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SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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SUPERSYMMETRIC GUTS

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Please note: These are preliminary notes intended for internal distribution only.

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At the moment we think that at the energies below $M_W = 100$ GeV, the nature of the strong + electroweak interactions is described by the standard model with $G_W = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge symmetry.

Gauge bosons:

gluons $g^a_\mu = (8.1.0)$.

$SU(2)_L$ — $W^a_\mu = (1.3.0)$

$U(1) = B_\mu = (1.1.0)$.

quarks and leptons: each family

$$Q^\alpha = \begin{pmatrix} u^\alpha \\ d^\alpha \end{pmatrix} = (3.2.\tfrac{1}{3})$$

$$u^\alpha_c = (\bar{3}.1.-\tfrac{4}{3}); \quad d^\alpha_c = (\bar{3}.1.\tfrac{2}{3})$$

$$L^\alpha = \begin{pmatrix} e \\ \nu \end{pmatrix} = (1.\bar{2}.-1)$$

$$e_c = (1.1.2)$$

In the standard model (as you know) the one Higgs doublet gives masses to both up and down fermions trough the interaction

$$\overline{Q}^\alpha u_c H_\alpha + \overline{Q}^\alpha d_c \varepsilon_{\alpha\beta} H^{+\beta}. \qquad Q=\begin{pmatrix}u\\ d\end{pmatrix} \qquad H=\begin{pmatrix}H^0\\ H^-\end{pmatrix}$$

However among the superfields such type couplings are forbidden, since H^+ is an superfield of the opposite chirality and therefore $[Q d_c \varepsilon H^+]_F$ is not a supersymmetric quantity. (in fact it is not an even ~~vector~~ scalar (chiral) superfield).

So together with susy we are forced to introduce one more Higgs doublet (in fact antidoublet $\overline{H}$) with an conjugated $SU(2)_L \otimes U(1)_Y$ quantum numbers.

Higgs doublets: $H \equiv (1.2.1)$
$\overline{H} \equiv (1.2.-1)$

In the supersymmetric extension all the particles require their ~~superpartners~~ fermions (bosons) of the standard model bosonic (fermionic) superpartners. **At the** moment there is no direct experimental evidence for the supersymmetry, However, this is the only known extention that gives natural explenation for the small mass of the elementary Higgs scalar. **(Tecnical solution to the M_P/M_W** hierarchy **problem).**

Imortant point is that SUSY introduces one extra doublet $\overline{H}$ with conjugate quantum numbers. this is related to ~~the fact~~ two facts:

① cancellation of the Higgsino anomalies.

② one doublet can not give masses to both up and down fermions superfields since H^+ is no more leftchiral

$$\alpha_1 = \frac{5}{3}\,\frac{g'^2}{4\pi}$$

$$\alpha_2 = \frac{g^2}{4\pi}$$

$$\alpha_3 = \frac{g_s^2}{4\pi}$$

$$\mu \frac{\partial}{\partial \mu} \alpha_i(\mu) = \frac{1}{2\pi}\left(b_i + \frac{b_{ij}}{4\pi}\alpha_j(\mu) + \frac{b_{ik}}{4\pi}\alpha_k(\mu)\right)\alpha_i^2(\mu) + \frac{2}{(4\pi)^2}\, b_{ii}\,\alpha_i^3(\mu)$$

$\underline{i \neq j \neq k}$

~~standard~~ SM

$$b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{22}{3} \\ -11 \end{pmatrix} + N_{FAM}\begin{pmatrix} \frac{4}{3} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix} + N_{Higgs}\begin{pmatrix} \frac{1}{10} \\ \frac{1}{6} \\ 0 \end{pmatrix}$$

$$b_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\frac{136}{3} & 0 \\ 0 & 0 & -102 \end{pmatrix} + N_{FAM}\begin{pmatrix} \frac{19}{15} & \frac{3}{5} & \frac{44}{15} \\ \frac{1}{5} & \frac{49}{3} & 4 \\ \frac{11}{30} & \frac{3}{2} & \frac{76}{3} \end{pmatrix} + N_{Higgs}\begin{pmatrix} \frac{9}{50} & \frac{9}{10} & 0 \\ \frac{3}{10} & \frac{13}{6} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

SSSM

$$b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_{FAM}\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + N_{Higgs}\begin{pmatrix} \frac{3}{10} \\ \frac{1}{2} \\ 0 \end{pmatrix}$$

$$b_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -24 & 0 \\ 0 & 0 & -54 \end{pmatrix} + N_{FAM}\begin{pmatrix} \frac{38}{15} & \frac{6}{5} & \frac{78}{15} \\ \frac{2}{5} & 14 & 8 \\ \frac{11}{15} & 3 & \frac{68}{3} \end{pmatrix} + N_{Higgs}\begin{pmatrix} \frac{9}{50} & \frac{9}{10} & 0 \\ \frac{3}{10} & \frac{7}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

~~Unificati~~
Unification of Couplings.

Analyses of ~~the~~ LEP data [Langacker, Ellis et al, Amaldi et al.] indicates grand unification of the coupling constants.

The coupling constants $\alpha_1, \alpha_2, \alpha_3$ at scale M_Z:

$$\alpha_1(M_Z) = \frac{5}{3}\alpha_Y = 0.016887 \pm 0.000040$$

$$\alpha_2(M_Z) = 0.03322 \pm 0.00025$$

$$\alpha_3(M_Z) = 0.113 \pm 0.005$$

~~Two loop~~ 2-loop RG equations

$$\mu \frac{d}{d\mu}\alpha_i(\mu) = \frac{1}{2\pi}\Big[b_i + \frac{1}{4\pi}\sum_j b_{ij}\,\alpha_j(\mu)\Big]\alpha_i^2(\mu) + \frac{2b_{ii}}{(4\pi)^2}\alpha_i^2(\mu)$$

$$b_i = \begin{pmatrix} 0 \\ -22/3 \\ -11 \end{pmatrix} + N_{Fam}\begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix} + N_{Higgs}\begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix} ; \quad \text{SM.}$$

$$b_i = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_{Fam}\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + N_{Higgs}\begin{pmatrix} 3/10 \\ 1/2 \\ 0 \end{pmatrix} ; \quad \text{SUSM.}$$

①. Evolution of the $\alpha_i(\mu)$ ~~ruth~~ in MSM with one Higgs doublet does not leads to the unification. But if one adds 5 additional Higgs doublets unification ~~[illegible]~~ can be fitted at $M_G = 10^{14}$ GeV.

