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Lecture I

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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STANDARD BIG BANG MODEL OF COSMOLOGY

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STANDARD BIG-BANG MODEL of COSMOLOGY

For $t \gtrsim t_{Pl} \simeq 10^{-44}$ sec $(\equiv M_{Pl}^{-1})$ after the Big-Bang quantum fluct. of gravity seize to exist

→ Classical theory of gravity (Einstein's general relativity) is enough

— Strong, weak and electromagnetic interactions are described by Relativistic Quantum Field Theory (Gauge Theorie

Let us for the moment concentrate on gravity:

Einstein eqns ⟶ $R_\mu{}^\nu - \frac{1}{2}\delta_\mu{}^\nu R = 8\pi G\, T_\mu{}^\nu$

$R_\mu{}^\nu$ = Ricci tensor $(g_{\mu\nu}, \partial g_{\mu\nu})$, $R = g^{\mu\nu} R_{\mu\nu}$ = scalar curvatu

$T_\mu{}^\nu$ = energy momentum tensor for "matter"

Energy Momentum Conservation

$$T_\mu{}^\nu{}_{;\nu} = 0$$

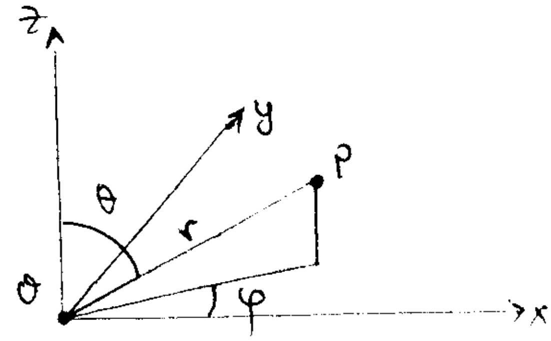
"Cosmological Principle": Universe is homogeneous and isotropic

(perhaps strongest evidence is BCR)

⟹ "Robertson-Walker" metric"

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right]$$

$r, \varphi, \theta \equiv$ "comoving polar coordinates" (fixed for objects which only follow the general exp.)



k = scalar curvature of 3-space:

$k=0$ flat universe
$k>0$ closed "
$k<0$ open "

$a(t)$ = "scale factor" of the universe $\Leftrightarrow$ descibes exp.

$\nwarrow\!\!\searrow$ dimensionless, normalization: $a(t_0) = 1$

$\Longrightarrow$ Phys. distance (istantaneous) $\quad R = a(t)\displaystyle\int_0^r \frac{dr}{(1-kr^2)^{1/2}}$

$$= a(t)\, r \quad (\text{for } k=0)$$

<u>Hubble expansion</u> (flat case):

$\vec{R} = a(t)\vec{r} \longrightarrow \quad \vec{V} = \dfrac{d\vec{R}}{dt} = \dfrac{d}{dt}\left(a(t)\,\bar{r}\right) =$

$$= \frac{\dot{a}}{a}\vec{R} + a\,\vec{v}$$

$$\vec{v} = a(t)\frac{d\bar{r}}{dt} = \text{"peculiar velocity"}$$

$\vec{v} = 0 \Longrightarrow \vec{V} = \dfrac{\dot{a}(t)}{a(t)}\vec{R} \equiv H(t)\vec{R}$ ("Hubble law") $\qquad$ <u>1^{rst} success</u>

H = Hubble parameter $\longrightarrow \quad H_0 = 100h$ km/sec Mpc
$0.4 \lesssim h \leq 1$

"most ambiguous"

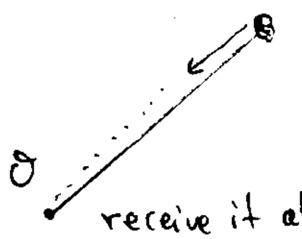
H_0 is measured by plotting velocities / distances

doppler shifting

Redshifting of light ($k=0$)

light path $\longrightarrow$ null geodesics $\longrightarrow$

$$ds^2 = 0 \rightarrow -dt = a(t)\,dr \quad \text{(radial direction)}$$



emit light at t_e, $t_e + \delta t_e$ $\qquad \delta t_e = \lambda_e$

receive it at t_o, $t_o + \delta t_o$

$$r = \int_{t_e}^{t_o} \frac{dt}{a(t)} = \int_{t_e+\delta t_e}^{t_o+\delta t_o} \frac{dt}{a(t)}$$

