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"College on Atmospheric Boundary Layer and Air Pollution Modelling"
16 May - 3 June 1994

"A 4-layer Model of Turbulent Air Flow over Low Hills"
(The Jackson Hunt Approach)

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Please note: These notes are intended for internal distribution only.

A 4 Layer Model of Turbulent Air Flow over Low Hills

(The Jackson and Hunt Approach)

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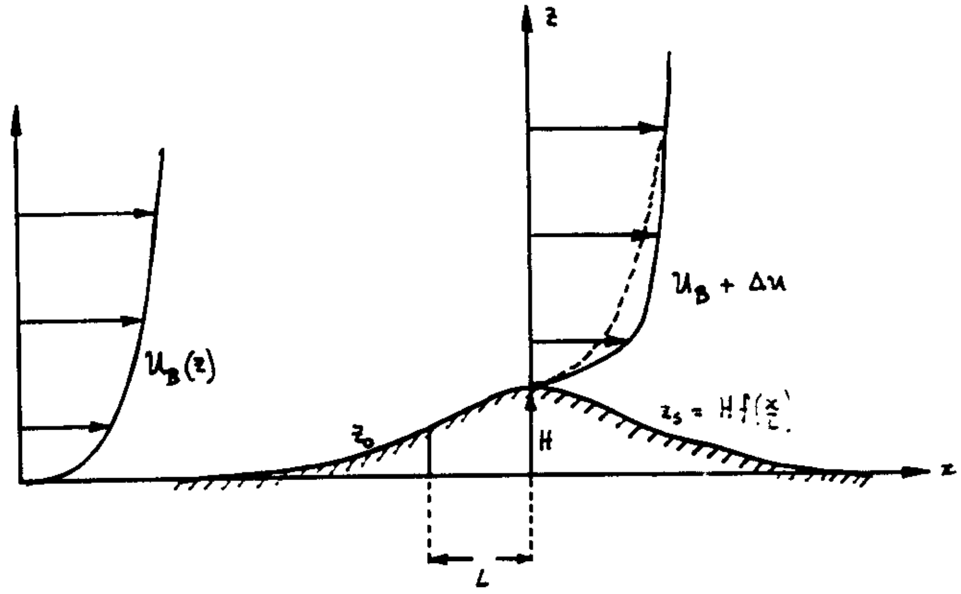
We shall develop a method for analysing how a turbulent boundary layer responds to changes in surface condition
eg. a hill; a change in surface roughness; a water wave.

Qualitative Questions:

1. How can these flows be modelled?
2. How much is the mean flow changed; where is it changed most?
3. How does the turbulence change?

Flow Geometry



Approach Flow: $U_B(z) = \frac{u_*}{K} \ln z/z_0$

$\tau_B(z) = \rho u_*^2$

$\Rightarrow$ 2 key parameters: u_* and z_0

Also have boundary layer depth δ

($\delta \sim 1$Km, $z_0 \sim 0.1$m, $U_{10} \sim 10-20$ ms^{-1}, $u_* \sim 0.3 - 0.8$ ms^{-1})

Hill: Two dimensional $z_s = Hf(x/L)$

Low slope, ie. $H \ll L$

$\Rightarrow |\Delta u| \ll U_B$

Then can analyse linear changes to U_B

Mathematical Model

Write flow variables on hill as approach flow plus perturbation

$$U = U_B + \Delta u, \qquad W = \Delta W,$$

$$p = P_B + \Delta p, \qquad \tau = \tau_B + \Delta\tau$$

Substitute into Reynolds-averaged Navier-Stokes equations and linearise (i.e. neglect products of perturbations)

$$\rho\left\{U_B \frac{\partial \Delta u}{\partial x} + \Delta W \frac{dU_B}{dz}\right\} = -\frac{\partial \Delta p}{\partial x} + \frac{\partial \Delta\tau_{xz}}{\partial z} + \cancel{\frac{\partial \Delta\tau_{xx}}{\partial x}}$$

$$\rho U_B \frac{\partial \Delta W}{\partial x} = -\frac{\partial \Delta p}{\partial z} + \frac{\partial \Delta\tau_{xz}}{\partial x} + \cancel{\frac{\partial \Delta\tau_{zz}}{\partial z}}$$

$$\frac{\partial \Delta u}{\partial x} + \frac{\partial \Delta W}{\partial z} = 0$$