② In MSSM with 2 Higgs doublets unification occures at $M_G = 10^{16.0 \pm 0.3}$ GeV. There is no problem with proton decay. Since

$$\tau_p \approx \frac{1}{\alpha^2_{GUT}} \frac{M_X^4}{M_p^5} = 10^{33.2 \pm 1.2} \text{ yr}.$$

~~M~~ Values of M_{SUSY} M_G α_G^{-1} are correlated

$$M_{SUSY} = 10^{3.0 \pm 1.0} \text{ GeV}$$

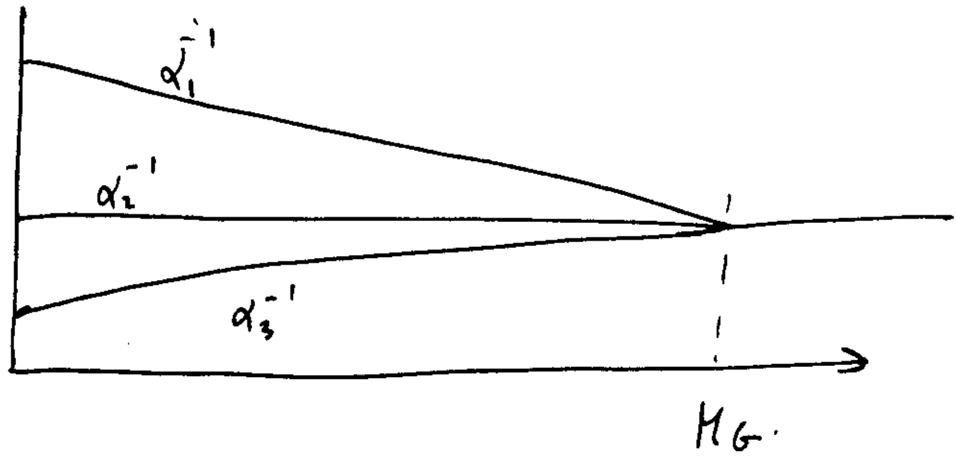
$$M_{GUT} = 10^{16.0 \pm 0.3} \text{ GeV}$$

$$\alpha^{-1}_{GUT} = 25.7 \pm 1.7$$

The best fit is for $M_S = 10^{3.0 \pm 1.0}$ GeV (error is due to uncertainties in α_s).

$$\sin^2\theta_{\overline{MS}} = 0.2336 \pm 0.0018$$

In SSSM with pair of Higgs doublets the couplings ~~must~~ get unified at ~~one point~~ $M_G = 10^{16.1 \pm 0.3}$ GeV. $\alpha_G^{-1} = 25.7 \pm 1.7$.



The skeptical view: the unification of couplings can be just an accident.

~~However, ...~~

However it is very likely that unification phenomena ~~provides a hint that~~ indicates ~~something nontrivial~~ something nontrivial happening at the scale M_G.

Namely, ~~crossing of the couplings may indicate~~ equality of the couplings $\alpha_i(M_G)$ at M_G, can have deep symmetric reason: three interaction are unified within the common gauge symmetry

group G ~~that~~ which containes ~~$SU(3)\otimes SU(2)\otimes U(1)$~~
$G_W = SU(3)_C \otimes SU(2)_L \otimes U(1)$ as an subgroup.
The symplest and straightforward candidate is $G = SU(5)$ which is a minimal simple group that containes $SU(3)_C \otimes SU(2)_L \otimes U(1)$ as an ~~subgroup~~ maximal subgroup.

Some Group Theory for model building:

The $SU(5)$-group can be defined as the group of all possible unitary, unimodular (det = 1) transformations

$$\varphi \to U\varphi$$

in the five dimentional complex space. ~~The vectors~~

~~this five dimentional complex spinors~~

The vectors which form this space are five dimentional complex spinors.

$$\varphi_\alpha = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \vdots \\ \varphi_5 \end{pmatrix} \qquad \alpha = 1, 2 .. 5$$

and they form the 5-dimentional fundamental representation of $SU(5)$. In this space the elements of $SU(5)$ are represented by

5x5 unitary matrices. $U^\dagger U = 1$ with

det $U = 1$.

The conjugate spinors $\phi^{+\alpha}$ define antifundamental representation ~~of~~ $\bar{5}$. Higher representations of SU(5) ~~we~~ can be extracted from the direct products of 5 and $\bar{5}$.

For example.

$$\bar{5}_i \otimes \bar{5}_k = 10_{[ik]} + 15_{\{ik\}}.$$

$$5_i \bar{5}^k = 24_i{}^k + 1.$$

Embedding of $SU(3)_c \otimes SU(2)_L \otimes U(1) \subset SU(5)$.

$$\begin{pmatrix} & & \\ & SU(5) & \\ & & \end{pmatrix} \supset \left(\begin{array}{c|c} SU(3)_c & 0 \\ \hline 0 & \begin{matrix}1 & \\ & 1\end{matrix} \end{array}\right) + \left(\begin{array}{c|c} \begin{matrix}1 & & \\ & 1 & \\ & & 1\end{matrix} & 0 \\ \hline 0 & SU(2)_L \end{array}\right) + \cancel{U(1)_Y}$$

$$+ \; U(1)_Y = \begin{bmatrix} e^{i2\theta} & & & & \\ & e^{i2\theta} & & 0 & \\ & & e^{i2\theta} & & \\ & 0 & & e^{-i3\theta} & \\ & & & & e^{-i3\theta} \end{bmatrix} \quad \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix} c$$

this is the Y-generator of $U(1)_Y$ phase transformations.

decomposition of the SU(5) representations into G_W ones.

$\bar{5}^i$ $\quad A = i$ (for $i = 1,2,3$) $\leftarrow$ color.
$\quad a = \cancel{i\text{-}4}$ (for $i = 4,5$) $\leftarrow$ $SU(2)_L$.

[redacted]

$$\bar{5} = \begin{pmatrix} \bar{5}^1 \\ \bar{5}^2 \\ \bar{5}^3 \\ \hline \bar{5}^4 \\ \bar{5}^5 \end{pmatrix} \begin{array}{l} \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} \text{color triplet.} \\ \left.\begin{array}{c} \\ \\ \end{array}\right\} \leftarrow \text{weak doublet.} \end{array}$$

[redacted]

$$\bar{5} = \left(\bar{3}, 1, \tfrac{1}{3}\right) + \left(1, \bar{2}, -\tfrac{1}{2}\right)$$
$$\uparrow d_c^A \qquad\qquad L_a^a$$

$$10_{ik} = \left[\begin{array}{c|c} u_c & Q \\ \hline - & e^c \end{array}\right] \begin{array}{l} \leftarrow (3.2) \\ \leftarrow (1.1) \end{array}$$

$$\left[\begin{array}{ccc|cc} 0 & u_c^1 & u_c^2 & u^1 & d^1 \\ & 0 & u_c^3 & u^2 & d^2 \\ & & 0 & u^3 & d^3 \\ \hline & & & 0 & -e_c \\ & & & e_c & 0 \end{array}\right]$$

$$[3 \times 3] = \frac{3(3-1)}{2} = \bar{3}.$$

[redacted]

$$10 = \left(\bar{3}, 1, -\tfrac{2}{3}\right) + \left(3, 2, \tfrac{1}{6}\right) + (1, 1, 1).$$
$$\uparrow u_c^A \qquad\qquad Q_{A,a} \qquad\qquad \uparrow e_c^a$$

Gluons

$$24_i^{\,K} = \left[\begin{array}{ccc|cc} & & & X & Y \\ & G_\mu^a & & X & Y \\ & & & X & Y \\ \hline X & X & X & \frac{W^3}{\sqrt{2}} & W^+ \\ Y & Y & Y & W^- & -\frac{W^3}{\sqrt{2}} \end{array}\right] + \frac{B}{\sqrt{30}} \begin{bmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{bmatrix}$$

$$24_i^{\,K} = (3\times\bar{3})_A^B + (3\times\bar{2})_A^a + (\bar{3}\times 2)_a^A +$$

$$+ C\left(\delta_A^B 2 - \delta_a^b 3\right) \Longrightarrow$$

$$\Downarrow$$

$$C\begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}$$

$$24 = (8.1.0) + (3.\bar{2}.-5) + (\bar{3}.2.5) + (1.1.0)$$

X, Y - bosons

X, Y - bosons are the ones which can convert quark-into -quark and quark-into lepton. So their exchange can mediate barion number violating interaction and in particular the proton decay.

