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SMR.762 - 14

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

13 June - 29 July 1994

THE COSMIC STRING THEORY

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The Cosmic String Theory

A. Basics, Defect Formation

B. String Evolution – Gravitational Effects

C. Structure Formation – Direct Signatures

D. Microwave Background Signatures

Lecture A

1. Big Bang Theory
   (a) Input
   (b) Predictions
2. Observations
3. Models
4. Defect Formation
5. String Fields

Big Bang

Input

(1) Universe → Homogeneous
→ Isotropic.

(2). General Relativity

$$ds^2 = dt^2 - a(t)^2\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = 8\pi G T_{\alpha\beta} \rightarrow a(t)$$

(3). $T_{\alpha\beta}$ → perfect fluid

$$T_{\alpha\beta} = \mathrm{diag}\left[\rho, -p, -p, -p\right]$$

(2), (3) $\Rightarrow$

$$\left.\begin{aligned} \left(\frac{\dot{a}}{a}\right)^2 - \frac{k}{a^2} &= \frac{8\pi G}{3}\rho \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(\rho + 3p) \end{aligned}\right\} \text{FRW eqns}$$

$$\text{FRW} + p = p(\rho) \Longrightarrow a(t)$$

A3

Examples:

$$p = \frac{1}{3}\rho \text{ (radiation)} \xrightarrow[k=0]{\text{FRW}} a(t) \sim t^{1/2}$$

$$p = 0 \text{ (matter)} \xrightarrow[k=0]{\text{FRW}} a(t) \sim t^{2/3}$$

$$\text{Observations} \Rightarrow \rho_{mat}(t_0) \gg \rho_{rad}(t_0)$$

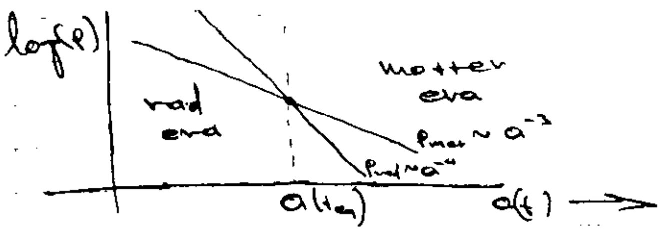
t_0: present time

$$\Rightarrow a(t_0) \sim t_0^{2/3}$$

But:

$$\text{FRW} \Rightarrow \frac{d}{da}(\rho a^3) = -3a^2 p \Rightarrow \begin{cases} \rho_{mat} \sim a^{-3} \\ \rho_{rad} \sim a^{-4} \end{cases}$$

$$\Rightarrow \exists\, t_{eq} < t_0 : \rho_{mat}(t_{eq}) = \rho_{rad}(t_{eq})$$



Also:

$$\left.\begin{matrix} \rho_{rad} \sim a^{-4} \\ \rho_{rad} \sim T^4 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} T(t) \sim a(t)^{-1} \\ T(t_0) \simeq 2.7^\circ k \end{matrix}\right\}$$

$T(t_0) \simeq 2.7^\circ k$ ↳ observed

$$\Rightarrow T(t) = 2.7^\circ k \, \frac{a(t_0)}{a(t)}$$

A4

Predictions:

1. Hubble's law:

$$k=0 \Rightarrow ds = a(t)\, dr \Rightarrow s = a(t)\, r$$

s: physical, r: comoving

$$\Rightarrow \underbrace{\dot{s}}_{\text{physical velocity}} = \underbrace{\dot{a} r}_{\text{Hubble flow}} + \underbrace{a \dot{r}}_{\text{peculiar velocity}} \Rightarrow$$

$$\xRightarrow{r \gg} \dot{s} \simeq \frac{\dot{a}}{a} s \equiv H(t)\, s \Rightarrow$$

$$\Rightarrow \boxed{cz \simeq v = H_0 s}$$

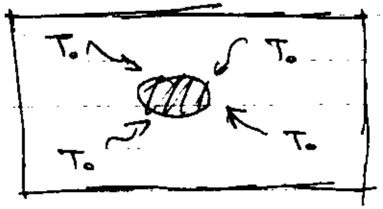
Hubble's law

$$H_0 = 100\, h\, \text{km} / (\text{sec} \cdot \text{Mpc}) \quad (0.5 \lesssim h \lesssim 1)$$

$$(1\, \text{pc} \simeq 3\, \text{ly})$$

[Fig. A1]

2. Microwave Background (CMB)

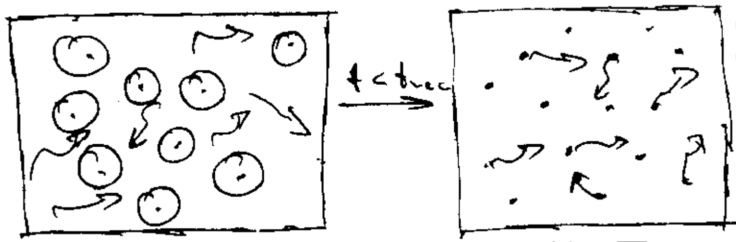


Q: What thermalized the photons?

Big Bang: $T(t) = 2.7^\circ K \frac{a(t_0)}{a(t)}$

$\Rightarrow \exists\ t_{rec} < t_0 : T(t_{rec}) = T_{atom} \sim 10^{4}\,^\circ K$

$\Rightarrow t < t_{rec}$ Universe Ionized Photons Scatter

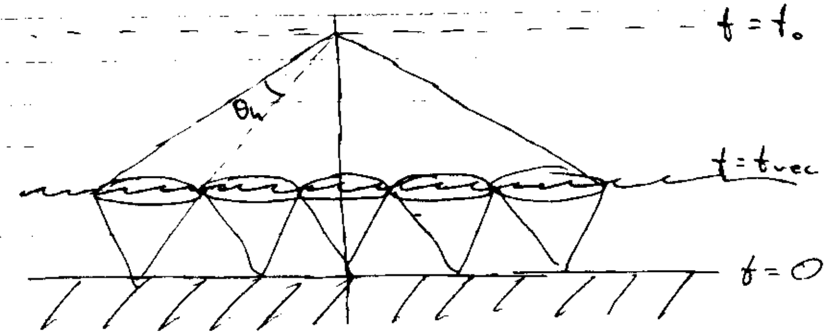


$T(t_0) \simeq 2.7^\circ K$ $\quad$ $T > T_{atom}$

$$1 + z(t_{rec}) \simeq \frac{\lambda(t_0)}{\lambda(t_{rec})} = \frac{a(t_0)}{a(t_{rec})} = \frac{T(t_{rec})}{T(t_0)} \simeq$$
$$\simeq 1500$$

[Fig. A2]

Primordial Fluctuations



$\theta_h(t_{rec}) \simeq 2^\circ$ (horizon scale):

COBE $\Rightarrow \theta_{res} \simeq 10^\circ > \theta_h(t_{rec})$

$\Rightarrow$ Primordial Fluctuations

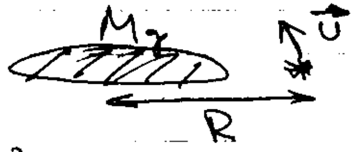
[Fig A3]

[Fig A4]

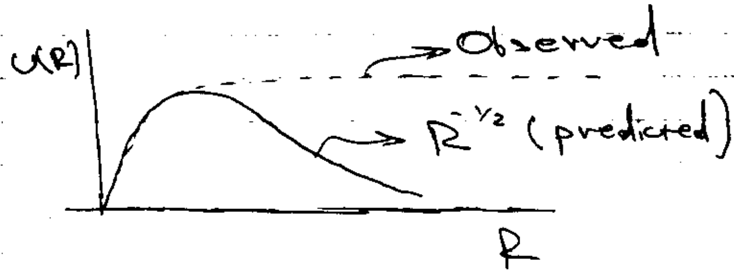
3. Nucleosynthesis

[Fig A5]

Dark Matter



$$\frac{v^2}{R} = \frac{G M_g}{R^2} \Rightarrow v(R) \sim R^{-1/2}$$



$$M(R) \sim R \Rightarrow \rho(R) \sim \frac{1}{R^2} > 0$$

$\Rightarrow$ There is dark matter

How much?

$$\Omega_{dm} \equiv \frac{\rho_{dm}}{\rho_c}, \quad \rho_c(t_0) = \frac{3}{8\pi G} H(t_0)^2$$

$$\Omega_{lum} \equiv \frac{\rho_{lum}}{\rho_c} \lesssim 0.01$$

$$\Omega_{dm} \in [0.1, 1]$$

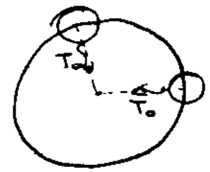
What is the dark particle?

Nucleosynthesis $\Rightarrow \Omega_{baryon} \in [0.01, 0.1]$

[Fig. A6]

Open issues in Big Bang theory

1. Horizon Problem



$$r_h \overset{k=0}{=} \int_0^{t_{rec}} \frac{dt}{a(t)} \Rightarrow$$

$$\Rightarrow \theta_h \simeq 2^\circ$$

Why is CMB so isotropic?

2. Flatness problem

$$\text{FRW} \Rightarrow \left.\begin{array}{l} \left(\frac{\dot a}{a}\right)^2 - \frac{k}{a^2} = \frac{8\pi G}{3}\rho \\ \frac{8\pi G}{3}\rho_c(t) \equiv \left(\frac{\dot a}{a}\right)^2 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left(\frac{\rho(t)}{\rho_c(t)} - 1\right)\dot a^2 \equiv (\Omega(t)-1)\dot a^2 = k = \text{const}$$

$$\Rightarrow \Omega(t_0) - 1 = \left(\frac{\dot a(t)}{\dot a(t_0)}\right)^2 (\Omega(t)-1)$$

Thus iff $a(t) \sim t^\alpha$, $\alpha < 1$ then $\Omega(t) = 1$ is unstable

Why is $\Omega(t_0) \simeq 1$?

A9

3. Structure Formation

Sheets
Filaments
Voids

$\left.\right\} \longrightarrow 50\, h^{-1}$ Mpc

[Fig. A7]

<u>But</u>

$d r_{max} \simeq \delta v\, t_0 \simeq \delta v\, H_0^{-1} \simeq$

$\simeq 600$ km/sec $\times$ (100 h km/sec Mpc)$^{-1}$

$\simeq 6\, h^{-1}$ Mpc $\ll 50\, h^{-1}$ Mpc

$\Longrightarrow$ Primordial Perturbations

What caused these perturbations?

[Fig. A8]

Models

A10

1. Types of dark matter:

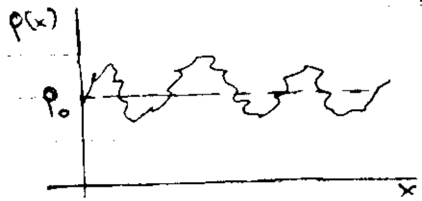
CDM $\longrightarrow v(t_{eq}) \ll 1$ (axion)

HDM $\longrightarrow v(t_{eq}) \simeq 1$ (25 eV neutrino)

HDM $\Longrightarrow$ Free Streaming
($l \lesssim l_{fs} \equiv v(t)\, t \Longrightarrow$ no growth of perturbations)

2. Types of Perturbations

<u>Inflation</u>

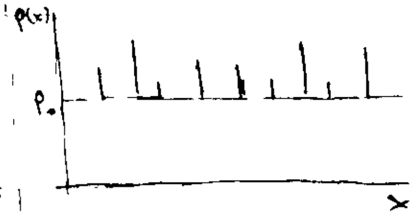


$\delta \equiv \frac{\delta\rho}{\rho}(x) = \sum_k |\delta_k| e^{i\theta_k} e^{ikx}$ (θ_k: random)

$\Rightarrow P(\delta) \sim e^{-\delta^2/\sigma^2}$

HDM ✗ CDM ✓?

<u>Topological Defects</u>



$\delta = \sum_i \frac{\delta\rho}{\rho}(x - x_i)$

$\Rightarrow P(\delta) \neq e^{-\delta^2/\sigma^2}$

HDM ✓? CDM ✓?

[Fig A9]

A11 **Focus on Cosmic Strings**

- Formation
- Evolution
- Observational Effects

Motivation of Scalar Fields:

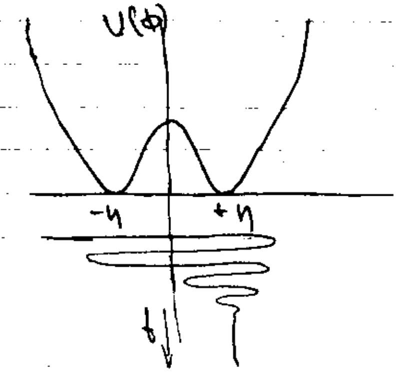
Symmetry Breaking:

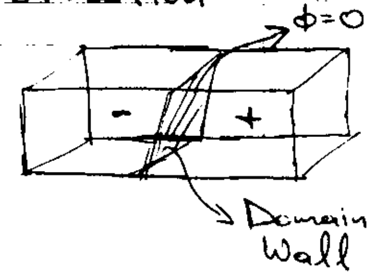
$$\phi = \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_N \end{bmatrix} : V(\phi) = \frac{\lambda}{4}\left(\phi^{+}\phi - \eta^2\right)^2$$

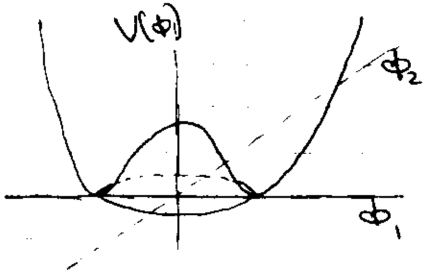
$\Rightarrow$ a) $M_W \neq 0$

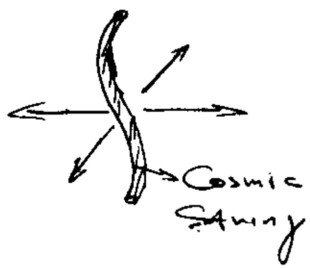
b) Renormalizable theory

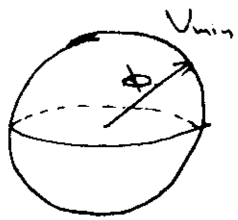
Defect Formation A12

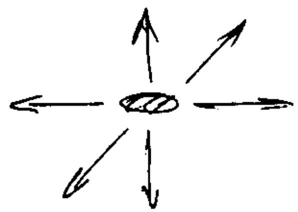






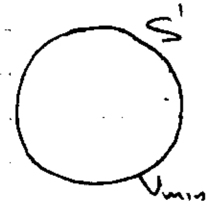






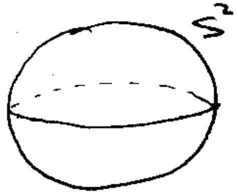
Monopole

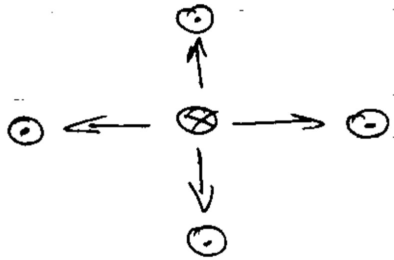
A13. # Nonlocal Defects: Textures



$\uparrow \;\rightarrow\; \downarrow \;\leftarrow\; \uparrow$

1d texture





2d texture

3d texture

$3d \Rightarrow E \xrightarrow{r\to\alpha r} \alpha^{-1}E \Rightarrow$ collapse

$\Rightarrow$ unwinding

String Fields

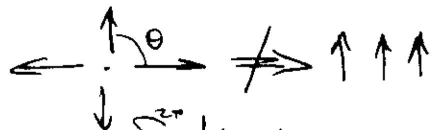
A14

$U(1)\to 1 \quad \left(\phi \to e^{i\theta}\phi \Rightarrow \begin{cases} \mathcal{L}\to\mathcal{L} \\ \phi_{vac}\to\phi'_{vac}\end{cases}\right)$

$\mathcal{L} = \frac{1}{2} D_\mu\phi D^\mu\phi - V(\phi) + \frac{1}{4} F_{\mu\nu}F^{\mu\nu}$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad D_\mu \equiv \partial_\mu - ieA_\mu$

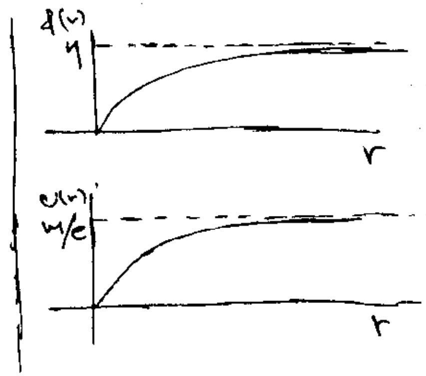
$V(\phi) = \lambda/4\,(|\phi|^2-\eta^2)^2, \quad \phi = \phi_1 + i\phi_2$



$m \equiv \int_0^{2\pi} \frac{d\alpha}{d\theta}\, d\theta = 1$

$\phi = f(r)\, e^{im\theta}$

$\vec{A} = \frac{v(r)}{r}\,\hat{e}_\theta$



A15

Energy

$$\mu = \frac{E}{L} = \int d^2x \left(f'^2 + \frac{v'^2}{r^2} + \frac{(ev-m)^2}{r^2} f^2 + \frac{\lambda}{4}(f^2-\eta^2)^2 \right)$$

