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INTERNATIONAL CENTRE FOR SCIENCE AND HIGH TECHNOLOGY

c/o INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS 34100 TRIESTE (ITALY) VIA GRIGNANO, 9 (ADRIATICO PALACE) P.O. BOX 586 TELEPHONE 040-224572 TELEFAX 040-224575 TELEX 460449 APH I

SMR/760-43

**"College on Atmospheric Boundary Layer
and Air Pollution Modelling"
16 May - 3 June 1994**

"Statistical Analysis of High Return Period Wind Speeds"

G. SOLARI
Institute of Construction Science
University of Genoa
Genoa, Italy

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Statistical analysis of high return period wind speeds

S. Lagomarsino^a, G. Piccardo^a and G. Solari^b

^aInstitute of Science of Constructions, University of Genoa, Via Montallegro 1, 16145 Genoa, Italy

^bDepartment of Structures, University of Calabria, 87036 Arcavacata di Rende, Cosenza, Italy

Abstract

This paper demonstrates that in the range of high return periods different extreme wind speed distributions tend to agree or to diverge due to the parameters of the parent distribution. General criteria are therefore established to select a priori the most reliable or prudential model. Approximate relationships are also derived, for evaluating the parameters of a given model assuming as known the parameters of another.

1. INTRODUCTION

The statistical analysis of wind speed in well behaved climates is traditionally carried out in two subsequent steps. In the first step the probability distribution of the parent population is determined. In the second step the statistical analysis of extreme wind speeds is performed usually applying three different methods herein referred to as process analysis, population analysis and asymptotic analysis; this last method does not require the preliminary evaluation of the parent distribution.

The practical comparison of these methods reveals that, when correctly applied, they furnish almost coincident results in the range of the return periods not largely exceeding the number of years for which measurements are available. However, they often lead to increasing divergencies when considering return periods well above this range. This situation becomes important whenever exceptional structures are of concern. In these cases it is difficult and often impossible to establish, in general terms, which of these methods is the most reliable. In fact, from a theoretical point of view, every kind of approach has merits and defects; on the other hand, experimental judgements cannot be given, since no data base is at present large enough to allow such evaluations.

Starting from these considerations, the reciprocal limit behaviour of extreme wind speed distributions is analyzed. In this way it is demonstrated that different models tend to agree or to diverge due to the parameters of the parent distribution; general criteria are therefore established to select a priori the most reliable or prudential method. Furthermore, several approximate but conceptually consistent relationships are derived, for evaluating the parameters of a given model assuming as known the parameters of another; some formulae previously proposed in the technical

literature are in this way obtained again, other formulae are judged on the contrary to be too approximate or, in some cases, definitely unjustified.

Numeric examples are developed by means of a computer program implemented by the Authors. The data used relates to wind speeds, averaged over 10 minutes, measured by ITAV (Ispettorato delle Telecomunicazioni ed Assistenza al Volo) in the meteorological stations of the Italian Air Force. Model parameters are evaluated, for every case, according to the most efficient estimation criterion among those discussed in Refs. [1,2].

2. PARENT DISTRIBUTION

The use of Weibull's distribution [3] as a probabilistic model of the mean wind speed arises from pure empiricism but is by now accepted by all the sectors interested in wind phenomena [4,5,6,7]. It is defined by:

$$f_V(v) = \frac{k'}{c'} \left(\frac{v}{c'}\right)^{k'-1} \exp\left[-\left(\frac{v}{c'}\right)^{k'}\right] \quad (v \geq 0) \quad (1)$$

$$F_V(v) = 1 - \exp\left[-\left(\frac{v}{c'}\right)^{k'}\right] \quad (v \geq 0) \quad (2)$$

Eqs. (1,2) coincide with the exponential distribution for $k'=1$, with the Raileigh's distribution for $k'=2$.

It must be observed that Weibull's distribution imposes conditions $f_V(0)=F_V(0)=0$. The anemometric recordings, on the other hand, give, according to the site examined, more or less long periods of wind calms. This incongruence can be eliminated by applying the censored technique [8] or the hybrid technique [9]. The former considers velocities v below the lower threshold v_s of the anemometric instrument as not reliable, modifies the values in interval $[0, v_s)$ attributing an appropriately varied distribution to them; the model parameters k', c' are regressed to modified data. The latter accepts the instrument response substituting Eqs. (1,2) by the formulae:

$$f_V(v) = (1-A) \delta(v) + A \frac{k''}{c''} \left(\frac{v}{c''}\right)^{k''-1} \exp\left[-\left(\frac{v}{c''}\right)^{k''}\right] \quad (v \geq 0) \quad (3)$$

$$F_V(v) = 1 - A \exp\left[-\left(\frac{v}{c''}\right)^{k''}\right] \quad (v \geq 0) \quad (4)$$

