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SMR/760-10

"College on Atmospheric Boundary Layer and Air Pollution Modelling"
16 May - 3 June 1994

"Applied Dispersion Modelling for Ground-Level Concentrations from Elevated Sources"

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Pergamon

Atmospheric Environment Vol. 28, No. 4, pp. 611–615, 1994
Elsevier Science Ltd
Printed in Great Britain.
1352–2310/94 \$6.00 + 0.00

APPLIED DISPERSION MODELLING FOR GROUND-LEVEL CONCENTRATIONS FROM ELEVATED SOURCES

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(*First received 20 March 1993 and in final form 26 September 1993*)

Abstract A practical short-range model evaluating ground-level concentrations from elevated sources is presented, which utilizes a Fickian-type formula, where the source height and mixing layer height are simple functions of the wind velocity and eddy diffusivity profiles. The model performances are evaluated against an exact solution of the advection diffusion equation and against experimental ground-level concentrations, using meteorological data collected near the ground and wind and eddy diffusivity profiles predicted by the Similarity Theory.

Key word index: Air pollution, Gaussian models, analytical model, diffusion equation.

INTRODUCTION

In practice most of the estimates of dispersion from continuous point sources are based on the Gaussian approach. A basic assumption for the application of this approach is that the plume is dispersed by homogeneous turbulence. However, due to the presence of the ground, turbulence is usually not homogeneous in the vertical direction. Moreover, the input parameters of the Gaussian plume model are often related to simple turbulence typing schemes or stability classes. The problem with such stability classes is that each covers a broad range of stability conditions; they are also very site-specific and biased towards neutral stability when unstable or convective conditions actually exist (Weil, 1983). In addition, the influence of these factors on the calculated ground-level concentration (glc) is considerable (Kretzschmar and Mertens, 1984).

In this paper we present and validate a practical model for evaluating the glc from elevated sources that applies a new Gaussian formulation for transport and vertical diffusion. The model has previously been described in Lupini and Tirabassi (1981) although, unfortunately, the cited paper contains many typographical errors and some incorrect formulae. Thus, a brief presentation of the model is made here and the formulae in the present paper should be taken as the correct ones. In the presented model, the source height and mixing height (or better, the virtual source height and mixing layer height) are expressed by simple functions of the vertical profiles of wind and turbulent diffusivity. The model accepts experimental profiles of the above parameters, as well as the theoretical profiles proposed in the scientific literature, such as the vertical profiles of the wind and eddy diffusion coefficients predicted by the Similarity Theory. In this last case, the model can be applied routinely using as input simple ground-level meteorological data acquired by an automatic network. In fact, in recent years, with the works of De Bruin and Holtslag (1982), Holtslag and van Ulden (1983), Weil and Brower (1984), van Ulden and Holtslag (1985), Trombetti *et al.* (1986) and Hanna and Paine (1989), it turns out that fundamental parameters for describing the characteristics of the atmospheric surface and boundary layers can be evaluated by measurements at ground level. Moreover, the evaluation of the model performances will be presented against an exact solution of the advection diffusion equation, that allows the mean velocity and vertical eddy diffusivity profiles to vary independently as power law functions, and against SF_6 tracer glc released at a height of 115 m.

THE MODEL

The advection–diffusion equation of air pollution in the atmosphere is essentially a statement of conservation of the suspended material; in a steady-state boundary layer, for a continuous line source, it can be written as:

$$u\frac{\partial C}{\partial z} = \frac{\partial}{\partial z}\left(K\frac{\partial C}{\partial z}\right) \qquad (1)$$

$$C \to \delta(z-1) \quad \text{as } x \to 0$$

$$K\frac{\partial C}{\partial z} \to 0 \quad \text{as}: z \to 0$$

$$z \to H$$

where we have introduced nondimensional variables for the wind velocity (u), pollutant concentration (C), downwind distance (x), vertical coordinate (z), eddy diffusivity (K) and mixing-layer height (H). These are related to their dimensional counterparts indicated by prime as follows:

$$z' = h_s z$$

$$x' = \frac{u_s h_s^2}{K_s} x$$

$$C' = \frac{Q}{u_s h_s} C$$

$$u' = u_s u$$

$$K' = K_s K$$

$$H' = h_s H$$

where Q is the source emission rate, h_s is the emission height and the subscript s indicates values at the emission height.

Gaussian models, which are the best-known and most widely used, are based on a solution of the above equation where both the wind and exchange coefficients are assumed constant. The Gaussian model solution is forced to represent an inhomogeneous atmosphere through empirical parameters of dispersion, the so-called "sigmas".

