INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

SMR.762 - 2

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

13 June - 29 July 1994

INTRODUCTION TO THE STANDARD MODEL AND TO SOME OF ITS MOST RELEVANT PREDICTIONS

C. VERZEGNASSI
Trieste University

Please note: These are preliminary notes intended for internal distribution only.

An introduction to the Standard Model

and to some of its most relevant Predictions.

C. Verzegnassi / CERN + / INFN, TRIESTE.

Lemma:

For every beautiful theory there exists at least one "special" experimental test.

The latter is supposed to be able to verify whether one is dealing with a satisfactory "effective" description or with something "more".

Definition:

A beautiful theory is one that makes definite unambiguous predictions that can be experimentally tested ("provando e riprovando").

By definition, a "special" test will be one where both the experimental and the theoretical sides are pushed to their technical limits.

Since the latter ones can vary with time (developements of new techniques.....), the speciality property is generally time-dependent (with a ≤ 0 derivative).

In practice, (the) available predictive theories are based on perturbative expansions. Therefore, a "special" test must be able to verify the theory <u>at least</u> to the one loop level.

Implicit assumption: the (some of the) <u>specific</u> features of the theory appear at the perturbative (≠tree) level(s) that is (are) tested.

("Good" situation: the theoretical prediction at tree level is "bad").

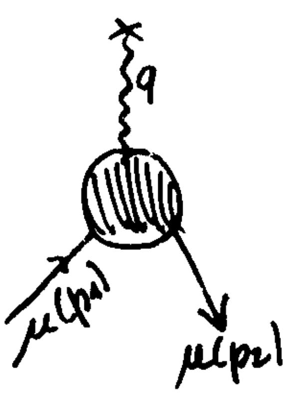
It may happen that some of the specific one (or more) loop(s) features are still unknown. Then the "special" test can even become a predictive one. (In this case, some extra supporting measurement(s) is (are) necessary).

Illustrative example: QED and the muon's (g-2).

The main features of the theory are supposed to be known.
The relevant coupling $= \alpha(0)$ is defined and measured from a previous independent experiment (e.g. Thomson scattering/or Hall.... or other ~ zero momentum transfer measurements). This fixes $\alpha(0)$:

e.g. $$\alpha(0) = \frac{1}{137.03599979(32)}$$

The muon's (g-2) is defined from:



$$M \sim \bar{u}(p_2)\Lambda_\sigma u(p_1) . A^\sigma(q)$$

$$\Lambda_\sigma = F(q^2)\gamma_\sigma - \frac{1}{2m_\mu}\sigma_{\sigma\lambda} q^\lambda G(q^2)$$

$$a_\mu = G(0) \equiv \frac{1}{2}(g-2)_\mu$$

Experiment (CERN 1977):

(J. BAILEY et al; F. J. M. FARLEY + E. PICASSO..)

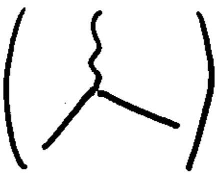
$$a_{\mu^+} = 1.165.937\,(12)\ 10^{-9}$$

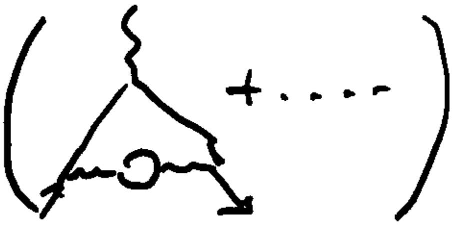
$$a_{\mu^-} = 1.165.911\,(11)\ 10^{-9}$$

Guess: $a_\mu \Rightarrow 1.165.924\,(\sim 10)\ 10^{-9}$

Theory (T. KINOSHITA ..... ENDLESSLY)

QED: (e, μ, τ)

$a_\mu = 0$  $+ \frac{\alpha}{2\pi}$  $+$

$+ 0.76578223 \left(\frac{\alpha}{\pi}\right)^2$ 

$+ \ldots\ldots$

$$\Longrightarrow \left(\frac{\alpha}{\pi}\right)^4$$

$\left(\left(\frac{\alpha}{\pi}\right)^5\right.$ estimated in 1990.... $\left.\right)$

Summing up the available QED ($\equiv$ leptons) contributions gives:

$$
\begin{array}{lrl}
10^9 a_\mu^{QED} & 0 & \\
 & 1.\,161417 & \alpha \\
 & \sim \boxed{4}\square\square\square & \alpha^2 \\
 & \sim \boxed{2}\square\square & \alpha^3 \\
 & \sim \boxed{3}\square & \alpha^4 \\
 & \overset{?}{\Longrightarrow} \sim \square & \alpha^5
\end{array}
$$

$$10^9 a_\mu^{QED} = 1.\,165.\,847\,(0.0?)$$

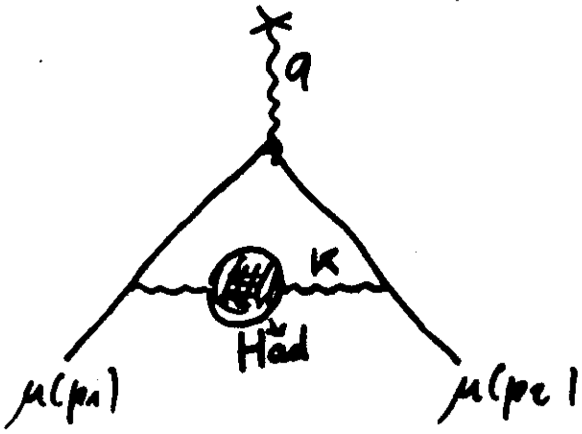
$$10^9 a_\mu^{EXP} = 1.\,165.\,924\,(10)$$

One still needs some ~ 70-80 10^{-9} pieces!

QED α^6 unlikely ... Z, W (H) can give ~

Luckily, hadrons contribute!