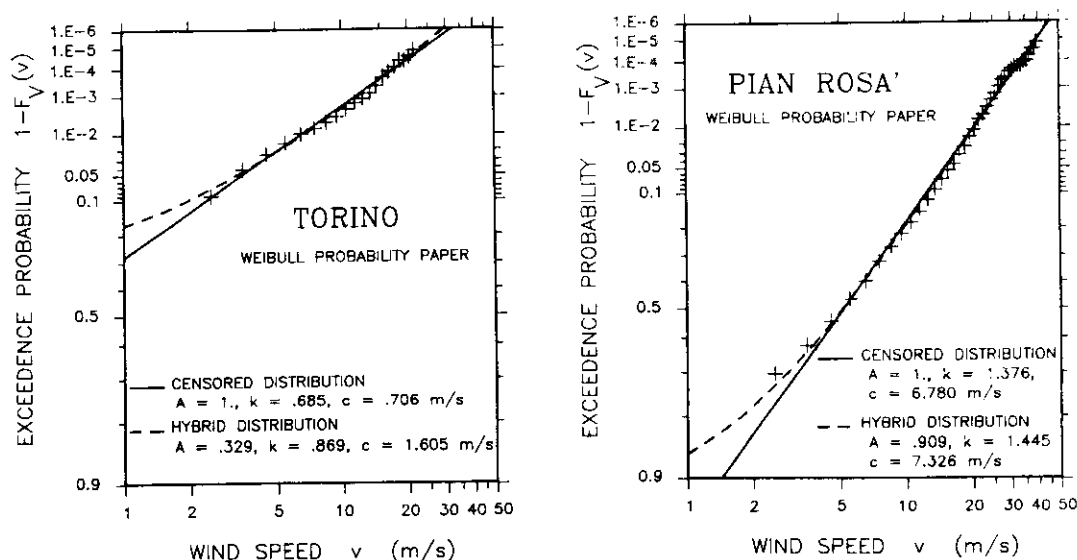
in which A is the probability that $V > 0$, $\delta(\cdot)$ is the Dirac's operator; parameters k'', c'' are estimated on the basis of the only values $v > 0$.

It is demonstrated [2] that $k'' \geq k'$ and $c'' \geq c'$; furthermore the smaller A , the higher k''/k' and c''/c' ; naturally $k'=k''$ and $c'=c''$ for $A=1$; in this case Eqs. (3,4) coincide with Eqs. (1,2). The application of Eqs. (1-4) to the Italian anemometric data shows that the hybrid technique is definitely better than the censored technique since it gathers, more confidently, even the values in the tails of distributions (Figs. 1).

3. EXTREME DISTRIBUTIONS

The probability distribution of the maximum wind speed, M , over a time period T assumed as unitary, is usually calculated on the basis of three alternative methods defined in order as process analysis, population analysis and asymptotic analysis.

Process analysis treats the wind velocity according to a stochastic stationary process [10]. The average number \bar{N} of the up-crossings of the threshold v in the unit time T is given by the equation [11,12]:



Figures 1. Examples of parent probability distributions.

$$\bar{N}(v) = \int_0^{\infty} \dot{v} f_{V\dot{V}}(v, \dot{v}) d\dot{v} \quad (5)$$

in which \dot{V} is the derivative process of V ; $f_{V\dot{V}}(v, \dot{v})$ is the joint density function of V and \dot{V} ; v and \dot{v} are the state variables of the distribution. Eq. (5) takes a particularly simple expression assuming V and \dot{V} as statistically independent [2,10]; in this case $f_{V\dot{V}}(v, \dot{v}) = f_V(v) f_{\dot{V}}(\dot{v})$, $f_{\dot{V}}(\dot{v})$ being the density function of \dot{V} . Eq. (5) for this reason becomes [1,2]:

$$\bar{N}(v) = \lambda f_V(v) \quad (6)$$

$$\lambda = \int_0^{\infty} \dot{v} f_{\dot{V}}(\dot{v}) d\dot{v} \quad (7)$$

Introducing the further hypothesis that v is a sufficiently high threshold, its up-crossings can be considered as rare and independent events and treated therefore as a Poissonian process. The cumulative distribution of maximum, $F_M(v)$, is in this case:

$$F_{M-\lambda}(v) = \exp[-\lambda f_V(v)] \quad (8)$$

Population analysis considers data bases composed of N^* values from which N samples formed by $n=N^*/N$ values corresponding to time T are extracted according to a principle equivalent to casuality. Assuming these samples as statistically independent, the cumulative distribution of maximum, $F_M(v)$, is equal to the cumulative parent distribution, $F_V(v)$, raised to the n -th power. In reality, the hypothesis of statistical independence is wholly unacceptable. It can be removed by expressing $F_M(v)$ through formula [4,5]:

$$F_{M-n'}(v) = [F_V(v)]^{n'} \quad (9)$$

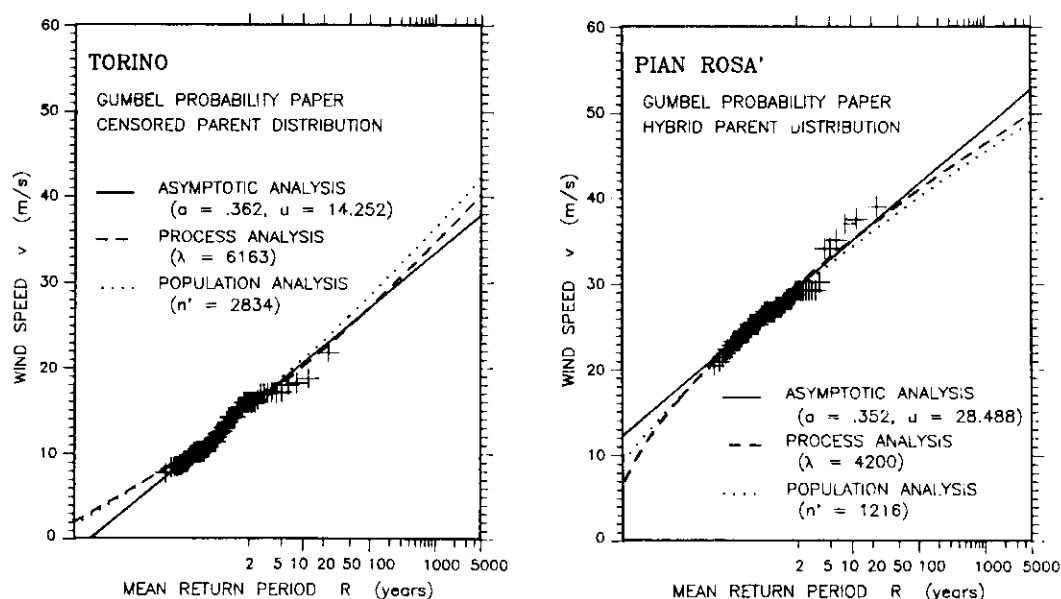
n' being the number of data effectively independent in the course of T .