Field Equations $\Rightarrow$

$$f(r) \xrightarrow{r\to\infty} \eta - C_f \frac{1}{\sqrt{r}} e^{-\sqrt{\lambda}\eta r}$$

$$v(r) \xrightarrow{r\to\infty} \frac{m}{e} - C_v \sqrt{r}\, e^{-e\eta r}$$

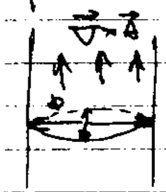
$$\Rightarrow \quad w \sim \eta^{-1} \quad \Rightarrow$$

$$\Rightarrow \mu \simeq \int_w d^2x\, V(0) \sim \eta^{-2}\eta^4 \sim \eta^2$$

$$\text{GUTs} \Rightarrow \eta \simeq 10^{16}\ \text{GeV}$$

$$\Rightarrow w \simeq 10^{-30}\ \text{cm} \simeq 10^{-16}\ r_{prot}$$

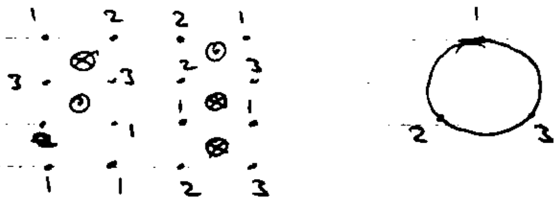
$$\mu \sim \eta^2 \simeq 10^{14}\ \text{tons/mm}$$



Lecture B

1. String Network Evolution
2. The Benefit of Scaling
3. Gravitational Effects
   a. Velocity Perturbations
   b. CMB Perturbations
   c. Lensing

✓ • Formation
→ • Evolution
• Observational Effects



[Fig. B1]

String Evolution

Nambu Action

Point Particle:

$\overset{A}{\underset{m}{\longrightarrow}} \vec{v} \quad - - - - - - - - \quad B$

$$S = -m\int_A^B d\tau = -m\int dt\,(1-v^2)^{1/2} \simeq$$

$$\simeq \int dt\ \tfrac{1}{2} m v^2 + \text{const}$$

String:

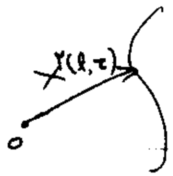
$A \ \vec{v}_s \quad - - - - - - - - \quad B$

$$S = -\underbrace{\frac{dm}{dl}}_{\mu}\int_A^B dl\, d\tau \sqrt{-g^{(2)}} + O\!\left(\tfrac{w}{R}\right)$$

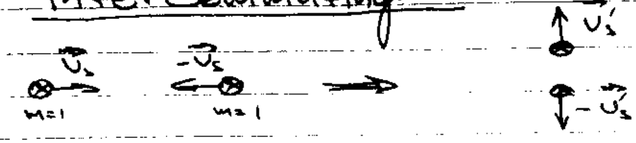
$$g^{(2)} = \det g^{(2)}_{ab}\,, \quad g^{(2)}_{ab} \equiv X^{\mu}_{,a} X^{\nu}_{,b}\, g_{\mu\nu}$$

$a, b = 1, 2 \to l, \tau$

$\mu, \nu = 1 \ldots 4$



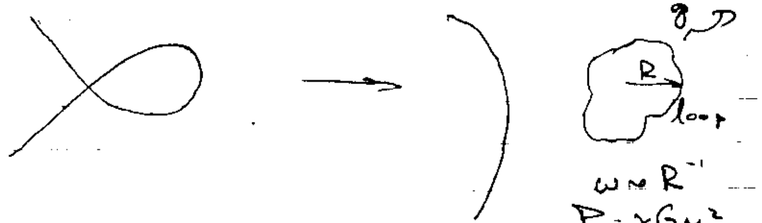
Intercommuting



$(v_s \lesssim 0.9\,sc)$

[Fig. B2]

[Fig. B3]



$\omega \sim R^{-1}$

$P = \gamma G \mu^2$

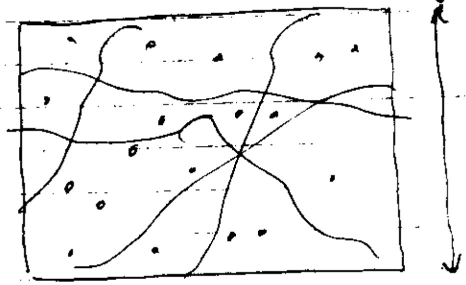
Thus: $\varepsilon_{str} \sim a^{2} \longrightarrow \varepsilon_{rad} \sim a^{-4}$

Simulations

1. Isolated String $\longrightarrow$ Nambu Action
2. Intersecting " $\longrightarrow$ Intercommuting
3. Loops $\longrightarrow$ Gravit. Radiation

B4

① , ② , ③ $\Longrightarrow$ Scaling Solution



[Fig. B4]

1. $M_{ls} \simeq 10$ (10 long strings)
2. $R_{loop} \simeq 10^{-4}\, t$ (small loops)
3. $\mu_{eff} \simeq 1.4\, \mu$ (wiggly strings)

[Movie]

Benefit of Scaling

B5

Strings:

$$\rho_{str} \sim \frac{M\, \mu\, t}{t^3} \sim \frac{\mu}{t^2} = \frac{G\mu}{G t^2} \sim$$

$$\rho_c \sim \left(\frac{\dot a}{a}\right)^2$$

$$\sim G\mu\, \rho_c \Rightarrow \boxed{\frac{\rho_{str}}{\rho_c} \simeq G\mu = \text{fixed}}$$

Monopoles $\xrightarrow{\text{no scaling}}$

$$\left.\begin{array}{l} \rho_{mon} \sim a^{-3} \\ \rho_{rad} \sim a^{-4} \end{array}\right\} \frac{\rho_{mon}}{\rho_r} \sim a(t)$$

$$\Longrightarrow \rho_m(t_0) >> \rho_c(t_0)$$

Monopole Problem

$$G \longrightarrow H \Longrightarrow \overset{\text{vacuum}}{M} = G/H$$

$$\pi_2(G/H) = \pi_1(H) = \pi_1(U(1)_{em}) = Z$$

$\Longrightarrow$ Monopoles form

Walls

$$\rho_w \sim \frac{V(0)\, \eta^{-1}\, t^2}{t^3} \sim \eta^3 t^{-1} \sim$$

$$\sim (G\eta^2)\, \eta t \,/\, G t^2 \xrightarrow[\eta = 10^{16}\,\text{GeV}]{t = t_0}$$

$$\Longrightarrow \boxed{\frac{\rho_w}{\rho_c}(t_0) \simeq 10^{52}}$$

Gravitational Effects

Lorenz Invariance $\Rightarrow$

$$T^z_z(\rho) = T^0_0(\rho) \simeq \mu\,\delta(x)\,\delta(y)$$

Also: $T^\nu_{\mu,\nu} = 0 \Rightarrow \frac{d}{dx} T^x_x(\rho) = 0$

$$\Rightarrow T^x_x = \text{const} \xRightarrow{T^x_x \xrightarrow{r\to\infty} 0} \boxed{T^x_x = T^y_y = 0}$$

Thus

$$T_{\mu\nu} \approx \delta(x)\,\delta(y)\,\mathrm{diag}(\mu, 0, 0, -\mu)$$

$$\equiv \mathrm{diag}(\rho, P_x, P_y, P_z) \Rightarrow$$

$$P_x = P_y = 0, \quad \boxed{P_z = -\rho = -\mu}$$

Newtonian limit:

$$\nabla^2 \Phi = 4\pi G \left(\rho + \sum_i P_i\right) = 0$$

$$\Rightarrow \boxed{\vec{F}_N = 0}$$

No Newtonian Force

Wiggly Strings:

$\hookrightarrow$ Wiggles $\Rightarrow$

$\Rightarrow$ No Lorenz Invariance

$$\Rightarrow \left\{ \begin{array}{l} T = -P_z < \mu \\ \mu_{eff} > \mu \end{array} \right\} \rightarrow T_{\mu\nu} = \delta(x)\delta(y)(\mu, 0, 0, T) \quad (\mu_{eff} T = \mu^2)$$

Special cases:

$\mu_{eff} = T \Rightarrow$ no wiggles

$T \to 0 \Rightarrow \mu_{eff} \to \infty$

Metric:

Einstein eqns $\Rightarrow$

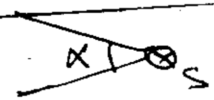
$$\Rightarrow ds^2 = (1 + h_{00})\left(dt^2 - dz^2 - (1 - 4G\mu_{eff})^2 r^2 d\varphi^2\right)$$

with $h_{00} = 4G(\mu_{eff} - T)\ln(r/r_0)$, $r_0 \hookrightarrow$ const

Define $\varphi' = (1 - 4G\mu_{eff})\,\varphi$

$$\varphi = 2\pi \Rightarrow \varphi' = 2\pi - 8\pi G\mu$$

$$\Rightarrow \boxed{\text{Deficit Angle: } \alpha = 8\pi G\mu_{eff}}$$



Velocity Perturbations

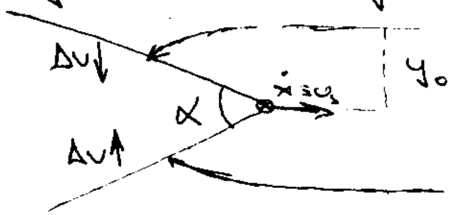
Cartesian Coordinates

$$y' = r\sin\varphi' \simeq y - r\cos\varphi\, 4G\mu_{eff}\,\varphi$$
$$x' = r\cos\varphi' \simeq r\cos\varphi = x$$

$$\boxed{\begin{aligned} ds^2 &\simeq (1+h_{00})(dt^2 - dz^2 - dx^2 - dy'^2) \\ y' &= y - 4G\mu\varphi x \end{aligned}}$$

Geodesics

$$2\ddot{x} = -(1-\dot{x}^2-\dot{y}'^2)\,\partial_x h_{00}$$
$$2\ddot{y}' = -(1-\dot{x}^2-\dot{y}'^2)\,\partial_y h_{00}$$



$$\partial_x h_{00} = 4G(\mu_{eff} - T)\frac{x}{r^2}$$

$$\left.\begin{aligned} x &= v_s t \\ y' &\simeq y_0 \end{aligned}\right\} \Longrightarrow \left\{\begin{aligned} \ddot{x} &= -\tfrac{1}{2}(1-v_s^2)\,4G(\mu_{eff}-T)\frac{x}{r^2} \\ \ddot{y} &= -\tfrac{1}{2}(1-v_s^2)\,4G(\mu_{eff}-T)\frac{y'}{r^2} \end{aligned}\right.$$

Heuristic Solution

$$\left.\begin{aligned} F &= 2mG(\mu_{eff}-T)/(\gamma_s r) \\ \Delta t &\simeq r/v_s \end{aligned}\right\} \Longrightarrow$$

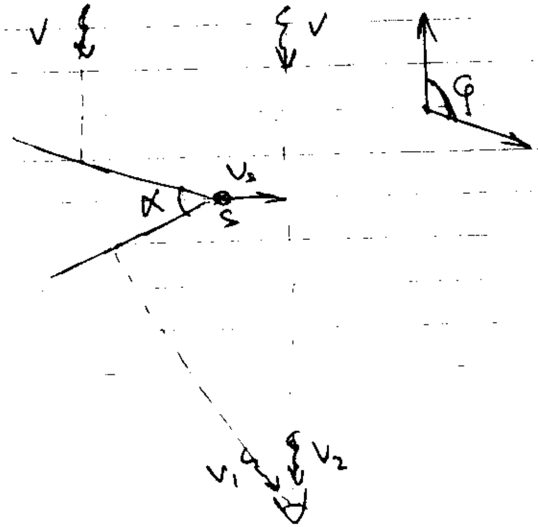
$$\Longrightarrow \dot{y}' \simeq \frac{F}{m}\Delta t \simeq \frac{2G(\mu_{eff}-T)}{v_s\gamma_s}$$

Exact Solution

$$\dot{y}' = \underbrace{\dot{y}}_{\Delta v} - 4G\mu\pi v_s\gamma_s = \frac{2\pi G(\mu_{eff}-T)}{v_s\gamma_s}$$

$$\Longrightarrow \boxed{\Delta v = \underbrace{\frac{2\pi G(\mu_{eff}-T)}{v_s\gamma_s}}_{\text{wiggles}} + \underbrace{4\pi G\mu_{eff} v_s\gamma_s}_{\text{string}}}$$

CMB Perturbations



$$ds^2 = (1+h_{00})(dt^2 - dx^2 - dy'^2)$$
$$dy' \simeq dy - 4G\mu_{eff}\,\varphi\, dx =$$
$$= dy - \underbrace{4G\mu_{eff}\,\varphi\, v_s}_{V(\varphi)}\, dt$$

Photons $\Rightarrow ds^2 = 0 \Rightarrow h_{00}$ no effect

Thus:

String $\Leftrightarrow$ Lorentz Boost $v_y = V(\varphi)$

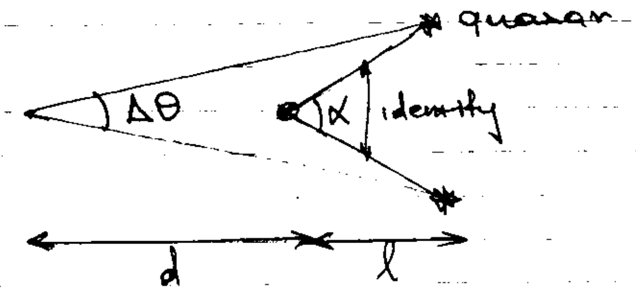
Thus

$$\left(\frac{\delta T}{T}\right)_1 = \left(\frac{\delta \nu}{\nu}\right)_1 =$$
$$= V(\varphi = 2\pi) - V(\varphi = \pi) =$$
$$= +4\pi G\mu_{eff}\, v_s \gamma_s$$

$$\left(\frac{\delta T}{T}\right)_2 = \left(\frac{\delta \nu}{\nu}\right)_2 =$$
$$= V(\varphi = 0) - V(\varphi = \pi)$$
$$= -4\pi G\mu_{eff}\, v_s \gamma_s$$

$$\boxed{\frac{T_1 - T_2}{T} = \frac{\nu_1 - \nu_2}{\nu} = 8\pi G\mu_{eff}\, v_s \gamma_s}$$

Lensing



$$(l+d)\sin\left(\frac{\Delta\theta}{2}\right) = l \sin\left(\frac{\alpha}{2}\right)$$

$$\Rightarrow \Delta\theta \simeq 8\pi G\mu_{\text{eff}} \frac{l}{l+d} \lesssim 5''$$

Lecture C

1. Structure Formation
2. Velocities
3. Constraints from the msec pulsar
4. A String Detection?

Cosmic Strings

✓ • Formation

✓ • Evolution

→ • Observational Effects

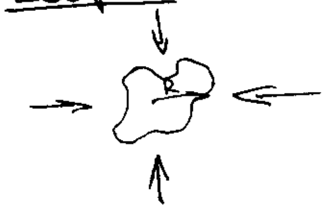
C2 Structure Formation

Two Mechanisms

a. Loops $\to$ galaxies, clusters

b. Long Strings $\to$ Sheets

Loops

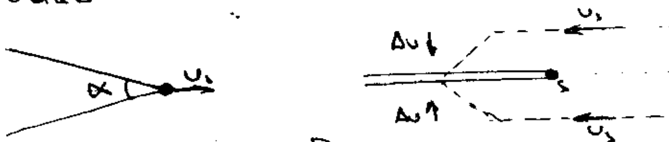


Matter Accretion

$$M_{loop} = \underset{\simeq 2\pi}{\beta} R \mu$$

Long Strings

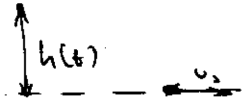
Wake formation

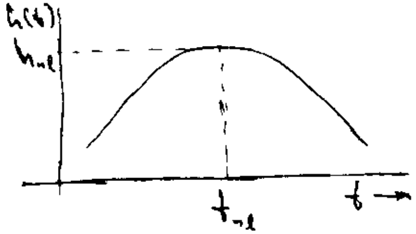


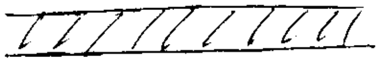
$$\Delta v = 4\pi G\mu_{eff} v_s \gamma_s F$$

$$F = \left(1 + \frac{1 - \frac{T}{\mu_{eff}}}{2(v_s\gamma_s)^2}\right)$$

Gravitational Growth C3







$h_{nl}(t) \Rightarrow \frac{\delta\rho}{\rho} \simeq 1$

$\Rightarrow$ galaxies form

Observations: $h_{nl}(t_0) \simeq 5h^{-1}\,\mathrm{Mpc}$

Q: $h_{nl}^{string}(t_0, G\mu) \simeq 5h^{-1}\,\mathrm{Mpc}$

$$\Longrightarrow \quad G\mu = ?$$

Method: Zeldovich approximation

- Define $\vec{r}(\vec{q}, t) = a(t)\left(\vec{q} - \vec{\psi}(\vec{q}, t)\right)$ (1)
- Find $\vec{\psi}(\vec{q}, t) \xRightarrow{\dot{\vec{r}} = 0} q_{nl}(t)$

C4 Newtonian approximation:

$$\ddot{\vec r}(\vec q,t) = -\frac{\partial}{\partial \vec r}\Phi(\vec r,t) \qquad (2)$$

$$\frac{\partial^2}{\partial \vec r^2}\Phi(\vec r,t) = 4\pi G\rho(\vec r,t) \qquad (3)$$

