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SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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LECTURE ON TRINIFICATION

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Please note: These are preliminary notes intended for internal distribution only.

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Supersymmetric Unification

GOALS

- 'Naturally' incorporate unification of the three gauge couplings at scales $\sim 10^{16}$ GeV (as 'observed' at LEP), consistent with $\sin^2\theta_W$;
- no Gauge Hierarchy Problem;
- Explain the stability of the proton;
- Origin of GUT scale;
- Origin of fermion masses, mixings, $\cancel{CP}$;
- Higgs & sparticle masses of MSSM;
- Applications in Cosmology (e.g. inflation, dark matter, ...)
- Merger in superstrings, susy breaking, Λ, ...
- ⋮

Beyond SU(5) & SO(10) SUSY GUTS
& Eventual Simplification

- Minimal GUT models typically do not resolve the 'gauge hierarchy' problem.

In SU(5):

$$W \supset \bar{H}(\bar{5})\,\Sigma(24)\,H(5)$$

$$\Rightarrow \begin{pmatrix} \bar{H}(3) & \bar{H}(2) \end{pmatrix} \begin{pmatrix} a & & & & \\ & a & & & \\ & & a & & \\ & & & b & \\ & & & & b \end{pmatrix} \begin{pmatrix} H(3) \\ H(2) \end{pmatrix}$$

- $\Rightarrow \quad m_{\bar{H}(3)} \sim a \quad , \quad m_{\bar{H}(2)} \sim b$

$\uparrow \sim M_{GUT}$ $\qquad$ $\uparrow$ Also $\sim M_{GUT}$ since $b = -\frac{3}{2}a$ $\quad$ ($\mathrm{Tr}\,\Sigma = 0$)

- ∴ Add $m\,\bar{H}(\bar{5})\,H(5)$ and fine tune (or Try 75, $50 + \overline{50}, \cdots$)

What about $SO(10)$?

Has a number of attractive features.

- $10 + \bar{5} \subset \underset{\sim}{16}$ (unifies a single family)
- Automatically anomaly free
- $m_\nu \neq 0$ (MSW? hot dark matter?)
- Automatic 'Matter Parity' provided gauge breaking by tensor repsns.
- More restrictive mass matrices

How about Gauge Hierarchy?

$$W \supset T_1(10)\ A(45)\ T_2(10)$$

$$\langle A \rangle = \begin{pmatrix} a & & & & \\ & a & & & \\ & & a & & \\ & & & b & \\ & & & & b \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$m_{H(2)} \sim b \quad , \quad m_{H_3} \sim a$$

$a \neq 0$, $b = 0$ possible in $SO(10)$

since Trace of $U(5)$ Generators

are non zero.

Can one exploit this and construct

sensible models ?

- Ignoring Planck scale corrections you need the following:

  3 $\underset{\sim}{45}$'s

  1 $\underset{\sim}{54}$

  2 $\underset{\sim}{10}$'s

  $126 + \overline{126}$ pair

- In addition you will need some discrete symmetries.

- With Planck scale corrections (non-renormalizable terms) need additional symmetries (even local $U(1)$??)

$$G \equiv SU(3)_c \times SU(3)_L \times SU(3)_R$$

(Trinification)

Chiral fermions/bosons (superfields) transform as follows:

$$\lambda_i \equiv (1, \bar{3}, 3)_i = \begin{pmatrix} H^u & H^d & L \\ e^c & \nu^c & N \end{pmatrix}_i$$

$$Q_i = \begin{pmatrix} u \\ d \\ g \end{pmatrix}_i \equiv (3, 3, 1)$$

$$Q_i^c = (u^c \;\; d^c \;\; g^c)_i \equiv (\bar{3}, 1, \bar{3})$$

$SU(3)_L$ acts vertically

$SU(3)_R$ " horizontally

Need Higgs superfields for gauge symmetry breaking. Two supermultiplets needed to break G to $SU(3)_c \times SU(2)_L \times U(1)$. These transform as λ_i and denoted by $\lambda\ (\bar{\lambda})$ and $\lambda'\ (\bar{\lambda}')$ [Note the simplicity. Cf: $SU(5)$, $SO(10)$]

