INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P,P.O. BOX 586,34100 TRIESTE,ITALY,CABLE:CENTRATOM TRIESTE

SMR. 758 - 4

SPRING COLLEGE IN CONDENSED MATTER
ON QUANTUM PHASES
(3 May - 10 June 1994)

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SIMPLE (BUT IMPORTANT) IDEAS ABOUT THE QUANTUM MANY-BODY PROBLEM

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These are preliminary lecture notes, intended only for distribution to participants.

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SIMPLE (BUT IMPORTANT) IDEAS
about the quantum many-body problem

- Subject of this school:

  Physics of large number of "objects" interacting with one another when quantum effects are important

  Here "object" may stand for:

  - FERMIONS (spin + charge/translation d.o.f)

    electrons in a metal

    ^{3}He atoms, ...

  - BOSONS

    ^{4}He atoms, ...

  - LOCALISED SPINS on some lattice

    electrons in an insulator

  ... OTHER ANIMALS

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Spring College in Condensed Matter on Quantum Phases , 3 May - 10 June 1994

TENTATIVE PROGRAMME *as of 26 April 1994*

WEEK 1 - Tuesday 3 May to Saturday 7 May :

A. Georges, R. Shankar, V. Kravtsov, P. Wölfle, G. Thomas

WEEK 2 - Monday 9 May to Friday 13 May :

P. Nozieres, G. Thomas, S. Sachdev, B. Jones, P. Wölfle

WEEK 3 - Monday 16 May to Friday 20 May :

J. Jolicoeur, A. MacDonald, S. Sachdev, M. Shayegan, G.G. Lonzarich

WEEK 4 - Monday 23 May to Friday 27 May :

D. Fisher, R. Singh, M. Shayegan, T. Giamarchi, S. White, M. Paalanen

WEEK 5 - Monday 30 May to Friday 3 June :

M. Paalanen, S. White, R. Bhatt, G. Kotliar, A. Georges, S. Girvin

WEEK 6 - Monday 6 June to Friday 10 June :

J. Chalker, R. Bhatt, N. Read, S. Girvin

BHATT, R.	Metal-insulator transition in disordered materials (theory); the quantum Hall effect (numerical studies)
CHALKER, J.	The quantum Hall effect: numerical methods
FISHER, D.S.	Random quantum magnets
GEORGES, A.	Fermi liquid theory of strongly correlated fermions
GIAMARCHI, T.	Interacting fermions in one dimension. Disordered bosons
GIRVIN, S.	Superconductor-insulator transition. Scaling in the quantum Hall effect
JOLICOEUR, J.	Quantum antiferromagnets: field-theory methods
JONES, B.	Numerical renormalization Group studies of Kondo problems
KOTLIAR, G.	Soluble limits of itinerant electron problems: large N and large D methods Scaling approach to metal-insulator transition with electron interactions
KRAVTSOV, V.	Quantum coherence effects in disordered metals: Anderson localization and mesoscopic fluctuations
LONZARICH, G.G.	Magnetic phase transitions at low temperatures
MACDONALD, A.	The quantum Hall effect: theory
NOZIERES, P.	Infrared singularities: X-ray edge, Kondo effect, heavy particles etc.
PAALANEN, M.	The metal-insulator and the superconductor-insulator transitions in disordered systems: experiment
READ, N.	Quantum Hall effect
SACHDEV, S.	Quantum phase transitions. Quantum spin systems
SHANKAR, R.	Introduction to renormalization Group and Fermi-liquid theory
SHAYEGAN, M.	Magnetotransport experiments in quantum Hall effect
SINGH, R.	Series expansions for quantum spin systems
THOMAS, G.	Optical (and transport) studies across the metal insulator transition in semiconductors, high Tc oxides and vanadium oxide
WHITE, S.	The density matrix renormalization group
WÖLFLE, P.	Fermi liquid theory, superconductors and superfluids