The coupling of the X and Y bosons with the matter fields can be easily obtained from the $G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ - invariant decomposition of the gauge covariant derivatives of the matter fermions 5^i 10^{ik} ... In the covariant derivatives gauge field are coupled in the way

$$\overline{10}^{ik} A^a T^{a\,mn}_{ik} 10_{mn} = A^a \left(\overline{10}^{ik}\left(\delta_i^m T^{an}_k + \delta_k^n T^{am}_i\right) 10_{mn}\right) =$$

$$= A^a\left(\overline{10}^{ik} T^{an}_k 10_{in} + \overline{10}^{ik} T^{am}_i 10_{mk}\right) =$$

$$= A^a\left(-\overline{10}^{ik} T^{an}_k 10_{ni} - \overline{10}^{ki} T^{am}_i 10_{mk}\right) =$$

$$= -2\, \overline{10}^{ik} A^a T^{an}_k 10_{ni} = -2\, \mathrm{tr}\, \overline{10} A^a T^a 10 =$$

$$= g\, \mathrm{tr}\, \overline{10} \gamma_\mu G^\mu 10 .$$

Couplings are:

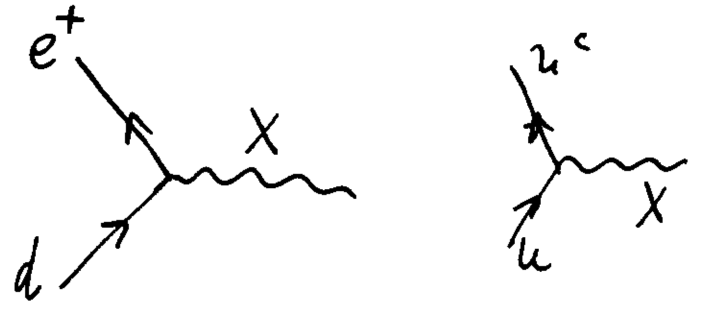
$$g\, \mathrm{tr}\left[\overline{10} \gamma_\mu G^\mu 10\right] + g\, \overline{5} \gamma_\mu G^\mu 5$$

where G_μ

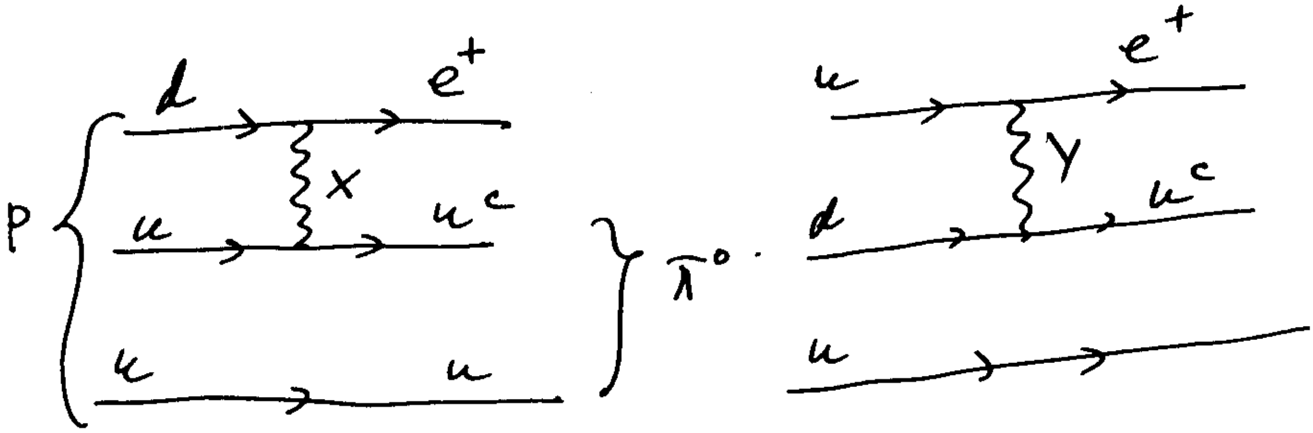
$$\begin{array}{c} \\ G_\mu = \end{array} \left[\begin{array}{c|c} \cancel{g^{\,B}_{\mu A}} & \overbrace{\begin{matrix} \bar{X}^A_\mu & \bar{Y}^A_\mu \end{matrix}}^{SU(2)_L} \Bigg\} SU(3)_c \\ \hline \begin{matrix} X^A_\mu \\ Y^A_\mu \end{matrix} & \begin{matrix} \frac{W^3_\mu}{\sqrt{2}} & W^+ \\ W^- & -\frac{W_3}{\sqrt{2}} \end{matrix} \end{array}\right] \qquad A, B = 1,2,3 \quad a = 1,2,$$

$$G_\mu = \left[\begin{array}{c|cc} & \bar{X}^1 & \bar{Y}^1 \\ \text{gluons}\; g^{\,A}_{\mu\, B} & \bar{X}^2 & \bar{Y}^2 \\ & \bar{X}^3 & \bar{Y}^3 \\ \hline X_1 \quad X_2 \quad X_3 & \frac{W^3_\mu}{\sqrt{2}} & W^+ \\ Y_1 \quad Y_2 \quad Y_3. & W^- & -\frac{W^3_\mu}{\sqrt{2}} \end{array}\right] \Bigg\}\,\text{Color} + \frac{B_\mu}{\sqrt{30}} \begin{bmatrix} 2 & & & & 0 \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ 0 & & & & -3 \end{bmatrix}$$

Decomposing this coupling, we can find that X_μ and Y_μ boson have following lepto-quark and diquark ~~vertex~~ interaction