Mass Conservation:

$$\rho(\vec r,t)\,d^3r = a(t)^3\rho_0(t)\,d^3q$$

$$\Rightarrow \rho(\vec r,t) = a(t)^3\rho_0(t)\frac{1}{\det\left|\frac{d\vec r}{d\vec q}\right|} \simeq$$

$$\simeq \rho_0(t)\left(1+\frac{\partial}{\partial \vec q}\vec\Psi(\vec q,t)\right) \qquad (4)$$

(3),(4) $\Rightarrow \frac{\partial}{\partial \vec r}\Phi(\vec r,t) \simeq 4\pi G\left[\frac{1}{3}\rho_0(t)\vec r + \rho_0(t)\,a(t)\,\vec\Psi(\vec q,t)\right]$ (5)

Also FRW equs $\Longrightarrow$

$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}\rho_0(t) \qquad (6)$$

(1)$\to$(2), (5),(6)$\to$(2) $\Rightarrow$ $\boxed{\ddot{\vec\Psi} + 2\frac{\dot a}{a}\dot{\vec\Psi} + 3\frac{\ddot a}{a}\vec\Psi = 0}$ (7)

Initial Conditions C5

$$\left.\begin{array}{l} \vec\Psi(t_i) = 0 \qquad (t_i = t_{eq}) \\ \dot\Psi_x(t_i) = \dot\Psi_y(t_i) = 0 \\ \Delta v \equiv a(t_i)\,\dot\Psi_z(t_i) = 4\pi G\mu_{eff}\, v_s\gamma_s R \end{array}\right\} (8)$$

(7),(8) $\xrightarrow[\Psi\sim t^{\alpha}]{a\sim t^{2/3}}$ $\boxed{\Psi(t) \simeq \frac{3}{5}\Delta v\left(\frac{t}{t_i}\right)^{2/3}\left(\frac{t_0}{t_i}\right)^{1/3} t_i}$ (9)

Find $q_{nl}(t)$:

$$\frac{d}{dt}\left(a\left(q_{nl} - \Psi(t)\right)\right) = 0 \Rightarrow$$

$$\left.\begin{array}{l} \Longrightarrow q_{nl}(t) = 2\Psi(t) \\ q_{nl}(t_0) \gtrsim 5\,h^{-1}\,\mathrm{Mpc} \end{array}\right\} \xRightarrow[t_i = t_{eq}]{(9)}$$

$$\boxed{G\mu \gtrsim 0.7\,(v_s\gamma_s R)^{-1}\times 10^{-6}}$$

Also

$$\boxed{(G\mu)_{GUT} \simeq (G\eta^2)_{GUT} \simeq 10^{-6}}$$

C6 **Thus:**

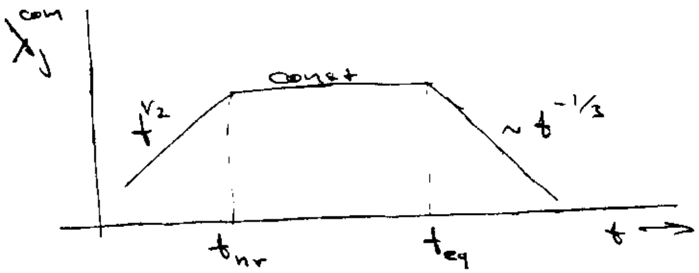
String Model $\xRightarrow{G\mu \simeq 10^{-6}}$ Sheets of galaxies:

$$\xi \underbrace{t^c_{eq}}_{R_{string}(t_{eq})} \times v_s \gamma_s t^c_{eq} \times a_{nr}(t_0) \simeq$$

$$\simeq \xi\, 40 \times v_s\, 40 \times 5 \text{ Mpc}^3$$

The effects of HDM

Free Streaming $\Rightarrow$ No growth for $\lambda \lesssim \lambda_J^{com} = v_\nu(t)\, t / a(t)$



$$\lambda_J^{max} = \lambda_J^c(t_{eq}) \underset{\substack{v_{eq} \simeq 0.1 \\ m_\nu \simeq 25\,eV}}{\simeq} 6\, h_{50}^{-2} \text{ Mpc}$$

$\xRightarrow{\text{delay}} (G\mu)_{HDM} > (G\mu)_{CDM}$

C7

Thus:

HDM $\Rightarrow$ power on large scales ✓

But

✗ Inflation + HDM $\Rightarrow$
$\Rightarrow$ small scales <u>erased</u>

Seeds + HDM $\Rightarrow$
$\Rightarrow$ small scales <u>delayed</u>
$\Rightarrow$ galaxies <u>can</u> form

C8

Peculiar Velocities

$$u(t_0,t_i) = \dot{\psi}(t_0,t_i) = \frac{2}{5}\,\Delta v \left(\frac{t_0}{t_i}\right)^{1/3} \qquad (t_i \geq t_{eq})$$

$$\Delta v = 4\pi G \mu_{eff}\, v_s \gamma_s\, t$$

Thus:

$$u(t_0, L) \simeq 300\, \mu_6\, h\, \frac{L_{eq}}{L}\ \mathrm{km/sec}$$

where $L(t_i) \equiv t_i^{com} \simeq \int_{t_i} \frac{dt}{a(t)}$

$$L_{eq} = t_{eq}^{com} = 40\, h_{50}^{-2}\ \mathrm{Mpc}$$

$$\mu_6 \equiv \mu \times 10^{6}$$

$$H_0 = 100\, h\ \mathrm{km/(sec \cdot Mpc)}$$

[Fig. C1]

Power Spectrum

C9

$$\frac{\delta\rho}{\rho}(\vec{x}) = \int d^3k\, \frac{\delta\rho}{\rho}(\vec{k})\, e^{i\vec{k}\vec{x}}$$

$$\boxed{P(\vec{k}) \equiv \left\langle \left|\frac{\delta\rho}{\rho}(\vec{k})\right|^2 \right\rangle}$$

$$C(\vec{x}) \equiv \left\langle \frac{\delta\rho}{\rho}(\vec{x}_1)\,\frac{\delta\rho}{\rho}(\vec{x}_1+\vec{x}) \right\rangle_{x_1} = \int d^3k\, P(k)\, e^{i\vec{k}\vec{x}}\, W(k-k_0)$$

$$\Rightarrow C(0)_{k_0} = \left\langle \left(\frac{\delta\rho}{\rho}(x)\right)^2 \right\rangle_{k_0} \equiv \delta(k_0,t)^2 \simeq k_0^3\, P(k_0)$$

Scaling Solution $\xrightarrow{\lambda_k = a(t)\frac{2\pi}{k}}$

$$\delta(\lambda_k = t,\ t) = \mathrm{const}$$

$$\left(\text{since } \rho_s(k_0 = t, t)/\rho \simeq \frac{\mu t / t^2}{1/G t^2} = \mathrm{const}\right)$$

Thus: $\left.\begin{array}{l} \delta(k,t) = \left(\frac{t}{t_i}\right)^{2/3} \delta(\lambda_k = t_i, t_i) \\ t_i = \lambda_k \sim \frac{a(t_i)}{k} \sim t_i^{2/3}/k \Rightarrow t_i \sim k^{-3} \end{array}\right\} \Rightarrow$

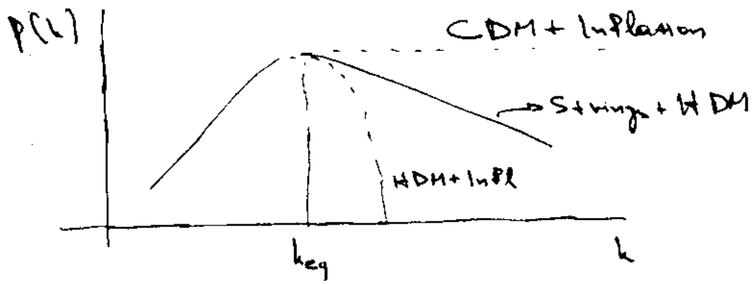
$$\delta(k,t) \sim k^2 \xrightarrow{\delta(k,t) = k^3 P(k)} \boxed{P(k) \sim k}$$

Zeldovich Spectrum

C10 Growth starts at t_{eq} $\Rightarrow$

$\lambda_k > t_{eq} \Rightarrow P(k) \sim k$

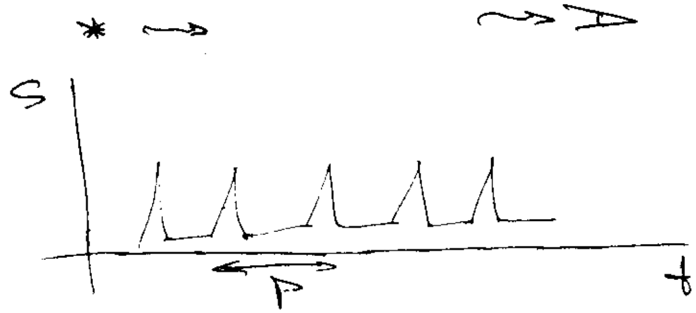
$\lambda_k < t_{eq} \Rightarrow P(k) \sim k^n,\ n<1$
(delayed growth)



[Fig. C2]

[Fig. C3]

Constraints from the msec pulsar C11



$$\varphi(t) = \varphi_0 + \dot\varphi t - \frac{1}{2}\ddot\varphi t^2 + \varphi_R(t)$$

$\varphi_R(t)$ → $\Omega_g(\omega) = \frac{\omega \rho_g(\omega)}{\rho_c}$

$\varphi_R(t)$ → other

Strings $\Rightarrow \Omega_g(\omega) = \text{const}$

If $\Omega_g \longleftrightarrow \langle \varphi_R(t)^2 \rangle \equiv \langle \frac{1}{T}\int_t^{t+T} \varphi_R^2\, dt \rangle$

then $\Omega_g(G\mu) \lesssim \frac{10^{-7}}{4} \frac{P^2}{2\pi} \langle \varphi_R^2(T) \rangle h^{-4} \left(\frac{2\pi}{T}\right)^4$ $\sim 1.5\,\mu s$

$\Rightarrow$ Ben-Bouch: $G\mu \lesssim 8 \times 10^{-6}\, h^{-2}$

Alb-Turok: $G\mu < 5 \times 10^{-7}\, h^{-8/3}$

C12 A Cosmic String Detection?

Lensing by a Loop

Expect:

1. $N \gtrsim 3$ galactic twin pairs in $50'' \times 50''$ region
2. $\delta\varphi \sim 1'' =$ fixed
3. No magnification
4. Similar properties (redshifts, spectra ...)
5. $z \gtrsim 0.1$ (large sample)

[Fig. C4]

[Fig. C5]

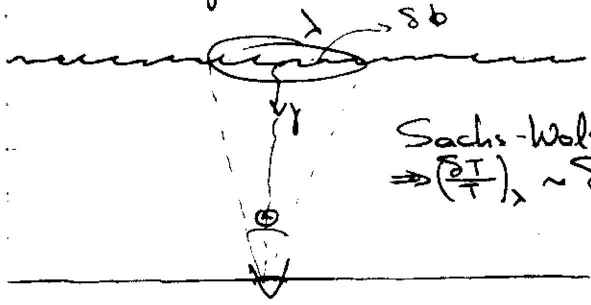
Lecture D

CMB Signatures

1. Angular Spectrum Basics
2. String CMB spectrum
3. Non-gaussian features
4. Conclusion

D2

Angular Spectrum



Sachs-Wolfe effect
$\Rightarrow \left(\frac{\delta T}{T}\right)_\lambda \sim \delta\Phi_\lambda$ ①

$$\delta\Phi_\lambda \sim \frac{G\,\delta M}{\lambda} \sim G\,\delta\rho_\lambda\,\lambda^2 \sim$$

$$\sim G\,\lambda^{-(3+n)/2}\,\lambda^2 \sim \lambda^{(1-n)/2} \quad ②$$

(since $\delta\rho_\lambda \sim (k^3 P(k))^{1/2} \sim k^{(3+n)/2} \sim \lambda^{-(3+n)/2}$)

$$①, ② \Rightarrow \left(\frac{\delta T}{T}\right)_\lambda \sim \delta\Phi_\lambda \sim \theta^{(1-n)/2} \quad ③$$

$$C(\Delta\theta = 0) \equiv \left\langle \frac{\delta T}{T}(\theta_1 + \overset{\to 0}{\Delta\theta})_\Theta \, \frac{\delta T}{T}(\theta_1)_\Theta \right\rangle_{\theta_1}$$

$$\sim \Theta^{1-n} \quad ④$$

$$n = 1 \Rightarrow \left(\frac{\delta T}{T}\right)_{rms} = \text{const} \Rightarrow$$

$\Rightarrow$ Scale Invariance

The angular spectrum C_ℓ

D3

$$\frac{\delta T}{T}(\hat{q}) = \sum_{\ell m} a_\ell^m\, Y_\ell^m(\theta, \varphi)$$

$$C_\ell = \langle |a_\ell^m|^2 \rangle \Rightarrow C(\theta) \underset{\hat{q}_1 \cdot \hat{q}_2 = \cos\theta}{\equiv} \left\langle \frac{\delta T}{T}(\hat{q}_1)\frac{\delta T}{T}(\hat{q}_2) \right\rangle =$$

$$= \frac{1}{4\pi} \sum_{\ell=0}^{\infty} (2\ell+1)\, C_\ell\, P_\ell(\cos\theta) \simeq$$

$$\overset{\ell \gg 1}{\simeq} \frac{1}{2\pi} \sum_\ell \ell\, C_\ell\, J_0(\ell\theta) \Longrightarrow$$

$$\Rightarrow C(\theta)_{\Theta_{\ell_0}} \simeq \frac{1}{(2\pi)^2} \int d^2\ell\; C_\ell\, e^{i\vec{\ell}\cdot\vec{\theta}}\, W(\ell - \ell_0)$$

Let $C_\ell \sim \ell^\alpha$, $\Theta_\ell = \frac{2\pi}{\ell}$

$$C(0)_{\Theta_{\ell_0}} \sim \ell_0^2\, C_{\ell_0} \sim \Theta_{\ell_0}^{-\alpha - 2}$$

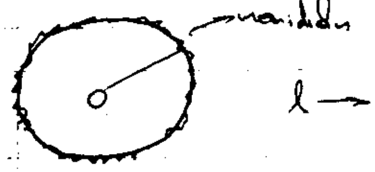
$$④, ⑤ \Rightarrow -\alpha - 2 = 1 - n \Rightarrow \boxed{\alpha = n - 3}$$

$$\boxed{C_\ell \sim \ell^{n-3}}, \ \ell \gg 1$$

[Fig. D1]

[Fig. D2]

D4 1 dim angular spectrum



$\ell \to k$, $C_\ell \to P(k)$

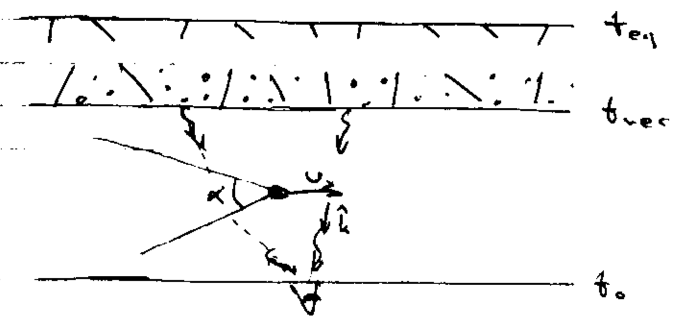
$$C(\theta)_{k_0} \simeq \frac{1}{2\pi} \sum_k P(k)\, e^{ik\theta}\, W(k-k_0)$$

$$C(0)_{k_0} = C(0)_{\ell_0} \xrightarrow{\ell \simeq k}$$

$$\boxed{\ell^2 C_\ell \simeq k P(k) \sim k^{n-1}}$$

$$\left(\frac{\delta T}{T}\right)_{rms} = C(0)_{k_0}$$

String CMB Spectrum D5

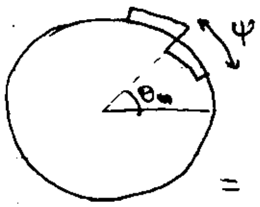


$$\left(\frac{\delta T}{T}\right)_{tot} = \left(\frac{\delta T}{T}\right)_{ks} + \left(\frac{\delta T}{T}\right)_{\phi} + \left(\frac{\delta T}{T}\right)_{Dop}$$

Q: 1. How to superpose seeds?
2. What seeds to superpose?