These are p.d.e's. To solve them :-

1. Take Fourier Transform in x : $\Delta\tilde{u} = \int_{-\infty}^{\infty} \Delta u \, e^{-ikx} dx$

   Then p.d.e.'s $\mapsto$ o.d.e.'s because $\partial/\partial x \mapsto ik$

   $\Rightarrow$ Fundamental solutions are for cosinusoidal hill.

2. Need a model for Reynolds stress tensor

   $\Delta\tau_{ij} = -\rho\Delta\overline{u_i' u_j'}$

   In fact only $\Delta\tau_{xz} \equiv \Delta\tau$ is dynamically significant.

Preliminary Scaling : Try to simplify equations

$$\tau = \tau_B + \Delta\tau = \rho u_*^2 + \Delta\tau = \rho u_*^2\left(1 + \frac{\Delta\tau}{\rho u_*^2}\right)$$

We expect $\Delta\tau \sim \frac{H}{L}\rho u_*^2$

In x-momentum equ. we have : $\frac{\partial\Delta\tau}{\partial z} \sim \frac{H/L\,\rho u_*^2}{L}$

also have inertial terms : $\rho U_B \frac{\partial \Delta u}{\partial x} \sim \rho U_0 \frac{H/L\,U_0}{L}$

$\therefore$ Compare these terms :

$$\frac{\partial\Delta\tau/\partial z}{\rho U_B\,\partial\Delta u/\partial x} \sim \frac{u_*^2/L}{U_0^2/L} = \frac{u_*^2}{U_0^2} \ll 1$$

Typically, in atmosphere $\frac{u_*}{U_0} \sim 0.03 - 0.07$

So we can ignore the Reynolds stress gradients ?

x-momentum : $\rho\left\{U_B \frac{\partial \Delta u}{\partial x} + \Delta W \frac{dU_B}{dz}\right\} = -\frac{\partial \Delta p}{\partial x} + \cancel{\frac{\partial\Delta\tau}{\partial x}}$

Inviscid irrotational flow

We have ignored Reynolds stress gradients, now also ignore shear in approach flow, $dU_B/dz = u_*/\kappa z$

i.e. $\dfrac{dU_B}{dz} \approx 0$, $U_B \approx U_0$ constant. when z big

$$\Rightarrow \quad \rho\, U_0 \frac{\partial \Delta u}{\partial x} \approx -\frac{\partial \Delta p}{\partial x}$$

$$\rho\, U_0 \frac{\partial \Delta w}{\partial z} \approx -\frac{\partial \Delta p}{\partial z}$$

$$\frac{\partial \Delta u}{\partial x} + \frac{\partial \Delta w}{\partial z} = 0$$

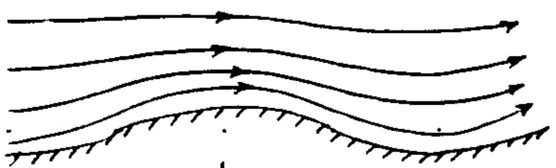
Boundary conditions :

(i) at surface, $z_s = H f\left(\frac{x}{L}\right)$: $\quad \Delta w(0) = U_0 \dfrac{\partial z_s}{\partial x}$
ie streamline follows surface $\quad = U_0 \dfrac{H}{L} f'$

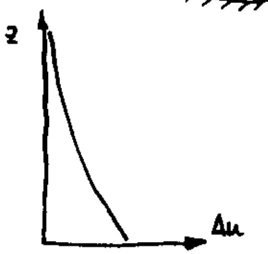
(ii) High above hill, $z \to \infty$: $\quad \Delta w \to 0$

