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Lecture II, III and IV

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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STANDARD BIG BANG MODEL OF COSMOLOGY

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$$\Longrightarrow \quad \ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

Inflation is <u>by definition</u> when $\ddot{\phi}$ is subdominant
(friction term dominates)

$$\Longrightarrow \quad 3H\dot{\phi} = -V'(\phi)$$

$$\Longrightarrow \quad \dot{\phi} = -\frac{V'(\phi)}{3H(\phi)}$$

$$\Longrightarrow \quad \ddot{\phi} = -\frac{V''(\phi)\,\dot{\phi}}{3H(\phi)} + \frac{V'(\phi)}{3H^2(\phi)}\,H'(\phi)\,\dot{\phi}$$

For inflation to hold:

$$\left|\frac{V''(\phi)\dot{\phi}}{3H(\phi)}\right| \leq 3H(\phi)\dot{\phi} \quad \Longrightarrow \quad |V''(\phi)| \leq 9H^2(\phi) = 24\pi V(\phi) G$$

<u>and</u> $$\left|\frac{V'(\phi)H'(\phi)\dot{\phi}}{3H^2(\phi)}\right| \leq 3H(\phi)\dot{\phi} \Longrightarrow \left|\frac{V'(\phi)M_{Pl}}{V(\phi)}\right| \leq \sqrt{48\pi}$$

The end of slow roll-over occurs when either of these ineq. is saturated

$$\Rightarrow \quad \phi_f = \text{field value at the "end" of inflation}$$
$$\longrightarrow \quad t_f \sim H^{-1}(\phi_f)$$

<u># of e-foldings</u>: $\dot{\phi} = -V'(\phi)/3H \Rightarrow \frac{d\phi}{dt} = -\frac{V'(\phi)}{3H}$

$$\Rightarrow \quad H\,dt = -\frac{3H^2}{V'(\phi)}\,d\phi \Rightarrow \ln\frac{a(t_f)}{a(t_i)} = \int_{t_i}^{t_f} H\,dt = -\int_{\phi_i}^{\phi_f}\frac{3H^2(\phi)\,d\phi}{V'(\phi)} = N(\varphi_i \to \varphi_f) = N(\varphi_i)$$

$$\Rightarrow \quad \frac{a(t_f)}{a(t_i)} = \exp\int_{t_i}^{t_f} H\,dt$$

We can shift the ϕ-field so that $\phi = 0$ at its "global minimum"!

We can also assume $V(\phi) = \lambda \phi^{\nu}$ during inflation

$$\Rightarrow \quad N(\phi_i) = -\int_{\phi_i}^{\phi_f} \frac{3H^2 d\phi}{V(\phi)} = -8\pi G \int \frac{V(\phi)\, d\phi}{V'(\phi)} = -\frac{4\pi G}{\nu}\left(\phi_f^2 - \phi_i^2\right)$$

$$= \frac{4\pi G}{\nu}\phi_i^2 \quad \text{"assuming } \phi_i \gg \phi_f \text{ !! "}$$

$$\Rightarrow \qquad N(\phi) = \frac{4\pi G}{\nu}\phi^2$$

<u>Field oscillations:</u>

After inflation $\ddot{\phi}$-term takes over

$$\Longrightarrow \qquad \ddot{\phi} + V'(\phi) = 0 \quad \Leftrightarrow \quad \phi \text{ oscillates about its global min.}$$

$$\longrightarrow \frac{d\rho}{dt} = \frac{d}{dt}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) = 0 \quad \Leftrightarrow \text{ the energy in } \phi \text{ remain const.}$$

$\longrightarrow$ Not quite true. In reality, the energy density ρ decreas gradually due to exp.

$$\frac{d}{dt}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) \equiv \frac{d\rho}{dt} = -3H\dot{\phi}^2 \equiv -3H(\rho + p)$$