$$\Rightarrow \quad \frac{\delta t_e}{a(t_e)} = \frac{\delta t_o}{a(t_o)} \quad \Rightarrow \quad \frac{\lambda_e}{a(t_e)} = \frac{\lambda_o}{a(t_o)}$$

$$\text{Redshift} = z = \frac{\lambda_o - \lambda_e}{\lambda_e} \equiv \frac{\delta\lambda}{\lambda} = \frac{a(t_o)}{a(t_e)} - 1$$

$$\Longrightarrow \quad 1 + z = \frac{a(t_o)}{a(t_e)}$$ "often used"

Friedmann eqns (Einstein eqns for RW metric):

Homog. and isotropic univ. $\longrightarrow$ $T_\mu{}^\nu = \mathrm{diag}(-\rho, p, p, p)$

$\left.\begin{array}{l}\rho = \text{energy density} \\ p = \text{pressure}\end{array}\right\} \Longrightarrow T_\mu{}^\nu = (\rho + p) u_\mu u^\nu + p\,\delta_\mu{}^\nu$

$u^\nu \equiv (1, 0, 0, 0)$

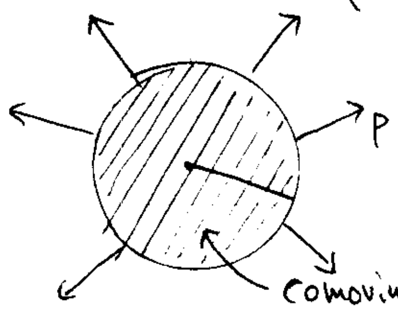
Energy-Momentum Conservation ($T_{\mu}{}^{\nu}{}_{;\nu} = 0$)

$$\longrightarrow \quad \frac{d\rho}{dt} = -3\frac{\dot{a}}{a}(\rho + p)$$

work done by pressure during exp.

dilution of energy due to exp.

$$\Longleftrightarrow \quad d\left(\frac{4\pi}{3}a^3\rho\right) = -p4\pi a^2 da$$



comoving volume with radius $\propto a(t)$

Einstein equs $\longrightarrow$ Friedmann equs:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$ $\leftrightarrow$ Not independent ($\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - k$

$$\longrightarrow \quad 2\dot{a}\ddot{a} = \frac{8\pi G}{3}\dot{\rho}a^2 + \frac{16\pi G}{3}\rho\, a\dot{a} = -8\pi G a\dot{a}(\rho + p) + \frac{16\pi G}{3}\rho a\dot{a}$$

$$\longrightarrow \quad \ddot{a} = -\frac{4\pi G}{3}a(\rho + 3p)$$)

<u>Expansion Law</u> ($a = a(t)$):

$$\rho + p = \gamma\rho \implies \dot{\rho} = -3\frac{\dot{a}}{a}\gamma\rho \Rightarrow \frac{d\rho}{\rho} = -3\gamma\frac{da}{a}$$

$$\Rightarrow \ln(\rho/\rho_0) = -3\gamma\ln(a/a_0) \Rightarrow \underline{\rho \propto a^{-3\gamma}}$$

"matter" : $p=0 \longrightarrow \gamma=1 \longrightarrow \rho \propto a^{-3}$ (dilation of a fixed no of particles in a comov. volume)

"radiation" : $p=\frac{1}{3}\rho \Rightarrow \gamma = 4/3 \rightarrow \rho \propto a^{-4}$ (extra redshifting of wave-length)

dominates in the early univ

$\rho \propto a^{-3\gamma}$ in Friedmann ($k=0$)

$$\rightarrow \frac{\dot a}{a} \propto a^{-\frac{3\gamma}{2}} \rightarrow a^{\frac{3\gamma}{2}-1} da \propto dt$$

$$\Longrightarrow a \propto t^{2/3\gamma} \quad (\gamma \neq 0) \Leftrightarrow a(t) = \left(\frac{t}{t_0}\right)^{2/3\gamma}$$

"matter" : $a(t) = (t/t_0)^{2/3}$

"radiation" : $a(t) = (t/t_0)^{1/2}$

no. of b(f) degrees of freedom.