For cosine hill $f = \cos 2\pi x/L$ solution is

$$\left.\begin{aligned} \Delta u &= U_0\, e^{-2\pi z/L} \cos 2\pi x/L \\ \Delta w &= -U_0\, e^{-2\pi z/L} \sin 2\pi x/L \end{aligned}\right\}$$

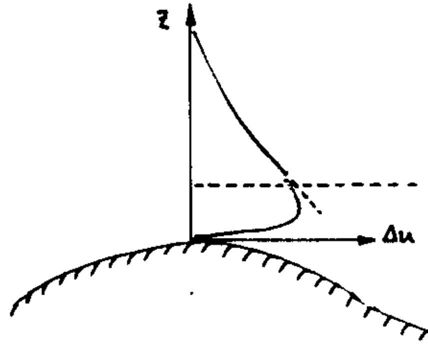


streamlines



But Δu does not satisfy the b.c. $\Delta u = 0$ at surface!
$\Rightarrow$ Need to bring back $\partial \Delta \tau / \partial z$ near surface.

The need for inner and outer regions :-



Outer region : neglect $\Delta\tau$

Inner region : retain $\Delta\tau$

1. Outer region

The Reynolds stress gradients can be neglected over most of the flow. The potential flow sol^n is valid through most of this region.

2. Inner Region

Within a narrow region close to the surface, the velocity perturbation is reduced to zero at surface.

This is mediated by Reynolds stress gradients in the inner region.

The inner region is like a boundary layer that lies within the upstream boundary layer, and so the inner region is sometimes called an internal boundary layer.

$\Rightarrow$ We need a model for $\Delta\tau$.

Effects of hill on turbulence

1. Consider approach flow, ie no hill.

In surface layer: $\tau_B = \kappa^2 z^2 \left|\frac{\partial U_B}{\partial z}\right| \frac{\partial U_B}{\partial z}$

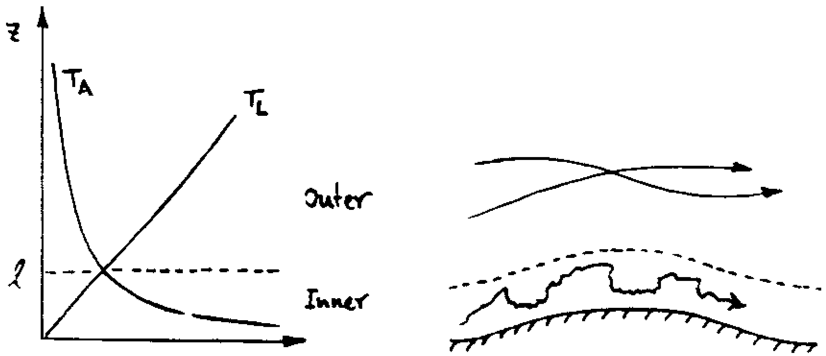
mixing-length model is a good model.

But care needed for $\Delta\tau$

2. Stress perturbation $\Delta\tau$

2 timescales: $T_A = L/U_B(z)$ = advection timescale

$T_L = z/u_*$ = eddy turnover timescale.



$T_A \sim T_L$ when $z \sim l$ where $l \ln l/z_0 \sim L$

Then $\frac{l}{L} \sim \frac{u_*}{U_0} \ll 1$

Turbulence Models

Inner Region: when $z \ll l$ $\Rightarrow$ $T_L \ll T_A$

$\Rightarrow$ Turbulence in local equilibrium

$\Rightarrow$ Can use mixing length model:

$$\tau + \Delta\tau = \kappa^2 z^2 \left(\frac{dU_B}{dz} + \frac{\partial \Delta u}{\partial z} \right)^2$$

$$\Rightarrow \Delta\tau = 2\kappa z u_* \frac{\partial \Delta u}{\partial z}$$

Outer Region: when $z \gg l$ $\Rightarrow$ $T_A \gg T_A$

$\Rightarrow$ changes to turbulence are rapid

$\Rightarrow$ can compute changes using rapid distortion theory

RDT shows that $\Delta\tau \sim O(u_*^2)$ as used above.

