UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

UNITED NATIONS INDUSTRIAL DEVELOPMENT ORGANIZATION

INTERNATIONAL CENTRE FOR SCIENCE AND HIGH TECHNOLOGY
c/o INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS 34100 TRIESTE (ITALY) VIA GRIGNANO, 9 (ADRIATICO PALACE) P.O. BOX 586 TELEPHONE 040-224572 TELEFAX 040-224575 TELEX 460449 APH I

SMR/760-59

"College on Atmospheric Boundary Layer and Air Pollution Modelling"
16 May - 3 June 1994

"Atmospheric Planetary Boundary Layer Similarity Theory"

G.D. DJOLOV
University of Zimbabwe
Department of Physics
Mount Pleasant, Zimbabwe

Please note: These notes are intended for internal distribution only.

ATMOSPHERIC PLANETARY BOUNDARY LAYER SIMILARITY THEORY

G.DJOLOV
UNIVERSITY OF ZIMBABWE
DEPARTMENT OOF PHYSICS
MOUNT PLEASANT, P.O.BOX 167
HARARE, ZIMBABWE
E-MAIL DJOLOV@ZIMBIX.UZ.ZW

1. ATMOSPHERIC BOUNDARY LAYER

Atmospheric planetary boundary layer (APBL) is defined as the layer formed in fluid environment (gas or liquid) of rotating planets as a result of the combined action of the pressure gradient, Coriolis and turbulent friction forces. This conditions are usually realized in the immediate vicinity of a material surface where significant exchange of momentum, heat, or mass takes place between the surface and the fluid. Our attention will be focused on the atmospheric boundary layer in regions away from the equator (where the Coriolis force is negligibly small) which is usually called Ekman layer. Some of the approaches and results are , however, applicable for the ocean bottom layer as well for the boundary layers of other planets. The vertical scale of the APBL, which depends on the turbulent friction expressed by the frictional velocity u_* and the Coriolis parameter, can be estimated using similarity arguments as a proportional to the ratio $u_*/|f|$. More precise evaluations show, that the height h of the APBL is approximately ten times less than this ratio, which is about 1 km away from the equatorial region. This value is considerably less than the effective height of the atmosphere (≈ 10 km) and the processes taking place in the APBL are of micrometeorological interest. The height of the APBL varies, however, over a wide range (several tens of meters to few kilometers) and depends on the rate of heating/cooling, strength of winds, the orography and roughness, large scale weather and other factor. Most often in the air pollution applications the APBL height is associated with the mixing height.

2. EXPERIMENTAL DATA

The regime of the meteorological elements in APBL is characterized with relatively slow changes and specific diurnal variations of their vertical gradients. In Figs 1, 2, 3, and 4 the diurnally averaged profiles of the wind, temperature and humidity are shown. The magnitude of the arrows is a measure of the nonstaionarity - the differences between 13 and 01 hours at different heights i.e. they present some characteristic amplitude of the diurnal variation. The common feature is that the amplitude is bigger close to the earth surface with the exception of the wind velocity which is zero.

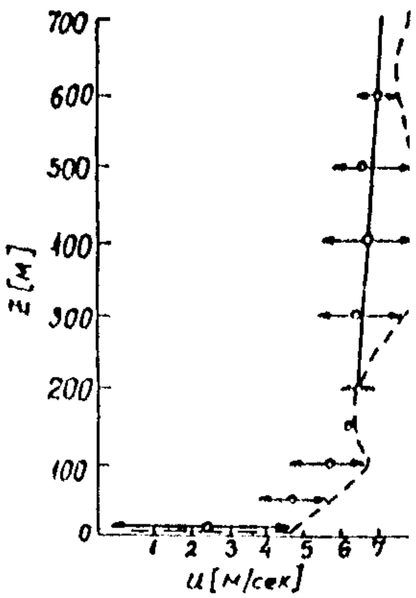


FIG.1 Diurnal wind profile.

The diurnal wind variation is caused by the turbulent regime which has maximum during the day and minimum during the night. This is reflected in the two maximums in the vertical distribution of the amplitude of wind velocity. The wind velocity increases with the height and during periods of intensive turbulence the wind speed at the layers close to the earth surface may be bigger than at the altitudes close to the APBL height. The reverse picture is true in periods of weak turbulence which fact requires at certain level the wind speed to show little variation. It can be seen from Fig.1 that this height is about 200 m. The wind velocity follows a logarithmic law to about 200 m and than slowly increases to the geostrophic wind speed at the height of APBL. Typical variations of the monthly wind averages is shown in Fig.2.

The influence of the underlying surface on the average temperature profiles is not so clearly pronounced (Fig.3). This can be explained with the difference in the interaction between the surface and air masses with different properties (cold or hot).

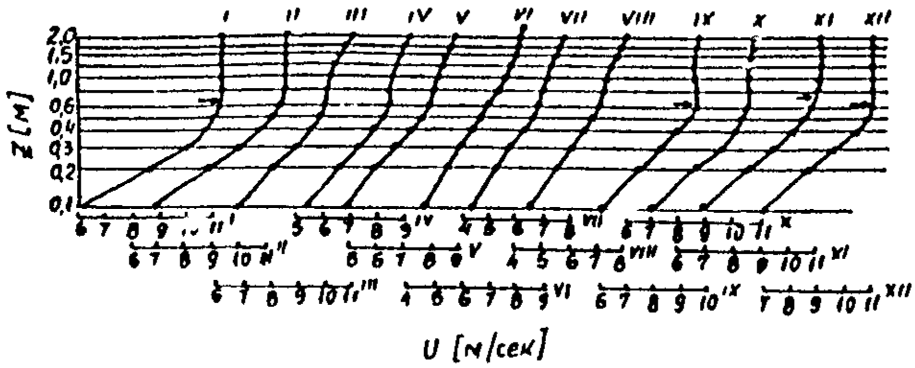


FIG.2 Vertical profile of the monthly mean wind velocity (the arrows show the height of APBL).

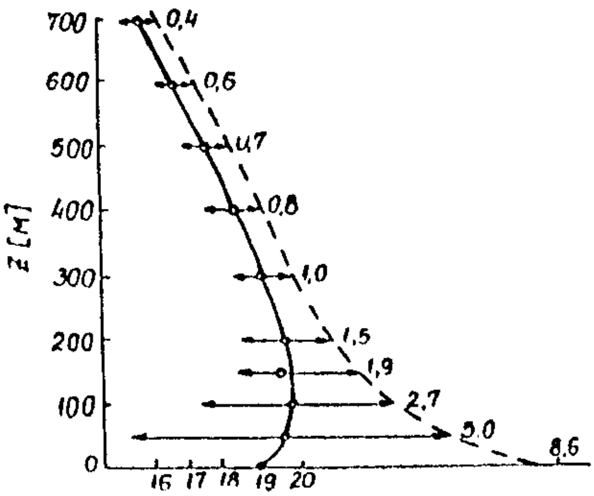


FIG.3 Diurnal profile of the temperature.

The influence of the earth surface as a source of humidity explains its monotonic decrease with altitude (see Fig.4).

In Fig.5 the distribution of two commonly exploited turbulent characteristics K_m (turbulent coefficient for momentum) and K_ϑ (turbulent coefficient of heat exchange). The considerable variations of these quantities is apparent which sends a clear message to the modelers for the inherent difficulties when using K approach for the APBL system of equations closure.

The experimental data for the turbulent energy dissipation ε are presented in Fig.6. It can be noted that ε changes generally as z^{-1} and that the dissipation decreases quicker with height during stable condition, than in unstable APBL.