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- ✱ FULL SPECTRUM (eigenenergies/wave functions) is:
  - huge : $\# \propto e^N$
  - generally unknown
  - sensitive to all sorts of details
- ✱ INSTEAD, one is interested in:
  - Some (often approximate) understanding of the ground-state $|\phi_0\rangle$
  - Most importantly, some description of the low-energy excited states
- ✱ WHY?
  - controls low-T thermodynamics
  - controls response to (weak) external fields
  - have some degree of universality
- ✱ AN IMPORTANT DISTINCTION

E_1 energy of 1st excited state, finite volume V

$$? \left\langle \begin{array}{lll} E_1 - E_0 \longrightarrow 0 & \text{as } V \longrightarrow \infty & \text{GAPLESS} \\ E_1 - E_0 \longrightarrow \Delta_g \neq 0 & \text{"} & \text{GAP} \end{array} \right.$$

* "UNIVERSALITY"

Low-energy excited states are unsensitive to 'details' (of the interactions, lattice, ...) because they probe large distance / long time properties -

dispersion relation : $(E_n - E_0 = \hbar\omega)$

$$\omega \sim k^z \quad (z > 0)$$

* EFFECTIVE LOW-ENERGY THEORY

[Masless] field theory describing low-energy excitations, derived by integrating out high energy / short distance degrees of freedom

→ RENORMALIZATION GROUP

R. SHANKAR
P. Nozieres
Th. Giamarchi
B. Jones
S. White

$\left.\begin{array}{l}\text{B. Jones}\\ \text{S. White}\end{array}\right\rangle$ Numerical implementation

* 'BARE' STRENGTH OF INTERACTIONS

vs. EFFECTIVE INTERACTIONS AMONG 'QUASIPART.

SOME EXAMPLES [OVERVIEW]

• LANDAU - FERMI LIQUIDS

Interacting fermions in 3D (also 2D?)

$$\hat{H} = \text{K.E} + \text{interactions}$$

- Low energy excitations are 'quasiparticles' ('quasiholes') carrying spin AND charge degrees of freedom.
- dispersion relation $\omega \propto v_F (k-k_F)$
- Effective theory is a free-field theory (gaussian fixed point)
- Low ω, T behaviour qualitatively = free Fermi gas, with quantitative changes of parameters.

$$C \sim \gamma T \qquad \chi \sim \text{const.} \qquad \dots$$

(note: changes may be BIG cf 'heavy-fermions' compounds)

P. WÖLFLE
R. SHANKAR

$\left.\begin{array}{l}\text{G. Kotliar}\\ \text{A. G}\\ \text{G. THOMAS}\end{array}\right\}$ application to strongly correlated systems

A celebrated example of a Fermi-liquid state:

- THE KONDO EFFECT -

reavy fermion compounds?

{ NOZIERES
{ JONES

$$\hat{H} = \underbrace{\sum_{k\sigma} \varepsilon_k c^\dagger_{k\sigma} c_{k\sigma}}_{\text{conduction electron bath}} + J\, \underset{\substack{\uparrow \\ \text{spin-}\frac{1}{2} \\ \text{impurity (localised at } R=0)}}{\vec{S}} \cdot \left[c^\dagger_\alpha(\vec{R}=\vec{0}) \frac{\vec{\sigma}_{\alpha\beta}}{2} c_\beta(\vec{R}=0) \right]$$

Under Renormalization:

J_{eff} (scale) $\rightarrow \infty$ as scale $\rightarrow \infty$

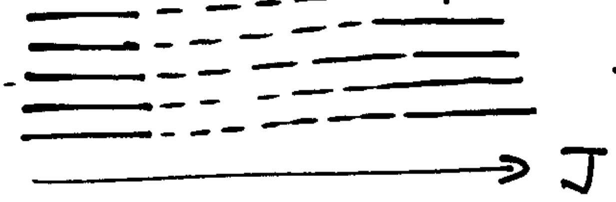
(strong coupling fixed point).