So this bosons can mediate proton decay trough the diagrams

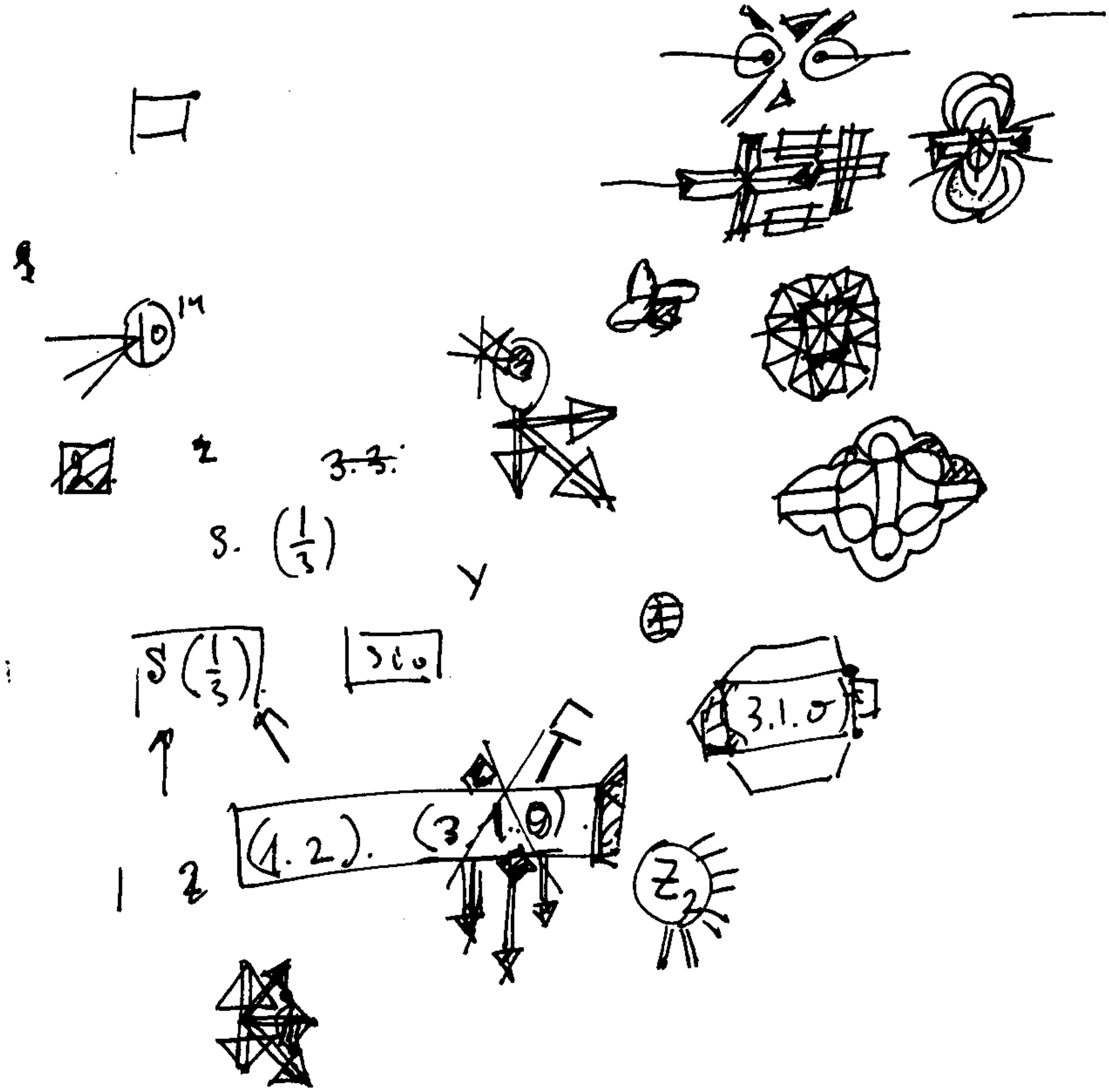


this diagrams give four fermion interaction of the strenght

$$\alpha_{GUT} / M_X^2$$

τ_p and the proton lifetime is about

$$t_p \sim \frac{1}{\alpha_G^2} \frac{M_X^4}{m_p^5} \sim \begin{array}{l} 10^{38} \text{ yr for } M_X \sim 10^{16} \text{ GeV.} \\ 10^{30} \text{ yr for } M_X \sim 10^{14} \text{ GeV.} \end{array}$$


10 14
3.3.
S. (1/3)
Y
S (1/3)
310
(3.1.0)
(1.2). (3.1.0).
1 2
Z2

General rule:

Consider the generator of $SU(5)$ which in the fundamental representation ~~[illegible]~~ is given by 5x5 matrix T_i^k and act on 5_i as $(T5)_i = T_i^k 5_k$

Then in the representations contained in the ~~prod~~ direct product

$$5_i \times 5_k = 10_{[ik]} + 15_{\{ik\}}$$

it will have a form:

$$T_{ik}^{mn} = \delta_i^m T_k^n + \delta_k^n T_i^m$$

and for

$$5_i \times \bar{5}^k$$

$$\delta_i^k T$$

$$(\delta_m^k T_i^n - \delta_i^n T_m^k)$$

this rule one can easily obtain from the performing of the infinitesimal rotation in $5_i \times 5_k$ and $5_i \otimes \bar{5}^k$ spaces.

$$U(\theta T)(5_i \otimes \bar{5}^k) = (U5_i)_i \otimes (\bar{5}U^+)_k =$$

$$U = \exp(i\varepsilon T) = (1+i\varepsilon T)5 \otimes \bar{5}(1-i\varepsilon T) = 8\varepsilon$$

$$(1+i\varepsilon T)5\otimes \bar{5}(1-i\varepsilon T) =$$

$$= 5\otimes\bar{5} + i\varepsilon\left(T5\otimes\bar{5} - 5\otimes\bar{5}T\right) =$$

$$T_{5\otimes\bar{5}} = (T\otimes 1 - 1\otimes T).$$

Using this rule we can obtain the ~~quantum numbers~~ of Y hypercharges of the different fragments of 24, 10.

Supersymmetry

With supersymmetry all this particles ~~appropria~~ aquire appropriate superpartners: bosons - fermions and fermions bosons.

The superpatners are precisely in the [~~...~~] same representations of the internal gauge group as their „usual" conterparts.

~~so that~~ ~~Matter and Higgs multiplets~~

Matter fermion (quarks and leptons) aquire scalar patners (squarks and sleptons) and the Higgs scalars aquire fermionic conterparts Higgsinos.

All this particles ~~to~~ being paired with their superpartners now form the ~~[illegible]~~ irreducible representations under Gauge symmetry × SUSY. This are the chiral superfields:

~~[illegible]~~

~~[illegible]~~

$\widehat{quark} = (sq, q)$

$\widehat{Higgs} = (sq H, \tilde{H}iggsino)$.

~~[illegible]~~ All chiral superfields are bosons. So in SUSY theory Higgses and matter multiplets enter on the equal **<u>Bases</u>**.

[upset of the fact that matter superfields should be in the chiral representation under SU(5) group ⇒ they are forbidden to have gauge invariant masses].

In the similar way the Gauge fields require fermionic partners (gauginos) and form the vector supermultiplets.

Now let us consider the ~~supersymmetric~~ minimal ~~sy~~ supersymmetric ~~of~~ SU(5).

The Higgs and matter ~~multi~~ superfields are:

Higgses. $\Sigma_i^k \equiv 24$

$H_i + \bar{H}^i \equiv 5 + \bar{5}$

matter $10^a + \bar{5}^a$ $\quad (a = 1, 2, 3$ - family index$)$

Let us ~~ah~~ write the most general renormalizable (up to a ~~~~) ~~~~ cubic ~~~~ superpotential SU(5)-symmetric ~~~~ of this superfields.