Radiation dominated era (early univ.) : $\rho = \frac{\pi^2}{30}\left(N_b + \frac{7}{8}N_f\right)T^4 \equiv c\,T^4$

$$\text{entropy density} = s = \frac{2\pi^2}{45}\left(N_b + \frac{7}{8}N_f\right)T^3$$

Adiabatic evolution ($S = s\,a^3 = \text{const.}$)

$\rightarrow \quad aT = \text{const.}$

"Temperature-time relation during radiation dom." (F-eq, $k=0$)

$$\rightarrow \quad T^2 = \frac{M_{Pl}}{2(8\pi c/3)^{1/2}\,t}$$

$\rightarrow$ Univ. starts exp. from $t=0$ with $T=\infty$, $a=0$ ($t \geq 10^{-44}$ sec.)

Important parameters : (i) H_0

(ii) $K=0$ in F-equ. $\rightarrow$

$\rho = \rho_{crit.} = 3H^2/8\pi G \Rightarrow$ flat univ.

Define $\Omega = \rho/\rho_{crit} = 1 + \frac{k}{a^2H^2}$

$\Omega = 1 \rightarrow$ flat univ. $(k=0)$

$\Omega > 1 \rightarrow$ "closed" " $(k>0)$

$\Omega < 1 \rightarrow$ "open" " $(k<0)$

present value of Ω : $\Omega_0 = 1$ (inflation)
"Most matter is dark" " $0.1 \lesssim \Omega_0 \lesssim 0.3$ "
"Baryons 1% – 10% of ρ"

(iii) deceleration parameter :

$$q = -\left(\frac{\ddot{a}}{\dot{a}}\right) / \left(\frac{\dot{a}}{a}\right) = \frac{1}{2}\,\frac{\rho + 3p}{\rho_{crit.}}$$

For "matter" $(p=0) \rightarrow \quad q = \frac{1}{2}\Omega$

Inflation $\rightarrow q_0 = 1/2$

Particle Horizon :

The light travels only a finite distance $d_H(t)$ from big-bang $(t=0)$ to some cosmic time t :

$a(t)\, dt = dr \quad \rightarrow \quad d_H(t) = a(t) \int_0^t \frac{dt'}{a(t')}$ = particle horizon at t.

= size of the universe that we have already seen at t.

= distance at which causal contact has been established at t.

$$a(t) = \left(\frac{t}{t_0}\right)^{2/3\gamma} \Rightarrow \quad d_H(t) = \frac{3\gamma}{3\gamma-2}\, t \qquad (\gamma > 2/3)$$

$$\Rightarrow H(t) = \frac{2}{3\gamma}\, t^{-1}$$

$$\Rightarrow \qquad d_H(t) = \frac{3\gamma}{3\gamma-2}\, t = \frac{2}{3\gamma-2}\, H^{-1}(t)$$

"matter dom." $(\gamma = 1)$: $d_H(t) = 3t = 2H^{-1}(t)$

"radiation dom." $(\gamma = 4/3)$: $d_H(t) = 2t = H^{-1}(t)$

present horizon: $d_H(t_0) = 2H_0^{-1} = 6.000 h^{-1}$ Mpc

" time $t_0 = \frac{2}{3} H_0^{-1} = 6.7 \times 10^9 h^{-1}$ years

" critical dens: $\rho_{crit} = \frac{3H_0^2}{8\pi G} = 1.9 \times 10^{-29} h^{-2}$ gm/cm^3

<u>A brief history of the universe in accordance with GUTs</u>:

- Suppose we have a GUT based on G (= $SU(5)$, $SO(10)$, $SU(3)^3$, ...) SUSY or non-SUSY:
- Suppose $G \xrightarrow[M_X]{\langle\phi\rangle} G_S = SU(3)_c \times SU(2)_L \times U(1)_Y \xrightarrow[M_W]{\langle H\rangle} SU(3)_c \times U(1)$

GUTs + Standard Cosmological Model (based on classical grav.)

can describe the early Univ. for $t \gtrsim 10^{-44}$ sec.