Physical picture: $(J > 0)$

Impurity spin 'binds' to conduction electron to form a singlet state with $\simeq 0$ residual scattering → local Fermi liquid

Hence:

- $\delta C_{imp} \propto T$ $\quad \delta \chi_{imp} \propto$ const.
- Universal form of finite size spectrum:



$$\frac{L\Delta E}{\pi v_F} = 0, \frac{1}{2}, 1, \frac{3}{2}, \dots$$

COMPLICATIONS OF THE MODEL LEAD TO NON-TRIVIAL FIXED POINTS.

• 'LUTTINGER' LIQUIDS'

Interacting fermions in 1D

Quasi one-dimensional conductors

e.g TTF-TCNQ, ...

→ Th. GIAMARCHI (week 4)

Elementary excitations of 2 kinds:

- spin $\frac{1}{2}$, no charge $\omega \propto v_s \delta k$
- charge $\pm e$, no spin $\omega \propto v_c \delta k$

Hence spin-charge is deconfined, and the effective theory is no longer that of a free fermionic theory.

Instead, continuous family of free bosonic theories:

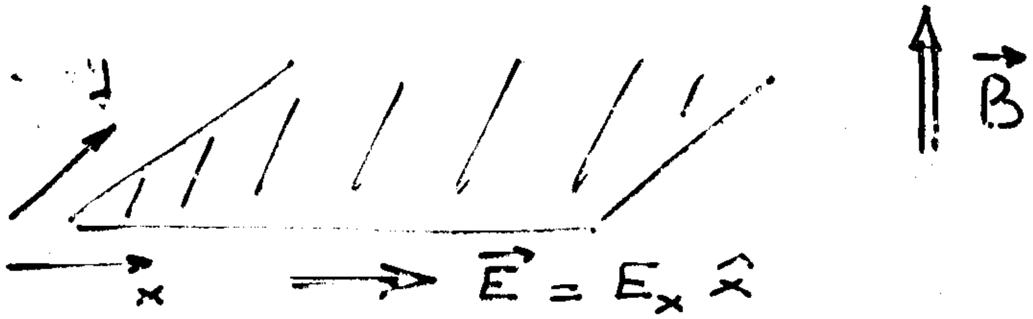
$$\Psi \propto e^{i\left[\phi - \int_{-\infty}^{x} \frac{\partial \phi}{\partial \tau}\right]}$$ Bosonisation

$$S_{eff} = \int dx \int dz \; v_c\left[K_c(\partial_\tau \phi_c)^2 + \frac{1}{K_c}(\nabla\phi_c)^2\right] + v_s\left[K_s(\partial_\tau \phi_s)^2 + \frac{1}{K_s}(\nabla\phi_s)^2\right]$$

exponents depend continuously on K_c, K_s.

• QUANTUM HALL EFFECT

A 2-DIMENSIONAL gas of electrons submitted to a strong magnetic field



Hall current $J_y = -\sigma_{xy} E_x$

$$\sigma_{xy} = -\frac{n}{m}\frac{e^2}{h}$$

→ R. Bhatt, J. Chalker, S. Girvin
A. Mc.Donald, N. Read, M. Shayegan

Incompressible fluid state. Quasiparticle excitations have fractional charge and fractional statistics ("anyons").

Effective field theory involves a gauge field w/ topological (C-S) term.

