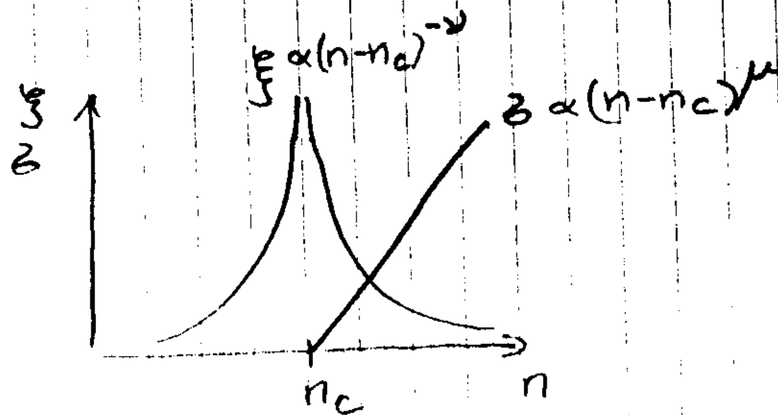
II.5.1. First scaling ideas for noninteracting electrons:

1974 Thouless Phys. Rep. 13, 93

1976 Wegner Z. Phys. B 25, 327

1979 Abrahams, Anderson, Licciardello, Ramakrishnan Phys. Rev. Lett. 42, 673 "Gang of 4"

3D
$T=0$



- ξ is correlation length (localization length in ins)
- $n=n_c$, $T=0$ is a critical point
- correlation lenght diverges at n_c !
- σ is smooth (no σ_{min})
- scaling means that all critically behaving quantities obey scaling around the critical point.

II.5.2. Including e-e interactions:

Perturbation calculations {
1979 Altshuler & Aronov Solid St. Comm 39, 115
1980 Altshuler, Aronov & Lee PRL 44, 1288
}

1980 – 1994

Further development of both perturbation calculations and scaling ideas

{ Finkelstein
Castellani, DiCastro & Lee
Sachdev & Bhatt
Kirkpatrick & Belitz }

→ Hard theoretical work trying to include both e-e interaction and single electron interference effects simultaneously in the scaling. (Also very hard to follow by experimentalists)

The tables on pages (18) a & b summarizes the critical quantities and their scaling exponents at the critical point $n = n_c$, $T = 0K$.

In the following I will present selected experimental highlights, which have helped the above theoretical developments to the right path. (Physics is often all an experimental science)

Scaling theory for Metal - Insulator Transition

Disorder + Interactions
(From review article of Kirkpatrick & Belitz)

universality classes!

symbol	symmetry breaker	diffusive modes	interaction
MF(SR)	magnetic field	$r = 0,3$ $i = 0,3$	short-range
MF(LR)			Coulomb
MI(SR)	magnetic impurities	$r = 0,3$ $i = 0$	short-range
MI(LR)			Coulomb
SO(SR)	spin-orbit scattering	$r = 0,1,2,3$ $i = 0$	short-range
SO(LR)			Coulomb
G(SR)	none	all	short-range
G(LR)			Coulomb

ritical ponents

physical quantity	scaling behavior	scaling equality
electrical conductivity	$\sigma(t\to 0,\ T=0) \sim t^{s}$	$s = \nu(d-2)$
density of states	$N_F(t\to 0,\ T=0) \sim t^{\beta}$	$\beta = \nu(d-y_h)$
order parameter density susceptibility	$\chi_0(t\to 0,\ T=0) \sim t^{-\gamma}$	$\gamma = \nu(2y_h-d)$
specific heat	$C_V(t=0,\ T\to 0) \sim T^{1-\kappa/z}$	$\kappa = z-d$
order parameter density correlation function	$G(k\to 0;\ t=0,\ T=0) \sim k^{-2+\eta}$	$\eta = 2+d-2y_h$
heat diffusion coefficient	$D_h(t\to 0,\ T=0) \sim t^{s_h}$	$s_h = s+\nu(z-d)$
spin diffusion coefficient	$D_s(t\to 0,\ T=0) \sim T^{c_s}$	$s_s = s+\nu\kappa$

(18) b

(From review article of Kirkpatrick & Belitz)

universality class / exponent	MI		MF		SO		G	
	SR	LR	SR	LR	SR	LR	SR	LR
ν	$\frac{1}{2\epsilon} - \frac{3}{4}\epsilon + O(\epsilon^3)$	$\frac{1}{\epsilon} + O(1)$	$\frac{1}{\epsilon} + O(1)$	$\frac{1}{\epsilon}+O(1)$	?	$\frac{1}{\epsilon} + O(1)$	?	$\cong 0.75$
z	d	$2+\frac{\epsilon}{2}+O(\epsilon^2)$	d	d	d	$2+O(\epsilon^2)$	?	$\cong 5.91$
β	0	$\frac{1}{\epsilon} + O(1)$	$-(1/2f^*) \times \ln[1-(\gamma_t^*)^2] + O(\epsilon)$	$\frac{1/2\epsilon}{1-\ln 2} + O(1)$	0	$\frac{2}{\epsilon} + O(1)$	?	$\cong 0.50$

TABLE III. Values for the three independent exponents ν, z, and β for the eight universality classes of Table 3.1. Values are given for $d=2+\epsilon$ dimensions except for class G, where approximate values for d=3 based on a 2-loop approximation are shown. β for class MF(SR) depends on the nonuniversal quantities γ_t^* and $f^* \equiv f(\gamma_t^*)$, cf. Eqs. (5.25). The critical behavior for class SO(SR) is not understood, and that for class G(SR) has not been considered.

III HIGHLIGHTS OF TRANSPORT EXPERIMENTS:

III 1. Weak localization:

III 1.1. T-dependence

The first experiment (Dolan & Osheroff PRL 43, 721) is presented in Fig. 4.

- Shows a new type of low T correction $\Delta\rho \propto \ln T$ in thin PdAu films, which are effectively 2D
- Experiment was testing ideas presented by Thouless (1977) and gang of 4 1979.

Perturbation theory

$$(2D) \quad \Delta\sigma_{2D} = \frac{p}{2}\frac{e^2}{\hbar\pi^2}\ln(T/T_0) + (1-\tfrac{3}{4}\tilde{F}_2)\ln T\tau$$

single electron localization term; e-e-interaction term

- dimensionality is important: thin films < 1000 Å are effectively 2-dimensional. Similar results in 1D and 3D cases
- both single electron localization and e-e interactions contribute. contributions have similar T dependence but different magnetic field B-dependence

factor p? – $\tau_\varphi \propto T^{-p}$; τ_φ is the dephasing time of Bloch wave.

