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Background information for lectures on

SOLUBLE LIMITS OF ITINERANT ELECTRON PROBLEMS: LARGE N AND LARGE D METHODS

SCALING APPROACH TO METAL-INSULATOR TRANSITION WITH ELECTRON INTERACTIONS

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These are preliminary lecture notes, intended only for distribution to participants.

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Landau Fermi Liquid Theory for Disordered Systems and the Scaling Theory of the Metal-Insulator Transition

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We pedagogically outline a Fermi liquid framework to describe interacting electrons in a random potential in complete analogy with Landau's theory for translationally invariant system and use this language to interpret the different physical scenarios for the metal insulator transition.

1. LANDAU THEORY: CLEAN CASE

Landau Fermi liquid theory provides a rigorous description of the low energy long wavelength properties of strongly interacting Fermi systems. The description of strongly interacting Fermions in terms of weakly interacting quasiparticles (QP), originally formulated by Landau[1] for translationally invariant systems has also been used to describe finite systems like atomic nuclei[2]. In this talk we will outline a Fermi liquid framework to describe interacting electrons in a random potential which closely parallels Landau's construction. This framework is useful to parametrize all the response functions of a strongly disordered metal and naturally leads to the scaling variables relevant to the metal insulator transition originally proposed by Finkelstein[3] and developed by Castellani, et al.[11,12] The purpose of these notes is pedagogical. We shall first review the key concepts of the Landau approach in the translational invariant case, and then we will indicate how a very similar construction can be carried out for the disordered electron gas. We conclude by interpreting the different scenarios for the metal insulator transition (MIT) in terms of the QP parameters. This exposition is based on refs. 4 and 5. Refs. 6 and 7 and in particular ref. 8, contain earlier references, and stress the relevance of Fermi liquid ideas to disordered Fermi systems. Ref. 9 contains excellent reviews of Landau theory.

Basic to Landau's theory is the idea of quasiparticles which are well defined low lying excitations of a many body system. In a translational invariant system the QP states are labelled by their momenta k, their energy is given by $\varepsilon_k = k^2/2m^*$. The interactions are taken to invariant under spin rotations and σ is a spin index. The energy of a weakly excited state characterized by a definite number of QP excitations $n_{p\sigma}$ in a unit volume, is written as

$$F = \sum_{p\sigma} (\varepsilon_p - \mu)\, \delta n_{p\sigma} + 1/2 \sum_{pp'\sigma\sigma'} f_{p\sigma,p'\sigma'} \delta n_{p\sigma} \delta n_{p\sigma'} \qquad (1)$$

$f_{p\sigma,p'\sigma'}$ describes the interactions between the QP while $\delta n_{p\sigma} = n_{p\sigma} - n^{o}_{p\sigma}$ is the deviation of the QP occupation number from the equilibrium value $n^{o}_{p\sigma}$ which is given by the Fermi distribution. In the presence of an external field V_σ it obeys a linearized Boltzman equation which in the collisionless regime becomes

$$\frac{\partial \delta n_{p\sigma}}{\partial t} + v_p \frac{\partial \delta n_{p\sigma}}{\partial r} + F_{p\sigma} \frac{\partial n^{o}_{p\sigma}}{\partial p} = 0 \qquad (2)$$

which in turn can be derived from the equations of motion $p/m^* = v_p = \partial H/\partial p$, $F_{\sigma p} = -\partial H_\sigma/\partial r$ for the semiclassical Hamiltonian

$$H_\sigma = \frac{p^2}{2m^*} + \sum_{p'\sigma'} \delta n_{p'\sigma'} f_{p\sigma,p'\sigma'} + V_\sigma(r). \qquad (3)$$

The kinetic equation can be used to derive the spin-spin, density-density and heat-heat correlation functions. Defining the Landau paramters F_o^s, F_o^a by

$$2\rho_Q f_{p\sigma p'\sigma'} = \sum_\ell P_\ell(\cos\theta)\,(F_\ell^s + \sigma\sigma' F_\ell^a) \qquad (4)$$

with $\rho_Q = m^* k_F/2\pi^2$ the QP density of states per unit spin in 3-dimensions. Solving (2) for the response functions one gets in the static the well known results for the compressibility, susceptibility and specific heat.