Main hadronic contribution:



$$\equiv O(\alpha^2)$$

$$\sim\!\!\bullet\!\!\sim \;\Rightarrow (K_\mu K_\nu - g_{\mu\nu}K^2)\Pi$$

$$a^{(2)}_{\mu,\text{Had}} \simeq \alpha^2 \int dK \;(\bar{f}\ldots)\;\Pi^{\text{Had}}(K^2)$$

$$\Rightarrow \quad \Pi^{\text{Had}}(K^2) = \frac{K^2}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\text{Im}\,\Pi^{\text{Had}}(t)\,dt}{t(t-K^2)}$$

$$\text{Im}\,\Pi^{\text{had}}(t) \equiv R = \frac{\sigma_{\text{tot}}(e^+e^-\to \text{had})}{\sigma_{\text{tot}}(e^+e^-\to\mu^+\mu^-)}$$

Therefore

$$a^{(2)}_{\mu,\text{Had}} = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} dt\, f(t)\, R(t)$$

$$f(t) = \frac{1}{t}\int_0^1 \frac{dx\; x^2(1-x)}{x^2+(1-x)t/m_\mu^2} \sim \begin{array}{ll} 1/t^2 & t\to\infty \\ 1/t & t\to 0 \end{array}$$

(<u>very</u> low-energy part $-\sim\rho,\omega-$ of the integrand dominates)

TABLE III. Hadronic contributions to the muon anomalous magnetic moment arising from Fig. 1(a). The first error is statistical and the second is systematic.

Contributing process and energy range	Contribution to $10^{10} a_\mu$	Reference
$\rho,\omega\to\pi^+\pi^-$ ($2m_\pi \le \sqrt{s} \le 1.1976$ GeV)	506.39(2.15)(15.0)	14,29,33,34,35
$\omega\to3\pi$ ($3m_\pi \le \sqrt{s} \le 2.0$ GeV)	46.04(4.73)(1.49)	36
ϕ ($2m_K \le \sqrt{s} \le 2.0$ GeV)	40.17(1.80)(1.29)	36
$J/\psi(3.100)$	5.64(71)(69)	37
$\psi(3.685)$	1.47(21)(22)	37
$\psi(3.770)$	0.18(4)(4)	36
$\Upsilon,\Upsilon',\Upsilon'',\Upsilon'''$	0.085(3)(5)	38
	Background	
$e^+e^-\to\pi^+\pi^-$ ($1.1976 \le \sqrt{s} \le 2.0$ GeV)	3.05(28)(31)	33,39
$e^+e^-\to\pi^+\pi^-\pi^0$ ($0.8432 \le \sqrt{s} \le 1.002$ GeV)	2.92(81)(<81)	40
$e^+e^-\to K^+K^-$ ($1.05 \le \sqrt{s} \le 3.0$ GeV)	4.32(32)(46)	33,34,41,42
$e^+e^-\to K_S^0K_L^0$ ($1.008 \le \sqrt{s} \le 2.15$ GeV)	0.90(47)(10)	34,43,44
$e^+e^-\to p\bar{p}$ ($1.9 \le \sqrt{s} \le 2.2375$ GeV)	0.17(3)(<3)	41,45
$e^+e^-\to\pi^+\pi^-\pi^0$ ($1.42 \le \sqrt{s} \le 2.05$ GeV)	0.96(7)(10)	41
$e^+e^-\to K_S^0K^\pm\pi^\mp$ ($1.4415 \le \sqrt{s} \le 2.05$ GeV)	1.12(9)(<9)	46
$e^+e^-\to\pi^+\pi^-\pi^0\pi^0$ ($0.99 \le \sqrt{s} \le 2.05$ GeV)	23.95(79)(3.00)	47
$e^+e^-\to\pi^+\pi^-\pi^+\pi^-$ ($0.986 \le \sqrt{s} \le 2.05$ GeV)	14.02(35)(1.12)	34,41,47
$e^+e^-\to K^+K^-\pi^+\pi^-$ ($1.45 \le \sqrt{s} \le 2.05$ GeV)	1.39(9)(18)	48
$e^+e^-\to\pi^+\pi^-\pi^+\pi^-\pi^0$ ($1.202 \le \sqrt{s} \le 2.05$ GeV)	1.72(13)(21)	34,49
$e^+e^-\to\pi^+\pi^-\pi^+\pi^-\pi^0\pi^0$ ($1.44 \le \sqrt{s} \le 2.05$ GeV)	5.05(46)(1.00)	50
$e^+e^-\to\pi^+\pi^-\pi^+\pi^-\pi^+\pi^-$ ($1.45 \le \sqrt{s} \le 2.05$ GeV)	0.43(4)(12)	51
$e^+e^-\to$ more than two hadrons ($2.05 \le \sqrt{s} \le 3.15$ GeV)	21.63(81)(4.33)	52
$e^+e^-\to$ hadrons		
($3.15 \le \sqrt{s} \le 7.8$ GeV)	19.81(27)(2.77)	53
($7.8 \le \sqrt{s} \le 30.8$ GeV)	4.27(26)(26)	54
($\sqrt{s} \ge 30.8$ GeV) (Asymptotic freedom with 6 quarks)	0.4	
Total	[illegible](5.9)(16.4)[a]	

[a]The second error is obtained by treating systematic errors by the least-squares analysis. Simple addition of these errors will give a value of about 34.

Note that for the contributions from the processes $e^+e^-\to\pi^+\pi^-\pi^0$ (0.8432 GeV $\le \sqrt{s} \le$ 1.002 GeV), and $e^+e^-\to K^+K^-$ (1.05 GeV $\le \sqrt{s} \le$ 3.0 GeV), we have subtracted the Breit-Wigner tails of ω and ϕ resonances, and the ϕ resonance itself, respectively, in order to avoid double counting. Finally, the contribution from the region $\sqrt{s} \ge 30.8$ GeV was estimated by the lowest-order QCD with six quarks. The results are summarized in Table III. The total contribution from this diagram [Fig. 1(a)] is given by (1.4).

APPENDIX B: CLARIFICATION OF THE PARAMETRIC METHOD IN THEORIES WITH DERIVATIVE COUPLINGS

In this appendix we present a minor simplification of the parametric method in theories with derivative cou-

Numerically:

$$a^{(2)}_{\mu,\mathrm{Had}} \simeq [71 \pm (2?\,3?)]\,10^{-9}$$

Therefore:

$$10^{9}[a_\mu^{QED} + a^{(2)}_{\mu,\mathrm{Had}}] = 1.165\,918(03?)$$

$$10^{9} \quad a_\mu^{\exp} = 1.165\,924(10.$$

(extra(+)copper from Z, W, $a^{(4)}_{\mu,\mathrm{Had}}$ -?-)

$\Longrightarrow$ O.K. for QED.

Two important facts:

I) The contribution from $\Pi^{\mathrm{Had}} \Longrightarrow$ $\sigma_{e^+e^- \to \mathrm{had}}$ is essential (~ 7 times larger than the experimental error).