Hence stress perturbations are indeed negligible in outer.

N.B. If mixing length is used in outer region then

$$\Delta\tau = O(u_* U_0) = O\left(u_*^2 \frac{U_0}{u_*}\right)$$

ie too big by factor of $U_0/u_* \gg 1$.

This does not affect the mean flow, but is important for $\Delta\tau$ calculations and also affects force on hill.

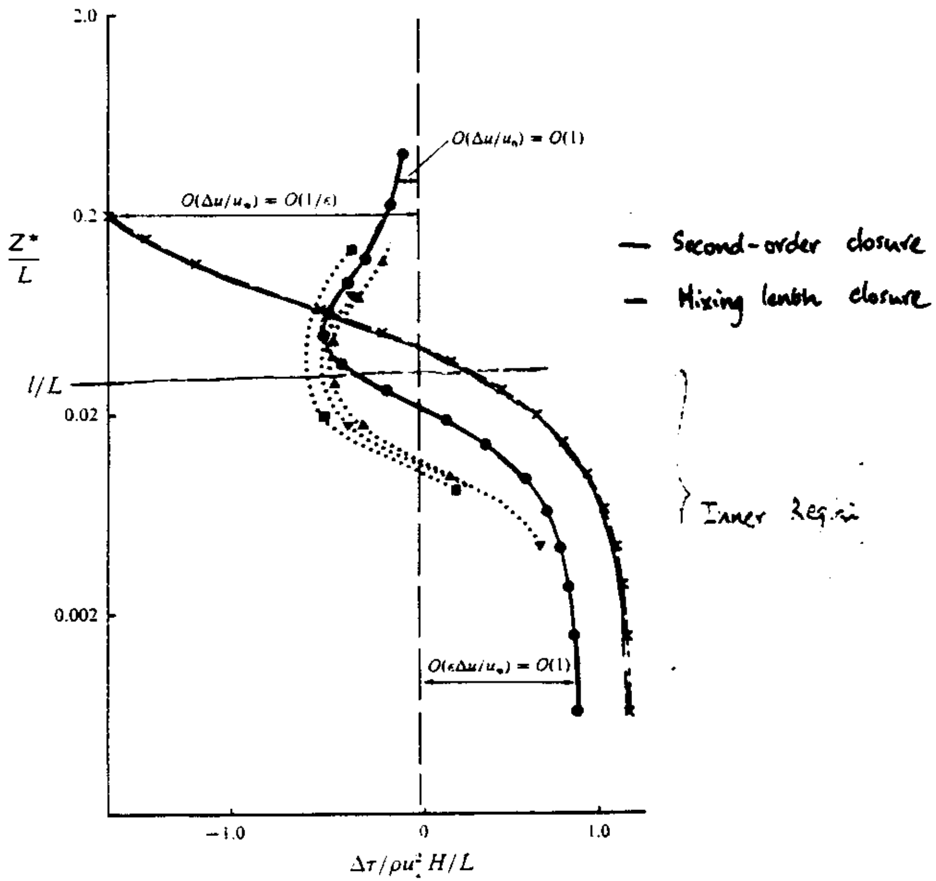
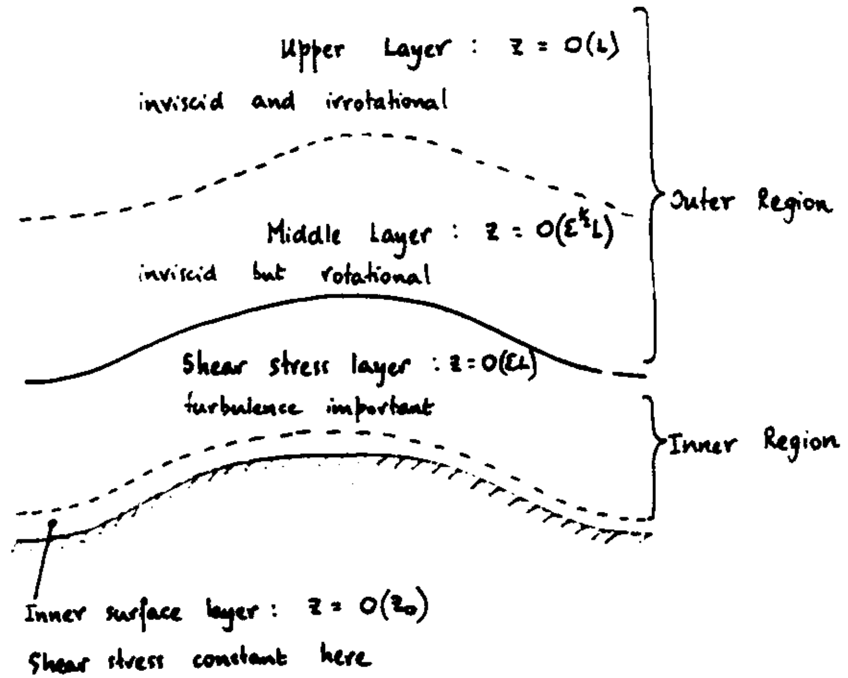