$$\frac{dn}{d\mu} = \frac{2\rho_Q}{1+F_o^s} \qquad \chi = \frac{2\rho_Q \mu_B^2}{1+F_o^a} \qquad C_V = \frac{2\pi^2 \rho_Q T}{3}. \qquad (5)$$

The microscopic justification of Landau theory is based on two key insights. The first focuses on the pole structure of the one particle Green's function for small ω and $\varepsilon_k - \mu$:

$$G(k,\omega) = \frac{a}{\omega - (\varepsilon_k - \mu) + i\ \mathrm{sign}\,\omega/2\tau_{in}}. \qquad (6)$$

The location of the pole ε_k defines the quasiparticle energy, the residue a is identified as the quasiparticle weight while τ_{in} is the inverse width of the level and diverges as $\varepsilon_k - \mu$, ω and the temperature goes to zero. The second insight is that the pole structure in G induces a singular dependence of the response functions and four point functions in the ratio $r = \Omega/qv_F$. This arises because the skeleton structure of these quantities contain a singularity from the free propagation of a particle hole pair represented by the product of two Green functions. For small Ω and q[9].

$$G(k+q/2,\ \omega+\Omega/2)\ G(k-q/2,\ \omega-\Omega/2) = \lim_{r\to 0} [G^2] + \tilde{R}(k\omega, q\Omega) \qquad (7)$$

with all the Ω and q dependence contained in the singular function:

$$\tilde{R} = \frac{2\pi i a^2\ \delta(\varepsilon_k - \mu)\ \delta(\omega)\ \Omega}{\Omega - q\cdot v_k + i\ \mathrm{sig}\ \Omega/\tau_{in}}. \qquad (8)$$

The singular factor $\tilde{R}$ enters as an intermediate state in the skeleton decomposition of the scattering amplitude (four-point function with amputated external legs) with small momentum transfer in the particle hole channel $\Gamma^{\gamma\delta}_{\alpha\beta}(P_1\varepsilon_1,\ P_2\varepsilon_2,\ q\Omega)$ (see Figs.1,2).

As a result Γ has two different values $\tilde{\Gamma}^{\omega}$ and $\tilde{\Gamma}^{q}$ in the limit $\omega\to 0$ $q=0$ and $q\to 0$ $\omega=0$ respectively. They identify the Landau parameters F_ℓ^s, F_ℓ^a, A_ℓ^s, A_ℓ^a microscopically via the relations

$$2\rho_Q a^2 \tilde{\Gamma}^{\omega}_{\alpha\beta\gamma\delta}(P_1P_2) = \sum_{\ell=o}^{\infty} P_\ell(\cos\theta)\,(F_\ell^s \delta_{\alpha\gamma}\delta_{\beta\delta} + F_\ell^a \vec{\sigma}^{i}_{\alpha\gamma}\vec{\sigma}^{i}_{\beta\delta})$$
$$2\rho_Q a^2 \tilde{\Gamma}^{q}_{\alpha\beta\gamma\delta}(P_1P_2) = \sum_{\ell=o}^{\infty} P_\ell(\cos\theta)\,(A_\ell^s \delta_{\alpha\gamma}\delta_{\beta\delta} + A_\ell^a \vec{\sigma}^{i}_{\alpha\gamma}\vec{\sigma}^{i}_{\beta\delta}) \qquad (9)$$

with P_1P_2 on the Fermi surface and $P_F^2 \cos\theta = P_1 \cdot P_2$. All the Ω and q dependence of the response functions which enters via the singular factor $\tilde{R}$ is extracted