[Fig D3]

D6 Method



$$f(\theta) = \sum_{n=1}^{N} a_n f_1^{\psi}(\theta - \theta_n) =$$

$$= \sum_{n=1}^{N} a_n \frac{1}{2\pi} \sum_{k=-\infty}^{+\infty} \tilde{f}_1^{\psi}(k)\, e^{ik(\theta-\theta_n)}$$

$$\Rightarrow \tilde{f}(k) = f_1^{\psi}(k) \sum_{n=1}^{N} a_n e^{-ik\theta_n} \Rightarrow$$

$$\Rightarrow P_0(k) = \langle |\tilde{f}(k)|^2 \rangle =$$

$$= N \langle |a_n|^2 \rangle \, |\tilde{f}_1^{\psi}(k)|^2$$

Realistic case: Seed size growth

$$f^{com} \to \alpha f^{com} \Rightarrow \theta_h \to \alpha \theta_h \begin{cases} \psi \to \alpha\psi \\ N \to N/\alpha \end{cases}$$

Q steps $\Rightarrow$

$$\boxed{P_Q(k) = \sum_{q=0}^{Q} \frac{N}{\alpha^q} \left| f_1^{\alpha^q \psi}(k) \right|^2 \langle |a_n|^2 \rangle}$$

$$\psi_{max} = \alpha^Q \psi_{min}, \quad N = M \frac{2\pi}{\theta_{min}}$$

[Fig. D4]

D7

$$P_{tot} = P_{ks}(k) + P_w(k) + P_L(k) + P_D(k)$$

Parameters:

$$\left.\begin{array}{l} b \equiv M \langle (v_s \gamma_s)^2 \rangle \\ \xi : \ \psi = \xi\, \theta_h(t)/2 \\ f \equiv 1 + \dfrac{1 - T/M_{ass}}{2 (v_s \gamma_s)^2} \end{array}\right\} \to \text{Simulations}$$

$$G\mu_{eff} \longrightarrow \text{COBE}$$

$$C(\theta) = \frac{1}{2\pi^2} \sum_{k=0}^{\infty} P_{tot}(k)\, W(k-k_0)\, e^{ik\theta}$$

$$\left(\frac{\delta T}{T}\right)_{rms}^{COBE} = (C(0))^{1/2} = 1.1 \times 10^{-5} \Rightarrow$$

$$\Rightarrow G\mu_{eff} = 1.6 \times 10^{-6}$$

[Fig. D5] → COBE

[Fig. D6]
[Fig. D7] } → Spectrum

[Fig. D8] → Table

Non-gaussian Signatures

Statistics Basics

Moments: $\langle \delta^n \rangle \equiv \int d\delta \, \delta^n P(\delta)$

$\langle \delta \rangle = \mu$ (mean)
$\langle \delta^2 \rangle = \sigma^2$ (variance)
$\langle \delta^3 \rangle / \langle \delta^2 \rangle^{3/2} = a_3$ (skewness)
$\langle \delta^4 \rangle / \langle \delta^2 \rangle^2 = a_4$ (kurtosis)

Generating function:

$$M_\delta(t) \equiv \int d\delta \; e^{t\delta} P(\delta)$$

$$\Rightarrow \langle \delta^n \rangle = \frac{d^n}{dt^n} M(t) \Big|_{t=0}$$

Example: Gaussian $P(\delta)$

$$P(\delta) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\delta-\mu)^2}{2\sigma^2}} \xrightarrow{\delta \to \delta' = \frac{\delta-\mu}{\sigma}}$$

$$\rightarrow P(\delta') = \frac{1}{\sqrt{2\pi}} e^{-\delta'^2/2}$$

$$M_{\delta'}(t) = e^{t^2/2} \Rightarrow a_3 = 0, \quad a_4 = 3$$

Theorem: $M_{\delta_1+\delta_2} = M_{\delta_1} M_{\delta_2}$

Central limit theorem:

$$\left.\begin{array}{l} \delta_n \equiv x_1 + \dots + x_n \\ P(x_1) = \dots = P(x_n) \end{array}\right\} \Rightarrow$$

$$\Rightarrow P(\delta_n) \xrightarrow{n\to\infty} \frac{1}{\sqrt{2\pi}} e^{-\delta_n^2/2}$$

[Proof: $\langle e^{t\delta_n} \rangle = \left(\langle e^{tx/\sigma\sqrt{n}} \rangle\right)^n$

$\simeq \left(1 + \frac{t^2}{2n}\right)^n \xrightarrow{n \gg 1} e^{t^2/2}$]

D10

CMB Fluctuations

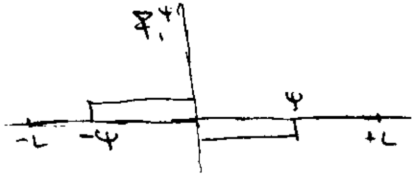
Inflation:

$$\delta = \frac{\delta T}{T}(\theta) \sim \sum_k |\delta_k| \underbrace{e^{i\theta_k}}_{\text{random}} e^{ikx}$$

$$\xRightarrow{CLT} \quad P(\delta) \to \text{gaussian}$$

Strings:

$$\delta = \frac{\delta T}{T}(\theta) \sim \sum_{n=1}^{N} f_1^{\Psi}(\theta - \theta_n)$$



$$P(\delta) = ?$$

$$M_{x_1}(t) = \langle e^{tx_1} \rangle \overset{p = \frac{\Psi}{L}}{=}$$

$$= \left(pe^{t} + pe^{-t} + (1-2p)\right)$$

$$M_{\delta_n = x_1 + \dots + x_n} = M_{x_1} \cdots M_{x_n} =$$

$$= \left(2p\cosh t + (1-2p)\right)^n \simeq$$

$$\underset{n \gg 1}{\overset{t \ll 1}{\simeq}} \left(1 + \frac{\overbrace{(2pn)}^{\sigma^2} t^2}{2n}\right)^n \xrightarrow[n \gg 1]{} e^{\sigma^2 t^2/2}$$

D11

Moments:

$$\sigma^2 = \frac{d^2}{dt^2} M_\delta(t)\Big|_{t=0} = 2np$$

$$a_3 = \frac{d^3}{dt^3} M_\delta(t)\Big|_{t=0} = 0$$

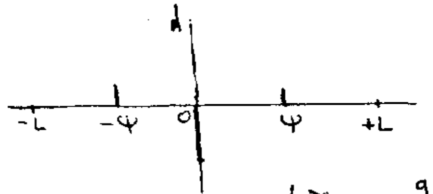
$$a_4 = \frac{d^4}{dt^4} M_\delta(t)\Big|_{t=0} = 3 + \frac{1-6p}{2np} \xrightarrow{np \gg 1} \underbrace{3}_{\text{gaussian value}}$$

$$np = ?$$

$$p \simeq \frac{\Psi}{L}, \quad n \simeq M\frac{L}{\Psi} \Rightarrow pn = M \simeq 10$$

$$\Rightarrow \frac{a_4 - a_4^{(gauss)}}{a_4} \lesssim \frac{\frac{1}{2M}}{3} \simeq \frac{1}{60} \ll 1$$

But: $P(d = \delta_{i+1} - \delta_i) = ?$



$$M_{x_1} = \langle e^{tx_1} \rangle \overset{q = \frac{1}{L}}{=}$$

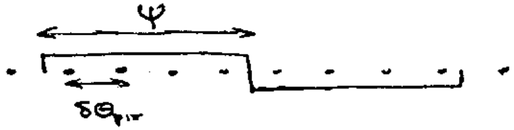
$$= \left(2qe^{t} + 2qe^{-t} + qe^{-2t} + qe^{2t} + (1-6q)\right)$$

$$\Rightarrow \dots \Rightarrow a_3 = 0, \quad a_4 = 3 + \frac{1-12q}{4nq}$$

$$q = \frac{1}{L}, \quad n = M\frac{L}{\Psi} \Rightarrow nq \sim \frac{M}{\Psi} \ll M \simeq np$$

D12

$$\frac{a_4 - a_4^{(gaus)}}{a_4} \simeq \frac{\frac{\Psi}{4M}}{3} = \frac{\Psi}{12M} >> \frac{1}{6M}$$

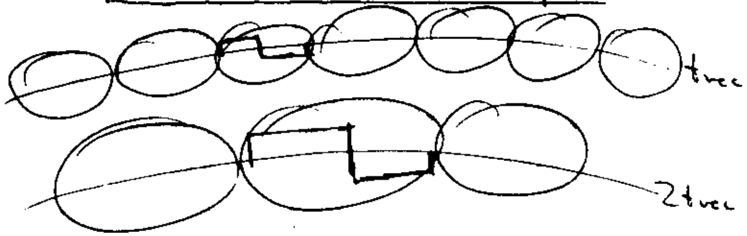


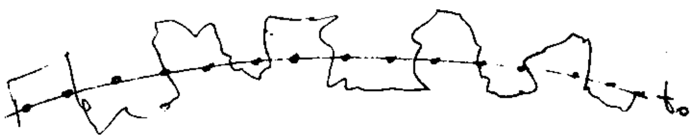
Test is more effective if:

$\Psi >> 2 \Rightarrow \delta\Theta^{\circ}_{pix} << \Psi^{\circ}_{min} \Rightarrow$

$\Rightarrow \delta\Theta^{\circ}_{pix} << 2^{\circ}$.

<u>Monte Carlo Maps</u>





D13

$$\frac{\delta T}{T}(\theta) = \sum_{q=1}^{Q}\sum_{i=1}^{N/\kappa^q} a_n f_i^{\kappa^q \Psi}(\theta-\theta_i)$$

$$\frac{\widetilde{\delta T}}{T}(k) = \frac{1}{2L}\int_{-L}^{+L} d\theta \; e^{ik\pi\theta/L}\,\frac{\delta T}{T}(\theta)$$

$$\frac{\delta T}{T}(\theta)\Big|_{str} = \sum_{k=-\infty}^{+\infty}\frac{\widetilde{\delta T}}{T}(k)\, e^{ik\pi\theta/L}$$

$$\frac{\delta T}{T}(\theta)\Big|_{gaus} = \sum_{k=-\infty}^{+\infty}\left|\frac{\widetilde{\delta T}}{T}(k)\right| e^{\overset{\text{random}}{i\theta_k}}\, e^{ik\pi\theta}$$

$$\frac{\delta T}{T}\Big|_{str} \longleftrightarrow \frac{\delta T}{T}\Big|_{gaus}$$

[Fig. D9]

[Fig. D10]

[Fig. D11]

[Fig. D12]

[Fig. D13]

[Fig. D14] → Conclusion

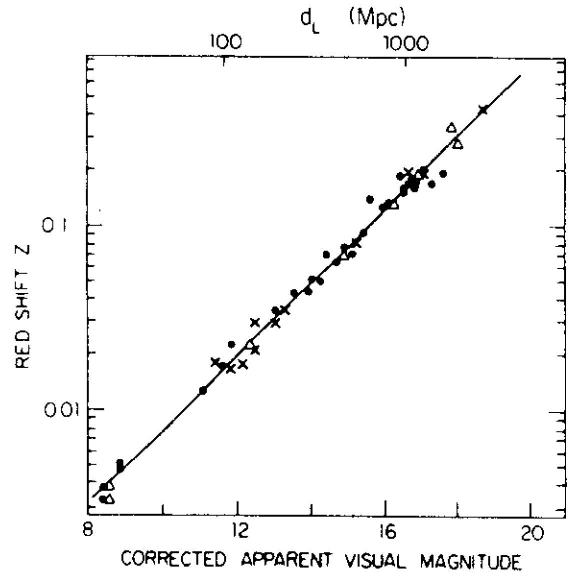
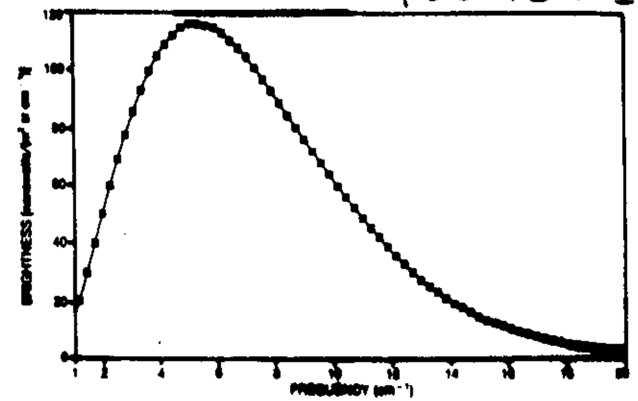
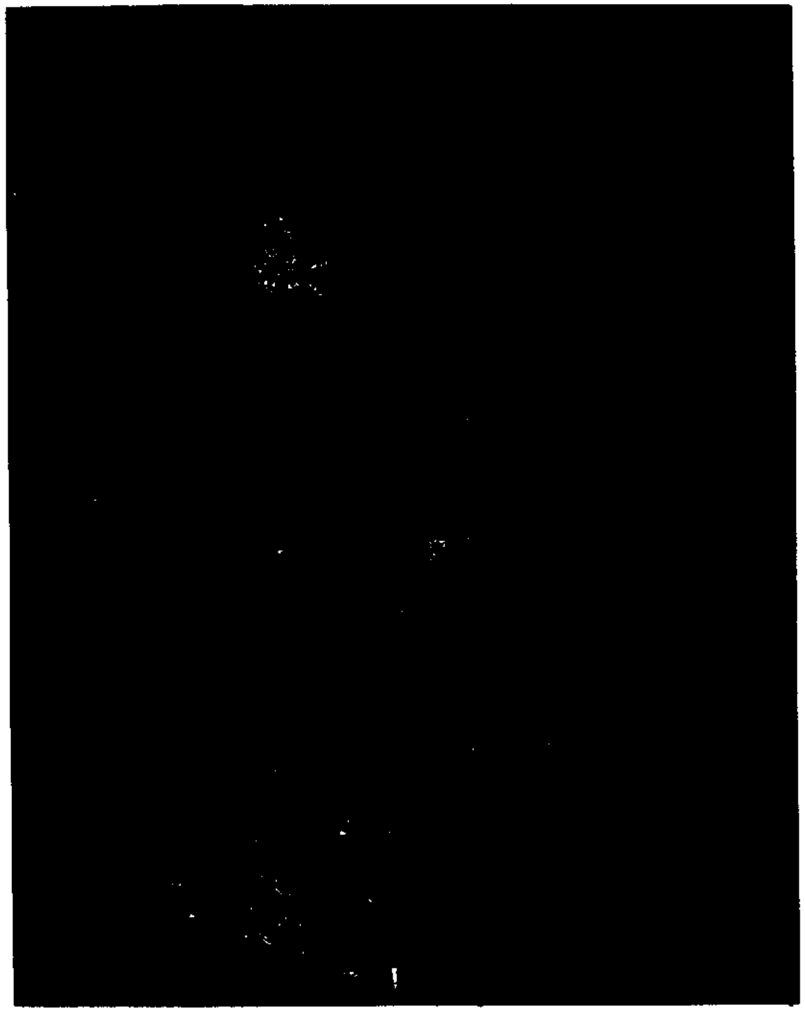


Fig. 1.1: The Hubble diagram. The corrected apparent magnitude is proportional to the logarithm of the luminosity distance. The straight line indicates the theoretical relationship for $q_0 = 1$ (from [7]).

FIRAS instrument of COBE



A3



A4 Aspen 8/93

O CMBR

An Anisotropy Scoresheet

Compiled by Mark Dragovan

Experiment	θ_x	Switch	Beam	Ω	Detectors	Sensitivity	95% Limits	Comments
COBE	> 5°	Full sky	7°	4π	Schottky diode	$30\mu K$	$1 \times 10^{-}$ D	20, 50, 90 GHz Space
Relikt								Space
FIRS	> 2°	Northern Hemisphere	4°	π	250 mK bolom	$60\mu K$	$< 2 \times 10^{-5}$	5.6,9,16.5,22 c Balloon
BCCF	> 1°	N and S hemisphere	3°	3.84π	MASER	$800\mu K$	$< 2 \times 10^{-4}$	19.2 GHz
Tenerife		8.1°	5.6°				1.8×10^{-5} L	10.5,14.9 GHz
Ulisse		6°	2.2°		1.5 K bolom	$30\mu K$	1.2×10^{-5}L	$< 30cm^{-1}$, 52c balloon
ACME-HEMT	1.2°	2.1°pk-pk sine	1.4°	13.8 deg^2	HEMT	$30\mu K$ $12\mu K$	$1.4 \times 10^{-5}L$ $1 \times 10^{-}$ D	25-35 GHz Ground SP
T+W	1.1°	±2.25°	2° × 10°	204 deg^2	SIS Interferometer	$130\mu K$	$1.1 \times 10^{-4}L$	42 GHz Gnd.Saskatoo
Python	1°	±2.75° 4 beam sq	.75°	8 deg^2	Single Mode 50 mK bolom	$20\mu K$	$3 \times 10^{-}$ D	90 GHz Gnd S 4 pixel array
MAX	0.42°	1.3° pk-pk sine	0.5°	6 deg^2	300 mK bolom	$35\mu K$	$4 \times 10^{-}$ D	$6,9,12 cm^{-1}$ Balloon
MSAM	.3°	±.66°	.47°	6 deg^2	250 mK bolom	$25\mu K$	$1.4 \times 10^{-}$ D	FIRS with small beam
ACME-SIS		sine		2.5 deg^2	SIS	$50\mu K$	$4 \times 10^{-5}L$	90 GHz Balloon and G
White Dish	0.27°	0.83° circ scan	0.20°		Single mode 50 mK bolom	$9\mu K$	$8 \times 10^{-6}L$	90 GHz Ground SP
OVRO NCP	2.5′	7′	1.8′	0.03 deg^2	MASER	$30\mu K$	$1.9 \times 10^{-5}L$	20 GHz Gnd Owens V
OVRO RING	2.5′	7′	1.8′	0.1 deg^2	MASER now HEMT	$100\mu K$	4.5×10^{-5} D	20 GHz Gnd Owens V
U+W	2′	4′.5	1′.5	0.02 deg^2	MASER	$300\mu K$	4.5×10^{-5}L	19.5 GHz Gnd Greenba
CAT			.5°		Interferometer			
BAM	1°	4°	1°		Martin-Pulpet Interferometer			$2 - 10cm^{-1}$ Balloon
SK 93	1.2°	±2.45°	1.45°		HEMT			Gnd Saskatoo
OVRO 5.5m	10′	22.16′	7′		HEMT			29-35 GHz Gnd Owens V

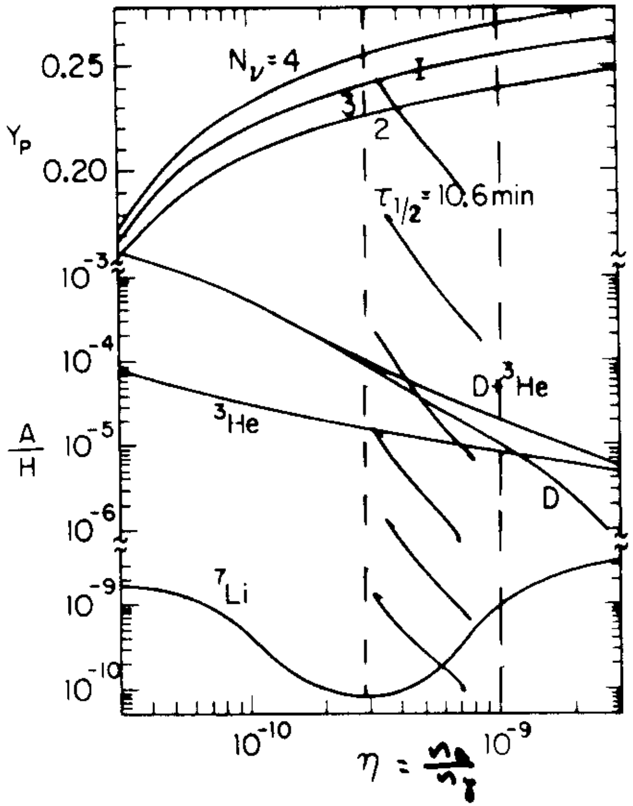


Fig. 4.4: The predicted primordial abundances of the light elements as a function of η. The error bar indicates the change in Y_P for $\Delta\tau_{1/2} = \pm 0.2$ min.