II) The dominant theoretical error comes from the experimental error on $\sigma_{e^+e^- \to \mathrm{had}}$ ~ in the ρ, ω region

[illegible]

be considered a "predictive" one?

Not for the moment (<u>but</u> a spectacular reduction of the experimental error, and a suitable reduction of the theoretical one, perhaps ......)

The interpretation <u>would</u> have been different if hadrons (in particular, $\rho \rightleftarrows \pi^+(\pi^+)$) had not yet been known!

Still, one can say that:

the consistency between QED and the muon's $(g-2)$ <u>requires</u> the existence of a couple of pions of "effective" mass ~ a few $\sqrt{q^2} \simeq m_\mu$, and of a "loop" term ~ 

Question:

do (shall) we have similar tests for the

Minimal Standard Model? (MSM)

To answer the question, a (quick) review of the main properties of the MSM is necessary.

The MSM at tree level.

The MSM is a renormalizable quantum field theory of strong and electroweak interactions whose Lagrangian is invariant with respect to local gauge transformations belonging to the group $SU(3)_c \times SU(2)_L \times U(1)_Y$:

$$g \equiv e^{i\sum_{a=1}^{8} \eta_a(x) T_a^{(3)}} \, e^{i\sum_{j=1}^{3} \varepsilon_j(x) I_j} \, e^{i\omega(x) Y}$$

where T_a, I_j and Y are the generators of $SU(3)_c$, $SU(2)_L$, $U(1)_Y$ and

$$Q = I_{3L} + \frac{Y}{2} .$$

The particle content of the model is the following:

a) 3 families of left-handed and right-handed leptons: $\left(f_{L,R} \equiv \frac{1}{2}(1 \mp \gamma_5) f\right)$,

$$L_L \equiv \begin{vmatrix} \nu_L \\ e_L \end{vmatrix} \begin{matrix} \Rightarrow I_{3L} = +1/2 \\ \Rightarrow I_{3L} = -1/2 \end{matrix} \Rightarrow y = -1$$

<u>COLOR = 0.</u>

e_R $(I_{3L} = 0;\ y = -2)$, <u>no ν_R</u>.

b) 3 families of left-handed and right-handed quarks:

$$Q_L \equiv \begin{vmatrix} u_L \\ d_L \end{vmatrix} \begin{matrix} \Rightarrow I_{3L} = +1/2 \\ \Rightarrow I_{3L} = -1/2 \end{matrix} \Rightarrow y = \frac{1}{3}$$

u_R, d_R $\left(I_{3L} = 0;\ y = \frac{4}{3}, -\frac{2}{3}\right)$.

<u>COLOR = "3".</u>

(In each family), quarks are <u>requested</u> to make

$$\sum_i Q_{f_i} = \underset{e_L}{-1} \ \underset{e_R}{-1} + 3\left[\underset{u_L}{\frac{2}{3}} \underset{d_L}{-\frac{1}{3}} + \underset{u_R}{\frac{2}{3}} \underset{d_R}{-\frac{1}{3}}\right] =$$

$= 0$ (to avoid <u>anomalies</u>) (next order....).

c) 12 Gauge vector bosons:

$$8 \text{ gluons} \Rightarrow SU(3)_c$$
$$3 \ W_s \Rightarrow SU(2)_L$$
$$1 \ B \Rightarrow U(1)_Y$$

to produce gauge-invariance.

With a), b), c) one could make a gauge-invariant theory of <u>massless</u> leptons, quarks, gauge-bosons.

$\Rightarrow$ Magic trick: to introduce

d) a $SU(2)_L$ doublet of complex scalar fields

$$\Phi \equiv \begin{vmatrix} \phi^+ \\ \phi^0 \end{vmatrix} \begin{matrix} \Rightarrow I_{3L} = +1/2 \\ \Rightarrow I_{3L} = -1/2 \end{matrix} \Rightarrow Y = 1.$$

[...] of the Higgs doublet in the MSM is that of Santa Klaus

it gives mass to whomever asks for it ...

(to the charged leptons, to the quarks, to three not colored gauge bosons)

and leaves massless those who so wish

(the gluons, the photon, the neutrinos(?))

This operation requires the simultaneous presence (and collaboration) of two distinct tricks:

a) The spontaneous symmetry breaking

b) The Higgs mechanism

The spontaneous symmetry breaking

This happens when a symmetry of the Lagrangian is <u>not</u> a symmetry of the minimum energy state ("vacuum

Old classical example: ferromagnetics near the Curie temperature T_c.
For T close to T_c one (Ginzburg-Landau) expands the free energy in powers of the magnetization $\vec{M}$:

$$u(\vec{M}) = (\partial_i \vec{M})^2 + V(\vec{M})$$

$$V(\vec{M}) = \alpha_1(T)\, \vec{M}.\vec{M} + \alpha_2 (\vec{M}.\vec{M})^2 \quad (\alpha_2 > 0$$

(rotational invariance)

$$\alpha_1 = \alpha(T - T_c) \qquad (\alpha > 0)$$

Min. $\frac{\partial V}{\partial M_i} = 0 \Rightarrow \vec{M}\left(\alpha_1 + 2\alpha_2 \vec{M}.\vec{M}\right) = 0$

<u>Vacuum</u>: $\alpha_1 > 0 \ (T > T_c) \Rightarrow \vec{M} = 0$ (rot. inv

$\alpha_1 < 0 \ (T < T_c) \Rightarrow |\vec{M}| = \sqrt{\frac{-\alpha_1}{2\alpha_2}}$ (<u>no</u> rot. inv

$(\mathcal{R}\,|0\rangle \neq |0\rangle)$

In QFT, one has the Goldstone theorem:

Spontaneous breakdown of a global continuous symmetry implies the existence of massless spinless particles.
(true to all orders of perturb. theory).

$U \equiv e^{i \varepsilon_\alpha A^\alpha}$ (element of sym. group)

$U|0\rangle \neq |0\rangle \Rightarrow$ at least one $A_\alpha|0\rangle \neq 0$.

