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SMR. 758 - 35

SPRING COLLEGE IN CONDENSED MATTER
ON QUANTUM PHASES
(3 May - 10 June 1994)

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INTERACTING FERMIONS IN ONE DIMENSION

DISORDERED BOSONS

Part II

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These are preliminary lecture notes, intended only for distribution to participants.

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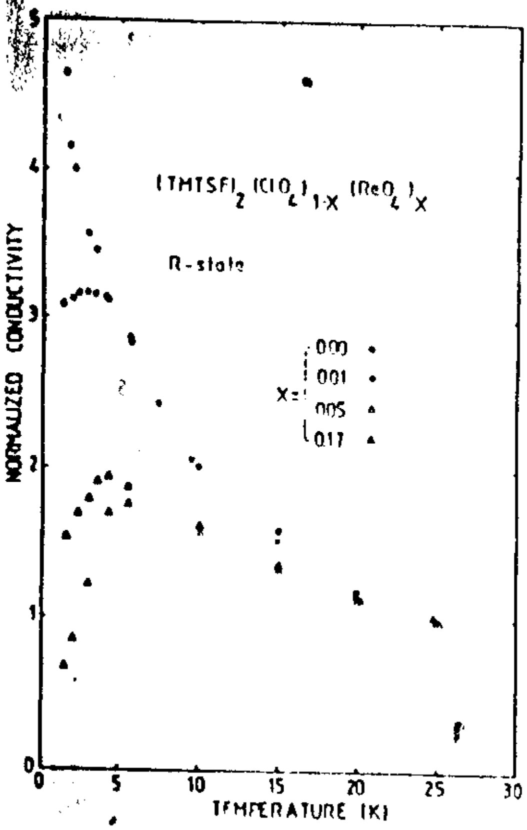


[S. Tomic et al.
J. Physique C3 - 1075 (83)]

(ClO_4) supra à T ≤ 1.2 K.

"SDW" ground State

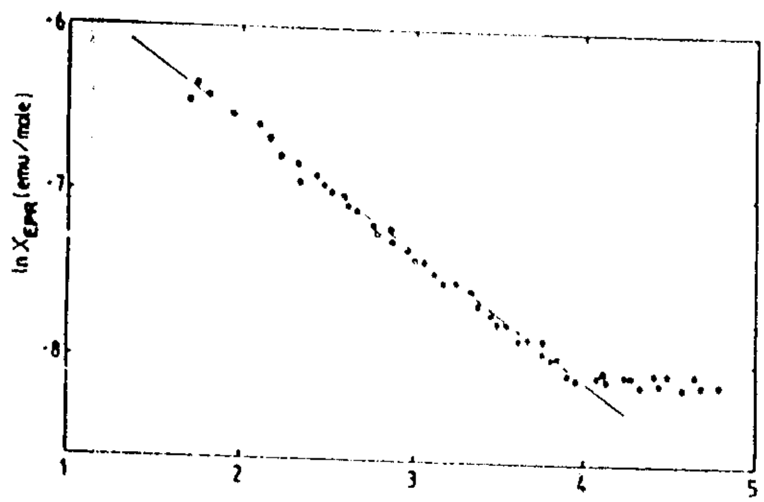


Fig. 6. — Logarithm of the EPR susceptibility *versus* logarithm of temperature below 50 K. Fitting to $\ln \chi = - \alpha \ln T$ gives $\alpha = 0.72 \pm 0.07$. The solid line is a guide for the eye.

[S. Tomic et al.
J. Physique <u>44</u> 375
(83)]

$(TMTSF)_2\ BrO_4$

$(BEDT-TTF)_3\ Cl_2 2H_2O$ T_c ↘ with disorder !

IV] Effects of the Lattice: the Metal-Insulator ("Mott") transition

1) Umklapp:

- Something is missing:

  e.g. xxz chain $\quad H = J_{xy} \sum_i (S^x_i S^x_{i+1} + S^y_i S^y_{i+1}) + J_z \sum_i S^z_i S^z_{i+1}$

  if $J_z > 1 \quad \rightarrow$ ising order ! (How to get it ?)