$$|\lambda| = |\bar{\lambda}^*| = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & N \end{pmatrix} \rightarrow SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$\downarrow$$

$$SU(2)_L \times U(1)_Y$$

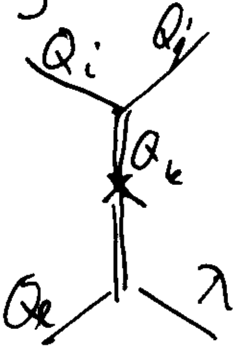
$$|\lambda'| = |\bar{\lambda}'^*| = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \nu^c & 0 \end{pmatrix}.$$

G-invariant couplings include

$$\underbrace{\lambda^3} \qquad Q_i Q_j \underbrace{Q_k \qquad Q_i Q_j^c} \lambda$$

contains the term $H^u H^d \langle N \rangle$

dim 5 p decay



$$\left(\lambda^3 \equiv \epsilon^{\alpha\beta\gamma} \epsilon_{ABC} \lambda_\alpha^A \lambda_\beta^B \lambda_\gamma^C \ , \text{etc} \right)$$

Eliminate the λ^3 term $\Rightarrow$ $H^u - H^d$ pair (in λ) is 'massless'

Simplest way: $\lambda \rightarrow -\lambda$ (under Z_2)

Would like to retain $Q_i Q_j^c \lambda$ (at least for the 3rd family;) (gives masses to add'l fields)

<u>Really need a Z_4</u>:

$$Q_i, Q_j^c \rightarrow i(Q_i, Q_j^c)$$
$$\lambda \rightarrow -\lambda$$

What have we achieved so far?

Supplement G with a discrete Z_4:

- 'Massless' electroweak doublet pair H^u-H^d;
- Dimension five p decay absent;
- Z_2 (subgroup of Z_4) unbroken and acts as 'matter parity';

Are we done?

- What about non-renormalizable intcns?
- Couplings of λ to the remaining fields?

$\vdots$

Gauge Hierarchy & Planck Scale Corrections

Consider the superpotential λ term $\bar{\lambda}^3 (\lambda \bar{\lambda})/M_p^2$ $\boxed{\mu H^u H^d}$

This would lead to $\mu \sim \mathcal{O}(10^{-4}) M_{GUT}$,

assuming $\lambda/M_p \sim 10^{-2}$, $\lambda \sim M_{GUT} \sim 10^{16}$ GeV.

$\therefore$ should be eliminated.

Should $\bar{\lambda} \to -\bar{\lambda}$ under Z_4?

No! Otherwise wont have $\bar{\lambda}^3$ term

$\Rightarrow$ 'massless' doublet pair which we dont need

Need something which allows $\bar{\lambda}^3$ but not above term $\Rightarrow Z_3$ $(: \bar{\lambda} \to \alpha \bar{\lambda})$

In this case get $\mu \sim \lambda^3 \frac{(\lambda \bar{\lambda})^3}{M_p^6}$

$\Rightarrow$ right order of magnitude

In order to resolve the gauge hierarchy problem to all orders (including the $\lambda' - \bar{\lambda}'$ sector) we need one more ingredient:

Discrete R parity such that the superpotential as well as the chiral superfields change sign (under the action of R)

Proof:

Any invariant takes the 'form'

$$\epsilon_{\alpha\beta\gamma}\,\epsilon_{ABC}\;\underbrace{f_{\alpha\beta\gamma,\,ABC}}_{\text{odd product of } \lambda,\bar{\lambda},\lambda',\bar{\lambda}'}$$

Any such invariant as well as its derivatives w.r.t. $\lambda, \bar{\lambda}, \lambda', \bar{\lambda}'$ vanish along the vacuum configuration

$$\frac{\partial W}{\partial \lambda} = \frac{\partial W}{\partial \bar{\lambda}} = \frac{\partial W}{\partial \lambda'} = \frac{\partial W}{\partial \bar{\lambda}'} = 0$$

The 'correct' vacuum corresponds to a flat direction (to all orders in M_P^{-1}!!)