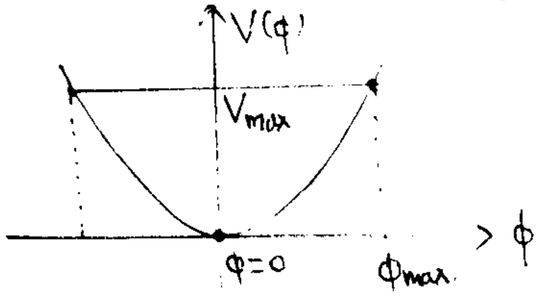
$$\Rightarrow \text{"}\rho + p = \dot{\phi}^2\text{"} \qquad \left\{ \begin{aligned} \rho &= \tfrac{1}{2}\dot{\phi}^2 + V(\phi) \\ p &= \tfrac{1}{2}\dot{\phi}^2 - V(\phi) \end{aligned} \right\}$$

We have already solved this eqn. $\Longrightarrow$ Put

$\rho + p = \dot{\phi}^2 = \gamma\rho$ (after averaging the oscillating component)

$$\Rightarrow \quad \rho \propto a^{-3\gamma} \qquad a(t) \propto t^{2/3\gamma}$$

How do we find γ for an oscillating field?



assuming sym. of pot. $V(\phi) = V(-\phi)$

$$\rho + p = \dot\phi^2 \quad \Rightarrow \quad \gamma = \frac{\int_0^T \dot\phi^2 dt}{\int_0^T \rho\, dt} = \frac{\int_0^{\phi_{max}} \dot\phi\, d\phi}{\int_0^{\phi_{max}} \frac{\rho}{\dot\phi} d\phi}$$

$$\rho = \frac{1}{2}\dot\phi^2 + V(\phi) = V(\phi_m) = V_{max} \quad \Rightarrow \quad \dot\phi = \sqrt{2(V_{max} - V(\phi))}$$

$$\Rightarrow \quad \gamma = 2\,\frac{\int_0^{\phi_m} (1 - V/V_m)^{1/2} d\phi}{\int_0^{\phi_m} (1 - V/V_m)^{-1/2} d\phi}$$

For a potential $V(\phi) = \lambda \phi^\nu \Rightarrow \gamma = \frac{2\nu}{\nu+2}$

$$\Rightarrow \quad \rho \propto a^{-\frac{6\nu}{\nu+2}} \quad , \quad a(t) \propto t^{\frac{\nu+2}{3\nu}}$$

Some examples:

$\nu = 2 \quad : \quad \gamma = 1$ (pressureless "matter") $\Rightarrow \rho \propto a^{-3}$, $a(t) \propto t^{2/3}$

→ this is expected since a "coherent" oscillating massive free field is like a distribution of static massive particles!!

$\nu = 4$: $\gamma = 4/3$ ("radiation") $\Rightarrow$ $\rho \propto a^{-4}$, $a(t) \propto t^{1/2}$

$\nu = 6$: $\gamma = 3/2$ $\Rightarrow$ $\rho \propto a^{-4.5}$, $a(t) \propto t^{4/9}$

($\gamma - 1 = 1/2$ more pressure than "radiation") (slower than "rad")

N.B:

For $\nu = 4, 6, \ldots$ there is enough pressure <u>not</u> to let perturbations grow!!

For $\nu = 2$ perturbations grow like in "matter"

Thus "coherent oscillation" period starts right after inflation and stops when ϕ decays ($t_d = \Gamma_\phi^{-1}$).

$\Rightarrow$ The total exp. during this period is

$$\frac{a(t_d)}{a(t_f)} = \left(\frac{t_d}{t_f} \right)^{\frac{\nu+2}{3\nu}}$$

<u>Decay of ϕ-field:</u>

We must reintroduce Γ_ϕ-term

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma_\phi \dot{\phi} + V'(\phi) = 0$$

$$\Longleftrightarrow \quad \frac{d}{dt}\left(\frac{1}{2}\dot{\phi} + V(\phi) \right) = -(3H + \Gamma_\phi)\dot{\phi}^2$$

$$\Longleftrightarrow \quad \dot{\rho} = -(3H + \Gamma_\phi)\gamma\rho$$

$$\Longrightarrow \quad \rho(t) = \rho_f \left(\frac{a(t)}{a(t_f)} \right)^{-3\gamma} \exp\left[-\gamma\Gamma_\phi (t - t_f) \right]$$