  ↑↓↑↓↑↓↑ $\quad \rightarrow \quad$ ●×●×●×●× $\quad V = J_z$

  Metal-Insulator transition at $n = 1/2$

$\Rightarrow$ effects of the Lattice

- transport: interactions $\sum_i k_i = 0 \quad$ J conserved !

  but in fact $\sum_i k_i = G \quad$ G vector of reciprocal lattice

  Umklapp: transfers momentum to (and from) the lattice

  Boltzmann: $\rho_{3D} \equiv T^2 \qquad \rho_{2D} \equiv T^2 \operatorname{Log} T^{-1} \qquad \rho_{1D} \equiv T$

2) Hubbard "like" models:

$$\sum_i n_{i\uparrow} n_{i\downarrow} = (\text{Interactions})_{\sum k = 0} + e^{i4k_F x} (\psi_+^\dagger \psi_-)_\uparrow (\psi_+^\dagger \psi_-)_\downarrow + \text{h.c}$$

$$k_F = \frac{\pi}{2a} \quad \rightarrow \quad e^{i4k_F x} = 1 \qquad (x = na)$$

$$H_\rho = \frac{1}{2\pi} \int dx \; (uK)_\rho (\pi \Pi)^2 + \left(\frac{u}{K}\right)_\rho (\nabla \phi_\rho)^2 + \frac{2 g_3}{(2\pi\alpha)^2} \int dx \cos(\sqrt{8}\phi + \delta x)$$

where $\delta = [4k_F - 2\pi]$

another possibility:

$$H = H_\rho^0 + \frac{2g}{(2\pi\alpha)^2} \int dx \cos(\sqrt{8}\phi_\rho) + \mu \underbrace{\int dx \frac{1}{\pi} (\nabla \phi_\rho)}_{(\rho_+ + \rho_-)}$$

For Hubbard $g_3 = U$

organics:

$$g_3 \sim U \left(\frac{\Delta}{E_F}\right)$$

Δ changes with pressure $\Rightarrow$ g_3 varies with pressure !!

3) <u>Current, conductivity, etc:</u>

$$\rho = -\frac{\sqrt{2}}{\pi}\nabla\phi_\rho \qquad \frac{\partial\rho}{\partial t} + \mathrm{div}\, j = 0 \qquad \to \qquad j = \frac{\sqrt{2}}{\pi}(\partial_t\phi_\rho) = \sqrt{2}(u_\rho K_\rho)\Pi_\rho$$

Rq: $j = v_F(n_+ - n_-) \equiv \sqrt{2}\, v_F\, \Pi$ (but if $[\rho, H]$ $v_F = u_\rho K_\rho$)

$$\sigma(\omega) = \frac{i}{\omega}\left[\frac{2uK}{\pi} + \chi(\omega)\right]$$

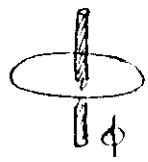
$$\chi(\omega) = \langle j ; j\rangle_\omega = -\frac{i}{L}\int dx \int_0^{+\infty} dt\, \langle [j(x,t), j(0,0)]\rangle\, e^{i\omega t}$$

if $g_3 = 0$

$$\sigma(\omega) = 2uK\left[\delta(\omega) + \frac{i}{\pi}\,\mathcal{J}\left(\frac{1}{\omega}\right)\right]$$

in general: $\sigma(\omega) = D\,\delta(\omega) + \sigma_{reg}(\omega)$

$$D = \frac{d^2 E_0}{d\phi^2}$$



ϕ

$$D = 2u_\rho^* K_\rho^*$$

D can be computed from Bethe-Ansatz

$$\sigma(\omega, T) \equiv \frac{i\,2uK}{\pi}\,\frac{1}{\omega + M(\omega)} \qquad M(\omega) = \frac{\omega\,\chi(\omega)}{\chi(0) - \chi(\omega)}$$