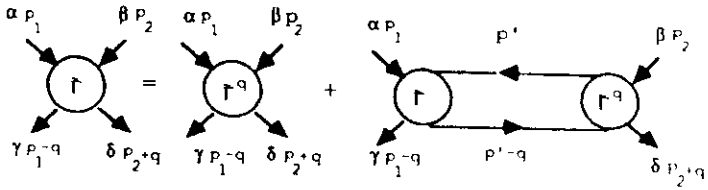

FIG. 1 The full scattering amplitude $\tilde{\Gamma}$ is expressed in terms of the static amplitude $\tilde{\Gamma}^q$ and the singular function $\tilde{R}$. (Product of two QP Green's functions). In the disordered case R is replaced by L

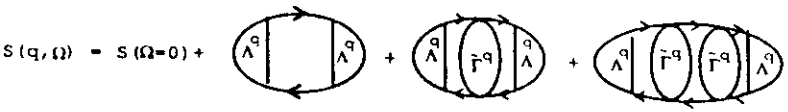

FIG. 2 Skeleton expansion of a response (spin-spin, density-density, heat-heat) function $S(q,\Omega)$. The Ω,q dependence is contained in the product of two intermediate Green's function lines, i.e., the singular function R in the clean case or L in the disordered case. The first blob is the static (singlet, triplet, or energy) vertex A^q. The blob $\tilde{\Gamma}^q$ denotes the static (singlet for the density or heat for the spin triplet) amplitude

using skeleton decompositions in vertex parts irreducible with respect to $\tilde{R}$. As shown in Fig.2.

The physical fact that QP carries unit charge, spin and heat is reflected in the Ward identities for the charge (singlet) Λ_s^{ω}, spin (triplet) Λ_t^{ω}, and heat Λ_H^{ω} vertices:

$$a\Lambda_s^{\omega} = 1, \quad a\Lambda_t^{\omega} = 1, \quad a\Lambda_H^{\omega} = 1. \tag{10}$$

The previous discussion applies to short range interactions. In the presence of long range forces one separates the long range part of the interaction which is treated as a self consistent electric potential from the short range part which is treated as above. The short range part of the interaction gives rise to $f_{p\sigma,p'\sigma'}$ and to the Landau parameters while the self-consistent potential which arises from the long range part of the interaction is added to V_σ in the kinetic equation. Diagramatically the short range part of the interaction $\Gamma_{S,R}$ is identified as the sum of all diagrams contributing to the scattering amplitude which are irreducible with respect to a coulomb line . They sum up to $(A^q_s)^2/(dn/d\mu)$. Adding the long range and the short range part to construct the total singlet static amplitude one finds the general result $2a^2 \rho_Q \Gamma_s^{tot} = 1$.

2. LANDAU THEORY: DISORDERED CASE

Fermi liquid theory survives the introduction of disorder and is valid up to the metal insulator transition.

As a result, the QP residue a disappears from all the response functions[9] and one obtains (5). In the absence of translation invariance k is not conserved and cannot characterize quasiparticle eigenstates. We will use ε_n the energy of a quasiparticle in isolation (analogous to $\varepsilon_k = k^2/2m^*$) to label QP

states. The free energy as a functional of the QP distribution function $n_\sigma(\varepsilon_n, r)$ is written by analogy to (1)

$$F(n_\sigma) = \sum_\sigma \int dr\, n_\sigma(\varepsilon_n r)(\varepsilon_n - \mu) + 1/2 \int d^d r \quad , \delta N_\sigma(r)\, \delta N_{\sigma'}(r^*)\, f_{\sigma\sigma'} . \tag{11}$$

$N_\sigma(r) = \sum n_\sigma(\varepsilon_n r)$ is the total density per spin and $f_{\sigma\sigma'} = f^s + \sigma\sigma' f^a$ is the QP interaction function.