How does M_{GUT} arise ??

M_{GUT} will be fixed only after SUSY breaking has occurred.

$\Rightarrow$ exciting possibility of explaining where 10^{16} GeV comes from.

But M_{GUT} is now linked with the SUSY breaking scale.

Origin of GUT scale

Suggestion:

Spontaneous breaking of R-parity (& SUSY) in the hidden sector induce R-parity noninvariant terms in the observable sector which help generate the GUT scale.

Consider for instance the coupling

$$\frac{R\,(\lambda'\bar{\lambda}')^2}{M_P^2}$$

$\sim M_{SUSY}^{(hidden)}$

which can stabilize the $\lambda'-\bar{\lambda}'$ vev ($\langle R\rangle \neq 0$)

Similar terms arise in the $\lambda - \bar{\lambda}$ sector.

With $\langle R \rangle \sim (m_{3/2} M_P)^{1/2}$, induce a term in the potential

$$\frac{m_{3/2} \, |\lambda'|^6}{M_P^3}$$

which leads to $|\langle \lambda' \rangle| \sim (m_{3/2}/M_P)^{1/4} M_P$

$$\sim M_{GUT} \; (\ll M_P)$$

Similarly for $(\lambda - \bar{\lambda})$ vevs.

An alternative approach relies on R- symmetry. In this case the GUT scale is put in by hand from the begining.

Consider the terms

$$W \supset (\kappa \lambda \bar{\lambda} - \mu^2) S + a \bar{\lambda}^3 + \dots$$

$$\boxed{M_{GUT} = \mu / \sqrt{\kappa}}$$

Under R:

$$S \to e^{iR} S$$

$$\lambda \bar{\lambda} \to \text{invariant}$$

$$\lambda \to e^{-iR/3} \lambda, \quad \bar{\lambda} \to e^{iR/3} \bar{\lambda}$$

$$W \to e^{iR} W$$

$$\lambda' \to e^{iR/3} \lambda'$$
$$\bar{\lambda}' \to e^{iR/3} \bar{\lambda}'$$

"

What about the 'μ term'?

It arises from $\lambda^3 (\lambda' \bar{\lambda}')^3 / M_p^6$

(as before!)
(almost)

$\therefore \quad \mu \sim \text{TeV}$ (as desired)

R-symmetry also eliminates dimension five p decay:

$$\left.\begin{array}{ll} \text{Retain} & Q_i Q_j^c \lambda \\ \text{Eliminate} & Q_i Q_j Q_k \end{array}\right\} \Rightarrow Q_i \to e^{\frac{i 2R}{3}} Q_i$$

Summary

SUSY GUTS based on $[SU(3)]^3$:

- Retain unification of the gauge couplings at $M_X \sim 10^{16}$ GeV, consistent with $\sin^2\theta_W \simeq 0.23$;
- Give rise to the 'light' doublet pair, with $\mu \sim$ TeV;
- May explain how M_X arises from M_P and M_S (SUSY scale);
- Make the proton stable;
- May ~~[illegible]~~ Arise from superstrings;
- Interesting possibility of implementing inflation with $\delta\rho/\rho \sim (M_X/M_P)^2$
- Mixing angles vanish in lowest order

Unification of the Three Gauge Couplings in $SU(3)_c \times SU(3)_L \times SU(3)_R$

Introduce suitable discrete symmetries such that

- Couplings unify (presumably) above the GUT scale;
- Additional symmetries retain the solution to the Gauge Hierarchy Problem;
- Do not induce new 'undesirable' couplings;

It turns out that this can be done.

Moreover one achieves a sort of matter-higgs unification.