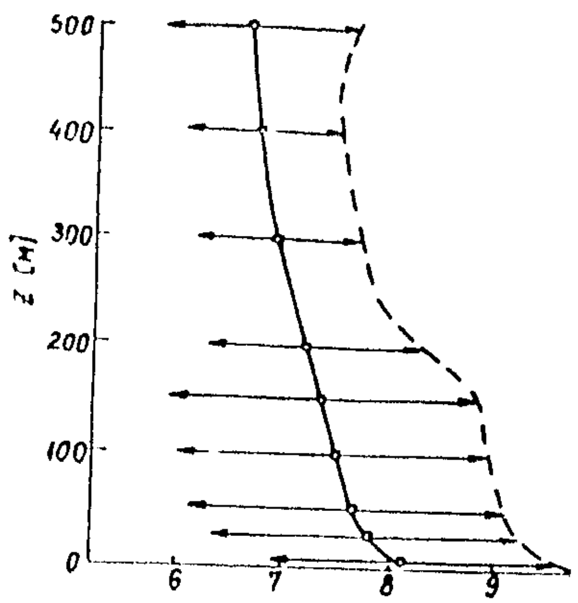


FIG.4 Diurnal humidity profile

The data presented, even averaged diurnally or monthly, exhibits the complicated structure and considerable variability of conditions which can be realized in the APBL. This is an apparent challenge to the theoretical investigations and modelling attempts.

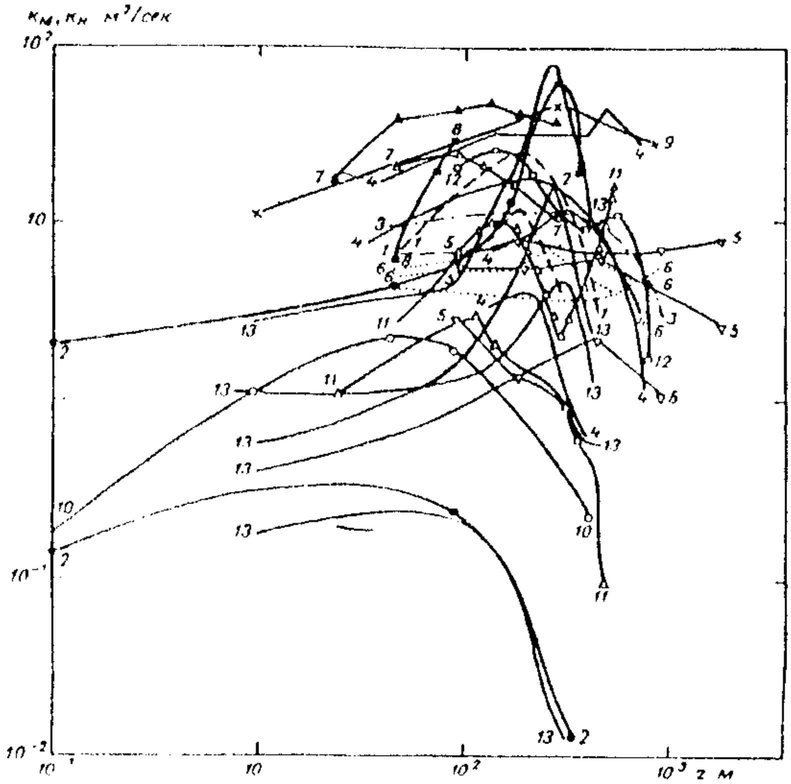


FIG.5 Empirical data for K_m and K_h

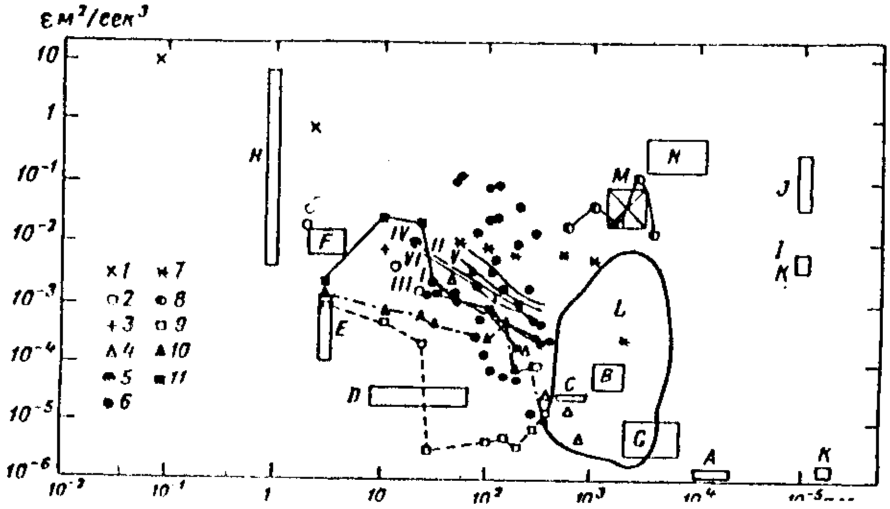


FIG.6 Empirical data on the vertical distribution of energy dissipation

3. Similarity Theory for APBL

The turbulent regime in the APBL is not universal and depends essentially on the weather conditions and the properties of the underlying surface. This is reflected in the considerable variations of the basic turbulent statistical characteristics. There is, however, a hope that using relatively simple and physically sound hypothesis, may explain considerable part of these variations with the variation of properly chosen scales. Using these scales the nondimensionilized turbulent characteristics, hopefully, will not exhibit considerable changes for the different conditions in the APBL. This is the basic philosophy of the similarity approach.

2.1. Basic Similarity Criteria (Theory of Kazanskii and Monin)

The two authors extend further the approach taken in the development of the already famous and well proven Obukhov-Monin similarity theory for the atmospheric surface layer for the case of the APBL. There are two essential complications connected with nonstationarity of the meteorological elements and the effect of the Coriolis parameter which plays a major role in the APBL dynamics.

Assuming stationary conditions, ignoring the radiative heat exchanges (which are usually small) one can accept that the turbulent heat flux in the APBL is constant with altitude. Further if the basic idea of the automodelity is used, i.e. the independence of the turbulent regime on the molecular constants and the characteristics of the earth surface, one can use the Kazanskii and Monin hypothesis which reads: the statistical characteristics of the APBL are fully determined by the values of the physical parameters u_*-turbulent friction velocity, β=buoyancy parameter, c_p-constant of heat capacity at constant pressure, ρ-density of the air and f-Coriolis parameter. Form these parameters one can devise two independent length scales:

$$L = -\frac{(\tau/\rho)^{3/2}}{\kappa\beta H/c_p\rho} = \frac{u_*^2}{\kappa^2\beta T_*} \qquad (1)$$

which is the Obukhov-Monin length scale and

$$\lambda = \kappa u_*/f \qquad (2)$$

which is usually referred to as height scale of the APBL. Therefore, all nondimensional statistical characteristics of the hydrodynamical fields, which are normalized by using the u_* as a velocity scale, T_* as a temperature scale and λ as a length scale, must be universal functions of the nondimensional height

$$\xi = z/\lambda \qquad (3)$$

and the stratification parameter

$$\mu = \lambda/L \qquad (3')$$

Thus, the general expressions for the vertical profiles of horizontal wind components and the potential temperature will be

$$u(z_2) - u(z_1) = \frac{u_*}{\kappa}\left[\psi_u(\frac{z_2}{\lambda},\mu) - \psi_u(\frac{z_1}{\lambda},\mu)\right],$$

$$v(z_2) - v(z_1) = \frac{u_*}{\kappa}\left[\psi_v(\frac{z_2}{\lambda},\mu) - \psi_v(\frac{z_1}{\lambda},\mu)\right] signf, \qquad (4)$$

$$\vartheta(z_2) - \vartheta(z_1) = \frac{u_*}{\kappa}\left[\psi_\vartheta(\frac{z_2}{\lambda},\mu) - \psi_\vartheta(\frac{z_1}{\lambda},\mu)\right],$$

where z_1 and z_2 are arbitrary levels; $\psi_u, \psi_v, \psi_\vartheta$ are nondimensional universal functions, which are determined with accuracy to an additive constant. The *signf* multiplier in the second of the formulae in Equation (4) takes into account the difference in the wind rotation with height in the North and South hemisphere.