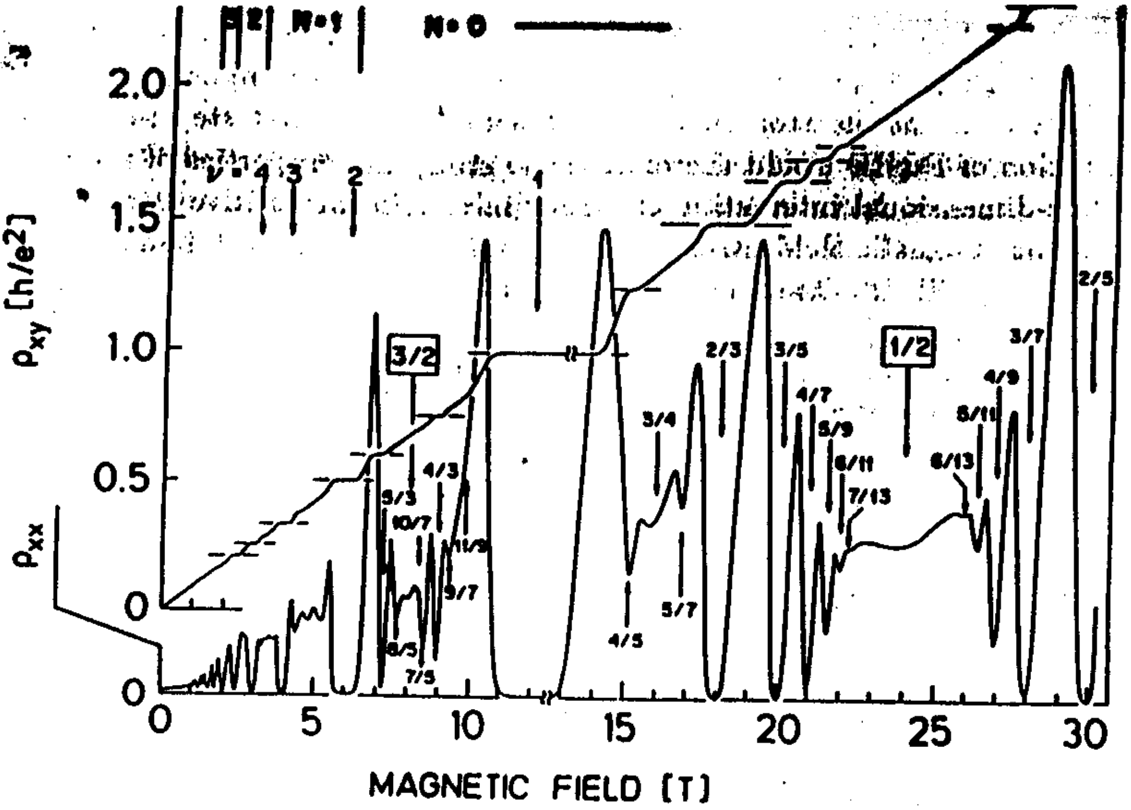


Fig. 1.3. Overview of the observed fractions in the FQHE measurements. The Landau level filling factor ν has been defined in the text [1.18]

excitation spectrum. The striking feature of the above results is that all fractional fillings appear with *odd* denominators. In Fig. 1.3, we have reproduced the latest results on the FQHE, where several new (as well as established) fractions have been resolved. FQHE has also been observed in high-mobility Si-MOSFET samples.[3]

In explaining the FQHE, the system of noninteracting electrons is, however, inadequate. According to our present understanding of FQHE, electron correlations play a major role in this effect, and there have been a variety of theoretical attempts to understand this unique many-electron phenomenon. In this volume, we have attempted to survey most of these theoretical approaches, and have tried to present in detail the current state of our understanding of this fascinating effect.

There has been another surprise recently in FQHE. In the higher Landau level (n=1), *Willet* et al. [1.18] have discovered a fractional Hall plateau at

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[3] We have not attempted to review the experimental work on FQHE by various groups; these can be found e.g. in [1.6] and [1.17].

• QUANTUM MAGNETS

- Localised spin degrees of freedom on some lattice

  Prototype: Heisenberg model

$$\hat{H} = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$[S_i^x, S_i^y] = i\hbar S_i^z$$

- Breaking of a continuous symmetry
- ⟹ gapless excitations

  SPIN-WAVES (Goldstone modes)

  $\omega \propto k^2$ ferromagnet

  $\omega \propto ck$ antiferromagnet

- Effective field theory of AF spin waves:

  non-linear σ-model

$$S = \frac{1}{g} \int d^d x \int_0^\beta d\tau \left[(\nabla \vec{n})^2 + \frac{1}{c^2} \left(\frac{\partial \vec{n}}{\partial \tau} \right)^2 \right]$$

→ T. JOLICOEUR
S. SACHDEV
R. SINGH
S. WHITE
D. S. FISHER

- QUANTUM PHASE TRANSITIONS -

- Go from one phase to another as a function of :

  Temperature
  Magnetic field
  Doping / alloying
  etc...