$\Rightarrow$ A series of phase transitions during which the initial Gauge Symmetry is gradually reduced:

$t=0$ BIG BANG — G — $t \sim 10^{-37}$ s — G_S — $t \sim 10^{-10}$ s

G: A hot soup of massless q, ℓ, W, Z, gluons, X, Y, ... $\equiv$ "light"

$t \sim 10^{-37}$ s: $T \sim 10^{16}$ GeV ($\equiv 10^{28}$ K)

(i) BAU (success)
(ii) Magnetic Monopoles (+ other extended objects)

G_S: $X, Y \to$ massive

$t \sim 10^{-10}$ s: $T \sim 100$ GeV

$SU(3)_c \times U(1)_{em}$ — $t \sim 10^{-4}$ s — $t \sim 180$ s

$t \sim 10^{-4}$ s: $T \sim 1$ GeV ($= 10^{12}$ K)

Confinement (quarks $\to$ Hadrons)

$t \sim 180$ s: $T \sim 1$ MeV

"nucleosynthesis" (Abundances of light elements) 2^{nd} success

$\Updownarrow$

depends on (i) b/γ
(ii) no. of light ($m \lesssim$ MeV) particle species $\Leftarrow$ N_ν = no. of neutrino species

Agreement D, ^{3}He, ^{4}He, ^{7}Li
if $b/\gamma \cong 4 \times 10^{-10}$, $N_\nu \lesssim 3.4$

$t_{eq.} = 3.000$ yrs || "Matter dom."

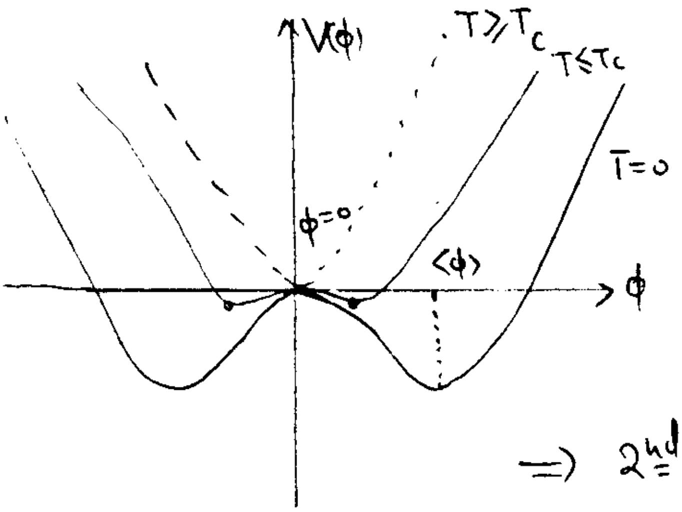
$t = 200.000 h^{-1}$ yr || $T = 3.000°K$
decoupling of matter and radiation

$t \sim 200.000.000$ yr || „Galaxy formati"

($\equiv$ recombination of atoms) $\Leftarrow$ last scattering surface.
$\downarrow$
BR ($T_0 = 2.735 \pm 0.016°K$)
3^{rd} success

GUT-transition :

$$G \xrightarrow{\langle\phi\rangle} G_S$$



temperature corrections:

$$V(\phi) \Longrightarrow V(\phi) + \sigma^2 T^2 \phi^2 + \cdot$$

$$\langle\phi\rangle(T) \simeq \langle\phi\rangle(T=0) \left(1 - \frac{T^2}{T_c^2}\right)^{1/2}$$

$$T \leq T_c$$

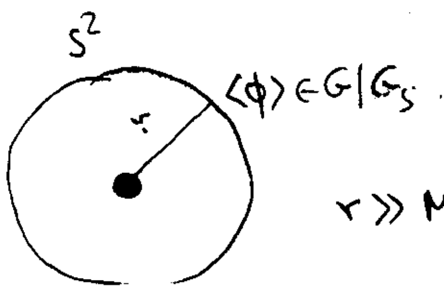
$$m_H(T) \sim h \langle\phi\rangle(T)$$

$\Longrightarrow$ 2^{nd} order phase transition

Shortcomings of Standard Model :

(α) Magnetic Monopole Problem :