(for Drude $M(\omega, T) \equiv 1/\tau$ τ scattering time)

$M(\omega)$ can be computed "perturbatively" in g_3

⚠ o.k for ω dependence
might cause problems for T dependence

$$M(\omega, T) \approx \frac{\frac{1}{\omega}\left[\langle F; F\rangle_\omega^{H_0} - \langle F; F\rangle_{\omega=0}^{H_0}\right]}{-\chi(0)}$$

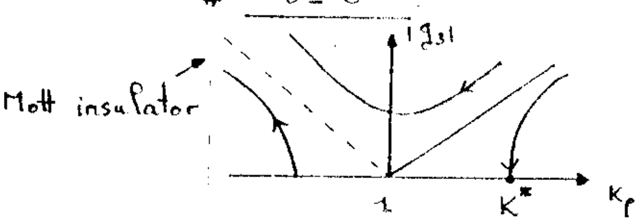
$$F = [j, H]$$

4) Mott transition :

Can be done in two ways: $\begin{cases} \delta = 0 \text{ fixed} & g_3 \text{ varies } (1) \\ \delta \text{ varies} & g_3 \text{ fixed } (2) \end{cases}$

(1) : organics ; [Vanadium Ox.] (2): [High T_c]

$\delta = 0$:



$$\begin{cases} \dfrac{dK}{d\ell} = -\dfrac{1}{2} y_3^2 K^2 \\ \dfrac{dy_3}{d\ell} = (2-2K) y_3 \end{cases} \qquad y = \frac{g}{\pi u}$$

$\cos(\phi) \approx 1 - \frac{\phi^2}{2}$ $\Rightarrow$ field ϕ becomes massive

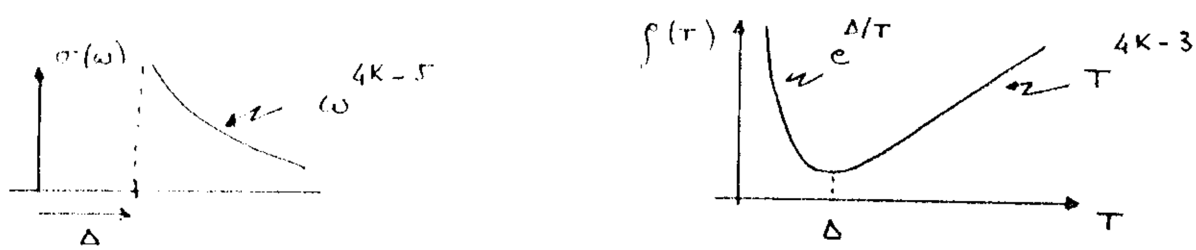
$\omega^2 + q^2 \rightarrow \omega^2 + q^2 + m^2$

$\Rightarrow$ Quantum phase transition

(I) $\sigma(\omega) = 2u^* K^* \delta(\omega) + \sigma_{reg}(\omega) \leftarrow \omega^{4K^*-5}$

at the transition : $\begin{cases} \text{jump of } D : 2u^* \rightarrow 0 \\ \sigma_{reg}(\omega) = \dfrac{1}{\omega \operatorname{Log}^2\left(\frac{2u}{\alpha\omega}\right)} \end{cases}$

(II)



$$\langle F; F\rangle(\omega) \sim \langle \sin(\sqrt{8}\phi) \sin(\sqrt{8}\phi) \rangle \sim \left(\frac{1}{r}\right)^{4K}$$

$$M(\omega) \sim \int dx \int_0^\beta d\tau \, (e^{i\omega\tau} - 1) \frac{1}{\omega} \langle F; F \rangle \sim [\omega \text{ or } T]^{4K-3}$$

Rq : $K = 1$ $\rho(T) \sim T$ idem Boltzmann.