FIGURE 8. Values of the shear-stress perturbations at the crest of an isolated hill computed using the second-order-closure and mixing-length models (with the same hill shape and parameters as the numerical results in figure 4c), together with comparisons with field data. Note how the second-order closure model, which is in good agreement with the field data, shows a small perturbation in the outer region, whereas the mixing-length model erroneously predicts a large negative perturbation: ----, upwind profile: —○—, numerical results using the second-order-closure model: —×—, numerical results using the mixing-length model: ▲, field data for Askervein (reported in Zeman & Jensen 1987): ▼, field data for Nyland hill (Mason 1986b); ■, field data for Blashaval (Mason & King 1985).

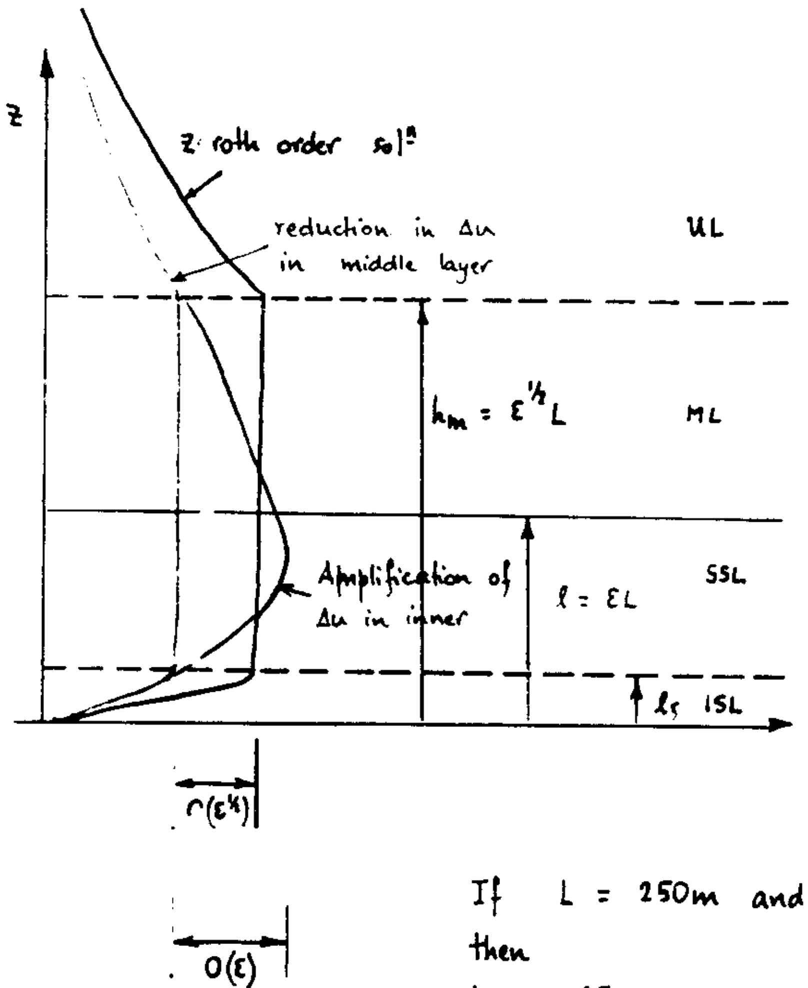
Overall structure of the perturbed boundary layer