In correspondence to that A^α:

$\sim A^\alpha \Rightarrow \phi_i \; : \quad \langle 0|[A^\alpha, \phi_i]_-|0\rangle \neq 0$

and there exists a massless physical state $|n_\alpha\rangle$ such that: ($A^\alpha \overset{?}{\sim} \int d\vec{x}\, J_0^\alpha(\vec{x},t)$)

$$\langle 0|J_0(0)|n_\alpha\rangle \neq 0 \; (\Rightarrow \text{spinless})$$

$$\langle 0|\phi_i(0)|n_\alpha\rangle \neq 0 \; (\Rightarrow \phi_i \;\; " \;\;)$$

Also, if $[A^\alpha, \phi_i]_- = i t^\alpha_{ij} \phi_j$

at least one ϕ_j must exist such that

$$\langle 0|\phi_j(0)|0\rangle \equiv v_j \neq 0.$$

Rule: (for <u>global</u> cont. symm.)

to every generator of the symm. group that does <u>not</u> annihilate the vacuum there corresponds a massless spinless particle (Goldstone boson).

This is a <u>physical</u> state

($\sim$ the pion in the σ-model ...).

If the spont. broken symmetry is a (local) gauge symmetry, this is no longer true since the <u>Higgs phenomenon</u> appears.

The Higgs phenomenon.

This is typical of spontaneously broken gauge symmetries.

Simplest example: an Abelian U(1) gauge theory

$$L = (D_\mu \phi)^\dagger D^\mu \phi - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$D_\mu \equiv (\partial_\mu - i g A_\mu), \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$(\phi \to \phi' = e^{-i\alpha(x)} \phi \; \ldots\ldots)(A'_\mu = A_\mu - \frac{1}{g} \partial_\mu \alpha(x))$$

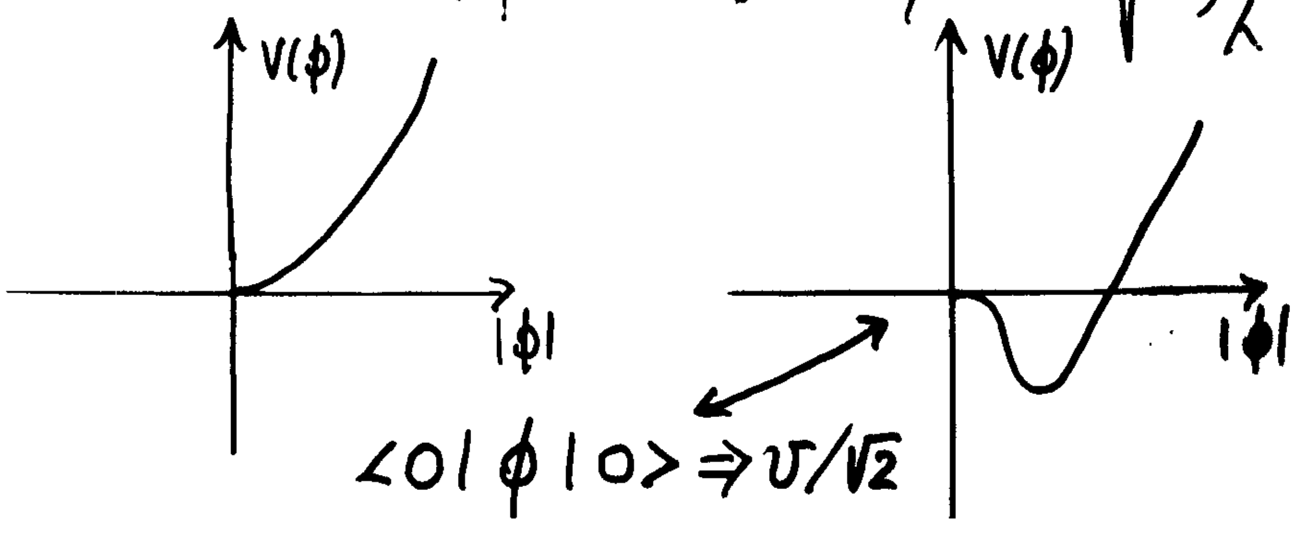
$$[(D_\mu \phi)' = e^{-i\alpha(x)} D_\mu \phi]$$

The potential

$$V(\phi) = +\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \; ; \; \lambda > 0$$

for $\mu^2 < 0$ has a minimum at

$$|\phi| = v/\sqrt{2} \; , \; v = \sqrt{\frac{-\mu^2}{\lambda}}$$



Natural choice: $\left(\phi \equiv \frac{\phi_1 + i\phi_2}{\sqrt{2}}\right)$

$$\langle 0|\phi_1|0\rangle \equiv v; \quad \langle 0|\phi_2|0\rangle = 0.$$

Then: "shifted" fields $\phi_i' \equiv \phi_i - \langle 0|\phi_i|0\rangle$

$$\phi_1' \equiv \phi_1 - v; \qquad \phi_2' = \phi_2$$

The Lagrangian becomes:

$$\begin{aligned}
L = {} & \underline{\mu^2 \phi_1'^2} + \frac{\mu^2}{4v^2}\left[O(\phi_i'^3, \phi_i'^4)\right] \\
& - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \left[O(g^2 A_\mu^2 \phi_i'^2, g A_\mu \phi_i \phi_j' (\partial\phi'^2))\right] \\
& - g v A^\mu \left[g A_\mu \phi_1'\right] \\
& + \underline{\frac{g^2 v^2}{2} A^\mu A_\mu} \qquad \underline{- g v A^\mu \partial_\mu \phi_2'}.
\end{aligned}$$

$\Longrightarrow$ ϕ_2' is massless (expected)

$\phi_1' \Rightarrow m_1^2 = -2\mu^2 \; (>0)$.

<u>A_μ has acquired a mass $M_A = gv$</u>

<u>a very disturbing term</u> $\sim A^\mu \partial_\mu \phi'$

has appeared.

The disturbing term $\sim H^\mu \partial_\mu \phi_2'$ can be removed if ϕ_2' is washed out (drastic solution).

This can be done parametrizing the fields as

$$\phi(x) = \frac{1}{\sqrt{2}} \left[\eta(x) + v \right] e^{i \xi(x)/v}$$

$(\eta \sim \phi_1', \ \xi \sim \phi_2' \ldots)$

and performing a gauge transf.

$$\phi''(x) = e^{-i \xi(x)/v} \phi = \frac{1}{\sqrt{2}} \left[\eta(x) + v \right]$$

$$B_\mu(x) = A_\mu - \frac{1}{gv} \partial_\mu \xi$$

that fixes the choice of the gauge (≡ Unitary gauge)

In this gauge we are left with:

One massive scalar $\eta \to \phi_1'$, $m_\eta^2 = -2$

One massive g.b. B_μ, $M_B^2 = (gv)^2$.

no more ϕ_2' ("eaten" by B_μ).