factor $\tilde{F}_2$ – $\tilde{F}_2$ is nonuniversal screening term

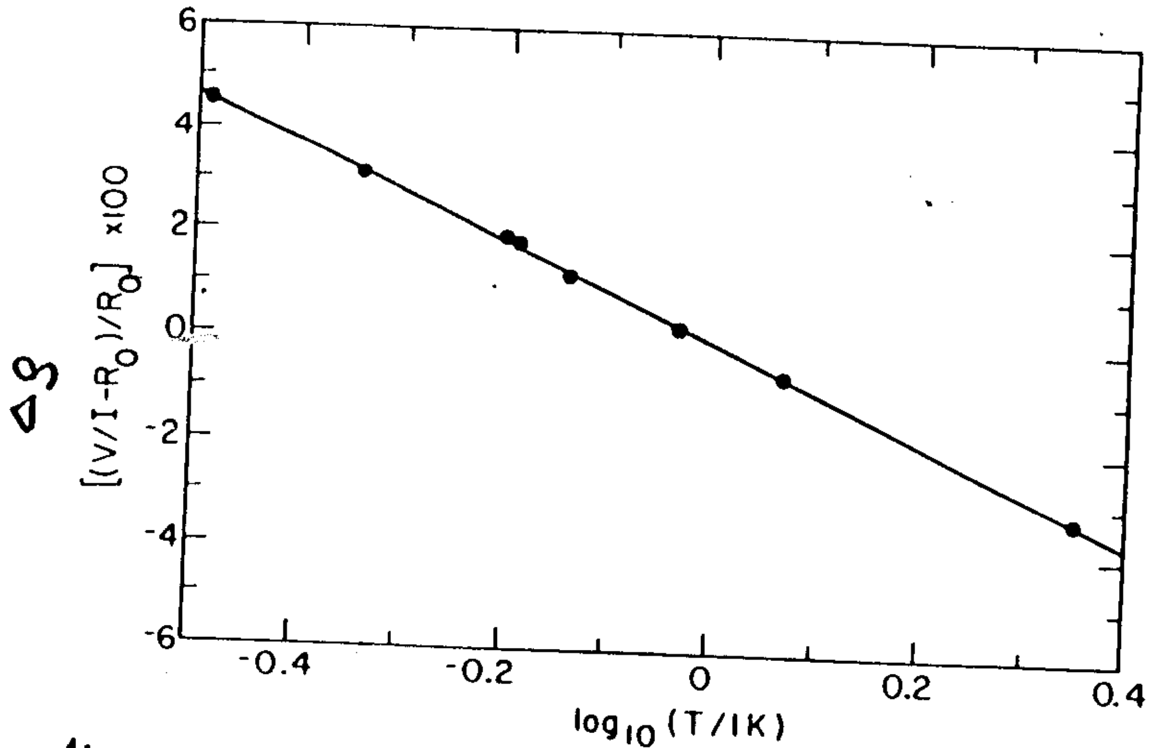


FIG. 4. Resistivity rise plotted vs $\ln T$ for PdAu film (from Dolan and Osheroff, 1979).

$$\frac{d\rho}{dT} \neq 0 \quad \text{at } T = 0$$

There is a non trivial conductivity correction at low temperatures

$$\Delta\rho = \frac{\Delta\sigma}{\sigma_0^2} \propto \log T$$

The correction for <u>2-dimensional</u> thin metallic films is <u>logorithmic</u>!

This correction is called <u>weak localization correction.</u>

- In 3D the weak localization corrections are $\propto \sqrt{T}$ and in 1D wires $\propto 1/\sqrt{T}$ (diverging ?!)

Since 1979 several experiments in 1D, 2D & 3D samples show similar weak localization results. Finally we found universal behavior in disordered world !!!

III 1.2 Weak localization : B - dependence

Most elegant work was done 1982 by Gerd Bergmann (PRL 48, 1046)

Fig 5 shows the magnetoresistance $\Delta \rho_{xx}(B)$ for a thin Mg film (2D) B ⊥ to the film.

The magnetoresistance is negative, but addition of strong spin-orbit scatterer gold (Au) changes the magnetoresistance positive.

Similarly, adding strong spin-flip scatterer (Fe), makes the negative magnetoresistance smaller.

The solid lines in Fig 5 are theory fits (Hikami, Larkin & Nagaoka, Prog Theor Phys 63, 707). Fits are excellent and we seem to understand the weak localization quite well. It is a rich field of different phenomena

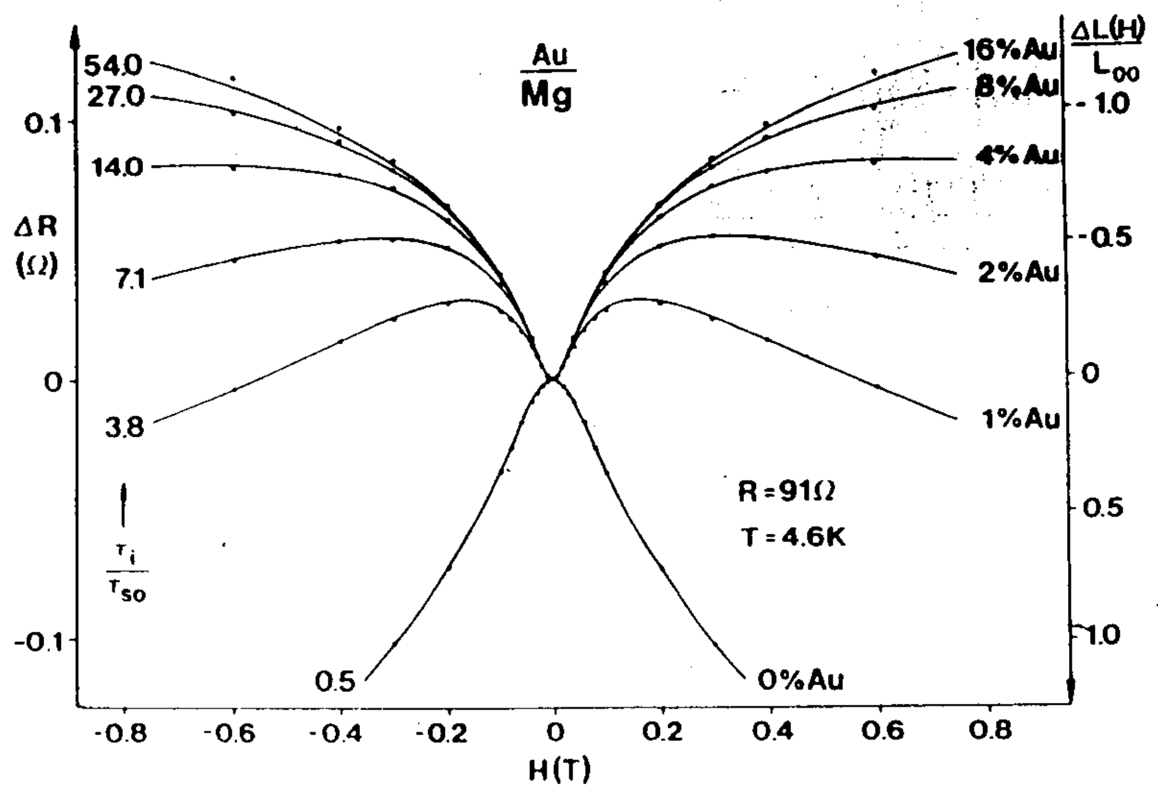
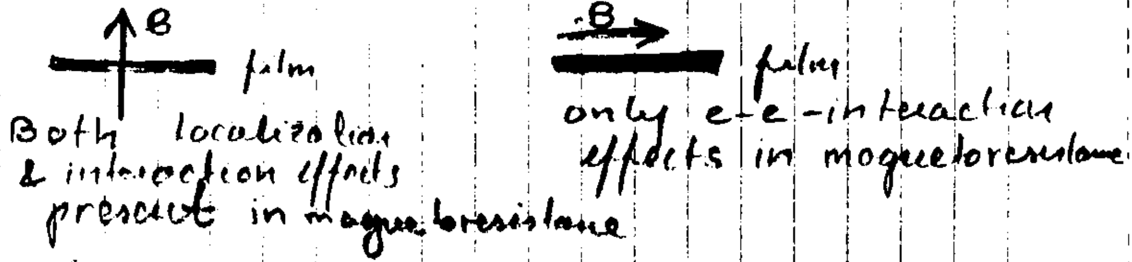


FIG. 17. The magnetoconductance curve of a Mg film with different coverages of Au. [$\Delta L(H)$ is the magnetoconductance, and $L_\infty = e^2/2\pi^2\hbar$.] The coverages shown are in percent of an atomic layer. Increasing Au coverage converts the positive magnetoconductance to negative. Full curves through the data points are fits using the theory of Hikami, Larkin, and Nagaoka (1980). Figure is taken from Bergmann (1982b).