The usual term due to exp. exp. decay law of ϕ

Assuming the decay products are "photons" ("new rad")

$$\Longrightarrow \quad \dot{\rho}_r = \underbrace{-4H\rho_r}_{\text{dilution due to exp.}} + \underbrace{\gamma\Gamma_\phi\rho}_{\text{"energy transfer from } \phi \to \chi\text{"}}$$

Assuming also $\rho_r(t_f) = 0$, we solve this equ.

$$\longrightarrow \quad \rho_r(t) = \rho(t_f)\left(\frac{a(t)}{a(t_f)}\right)^{-4}\int_{\gamma\Gamma_\phi t_f}^{\gamma\Gamma_\phi t}\left(\frac{a(t')}{a(t_f)}\right)^{4-3\gamma} e^{u_0-u}\,du$$

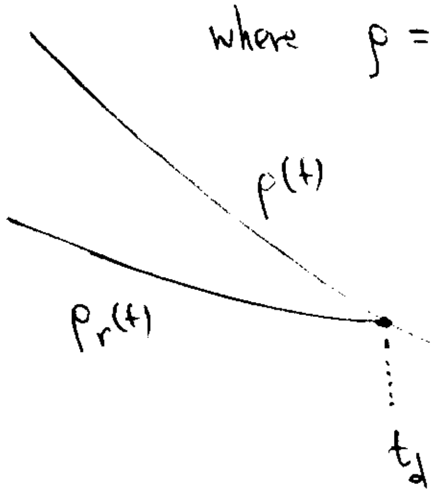
Since $t_f \ll t_d$ (also concentrate on $\nu = 2$ case)

$$\text{Approximate} \longrightarrow \quad \rho_r(t) = \rho(t_f)\left(\frac{t}{t_f}\right)^{-8/3}\int_0^t\left(\frac{t'}{t_f}\right)^{2/3} e^{-\Gamma_\phi t'}\,\Gamma\,dt'$$

Use formula: $\displaystyle\int_0^u x^{p-1}e^{-x}\,dx = e^{-u}\sum_{k>0}^{\infty}\frac{u^{p+k}}{p(p+1)\cdots(p+k)}$

$$\longrightarrow \quad \rho_r = \frac{3}{5}\rho\,(\Gamma_\phi t)\left[1 + \frac{3}{8}(\Gamma_\phi t) + \frac{9}{88}(\Gamma_\phi t)^2 + \cdots\right]$$

where $\rho = \rho(t_f)\left(\frac{t}{t_f}\right)^{-2} e^{-\Gamma_\phi t}$



$$\rho(t_d) = \rho_r(t_d) \iff \Gamma_\phi t_d = 1$$

$$1 + \frac{3}{8} + \frac{9}{88} + \cdots \simeq 3/2$$

$$\longrightarrow \quad t_d = \Gamma_\phi^{-1}$$

So for $t \geq t_d$ we have "radiation" $\longrightarrow$ "Normal developm.

The temperature at t_d

$$T(t_d) = T_r = \text{"reheat temperature" (historic)}$$

Can be calculated : $$T_r^2 = \frac{M_{Pl}}{2(8\pi/3)^{1/2}\, t_d} = \frac{M_{Pl}\,\Gamma_\phi}{2(8\pi/3)^{1/2}}$$

$$\longrightarrow \quad T_r = \left(\frac{32\pi c}{3}\right)^{-1/4} (M_p \Gamma_\phi)^{1/2} \;\propto\; (M_p \Gamma_\phi)^{1/2}$$