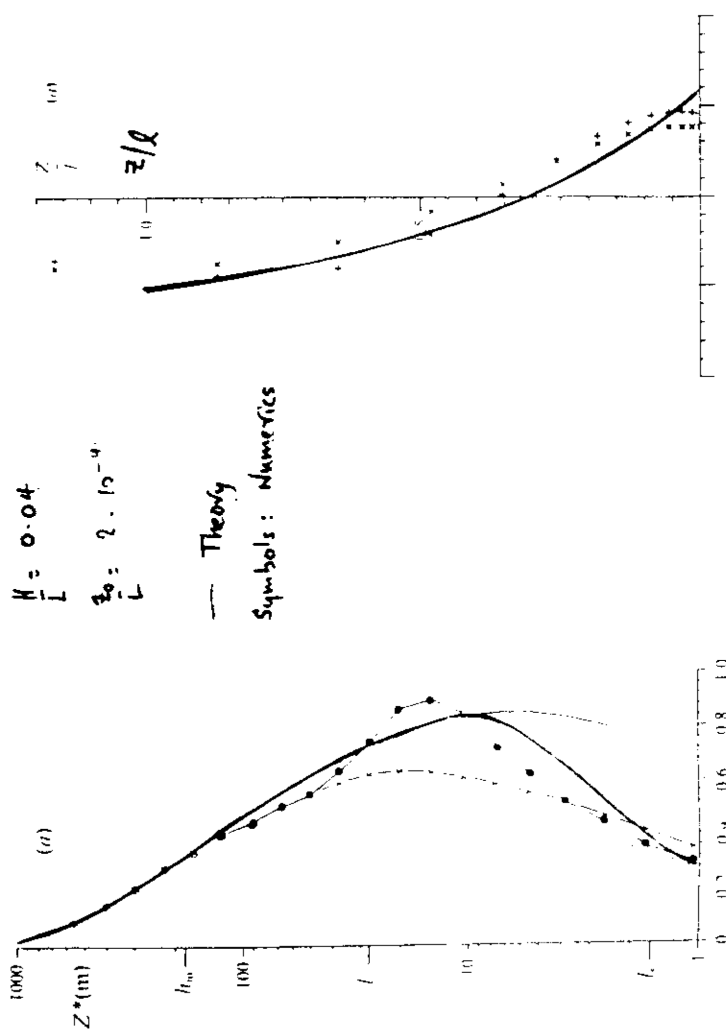
ie 4 layers!

Solutions have been coded up into FLOWSTAR which calculates air flow over arbitrarily shaped hills.