3.2 External Parameters

The similarity approach requires to study the automodelity region thereby the boundary conditions for the wind components and temperature at its lower boundary will be

$$u = 0, \quad v = 0, \quad \vartheta = \vartheta_0 = \vartheta_s + \delta s \quad \text{at } z = z_0$$

where ϑ_s is the mean potential air temperature at the real surface, ϑ_0 is the extrapolated ϑ at the level z_0 using the logarithmic temperature profile.

We can now formulate the similarity conditions for the turbulent regime using the external to APBL parameters. Those are the wind velocity above the height of APBL ,i.e. the geostrophic wind velocity $\vec{G}$, the difference between potential temperature at the height of APBL and at z_0 $\delta\vartheta = \vartheta_h - \vartheta_0$ (where $\vartheta_h = \vartheta|_{z=h}$) and the z_0 $\vartheta = \vartheta_0$. Moreover in the number of the external parameters are included the Coriolis parameter and the buoyancy parameter, which are part of the dynamic equations and practically do not change from case to case. The main hypothesis will be that the turbulent regime in temperature stratified APBL is uniquely defined by this five parameters. From these one can construct two nondimensional combinations, namely, the Rosby number

$$Ro = G/|f|z_0 \tag{5}$$

and temperature stratification parameter

$$S = \beta\delta\vartheta/|f|G \tag{6}$$

In accordance with the similarityhypothesis, the nondimensional statistical characteristics of the wind and temperature fields, obtained using: the length scale $\Lambda = G/|f|$, the velocity scale G and the temperature scale $\delta\vartheta$, can depend on the external conditions only through the nondimensional parameters Ro and S. In particular, this means, that the "internal" APBL turbulent characteristics, for example, the turbulent friction velocity u_*, the angle between the isobars and the direction of the turbulent stress at the earth surface α (negative in the Northern and positive in the Southern hemisphere) and temperature scale T_* (proportional to the vertical turbulent heat flux) must obey the following relationships:

$$\frac{u_*}{G} = \eta_u(Ro,S), \quad \alpha = \eta_\alpha(Ro,S)\,signf, \quad \frac{T_*}{\delta\vartheta} = \eta_T(Ro,S), \qquad (7)$$

where $\eta_u, \eta_\alpha, \eta_T$ are universal functions of two arguments, u_*/G is the geostrophic drug coefficient, and the ratio $T_*/\delta\vartheta$ is an integral characteristic of the heat exchange.

In many cases in the formulae of this type instead using the S stratification parameter one can use the μ parameter since they are functionally dependant

$$\mu = \kappa^3 S \frac{T_*/\delta\vartheta}{u_*/G} \qquad (8)$$

2.3 Dynamic and Heat Resistance Laws (Zilitinkevich approach)

The dependence of the variables u_*, α and T_* on the external parameters is expressed by the general Equation (4). Zilitinkevich (1967) proposes the following way to find out more concrete form of these equations. Consider APBL with finite height and boundary conditions

$$u|_{z=h} = G\cos\alpha, \quad v|_{z=h} = G\sin\alpha, \quad \vartheta|_{z=h} = \vartheta_h$$

then formulae (4) can be written in the form:

$$u(z) - G\cos\alpha = \frac{u_*}{\kappa}\psi_u(\frac{z}{\lambda},\mu),$$
$$v(z) - G\sin\alpha = \frac{u_*}{\kappa}\psi_v(\frac{z}{\lambda},\mu)\,signf, \qquad (9)$$
$$\vartheta(z) - \vartheta_h = T_*\psi_\vartheta(\frac{z}{\lambda},\mu).$$

The additive constants, which are part of ψ_u, ψ_v and ψ_ϑ, are determined in such a way that they have limit zero at $z \to h$. These equation are valid for every z including small z. We shall make use of the fact that at small z the stratification is always neutral and the following asymptotic relationships hold

$$u(z) = \frac{u_*}{\kappa}\ln\frac{z}{z_0}, \quad v(z) = 0, \quad \vartheta(z) - \vartheta_0 = \frac{T_*}{\alpha_H^0}\ln\frac{z}{z_0} \qquad (10)$$

The Equations (9) and (10) should be both satisfied at small z which gives the following relationships

$$\ln\frac{\lambda}{z_0} - \frac{\kappa G\cos\alpha}{u_*} = \psi_u(\frac{z}{\lambda},\mu) - \ln\frac{z}{\lambda},$$
$$-\frac{\kappa G\sin\alpha}{u_*} = \psi_v(\frac{z}{\lambda},\mu)\,signf,$$
$$\ln\frac{\lambda}{z_0} - \frac{\alpha_H^0(\vartheta_h - \vartheta_0)}{u_*} = \alpha_H^0\psi_\vartheta(\frac{z}{\lambda},\mu) - \ln\frac{z}{\lambda},$$

which are valid only asymptotically in the logarithmic boundary layer. Now the left-hand sides of the above equations do not depend on z and we should assume that the right hand sides do not also depend on z and therefore they are functions of μ only. Furthermore if we denote

$$\lim_{\xi \to 0} [\psi_u(\xi,\mu) - \ln\xi] = B(\mu) + \ln\kappa,$$

$$\lim_{\xi \to 0} \psi_v(\xi,\mu) = A(\mu)$$

$$\lim [\alpha_H^0 \psi_\vartheta(\xi,\mu) - \ln\xi] = C(\mu) + \ln\kappa$$

where A, B and C are nondimensional universal function of the argument μ, after simple derivation one gets

$$\ln Ro = B(\mu) - \ln \frac{u_*}{G} + \sqrt{\frac{\kappa^2}{(u_*/G)^2} - A^2(\mu)} ,$$

$$\sin \alpha = - \frac{A(\mu)}{\kappa} \frac{u_*}{G} \, \mathrm{sign} f, \qquad (11)$$

$$T_*/\delta\vartheta = \alpha_H^0 \Big/ \left[\ln(Ro \frac{u_*}{G}) - C(\mu) \right].$$

These relationships (first of which at μ=0 had been derived by Kazanskii and Monin in 1961) are the laws of resistance and heat exchange for APBl, which physical meaning is analogous to the laws derived by Prandtl and Karman for flows in pipes.

Equations (11) , if the analytical form of the universal functions A, B and C is known, give the opportunity to establish the dependence between u_*/G, α and $T_*/\delta\vartheta$ and Ro, S (μ).

3.4 Universal Functions for the Resistance and Heat exchange Laws

The empirical determination of the universal functions A, B, C and D requires data for the micrometeorological characteristics u_*, α, T_*, q_* as well as for the external characteristics G, $\delta\vartheta$, δq, z_0. The determination of the parameters f, g and β does not require measurements. The experimental material should have been obtained under conditions which satisfy the major assumptions of the similarity theory - horizontal homogeneity and stationarity of the wind and temperature fields.

Fig 7 shows systematization of the experimental data. They exhibit in general the universal character of the investigated functional dependence. Later investigations confirmed these results and made the use of the universal functions A, B, C and D more reliable.