A "comoving" (present) scale $\ell \xrightarrow[\text{phys}]{\text{"}T_r\text{"}} \ell \frac{a(t_d)}{a(t_0)} = \ell \left(\frac{T_0}{T_r}\right) \xrightarrow[\text{phys}]{t_f} \ell \left(\frac{T_0}{T_r}\right) \frac{a(t_f)}{a(t_d)} =$
$= \ell \left(\frac{T_0}{T_r}\right)\left(\frac{t_f}{t_d}\right)^{\frac{\nu+2}{3\nu}} = \ell_{phys}(t_f) \Rightarrow$ "Crosses outside the horizon" $H^{-1}(\phi_\ell)\, e^{N(\phi_\ell)} = \ell_{phys}(t_f)$
$\Rightarrow \phi_\ell \Rightarrow N(\phi_\ell) = \#$ of e-foldings that ℓ suffered during Inflation
$\Rightarrow$ Our present Horizon $2H_0^{-1} \simeq 10^4\,\text{Mpc} \rightarrow N_{H_0} \sim 50-55$

DENSITY PERTURBATIONS

Inflation also $\longrightarrow$ dens. flucts $\longrightarrow$ Structure Formation !!!
To understand the origin of $\delta\rho/\rho$ we must first introduce the "event horizon"

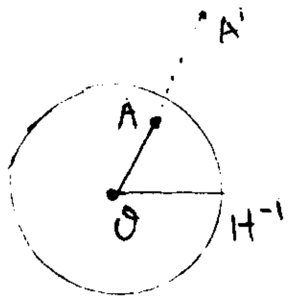
The "Event horizon" at time t includes all point with which we will eventually communicate sending signals now $\longrightarrow$

Instantaneous (at t) radius of "event horizon"

$$d_e(t) = a(t) \int_t^\infty \frac{dt'}{a(t')}$$

This "event horizon" is ∞ for "matter" or "radiation" but not for "inflation" $\longrightarrow$

$d_e(t) = \frac{1}{H} < \infty$ and slowly varying



Points in the "event hor." at t with which we can still communicate sending signals at t are eventually pulled out by exp. and we cannot communicate with them again at later times $t' \gg t$.

$$OA(t) \longrightarrow OA(t') = e^{H(t'-t)} \, OA(t) \gg H^{-1}$$

$\longrightarrow$ We say they crossed outside the "horizon".

The situation is very similar to that of a "black hole" with the only difference that it is up-side-down: We are inside and the "black hole" surrounds us from all sides!!

Exactly as in the case of a black hole there are "thermal" fluct. governed by the Hawking temperature

$$T_H = \frac{H}{2\pi}$$

It turns out that, in de Sitter space, quant. fluct. of all massless fields (inflaton is nearly massless $\leftrightarrow$ "flat" potential) are given by

$$\delta\phi = \frac{H}{2\pi} = T_H$$

Fluct. of ϕ lead to density fluct.

$$\delta\rho = V'(\phi)\,\delta\varphi$$

and as the scale of this pert. crosses outside the horizon they become classical metric perturb

The description of the evolution of these pert outside the horizon is quite subtle due to gauge freedom in gen relativity

However, there is a simple gauge inv quantity

$$\zeta \simeq \frac{\delta\rho}{\rho + p}$$

which remains <u>const</u> outside the horizon

$$\longrightarrow \quad \frac{\delta\rho}{\rho} \quad \left(\begin{array}{l} \text{when the scale crosses inside the} \\ \text{post inflationary horizon} \end{array} \right)$$

$$= \zeta \text{ (when the scale crosses outside the inflationary horizon)}$$

$$\Longrightarrow \quad \left(\frac{\delta\rho}{\rho}\right)_\ell = \left(\frac{\delta\rho}{\dot\phi^2}\right)_{\ell\sim H^{-1}} = \left(\frac{V'(\phi)\,\delta\phi}{\dot\phi^2}\right)_{\ell\sim H^{-1}}$$

$$= \left(\frac{V'(\phi)\,H(\phi)}{2\pi\,\dot\phi^2}\right)_{\ell\sim H^{-1}} = -\left(\frac{3H^2(\phi)}{2\pi\dot\phi}\right)_{\ell\sim H^{-1}} \Leftarrow \text{(almost scale ind.)}$$