Experiments show that the lateral profile of a plume resembles a Gaussian distribution (Silversten, 1978; Gryning *et al.*, 1978; Nieuwstadt and van Duuren, 1979); therefore, when the crosswind-integrated concentration at the surface C_y is known, we can confidently calculate the concentration at the surface at any point using the standard Gaussian model for lateral concentrations

$$C(x, y, 0) = C_y \frac{e^{-\left(\frac{y^2}{2\sigma_y^2}\right)}}{\sqrt{2\pi}\,\sigma_y} \quad (2)$$

where y is the crosswind distance and σ_y is the crosswind spread of the plume.

For the evaluation of C_y, we propose a Fickian-type formula where the source height and inversion height are expressed by simple functions of the vertical profiles of wind and turbulent diffusivity (see also Lupini and Tirabassi, 1981).

We now introduce two virtual source heights as:

$$\mu_s = \int_0^1 \left(\frac{u}{K}\right)^{1/2} dz \quad (3)$$

$$\zeta_s = \int_0^1 u\, dz \quad (4)$$

and two virtual boundary-layer heights

$$M = \int_0^H \left(\frac{u}{K}\right)^{1/2} dz \quad (5)$$

$$N = \int_0^H u\, dz. \quad (6)$$

The crosswind-integrated glc $C_y(x, 0)$ is approximated by means of a Fickian-type formula with a source placed at the geometric average of the two virtual source heights μ_s and ζ_s.

$$C(x, 0) = \frac{1}{\sqrt{\pi x}} e^{[-\zeta_s \mu_s / 4x]}. \quad (7)$$

In Lupini and Tirabassi (1981) it is shown that the glc admits a lower and upper bound that represents solutions at the ground level of two diffusion equations of Fickian-type with the two virtual sources μ_s and ζ_s, respectively. Moreover, for a general profile of the wind and eddy diffusivity we expect $\zeta_s < 1 < \mu_s$ and $N < H < M$. This fact explains the physical meaning of the two bounds and the relative importance of the u and K profiles in determining the solution of the advection diffusion equations at ground level.

On the basis of the above considerations, we used the interpolation between the two bounds (lower and upper bound), and selected the Gaussian formula (7), since the predicted maximum position

$$X_m = \frac{\zeta_s \mu_s}{2} \quad (8)$$

is exactly the same as that predicted by the solution of the advection–diffusion equation, which admits power law profiles of wind and eddy coefficients (Huang, 1979). In fact, the above analytical solution can be written as follows:

$$C(x, 0) = \frac{1}{\lambda^\eta \Gamma(\gamma) X^\gamma} e^{-\left(\frac{1}{\lambda^2 x}\right)} \quad (9)$$

with

$$u(z) = u(z_1)(z/z_1)^\alpha \quad K(z) = K(z_1)(z/z_1)^\beta$$

where

$$\lambda = \alpha - \beta + 2$$

$$\gamma = (\alpha + 1)/\lambda$$

$$\eta = (\alpha + \beta)/\lambda$$

Γ is the Gamma function.

From equation (9) we have

$$x_m = \frac{1}{(\alpha + 1)\lambda}. \quad (10)$$

The proposed approximated solution in the case of power law profiles gives

$$\zeta_s = 1/(\alpha + 1)$$

$$\mu_s = 2/\lambda$$

and thus

$$C(x, 0) = \frac{1}{\sqrt{\pi x}} e^{-\left(\frac{1}{2(\alpha+1)\lambda x}\right)}$$

and

$$x_m = \frac{1}{(\alpha+1)\lambda}$$

that is the same as equation (10).

The formulae are expected to deviate significantly from the solution of the advection–diffusion equation in the range $x \gg x_m$, where x_m is the position of the maximum of solution. In fact, in the case of the solution with power law profiles of wind and exchange coefficients, we have

$$C(x,0) \approx x^{-\gamma}$$

for $x/x_m \gg 1$, and

$$C(x,0) \approx x^{-1/2}$$

for the presented model.

We also expect deviations in a range near $x=0$. However, within these two ranges the ground-level concentration is small for tall sources and, in practical applications, a good prediction in a neighborhood of x_m is normally of primary concern for a single source. Moreover it allows not only power law profiles but general profiles of wind and eddy diffusivity.

Clearly, the Fickian-type approximation presented is given by equation (7) in the case of an infinite boundary-layer height (H), and by

$$\frac{1}{\sqrt{\pi x}} \exp(-h_s^2/4x) + \sum_{n=1}^{\infty} \{\exp[-(h_s-2nR)^2/4x] + \exp[-(h_s+2nR)^2/4x]\} \qquad (11)$$

where

$$h_s = (\zeta_s \mu_s)^{1/2}$$

in the case $R=(M/N)^{1/2} < \infty$ (Turner, 1969).