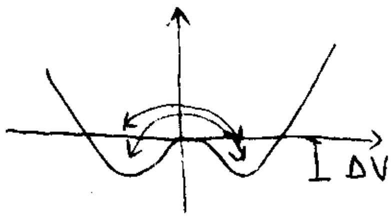
Magnetic Monopole $\equiv$ localized deviation from vacuum with radius $\sim M_X^{-1}$, energy $\sim M_X/\alpha_G$ and $\phi = 0$ at its centre



$r \gg M_X^{-1}$: $S^2 \longrightarrow G/G_S \Longrightarrow \pi_2(G/G_S)$
"homotopically non-trivial" $\longleftrightarrow$ topolog. stabili

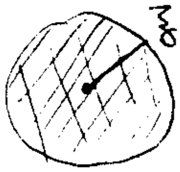
production of Monopoles:

At $T \lesssim T_c$ $\qquad \Delta V \sim h^2 \langle\phi\rangle^4$



ΔV = difference in free energy density between $\phi = 0$, $\phi = \langle\phi\rangle$

Higgs correlation length $\xi(T) = m_H^{-1}(T)$



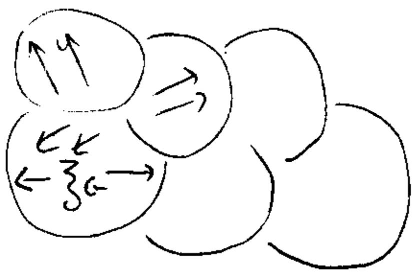
$\frac{4\pi}{3}\xi^3 \Delta V \lesssim T \rightarrow$ fluct. back and forth are allowed.

$\downarrow$

equality T_G

$\Rightarrow$ $T \lesssim T_G$ fluct. stop and $\langle\phi\rangle \in G/G_s$

At T_G:



$\xi_G \sim \frac{1}{h^2 T_c}$

$\rightarrow \quad n_M \sim p\, \xi_G^{-3} \sim p\, h^6 T_c^3 \quad \rightarrow \quad r_M = \frac{n_M}{T^3} \sim 10^{-6}$

$(p \sim 1/10)$

Causality bound: $\quad n_M \gtrsim \frac{p}{\frac{4\pi}{3}(2t_G)^3}$

$\Rightarrow \quad r_M \gtrsim 10^{-10}$

After T_G

$$\rightarrow \quad \frac{d n_M}{dt} = -D n_M^2 - 3 \frac{\dot{a}}{a} n_M$$

dilution due to expansion

"monopole-antimonopole annihilation"

Monopoles diffuse towards antimonopoles through the "plasma", capture each other in Bohr orbits $\rightarrow$ annihilate

Annihilation $\leftrightarrow$ mean free path $\lesssim$ capture distance
$\rightarrow \quad T \gtrsim 10^{12}$ GeV
After that essentially no annihilation

Final result: $\quad r_{in} \gtrsim 10^{-9} \longrightarrow r_{fin} \sim 10^{-9}$

$\qquad r_{in} \lesssim 10^{-9} \longrightarrow r_{fin} \sim r_{in}$

Causality bound $\rightarrow \quad r_{fin.} \gtrsim 10^{-10}$

"Observational Bound" from nucleosynthesis (they should not dominate)

$\rightarrow \quad r(T \simeq 1 \text{MeV}) \lesssim 10^{-19}$

$\rightarrow$ PROBLEM!!

(β) Horizon Problem:

The CBR photons we receive now where emitted at "decoupling" where $T_d = 3.000$ K

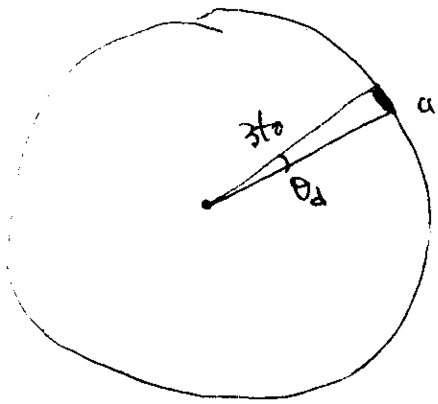
To calculate decoupling time t_d:

$$\frac{T_0}{T_d} = \frac{2.74}{3000} = \frac{a(t_d)}{a(t_0)} = \left(\frac{t_d}{t_0}\right)^{2/3}$$