Using $\dot\phi = -\frac{V'(\phi)}{3H} \Rightarrow$

$$= \left(\frac{9H^3(\phi)}{2\pi V'(\phi)}\right)_{\ell\sim H^{-1}}$$

$$= \frac{9}{2\pi}\left(\frac{8\pi G}{3}\right)^{3/2}\frac{V(\phi)^{3/2}}{V'(\phi)}\Bigg|_H = 24\left(\frac{2\pi}{3}\right)^{1/2}\frac{V^{3/2}(\phi_\ell)}{M_P^3\,V'(\phi_\ell)}\Bigg|_H$$

Let us take $V(\phi) = \lambda\phi^\nu$ $(\underline{\nu = 4})$

$$\left(\frac{\delta\rho}{\rho}\right)_\ell = 6\left(\frac{2\pi}{3}\right)^{1/2} \lambda^{1/2} \frac{\phi^3}{M_p^3}\Bigg|_H = 6\left(\frac{2\pi}{3}\right)^{1/2} \lambda^{1/2} \left(\frac{N_\ell}{\pi}\right)^{3/2}$$

$$= \frac{6}{\pi}\left(\frac{2}{3}\right)^{1/2} \lambda^{1/2} \left(N_\ell\right)^{3/2}$$

$\Rightarrow \left(\frac{\delta\rho}{\rho}\right)_{H_0} \sim 5\times10^{-6}$ (Measured from COBE)

$N_{H_0} \sim 50 \Rightarrow \lambda \sim 10^{-14} - 10^{-15}$

$\Rightarrow$ Inflaton must be a <u>very weakly coupled</u> field !!!

$\rightarrow$ "Gauge singlet" (?)

For $V(\phi) = \lambda\phi^\nu \longrightarrow \left(\frac{\delta\rho}{\rho}\right)_\ell \propto \phi^{\frac{\nu+2}{2}}$

$$N(\phi) \propto \phi^2$$

$$\Rightarrow \left(\frac{\delta\rho}{\rho}\right)_\ell \propto \left(N_\ell\right)^{\frac{\nu+2}{4}}$$

$$\Rightarrow \left(\frac{\delta\rho}{\rho}\right)_\ell = \left(\frac{\delta\rho}{\rho}\right)_{H_0} \left(\frac{N_\ell}{N_{H_0}}\right)^{\frac{\nu+2}{4}}$$

But $\frac{\ell\,(\mathrm{Mpc})}{10^4} = e^{N_\ell - N_{H_0}} \Rightarrow N_\ell = N_{H_0} + \ln\left(\ell/10^4\,\mathrm{Mpc}\right)$

$$\Rightarrow \left(\frac{N_\ell}{N_{H_0}}\right)^{\frac{\nu+2}{4}} = \left(1 + \ln\left(\ell/10^4\,\mathrm{Mpc}\right)^{1/N_{H_0}}\right)^{\frac{\nu+2}{4}} \simeq \left(\ell/10^4\,\mathrm{Mpc}\right)^{\frac{\nu+2}{4N_{H_0}}}$$

$\Rightarrow \left(\frac{\delta\rho}{\rho}\right)_\ell = \left(\frac{\delta\rho}{\rho}\right)_{H_0} \left(\ell/10^4\,\mathrm{Mpc}\right)^{\frac{\nu+2}{4N_{H_0}} \equiv \alpha_s}$ "Almost scale-inv"

$\longrightarrow \alpha_s(\nu=4) = 0.03 \longrightarrow n = 1 - 2\alpha_s = 0.94$

Density Fluct. in "matter".