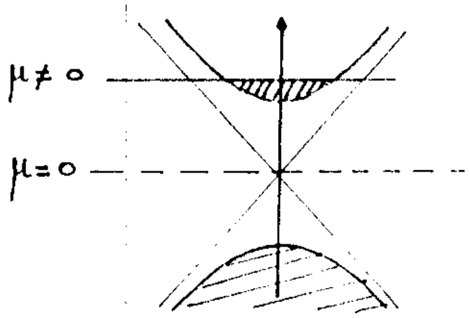
Problem : $K_{\text{transport}} \neq K_{\text{RMN}}$!!! ?

- exponents : $z = 1$ $\Delta \sim e^{-1/\sqrt{K - K_c}}$ $\nu = \infty$

$\delta \neq 0$:

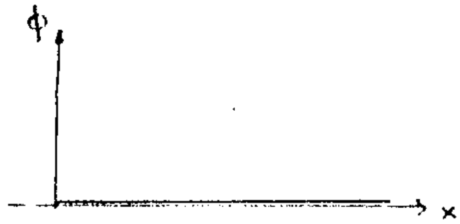
$\int dx \; \cos(\sqrt{8}\,\phi + \delta x)$ irrelevant (but needed to compute σ !)

• Luther - Emery :

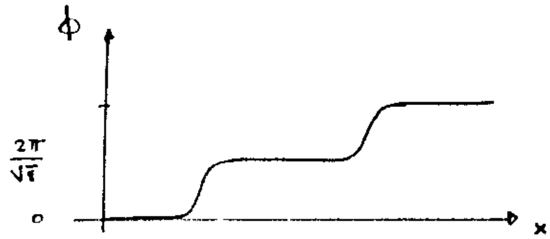


$$H = v_F \; \psi_+^\dagger \partial_x \psi_+ - \psi_-^\dagger \partial_x \psi_- + g_3(\psi_+^\dagger \psi_- + h.c) - \mu\,(\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-)$$

$$H = H^0 + g_3 \int dx \; \cos(\sqrt{8}\,\phi) - \mu\,(\nabla\phi)$$



$\mu < \Delta$

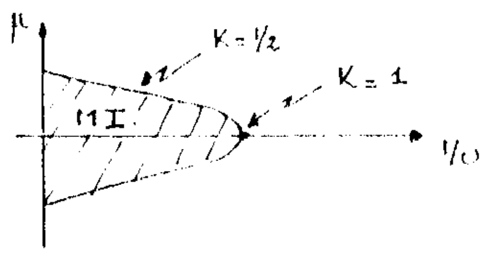


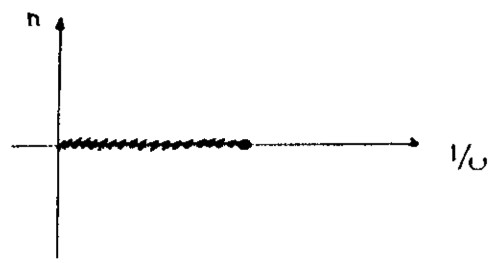
$\mu > \Delta$

$n \sim (\mu - \Delta)^{1/2}$

The ψ are solitons of ϕ

When $\delta \to 0$ $K \to 1/2$ (free spinless fermions)





• When $\delta \to 0$: $u \to 0$ $u \sim \dfrac{\delta}{\Delta}$

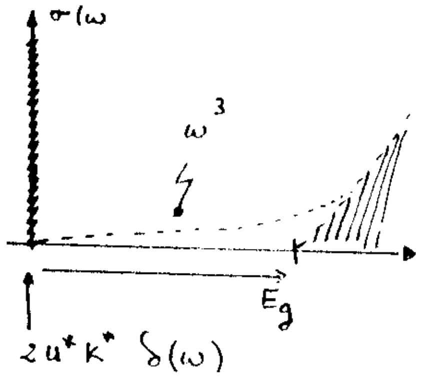
$D \sim \dfrac{\delta}{\Delta}$ (idem to B.A.)