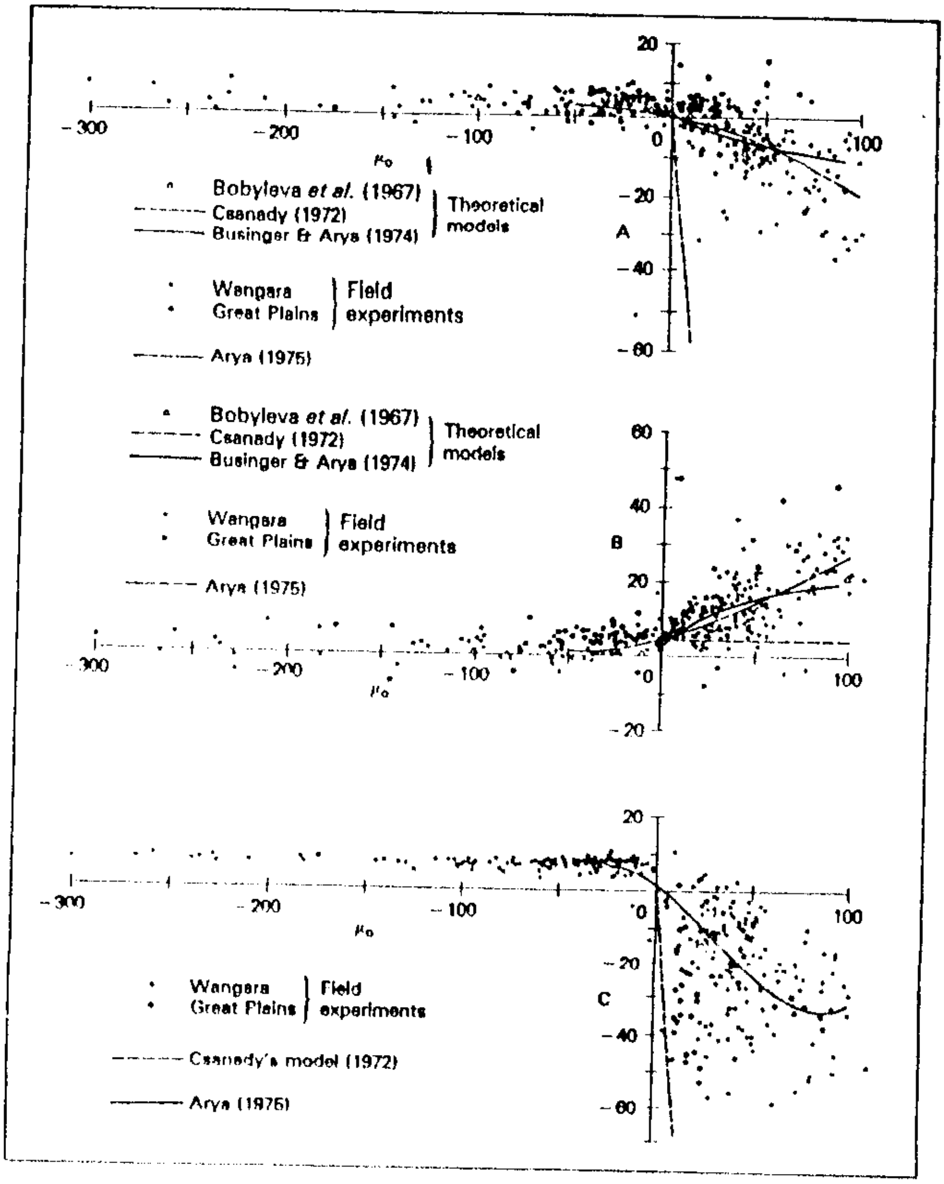


FIG.7 Similarity functions A, B, and C.

43. A Barotropic APBL Model

The application of the similarity theory will be demonstrated on the example of a simple but reliable barotropic APBL. The air pollution meteorology needs the wind velocity and turbulent exchange coefficients for the calculation of the concentration field.

The model presented here uses the contemporary understanding for the double structure of the APBL layer namely near surface boundary layer (which structure is elegantly described by the Obukhov-Monin similarity theory) and underlying Ekman layer . The use of the similarity theory in its development is two folded. First, the dynamics of the APBL is studied as a function of the 'external' parameters and second the similarity theory results are utilized for some needed meteorological variables

The following meteorological variables are taken as known:
(a) the geostrophic wind vector $\vec{V}_g$ at the top of the APBL,
(b) the potential temperature ϑ_h at the same level
(c) the surface temperature ϑ_0, which can be taken from the observations or calculated from the energy balance equation.

From these 'external' to APBL parameter together with the Coriolis parameter f, the roughness z_0 and the buoyancy parameter $\beta=g/\bar{\vartheta}$ the following external parameters can be formed:

$$Ro = \frac{\vec{V}_g}{f z_0}$$

the Rosby number and

$$S = \frac{\beta(\vartheta_h - \vartheta_0)}{f|\vec{V}_g|}$$

the external stratification parameter.

As we have already noted in the previous sections they uniquely determine the turbulent regime in stationary horizontally homogeneous APBL.

The determination of the wind velocity and vertical turbulent exchange distribution in APBL will be based upon the laws of conservation of momentum and energy. The closure of the system of equation will be at K level, where the results of the similarity theory can be readily exploited.

The nondimensionalized system of equations of motion is:

$$P = \frac{d}{dZ} K_m \frac{dQ}{dZ}$$

$$Q = \frac{d}{dZ} K_m \frac{dP}{dZ} \qquad (12)$$

where $P = \kappa(u-u_g)/u_*$ is the velocity defect of the u wind component, $Q = \kappa(v-v_g)/u_*$ is the defect of v wind component, $K_m = k_m f/\kappa^2 u_*^2$ is the vertical turbulent exchange coefficient, $Z=z/H$ is the height and $H = \kappa u_*/f$ the height scale.

The nondimensional equation of heat transfer for the surface boundary layer where there is a approximate constancy of the heat flux is:

$$K_\vartheta \frac{d\Theta}{dZ} = 1 \qquad 0 < Z \le h \tag{13}$$

In the Ekman layer the heat flux gradually decrease to zero at the top of APBL this equation is assumed to have the form:

$$K_\vartheta \frac{d\Theta}{dZ} = \left[1 - \frac{Z-h_\vartheta}{h_\vartheta - h}\right] \tag{14}$$

In Equations (13) and (14) $K_\vartheta = k_\vartheta f/\kappa^2 u_*^2$ is the nondimensional turbulent heat exchange coefficient, $\Theta = \vartheta/\vartheta_*$ the nondimensional temperature, $\vartheta_* = -q_0/c_p \rho \kappa k u_* = \mu u_* f/\kappa^2 \beta$ is the heat flux, h_ϑ is the thermal APBL upper boundary, n is properly chosen constant.

The closure of the system of equation is at K level which allows they to be solved separately. In accordance with the double structure of APBL in the surface layer with height h_* the similarity model of Zilitinkevich-Chalikov is used:

$$K_m = \begin{cases} Z(1 + \beta_1 \mu Z), & Z_0 \le Z < h \qquad \mu > 0 \\ Z & Z_0 \le Z \le \frac{\beta_2}{\mu} \\ \left(\frac{\beta_2}{\mu}\right)^{-1/3} . Z^{4/3} & \frac{\beta_2}{\mu} \le Z \le h \end{cases} \quad \mu < 0 \tag{15}$$

where $\mu = H/L$ is the internal stratification parameter, L is the Obukhov-Monin length, $h = h_*/H$ is the nondimensional height of the surface layer, $\beta_1 = 10$ and $\beta_2 = -0.07$.

In the Ekman layer ($Z > h$) the vertical turbulent coefficient is assumed to be constant with the altitude and equal to the value K_m at the top of the surface layer h:

$$K_{mh} = \begin{cases} h/(1 + \beta_1 \mu h) & \mu > 0 \\ \left(\frac{\beta_2}{\mu}\right)^{-1/3} h^{4/3} & \mu < 0 \end{cases} \qquad Z > h \tag{16}$$

If the axis x is directed along the surface layer stress the nondimensional velocity defects in the surface layer will have the form:

$$P = \kappa \frac{u-u_g}{u_*} = \begin{cases} \ln(Z/h)+\beta_1\mu(Z-h) - \frac{1}{\sqrt{2K_{mh}}} & , Z_0 \le Z \le h , \quad \mu \ge 0 \\ \ln(\mu Z/\beta_2)-3\{1-\beta_2(\mu h)^{1/3}\} - \frac{1}{\sqrt{2K_{mh}}} & , Z_0 \le Z \le \frac{\beta_2}{\mu} , \quad \mu < 0 \\ -3[(\beta_2/\mu Z)^{1/3}-(\beta_2/\mu h)]^{1/3} \quad \frac{1}{\sqrt{2K_{mh}}} & , \frac{\beta_2}{\mu} \le Z < H , \quad M < 0 \end{cases} \tag{17}$$

$$Q = \kappa \frac{v-v_g}{u_*} = \frac{1}{\sqrt{2K_{mh}}} ,$$

where u, v are the velocity components, $u_g = |\vec{V}_g|\cos\alpha$, $v_g = |\vec{V}_g|\sin\alpha$ are the geostrophic wind components, α is the angle between the surface and geostrophic wind.