These parameters control the relative amount of thermal / quantum fluctuations

- When $T=0$ quantum fluctuations are not too strong, lowering T generally drives the system in some broken-symmetry phase with long-range order :

  - spin density wave
  - charge density wave
  - superconductor / superfluid

  etc...

- Diverging length scale ξ / time scale ξ_τ

→ Field theory approach of critical behaviour (w/ anisotropic scaling).

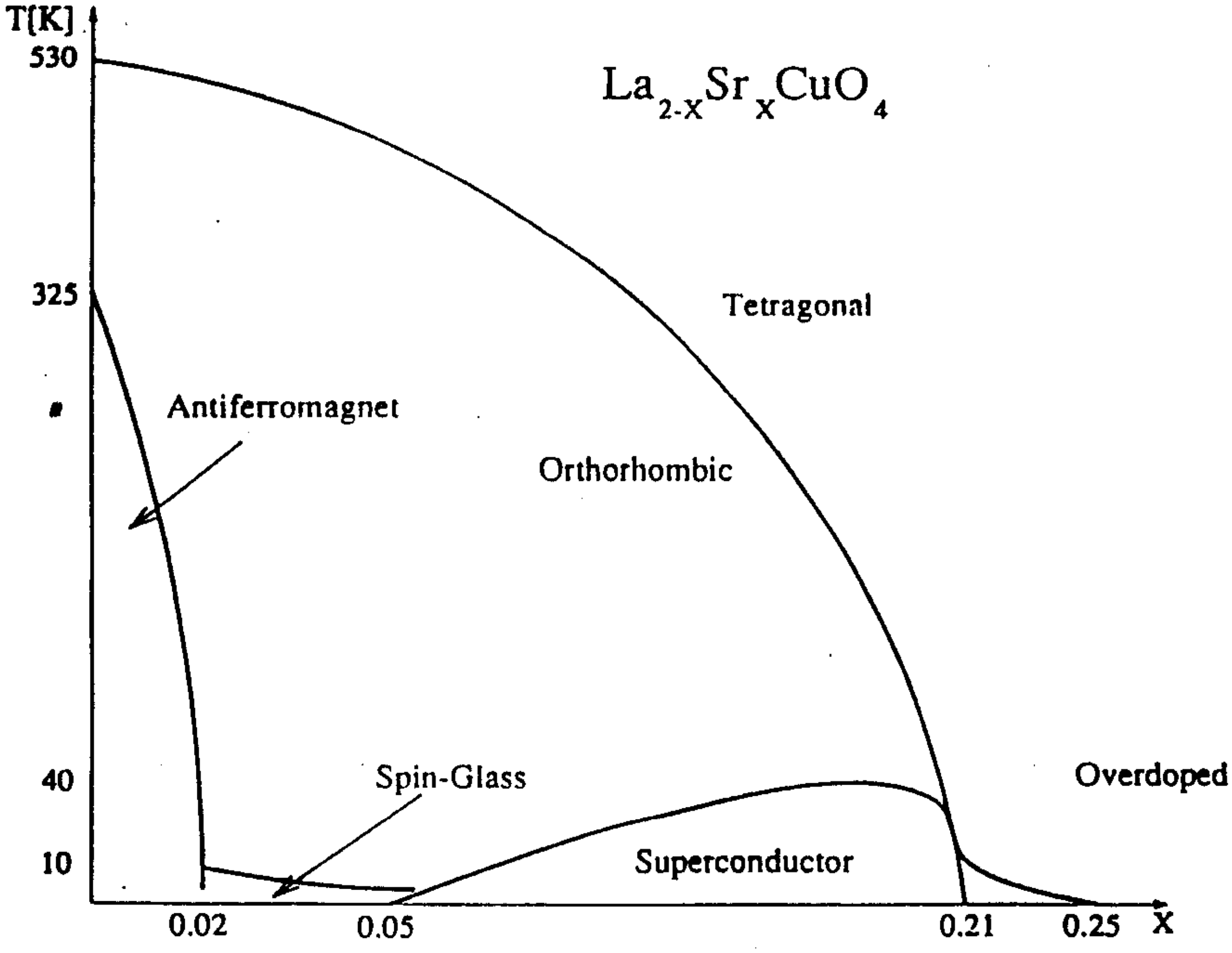

T[K]
530
325
40
10
$La_{2-x}Sr_xCuO_4$
Tetragonal
Antiferromagnet
Orthorhombic
Spin-Glass
Overdoped
Superconductor
0.02
0.05
0.21
0.25
X

Fig. 1

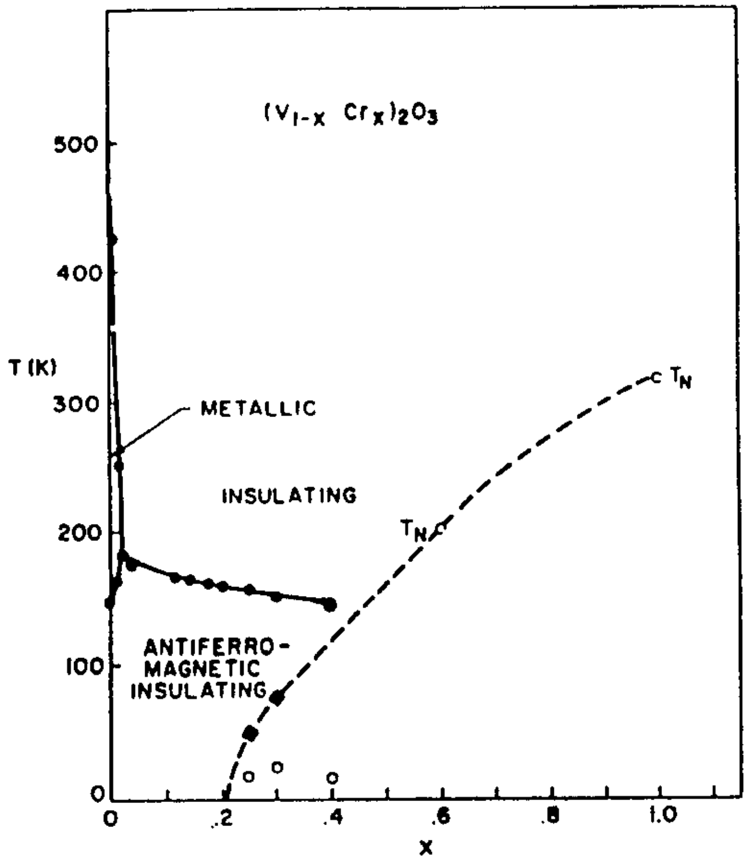


FIG. 3. Phase diagram of $(V_{1-x}Cr_x)_2O_3$ as deduced from magnetic susceptibility data shown in Fig. 1: closed circles, transition points (or midpoint of transition regions); open circles, maxima in the susceptibility as a function of temperature; and closed squares, temperature where the susceptibility starts to increase again with decreasing temperatures.

ponding to 2.69 μ_B per
the metallic phase (M)
rors a line calculated
$= -600$ °K, $p = 2.37\mu_B$
and Arnold and Mires.
anomaly, which appea
maximum in the susce
plot, is now clearly vi
strate, in agreement w
very clearly the existe
elevated temperatures
indicated by the work