Asymptotic analysis is the best known and most used procedure. It is demonstrated [13] that, for n' tending to infinity, the extreme distribution (9) tends, in accordance with rules associated to the tail of the parent distribution [14], to limit distributions, so called asymptotic. Assuming that the parent distribution is expressed by Weibull's model (Eqs. 1-4) it comes into the class of the exponential tail distributions whose extreme distribution tends towards the type I asymptotic distribution:

$$F_{M-I}(v) = \exp\{-\exp[-a(v-u)]\} \quad (10)$$

u being the mode and $1/a$ the dispersion. However, since Eq. (10) is unlimited, it is not completely adequate conceptually to represent the wind speed ($v \geq 0$). To this one can add that applying the asymptotic treatment to samples composed by a finite number of independent data, errors increase on increasing the return period of the estimates concerned.

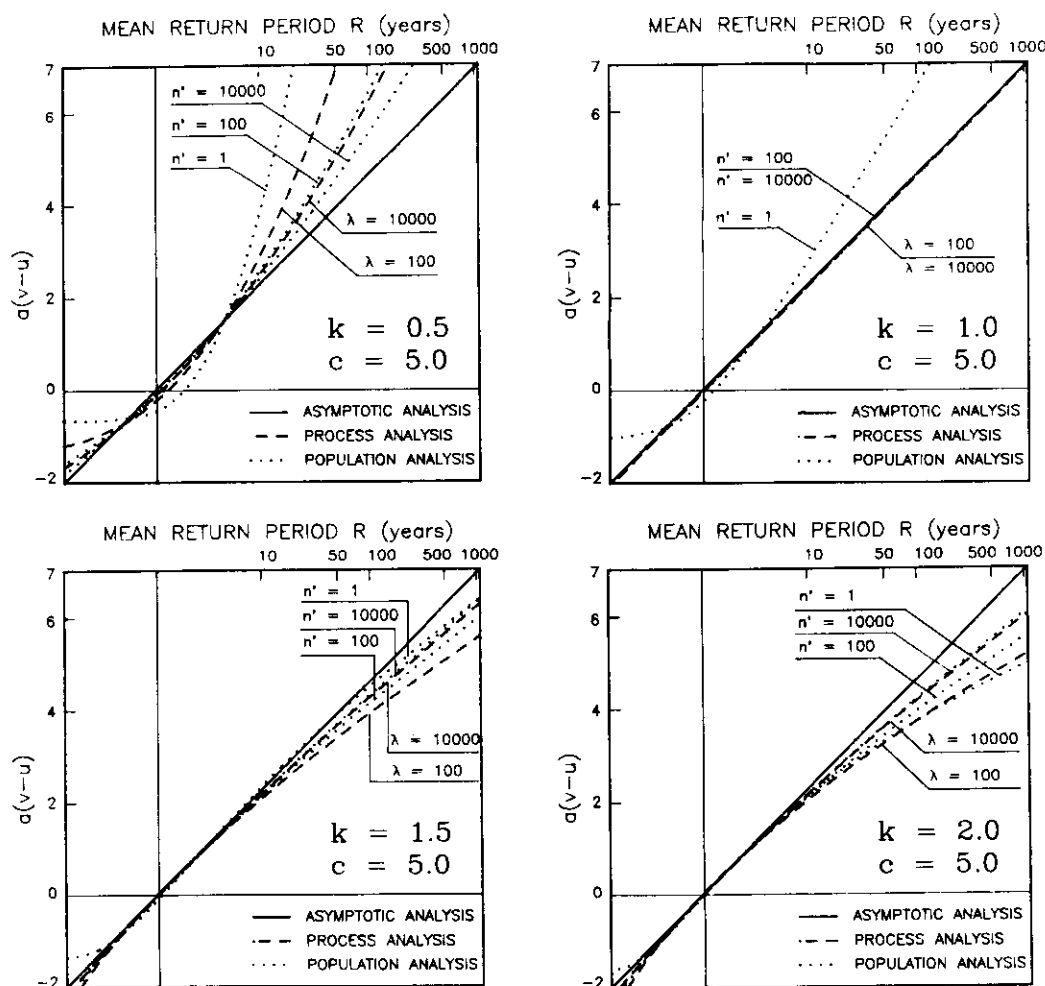
Comparing the results given by Eqs. (8,9,10) it is clear that these expressions, if correctly applied, give estimates of the maximum wind speed which are more or less coincident in the domain of the mean return period R not much greater than the time period for which records are available. On the other hand they often tend to diverge with the increase of R (Figs. 2).



Figures 2. Examples of extreme probability distributions.