In the Ekman layer the equations for the velocity defects take simpler form:

$$P = K_{mh} \frac{d^2Q}{dZ^2}$$

$$Q = K_{mh} \frac{d^2P}{dZ^2} \tag{18}$$

which are subject to the following boundary conditions:

$$Z \to h \qquad P = -\frac{1}{\sqrt{2K_{mh}}} , \quad Q = \frac{1}{\sqrt{2K_{mh}}}$$

$$Z \to \infty \qquad P = 0 , \quad Q = 0 \tag{19}$$

The solutions are:

$$P = -e^{-\psi}(\cos\psi - \sin\psi)/\sqrt{2K_{mh}} ,$$

$$Q = e^{-\psi}(\cos\psi + \sin\psi)/\sqrt{2K_{mh}} \tag{20}$$

where

$$\psi = \int_h^Z \frac{dZ'}{\sqrt{2K_{mh}}} = \frac{Z-h}{\sqrt{2K_{mh}}}$$

Equations (17) and (20) show that the calculation of the temperature profile requires the internal APBL parameters u_*, α and μ as well as the heights of the surface and APBL.

The internal APBL parameters may be determined through the external nondimensional parameters Ro and S using the similarity theory by solving the relevant resistance laws Equations ( ) which in this case will be written in slightly different form

$$\kappa \frac{\cos\alpha}{c_g} = \ln(c_g Ro) - B(\mu)$$

$$\kappa \frac{\sin\alpha}{c_g} = A(\mu) \qquad (21)$$

$$\alpha_\theta \frac{\kappa^3 S}{c_g \mu} = \ln(c_g Ro) - C(\mu)$$

where $c_g = u_*/|\vec{V}_g|$ is the geostrophic resistance coefficient A, B, C are universal functions of μ. These functions are determined on the basis of experimental data and similarity considerations for their dependence on μ. The following universal functions are used in the model:

$$A(\mu) = \begin{cases} N_1 \mu^{1/2} & \mu \geq 0 \\ N_2 |\mu|^{1/2} & \mu < 0 \end{cases}$$

$$B(\mu) = \begin{cases} \ln\mu - M_1 \mu^{1/2} - M_0 & \mu > 0 \\ \ln|\mu| - M_2|\mu| + M_3 & \mu < 0 \end{cases} \qquad (22)$$

$$C(\mu) = \begin{cases} \ln\mu - M_4 \mu^{1/2} + M_5 & \mu \geq 0 \\ \ln|\mu| - M_6|\mu| + M_7 & \mu < 0 \end{cases}$$

The constants in the above equations have the following values:

$M_0 = 0.7$ $M_1 = 2.6$ $M_2 = 4.2$ $M_3 = 0.4$ $N_1 = 2.6$

$N_2 = 10$ $M_4 = 3.9$ $M_5 = 1.3$ $M_6 = 4.2$ $M_7 = 3.2$

The Equations (21) with universal functions Equation (22) form a nonlinear algebraic system of equations for the internal parameters c_g, α and μ as a function of the external parameters. Its solution is approximated with polynomials, in order to facilitate its practical use, for the following range of values of the external parameters $Ro \in (10^4, 10^{10})$ and $S \in (-700, 700)$.

$$c_q = \sum_{j=0}^{3} \sum_{i=0}^{2} a_k \tilde{Ro}^i . \tilde{S}^j \qquad k = 3j + 1$$

$$\alpha = \sum_{j=0}^{3} \sum_{i=0}^{2} b_k \tilde{Ro}^i . \tilde{S}^j \qquad (23)$$

$$\mu = \begin{cases} 0.147S & S > 0 \\ \sum_{j=0}^{3}(c_j \tilde{Ro}^j).S & S < 0 \end{cases}$$

where $\tilde{Ro} = \lg Ro$, $\tilde{S} = 0.001S$. The coefficient in Equations (23) are presented in the table.

Table

	$\mu > 0$	$\mu > 0$	$\mu < 0$	$\mu < 0$
	a	b	a	b
0	0.5999E-01	0.1065E+01	0.1302E+00	0.1068E+01
1	-0.7083E-02	-0.1514E+00	-0.2333E-01	-0.1534E+00
2	0.2882E-03	0.6932E-02	0.1249E-02	0.7101E-02
3	-0.2601E+00	0.2608E+00	-0.9890E-00	0.5619E+01
4	0.4128E-01	0.3622E+00	0.1909E+00	-0.9635E+00
5	-0.1896E-02	-0.2396E-01	-0.9519E-02	0.5210E-01
6	0.5102E+00	-0.1260E+01	-0.2245E+01	0.1290E+02
7	-0.8635E-01	-0.5169E+00	0.4467E+00	-0.2305E+01
8	0.4109E-02	0.3911E-01	-0.2281E-01	0.1276E+00
9	-0.3389E+00	0.1049E+01	-0.1799E+01	0.9734E+01
10	0.5887E-01	0.2844E+00	0.3686E+00	-0.1789E+01
11	-0.2845E-02	-0.2326E-01	-0.1930E-01	0.1002E+00
c(i)	0.1057E+01	-0.2300E+00	0.2100E-01	0.6660E-03

One essential advantage of the model is that the height of the surface layer h and the thermal h_θ APBL are dependent on the stratification. The functions are:

$$h(\mu) = \begin{cases} d_1/\mu & \mu > 0 \\ d_2 & \mu = 0 \\ d_3|\mu|^{-1/2} & \mu < 0 \end{cases} \qquad (24)$$