Fig 5

The single electron localization effects are orbital effects and usually larger at small B fields than e-e interaction effects.

Experimentally in 2D one can separate them:



The size of the effect in 2D is:

$$\Delta \sigma \simeq \frac{e^2}{2\pi^2 \hbar}$$

The magnetic field scale is understood in terms of length scales

- dephasing length $L_\varphi = \sqrt{D\tau_\varphi}$
- magnetic length $L_B = \sqrt{\frac{\hbar}{eB}}$
- spin-flip scattering length $L_s = \sqrt{D\tau_s}$
- spin-orbit —"— $L_{so} = \sqrt{D\tau_{so}}$

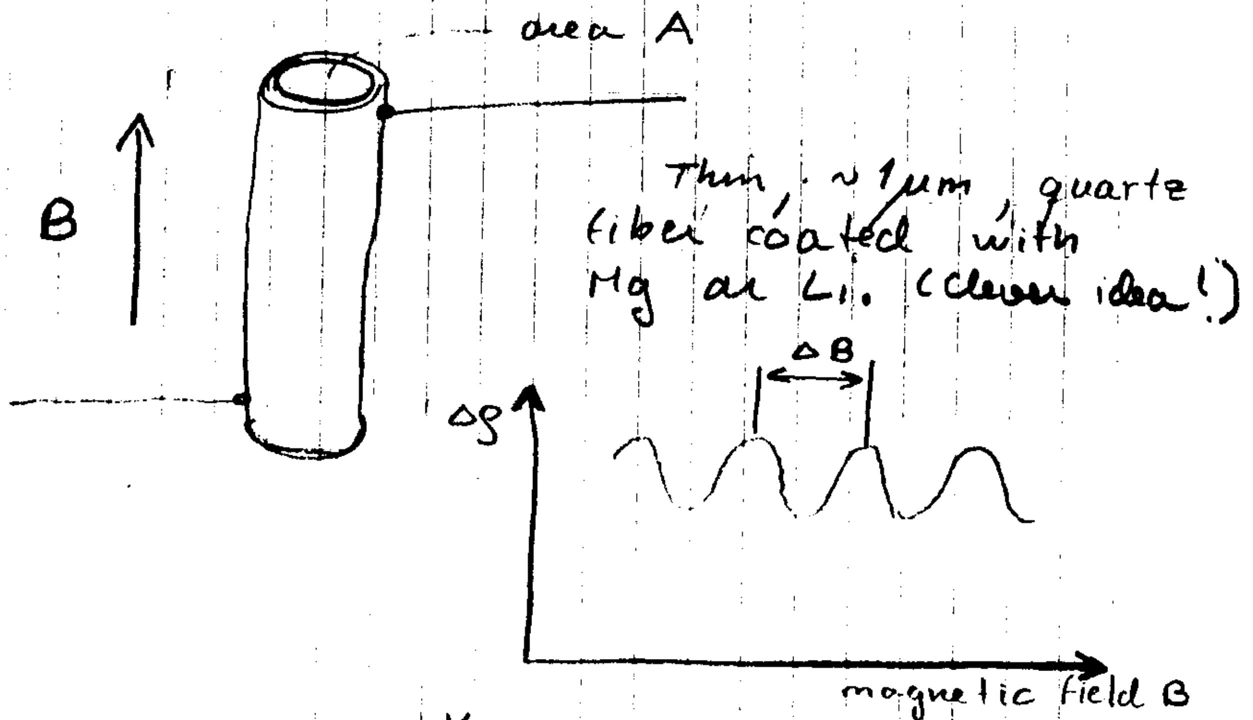
All these processes affect the phase of the electron wavefunction. Most effective dephasing wins and dominates the magnetoresistance

For example if $L_\varphi < L_B \longrightarrow$ $\boxed{B \leq \frac{\hbar}{eD\tau_\varphi}}$ This is small field!!

From the detailed theory fits to experimental data we can get the different scattering rates τ_φ, τ_s, τ_{so}. !!

For example the spin-orbit scattering cross section for an Au atom on Mg is 0.5×10^{-16} cm^{-2} from fig. 5.

III 1.3, Sharvin & Sharvin experiment:
JETP Lett 34 272 (1981)



$$\frac{\Delta \rho}{\rho} \sim 10^{-4}$$

$$\Delta B \simeq \frac{h}{2eA} \Rightarrow \frac{\Delta \phi}{\phi_0} = \frac{A \cdot \Delta B}{\phi_0} = 1$$

$\phi_0 = \dfrac{h}{2e}$ is flux quantum

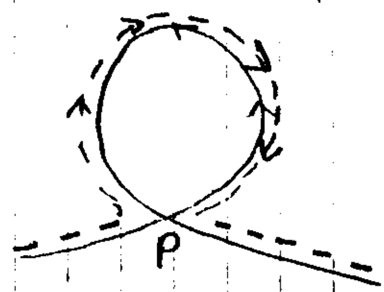
(φ, the phase of single electron, changes by π after it circles one flux quantum)

Simple and clever experiment showing dephasing of normal electron wavefunction

Done in sometimes and not trusted at the beginning in the U +!!

III 1.4. Bergmann's visualization of single electron localization effect
(understandable model for experimentalists)

Feynman path for some scattering events form a self-intersecting loop



Time reversal symmetry | Electron can travel clockwise or counterclockwise from P to P.
The clockwise and counterclockwise wave can interfere. Depending on the dephasing, the QM amplitudes A_{cw} and A_{ccw} are coherent or incoherent.

If L_φ < length of the loop the probability to come back to P is

$$P_{P\to P} = |A_{cw}|^2 + |A_{ccw}|^2 = 2|A|^2$$

If L_φ > Length

$$P_{P\to P} = |A_{cw} + A_{ccw}|^2 = 4|A|^2$$

The coherence increases backscattering probability and resistance. The coherence is destroyed by magnetic field (kills time reversal symmetry), and spin flip scattering.

Relative # of self-intersecting loops small in 3D, somewhat bigger in 2D and 1D. !
Localization effects bigger at lower dimensions !

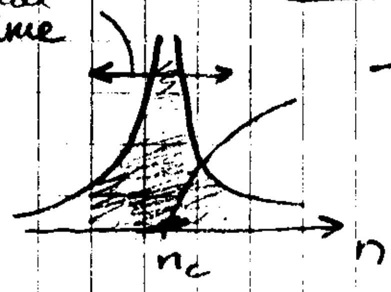
III 2. Strong localization: (Transport)

In principle all 1D wires are insulators (strongly localized) at $T=0$.