Figs. 3 show Gumbel non-dimensional probability papers in which diagrams of functions $v_x = v(R)$, $R = 1/[1-F_{M-x}(v)]$, $x = \lambda, n', I$, are constructed based upon the criterion formulated in [5] and therein applied limitedly to Eqs. (9,10) with $k=1,2$; observe that the extension of these diagrams to Eq. (8) makes them explicitly dependent on c ; A is assumed to be unitary. These diagrams fully confirm the divergent character of the estimates of v in the range of the high values of R , emphasizing the strict dependence of their tendencies upon k . It seems qualitatively clear, in particular, that (10)

underestimates v with reference to (8,9) if $k < 1$, while (8,9) furnish lower estimates of v compared to those given by (10) if $k > 1$; no judgement can be expressed according to the relative tendencies of (8,9). However, it is noteworthy to make an important specification. These diagrams do not give, as seems to emerge from some of their interpretations [15], the rate of convergence, with the increase of λ and n' , of Eqs. (8,9) to Eq. (10), but rather the tendency of Eqs. (8,9) to become straight lines as Eq. (10) rigorously is. In fact, in Gumbel real probability papers, when increasing λ and n' , $v_\lambda(R)$ and $v_{n'}(R)$ respectively move in parallel to themselves.



Figures 3. Theoretical extreme probability distributions.

4. LIMIT TENDENCIES

A generic cumulative distribution is monotone, not decreasing, and tending to 1 with the tending to infinity of the state variable. $F_{M-\lambda}(v)$, $F_{M-n'}(v)$, $F_{M-I}(v)$ thus admit the same unitary limit for v tending to

infinity. Transforming Eqs. (8,9,10) into equations $v = v(R)$, $R = 1/[1 - F_{M-x}^x(v)]$, $x = \lambda, n', I$, it is apparent that for R tending to infinity, v tends to infinity. The first objective of the present paper is to establish whether for v tending to infinity, $F_{M-x}^x(v)$ is greater, equal or less than $F_{M-y}^y(v)$, $x, y = \lambda, n', I$, $x \neq y$; these three conditions clearly correspond in the order, for R tending to infinity, to $v(R)$ less, equal or greater than $v(R)$. The analysis is initially made by studying the reciprocal tendencies of all couplings of the three extreme distributions herein examined.

Comparing Eqs. (9) and (10) one sees that $F_{M-n'}^n(v)$ is less, equal or greater than $F_{M-I}^I(v)$, and therefore $v_{n'}(R)$ is greater, equal or less than $v_I(R)$, if G^* is greater, equal or less than $G(v)$, being:

$$G^* = \frac{\exp(au)}{An'} \quad (11)$$

$$G(v) = \frac{-\ln\{1 - A \exp[-(\frac{v}{c})^k]\}}{A \exp[-\frac{v}{c}(ac)]} \quad (12)$$

It is demonstrated that $G^* = G(v)$ for any value of v and R , if and only if $k=1$, $ac=1$, $A \neq 0$, $\exp(au)/(An') \rightarrow 1$. Table 1 summarizes the different situations arising for v and R tending to infinity. Figs. 4 show the diagrams corresponding to $G^* = G(v)$ in the domain of the values of most practical interest; they can be used to evaluate the values of v and R above which the limit tendencies listed in Table 1 are satisfied; it is apparent the marginal role of A especially for high values of v and R .

Comparing Eqs. (8) and (10) one sees that $F_{M-\lambda}^\lambda(v)$ is less, equal or greater than $F_{M-I}^I(v)$, and therefore $v_\lambda(R)$ is greater, equal or less than $v_I(R)$, if H^* is greater, equal or less than $H(v)$, being:

$$H^* = \frac{\exp(au)}{\lambda a A} \quad (13)$$

$$H(v) = \frac{k (\frac{v}{c})^{k-1} \exp[-(\frac{v}{c})^k]}{(ac) \exp[-\frac{v}{c}(ac)]} \quad (14)$$

It is demonstrated that $H^* = H(v)$ for any value of v and R , if and only if $k=1$, $ac=1$, $\exp(au)/(\lambda a A) = 1$. Table 2 summarizes the different situations arising for v and R tending to infinity. Figs. 5 show the diagrams corresponding to $H^* = H(v)$; they can be used to evaluate the values of v and R above which the limit tendencies listed in Table 2 are satisfied.

Comparing finally Eqs. (8) and (9) one sees that $F_{M-\lambda}^\lambda(v)$ is less, equal or greater than $F_{M-n'}^n(v)$, and therefore $v_\lambda(R)$ is greater, equal or less than $v_{n'}(R)$, if J^* is greater, equal or less than $J(v)$, being:

$$J^* = \frac{\lambda}{n'c} \quad (15)$$

$$J(v) = \frac{-\ln\{1 - A \exp[-(\frac{v}{c})^k]\}}{Ak (\frac{v}{c})^{k-1} \exp[-(\frac{v}{c})^k]} \quad (16)$$

It is demonstrated that $J^* = J(v)$ for any value of v and R , if and only if $k=1$, $A \neq 0$, $\lambda/(n'c) = 1$. Table 3 summarizes the different situations arising for v and R tending to infinity. Fig. 6 shows the diagrams corresponding to $J^* = J(v)$; they can be used to evaluate the values of v and R above which

Table 1. Limit reciprocal tendencies of population and asymptotic analyses.