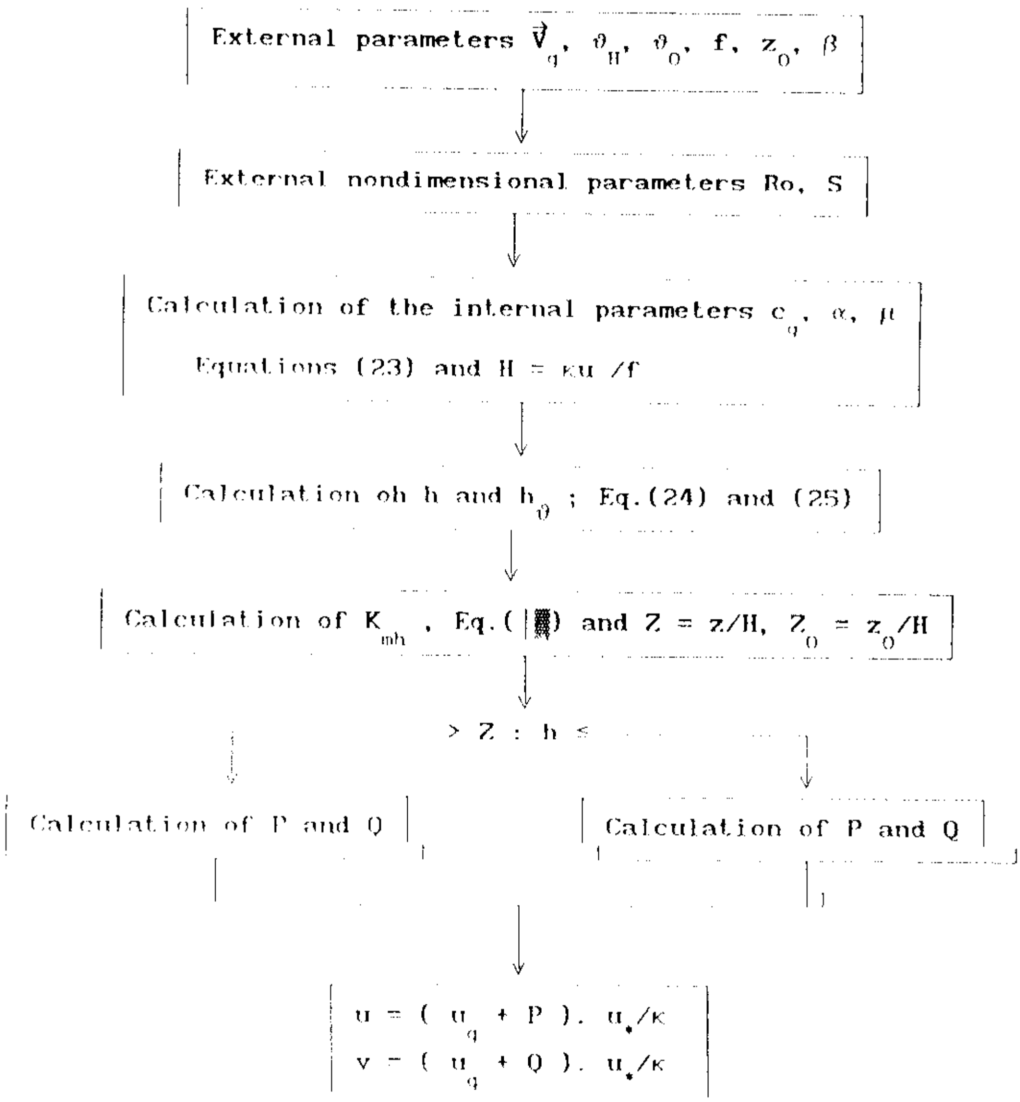
and

$$h_\theta(\mu) = \begin{cases} a_1\mu^{1/2} & \mu > 0 \\ a_2|\mu|^{1/2} & \mu < 0 \end{cases} \qquad (25)$$

The values of the constants in Equations ( ) and ( ) are $d_1 = 0.28$, $d_2 = 0.03$, $d_3 = 0.01$, $a_1 = 0.55$, $a_2 = 0.11$ and $n = 0.92$ are determined in such a way that at limit of $Z \to Z_0 = (\kappa c_g Ro)^{-1}$ the use of Equations ( ) and ( ) in the resistance law to secure coincidence with the constants in the universal functions.

The sequence of the calculation of the wind profile which is realized for a personal computer is presented in the following table.

Table



The model allows also to calculate the vertical wind component at the top of APBL which is needed for the large scale weather forecast as well as for the air pollution models. The divergence of the flow as a result of the turbulent friction may be expressed as

$$w = \frac{1}{f}\left[\frac{\partial}{\partial x}\vec{V}_g^2 c_g \sin\alpha - \frac{\partial}{\partial y}\vec{V}_g^2 c_g \cos\alpha\right]$$

All necessary parameters in the above equation are included in the outlined algorithm and therefore w is also calculated.

For the performance of the proposed APBL model many comparisons with the available experimental data have been performed. Here only the Leipzig universal profiles are presented together with the model results.

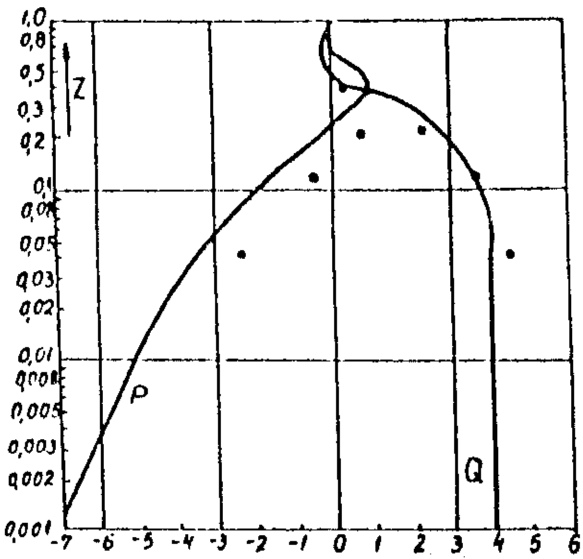


FIG.8 Universal profiles P(z) and Q(z) and Leipzig data points.

5. A Baroclinic APBL Model

The variation of the geostrophic wind the height due to the horizontal temperature inhomogeneities is usually referred to as a baroclinic APBL. To take into account the baroclinic effects on the distribution of the meteorological elements it is necessary to include a new parameters to the already used in the case of barotropic APBL and also to check their aplicability in the baroclinic conditions. For example it is not possible to use $\vec{V}_g$ as an external parameter since it will be varying with the height. Instead the surface value $\vec{V}_{g0}$ of the geostrophic wind may be included in the list of the external parameters. It can be easily calculated form the surface pressure distributiom by the formula:

$$\vec{V}_{g0} = \frac{1}{\rho f} \vec{k} \times \nabla p \qquad (26)$$

In addition to this new parameter it is necessary to include parameters which represent the effcet of baroclinicity on the turbulent regime in APBL. The mean horizontal temperature gradients are the natural choise : $\partial\bar{\vartheta}/\partial x_*$ and $\partial\bar{\vartheta}/\partial y_*$. Thhey are determined in a coordinate system which Ox_*axis is ditected along the vector of the surface geostrophic wind.

Now, the external parameters which determine the turbulent state of stationary baroclinic APBL are:

$$|\vec{V}_{g0}|,\ \delta\vartheta,\ \delta q,\ \partial\bar{\vartheta}/\partial x_*,\ \partial\bar{\vartheta}/\partial y_*,\ f,\ z_0 \qquad (27)$$

In the above list δq represents the humidity difference between the top and surface level of APBL.

From these parameters the following nondimensional parameters can be formed

$$Ro = |\vec{V}_{g0}|/fz_0, \quad S = \beta\delta\vartheta/f|\vec{V}_{g0}|, \quad \eta_{x*} = -\beta\frac{\kappa^2}{f^2}\frac{\partial\bar{\vartheta}}{\partial y_*}, \quad \eta_{y*} = \beta\frac{\kappa^2}{f^2}\frac{\partial\bar{\vartheta}}{\partial x_*} \qquad (28)$$

where in addition to Ro and S two new external baroclinicity parameters η_{x*} and η_{y*} are included. If ϑ does not depend on the height, or vertically averaged temperature is used, it can be shown that the geostrophic wind changes linearly with altitude, which is commonly used assumption and well supported by the experimental data. Therefore, we can write for the geostrophic wind components:

$$u_g(z) = |\vec{V}_{g0}| - \frac{\beta}{f}\frac{\partial\bar{\vartheta}}{\partial y_*} z, \qquad v_g(z) = \frac{\beta}{f}\frac{\partial\bar{\vartheta}}{\partial x_*} z.$$

The similarity theory allows the turbulent regime in APBL to be uniquely determined either by the external or by the internal parameters. If one uses the internal parameters they should include

$$u_* = (\tau/\rho)^{1/2}, \quad T_*, \quad q_*, \quad \lambda_x = -\beta\frac{\kappa^2}{f^2}\frac{\partial\vartheta}{\partial y}, \quad \lambda_y = \beta\frac{\kappa^2}{f^2}\frac{\partial\vartheta}{\partial x}. \qquad (29)$$

It should be noted here that the internal parameters are determined in a coordinate system with axis Ox directed along the surface turbulent stress. There is, however, a very simple geometric relationship between the external and internal baroclinicity parameters expressed by:

$$\lambda_x = \eta_{x*}\cos\alpha + \eta_{y*}\sin\alpha, \qquad \lambda_y = -\eta_{x*}\sin\alpha + \eta_{y*}\cos\alpha \qquad (30)$$

where α is the angle between the coordinate systems (x_*, y_*) and (x, y) i.e the angle between the surface turblent stress and the surface geostrophic wind directions.