• exponents : $\delta \neq 0$ $z = 2$

$\delta = 0$ $z = 1$

• $\sigma(\omega)$: $(T=0)$

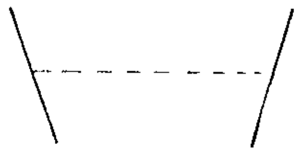
$\omega \gg \delta$ idem



$$\begin{cases} E_g \simeq u\delta & K>1 \\ E_g \simeq \left[\Delta^2 + (u\delta)^2\right]^{1/2} & K<1 \end{cases}$$

E_g : energy to make "interband" transitions

But:



Momentum $\Leftrightarrow$ current

$$\epsilon(k) = v_F\,(k-k_F) + \lambda_2\,(k-k_F)^2 + \lambda_3\,(k-k_F)^3$$

$$\sigma(\omega) \propto \lambda_3^2\,\left[K_\rho - K_\rho^{-1}\right]^2\,\omega^3 \qquad \rightarrow \text{pseudo-gap}$$

[Rq: 2D $\sigma(\omega) \propto \mathrm{Log}(\omega)$
$\sigma(\omega) \propto$ Cste]

• $\sigma(T)$ $(\omega = 0)$

$\$$If Umklapp $=0$ $\quad \Sigma\rho = 0 \quad \Rightarrow \quad \sigma(T) = \infty$

$\delta \neq 0$ Umklapp ineffective ("gaped") $\quad \rho \sim e^{-\delta/T}$

V) Disorder effects :

1) Motivations

- Exp. compounds :

organics $\begin{cases} \text{Disorder either natural (anions) or artificial} \\ T_c \searrow \text{ very fast with disorder} \\ \chi \sim T^{-\alpha} \end{cases}$

Spin chains : $H = J_{xy} \sum (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y) + J_z S_i^z S_{i+1}^z + \sum_i h_i S_i^z$

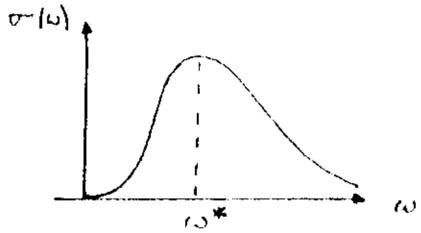
$$\rightarrow \quad H = -t \sum_{\langle ij \rangle} c_i^+ c_j + V \sum_i n_i n_{i+1} + \sum_i \mu_i n_i$$

Effect of disorder on interacting particles

- Theoretical motivations :

1D : non interacting problem solved

$\rightarrow$ Localized $\xi \sim \ell \sim 1/D$



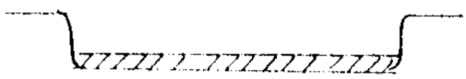
$\sigma(\omega) \sim \omega^2 \operatorname{Log}^2 \omega \qquad \omega \sim 0$

$\sigma(\omega) \sim 1/\omega^2 \qquad \omega \to \infty$

Effect of interactions ?

- What about bosons ? :

non interacting $\rightarrow$ pathological



$V = -V \qquad \rho \sim e^{-L}$

$E_c \sim (1/L)^2 \qquad E = -V \qquad \Rightarrow$ all bosons go in minimum

$\rho = \infty \quad \rightarrow$ unstable with interactions !

One needs interactions

2) <u>boson problem in 1D</u>

$$\psi_B^+(x) = [\rho(x)]^{1/2} e^{i\theta(x)}$$

$\rho(x) = \sum_i \delta(x - x_i)$ | introduce $\tilde{\phi}$ such that $\phi = n$ at each particle

$$\rho(x) \equiv \sum_n \delta(\tilde{\phi} - n) \equiv \sum_p e^{i 2\pi p \tilde{\phi}}$$

$$\Rightarrow \begin{cases} \rho(x) = \frac{1}{\pi}[\nabla\tilde{\phi}] \sum_{p=-\infty}^{p=+\infty} e^{i 2m \tilde{\phi}(x)} \\ \nabla\tilde{\phi} = \pi[\rho_0 + \nabla\Phi(x)] \end{cases} \qquad [\phi(x), \nabla\Theta_{x'}] = i\delta(x-x')\pi$$

$$\rho \equiv [\rho_0 + \nabla\phi] \sum_{p=-\infty}^{+\infty} e^{i 2m[\pi\rho_0 x + \phi(x)]}$$

$$H = \frac{1}{2\pi}\int dx\, (vK_b)(\nabla\theta)^2 + \left(\frac{v}{K_b}\right)(\nabla\phi)^2$$