2D samples are marginal (logarithmic localization) insulators at $T=0$.

Only bulk 3D-samples are predicted to be MIT as a function of disorder.
Both disorder and e-e interactions important at the transition.

Questions:



- What quantities behave critically ($\to 0$ or $\to \infty$) at the transition?
  - coherence length $\xi \propto (n-n_c)^{-\nu}$
  - conductivity $\sigma \propto (n-n_c)^{\mu}$
  - dielectric constant $\varepsilon \propto (n<n_c)^{x_\varepsilon}$?
  - Hall resistance ρ_{xy} ?
  - specific heat C ?
  - susceptibility χ ?
  - density of states at Fermi level $\frac{dn}{dE}$?

How do the critical quantities scale with the length scale ξ? (critical exponents)

- How wide is critical regime $\Delta n/n_c = \pm(n-n_c)/n_c$, where above critical behaviour is seen.

Experimental difficulties near MIT:

In measuring the critical exponents for small $\Delta n/n_c \gtrsim 1\%$ values:

- are the samples homogeneous enough?
- how can we make samples close to the critical doping n_c.
- can we measure $\Delta n = n - n_c$ accurately enough for $\Delta n/n_c < 1\%$

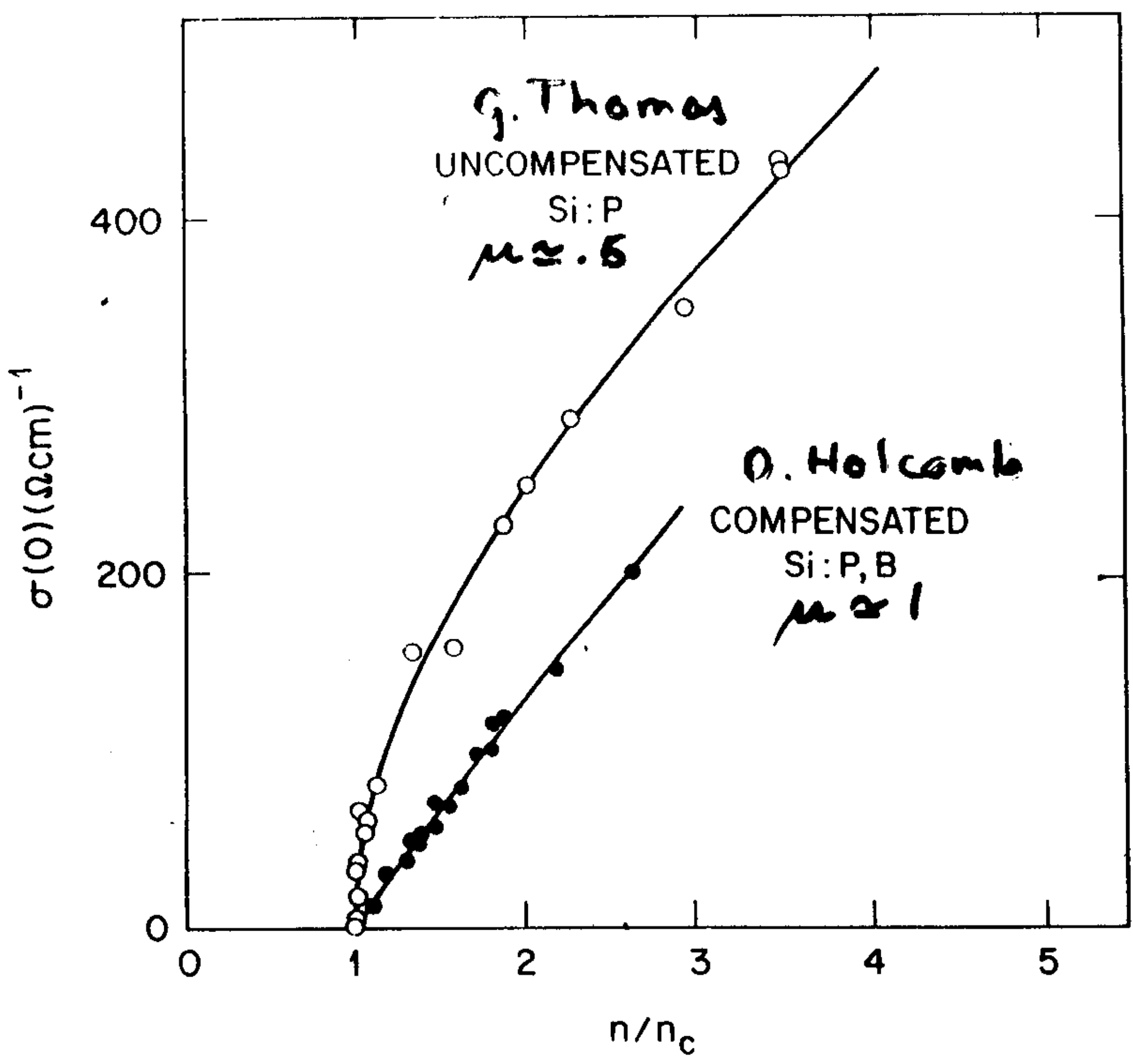
$\Rightarrow$ rounding of otherwise sharp transition

Example: In Fig 6 conductivity of Si:P is measured using samples with different doping density n. $\Delta n/n$ scattering is few %

Experimental solution is <u>continuous tuning of n_c</u> by uniaxial stress or magnetic field.

III 2.1. <u>Uniaxial stress</u> increases a_B^* and decreases n_c in Si:P (1982, PRL 48, 1284). The stress device is shown in Fig. 7. A needle like sample was stressed up to 12 kbar pressure. A slightly insulating sample ($n/n_c \sim 0.90$) was tuned continuously over the critical regime up to $n/n_c(S)$ value of ~ 1.03

This study of MIT of Si:P was done with large collection of separate samples. The relative accuracy of $\Delta n/n_c$ is only a few %. This makes it extremely difficult to study critical phenomena near $n - n_c$.



$$\sigma \propto (n - n_c)^{\mu}$$

Fig. 6

TOP
DO NOT AFFIX OVERLAYS ALONG THIS SURFACE

VG. NO.

TOP
DO NOT AFFIX OVERLAYS ALONG THIS SURFACE

SAMPLE

He

(30)

In Fig 8 we see the critical behavior of σ and $\varepsilon - \varepsilon_{Si} = 4\pi\chi$ (dielectric polarization) (PRL 51, 1896) extremely close to n_c.

From Figs 6 & 8 we get the critical exponents

$\sigma \propto (n-n_c)^{1/2}$
$\chi \propto |n-n_c|^{-1}$ } For uncompensated Si : P

$\sigma \propto (n-n_c)^{1}$ For compensated Si : P

In Fig. 9 we show the T-dependent conductivity $\sigma(T)$ near n_c. We observe tiny $+\sqrt{T}$ correction for conductivity, which changes into $-\sqrt{T}$ correction at higher concentrations (not shown, $\pm\sqrt{T}$ correction is qualitatively understood by perturbation theory). Near n_c the data is believed to have some rounding.