Rule:

~~~~ Out of the three irreps. $R_1 R_2 R_3$ one can construct invariant only if the direct product of the two contains conjugated of the third one:

e.g. $R_1 \otimes R_2 = \dots \overline{R_3} + \dots$

Using this rule and decomposition of the ~~~~ The direct products:
~~~~

$\bar{5}\times\bar{5} = \overline{10}_a + \overline{15}_s$ $\qquad$ $10\times 10 = \bar{5}_s + 45_a + 50_s$

$5\times\bar{5} = 24 + 1$ $\qquad$ $10\times 5 = \overline{10} + 40$

$\bar{5}\cdot 10 = \bar{5} + 45$ $\qquad$ $24\times 24 = 1_s + 24_s + \cdots$

This superpotential has the form:

$$W = \frac{M}{2}\,\mathrm{tr}\,\Sigma^2 + \frac{h}{3}\,\mathrm{tr}\,\Sigma^3 + \lambda \bar{H}\Sigma H + m\bar{H}H +$$

$$+\; g^u_{\alpha\beta}\, H\cdot 10^\alpha 10^\beta + g^d_{\alpha\beta}\,\bar{H}\cdot 10^\alpha \bar{5}^\beta +$$

$$+\left[\underbrace{m'_\alpha \bar{5}^\alpha H}_{1} + \underbrace{\lambda'_\alpha \bar{5}^\alpha \Sigma H}_{2} + \underbrace{g_{\gamma[\alpha\beta]}\, \bar{5}^\alpha 10^\gamma \bar{5}^\beta}_{3}\right]$$

$g_{\gamma[\alpha\beta]} = -g_{\gamma[\beta\alpha]}$ since $\bar{5}\times\bar{5} = \overline{10}_a + \cdots$

coupling 2 can be eliminated by means of redefinition of $\bar{H}$ and $\bar{5}^\alpha$ $\Rightarrow$ we can choose $\bar{H}' = (\lambda\bar{H} + \lambda'_\alpha \bar{5}^\alpha)\dfrac{1}{\sqrt{\lambda^2 + \lambda'^2_\alpha}}$

Coupling 3 $\equiv \bar{5}^\alpha 10^\gamma \bar{5}^\beta$ is <u>disaster</u>!

It will lead to the proton decay throug the operator dimention $d = 6$.

To see this let us decompose coupling 3 into $G_W = SU(3)_c \otimes SU(2)_L \otimes U(1)$ - invariant peices.

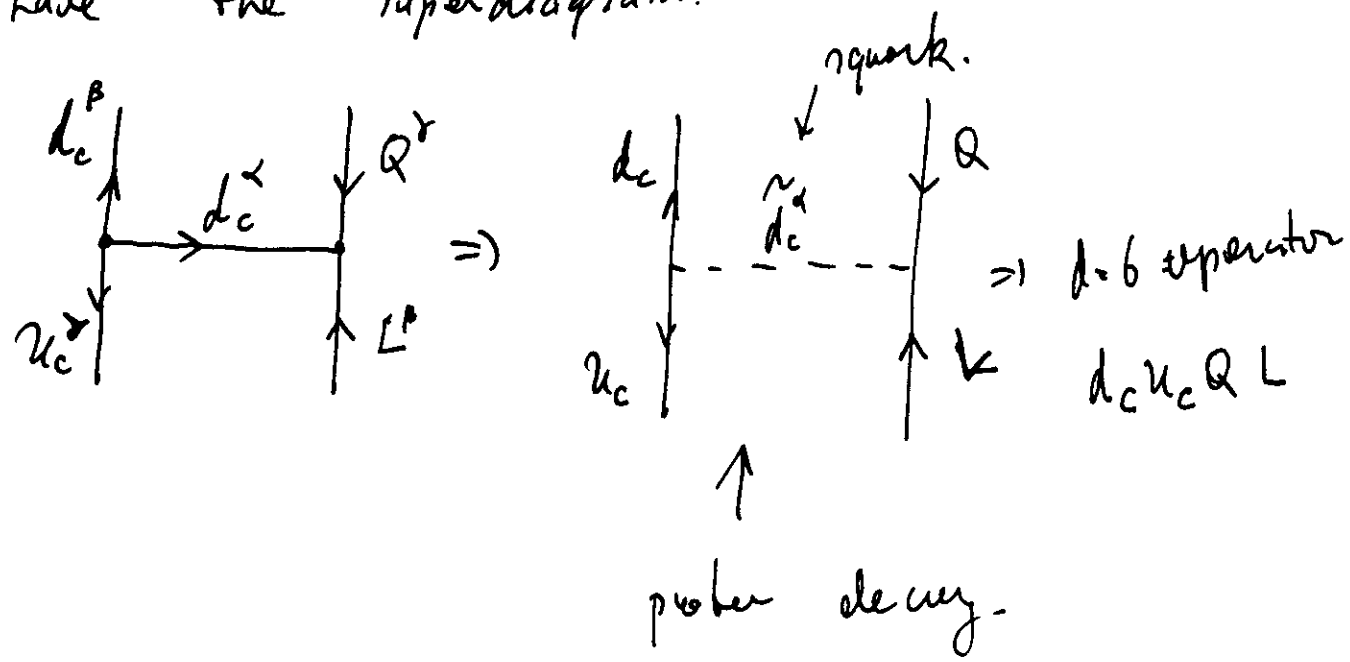
$$\bar{5}^*_\alpha 10^\gamma \bar{5}_\beta =$$

$$= d^A_{c,\alpha} 10^\gamma_{A,K} \bar{5}^K_\beta = \underbrace{d^A_{c,\alpha} u^\gamma_{c,AB} d^B_{c,\beta}}_{K=B} + \underbrace{d^A_{c,\alpha} Q^\gamma_{c,Aa} L^a_\beta}_{K=a} =$$

$$= d_c^\alpha u_c^\gamma d_c^\beta + d_c^\alpha Q^\gamma L^\beta$$

$\alpha \neq \beta$

So superfield d_c^α ~~[illegible]~~ can mediate ~~[illegible]~~ quark-quark and quark-lepton transitions simultaneously. So we have the superdiagram:



So trilinears of the matter fields should be forbidden,

Way out?

Matter parity Matter $\to$ $-$ (Matter)

Higgs $\to$ $+$ (Higgs).

So the ~~theory is~~ symmetry should be $SU(5) \otimes$ (matter parity).

And the superpotential is the one without the terms in [ ] - braket.

Now let us discuss the vacuum of the theory in the unbroken SUSY limit.
For this we have to minimize the scalar potential which as ~~the~~ we know has the form

$$V = \sum_{\alpha} |D^\alpha|^2 + \sum_i \left|\frac{\partial W}{\partial z_i}\right|^2$$

~~where~~ $D^\alpha = \sum_R R^+ T_R^\alpha R$ where ~~R~~

~~R are all the irreducible scalar~~
where R - runs over the all irreducible ~~p~~ representations of the Gauge group ~~which~~ (SU(5) in our case) ~~which contain~~ that are presented in the theory, and T_R^α are the generators in the representation R.

z_i - runs over the all scalar ~~posit~~ components of the chiral superfields.