- <u>Galilean invariance</u> : $\frac{vK_b}{\pi} = \frac{\rho_0}{m}$
- <u>Compressibility</u> $\pi\frac{v}{K_b} = \frac{K^{-1}}{\pi^{-2}\rho_0^2}$

v: velocity of "phonon" modes (sound waves)

$$\langle \psi_B^+(r)\, \psi_B(0)\rangle \quad B\, \rho_0\, (\rho_0 r)^{-K_b^{-1}/2}$$

$$\langle \rho(r)\rho(0)\rangle \equiv 2K_b (2\pi\rho_0 r)^{-2} + A\rho_0^2 \cos(2\pi\rho_0 r)(\rho_0 r)^{-2K_b}$$

very similar to spinless fermions

$$\langle \rho_F(x)\rho_F(0)\rangle \equiv \frac{1}{r^2} + A\cos(2k_F x)\, r^{-2K_F}$$

Fermions		**Bosons**	
$V=0$	$K_F = 1$	$V=0$	$K_B = \infty$
$V<0$	$K_F > 1$	$V \nearrow$	$K_B \searrow$
$V>0$	$K_F < 1$		

ex δ function potential $\quad V = \delta(x-x')\, V_0$

$V_0 \to \infty \qquad K_B \to 1 \qquad$ "non interacting spinless fermions"

bosons $\quad \xrightarrow{\text{increasing repulsion}}$

$K=\infty$ ———————— $K=1$

fermions $\quad \xleftarrow[\text{increasing attraction}]{}$

3) Disorder :

in 1D $\quad \to \times \to \quad$ forward $\qquad \rightleftarrows \times \quad$ backward.

$$H = \int V(x)\rho(x) \;:\; \int \mu(x)\,(\nabla\phi)_x\, dx \qquad \int dx\; V(x)\, e^{i2\pi\rho_0 x}\, e^{i2\phi(x)}$$

forward : $\quad (\nabla\phi)^2 + \mu(x)\,\nabla\phi \quad \to \quad \phi \to \tilde{\phi} = \phi + \int_{-\infty}^{x} dy\, \mu(y)$

trivial effects : $\quad \langle e^{i2\phi}\, e^{i2\phi} \rangle = e^{-|x|/\xi} \langle e^{i2\phi}\, e^{i2\phi} \rangle_{\mu=0}$

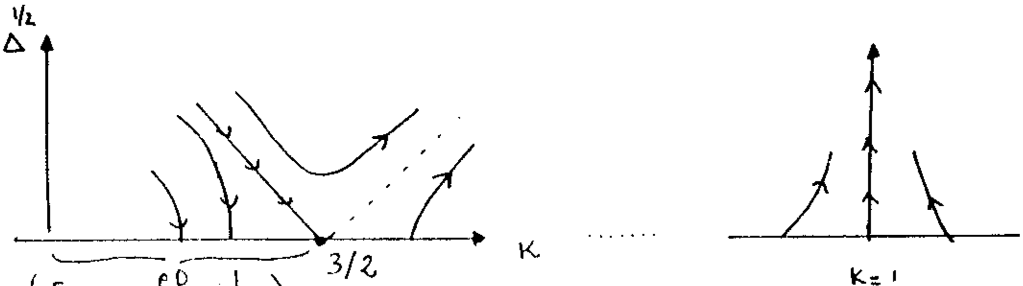
$\sigma \equiv \langle (\partial_\tau \phi)(\partial_\tau \phi) \rangle \quad$ unchanged

backward:

$$H = H_0 + \int dx\; \frac{\overbrace{V(x)}^{}\, e^{i2\pi\rho_0 x}}{\xi(x)}\, e^{i2\phi(x)} + h.c.$$

expand and do an RG calculation

$$\frac{dK}{dl} = -\frac{1}{2} K^2 \Delta \qquad \frac{d\Delta}{dl} = (3 - 2K)\,\Delta$$