Comparing (2) and (11) one sees that in the dirty case the angular dependence of f is neglected. This is because $n_\sigma(\varepsilon_n r)$ is intended to provide a course grained description of the system over distances longer than the mean-free path where only s-wave scattering is important. In an external spin dependent field v_σ the QP distribution function obeys a kinetic equation

$$0 = \frac{\partial}{\partial t} n_\sigma + (-D_Q \Delta^2) n_\sigma + \frac{\partial n_\sigma}{\partial \varepsilon} (+D_Q \nabla^2)[V_\sigma + \sum_{\sigma'} f_{\sigma\sigma'} N_{\sigma'}] = 0 . \tag{12}$$

D_Q is the QP diffusion constant to be identified microscopically below. Equation (12) is obtained from the kinetic equation of the clean system (2) by replacing $v_p \partial/\partial r$ by $-D_Q \partial^2/\partial r^2$. Physically, this substitution reflects the crossover from ballistic to diffusive transport as the mean free path is decreased. It can be derived from a semiclassical stochastic dynamics

$$\frac{dr}{dt} = \xi \qquad \frac{d}{dt}\,[\varepsilon_n + \sum_{\sigma'} f_{\sigma\sigma'}\, \delta N_{\sigma'}(r) + V_\sigma] = 0 \tag{13}$$

for the "coordinates" r and ε_n. ξ is a Brownian motion with covariance $\langle \xi(t)\xi(0)\rangle = D_Q \delta(t)$ while the second equation is the statement of conservation of energy.

Solving (12) for the density-density, spin-spin and heat-heat correlation function π, χ, χ_H, one find the standard diffusive form[11,12,5]

$$\pi(q,\omega) = \left(\frac{dn}{d\mu}\right) \frac{D_\rho q^2}{D_\rho q^2 - i\omega} \qquad \chi(q,\omega) = \frac{\chi D_s q^2}{D_s q^2 - i\omega} \qquad \chi_H(q,\omega) = \frac{TC_V D_H q^2}{D_H q^2 - i\omega} \tag{14}$$

with $dn/d\mu = 2\rho_Q/1+F_o^s$, $\chi = 2\rho_Q\mu_\beta^2 / 1+F_o^a$ and $C_V = 2\pi^2\rho_Q T/3$, $2\rho_Q f^s = F_o^s$, $2\rho_Q f^a = F_o^a$ as in the clean case and

$$D_\rho = D_Q\,(1 + F_o^s), \; D_\rho = D_Q\,(1 + F_o^a) \qquad D_H = D_Q \tag{15}$$

This structure has many interesting implications. As we approach the MIT the transport coefficients $\sigma = (dn/d\mu) D\rho$, $\sigma_s = \chi D_s$ and $\kappa = C_V D_H$ vanish but their ratio remains constant. In particular the Wiedeman Franz law holds up to the MIT[5]. As one approaches the MIT, χ is expected to diverge since it is infinite on the insulating side, while $dn/d\mu$ remains finite. Therefore, $D_s/D_Q = (dn/d\mu)/\chi$ goes to zero, i.e., spin diffusion is much slower than charge diffusion close to the MIT.

To justify the phenomenological picture we focus on the one and two particle Green's functions. In a disordered medium one lacks translational invariance and there is no basis (like plane waves in the ordered case) which a priori diagonalizes the one-particle Green's function $G(x,x',\omega)$. Nevertheless, as shown in ref.(6) at zero frequency $G(x,x',\omega=0)$ is an Hermitian matrix with eigenvectors $\varphi_n(x)$. In this basis the QP assumption states that for small frequency G has a pole structure

$$G(x,x',\omega) = \sum_n \frac{\varphi^*_n(x)\, \varphi_n(x')}{[\omega - (\varepsilon_n - \mu) + i/2\tau_{in} \operatorname{sig}\, \omega]} \tag{16}$$

which closely parallels (6). Equation (16) holds for any realization of the random potential. E_n and φ_n are the exact poles and eigenvectors of $G(\omega=0)$ and are very different from the exact eigenvalues and eigenvectors of the reference Anderson Hamiltonian obtained by turning off the interactions. In particular we will see that they lead to diffusion constants and density of states which behave very differently than that of the non-interacting case. To make further progress we consider disorder averaged, denoted by <>, quantities. We identify the QP density of states per unit spin ρ_Q as