~~Therefore the supersymmetric minimum~~

~~Of~~ Since all terms in V are positive, the SUSY minimum is defined by condition.

$$D^{\alpha} = 0 \quad \text{for } \alpha = 1, \ldots 24.$$

$$\frac{\partial W}{\partial z_i} = 0.$$

~~First of all~~ ~~we~~ note

Let us check whether our theory has an ~~f~~ "acceptable" vacuum in which gauge symmetry is ~~f~~ broken down G_W.

For this let's note that since matter field albums enter in W ~~as~~ in bilinear combinations (due to matter parity) the Eq $\frac{\partial W}{\partial z_i} = 0$ Always has solution with all matter scalars $= 0$. Farthermore the similar conclusion can be made for $H, \bar{H}$ (since the only terms they enter linearly are those with matter fields).

~~Thus~~ these equation ~~to~~ always have solutions with $\cancel{\frac{\partial w}{\partial \Sigma}}$ $H = \overline{H} =$ ~~matter~~ $\overline{5}^a = 10^a = 0$.

Note that ~~of cour~~ in general there are many other solutions with ~~H~~ this fields nonzero; but they do not correspond to the „right" ~~minimum~~ vacuum.

Now keeping $H = \overline{H} = \overline{5}^c = 10^c = 0$ we are left only with Σ-field and the vacuum condition become.

$$D^\alpha = [\Sigma^+ \Sigma]_i^k = 0$$

$$\frac{\partial w}{\partial \Sigma_k^i} = M \Sigma_i^k + h \left[\Sigma^2{}_i^k - \delta_i^k \frac{\mathrm{tr}\, \Sigma^2}{5} \right] = 0$$

~~this is the tag~~

we have extracted the trace in order to explicitly take into account the condition ~~Σ~~ $\mathrm{tr}\, \Sigma = 0$:

~~Harvey~~ As we know Σ transform under 24-representation which usually can be represented as an real (hermitian) 5×5 matric (with zero trace). ~~However, here~~

As it is well known such matric can be brought into the diagonal form by means of SU(5) - transformation.

$$U^{+} \Sigma U = \Sigma_{diagonal} .$$

However in SUSY case Σ is complex field! $\Sigma_i^{\,k} = A_i^{\,k} + i B_i^{\,k}$ where A and B are its hermitian and antihermitian parts respectively. (this is the price one pais for SUSY). So we can ~~note~~ ~~assume~~ ~~$\Sigma = \Sigma_{diag}$~~ assume that ~~Σ is~~ $\Sigma = \Sigma_{diag}$ only if $[A, B] = 0$. But this is given by $D^{\alpha} = 0$ condition!

So we assume

$$\Sigma_{\alpha} = diag(\Sigma_1 \Sigma_2 .. \Sigma_5).$$

Equation then becomes.

$$\frac{\partial u}{\partial \Sigma_{\alpha}} : M\Sigma_{\alpha} + \lambda\left(\Sigma_{\alpha}^{2} - \frac{tr\Sigma^{2}}{5}\right) = 0$$

We see that each Σ_α $(\alpha = 1 \ldots 5)$ satisfies one and the same <u>quadratic</u> equation $\Rightarrow$ So in ANY vacuum there can be ~~at the~~ only two Σ_α's different.

Since different $\Sigma_\alpha \neq \Sigma_\beta$ should satisfy

$$\Sigma_\alpha + \Sigma_\beta = -\frac{M}{h}.$$

and also we have another ~~to~~ $\operatorname{tr}\Sigma = 0$ condition

$$n\Sigma_\alpha + (5-n)\Sigma_\beta = 0.$$

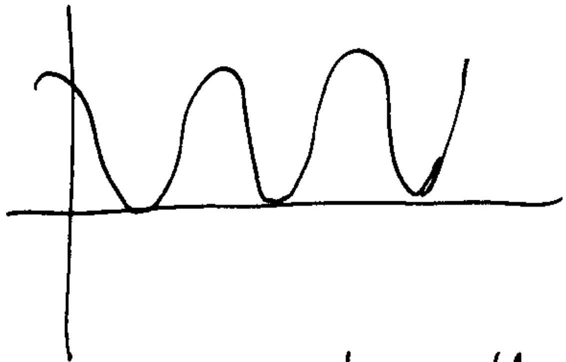
We immidiately arrive to three possible minimas.

$\Sigma = 0, \qquad SU(5)$

$$\Sigma = \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix} \frac{M}{h}.$$

$$\Sigma = \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & -4 \end{pmatrix} \frac{M}{3h}$$

All of them are discretely degenerate in the exact ~~SUPERSYM~~ SUS supersymetric limit. ~~After~~ However this degeneracy will be ~~spoiled after~~



lifted after supersymmetry breaking. And one should arrange the theory in such a way that our minimum, $\underbrace{\text{is the lowest one.}}$

$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ - symmetric

Now let us discuss the ~~phys~~ situation in the right ~~minimum.~~

~~First off all gauge bosons ~~ X, Y~~

First off all the gauge bosons that do not commute with the ~~supersymmetry~~ VEV. of Σ. (and their fermionic partners) will get masses of order $gM_G \sim g\frac{M}{h}$.

This are the superfields $\hat{X}, \hat{Y}$.

Fermions

~~Matter multiplets are decouple from~~

Since $SU(2)_L \otimes U(1)_C$ is unbroken the matter fermions can not get masses at this stage. Due to SUSY their superpartners (squarks, sleptons) stay <u>exactly</u> massless too.

Let us discuss the masses of the Higgs 5-plets. Supersymmetric mass matrix has the form.

$$\frac{\partial^2 W}{\partial H_i \partial \bar H^k} = \lambda \Sigma_i^k + \delta_i^k m =$$

$$M_{H_i \bar H^k} = \frac{\partial^2 W}{\partial H_i \partial \bar H^k} = \lambda \langle \Sigma_i^k \rangle + \delta_i^k m =$$

$$= m \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix} + \lambda \frac{M}{h} \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}$$

~~triplet~~

~~Triplet $M_T = m + 2\frac{\lambda}{h}M$~~

~~M_D~~

$$M_{triplet} = m + 2\frac{\lambda}{h}M$$

$$M_{doublet} = m - 3\frac{\lambda}{h}M.$$

To get a light doublet we have to fine tune the parameters. (hierarchy problem)

$$m - 3\frac{\lambda}{h}M = 0$$

$$\frac{M}{h} \sim M_G. \Rightarrow \quad m \sim M_G.$$

Then the Triplet mass becomes.

$$M_{Triplet} = 5\frac{\lambda}{h}M = 5\lambda M_G.$$

Triplet is superheavy! And this is good. because the triplet mediates proton decay.

~~Proton decay through the Higgs triplet~~

Higgs triplet superfield mediated proton decay.

Scalar channel ($d = 6$ operator).