Difficulties:

- The combined data of Figs. 8 & 9 cannot be fitted by one parameter scaling function $F[(n-n_c)^{\alpha} T^{\beta}]$, because the T-dependence is so complicated

- Exponent puzzle: The critical conductivity exponent $\mu \sim 0.5$ is smaller than $2/3$, which is argued to be the lower limit for $\mu = \nu$ (ν 2) [Chayes et al PRL 57, 2999 (1986)]

Si:P Conductivity close to MI transition

$n/n_c - 1$

−0.04 −0.02 0 0.005 0.01

STRESS

Si:P

χ

capacitance measurement of $\chi \propto \varepsilon - \varepsilon_{Si}$

INSULATOR METAL

$10^3/4\pi\chi$

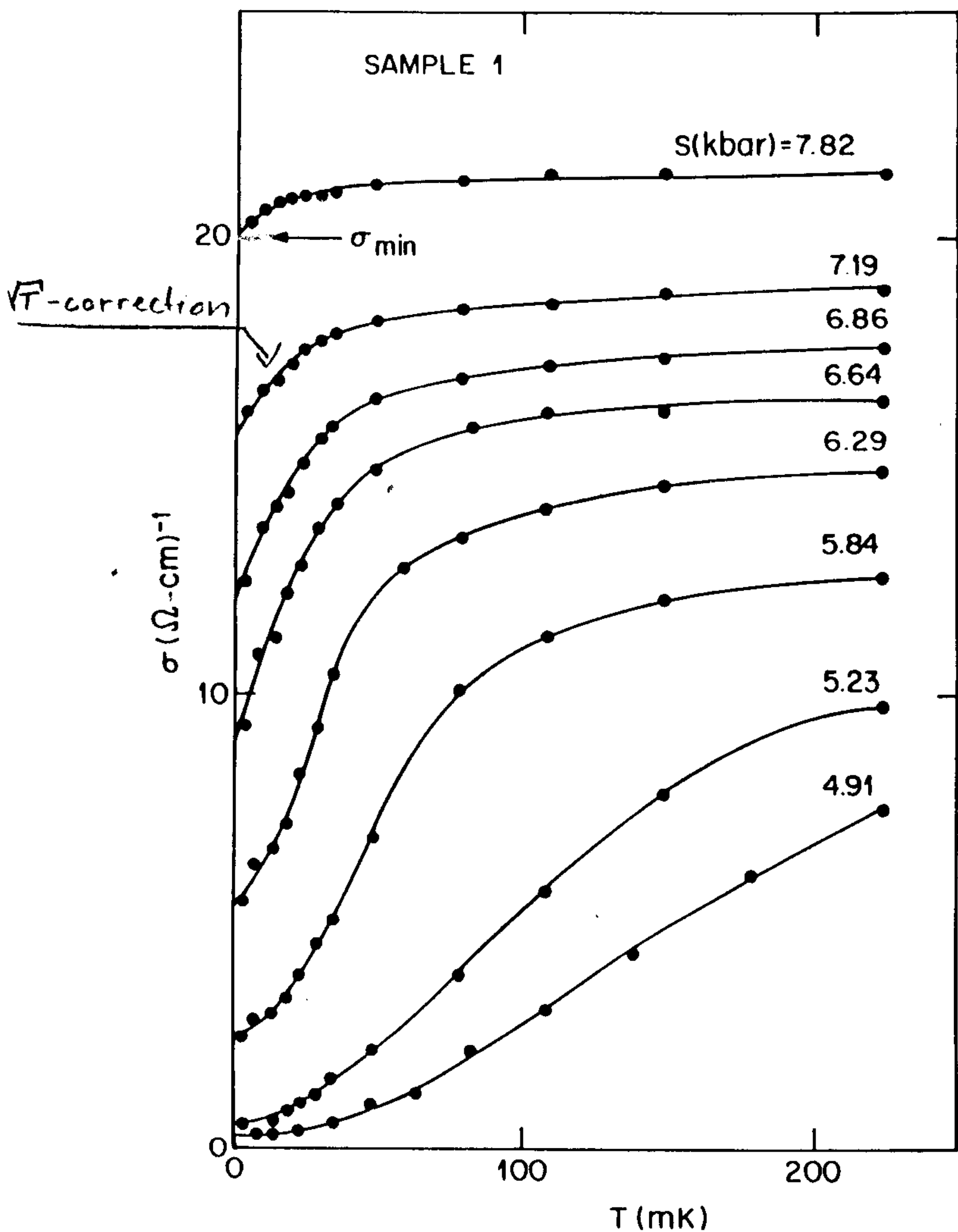
$\sigma^2\ (\Omega\text{-cm})^{-2}$

$S - S_c$ (k bar)

Fig. 8

Uncompressed Si:P. T = 0 K conductivity σ and polarization χ as a function on continuously tuned uniaxial stress. near MIT

Fig 9



Temperature dependence of conductivity near MIT

Possible explanations:

- spin orbit & spin flip scattering are weak in uncompensated Si:P and consequently it belongs at B=0 to so called general universality class. In this class the exponent μ is not well known.

- on page 34 it is shown that at high magnetic field $\mu \rightarrow 1$ (Dai et al). This is predicted by scaling theory (magnetic field universality class)

- The critical regime might be very narrow $(n-n_c) < 0.1$. At $n-n_c \sim 0.1$, $d\sigma/dT$ changes sign and only in regime $n-n_c < 0.1$ one can see $+\sqrt{T}$ conductivity correction and the critical exponent might be $\neq 0.5$, possible even 1.3 (rounding?) (Stupp et al, page 34)

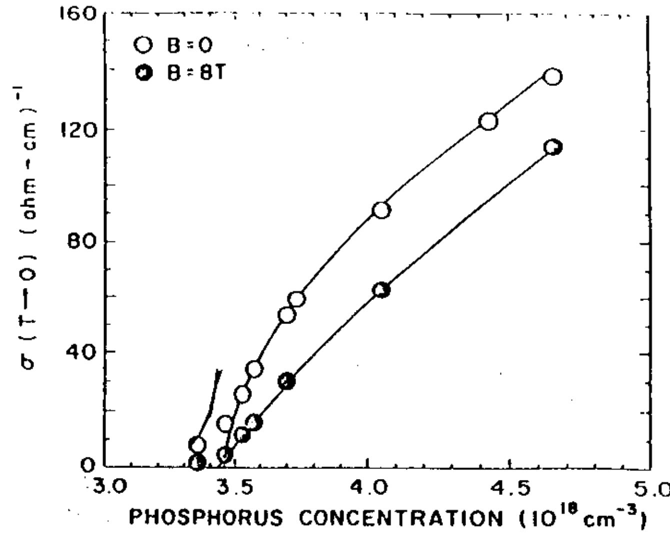
- The exponent 0.5 becomes from fits "over" wide critical regime $n-n_c < 5$. This is suspicious!

More work is needed in Si:P case. (Both theoretical & experimental)

Two recent studies of Si : P

Dai et. al. ?

$\sigma \propto (n - n_c)^{\mu}$, $\mu \to 1$ at high moguetic fields



Stupp et. al. PRL 71, 2634 (1993)

New study of Si:P for $|n - n_c| < .1$

Critical regime $|n - n_c| < 0.1$, small

Critical exponeut is not 0.5 close to n_c.