Susceptibility measu
single crystals show a
$\chi_{\parallel} = 0.22 \times 10^{-3}$ (cgs/mo
Silverstein and Jacobs
terms of a Van Vleck
value to evaluate our h
Cr_2O_3 according to Eq
the paramagnetic Curi
the effective Bohr mag
$p = 3.6\mu_B$. This shoul
only value of $3.87\mu_B$ fo
These values are repr
Foex and Graff[16] found
result for the Van Vle
constant correspondin
Since the Van Vleck su
and Cr_2O_3 are almost

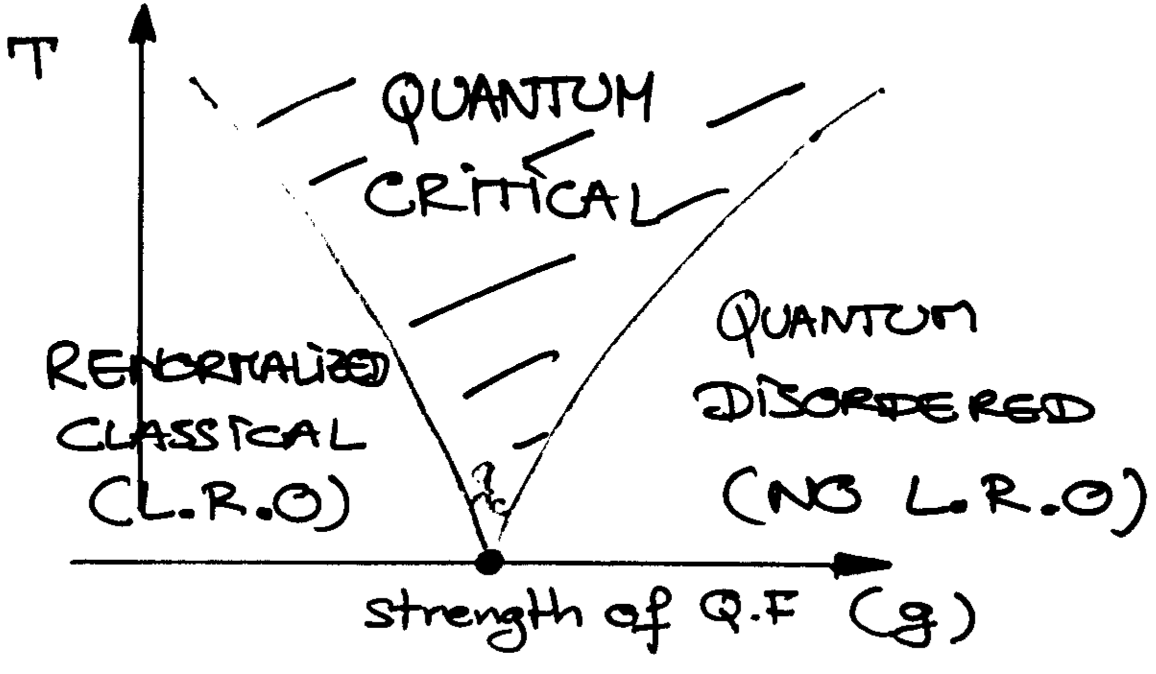
- ALSO, possibility of phase transitions <u>at T=0</u>, driven solely by the strength of quantum fluctuations.

e.g. phase with LRO / broken symmetry

$\Big\downarrow$ increasing Q.F

disordered ground-state at T=0

T=0 quantum critical point



→ S. SACHDEV
S. GIRVIN
T. JOLICOEUR
G. LONZARICH
R. SINGH
...

- EFFECT OF DISORDER

Impurities, Defects in lattice, substitutions, ...

- May induce radical changes already in one-particle wavefunctions.

e.g Anderson Localization
Metal / Insulator transition
→ Kravtsov
Kotliar
Paalanen
Bhatt

- May induce new type of long-range order (e.g spin-glass) -
→ DS Fisher

- Requires specific techniques
(Perturbation around ordered system often not recommended!)

- Problems w/ both strong disorder and strong interactions are still rather poorly understood.