$\Rightarrow \quad t_d = 200{,}000\, h^{-1}$ years

distance over which these photons traveled =

$$a(t_0)\int_{t_d}^{t_0} \frac{dt'}{a(t')} = 3t_0\left(1-\left(\frac{t_d}{t_0}\right)^{2/3}\right) \simeq 3t_0 = \text{present horizon} = 6000\, h^{-1}\,\text{Mpc}$$



horizon size at $t_d = 2H^{-1}(t_d) = 3t_d = 0.168\, h^{-1}$ Mpc

expands till now $= 0.168 \dfrac{a(t_0)}{a(t_d)} = 184\, h^{-1}$ Mpc today

θ_d = angle subtended by decoupling horizon now $= \dfrac{184}{6000} = 0.03$ rads (2°)

$\longrightarrow$ sky splits in $4\pi/(0.03)^2 = 14000$ patches that never communicated before sending light to us

$\longrightarrow$ how $\dfrac{\delta T}{T} \lesssim 10^{-4}$ $\longrightarrow$ PROBLEM!!

(γ) <u>Flatness problem</u>:

Today $\rightarrow$ $\quad 0.1\,\rho_{crit} \lesssim \rho \lesssim 2\,\rho_{crit}$.

$\uparrow$ virial theorem on galactic clusters

$\uparrow$ galactic # density at large distances $\rightarrow$ Volume exp. rate

F-eq. $H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} = \frac{8\pi G}{3}\rho_{crit}$.

$$\Rightarrow \quad \frac{\rho - \rho_{crit}}{\rho_{crit}} = \frac{3}{8\pi G \rho_{crit}} \left(\frac{k}{a^2}\right) \propto a \qquad (\rho_{crit} \propto a^{-3}$$

$\rightarrow$ in the early univ. $\frac{\rho - \rho_{crit}}{\rho_{crit}} << 1$

$\longrightarrow$ INITIAL CONDITION PROBLEM !!

(δ) For structure formation we need $\delta\rho/\rho$ at all scales with a "scale-invariant" spectrum $\leftarrow$ Where do they come fron

Also $\frac{\delta T}{T}$ observed at $\theta \gg \theta_d$

Since it comes from $\delta\rho/\rho$, how these fluct. violate causality?

Expand in plane waves:

$$\frac{\delta\rho}{\rho}(\vec{x},t) = \int d^3k\, \delta_{\vec{k}}(t)\, e^{i\vec{k}\vec{x}}$$

k = comoving wave # $\rightarrow$ $\lambda = 2\pi/k$ = comoving wave length

$\rightarrow$ $\lambda_{phys} = a(t)\lambda$

For $\lambda_{phys.} \lesssim H^{-1}$, Newtonian eq.

velocity of sou

$$\underbrace{\ddot{\delta}_{\vec{k}} + 2H\dot{\delta}_{\vec{k}}}_{} + \frac{v_s^2 k^2 \delta_{\vec{k}}}{a^2} = 4\pi G\rho\,\delta_{\vec{k}} \qquad \left(v_s^2 = \frac{dp}{d\rho}\right)$$

cosm. exp.   pressure term   gravit. attraction

Let us for the moment $H=0$

k_J = Jeans wavenumber $\Leftrightarrow$ $k_J^2 = \frac{4\pi G a^2 \rho}{v_s^2}$

$k \geq k_J$: pressure dom. $\longrightarrow$ pert. just oscillate

$k \leq k_J$: grav dom $\rightarrow$ " grow exp.

In particular, for $p=0$ (cold dark matter) $\rightarrow$ $v_s = 0$

$\rightarrow$ all scales are Jeans unstable !!

$\rightarrow$ $\delta_{\vec{k}} \propto \exp(t/\tau)$ $\quad \tau = (4\pi G\rho)^{-1/2}$

If we put $H \neq 0$ again since exp. "pulls particles apart" $\Rightarrow$

smaller growth $\delta_{\vec{k}} \propto a \propto t^{2/3}$ ("matter dom.")