Schematic of solution at crest of hill



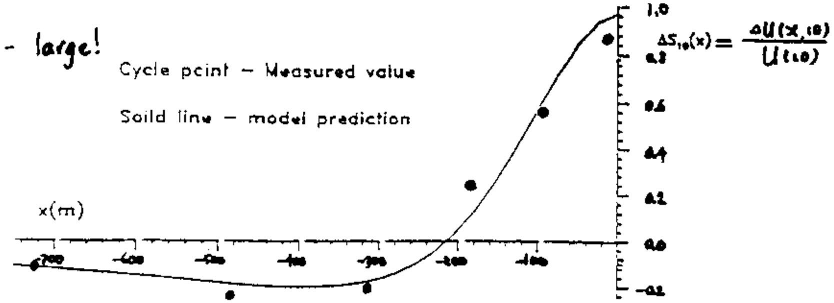
If $L = 250m$ and $z_0 = 0{\cdot}1m$
then
$h_m = 95m$
$l = 16m$
$l_s = 1.3m$



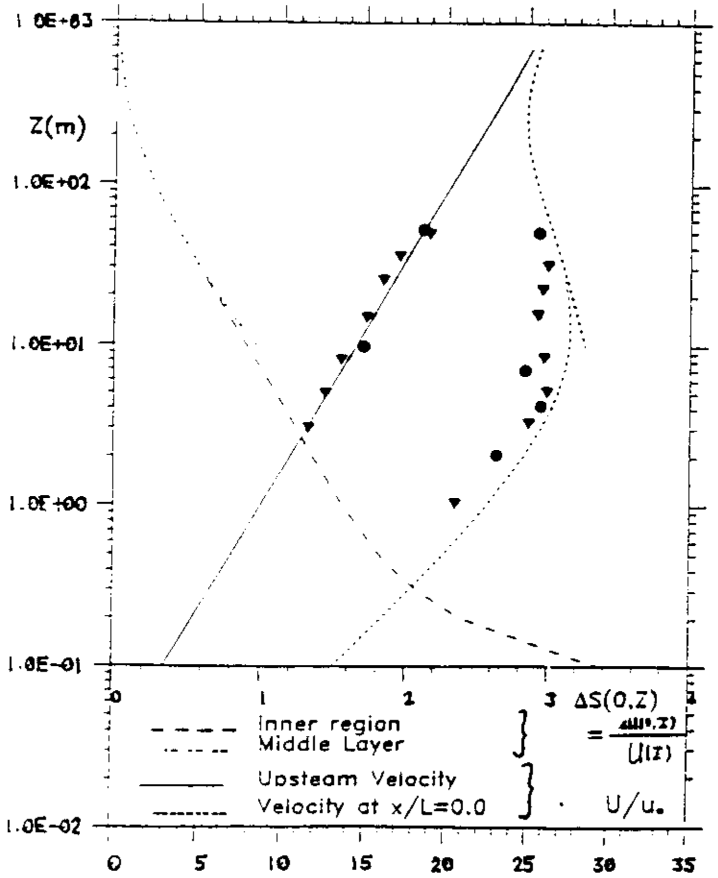
Comparison of the theory with a nonlinear numerical model with second-order closure (Newley 1986). Crest of sinusoidal terrain

Comparison with field data from Askervein experiment

$\frac{H}{L} = 0.3$ - large!



Speed-up at Z=10m along the x direction



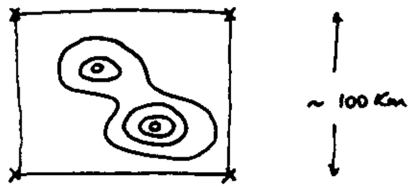
Force on the hill

Turbulent flow over a flat surface

⇒ frictional force on the surface.