| $k < 1$ | $k = 1$ | | | $k > 1$ |
|--|--|--|--|--|
| $G(v) \rightarrow +\infty$ | $ac > 1$ | $ac = 1$ | | $G(v) \rightarrow 0$ |
| | $G(v) \rightarrow +\infty$ | $G(v) \rightarrow 1$ | | $G(v) \rightarrow 0$ |
| $F_{M-n}(v) < F_{M-1}(v)$ $v_n(R) > v_1(R)$ | $F_{M-n}(v) < F_{M-1}(v)$ $v_n(R) > v_1(R)$ | $\frac{\exp(au)}{An'} < 1$ $F_{M-n}(v) < F_{M-1}(v)$ $v_n(R) > v_1(R)$ | $\frac{\exp(au)}{An'} = 1$ $F_{M-n}(v) = F_{M-1}(v)$ $v_n(R) = v_1(R)$ | $\frac{\exp(au)}{An'} > 1$ $F_{M-n}(v) > F_{M-1}(v)$ $v_n(R) < v_1(R)$ |
| | | | | $F_{M-n}(v) > F_{M-1}(v)$ $v_n(R) < v_1(R)$ |

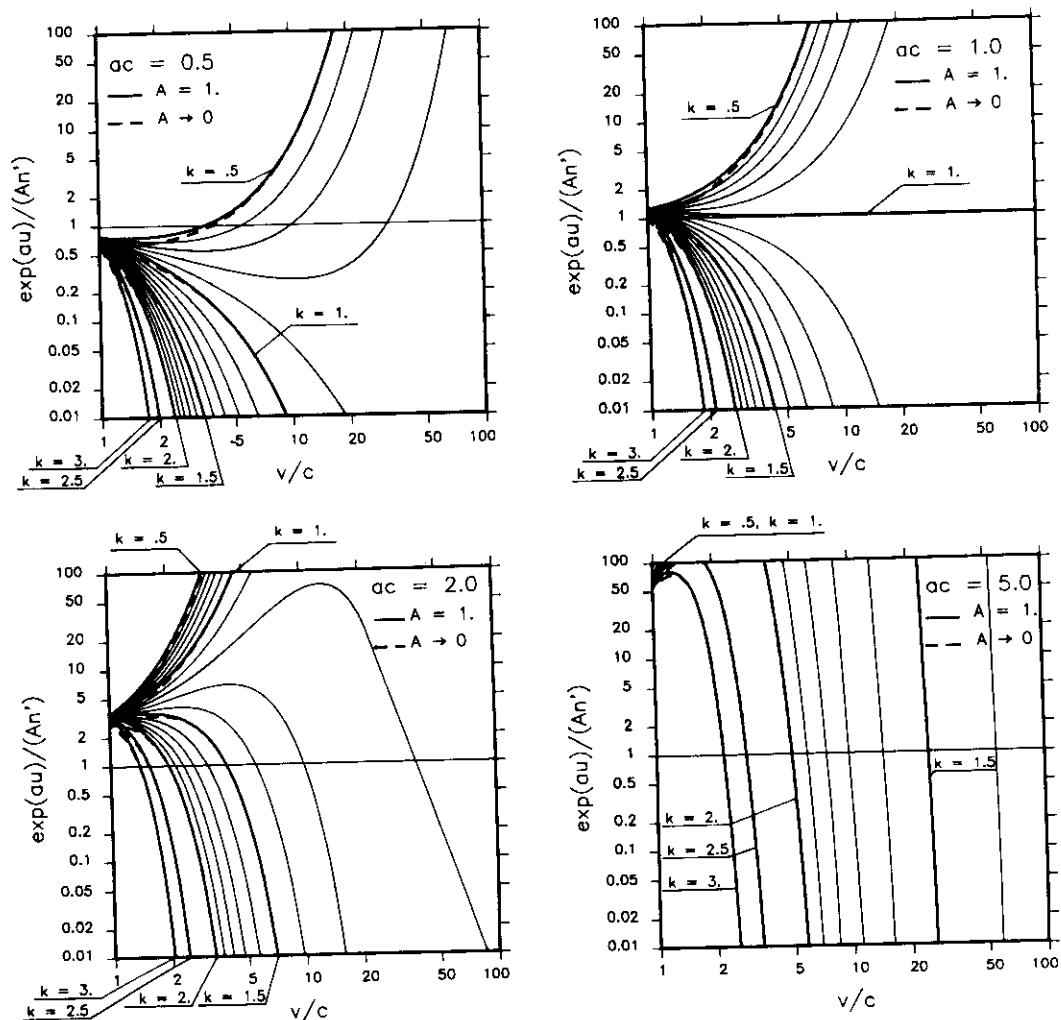
Figures 4. Parametric diagrams of $G^* = G(v)$.