ϕ_2' "pseudo" Goldstone boson.

Question:

What has all this to do with the SM (=Standard Model)?

To answer the question, one should (quickly) go back to the

BSM

("Before Standard Model")
(for most of the audience BSM $\simeq$ Paleozoic...)

hera.

In fact, old(er) people still remember that:

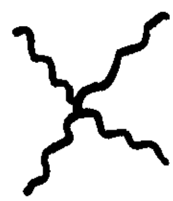
Once upon a time, there was the "Fermi theory" of weak interactions, where ~ everything was explained by the Lagrangian:

$$\mathcal{L}_F(x) = -\frac{G_F}{\sqrt{2}}\left[J_\lambda(x)\, J^{\lambda +}(x) + h.c.\right]$$

with the <u>charged</u> weak current:

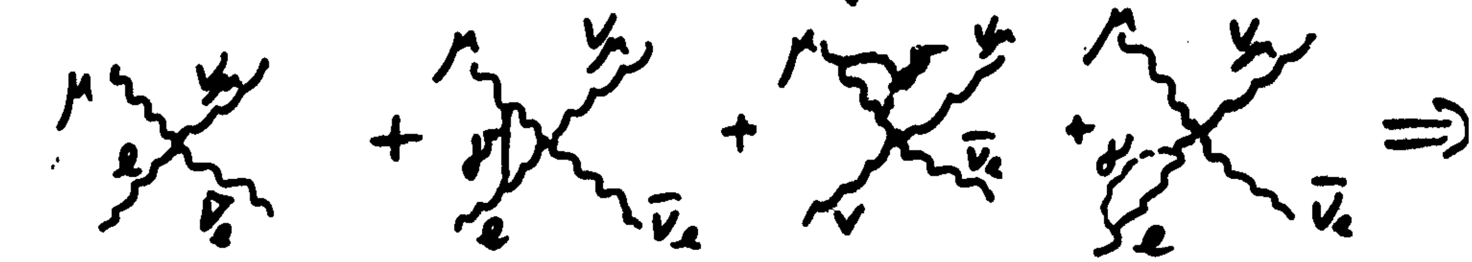
$$J_\lambda = J_{\lambda,\ell} + J_{\lambda,h}$$

and e.g. $J_{\lambda,\ell} = \bar{\psi}_{\nu_e}\gamma_\lambda(1-\gamma_5)\psi_e + \dots$

("four fermion" interaction $\Rightarrow$  ,

(<u>purely left-handed</u> fermions, $\psi_L = \frac{(1-\gamma_5)}{2}\psi$).

The coupling constant $G_F \to G_\mu$ was <u>defined</u> from the measured $\mu \to e \nu_\mu \bar{\nu}_e$ muon lifetime τ_μ:



$$\Rightarrow \frac{1}{\tau_\mu} \equiv G_\mu^2 \frac{m_\mu^5}{192\pi^3}\left[1 - \frac{8m_e^2}{m_\mu^2} + O_\mu(\alpha)\right]$$

(<u>definition</u> of G_μ...)

where $O_\mu(\alpha)$ is a (small) known finite R.C. (radiative correction) that describes the effect of the last 3 diagrams. $\left[O_\mu = \frac{\alpha}{2\pi}\left(\frac{25}{4}-\pi^2\right)\left(1+\frac{2\alpha}{3\pi}\ln\frac{m_\mu}{m_e}\right)\right]$

Numerically, one has:

$$G_\mu = 1.166389(22)\, 10^{-5}\,\mathrm{GeV}^{-2}$$

with $\frac{\delta G_\mu}{G_\mu}^{exp} \simeq 2.10^{-5}$

(not as good as $\frac{\delta\alpha(0)}{\alpha} \simeq 2.10^{-8}$, still

The Glashow-Salam-Weinberg model (tree level,

The Fermi theory of weak interactions as described by a Lagrangian

123) $$\mathcal{L}_F(x) \equiv -\frac{G_F}{\sqrt{2}}\left[J_\lambda^{+} J^{\lambda} + h.c. \right]$$

with the *charged* current

124) $$J_\lambda = J_{\lambda h} + J_{\lambda \ell} \qquad (G_F \simeq 10^{-5}/M_p^2$$

describes correctly a certain amount of weak interactions phenomene. But from the very beginning it is affected by two major deseases:

I) Lack of renormalizability (G_F has dimension $[mass]^{-2}$) $\Rightarrow$ meaningless at higher orders.

II) Violation of unitarity. Already at Born level *certain* processes cannot be described correctly. For instance, for the reaction $\nu_\mu e \to \mu \nu_e$, whose amplitude is purely $J = 1$, the high energy behaviour of the cross section would be

(125) $$\sigma \simeq G_F^2 s \qquad (s \equiv 2 m_e E, E \equiv E_{\nu_\mu}^{LAB})$$

(just by dimensional arguments)

and, from unitarity, $\sigma_{J=1}$ must be bounded by $\sim \frac{1}{s}$. This leads to the conclusion that eq. (125) can only be valid for energies such that

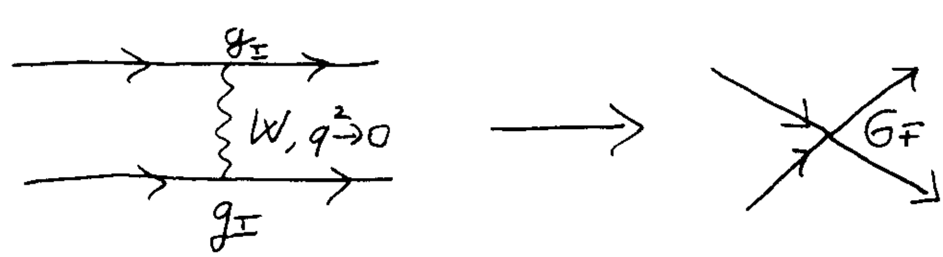
(126) $$\sqrt{s} \lesssim \frac{1}{\sqrt{G_F}} \simeq 300 \text{ GeV}$$

and, for higher energies, the Lagrangian eq. (123) provides a bad prediction already at tree level.