$$\rho_Q = < \sum_n \delta(E_n-\mu) > . \qquad (17)$$

The ratio of ρ_Q to ρ_o, the density of states of the non-interacting system plays the role of m^*/m in the translationally invariant case. The quasiparticle diffusion constant D_Q is identified by constructing a wave packet out of QP wave functions $\varphi_n(x)$ and finding its long time behavior. Assuming[10]

$$<|\int \varphi_m^*(x) e^{iqx} \varphi_n(x)|^2> = \frac{\rho_Q}{\pi}\left[\frac{D_Q q^2}{(D_Q q^2)^2 + |\varepsilon_m-\varepsilon_n|^2}\right] \qquad (18)$$

one finds that the wave packet diffuses with diffusion constant D_Q.

As in the clean case the pole structure in the single particle Green's function gives rise to a singular dependence in the ratio $r=\omega/D_Q q^2$ (which is analogous to the ratio $\omega/v_F q$ when one considers the propagation of a particle hole pair. This enters into intermediate states in the calculation of the scattering amplitude and the correlation factors. From (16) and (18) it follows

$$\int d(x-x') <G(x,x',\varepsilon+\omega)G(x,x'\varepsilon)> e^{iq(x-x')} = L(q,\omega,\varepsilon) + (\text{non-singular term}) \qquad (19)$$

with all the singular dependence in q and Ω contained in

$$L(q,\Omega,\varepsilon) = \frac{(2\pi)\ a^2\rho_Q\delta(\varepsilon)\Omega}{[D_Q q^2 - i\Omega+\tau_Q^{-1}]} . \qquad (20)$$

The analogy with the clean case becomes obvious once we integrate (7) and (8) over the fast momenta k which has no significance in the disordered case. The factor $\delta(\varepsilon)$ expresses the fact that L=0 unless $(\varepsilon+\omega)\varepsilon<0$. Once more the ballistic transport term $v_x.q$ is replaced by $D_Q q^2$ as in the kinetic equation. At this point is useful to indicate the expression for Finkelstein scaling variables in the perturbation theory framework of Castellani et al.[11,12] in terms of QP parameters: $D_Q=D/Z$, $a=\xi/Z$, $\rho_Q=\rho_o Z$, $\tau_Q=\tau_{ph}/Z$ with τ_{ph} the particle hole dephasing time introduced in ref.15.

The scattering amplitude (see Fig.1) has now a singular dependence in $\omega/D_Q q^2$. At this point the theory keeps only $\ell=0$ spherical harmonic

$$\tilde{\Gamma}_{\alpha\beta\gamma\delta} = \tilde{\Gamma}^s \delta_{\alpha\gamma}\delta_{\beta\delta} + \tilde{\Gamma}^t \sigma^1_{\alpha\gamma}\sigma^1_{\beta\delta} . \qquad (21)$$

In fact in perturbation theory in $1/k_F\ell$ the higher ℓ terms are less divergent than the $\ell=0$, and one assumes they are unimportant for the long distance behavior. The scattering amplitudes are related to the static amplitudes by the usual Bethe Salpeter equation, which in the disorder case, reduces to algebraic equations[11,12]. Fig.1 with the intermediate Green's function lines represented now by L, then gives

$$\tilde{\Gamma}^s(q,\omega) = \tilde{\Gamma}_s^\omega \frac{D_Q q^2 - i\omega}{[D_Q q^2 - i(1-A^\omega_s)\omega]} \quad \tilde{\Gamma}^t(q,\omega) = \tilde{\Gamma}_t^\omega \frac{D_Q q^2 - i\omega}{(D_Q q^2 - i(1-A^\omega_t)\omega)} . \qquad (22)$$