EVALUATION AGAINST RESULTS OF THE *K*-EQUATION AND EXPERIMENTAL DATA

Figure 1 shows the glc predicted by the proposed model against the two-dimensional analytical solution of the K-equation (equation (9)) proposed by Huang (1979) (for more about analytical solutions, see also Tirabassi *et al.* (1986) and Tirabassi (1989)).

The comparison was made among three cases corresponding to an unstable, neutral and stable atmosphere, respectively. In the case of an unstable atmosphere we utilized an exponent for the wind profile $\alpha=0.1$ and eddy coefficient profile $\beta=1.3$; in the case of a neutral atmosphere, $\alpha=0.14$ and $\beta=1$; while in a stable atmosphere $\alpha=0.4$ and $\beta=0.7$. In the figure it can be seen that the proposed analytical approximation represents a good estimate of the glc; in particular the value of the maximum glc is approximated with a percentage error of about 10%.

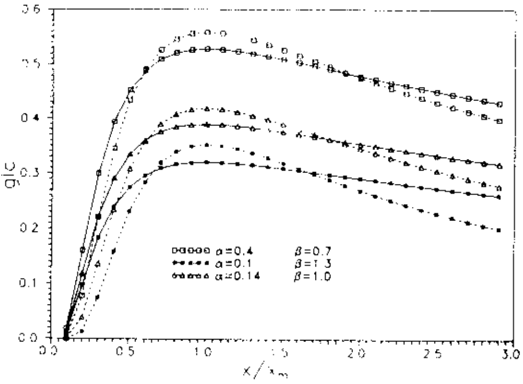

Fig. 1. Adimensional glc predicted by the proposed model (continuous line) and by analytical solutions of the K-equation (dashed line) as a function of the adimensional distance from the source normalized by the adimensional maximum glc position (x_m) in three different cases.

Moreover, the analytical approximation proposed in this paper has been evaluated using tracer SF_6 data from dispersion experiments carried out in the northern part of Copenhagen, described in Gryning and Lyck (1984). The tracer was released without buoyancy from a tower at a height of 115 m, and collected at the ground-level positions in up to three crosswind arcs of tracer sampling units. The sampling units were positioned 2–6 km from the point of release. Tracer releases typically started 1 h before the start of tracer sampling and stopped at the end of the sampling period; the average sampling time was 1 h. The site was mainly residential with a roughness length of 0.6 m.

Table 1 shows the data (from Gryning, 1981 and Gryning *et al.*, 1987) utilized for the validation of the proposed formula. The meteorological data used were collected near the ground, so the comparison can be said to simulate the values given by a routine use of the model.

In Table 2 the measured glc values are presented, together with the computed ones of the Fickian-type model. Two different parameterizations of wind and eddy exchange profiles have been used for the calculation. In fact, for 2/3 of the experiments (see Table 1) the boundary layer height is more than 1000 m, so that the source of emission can be considered to be at approximately the top of the surface layer. For this reason, we used two different wind and eddy exchange profiles calculated by means of the Similarity Theory: one valid in the surface layer and the other throughout the whole atmospheric boundary layer.

For the first one we used:

$$u = u_*/k[\ln(z/z_0) - \psi_m(z/L) + \psi_m(z_0/L)] \qquad (12)$$

$$K_z = k u_* z/\phi_h \qquad (13)$$

where the Monin–Obukhov length (L) is

$$L = \frac{-\rho C_p T u_*^3}{k g Q_h}$$

Table 1. Meteorological data used (from Gryning, 1981 and Gryning *et al.*, 1987)

Exp No.	u $(\mathrm{m\,s^{-1}})$	u_* $(\mathrm{m\,s^{-1}})$	L (m)	w_* $(\mathrm{m\,s^{-1}})$	H (m)	H/L
1	3.4	0.37	−46	1.7	1980	−43
2	10.6	0.74	−384		1920	−5
3	5.0	0.39	−108		1120	−10
4	4.6	0.39	−173		390	−2.3
5	6.7	0.46	−577		820	−1.4
6	13.2	1.07	−569		1300	−2.3
7	7.6	0.65	−136	2.1	1850	−14
8	9.4	0.70	−72	2.1	810	−11
9	10.5	0.77	−382		2090	−5.5

Table 2. Observed and estimated crosswind-integrated concentrations C_y/Q at different distances from the source. Model 1 uses equations (11) and (12), while Model 2 uses equations (12), (14) and (15)