Since the weak current in eq. (124) has the same Lorentz transformation properties at the electromagnetic current, one would be tempted to imitate QED introducing a new charged massive field W_μ and writing the Fermi interaction as a Yang-Mills one

127) $$\mathcal{L}_I \equiv g_I \left[J_\lambda W^\lambda + h.c. \right]$$

that would reproduce the low-energy ($\equiv$ zero momentum transfer) predictions of $\mathcal{L}_F$ at tree level

128) 

(e.g. $\mu \to \nu_\mu e \bar{\nu}_e$)

with the identification

129) $$g_I^2 / M_W^2 = G_F / \sqrt{2}$$ || **Fundamental!**

and with a dimensionless coupling g. However, even in this case the theory would not be renormalizable since the massive vector boson propagator $\Delta_{\mu\nu}(k) \sim (-)\, \frac{g_{\mu\nu} - k_\mu k_\nu / M_W^2}{k^2 - M_W^2}$ now behaves like a constant when $k \to \infty$, and not like $(k)^{-2}$. Also the unitarity problem remains, simply shifted to a different process ($\nu\bar{\nu} \to W^+ W^-$). Since the bad high-energy behaviour is related to the lack of renormalizability, one would hope that by renormalizing the theory unitarity is also restored. The problem would be in this approach that of introducing gauge boson mass terms without destroying renormalizability.

$\boxed{\text{N.B.}}$ The problem would remain for $M_{W^\pm} \to 0$!!

From the previous discussion, it appears that a possibly promising approach might be that of introducing a gauge symmetry and ~~of~~ breaking it spontaneously, to generate masses for the needed (massive) gauge bosons. Accepting that this possibility deserves to be investigated, a first problem is the choice of a proper gauge group.

To investigate this problem, suppose first that only <u>one</u> family of fermions (e, ν, u, d) exists and consider the expression of the weak charged <u>lepton</u> current to be coupled to the charged vector boson W as in eq. (127):

$$\text{(130)} \quad J_{\lambda,e} = \bar{\psi}_{\nu_e} \gamma_\lambda (1-\gamma_5) \psi_e$$

The candidate component of the Lagrangian eq. (127) contains J_λ and J_λ^+. If one wants to introduce a gauge symmetry group G, this will have to contain <u>at least</u> two charges (generators) associated to these currents (<u>and</u> to the two charged bosons W_μ^+, W_μ^-):

131) $$T_+ \equiv \frac{1}{2} \int d\vec{x}\; J_0(x)$$

and $T_- \equiv T_+^{\,+}$.

No meaningful groups are known with two generators. The "smallest" which is available i.e. $G \doteq SU(2)^{(*)}$ has in fact three generators, with an algebra:

132) $$[T_+, T_-] = 2\,T_3 .$$

Therefore, a candidate Lagrangian should contain at least one more gauge boson $\Rightarrow T_3$

(*) $\doteq$ means: "mathematically" equal to an $SU(2)$ group, to be still "physically" specified. [illegible] $G = SU(2)_W$

The simplest choice would be that of "adding" in a "unified" way the electromagnetic interaction. This introduces a term

133) $$\mathcal{L}^{e.m.} = e\, J_\lambda^{e.m.} A^\lambda \quad (e \simeq g)$$

with

134) $$J_\lambda^{e.m.} = \bar{\psi}_e \gamma_\lambda \psi_e$$

and an electric charge

135) $$Q = \int d\vec{x}\, J_0^{e.m.}(x) .$$

However, from the canonical a.commutation rules

136) $$\{ \psi_i^\dagger(\vec{x},t), \psi_j(\vec{x}',t) \}_+ = \delta_{ij}\, \delta(\vec{x}-\vec{x}')$$

one easily derives that

137) $$[T_+, T_-] \equiv 2T_3 = 2 \cdot \frac{1}{4} \int d\vec{x} \left[\psi_\nu^\dagger (1-\gamma_5) \psi_\nu - \psi_e^\dagger (1-\gamma_5) \psi_e \right]$$

and $T_3 \neq Q$ (~ obvious in the assumed picture of a fermion doublet $e, \nu \ldots$).
($SU(2)$ generators must be traceless $\Rightarrow$ sum of a multiplet "charges" $= 0 \ldots$).
The two simplest ways of solving the problem are now:

I) to add a minimal number of extra fermions to (e^-, ν_e) so that the new expressions of T_+, T_- and Q make up the correct $SU(2)$ algebra. In particular, at least one extra charged E^+ lepton must be contained in a (triplet) representation including (E^+, ν, e^-). In this solution, the only neutral current of the theory is the electromagnetic one.

II) To introduce another gauge boson W_3, different from the photon, and coupled to T_3 (therefore, electrically neutral). In this case, the Lagrangian describing both electromagnetic and weak interactions (≡ "electroweak" interactions) should contain at least four different gauge bosons ⟹ the symmetry group should have at least four generators.

The first choice (I), theoretically less appealing from the beginning (see discussion o Cheng + Li, page 341-342), was anyhow ruled out by the discovery (1973) of the existence of weak (i.e. not electromagnetic) neutral current effects ⟹ we are left with (II).

A (realistic) simple group G with four generators is not available. But one can consider a (minimal) not simple group i.e.:

$$G \equiv SU(2) \overset{w}{\times} U(1) \tag{138}$$

which has the requested property. Note that the generator associated to U(1) must commute with those associated to $SU(2) \equiv T_i$. Calling this generator Y, we shall therefore have to check that

$$[Y, T_i] = 0 \quad (i = 1, 2, 3). \tag{139}$$

At this point, a number of phenomenological considerations appear. They will influence drastically the construction of the <u>fermionic</u> sector of the Lagrangian

To fix the quantities $Y^{(K)}$, one writes the expression of the $SU(2)_L$ charges $T_\pm$, T_3 :

147) $$T_+ = \int d\vec{x}\, \left[\psi^+_{\nu_L} \psi_{e_L} + \psi^+_{u_L} \psi_{d_L} \right], \quad T_- \equiv T_+^{\,+}$$

148) $$T_3 \equiv \frac{1}{2} \int d\vec{x} \left[\psi^+_{\nu_L} \psi_{\nu_L} - \psi^+_{e_L} \psi_{e_L} + \psi^+_{u_L} \psi_{u_L} - \psi^+_{d_L} \psi_{d_L} \right]$$

and of the e.m. charge Q :

149) $$Q \equiv \int d\vec{x} \left[- \psi^+_{e_L} \psi_{e_L} - \psi^+_{e_R} \psi_{e_R} + \frac{2}{3}\left(\psi^+_{u_L} \psi_{u_L} + \psi^+_{u_R} \psi_{u_R} \right) - \frac{1}{3}\left(\psi^+_{d_L} \psi_{d_L} + \psi^+_{d_R} \psi_{d_R} \right) \right]$$

and $Q \neq T_3$, but $[Q, T_i] \neq 0$.