$\hat{H}_T = (\bar{T}, \tilde{T})$.

coupling of $\hat{H}_T$ with quark and lepton superfields.

$$\bar{H}^i 10_{ik} \bar{5}^k = \bar{H}_T^A 10_{AB} \bar{5}^B + \bar{H}_T^A 10_{Aa} \bar{5}^a =$$

$$= \bar{T}^a u_c^b d_c^k \epsilon_{abk} + \bar{T}^A Q_{Aa} L^a$$

(color indexes are antisymmetrized!)

$$H_i 10_{km} 10_{ns} \epsilon^{ikmns} = T_A 10_{km} 10_{ns} \epsilon^{Akmns} =$$

$$= T_A u_c^A e^c + T_A Q_{B,a} Q_{E,e} \epsilon^{ABE} \epsilon^{ae} =$$

$$= T u_c e^c + T Q Q$$

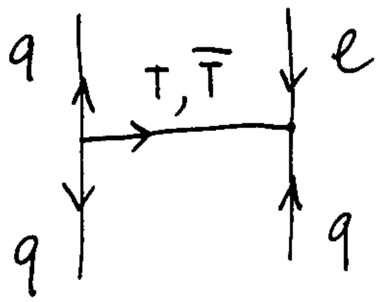
So couplings of the „Higgs" triplet superfields are

$\bar{T} u_c d_c + \bar{T} Q L$

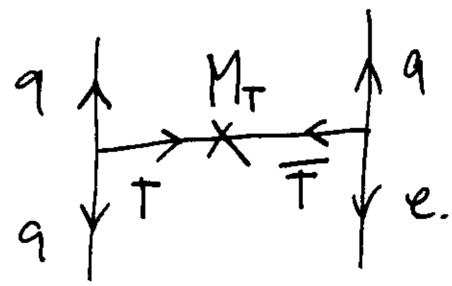
$T u_c e^c + T Q Q$

So we will have two ~~tipe~~ class of the superdiagrams that violate barion number (and are relevant for the proton decay).

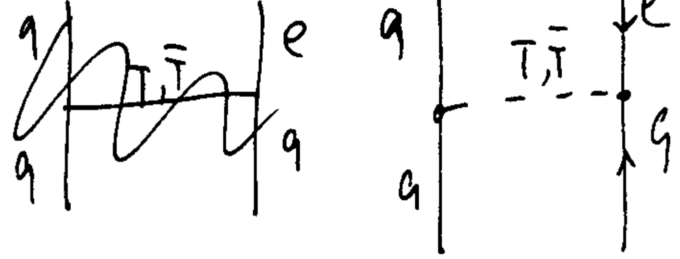
(6)



(5)



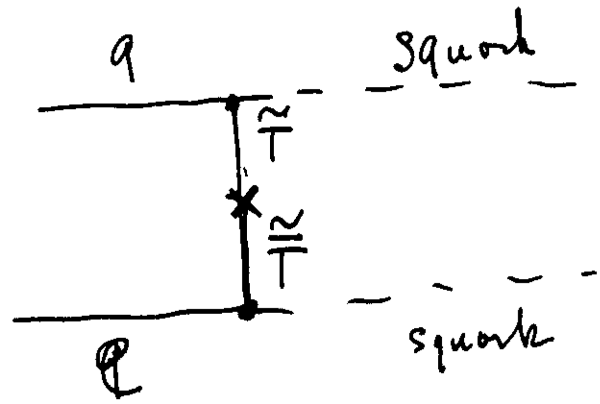
This are superdiagrams (with superfields propagating in the lines). To be relevant for proton decay we have to fixe outer legs to be fermions.



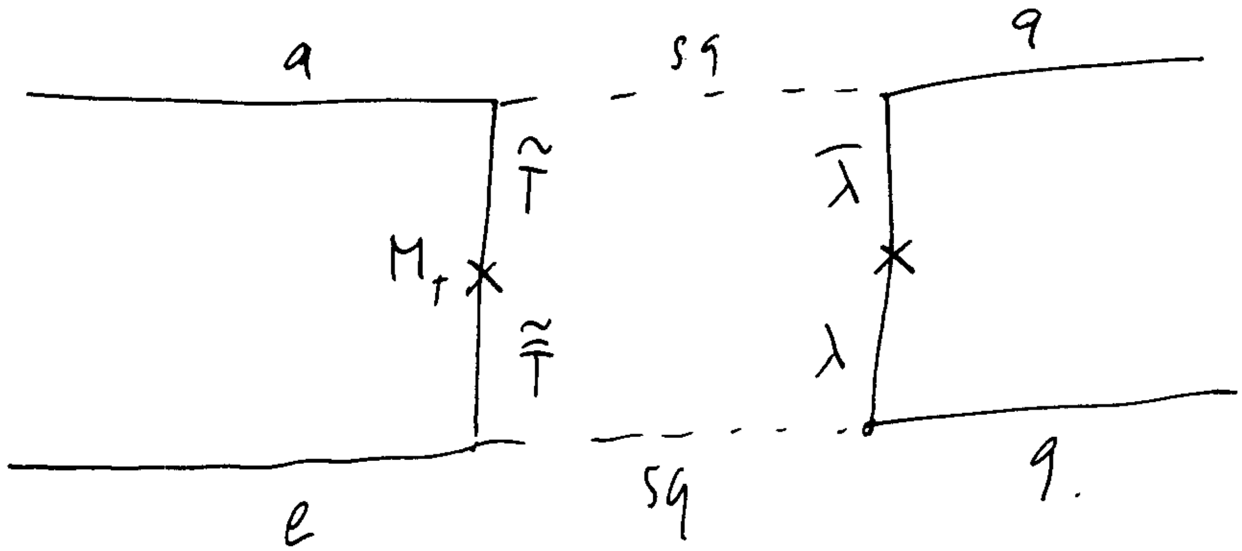
$\Rightarrow \sim \frac{g^2}{M_T^2}[qqql]$

$d=6$.

$d=5$.



~~to treat~~ |
to obtain the four fermionic operator
[qqqe) ~~want to~~ relevant for the proton decay
we have to ~~dress~~ ~~one more~~ exchange between
outcoming squarks some supersymmetric
particles (gaugino) in order to convert
them back to the normal states.



So $\dim = 5$ operators are suppressed only ~~with~~ by one inverse M_G^* scale and ~~are~~ ~~may produce a problem~~. can cause trouble.

~~How to suppress~~

How we can suppress the $\dim = 5$ operators? As we have seen the source of this operators are in couplings:

$\bar{H} \cdot \bar{5}\, 10 + H \cdot 10 \cdot 10 +$ (the large mass term for triplets) $M_T T \bar{T}$.