When surface has undulations = hills force increased

Motivation : Large scale weather forecast models



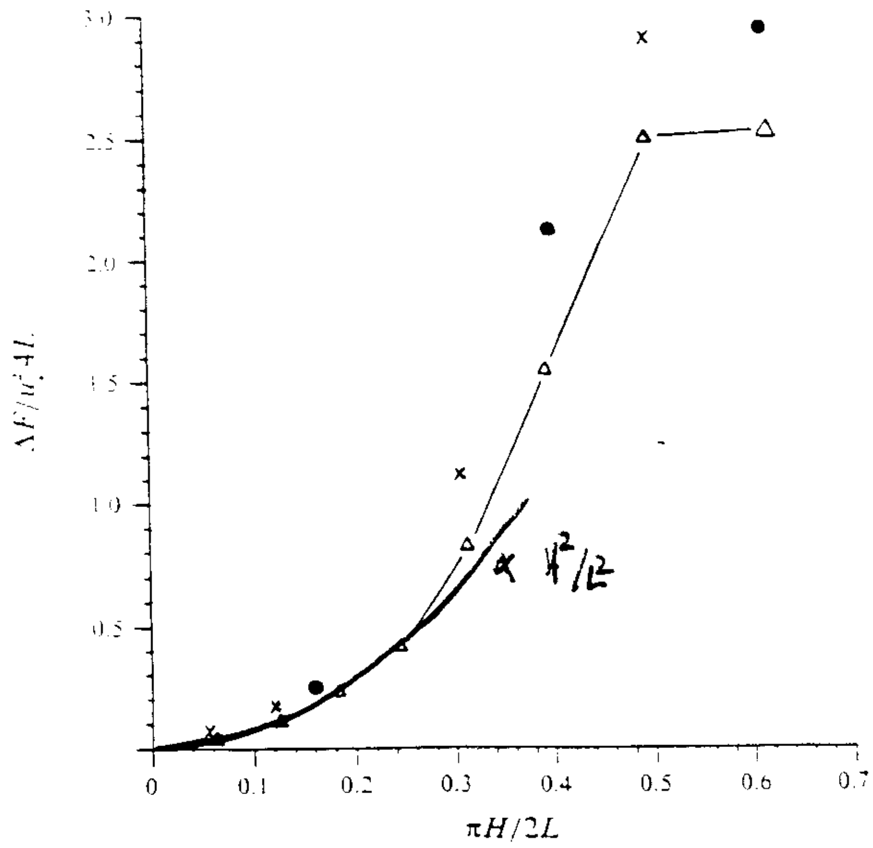
Effect of hills on momentum exchange must be parameterised

$$\Delta F = \frac{1}{L}\int_{-\infty}^{\infty} \Delta p \, f' \, dx + \frac{1}{L}\int_{-\infty}^{\infty} \Delta \tau \, dx$$

smaller

Theory gives $\quad \Delta F = u_*^2 \, \beta^4 \frac{1}{L} \int_{-\infty}^{\infty} \{f'\}^2 dx$

$$\beta = \frac{u_B(\varepsilon^{1/2} L)}{u_B(\varepsilon L)}$$



Variation of drag with slope

Review of requirement for theory to be valid

1. Low slope, $H/L \ll 1$

Then the governing equations can be linearised

Typically okay up to crest of hill if $H/L \lesssim 1/3$

2. $\varepsilon = u_*/U_0 \ll 1$

Then the Reynolds stress gradients can be neglected except in the inner region

In practice this can be determined by the ratio z_0/L

$$\frac{u_*}{U_0} \sim \frac{1}{1/\kappa \ln L/z_0}$$

3. Logarithmic velocity profile describes U_B in inner region

This is set explicitly in the solution procedure.

In the outer region U_B can have a more general form.

$U_B(z)$ logarithmic if $z \lesssim \delta/5$

$\therefore$ Require $\ell \lesssim \delta/5 \Rightarrow L \lesssim \frac{1}{5}\delta \frac{U_0}{u_*}$

~ 20 Km !

Brief Bibliography of papers using the 4 layer method.

1 Flow over hills

Jackson & Hunt (1975) Quart. J. Roy. Met. Soc 101 929 - 955

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2. Flow over roughness changes

Belcher, Xu & Hunt (1990) Q. J. R. Met. Soc. 116 611 - 635.

3. Flow over moving ocean - waves

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