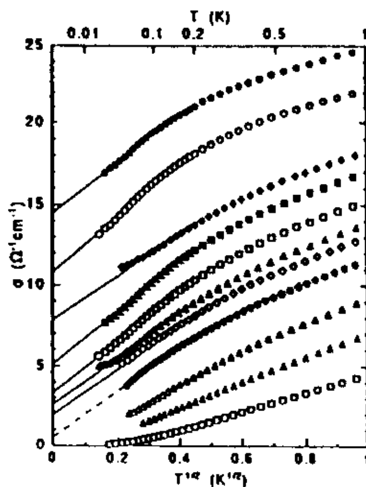


FIG. 1. Electrical conductivity σ vs square root of temperature $\sqrt{T}$ for Si:P samples with P concentration N close to the MIT. Solid lines indicate extrapolation to obtain $\sigma(0)$. The concentrations are (from top to bottom in units of 10^{18} cm^{-3}): 3.69, 3.67, 3.63, 3.60, 3.58, 3.56, 3.55, 3.52, 3.50, 3.45, and 3.38.

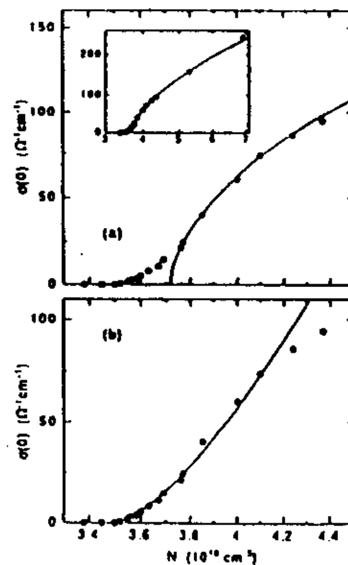


FIG. 2. Extrapolated conductivity $\sigma(0)$ for $T \to 0$ vs P concentration N. (a) Fit with $\mu = 0.55$, $N_c = 3.72 \times 10^{18}$ cm^{-3}. Inset shows the same fit over an extended N range. (b) Fit with 1, $N_c = 3.52 \times 10^{18}$ cm^{-3}.

III 2.2. Continuous tuning of AlGaAs thru n_c with photo-irradiation

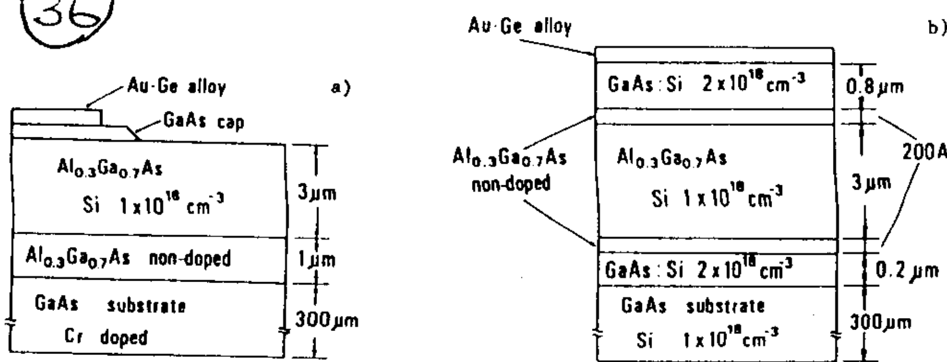
In AlGaAs one can use persistant photoconductivity to tune samples thru MIT at low temperatures. Photo-irradiation increases the number of conduction band electrons and positively charged metastable donor sites

Beautiful results by Dr. Katsumoto on page (36)

- $\Delta\sigma \propto \sqrt{T}$, in critical and outside critical regime.
- $\sigma \propto (n-n_c)^1$
- $\varepsilon - \varepsilon_{GaAs} \propto (n-n_c)^{-2}$

This all is understood to happen in spin-orbit universality class. (spin-orbit scattering is known to be strong in GaAs

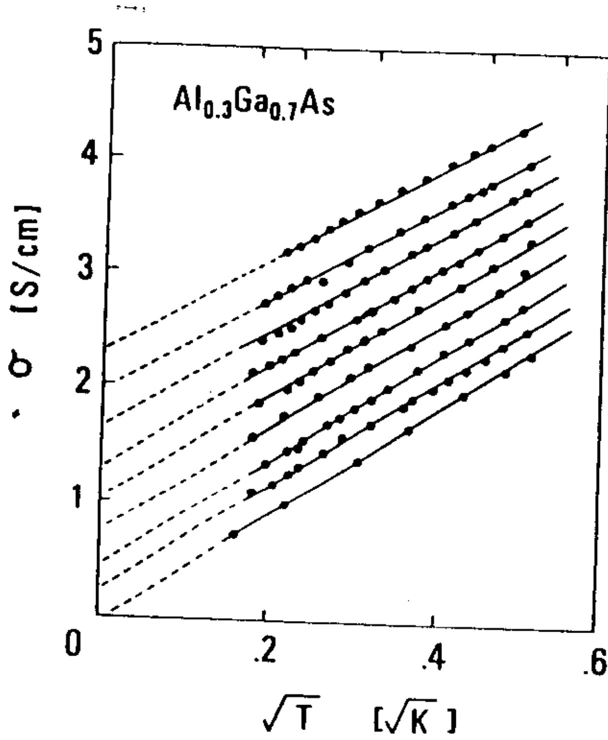
36



(a) Sample A for the conductivity measurement.

(b) Sample B for the dielectric susceptibility measurement.

Fig.1. Cross-sectional views of the epitaxially grown samples.



(b) $\sqrt{T}$ dependence of the conductivity below 300mK in the vicinity of the metal-insulator transition.

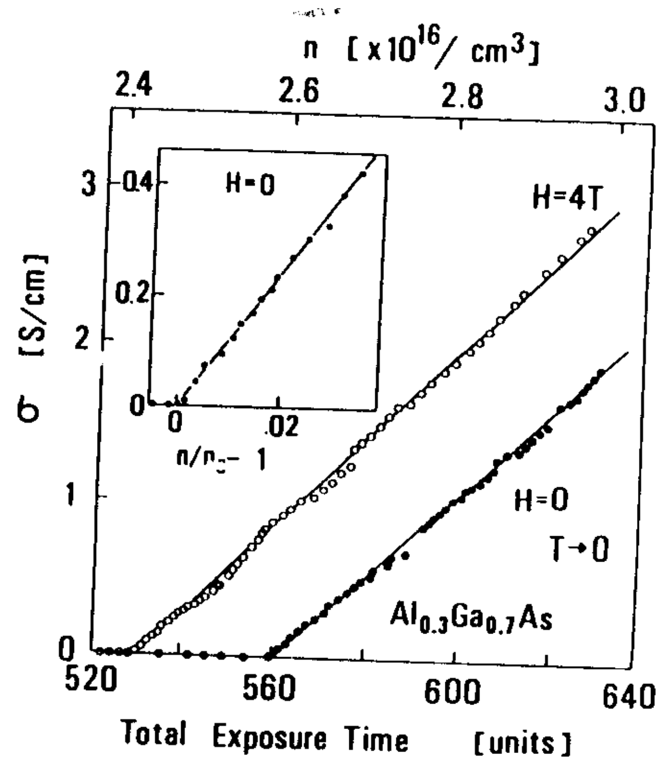


Fig.3 Zero temperature conductivity near the metal-insulator transition in the magnetic field of 4T and OT. The behavior very near the transition in zero field is shown in the inset.