Exp.	Distance (km)	Data $(10^{-4}\mathrm{sm^{-2}})$	Model 1 $(10^{-4}\mathrm{sm^{-2}})$	Model 2 $(10^{-4}\mathrm{sm^{-2}})$
1	1.9	6.48	5.99	7.35
	3.7	2.31	4.54	5.55
2	2.1	5.38	3.20	3.48
	4.2	2.95	2.72	2.93
3	1.9	8.20	5.79	6.53
	3.7	6.22	4.67	5.24
	5.4	4.30	4.02	4.50
4	4.0	11.66	4.92	6.64
5	2.1	6.72	5.25	5.91
	4.2	5.84	4.57	5.23
	6.1	4.97	4.04	4.66
6	2.0	3.96	2.49	2.50
	4.2	2.22	2.05	2.93
	5.9	1.83	1.82	2.60
7	2.0	6.70	3.63	4.23
	4.1	3.25	2.83	3.23
	5.3	2.23	2.55	2.90
8	1.9	4.16	3.04	4.23
	3.6	2.02	2.47	3.45
	5.3	1.52	2.11	2.97
9	2.1	4.58	3.16	3.39
	4.2	3.11	2.65	2.82
	6.0	2.59	2.33	2.48

and u_* is the friction velocity, $k = 0.4$ the von Karman constant, z the height, z_0 the roughness length, T is the air temperature, g the acceleration of gravity, ρ the air density, C_p the specific heat at constant pressure, Q_h the sensible heat flux ϕ_h and ψ_m are stability functions defined as follows:

for $L \leqslant 0$

$$\phi_h = (1-16\,z/L)^{-1/2}$$

$$\psi_m = 2\ln[(1+A)/2] + \ln[(1+A^2)/2] - 2\tan^{-1}A + \pi/2$$

where

$$A = (1-16\,z/L)^{1/4}$$

Table 3. Statistical evaluation of model results. Model 1 uses equations (11) and (12), while Model 2 uses equations (12), (14) and (15)

Model	NMSE	r	FB	FS
1	0.23	0.77	−0.24	−0.65
2	0.13	0.78	−0.06	−0.52

for $L > 0$

$$\phi_h = 1 + 5\,z/L$$

$$\psi_m = -5\,z/L.$$

In the second case, following Pleim and Chang (1992), during stable and near neutral conditions ($H/L \geqslant -1010$), we adopted

$$K_z = k u_* z (1 - z/H)^2 / \phi_h. \tag{14}$$

During convective conditions ($H/L \leq -10$) the friction velocity was replaced by the convective velocity (w_*) as scaling velocity to give (Pleim and Chang, 1992)

$$K_z = k w_* z (1 - z/H) \tag{15}$$

where the convective velocity is defined as follows:

$$w_* = \left(\frac{gQ_h H}{C_p\, gT}\right)^{1/3}.$$

Analysing Table 2, we can see that the model results adequately describe the experimental measurements in both parameterizations used, although better results are obtained with those proposed by Pleim and Chang (1992).

Moreover, Table 3 presents some statistical indices defined as follows:

$$\text{normalized mean square error (NMSE)} = \frac{\overline{(C_m - C_o)^2}}{\bar{C}_p \bar{C}_m}$$

$$\text{correlation coefficient } (r) = \frac{\overline{(C_m - \bar{C}_m)(C_o - \bar{C}_o)}}{\sigma_m \sigma_o}$$

$$\text{fractional bias (FB)} = 2\,\frac{\bar{C}_m - \bar{C}_o}{\bar{C}_m + \bar{C}_o}$$

$$\text{fractional standard deviation (FS)} = 2\,\frac{\sigma_m - \sigma_o}{\sigma_m + \sigma_o}$$

where C_o and C_m are the observed and model concentrations, respectively, while σ is the standard deviation.

Statistical indices confirm the reliability of the model results, in particular with the parameterization of Pleim and Chang (1992).

CONCLUSIONS

It has been shown that the glc predicted by an exact analytic solution of the advection–diffusion equation for a continuous elevated line source can be approximated by a Fickian-type formula where the source height and the mixing layer height are simple functionals of the wind and eddy diffusivity profiles. Moreover, the applicability of this formula can be extended to profiles other than those admitting an exact solution of the advection–diffusion equation.

A preliminary evaluation of the model performance, based on the above Fickian-type formula, using SF_6 tracer data and, as input, meteorological data collected near the ground alongside wind and eddy diffusivity profiles calculated by means of the Similarity Theory produced good results. Better results are obtained using the parameterizations proposed by Pleim and Chang (1992), although in these cases, both expressions for K_z are very sensitive to planetary boundary height which may be poorly estimated.

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