As in the clean case one considers the static and dynamic limits. The $\omega=0$ $q\to0$ gives the static amplitude from which we identify the Landau parameters. $A_o^s = 2\rho_Q a^2 \Gamma_s$, $A_o^t = 2\rho_Q a^2 \Gamma_t$. The $q=0$ $\omega\to0$ limit of Γ defines the Fermi liquid parameters F as in (4). Equation (22) then gives the relation

$$F_o^s = A_o^s/1-A_o^s \qquad F_o^a = A_o^a/1-A_o^a . \qquad (23)$$

The ward identities (10) which only rely on conservation laws remain valid in the dirty limit. Expressing them in terms of the static vertices via the relations $\Lambda^\omega_s = \Lambda^k_s\,0\ (1+F_o^s)$ $\Lambda^\omega_t = \Lambda^k_t(1+F_o^a)$ obtained from a Bethe Salpeter equation similar to (22) for the vertex functions, one finds the results[12,13]

$$a\Lambda^k_s = \frac{1}{(1+F_o^s)} , \quad a\Lambda^k_t = \frac{1}{(1+F_o^a)} , \quad a\Lambda^k_H = 1 . \qquad (24)$$

As in the clean case one studies the ω and q dependence of the response functions by isolating the singular factor L in the skeleton expansion of Fig.2. The density-density correlation function is expressed in terms of its static limit $dn/d\mu$, L, the singlet static vertex Λ^q_s and the singlet state scattering amplitude Λ^q_H. The spin-spin correlation function is expressed in terms of its static limit χ, and triplet static vertex Λ^q_t and the triplet scattering amplitude Λ^q_t. The heat-heat correlation function involves an energy vertex Λ^q_H and singlet quantities, but only the first two terms in the skeleton expansion are relevant, therefore, identifying the heat-diffusion constant with the QP diffusion constant without Fermi liquid corrections[5]. Using Fig.2 and (20)-(24) one justifies microscopically the fundamental relations (14)-(15).

The QP residue cancels against the vertex and disappear from all two particle observables. It appears only in the one paricle density of states ρ_1 which is proportional to the imaginary part of the one-particle Green's function. From (6) it follows that $\rho_1=a\rho_Q$.

3. THE MIT AS A BREAKDOWN OF FERMI LIQUID THEORY

The metal insulator transition can be viewed as a gradual breakdown of the Fermi liquid state. As one varies a control parameter n, i.e. the strength of the disorder, the donor concentration of uniaxial stress, the Fermi liquid parameters D_Q, ρ_Q, a, A^a_o, which describe the low energy physics, change continuously. As one approaches a critical value n_c some of these parameters go to zero or infinity. This behaviour is related to the divergence charactertized by a critical exponent ν of a length $\xi = 1/|n-n_c|^\nu$ which can be interpreted in the insulating side of the transition as the scale inside which a Fermi liquid description of the system holds.

It is a non trivial assumption, justified in the framework of Finkelstein's non linear σ model[3] and by lowest order perturbation theory[11] that $1+F_o^s$ scales with ρ_Q leaving $dn/d\mu$ unrenormalized. We are therefore left with two dimensionless scaling variables describing the physics at scale L close to the MIT: the dimensionless inverse conductance familiar from the Anderson localization problem $t(L) = (e^2/h)/\rho_Q D_Q L^{(d-2)}$ and the triplet Fermi liquid scattering amplitude $A_o^a = 2\rho_o a^2 \Gamma_t^q$. When the Cooper channel is present one also includes the corresponding scattering amplitude in this Channel $\gamma_c = 2\rho_Q a^2 \Gamma_c$. The renormalization-group equations for the scaling variables were derived and the critical exponents to lowest order in ε in $2+\varepsilon$ dimensions were calculated by Finkelstein and Castellani, et al., in refs.11-13-14 and several universality classes were found. It is illuminating to interpret them in terms of the behaviour of the QP paramenters in the different cases.