• Superfluid:
Power Law correlation functions
$\langle \psi^+ \psi \rangle \sim \left(\frac{1}{r}\right)^{K_i^{-1}/2}$

$\sigma(\omega) \equiv u_\rho^* K_\rho^* \delta(\omega) + \omega^{[2K-4]}$

at the transition:
$K^*_{b,F} \equiv 3/2$ universal $\qquad \sigma_{reg} \equiv \frac{1}{\omega \operatorname{Log}^2(\omega)}$

universal power law of correlation functions $\langle \psi^+ \psi \rangle \sim \left(\frac{1}{r}\right)^{1/3}$

$\eta = 1/3 \qquad z = 1 \qquad \rho_s = u^* K^*$ jumps

• Localized
→ localization length
integrate RG up to $\Delta \approx 1$
length scale $L_{loc} = e^{\ell} L^*_{loc} \approx e^{\ell} 1/\Delta \approx e^{\ell}$

ex: $\frac{\partial \Delta}{\partial \ell} = \Delta\,(3 - 2K) \Rightarrow \Delta = \Delta_0\, e^{(3-2K)\ell}$

$\Rightarrow \quad \xi \sim \left(1/\Delta_0\right)^{\frac{1}{3-2K}} \qquad K=1 \quad \xi \sim 1/\Delta$ o.k.

above ξ RG valid.

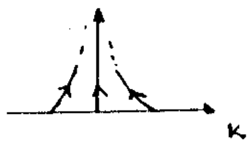
close to the transition $\xi \sim \exp\left[\frac{-1}{K-K_c}\right]$

- <u>what to do below ξ</u> :

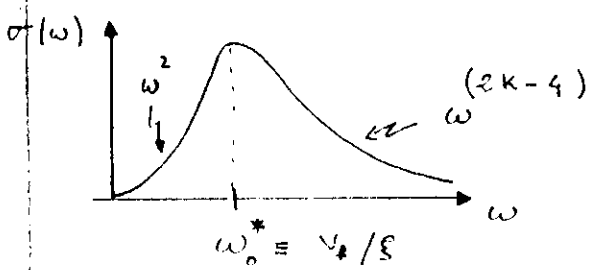
handwaving :

interactions not important.

RG :



if true $\sigma(\omega) \sim \omega^2 \left[\operatorname{Log}^2 \omega\right]$



better (?) :

<u>Replica</u> trick:

average over disorder

one computes : $\overline{\langle \mathcal{O} \rangle} = \int dv\, p(v) \dfrac{\int \mathcal{D}\phi\, e^{-S(\phi)}\, \mathcal{O}(\phi)}{\int \mathcal{D}\phi\, e^{-S(\phi)}}$

problems due to the denominator

$$\overline{\langle \mathcal{O} \rangle} = \lim_{n\to 0} \int dv\, p(v) \int \mathcal{D}\phi_1\, \mathcal{O}(\phi_1)\, e^{-S(\phi_1)} \left[\int \mathcal{D}\phi\, e^{-S(\phi)}\right]^{n-1}$$

$$= \lim_{n\to 0} \int dv\, p(v) \prod_{i=1}^{n} \int \mathcal{D}\phi_i\, e^{-\sum_{i=1}^{n} S(\phi_i)}\, \mathcal{O}(\phi_1)$$

can average over disorder

$$S = \sum_\alpha \int dx\, d\tau\, (\partial_\tau \phi_\alpha)^2 + (\partial_x \phi_\beta)^2 - \Delta \sum_{\alpha\beta} \int dx\, d\tau\, d\tau' \cos\left(2\phi_\alpha(x\tau) - 2\phi_\beta(\right.$$