The basic similarity hypothesis allows, when using the scales : $|\vec{V}_{g0}|$ for velocity, $\Lambda = |\vec{V}_{g0}|/f$ for length, $\delta\vartheta$ for temperature and δq for humidity, the nondimensional statistical characteristics of the velocity, temperature and humidity fields, to be expressed as a functions of the nondimensional external parameters Ro, S, η_{x*} and η_{y*}.

These relationships, which are usually called resistance laws, may be found in a similar way as in the case of barotropic APBL. The resistance laws for momentum, heat and humidity exchange for baroclinic APBL are :

$$\begin{aligned}
\kappa\frac{\cos\alpha}{c_g} &= \ln(c_g Ro) - B(\mu, \lambda_x, \lambda_y) \\
\kappa\frac{\sin\alpha}{c_g} &= A(\mu, \lambda_x, \lambda_y) \\
\frac{\alpha_{h0}}{C_h} &= \ln(c_g Ro) - C(\mu, \lambda_x, \lambda_y) \\
\frac{\alpha_{q0}}{C_q} &= \ln(c_g Ro) - D(\mu, \lambda_x, \lambda_y)
\end{aligned} \qquad (31)$$

where the stratification parameter μ is equal to

$$\mu = \frac{\kappa u_*}{fL} = \kappa^3 S \frac{T_*/\delta\vartheta}{u_*/|V_{g0}|}$$

and $\alpha_{h0} = K_h/K_m$ is the ratio of the turbulent exchange coefficients for heat K_h and momentum K_m at $z = z_0$ (or inverse Prandtl number), $\alpha_{q0} = K_q/K_m$ is the ratio between the turbulent exchange coefficients for humidity K_q and momentum K_m, $c_g = u_*/|\vec{V}_{g0}|$ is the geostrophic drag coefficient, $C_h = T_*/\delta\vartheta$ is the Stenton number, $C_q = q_*/\delta q$ is the Dalton number, A, B, C and are universal functions of the internal parameters μ, λ_x, λ_y , which can be determined from experimental data or from a proper APBL model.

The solution of the system of equations (31) can be simplified if the hypothesis for the similarity of the temperature and humidity profiles is assumed i.e. $C(\mu) = D(\mu)$ or $\alpha_{h0} = \alpha_{q0}$ from which the equality of the Stenton and Dalton numbers follows. This condition still allows one to calculate the humidity profiles if the modified internal and external stratifications parameters are used:

$$S^{*} = S\,(1 + 0.61T\delta q/\delta\vartheta)$$
$$\mu^{*} = \mu\,(1 + 0.61q_{*}/T_{*}) \qquad (32)$$

This approach allows the resistance laws obtained for dry atmosphere to be applied for moist APBL by using the modified stratification parameters as expressed by Equation (32).

The solution of the system of Equations ( ), which is non-linear , requires the analytical form of the universal functions A, B, C, D. We shall determine them with the aid of two-layer baroclinic APBL model presented in the next paragraph. This requires this model to be developed in accordance of the basic requirements of the similarity theory. If one uses more or less the same philosophy in the model development as this presented for the theory of the barotropic APBL, then the solutions for the baroclinic APBL should contain as a limiting case the barotropic case when the internal and external baroclinicity parameters are set equal to zero.

The baroclinic APBL is again considered to consist of two layers - surface layer and underlying Ekman layer. The system of equations which is nondimensionalized by the scales : u_{*}/κ for velocity, h (the surface layer height) and $H = \kappa u_{*}/f$ for length, $\kappa^{2}u_{*}^{2}/f$ for turbulent exchange, has the form:

$$P = K_{mh}\frac{d^{2}Q}{dZ^{2}} \qquad P = u - u_{q0} - \lambda_{x}(Z-h)$$
$$Q = -K_{mh}\frac{d^{2}P}{dZ^{2}} \qquad Q = v - v_{q0} - \lambda_{y}(Z-h) \qquad (33)$$

This system is subject to the following boundary conditions

$$P = P_{h} = u_{h} - u_{q0} \qquad Q = Q_{h} = v_{h} - v_{q0} \qquad \text{at } Z = h$$
$$P \to 0 \qquad Q \to 0 \qquad \text{at } Z = H \qquad (34)$$

The turbulent exchange coefficient K_{mh} is taken constant with values equal to the value of K_{m} at the height of the surface layer:

$$K_{mh} = \begin{cases} h / (1 + \beta_1 \mu h) & \mu \geq 0 \\ (\beta_2/\mu)^{-/3} h^{4/3} & \mu \leq 0 \end{cases} \qquad (35)$$

The solution of Equation (33) with boundary conditions Equation (34) and turbulent exchange coefficient Equation (35) is

$$P = P_b - \lambda_x K_{mh} P_b + \lambda_y K_{mh} Q_b$$
$$Q = Q_b - \lambda_x K_{mh} Q_b - \lambda_y K_{mh} P_b \qquad (36)$$

where P_b, Q_b are the velocity deffects for the case of barotropic APBL (see Equation (20)).

In the surface layer the velocity deffects are

$$P = P_b + \frac{K_{mh}}{\sqrt{2K_{mh}}} (\lambda_x + \lambda_y), \qquad Q = Q_b + \frac{K_{mh}}{\sqrt{2K_{mh}}} (\lambda_y - \lambda_x) \qquad (37)$$

where again P_b and Q_b are the velocity deffect for the barotropic case and are given by the Equations (17).

The analytical form of the universal functions in the resistance laws for momentum, heat and humidity exchange can be obtained by realizing the limit $Z \to Z_0 = (\kappa c_q Ro)^{-1}$ formula (20) and comparing the result with formula (31). The result is:

$$A\ (\mu, \lambda_x, \lambda_y) = A_b(\mu) + (\lambda_x + \lambda_y) \ /\ 2B(\mu)$$
$$B\ (\mu, \lambda_x, \lambda_y) = B_b(\mu) - (\lambda_x - \lambda_y) \ /\ 2B(\mu) \qquad (38)$$

where $A_b(\mu)$ and $B_b(\mu)$ are the universal functions valid for the barotropic case ($\lambda_x = \lambda_y = 0$) and are represented by Equations (22).

The temperature profile in the baroclinic model can be obtained from the equation of energy conservation:

$$q = -c_p \rho K_\vartheta \frac{d\vartheta}{dz} \qquad (39)$$

where q is the vertical turbulent heat flux, c_p is the specific heat capacity at constant pressure, ρ is the air density, K_ϑ is the turbulent exchange coefficient for heat, ϑ is potential temperature.

In surface layer the turbulent heat flux is constant with altitude and

Equation (39) in nondimensional form is

$$K_{\Theta} \frac{d\Theta}{dZ} = 1 \qquad (40)$$

where $K_{\Theta}=K_{\vartheta}f/\kappa^2u_*^2$, $\Theta=\vartheta/\vartheta_*$, $\vartheta_*=-q_0/c_p\kappa u_*=\mu u_* f/\kappa^3\beta$, $q_0=\overline{w'\vartheta}$ is the surface value of the turbulent heat flux.

In the integration of Equation (40) the relationship

$$K_{\Theta} = \alpha_{\vartheta} K_m$$

,where $\alpha_{\vartheta} = K_{\Theta}/K_m < 1$, is also used.