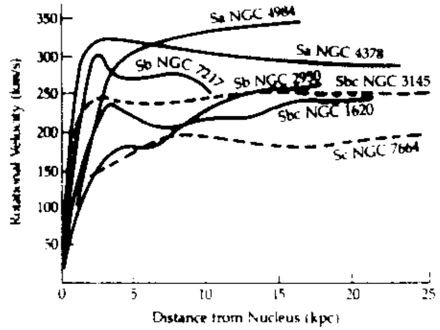


Figure 21 - 5

Rotation curves for spiral galaxies. Note how all flatten out at large distances from their centers. *[V. Rubin, W. K. Ford Jr., and N. Thonnard, Astrophysical Journal (Letters) 225:L107 (1978)]*

$$\eta = 2.68 \times 10^{-8}\, \Omega_B h^2 \qquad \left(\Omega_B = \frac{\rho_B}{\rho_c}\right)$$

$$\Rightarrow \quad 0.011 \leq \Omega_B h^2 \leq 0.037 \quad \Rightarrow$$

$$\xRightarrow{0.4 \leq h \leq 1} \quad 0.011 \leq \Omega_B \leq 0.23$$

$$\text{Observ} \Rightarrow \Omega_{lum} \leq 0.01$$

$\Big\} \Rightarrow$ Dark Matter

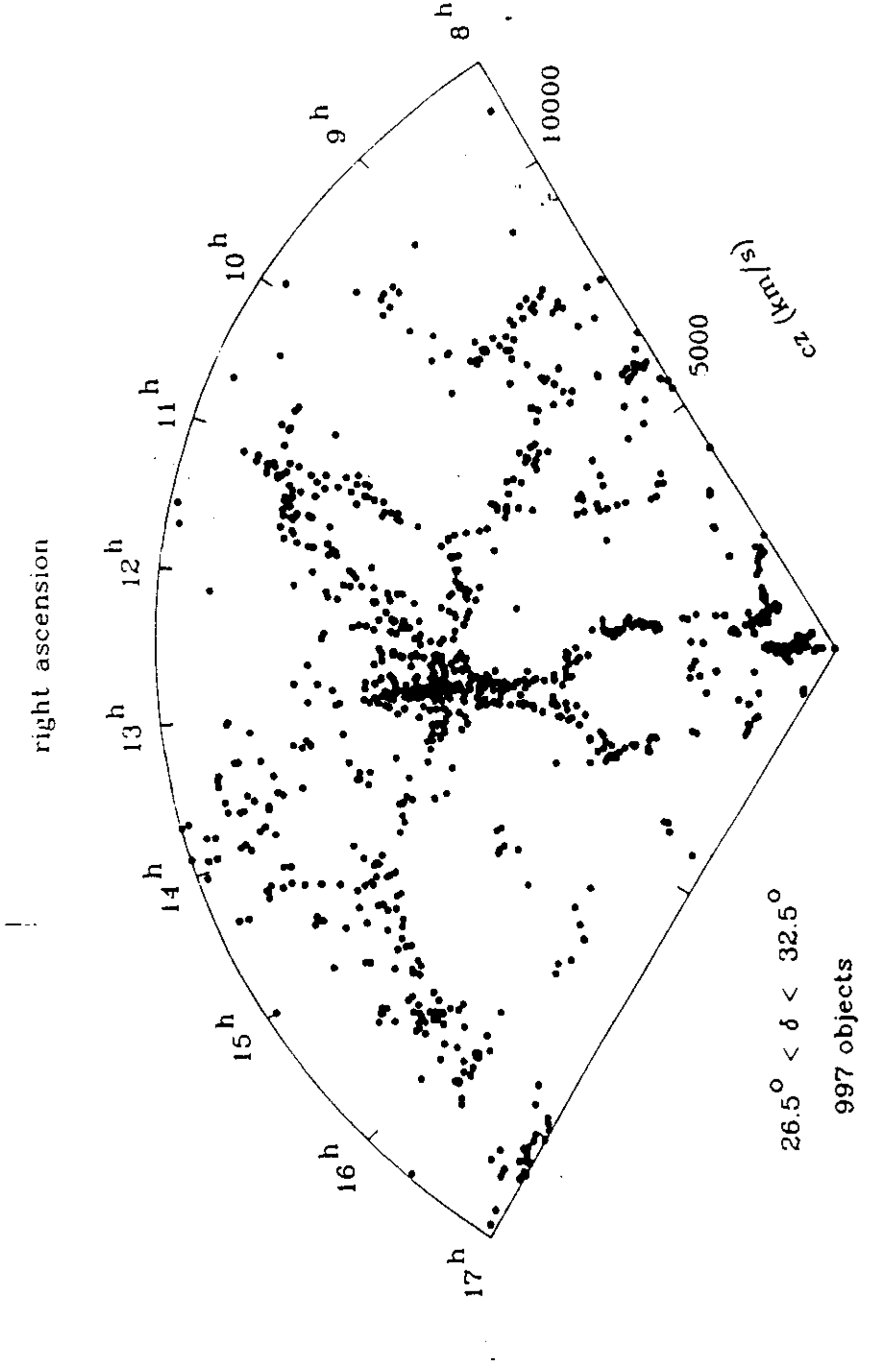


Figure 1a

Peculiar Velocities

$v_{pec} \equiv v - H r$

$v_{pec} \simeq 600$ km/sec

Coherent on scales $L \gtrsim 60 h^{-1}$ Mpc

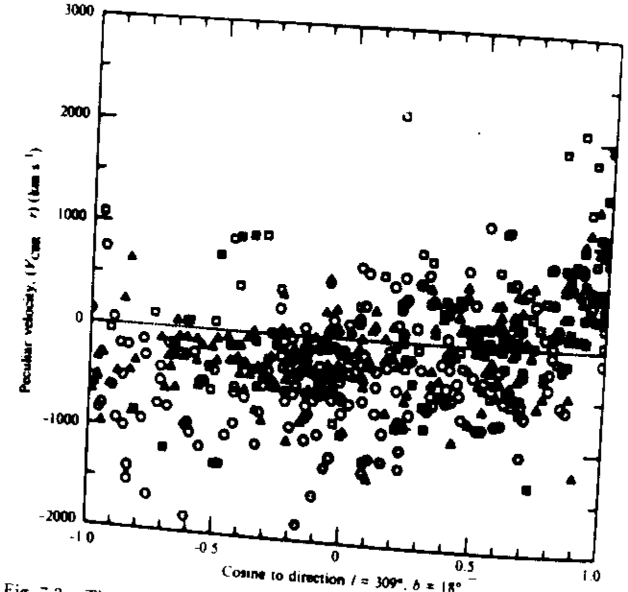
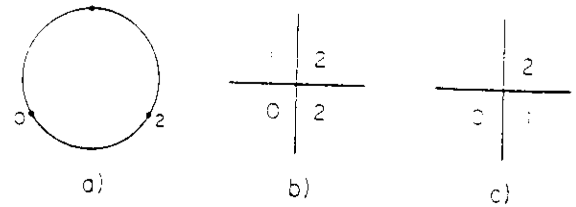


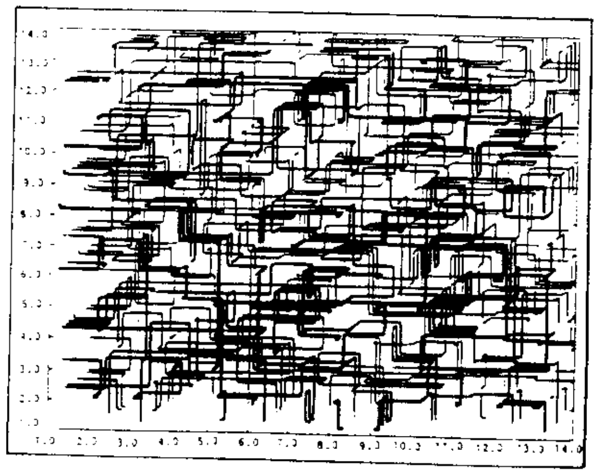
Fig. 7.3. The peculiar velocities of galaxies in the MBR frame is plotted against the cosine of the angle between the direction of that galaxy and direction of the dipole. (This figure is reproduced, with permission, from the review by D. Burstein, Rep. Prog. Phys., **523**, 421 (1990).)

Comparison of Models for Structure Formation

Scale (0-3) :	Inflation	Topological Defects
Addresses other cosmological problems	2/3	0/3
Consistent with known particle physics	1/3	2/3
Viable if $\Omega_0 < 1$	0/3	3/3
Difficulty of calculations	2/3	1/3
Consistency with COBE	3/3	3/3
Consistency with large scale structure obs	2/3	2/3
Conceptual Simplicity	1/3 (quantum → class)	2/3
Total	11/3	13/3



The approximation used in simulations of the formation of the simplest 'U(1)' cosmic strings (Section 2.8).
a) The space V_{min} (the circle) is approximated by 3 points. Each domain is assigned a value 0,1 or 2 corresponding to one of these points. The rule for the phase of the Higgs field is to move along the shortest path on V_{min} when going from one domain to the next.
b) an edge with four surrounding domains - this edge is a string.
c) this edge is not a string.



A box of strings at formation. The strings are formed on a cubic lattice with the algorithm described in Figure 2.6. The box is tilted and the strin

B2

L. Perivolaropoulos Nucl. Phys. B375(1992), p.665

Global Strings, t=0.0, v=0.00

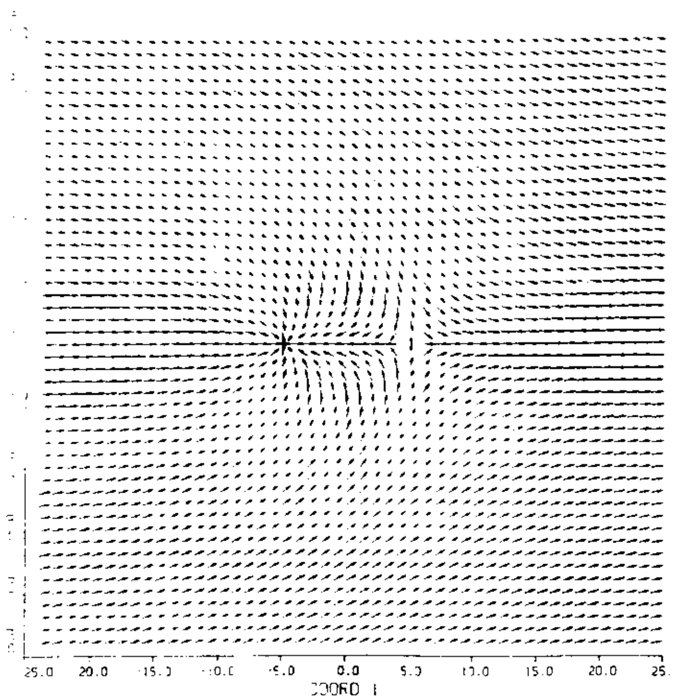


Fig. 4a

Global Strings, t=7.5, v=0.00

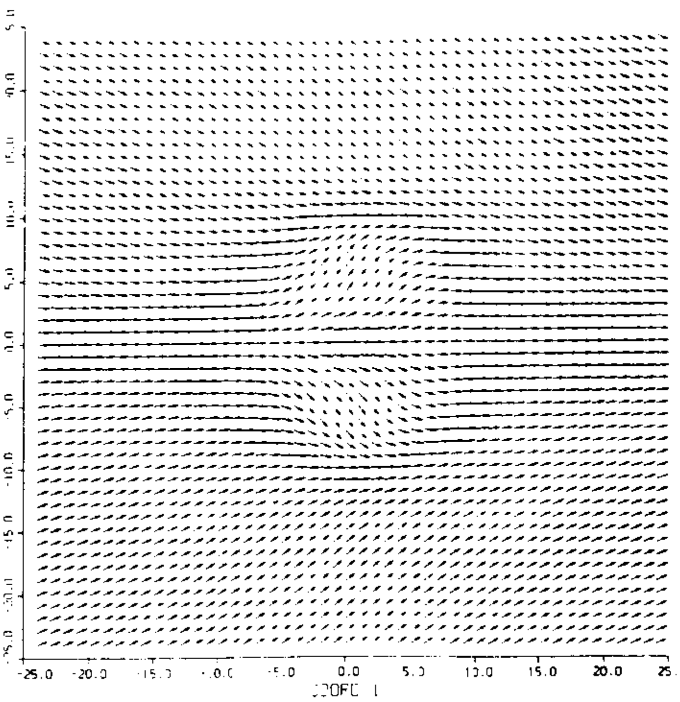


Fig. 4b

B3

Global Strings, t=0.0, v=0.95

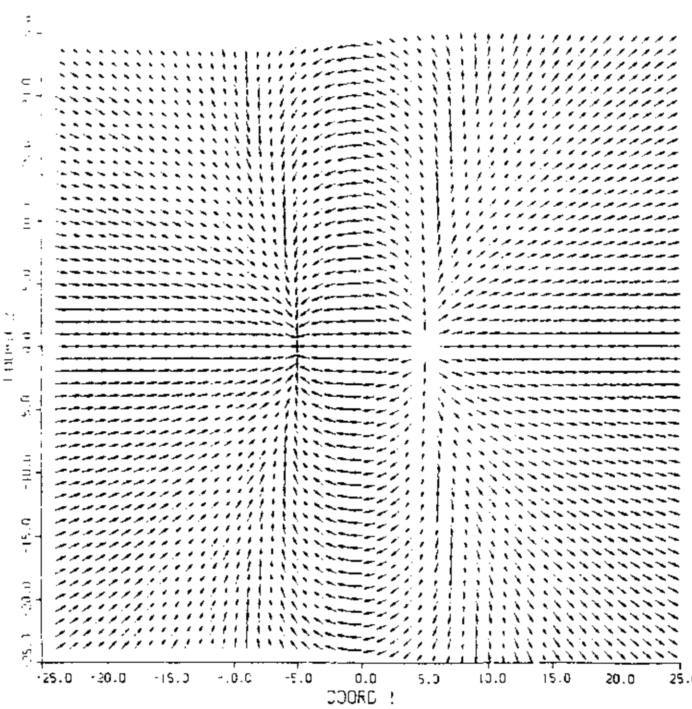


Fig. 10a

Global Strings, t=7.5 v=0.95

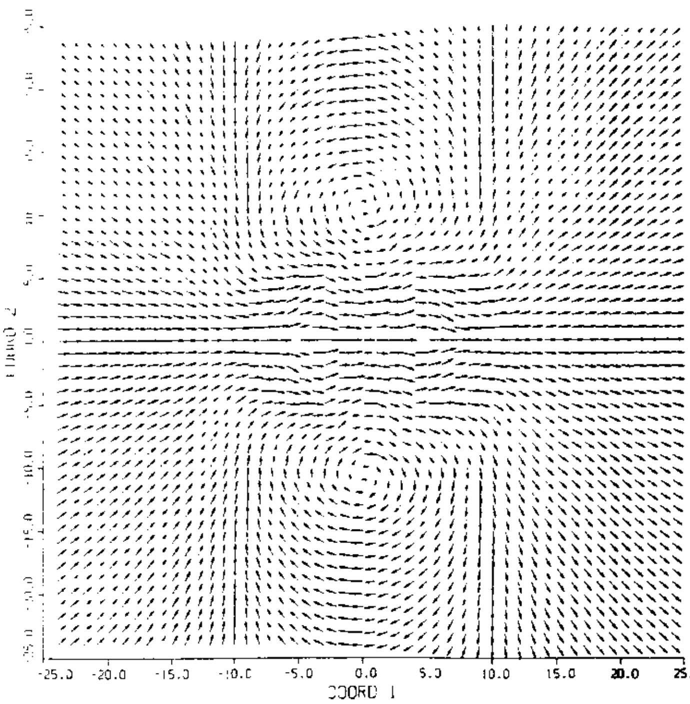
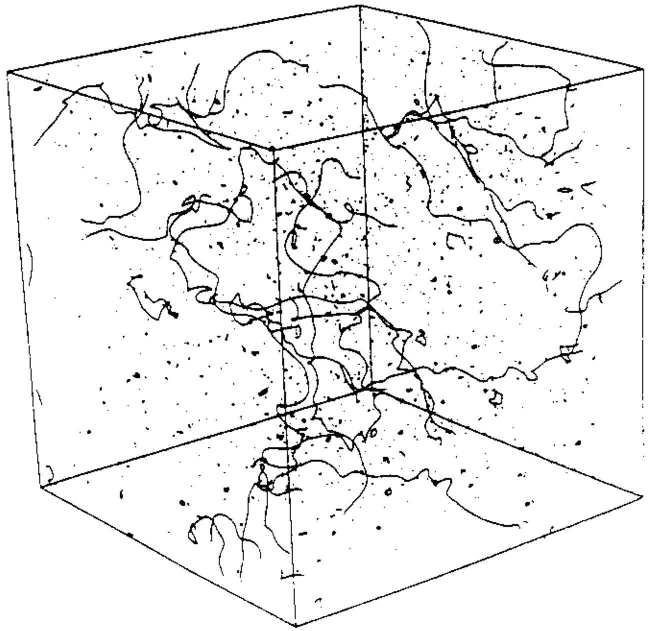
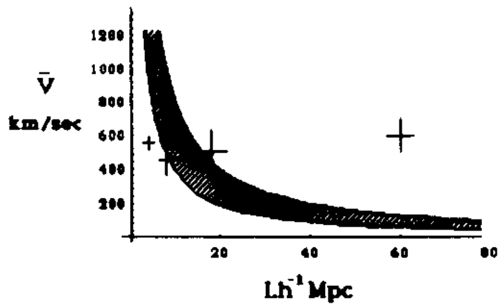