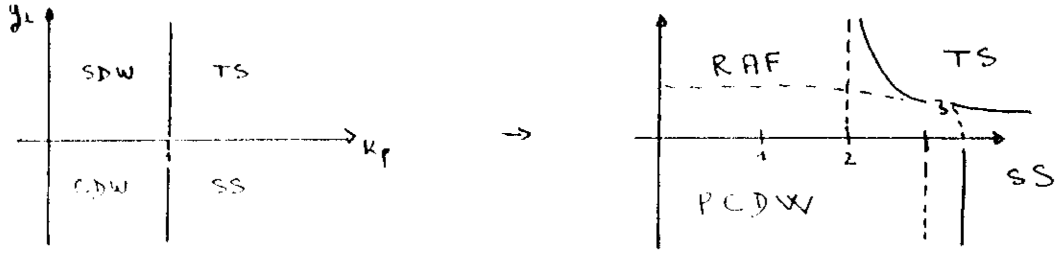
expand $\cos(\quad) \to (\quad)^2$ very bad !!

but variational ansatz $\int dq\, d\omega \sum_{\alpha\beta} G^{-1}_{\alpha\beta}(q,\omega)\, \phi^{*}_{\alpha,q\omega}\, \phi_{\beta,q\omega}$

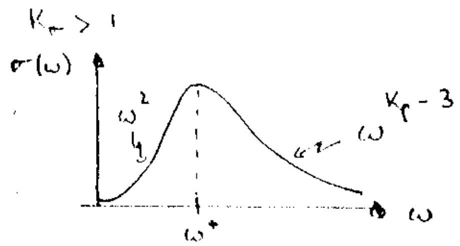
gives all properties of Loc. phase $[\,\sigma(\omega) \sim \omega^2$

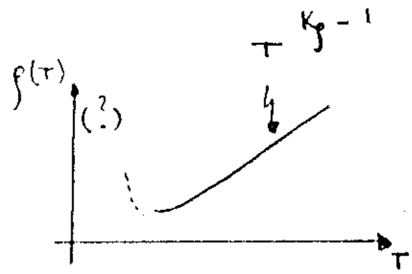
$\langle \rho(x)\rho(0)\rangle \sim e^{-|x|/\xi}$ etc $]$

4) Problem with spin:



$$\frac{\partial \Delta}{\partial e} = \Delta\,(3 - K_\rho - K_\sigma - y_\perp)$$

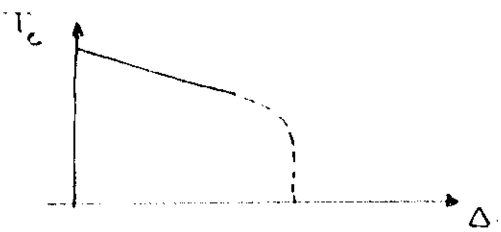




* Superconductivity:

$$H = H_{in} + J \sum_{\langle ij \rangle} O^{+}_{i\,SS}(x)\; O_{j\,SS}(x)$$

mean field



$\Delta T_c \propto \Delta$.

* References for Lectures III + IV:

Here again no tentative to be complete !

* M.I. transition :

B.S. Shastry and B. Sutherland PRL 65 243 (1990)
H.J Schulz PRL 64 2831 (1990)
N. Kawakami and S.K Yang PRL 63 3063 (1990)
R. Shankar Int. J. of Mod. Phys. B 4 2371 (1990)
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" PRB 46 342 (1992)
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* Disorder [+ bosons]: (1D)

F. D. M. Haldane PRL 47 1840 (1981)
T. Giamarchi and H.J Schulz PRB 37 325 (1988)
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* Bosons + disorder ($d \geq 2$)

M.P.A Fisher et al. PRB 40 546 (1989)
E.S. Sørensen et al. PRL 69 828 (1992)
S.M. Girvin et al. Prog. Theor. Phys. Sup. 107 135 (1992)
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and references therein.