The formulae for the temperature deffets in the surface layer are

$$\Theta_Z - \Theta_0 = \begin{cases} -\ln(Z/h)+\beta_1\mu(h-Z)+\frac{h_{\vartheta}-h}{(n+1)K_{mh}} & Z_0\le Z\le h \quad \mu>0 \\ -\ln(Z)+\ln(\beta_2/\mu)+3(1-\beta_2/\mu h)^{1/3}+\frac{h_{\vartheta}-h}{(n+1)K_{mh}} & Z_0<Z<\beta_2/\mu \\ 3((\beta_2/\mu Z)^{1/3}-(\beta_2/\mu h)^{1/3})+\frac{h_{\vartheta}-h}{(n+1)K_{mh}} & \frac{\beta_2}{\mu}\le Z\le h \end{cases} \quad \mu<0 \qquad (41)$$

In the Ekman layer the turbulent heat flux gradually decreases with the height to zero at the top of APBL. The nondimensional heat transfer equation which satisfies this condition is

$$K_{\Theta} \frac{d\Theta}{dZ} = (1 - \frac{Z - h}{h_{\vartheta} - h})^n \qquad h \le Z \le h_{\vartheta} \qquad (42)$$

where $n = 0.92$ is properly chosen constant.

The height of the thermal APBL is calculated using the results of the similarity theory:

$$h_{\vartheta}(\mu) = \begin{cases} a_1\mu^{-1/2} & \mu \ge 9.2 \\ a_2|\mu|^{1/2} & \mu \le -9.2 \end{cases} \qquad (43)$$

where $a_1 = 0.55$, $a_2 = 0.11$.

The solution for the temperature deffcets in the Ekman layer is:

$$\alpha_{\vartheta}\delta\Theta = \left\{ \frac{(h_{\vartheta} - Z)^{n+1}}{(n+1)K_{mh}(h_{\vartheta}-h)^n} \right. \qquad h \le Z \le h_{\vartheta}$$

Applying the same procedure as in the case of the resistance for momentum one can find the required analytical expression for the universal function for heat:

$$C(\mu,\lambda_x,\lambda_y) = \begin{cases} \ln\mu - c_1\mu^{1/2} + c_2 & \mu \geq 9.2 \\ \ln|\mu| - c_3|\mu|^{-1/2} + c_4 & \mu < -9.2 \end{cases} \qquad (44)$$

The solutions for the resistance laws is presented in the form:

$$c_q(\mathrm{Ro},S,\eta_x,\eta_y) = c_q(\mathrm{Ro},S) + \frac{\partial c_q}{\partial(\eta_{x*}+\eta_{y*})}(\eta_{x*}+\eta_{y*})$$

$$\alpha(\mathrm{Ro},S,\eta_x,\eta_y) = \alpha(\mathrm{Ro},S) + \frac{\partial \alpha}{\partial(\eta_{y*}-\eta_{x*})}(\eta_{y*}-\eta_{x*}) \qquad (45)$$

$$\mu(\mathrm{Ro},S,\eta_x,\eta_y) = \mu(\mathrm{Ro},S) + \frac{\partial \mu}{\partial(\eta_{x*}+\eta_{y*})}(\eta_{x*}+\eta_{y*})$$

i.e. as a sum of a barotropic part and baroclinic addition.

The practical use of the model is again fascilitated by approximating the solutions with appropriate polynomials which are function of Ro and S. The barotropic parts are expressed by Equations (23) and the baroclinic terms by the following approximating polynomials

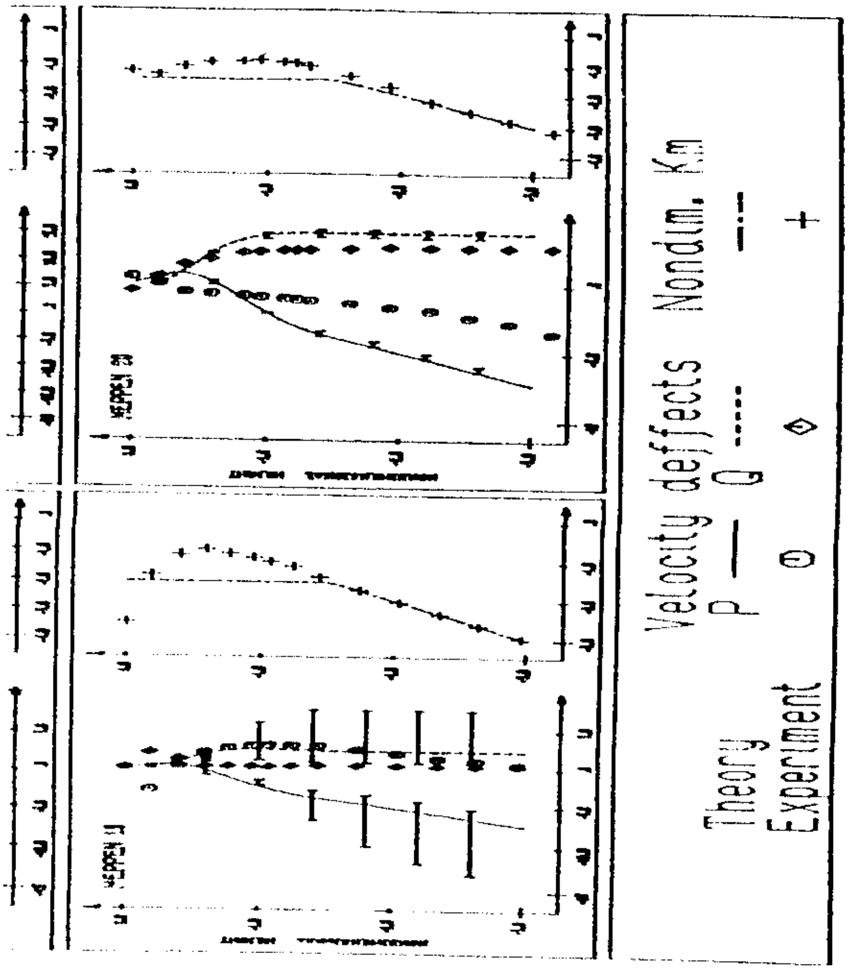
$$\frac{\partial c_q}{\partial(\eta_{x*}+\eta_{y*})} = \sum_{j=0}^{3}\sum_{i=0}^{2} a'_k \tilde{\mathrm{Ro}}^i.\tilde{S}^j \qquad k = 3j + 1$$

$$\frac{\partial \alpha}{\partial(\eta_{y*}-\eta_{x*})} = \sum_{j=0}^{3}\sum_{i=0}^{2} b'_k \tilde{\mathrm{Ro}}^i.\tilde{S}^j \qquad (46)$$

$$\frac{\partial \mu}{\partial(\eta_{x*}+\eta_{y*})} = \sum_{j=0}^{3}\sum_{i=0}^{2} c'_k \tilde{\mathrm{Ro}}^i.\tilde{S}^j$$

where $\tilde{\mathrm{Ro}} = \lg \mathrm{Ro}$, $\tilde{S} = 0.01S$ in formulae (23) and $\tilde{S} = 0.001S$ in formulae (46).

In the following Figure one randomly chosen result from the comparison of the model calculations with the available experimental data is presented (it should be pointed out the data is not as abandend as in the case of barotropic APBL).



F.G.9 Comparison of baroclinic model results with the Meppen data.

The model presentd here can be applied for APBL over land and sea. In the latter case some appropriate formula for Z_0 should be used and the roughness can not be considered any more as independent external parameter. In the case of APBL over sea surface the Rosby parameter can be changed with the Letau number:

$$Le = \frac{g}{f|V_{g0}|}$$

where g is the gravity acceleration

6. Conclusion

The examples of the application of similarity theory in obtaining the necessary meteorological parametrs necessary for the solution of the air pollution problem reveal its applicabilty. The root of the theory with the experimental data and the simplicity of the calculation secure good results in the practical applications. It also allows a better systematization of the concentration fields obtained in field experiments or by air pollution models.