Table 2. Limit reciprocal tendencies of process and asymptotic analyses.

| $k < 1$ | $k = 1$ | | | | $k > 1$ | |
|--|--|--|--|--|--|--|
| $H(v) \rightarrow +\infty$ | $ac > 1$ | $ac = 1$ | | | $ac < 1$ | $H(v) \rightarrow 0$ |
| $F_{M-\lambda}(v) < F_{M-I}(v)$ $v_\lambda(R) > v_I(R)$ | $H(v) \rightarrow +\infty$ | $H(v) = 1$ | | | $H(v) \rightarrow 0$ | $F_{M-\lambda}(v) > F_{M-I}(v)$ $v_\lambda(R) < v_I(R)$ |
| | $F_{M-\lambda}(v) < F_{M-I}(v)$ $v_\lambda(R) > v_I(R)$ | $\frac{\exp(au)}{\lambda a A} < 1$ $F_{M-\lambda}(v) < F_{M-I}(v)$ $v_\lambda(R) > v_I(R)$ | $\frac{\exp(au)}{\lambda a A} = 1$ $F_{M-\lambda}(v) = F_{M-I}(v)$ $v_\lambda(R) = v_I(R)$ $\forall v, R$ | $\frac{\exp(au)}{\lambda a A} > 1$ $F_{M-\lambda}(v) > F_{M-I}(v)$ $v_\lambda(R) < v_I(R)$ | $F_{M-\lambda}(v) > F_{M-I}(v)$ $v_\lambda(R) < v_I(R)$ | |

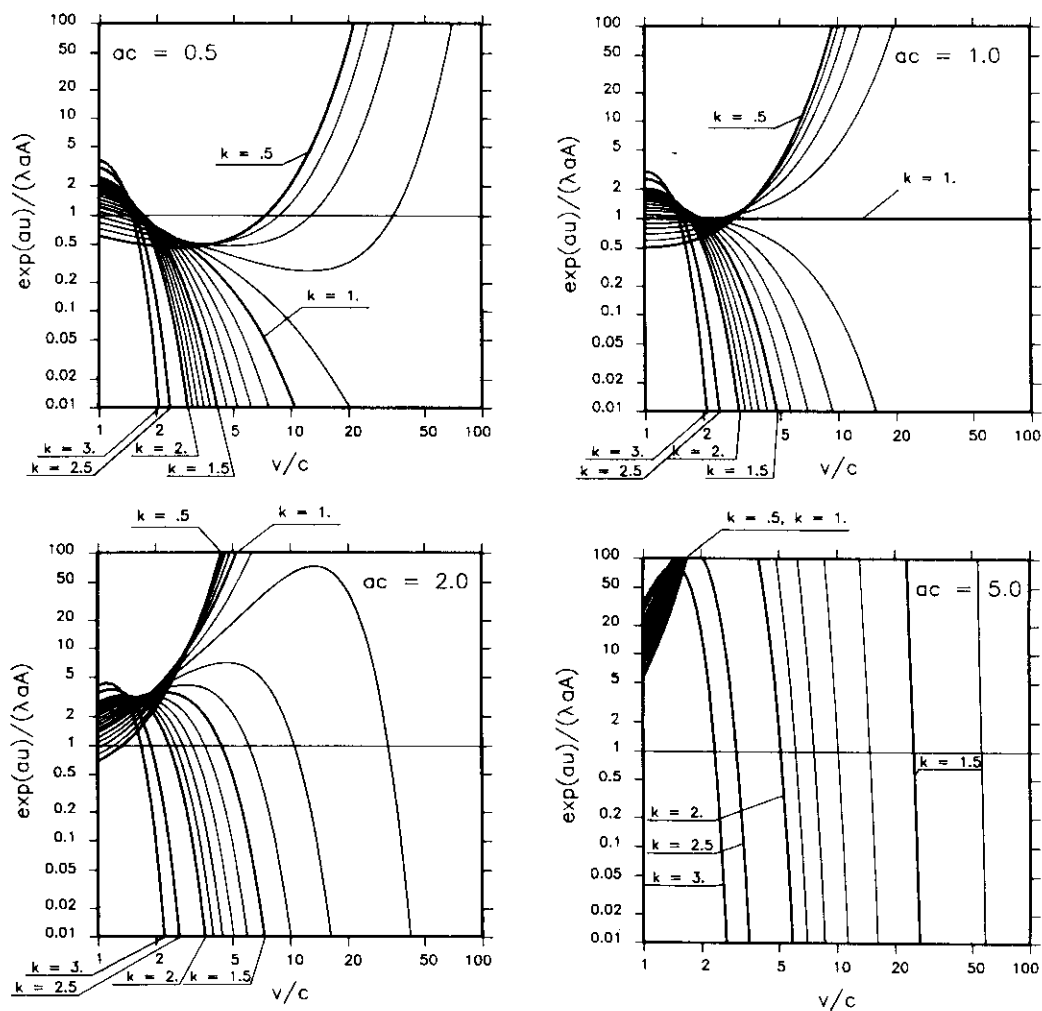
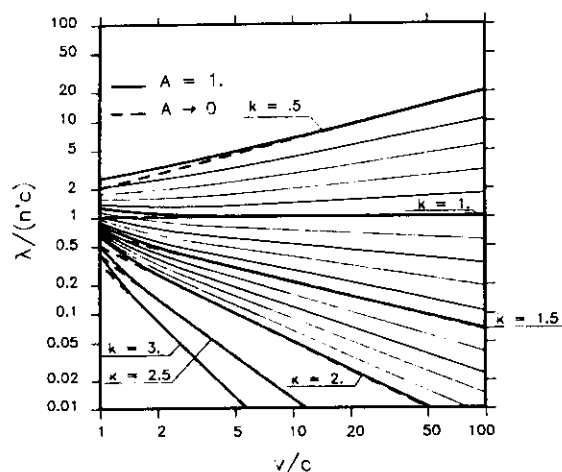
Figures 5. Parametric diagrams of $H^* = H(v)$.