($\Rightarrow$ Y cannot be Q ...)

and realizes that the quantity $(Q - T_3)$ commutes with all $SU(2)_L$ generators (and gives the same quantum numbers for the same $SU(2)_L$ doublet). One therefore makes the identification: (<u>conventional</u>)

150) $Y \equiv 2(Q - T_3)$

that generalizes to the electroweak generators T_3, Y the relation between charge and "strong" $SU(2), Y^s$ generators.

With the previous eq. (150) and the conventional electric charges choice (and $SU(2)_L$ quantum numbers as in conventional $SU(2)$ doublets i.e. $T_3 = +1/2, -1/2$) one has

151) $Y(\ell_L) = -1; \quad Y(e_R) = -2; \quad (T_3 = 0 \text{ for r.h.f.})$

$Y(q_L) = \frac{1}{3}; \quad Y(u_R) = \frac{4}{3}; \quad Y(d_R) = -\frac{2}{3}.$

The (gauge) fermionic sector.

This part of the Lagrangian contains the free fermionic and gauge boson components and the related gauge boson - fermion interaction. To reproduce correctly the known low-energy features of weak interactions, the charged part of the interaction purely described by the $SU(2)^W$ generators $T_\pm$ ($\Rightarrow W_\mu^\pm$) must only involve left-handed leptons e_L, ν_L and quarks u_L, d_L^W where in fact

140) $$d_L^W \equiv \cos\theta_c \, d_L^{(3)} + \sin\theta_c \, s_L^{(3)} \equiv \underline{\underline{d_L}}$$

where $d^{(3)}$ and $s^{(3)}$ have definite flavour $SU(3)$ transformation properties and

141) $$\psi^f_{\substack{L\\(R)}} \equiv \frac{1}{2}\left(1 \underset{(+)}{-} \gamma_5\right)\psi^f \quad (\text{two component fields})$$

The construction of the Lagrangian requires now a few prescriptions to generalise the U(1) QED rules to the non abelian SU(2) case.

Non Abelian gauge symmetry. Yang-Mills fields

The previous considerations can be generalised to the case of non Abelian gauge symmetries as shown by Yang and Mills in 1954. The simplest example is that of a $SU(2)$ (isospin) symmetry.

Consider a fermion isospin doublet

65) $$\Psi \equiv \begin{vmatrix} \psi_1 \\ \psi_2 \end{vmatrix}$$

Under a global $SU(2)$ this transforms as

66) $$\Psi'(x) = \exp\left[-i\,\frac{\vec{\theta}\cdot\vec{\tau}}{2}\right]\Psi(x) \equiv U(\theta)\Psi(x$$

with constant $\vec{\theta}$ and where τ_i are the three Pauli matrices:

67) $$\left[\frac{\tau_i}{2}, \frac{\tau_j}{2}\right] = i\,\varepsilon_{ijk}\,\frac{\tau_k}{2} \qquad (i,j,k = 1,2,3).$$

Under a global $SU(2)$ transformation, the free Lagrangian

68) $$\mathcal{L}_0 = \overline{\Psi}(x)\left(i\gamma^\mu \partial_\mu - m\right)\Psi(x)$$

remains invariant. Again, this is no longer true under local transformations $\vec{\theta} \to \vec{\theta}(x)$ since the derivative term becomes

69) $$\left[\overline{\Psi}(x)\,\partial_\mu \Psi(x)\right]' = \overline{\Psi}(x)\,\partial_\mu \Psi(x) + \\ + \overline{\Psi}(x)\,U^{-1}(\theta)\left[\partial_\mu U(\theta)\right]\Psi(x)$$

The construction of a gauge-invariant Lagrangian follows the (QED) previous procedure. One first introduces three vector gauge fields A_μ^i $(i = 1, 2, 3)$ (one for each $SU(2)$ generator) to form the covariant derivative :

70) $$D_\mu \Psi = \left(\partial_\mu - i g \frac{\vec{\tau} \cdot \vec{A}_\mu}{2} \right) \Psi$$

where g is the "SU(2) coupling" analogous to e in QED (with this sign convention $g > 0$), and imposes that $(D_\mu \Psi)$ transforms as Ψ i.e.

71) $$(D_\mu \Psi)' = U(\theta) D_\mu \Psi.$$

Eq. (71) fixes the transformation properties of the gauge fields A_μ^i. In particular, for infinitesimal transformations $|\vec{\theta}(x)| \ll 1$ one finds:

72) $$(A_\mu^i)' = \left[A_\mu^i - \frac{1}{g} \partial_\mu \theta^i(x) \right] + \varepsilon^{ijk} \theta^j(x) A_\mu^k$$

<u>Exercise</u>: derive eq. (72) from eqs. (66)-(71)

One sees from eq. (72) that under a *global* SU(2) transformation the A_μ^i fields are not "neutral", but transform like a triplet. They are SU(2) *charged*, which represents a major difference with respect to the QED case. This affects also the expression of the antisymmetric second-rank tensor of the gauge fields $F_{\mu\nu}^i$. To see this, it is convenient to start from the *definition*:

73) $$(D_\mu D_\nu - D_\nu D_\mu)\,\Psi \equiv i g \left(\frac{\vec{\tau}}{2} \cdot \vec{F}_{\mu\nu} \right) \Psi$$

and derive

74) $$\frac{\vec{\tau}.\vec{F}_{\mu\nu}}{2} = \partial_\mu \frac{\vec{\tau}.\vec{A}_\nu}{2} - \partial_\nu \frac{\vec{\tau}.\vec{A}_\mu}{2} - \\ - i g \left[\frac{\vec{\tau}.}{2}\vec{A}_\mu \, , \, \frac{\vec{\tau}.}{2}\vec{A}_\nu \right]$$

i.e.:

75) $$F_{\mu\nu}^i = [\partial_\mu A_\nu^i - \partial_\nu A_\mu^i] + g\,\varepsilon^{ijk} A_\mu^j A_\nu^k$$

One easily sees that, under infinitesimal transformations $|\vec{\theta}(x)| \ll 1$, $F_{\mu\nu}^{i}$ is not invariant but transforms ~ like A_μ^i:

76) $$(F_{\mu\nu}^{i})' = F_{\mu\nu}^{i} + \varepsilon^{ijk}\,\theta^{j}\,F_{\mu\nu}^{k}$$

However, the product

77) $$\mathrm{tr}\left[(\vec{\tau}.\vec{F}_{\mu\nu})(\vec{\tau}.\vec{F}^{\mu\nu})\right] \sim F_{\mu\nu}^{i}\,F^{\mu\nu,i}$$

is gauge-invariant. Thus, a complete gauge-invariant Lagrangian describing the interaction between the matter field Ψ and the gauge fields A_μ^i is:

78) $$\mathcal{L} = \bar{\Psi}(x)\left(i\gamma^{\mu}D_{\mu} - m\right)\Psi(x) - \frac{1}{4}F_{\mu\nu}^{i}F^{\mu\nu,i}$$

<u>Exercise</u>. Derive eq. (76).