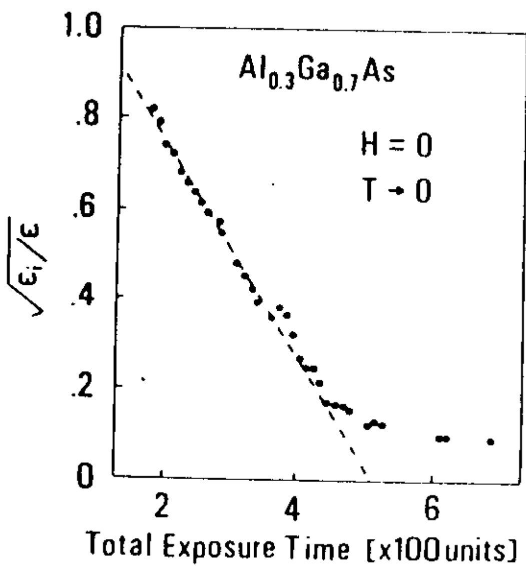


Fig 6 Zero-temperature dielectric susceptibility as a function of the

Continuous tuning of AlGaAs thru n_c with photo-irradiation [S. Katsumoto, Anderson Localization, eds. Ando & Fukuyama, Springer]

III 2.3. continuous tuning of $Nb_x Si_{1-x}$
thin MIT (Hertel et. al PRL 50, 743 (1983))

One can make $Nb_x Si_{1-x}$ samples by sputtering simultaneously Nb and Si sources. By geometrical separation of the sputtered sources one can build up a concentration gradient in x. Notice the preprepared Pb cross stripes, which are used as current and voltage contacts along the concentration gradient.
One can also prepare tunneling junctions for measuring the tunneling density of states at different points of the sample.

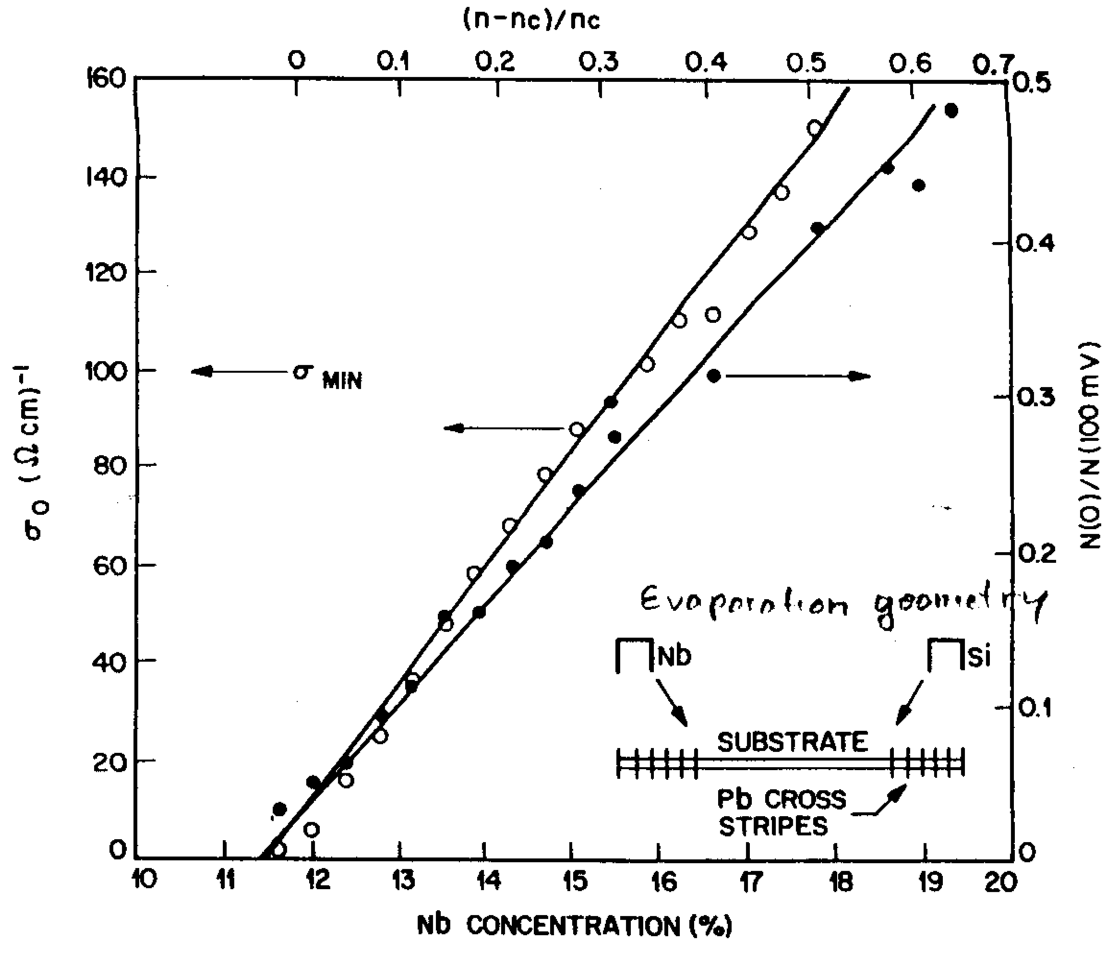
Results are shown on page 38:

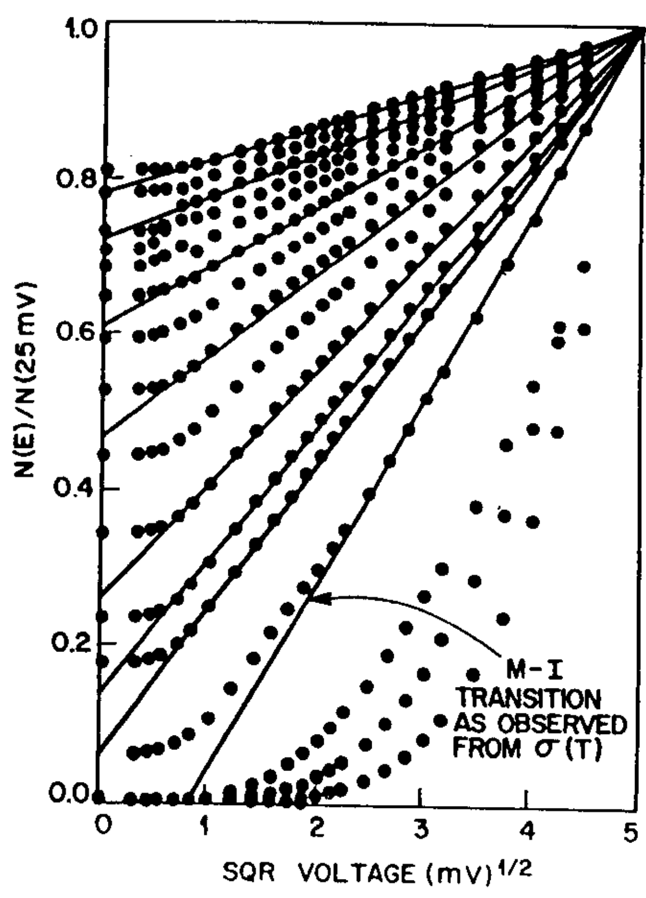
$$\sigma \propto (x-x_c)^1$$

Tunneling density of states $\quad N(0) \propto (x-x_c)^1$

This is in agreement with scaling theory in spin-orbit universality class.