Table 3. Limit reciprocal tendencies of process and population analyses.

| $k < 1$ | $k = 1$ | | | $k > 1$ |
|--|---|---|---|--|
| $J(v) \rightarrow +\infty$ | $J(v) \rightarrow 1$ | | | $J(v) \rightarrow 0$ |
| $F_{M-\lambda}(v) > F_{M-n'}(v)$ $v_\lambda(R) < v_{n'}(R)$ | $\frac{\lambda}{n'c} < 1$ $F_{M-\lambda}(v) > F_{M-n'}(v)$ $v_\lambda(R) < v_{n'}(R)$ | $\frac{\lambda}{n'c} = 1$ $F_{M-\lambda}(v) = F_{M-n'}(v)$ $v_\lambda(R) = v_{n'}(R)$ | $\frac{\lambda}{n'c} > 1$ $F_{M-\lambda}(v) < F_{M-n'}(v)$ $v_\lambda(R) > v_{n'}(R)$ | $F_{M-\lambda}(v) < F_{M-n'}(v)$ $v_\lambda(R) > v_{n'}(R)$ |

Figure 6. Parametric diagrams of $J^* = J(v)$.

the limit tendencies listed in Table 3 are satisfied; it is apparent, as in Figs. 4, the marginal role of A especially for high values of v and R .

Joining together the above listed results, it is apparent the existence of three distinct behaviours, first of all associated with the value of k :

(a) if $k=1$, then:

(1) $ac=1$, $\exp(au)=An'$ imply $F_{M-n'}(v)=F_{M-I}(v)$, $v_{n'}(R)=v_I(R)$, for large v, R ;

(2) $ac=1$, $\exp(au)=\lambda aA$ imply $F_{M-\lambda}(v)=F_{M-I}(v)$, $v_\lambda(R)=v_I(R)$, for any v, R ;

(3) $\lambda=n'c$ implies $F_{M-\lambda}(v)=F_{M-n'}(v)$, $v_\lambda(R)=v_{n'}(R)$, for large v, R ;

in all other cases the limit conditions listed in Tables 1,2,3 come into force, these having, however, a very limited importance, $k=1$ being a bifurcation point;

(b) if $k>1$, this being the case of major interest, then, for large v, R :

$$F_{M-I}(v) < F_{M-\lambda}(v) < F_{M-n'}(v) \quad (17a)$$

$$v_{n'}(R) < v_\lambda(R) < v_I(R) \quad (17b)$$

for any value of all parameters. In this situation Eq. (10) gives, in relation to the Eqs. (8,9) of higher level, more prudential estimates of v the more $k>1$; Eq. (8) is more prudential than Eq. (9);

(c) if $k < 1$, then, for large v, R :

$$F_{M-n'}(v) < F_{M-\lambda}(v) < F_{M-1}(v) \quad (18a)$$

$$v_I(R) < v_\lambda(R) < v_{n'}(R) \quad (18b)$$

for any value of all parameters. In this situation Eq. (10) gives, in relation to the Eqs. (8,9) of higher level, less conservative estimates of v the more $k < 1$; Eq. (9) is more prudential than Eq. (8).

These considerations clearly explain Figs. 2.

Remembering that the above limit tendencies are practically invariant with respect to A , it is clear that the use of the censured model (1,2) instead of the hybrid model (3,4) has no influence on the limit behaviour of the extreme distributions. What takes on instead an essential role is that the use of Eqs. (1,2) gives rise to k values, k' , less than or equal to the k values, k'' , given by Eqs. (3,4). From this it follows that extreme estimates by Eqs. (3,4), apart from being decidedly better, are also lower than estimates based on Eqs. (1,2), independently of using Eq. (8) or (9).

It is relevant to observe that $k < 1$ does not represent a case of pure theoretical interest. In Italy, for instance, the 15% of data collected by meteorological stations exhibits $k < 1$ if fitted by the hybrid technique; this percentage reaches the 35% when using the censured model.

5. APPROXIMATE RELATIONSHIPS

Having established the criteria to select a priori the most reliable or prudential extreme distribution, the opportunity becomes obvious of instituting simple but effective approximate relationships which permit the determination of the parameters of a given model on the basis of the parameters of another model. The assigning parameters λ and n' (of complex evaluation [1,2]) is clearly important in terms of parameters A, k, c, u, a (easier to be estimated): n' expresses the number of the independent repetitions of the mean wind speed during T and thus plays a central role in the sector of reliability analyses; the knowledge of λ , instead, makes it possible to extend the process analysis from the study of the wind to the study of its effects, according to global risk evaluations [16]. Also the calculation of λ from n' , and viceversa, can be relevant [10].

The problem is formulated here comparing couples of different models. The relationships linking the parameters of one to the parameters of another are based on three hypotheses: (a) the extreme distributions are more or less overlaid in the domain of the experimental data; (b) they coincide exactly for $v=u$; (c) u is much greater than c .

Comparing Eqs. (9) and (10) it follows that:

$$n' = \frac{1}{1 - F_V(u)} = \frac{1}{A \exp[-(\frac{u}{c})^k]} \quad (19)$$

This same comparison is formulated in [4] with the intention of evaluating u, a and therefore $\pi=ua$ in terms of k, c, n' being $A=1$. Generalizing these expressions to the case $A \neq 1$ [17], one obtains:

$$u = c [\ln(n'A)]^{1/k} \quad (20)$$

$$a = \frac{k}{c} [\ln(n'A)]^{1-1/k} \quad (21)$$

$$\pi = ua = k \ln(n'A) \quad (22)$$

Eq. (20) corresponds to imposition $F_{M-n'}(u) = F_{M-I}(u)$; Eq. (21) requests the additional condition $f_{M-n'}(u) = f_{M-I}(u)$, $f_M(v)$ being the density function of maximum M . In the light of this concept, the inversion of Eq. (22) [5,17]:

$$n' = \frac{1}{A \exp(-\frac{ua}{k})} \quad (23)$$

is thoroughly unjustified since it presupposes a criterion definitely uprooted from conditions $F_{M-n'}(u) = F_{M-I}(u)$, $f_{M-n'}(u) = f_{M-I}(u)$; these equations are in fact clearly conflicting, with the only exception of case $k=1$, $ac=1$, when applied both for determining the single parameter n' .