<u>Exercise</u>. Proof the gauge-invariance of eq. (77).

Thus, the covariant derivative "belonging" to the $SU(2)^W$ component must only act on left-handed fermions. To make this operation possible, the irreducible representations of $SU(2)^W$ in the fermionic sector must contain <u>doublets</u> of <u>left-handed fermions</u>

i.e.

142) $$\ell_L \equiv \begin{vmatrix} \nu_L \\ e_L \end{vmatrix} \quad , \quad q_L \equiv \begin{vmatrix} u_L \\ d_L \end{vmatrix}$$

transforming under the group in the "canonical" way (i.e. via Pauli matrices etc).
As a consequence, the $SU(2)^W$ group will be called "left $SU(2)$" and the conventional notation will be introduced:

143) $$G \equiv SU(2)_L \times U(1)_Y$$

In addition to the two-component left fermions, the Lagrangian will have to conta two-component right-handed fermions as well, since the electromagnetic interaction that one wants to incorporate contains them (and left-handed fermions as well). The fields must not be transformed by $SU(2)_L$

$\Rightarrow$ they will be $SU(2)_L$ singlets. But they will be transformed by $U(1)_Y$ (and the left-handed ones as well...).

Thus, the full (for one family) fermion spectrum will consist of 15 two-component fields, i.e. l_L (2 fields), q_L (3 . 2 fields since quarks can have three colours), e_R (one field), u_R (3 fields) and d_R (3 fields).

1.B. NO right-handed neutrino (not "requested")

$\Rightarrow m_\nu = 0$.

... to the MSM:
no need of the trick in the $SU(3)_c$ S...
(gluons remain massless).

In fact, $SU(3)_c$ can be separated
away and treated "canonically"
($SU(3)_c$ commutes with $SU(2)_L \times U(1)_Y$)
(only quark masses "borrowed"
from $SU(2)_L \times U(1)_Y$ Sp. Sy. Br.)

From now on: concentrate on
the E.W. component $SU(2)_L \times U(1)_Y$.

Of the four g. bosons, only one
(the photon) <u>must</u> remain massless.
Three must acquire a mass.....

$\Longrightarrow$ four real Higgs fields
(two complex fields)
<u>will do the job.</u>

"bonus": fermions too will become mass...

This is made of four pieces:

$$L = L_1 + L_2 + L_3 + L_4$$

$$L_2 = -\frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} - \frac{1}{4} G_{\mu\nu} G^{\mu\nu}$$

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g_0 \varepsilon^{ijk} A^j_\mu A^k_\nu \Rightarrow SU(2)$$

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \Longrightarrow U(1)_Y$$

$$L_1 = \sum_j \overline{\psi_j}\, i\, \gamma^\mu D_\mu \psi_j$$

$$j \Rightarrow (L)^{\dots}_{L,R} \;,\; Q^{1,2,3}_L \;,\; (u,d)^{1,2,3}_R$$

$$D_\mu \psi_j = \left(\partial_\mu - i g_0 \vec{T}^j \cdot \vec{A}_\mu - i g_0' \frac{Y^j}{2} B_\mu \right) \psi_j$$

$$(D_\mu L_L = \left(\partial_\mu - i g_0 \frac{\vec{\tau}}{2} \cdot \vec{A}_\mu + i g_0' \frac{1}{2} B_\mu \right) L_L$$

$$L_4 = \sum_i \Big[\overline{L}^i_L \Phi e^i_R h^i_e + \overline{Q}^i_L \Phi d^i_R h^i_d + \\ + \overline{Q}^i_L \tilde{\Phi} u^i_R h^i_u + h.c. \Big]$$

$$\Phi \equiv \begin{vmatrix} \phi^+ \\ \phi^0 \end{vmatrix} \;,\; \tilde{\Phi} \equiv i \tau_2 \Phi^* \Longrightarrow Y_{\tilde{\Phi}} = -1 .$$

$$\mathcal{L}_3 = (D_\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi)$$

$$D_\mu \Phi = \left(\partial_\mu - \frac{i}{2} g_0 \vec{\tau}\cdot\vec{A}_\mu - \frac{i}{2} g_0' B_\mu\right)\Phi$$

$$\underset{SU(2)_L}{\downarrow} \qquad \underset{U(1)_Y}{\downarrow}$$

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

$$\mu^2 < 0 \Rightarrow \text{Sp. Sy. Br. (SSB)}$$

$$\text{SSB}: \langle 0|\Phi|0\rangle = \begin{vmatrix} 0 \\ v/\sqrt{2} \end{vmatrix} \neq 0, \ v = \sqrt{-\mu^2/\lambda}$$

Then: $\Phi(x) \equiv e^{-i\left[\vec{\xi}(x)\cdot\frac{\vec{t}}{v}\right]} \cdot \begin{vmatrix} 0 \\ \frac{v+\eta(x)}{\sqrt{2}} \end{vmatrix}$

$$t_b \Rightarrow T_b : T_b |0\rangle \neq 0$$

$\xi_{1,2,3}$ are the "would-be" Golds. bosons that can be washed away (Unitary gauge).

The physical spectrum is then made by the following particles:

→ one physical massive scalar (the Higgs) ⇒ $\eta(x)$:

$$m_{o\eta}^2 = -2\mu^2$$

II) 3 families of fermions with a massless neutrino and massive electron and quarks:

$$m_{oe} = h_e \, v/\sqrt{2}$$
$$m_{ou,d} = h_{u,d} \, v/\sqrt{2}$$

III) 3 massive and one massless (≡ photon) gauge bosons.

The derivation of the g.b. masses is a "key-point" of the MSM. It contains predictions and features that will affect both the tree- and the one-loop level in the model