(1) The straightforward possibility is to suppress the direct mass term $M_T T\bar{T}$. ~~However~~ between the triplet ~~T and anti~~ T and antitriplet $\bar{T}$ which are coupled to the ~~quark~~ matter fermions. However in the same time we still keep them superheavy. This means that we should double Higgs $\bar{5}$-plet representation and introduce another pair $H, \bar{H}$ + $H' \bar{H}'$ so that only $H \bar{H}$ are

coupled to the fermions but they get masses through the ~~so~~ mixing with $\bar{H}'$ and H' respectively. So the relevant part of the potential now becomes:

$$\lambda \bar{H} \Sigma H' + \lambda' \bar{H}' \Sigma H + m \bar{H} H' + m' \bar{H}' H$$
$$+ \bar{H} \cdot 10^{\alpha} \bar{5}^{\beta} + H \cdot 10^{\alpha} \cdot 10^{\beta}.$$

The important point is that now there is no direct mass term between T and $\bar{T}$ and there is no chirality flipp in the $d=5$ operator.

However this approach has the number of problems (e.g. there are two pair of light doublets).

other terms are forbidden by discrete symmetry e.g. $(H, \bar{H}') \to -(H, \bar{H}')$.

② Another possibility is more radical.

Question: ~~Why the Higgs triplet should be heavy~~?

Does the proton stability necessarily requires the Higgs triplet to be heavy?

<u>Answer</u>: <u>No</u>! (if it is decoupled from quarks and leptons).

Let us assume that there is some discrete symmetry $\bar{H} \to -\bar{H}$, $\bar{5}^{\alpha} \to -\bar{5}^{\alpha}$,

All other fields are invariant:

Then the term

$\cancel{\lambda \Sigma}\ \lambda \bar{H} \Sigma H + M \bar{H} H$ are forbidden

and $\bar{H}, H$ - are ~~th~~ massless and decouple from superlarge VEVs. (so there is no ~~d-t~~ d-t splitting at all!)

~~Then proton is unstable.~~

~~Matter~~ If the only couplings between $H, \bar{H}$ and matter are

$\bar{H} \cdot \bar{5} \cdot 10 + H \cdot 10 \cdot 10$

There is disaster!

However this is no longer so if there ~~are~~ is some new physics between M_G and M_P. This new physics can ~~induce~~ induce effective operators of the form..

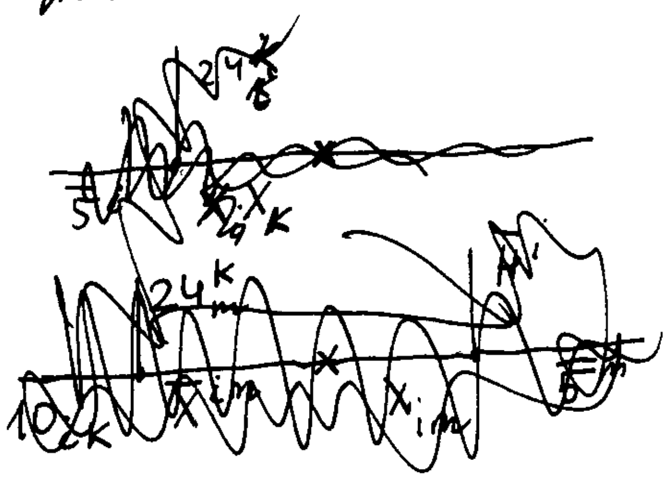
$$\frac{\overline{H}\cdot 24\cdot \overline{10}\cdot \overline{5}}{M} + \frac{H\,24\cdot 10\cdot 10}{M}$$

where M is some superlarge regulator scale ~~...~~ comparable with M_P.

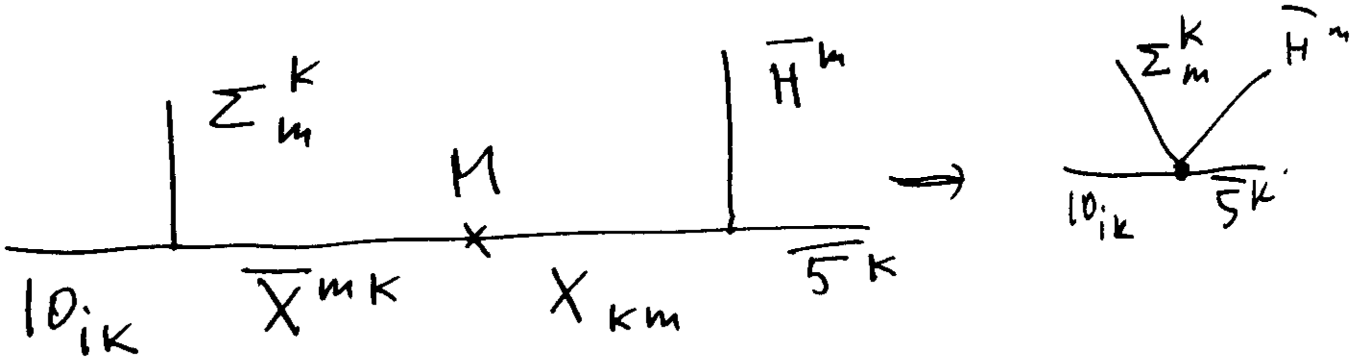
① ~~Some people~~ One (very unclear) possibility is that the ~~to~~ nonrenormalizable couplings like this can be induced by gravity. In fact there is some believe (but just believe) that gravity in general induces all possible effective couplings compatible with gauge symmetries.

②. However we will be more explicit and show that this ~~a~~ type operator can be induced (even without gravity) if there are some heavy (with masses $\sim M_G$) particles. The exchange of this particles can lead to an interaction which below M_0 will look like one we have presented above.

For example assume that there is heavy pair of multiplets $X_i \bar{X}^i$ $(i=1,..5)$ in 10, $\overline{10}$ - plet representation, which couple to $\bar{H}$, 5.10 and 24 in $SU(5)$ invariant way.



$$\bar{X}^{ik}\Sigma_k^m 10_{mi} + M\bar{X}^{ik}X_{ik} + X_{ik}\bar{H}^i\bar{5}^k$$



Below M_G this exchanges induce an effective couplings.

$$\bar{H}\Sigma\, 10\, \bar{5} \; + \; H \cdot 10 \cdot 10 \,.$$

Now let us combine this couplings with the old ones and find the <u>effective</u> couplings of the Higgs doublets and triplets with matter below the GUT symmetry breaking. One get e.g for $\bar{H}$. (just one family)

$$\bar{H}^i \left(g^d \delta_i{}^k + g^\Sigma \frac{\Sigma_i{}^k}{M_X} \right) 10^\alpha_{km}\, \bar{5}^{m\beta}$$

$$\downarrow$$

$$g^d \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix} + g^\Sigma \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix} \frac{M}{hM_X}.$$

~~So if we fine tune~~

So we find that the effective Yukawa coupling of the triplet ~~is~~ $\overline{T}$ and Higgs doublet ~~are~~ (with down quarks + electrons)

$$G_T = g^d + 2\frac{M}{M_X}\frac{g_\Sigma}{h}$$

$$G_T = g^d - 3\frac{M}{M_X}\frac{g_\Sigma}{h}.$$

So if we fine tune $G_T = 0$ then there are not $d=5$ and $d=6$ operators. In $SU(5)$ this mechanism needs fine tuning. But in higher GUTs like $SO(10)$, E_6, $SU(3)^3$ it can be done ~~without~~ <u>automatically</u>!