Fig. 10b

B4





C2

C3

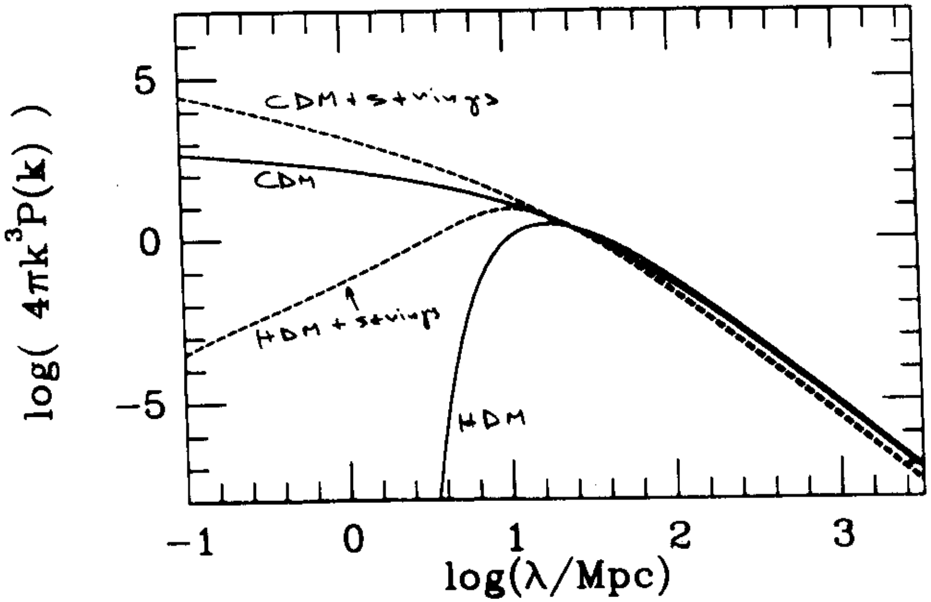


Figure 1. The logarithm of $4\pi(\lambda/2\pi)^{-3}P(\lambda)$ vs. $\log(\lambda)$ for I strings (dashed), and the Harrison-Zel'dovich spectrum (solid). In each case the curve with more power on small scales corresponds to CDM, the other to HDM. We use $h = \Omega_0 = 1$, normalised at $8h^{-1}$Mpc.

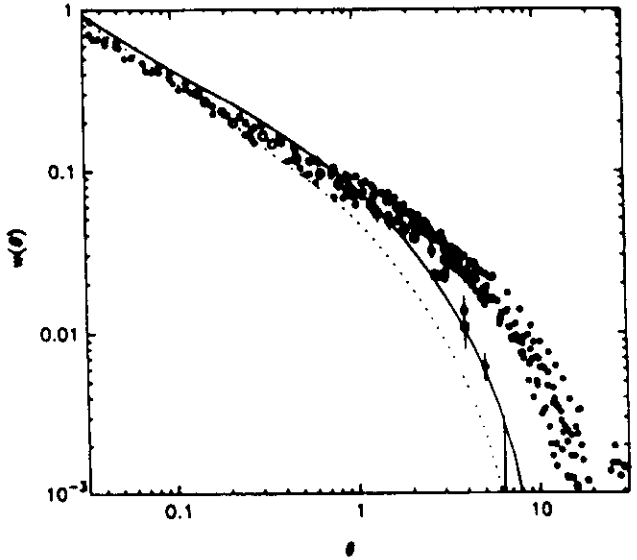


Figure 3. The filled circles show our estimates of $w(\theta)$ in six 0.5 mag slices (Fig. 2b) scaled to the Lick depth. The open symbols show $w(\theta)$ for the Lick catalogue from fig. 5 of Groth & Peebles (1977); the symbols have the same meaning as in their figure. The dotted and solid lines show computations of $w(\theta)$ based on the CDM model with $h=0.5$ and $h=0.4$, respectively, as discussed in the text.

C4

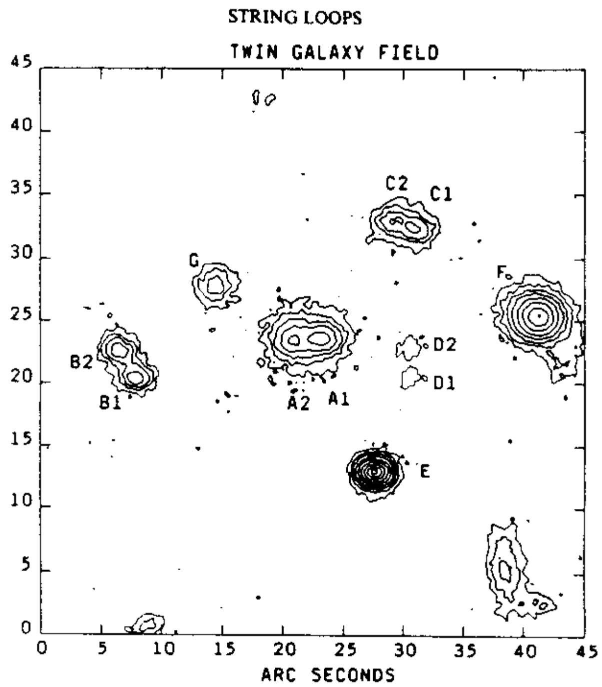


FIG. 1.—A 20 minute composite *R* band exposure of the twin group field showing the object labels used in the text and in Table 1. Contour levels are at roughly logarithmic spacing and have been chosen to highlight the twinned structure. Note the additional faint pair at the top of the field which became visible upon summing multiple exposures. Object coordinates may be read off relative to pair A, centered at $\alpha(1950)$: $2^h49^m23^s0$, $\delta(1950) = -18°25'32''$

C5

TABLE 1

GALAXY PROPERTIES

Galaxy	Relative *R* Magnitude	FWHM	Redshift	Angular Separation	P.A.
A1	0.49	2″1	0.426 ± 0.003	2″45	95°3
A2	0.62	2.9	0.431 ± 0.003		
B1	1.51	1.5	0.431 ± 0.002	2.56	33.3
B2	1.53	1.6	0.4335 ± 0.002		
C1	1.57	1.8	0.2003*e* ± 0.0005	2.04	72.6
C2	1.63	2.0	0.2000*e* ± 0.0005		
D1	3.12	1.9	...	2.57	162.6
D2	3.11	1.9	~ 0.4		
E	0.0	1.4	...	...	...
F	−0.63	2.5	0.1548*e* ± 0.0002	...	...
G	1.53	2.0	...	...	...

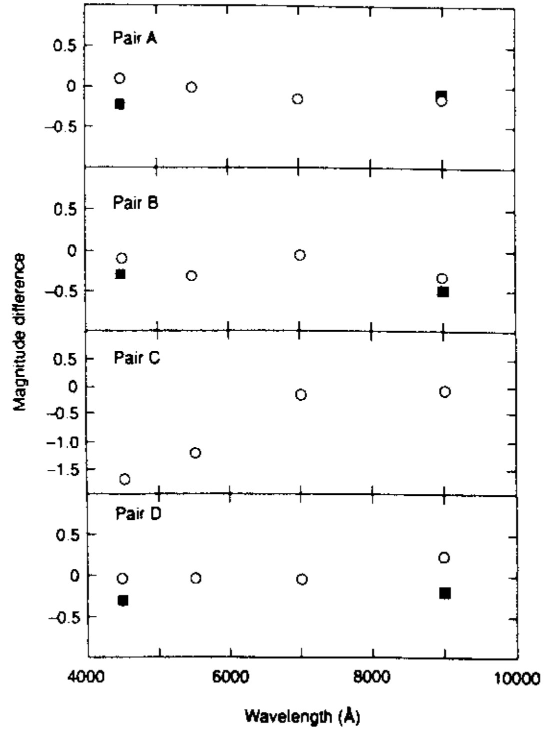
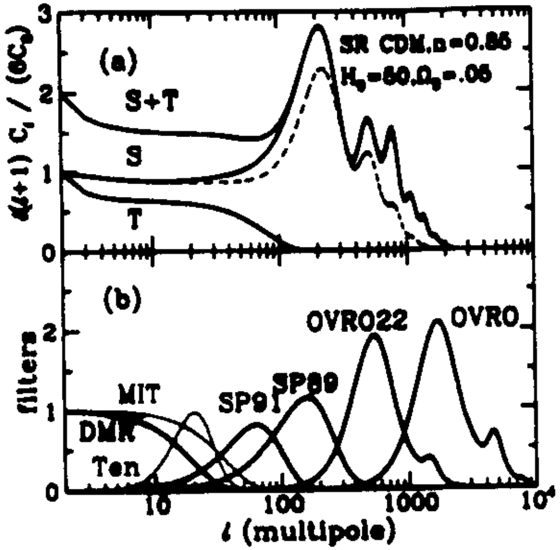


FIG. 3.—Differential magnitudes (Δm) between pairs shown as a function of wavelength for pairs A, B, C, and D (*open circles*). Where separate deep images are available, second measurements are shown (*squares*). A sense of the deconvolution errors may be obtained from the differences in these results.

Bond et al.
Phys. Rev. Lett
(1994)



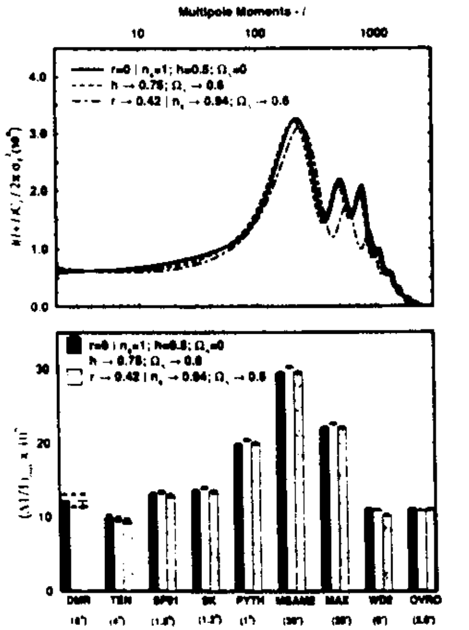


FIG. 3 Examples of different cosmologies with nearly identical spectra of multipole moments and $(\Delta T/T)_{rms}$. The solid curve is $(r = 0|n_s = 1, h = 0.5, \Omega_\Lambda = 0)$. The other two curves explore degeneracies in the $(r = 0|n_s = 1, h, \Omega_\Lambda)$ and $(r|n_s, h = 0.5, \Omega_\Lambda)$ planes. In the dashed curve, increasing Ω_Λ is almost exactly compensated by increasing h. In the dot-dashed curve, the effect of changing to $r = 0.42|n_s = 0.94$ is nearly compensated by increasing Ω_Λ to 0.6.

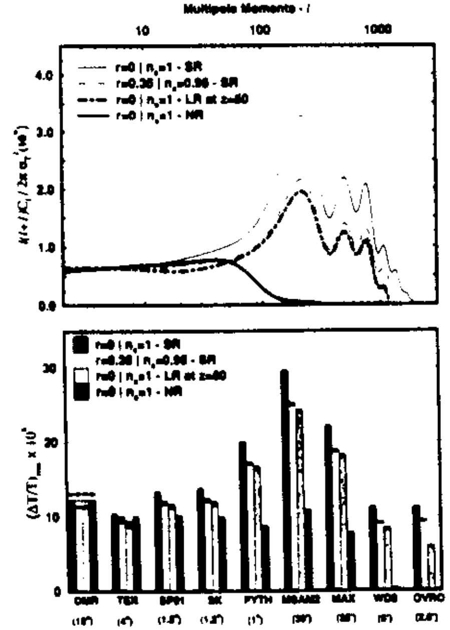


FIG. 4. Power spectra for models with standard recombination (SR), no recombination (NR), and "late" reionization (LR) at $z = 50$. In all models, $h = 0.5$ and $\Omega_\Lambda = 0$. NR or reionization at $z \geq 150$ results in substantial suppression at $\ell \geq 100$. Models with reionization at $20 \leq z \leq 150$ give moderate suppression that can mimic decreasing n_s or increasing h; e.g., compare the $n_s = 0.95$ spectrum with SR (thin, dot-dashed) to the $n_s = 1$ spectrum with reionization at $z = 50$ (thick, dot-dashed).

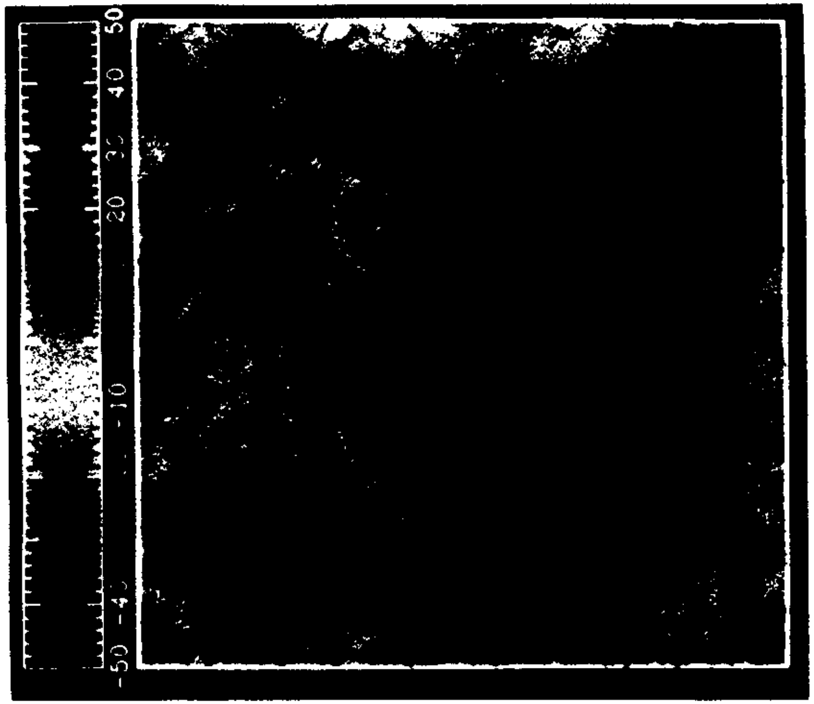
Monte Carlo Simulations

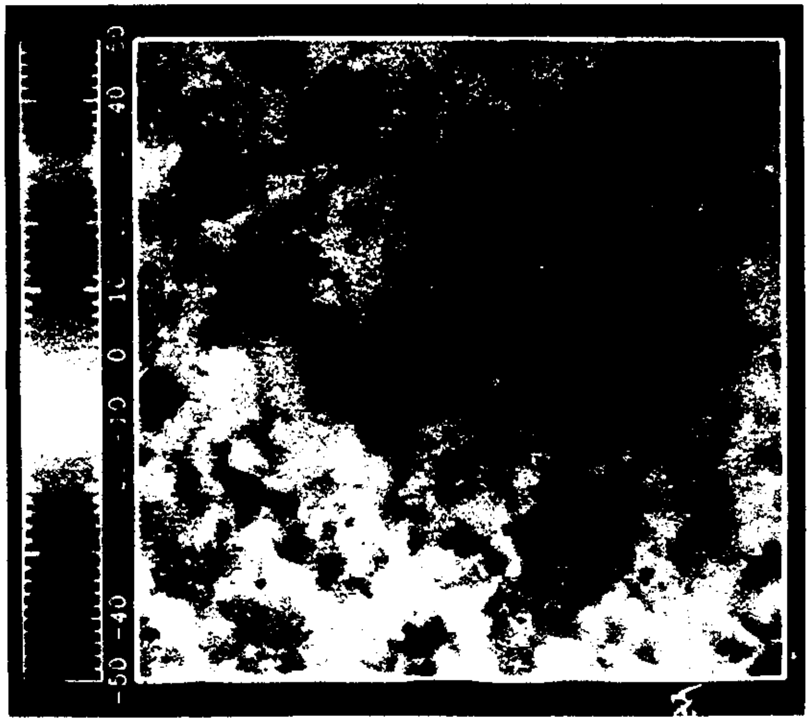
Scale Invariance + Gaussian Statistics
(Inflation)

Plate 1. Grey-scale photograph of the total temperature fluctuations in a 10°×10° patch of sky expected in a [illegible] adiabatic model with $\Omega = 1$, $\Omega_b = 0.03$, $h = 0.75$.