"radiation dom." $\rightarrow$ no essential growth. ($p \neq 0$)

So we need $(\delta\rho/\rho)_{eq.} \sim 4 \times 10^{-5} (\Omega_0 h)^{-2}$

because the total available growth $a_0/a_{eq} \simeq 2.4 \times 10^4 (\Omega_0 h)^2$

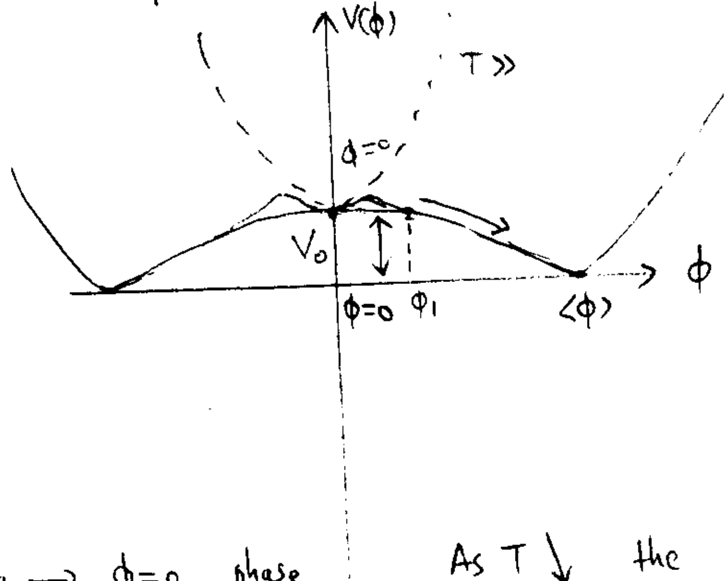
INFLATION

Inflation is idea which solves all the "cosmological puzzles"

in one go !!

The idea is the following:

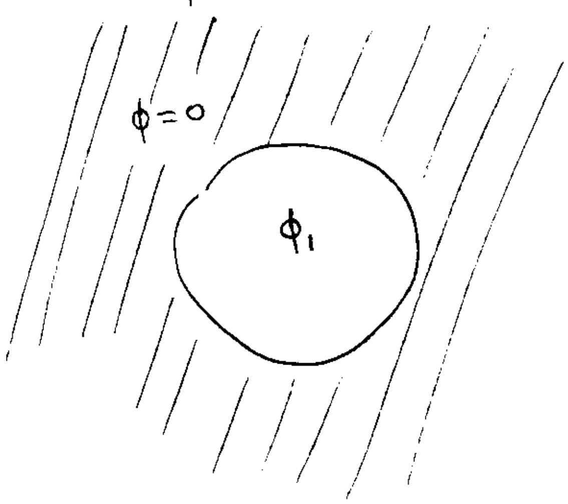
Suppose there is a scalar field ϕ = inflaton with a potential $V(\phi)$ which is quite "flat" near $\phi=0$ and has a minimum at $\phi=\langle\phi\rangle$



At T ≫ Universe → $\phi=0$ phase

As T ↓ the potential → T=0 potential but a little barrier still remains

→ At some point $\phi \xrightarrow{\text{tunnels}} \phi_1 \ll \langle\phi\rangle$ "NEW INFLATION"



and a bubble with $\phi=\phi_1$ is created in the universe ⇒

Then the field rolls over to its minimum very slowly ("flat")

⇒ $\rho \simeq V_0 = \text{const.}$ for quite some tim

Lagrangian density → $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$

$$\Rightarrow \quad T_\mu{}^\nu = -\partial_\mu\phi\partial^\nu\phi + \delta_\mu{}^\nu\left(\frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi - V(\phi)\right)$$

⇒ During the slow roll-over ⇒ $T_\mu{}^\nu \cong -V_0\,\delta_\mu{}^\nu$

$\Rightarrow \quad \rho = V_0 = -p$ (negative pressure equal in magnitude with ρ)

$\Leftrightarrow$ Consistent with $\dot\rho = -3H(\rho+p)$

$\Rightarrow \gamma = 0 \Rightarrow$ previous formulas do not hold

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}V_0 - \frac{k}{a^2}$$

$a(t)$ grows and, thus, the second term becomes subdominant

$$\Rightarrow \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}V_0 \Rightarrow a(t) = e^{Ht}, \quad H^2 = \frac{8\pi G}{3}V_0 = \text{const.}$$

$\Rightarrow$ exponential expansion of the bubble between t_i, t_f

$$\Rightarrow \frac{a(t_f)}{a(t_i)} = \exp H(t_f - t_i) = \exp H\tau$$

$H\tau = \#$ of e-folding $\Rightarrow$ With enough e-fold the cosmological problems !!