From a scaling analysis point of view there are two very different situations.

a) The transition takes place at t=0, $A^a_0=\infty$. The critical point occurs at a finite value of the product tA_0^a which can be interpreted as a ratio of the on-site repulsion to the bandwidth of a Hubbard model. Hence this transition has the character of a Mott transition in a weakly disordered enviroment. As one approaches the transition the QP density of states diverges with an exponent x, $\rho_Q \sim \xi^x$ while A_0^a scales with an exponent $A_0^a \sim \xi^\theta$ which also gives the rate at which the dangerously irrelevant variable t approaches 0. At the transition the conductivity scales as $\sigma \sim 1/\xi^{d-2-\theta}$, this modifies the usual relation between the conductivity and the localization length which now becomes

$$\sigma \sim 1/\xi^{d-2-\theta} \sim (n-n_c)^{\nu(d-2-\theta)}.$$

In this framework it is still possible to have minimum metallic conductivity and a continuous second order MIT provided that $\theta=d-2$. This seems to be the case, at least to lowest order in the $2+\epsilon$ expansion[13]. This strange behavior can be understood as a cancellation between the vanishing of the QP diffusion constant $D_Q \sim 1/\xi^{d-2+x-\theta}$ ($d-2+x-\theta=3\epsilon+0(\epsilon^2)$) and the divergence of the QP density of states $x=3\epsilon+0(\epsilon^2)$ in the product $\sigma=\rho_Q D_\theta$. The QP weight a scales to zero as $a \sim 1/\xi^\beta$, $\beta=3\epsilon+0(\epsilon^2)$ this fact together with the divergence of $\chi \sim \xi^{x+\theta}$ and $C_V/T \sim \xi^x$ suggest a physical picture of the MIT which takes place as a gradual conversion of QP degrees of freedom into spins.

It is interesting to contrast the different behavior of the 3 density of states C_V/T, $dn/d\mu$, and ρ_1. The linear coefficient of the specific heat is enhanced due to the divergence of the QP density of state. $\rho_1=a\rho_Q$ is driven to zero because a vanishes faster than ρ_Q diverges while $dn/d\mu$ stays finite. Notice that since $A^s_0=1$ while A^a_0 diverges the Landau sum rule

$$\sum_{\ell=0}^{\infty}(2\ell+1)\ (A^s_\ell+A^a_\ell) = 0$$

cannot be satisfied with just two amplitudes. Presumably, a large number (of the order of some power of ξ) of amplitudes A^s_ℓ, A^a_ℓ become large as one approaches the transition.

b) When spin is not conserved the MIT takes place at a finite value of the disorder parameter t. The usual scaling relation $\sigma \sim 1/\xi^{d-2}$ holds. This scenario is realized in the presence of magnetic impurities, strong magnetic fields, and spin orbit scattering occur at a finite value of the disorder parameter t. Since $\sigma=\rho_Q D_Q$, one could imagine a MIT situation driven by a vanishing ρ_Q, a vanishing D_Q or both. In $2+\epsilon$ dimensions, in all cases the QP lifetime τ_Q^{-1} vanishes faster than the temperature and the QP are well defined.

To lowest order in ϵ, in the $2+\epsilon$ expansion each of these possibilities is realized in some universality class. In the spin orbit case ρ_Q vanishes while D_Q stays finite. So to this order, this is not a localization transition at all. In the strong magnetic field case, D_Q vanishes and ρ_Q stays finite while the MIT in the presence of magnetic impurities is driven by the simultaneous vanishing of ρ_Q and D_Q.

Experimentally there is growing evidence that uncompensated semiconductors, having a conductivity exponent s=1/2, are in a different universality class than compensated semiconductors, amorphous alloys, and semiconductors in a magnetic field, which have a conductivity exponent s=1. Mott's bound $\nu \geq 2/d$ has been put on rigorous mathematical grounds[16]. If $\theta=0$, this gives a bound on the conductivity exponent $s>2/3$ in three dimensions suggesting that uncompensated semiconductors belong to case a and have a non-zero θ. A crucial test of the theory would be to check whether systems having s=1 have a spin lifetime which is short compared to the temperature.

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