Bond and Efstathiou
MNRAS (1987)

D3

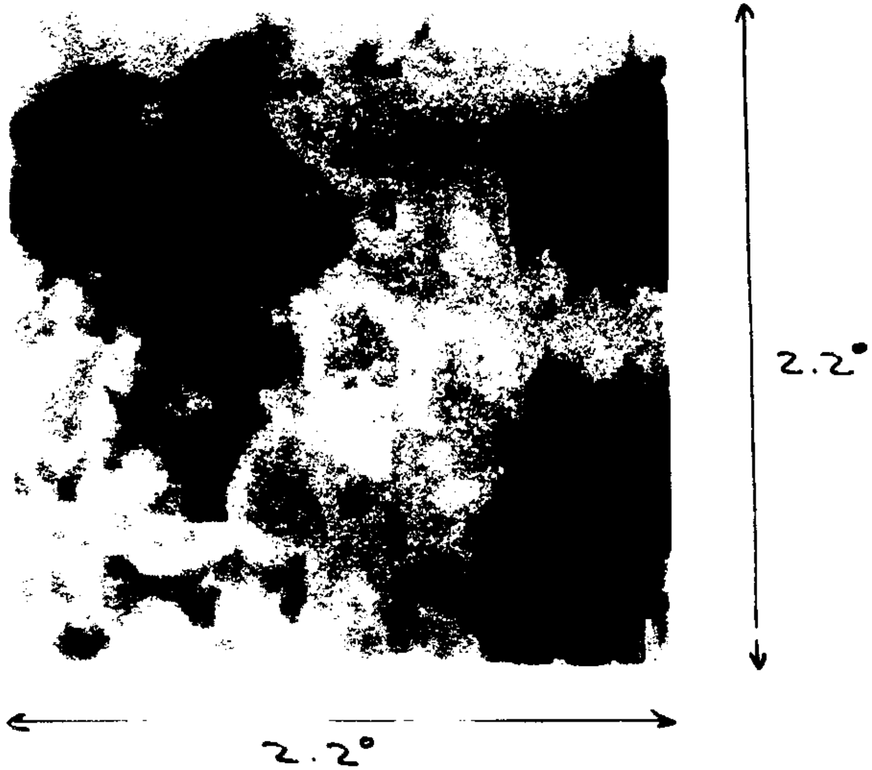




D4

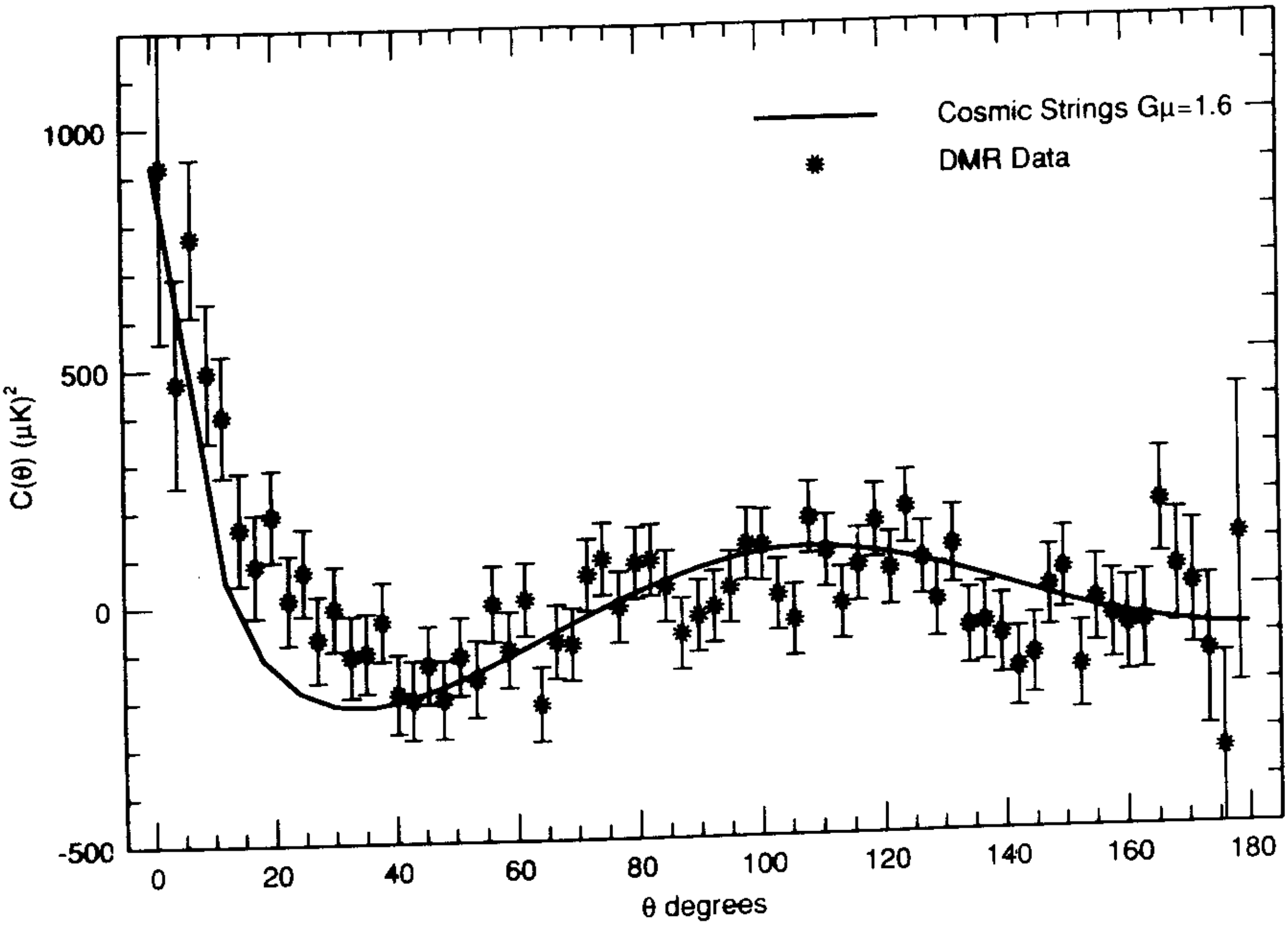
, $b = 4\pi G\mu \gamma_s v_s$

$\gamma_s \equiv (1 - v_s^2)^{-1/2}$



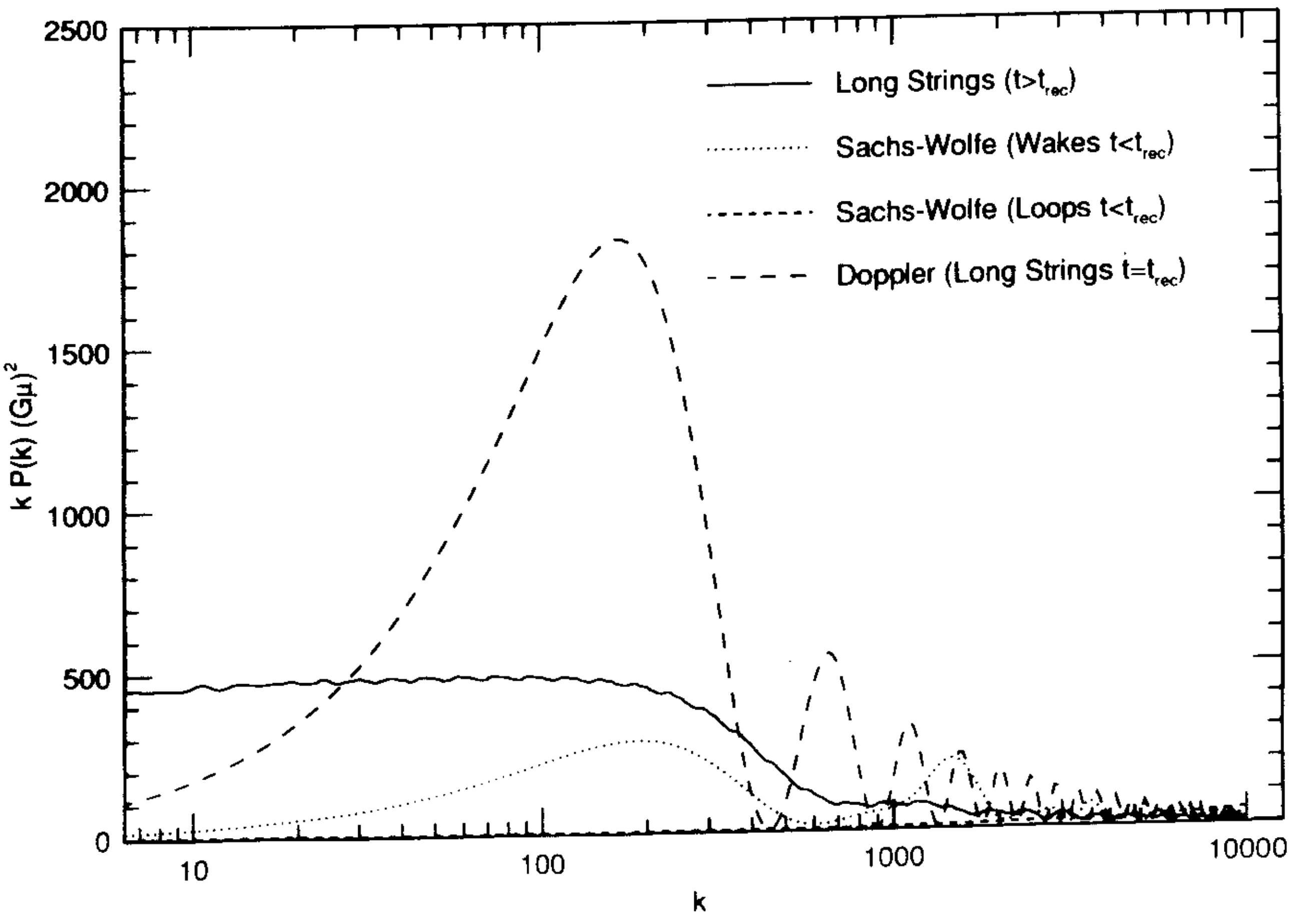
R. Moessner, L. Perivolaropoulos, and R. Brandenberger

Ap. J. v. 425, p. 365 (1994)



Cosmic Strings Gμ=1.6
DMR Data
C(θ) (μK)²
1000
500
0
-500
0
20
40
60
80
100
120
140
160
180
θ degrees

Figure 4



Long Strings (t>t_rec)
Sachs-Wolfe (Wakes t<t_rec)
Sachs-Wolfe (Loops t<t_rec)
Doppler (Long Strings t=t_rec)
k P(k) (Gμ)²
2500
2000
1500
1000
500
0
10
100
1000
10000
k

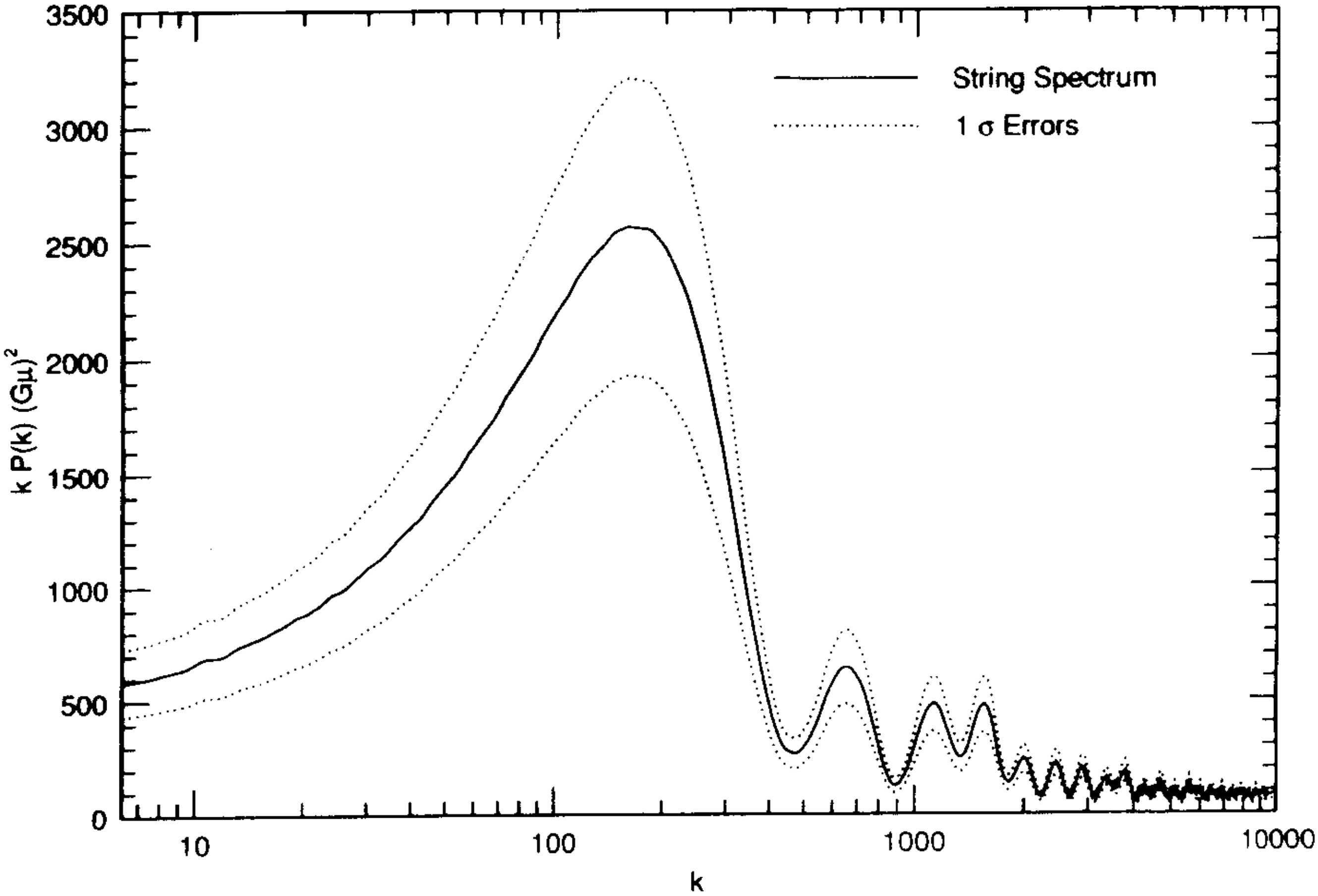


Figure 2b

Experiment	k_0	Δk	Detection	Strings	Inflation
COBE	0	18	11 ± 2	11 ± 3	11 ± 2
TEN	20	16	≤ 17	13 ± 3	9 ± 1
SP91	80	70	11 ± 5	20 ± 5	12 ± 2
SK	85	60	14 ± 5	19 ± 4	12 ± 3
MAX	180	130	≤ 30 (μPeg)	21 ± 5.5	16 ± 5
MAX	180	130	49 ± 8 (GUM)	21 ± 5.5	16 ± 5
MSAM	300	200	16 ± 4	19 ± 4	24 ± 6
OVRO22	600	350	-	13 ± 4	17 ± 7
WD	550	400	≤ 12	17.5 ± 4.5	7 ± [illegible]
OVRO	2000	1400	≤ 24	13.5 ± 3.5	7 ± 3