Comparing Eqs. (8) and (10) it follows that:

$$\lambda = \frac{1}{f_V(u)} = \frac{1}{A \frac{k}{c} (\frac{u}{c})^{k-1} \exp[-(\frac{u}{c})^k]} \quad (24)$$

Finally, comparing Eqs. (8) and (9) it follows that:

$$\frac{\lambda}{n'} = \frac{1 - F_V(u)}{f_V(u)} = \frac{c}{k} (\frac{u}{c})^{1-k} \quad (25)$$

Eq. (25) can be rewritten in terms of parameters A, k, c by substituting Eqs. (20,21) into Eq. (25). One has:

$$\frac{\lambda}{n'} = \frac{c}{k} [\ln(n'A)]^{1/k-1} \quad (26)$$

It is relevant to notice that applying Eqs. (25,26), if $k < 1$, then $F_{M-n'}(v) < F_{M-\lambda}(v)$ and $v_{n'}(R) > v_{\lambda}(R)$ for any $v > u$; analogously, if $k > 1$, then $F_{M-n'}(v) < F_{M-\lambda}(v)$ and $v_{n'}(R) > v_{\lambda}(R)$ for any $v > u$. It is finally interesting to observe that Eqs. (25,26), thoroughly deny the following formula derived from [10]:

$$\frac{\lambda}{n'} = \frac{c}{k} \quad (27)$$

However, it coincides with Eq. (25) when $k=1$, this being the case in which all approximate assumptions made in [10] become rigorously exact.

Table 4 summarizes mean values, standard deviations, maxima and minima of percent errors associated with formulae in this paragraph when applied to the 40 Italian meteorological stations studied in [18]; values between parentheses correspond to the same data base, having excluded the stations

Table 4. Errors associated to approximate relationships.

| EQUATION | (19) | (20) | (21) | (22) | (23) | (24) | (25) | (26) | (27) |
|--------------------|----------------------|--------------------|----------------------|---------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| MEAN ERROR | -5.3 % (-5.3 %) | 0.6 % (0.6 %) | -10.5 % (-14.0 %) | 16.0 % (19.2 %) | 36.6 % (-49.5 %) | -2.3 % (-0.9 %) | 4.6 % (6.2 %) | 4.4 % (5.9 %) | 57.6 % (59.8 %) |
| STANDARD DEVIATION | 10.0 % (10.4 %) | 1.2 % (1.3 %) | 16.5 % (11.5 %) | 20.5 % (17.7 %) | 351.4 % (45.8 %) | 7.2 % (5.6 %) | 16.1 % (15.7 %) | 15.8 % (15.4 %) | 50.9 % (51.9 %) |
| MAXIMUM ERROR | 14.8 % (14.8 %) | 3.1 % (3.1 %) | 40.5 % (7.4 %) | 63.3 % (63.3 %) | 1925.7 % (89.3 %) | 10.6 % (10.6 %) | 37.1 % (37.1 %) | 36.3 % (36.3 %) | 176.0 % (176.0 %) |
| MINIMUM ERROR | -26.0 % (-26.0 %) | -2.4 % (-2.4 %) | -37.7 % (-37.7 %) | -26.0 % (-8.7 %) | -94.6 % (-94.6 %) | -22.0 % (-14.4 %) | -19.6 % (-19.6 %) | -19.4 % (-19.4 %) | -45.4 % (-45.4 %) |

of Cozzo Spadaro, Sanremo and Venice. It is apparent the possibility of dividing the above relationships into three groups with different properties:

- (a) Eqs. (19), (20), (24), (25), (26) give excellent approximations being only based on condition $F_{M-x}(u) = F_{M-y}(u)$, $x, y = \lambda, n', I$, $x \neq y$;
- (b) Eqs. (21), (22) lead to rougher estimates involving the additional condition $f_{M-x}(u) = f_{M-y}(u)$, $x, y = \lambda, n', I$, $x \neq y$;
- (c) Eq. (27) and especially Eq. (23) are thoroughly misleading; observe in particular that Eq. (23) leads in some cases (the three stations quoted above) to exceptionally high errors.

These results are definitely independent of the values assumed by k .

6. CONCLUSIONS AND PROSPECTS

This paper has framed the limit tendencies of the best known models for representing extreme wind velocities. In particular it has demonstrated that these tendencies strictly depend on the k parameter of the parent distribution. If $k > 1$ the asymptotic analysis furnishes results on the safe side, while the process analysis is, with respect to the population analysis, more prudential. On the other hand, if $k < 1$ the asymptotic analysis becomes unconservative, while the population analysis is, at the same time, the most prudential and reliable one. A general frame has also been given of the most suitable approximate relationships aimed at estimating the parameters of a given model assuming as known the parameters of another. In light of the results obtained, the opportunity clearly emerges of going deeper into this subject by further considering the directional and seasonal effects as well as analyzing the role of statistical uncertainties.

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