Nb is thought to be strong spin-orbit scatterer





Continuous tuning of $Nb_x Si_{1-x}$ thru n_c with smooth consentration gradient in x.

[Hertel et. al. PRL 50, 743 (1983)]

conductivity:

$\sigma \propto (n-n_c)^1$

Tunneling density of state

$N(0) \propto (n-n_c)^1$

III 2.4. Few comments about Hall coefficient:

$$R_H = \frac{\rho_{xy}}{B} = \frac{\sigma_{xy}}{B\,\sigma_{xx}^2}$$

In classical picture

$$R_H \propto \frac{1}{ne}$$

Should it diverge at MIT because the "# of delocalized electrons" goes to zero?

No general prediction in scaling theory for interacting electrons (Perturbation calculations give σ_{xy} = constant $\Rightarrow R_H \propto (n-n_c)^{-2\mu}$)

Recent beautiful theoretical result by Wang et al [PRL 68, 2504 (1992)] in noninteracting case

Prediction: $\sigma_{xy} \propto (n-n_c)^{\nu(d-2)}$

$$\Rightarrow R_H \propto (n-n_c)^{-\nu(d-2)}$$

So in general we expect critical behavior for R_H.

Hall Coefficient R_H

Difficult to measure

[PRL 55, 1985]

Field & Rosenbaum

compensated Ge:Sb

$\sigma_{xx} \propto (n-n_c)^{0.9}$

$R_H \propto (n-n_c)^{-0.69}$

critical behavior

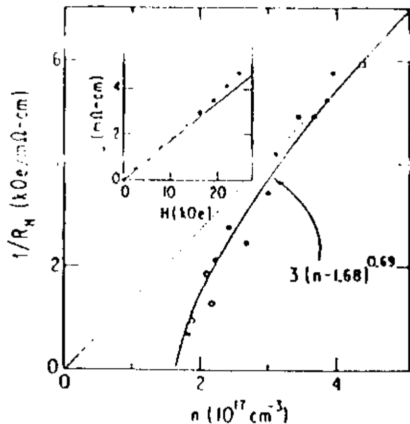


FIG. 3. Critical behavior of the inverse of the Hall coefficient R_H. Theory predicts no dependence on n. We determine R_H from the slope of the Hall resistivity ρ_H vs magnetic field H in the linear regime, as plotted in the inset for $n = 3.95 \times 10^{17}\ cm^{-3}$ sample.

[PRL 60, 1988]

Koon & Castner

uncompensated

Si:As, Si:P

$\sigma_{xx} \propto (n-n_c)^{0.6}$

$R_H \propto (n-n_c)^{0}$

non critical behavior

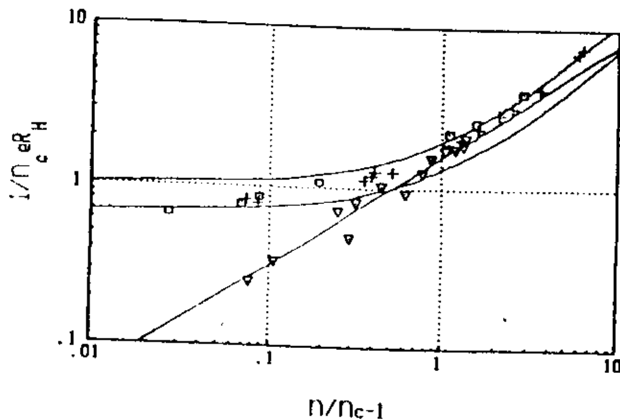


FIG. 3. $1/n_c e R_H$ vs $n/n_c - 1$ for Si:As (squares), Si:P (plusses) (after Ref. 11), and Ge:Sb (triangles) (after Ref. 6) samples. The two calculated lines flattening for small $n/n_c - 1$ are for Hall correction factors of 1 and 1.5, respectively. The straight line $1/R_H \propto (n/n_c - 1)^{0.69}$ is a best fit to the Ge:Sb data.

R_H critical:

Ge:Sb — compensated doped semic.

...orphous { Bi:Kr, Nb:Si, Pt:Si, (Ga.. Bi..) A..

R_H noncritical:

Si:As, Si:P } uncomp. doped semic

Ar:Ga, In_2O_3 } amorphous

Hall coefficient R_H

Situation is unclear

Speculations :

- R_H noncritical when $\mu = 1/2$
- R_H critical for strong SO scattering

New results :

[PRL 70, 1993]

1) Dai, Saraclik et al.

$\Longrightarrow$

R_H critical also when $\mu = 1/2$ (Si : B)

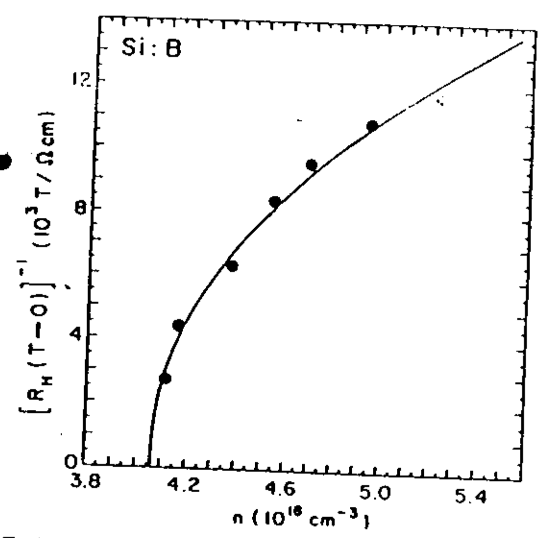


FIG. 3. Zero-temperature extrapolations of the Hall number plotted as a function of dopant concentration. The solid line is a best fit by Eq. (2) with $n_c = 4.06 \times 10^{18}$ cm^{-3} and $\mu_H = 0.45$.

[PRL 68 1992]

2) Wang, Wang, Kotliar & Castellani

R_H critical also for noninteracting single electron case !

Where is noncritical behavior of R_H coming from ?

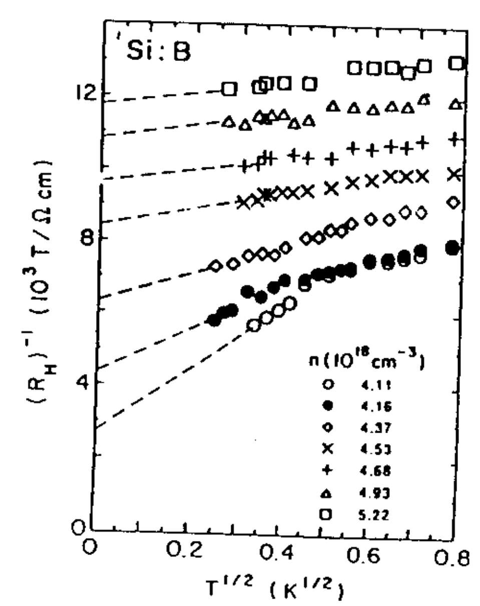


FIG. 2. The Hall number $(R_H)^{-1}$ as a function of $T^{1/2}$ for seven Si:B samples with dopant concentrations as labeled. The