∇8

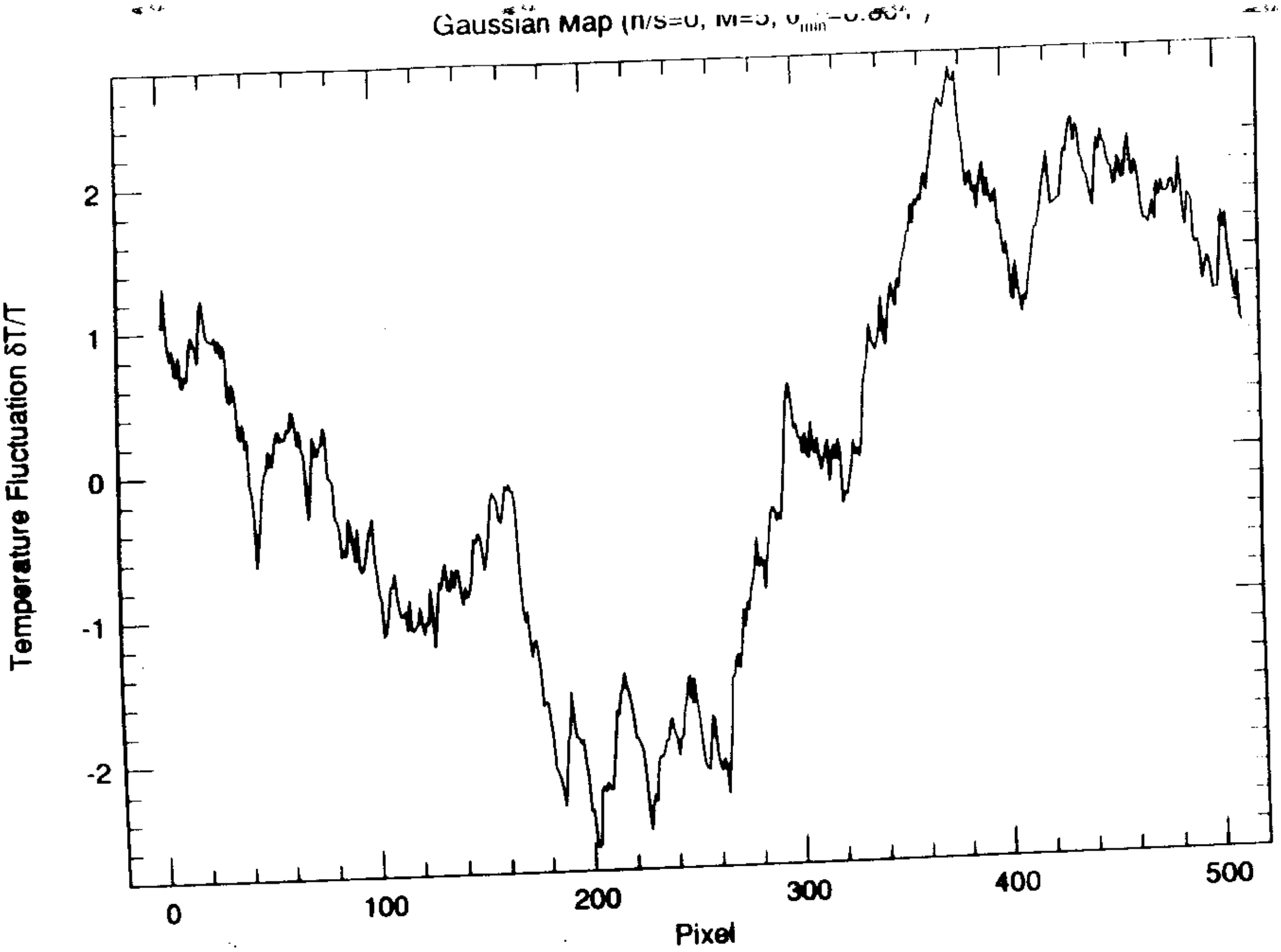


Figure 6b

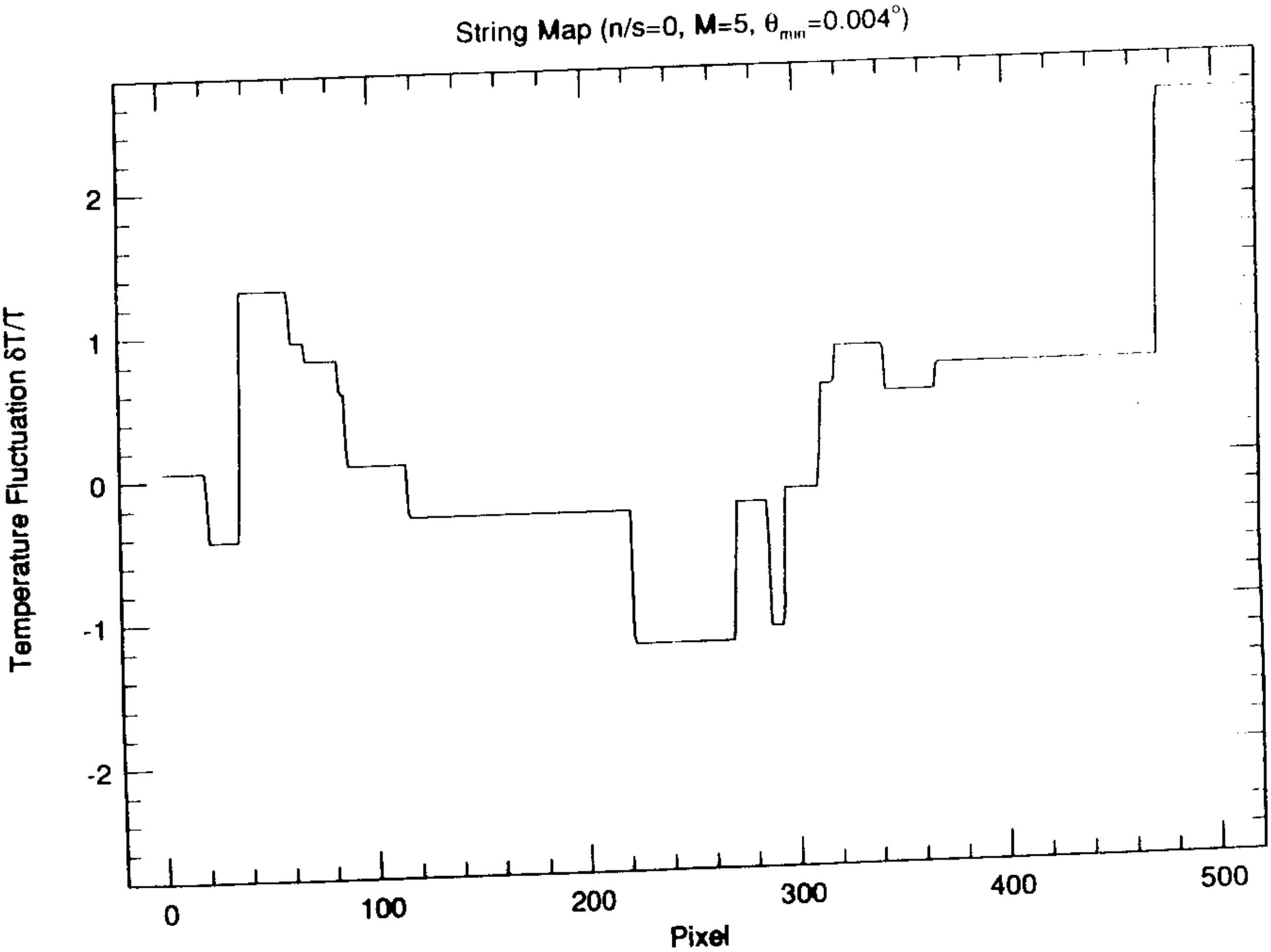


Figure 6a

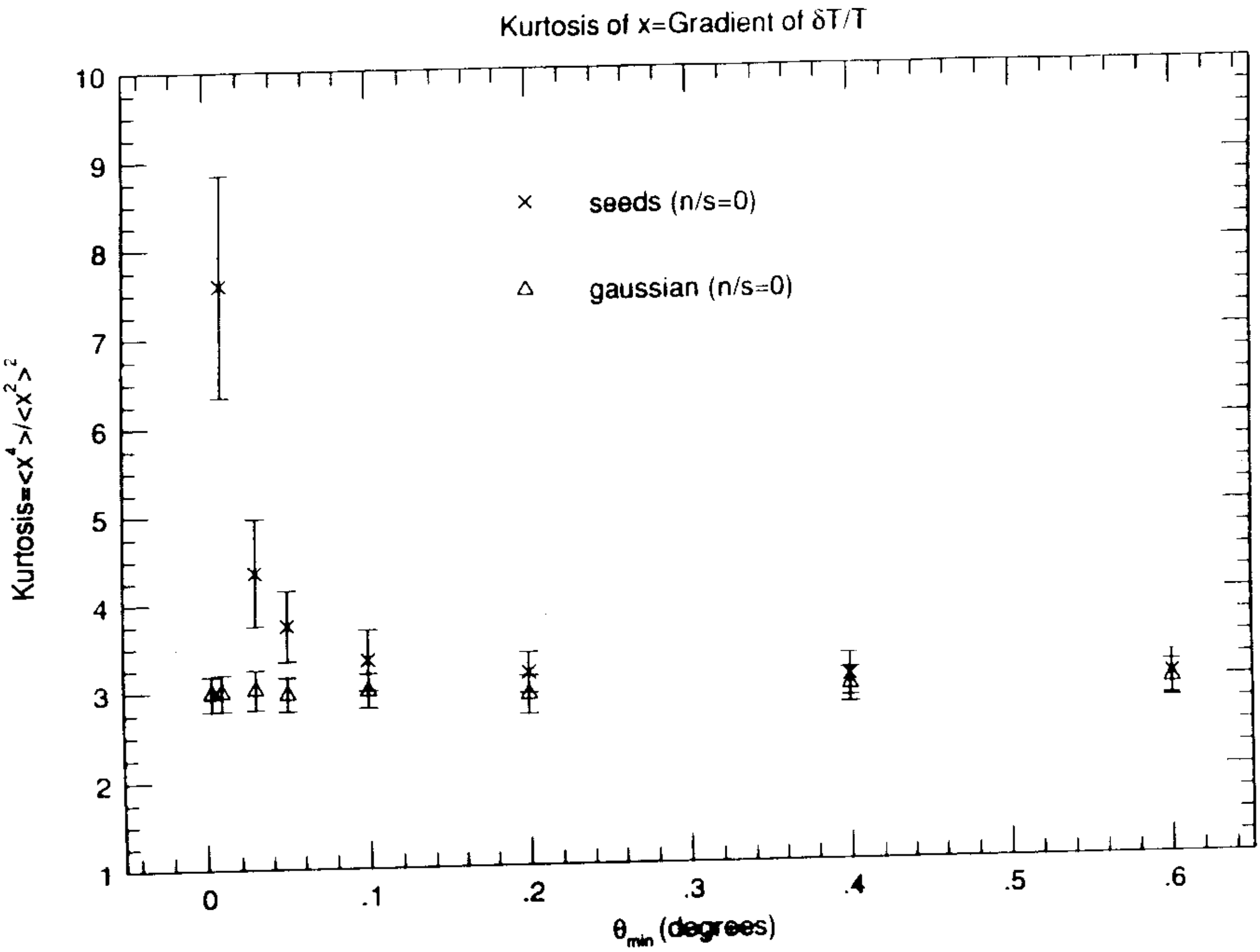


Figure 9

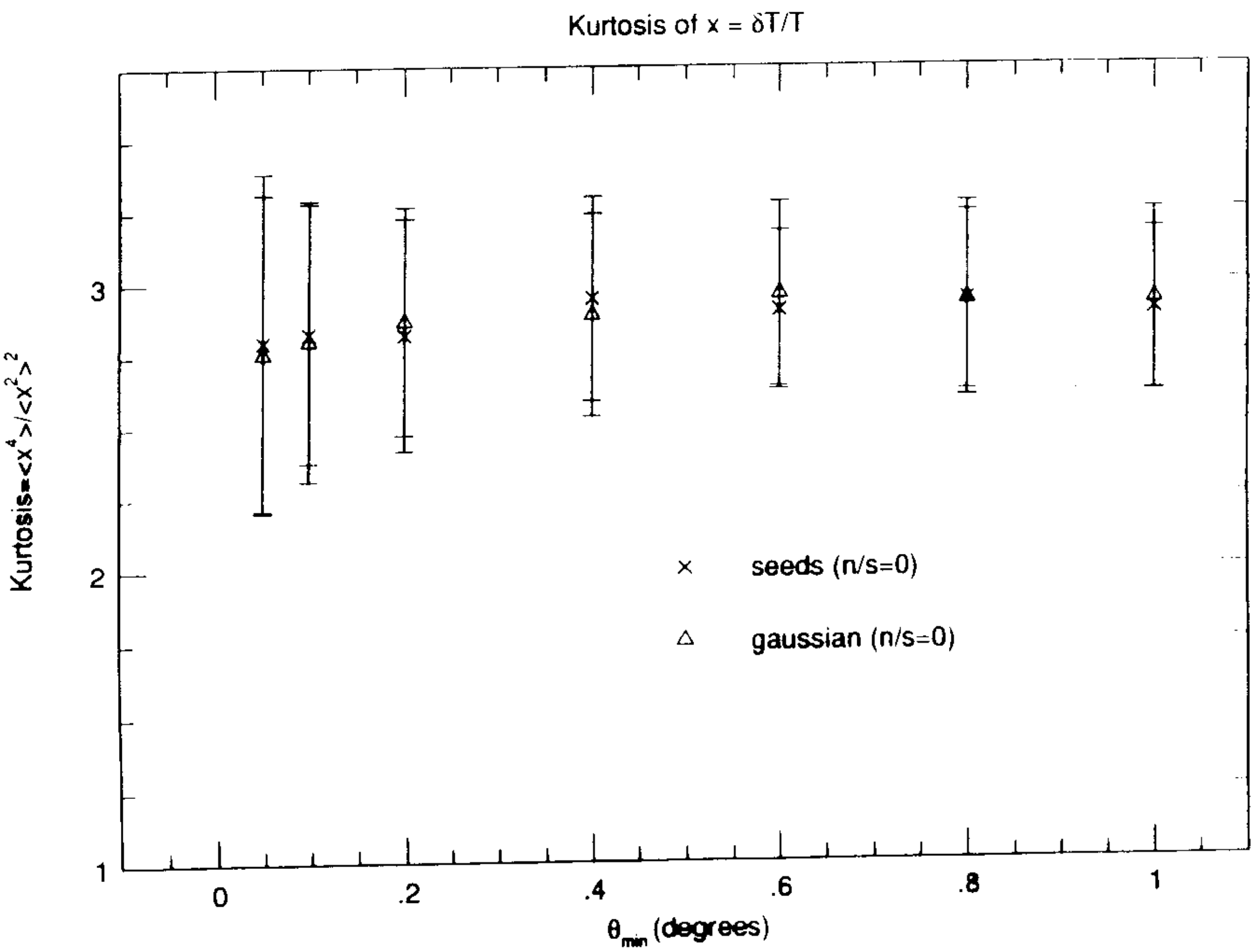
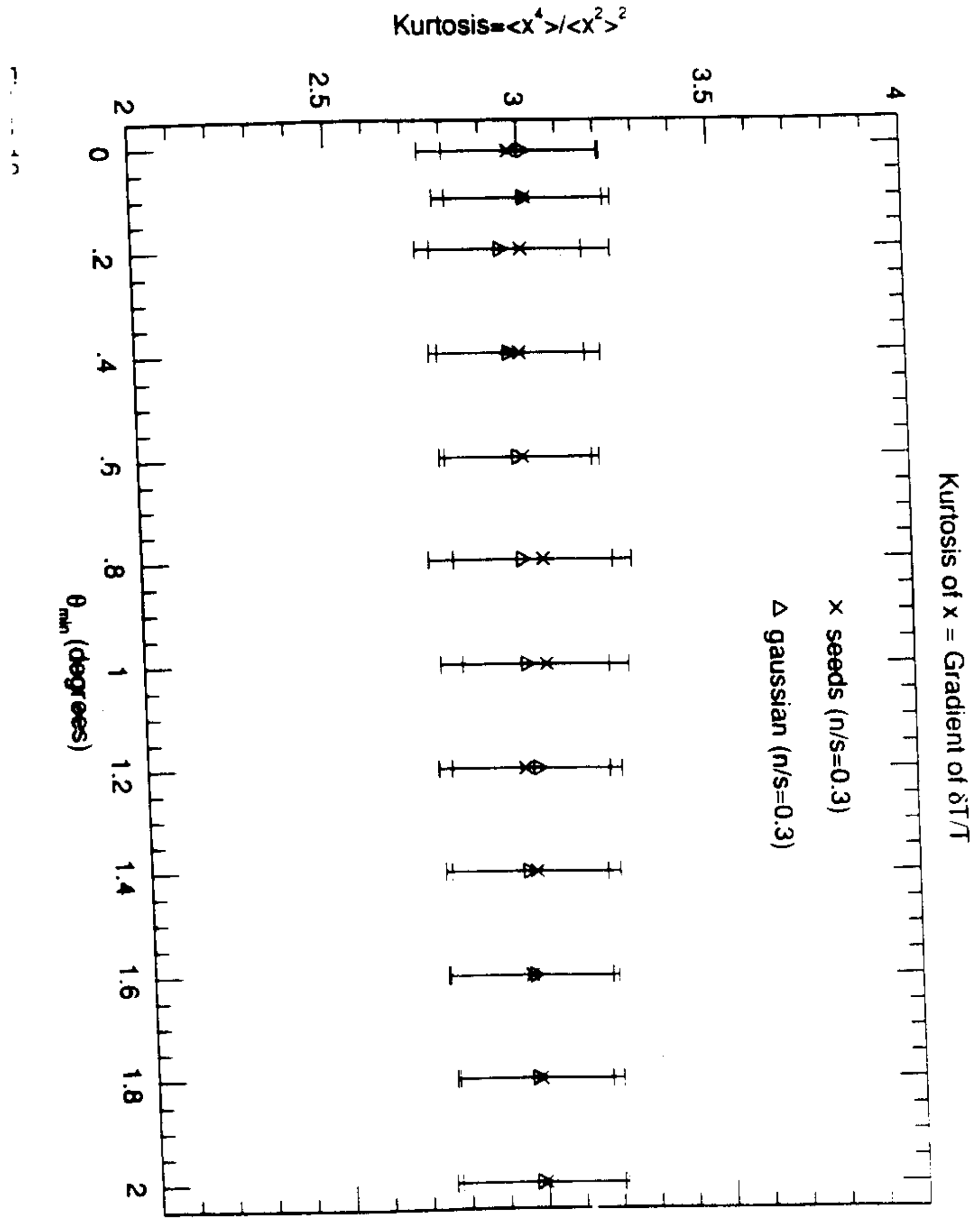


Figure 8



Conclusion

1. Cosmic Strings are linear concentrations of trapped energy predicted by GUTs to form during a phase transition in the early universe.

2. For a natural value of the single free parameter μ strings may provide the seeds for structure formation

Achievements	Challenges	Predictions
$(G\mu)_{LSS} \simeq (G\mu)_{CMB} \simeq (G\mu)_{GUT}$	Difficulty of simulations, calculations	Non-gaussian CMB sky ($\theta<1$
Sheets, filaments of galaxies	Scaling of velocity field	Linear Lensing
Power on large scales (with HDM)	msec pulsar constraints	
Consistent with CMB experiments		
Scaling! Consistency with B[illegible]		