(α) <u>Horizon problem</u>

With $a(t) = e^{Ht}$ the "particle horizon" $\rightarrow$

$$d(t) = e^{Ht}\int_{t_i}^{t}\frac{dt'}{e^{Ht'}} = \frac{1}{H}e^{Ht}\left(e^{-Ht_i} - e^{-Ht}\right) \simeq H^{-1}\exp(Ht), \quad t \gg H^{-1} \gg t_i$$

So we see that the "particle horizon" grows as fast as the "scale factor" $a(t)$

$$\Rightarrow d(t_f) = H^{-1}\exp(H\tau)$$

After the end of inflation at $t = t_f$, ϕ starts oscillating around its minimum at $\phi = \langle\phi\rangle$ for some time and then decays to "radiation" with temperature $T_r (\sim 10^{10}\,GeV)$ ("reheat temperature") and then we have "normal cosmology".

The horizon $d(t_f) = H^{-1}\exp(H\tau)$ is streched during the oscillation of ϕ by some factor ($\sim 10^5$) which depends on details and between Tr and now by Tr/T_0

$$\Rightarrow \quad H^{-1} e^{H\tau}\, 10^5 \frac{Tr}{T_0} \gtrsim \frac{2}{H_0}$$ to solve the "horizon problem"

$$H^2 = \frac{8\pi G}{\rho} V_0 \cong \frac{8\pi G}{3} M_X^4 \Rightarrow \quad H\tau \gtrsim 50$$

(β) Monopole problem:
With # of e-foldings ≥ 50, the primordial monopole density is also diluted to became totally irrelevant!! (by ≥ 60 orders of magnitude)
Also, there is no thermal production of monopoles since $T \lesssim T_r \ll m_M$

(γ) Flatness problem:
We know that the "curvature term" now

$$\frac{k}{a_0^2} = \left(\frac{k}{a^2}\right)_{\text{before inf.}} \underbrace{e^{-2H\tau}}_{\text{inf.}} \quad \underbrace{10^{-10}}_{\text{oscil.}} \quad \underbrace{\left(\frac{10^{-13}\,GeV}{10^{10}\,GeV}\right)^2}_{\text{after "reheat"}}$$

Assuming $\left(\frac{k}{a^2}\right)_{b.i} \sim \frac{8\pi G}{3}\rho \sim H^2 \quad (\rho = V_0)$

$$\Rightarrow \quad \frac{k}{a_c^2 H_0^2} \sim 10^{35} e^{-2H\tau}$$

F-eq $$1 - \Omega = \frac{k}{a_c^2 H_0^2} \lesssim 10^{-9} \qquad \text{for} \quad H\tau \gtrsim 50$$

$\Rightarrow$ Inflation predicts that the present Universe is "flat"!!!!

and $\rho = \rho_{crit}$.

DETAILS OF INFLATION

Hubble parameter is not const. during inflation as we naively assumed $\longrightarrow H(\varphi)$: $H^2(\varphi) = \frac{8\pi G}{3} (\phi)$

To find the evolution of φ during its slow roll-over, we vary

$$\Longrightarrow \quad \mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + M(\phi) \qquad (\text{remember } \sqrt{-\det g} = \text{volume element})$$

coupling to "light" matter causing decay of $\phi \to$ "massless particl

$$\Longrightarrow \qquad \ddot{\phi} + 3H\dot{\phi} + \Gamma_\phi \dot{\phi} + V'(\phi) = 0$$

Γ_ϕ = decay width of the inflaton and its decay time

$\Rightarrow \quad t_d = \Gamma_\phi^{-1} \gg H^{-1}$ = exp. time for inflation (slow roll-over)

$\Rightarrow$ Ignore for the moment $\Gamma_\phi \dot{\phi}$ - term