Introduce "conformal" time η

RW metric: $ds^2 = -dt^2 + a^2(t)\, d\vec{r}^2 \equiv a^2(\eta)\left(-d\eta^2 + d\vec{r}^2\right)$

"conf. exp Minkowski space"

$$\Rightarrow \quad H = \frac{\dot{a}}{a} = \frac{a'(\eta)}{a^2(\eta)}$$

Fried-equ. $\Rightarrow$ $\frac{1}{a^2}\left(\frac{a'}{a}\right)^2 = \frac{8\pi G}{3}\rho$

En-Mom cons $\Rightarrow$ $\rho' = -3H(\rho+p)$

"matter" $\xrightarrow[p \propto a^{-3}]{}$ $a = \left(\frac{\eta}{\eta_0}\right)^2$; $\frac{a'}{a} = \frac{2}{\eta}$

a^2 present

"Newtonian eq." $\longrightarrow$ $\delta''_{\vec{k}}(\eta) + \frac{a'}{a}\delta'_{\vec{k}}(\eta) - 4\pi G\rho a^2 \delta_{\vec{k}}(\eta) = 0$

2 is missing

Growing Mode $\rightarrow$ $\delta_{\vec{k}}(\eta) \propto \eta^2 \propto a(\eta)$

"Gaussian random variable"

$$\longrightarrow \quad \delta_{\vec{k}}(\eta) = \epsilon_H \left(\frac{k\eta}{2}\right) \hat{s}(\vec{k})$$

amplitude at "hor. cross"

Define: A phys scale $\ell_{phys.}$ crosses inside the horizon

iff $\frac{\ell_{phys}}{2\pi} = \frac{1}{H(\eta_H)}$

$$\rightarrow \quad \frac{a\ell}{2\pi} = \frac{1}{H} = \frac{a^2}{a'} \Rightarrow \quad \frac{\ell}{2\pi} \equiv \frac{1}{k} = \frac{a}{a'} = \frac{2}{\eta_H} \Rightarrow \frac{k\eta}{2}\Big|_H = 1$$

So at "horizon crossing"

$$\tilde{\delta}_{\bar{k}}(\eta_H) = \epsilon_H \, \hat{s}(\bar{k}) \longrightarrow \text{For scale inv. pert.} \Rightarrow \epsilon_H = \text{const}^{t}$$

The perturbation to the scalar "gravitational potential" $\Phi \to$

$$\Phi = -4\pi G \frac{a^2}{k^2} \rho \, \delta_{\bar{k}}(\eta) \qquad (\text{Poisson's eq.})$$

$\underrightarrow{\text{F-eq.}}$ $\Phi = -\frac{3}{2}\left(\frac{a'}{a}\right)^2 \frac{1}{k^2} \delta_{\bar{k}}(\eta) = \frac{3}{2}\left(\frac{2}{\eta k}\right)^2 \delta_{\underline{k}}(\eta)$

$= -\frac{3}{2}\epsilon_H \hat{s}(\bar{k})$ "always"

Gaussian rand. variable $\hat{s}(\bar{k})$:

$$\langle \hat{s}(\bar{k}) \rangle = 0$$

"deno fluct is dimensionless in both x, $\bar{k}$-space"

$$\langle \hat{s}(\bar{k}) \, \hat{s}(\bar{k}') \rangle = \frac{1}{k^3} \delta(\bar{k} - \bar{k}')$$

Power spectra:

$$\tilde{\delta}(\vec{x}, \eta) = \int d^3k \, \delta_{\underline{k}}(\eta) \, e^{i\bar{k}\bar{x}}$$

Def. correlation funct $\xi(r) = \langle \tilde{\delta}^*(\bar{x}, \eta) \, \delta(\bar{x} + \bar{r}, \eta) \rangle$

$$= \int d^3k \, d^3k' \, e^{-i\bar{k}\bar{r}} \epsilon_H^2 \left(\frac{k\eta}{2}\right)^2 \left(\frac{k'\eta}{2}\right)^2 \langle \hat{s}(k) \, \hat{s}(k') \rangle$$

$$= \int d^3k \, e^{-i\bar{k}\bar{r}} \epsilon_H^2 \left(\frac{k\eta}{2}\right)^4 \frac{1}{k^3}$$

$\longrightarrow$ Spectral fn. $P(k, \eta) = \epsilon_H^2 \frac{\eta^4}{16} k \quad (n = 1)$

In gen. $P \propto k^n \quad \boxed{n = 1 - 2\alpha_s}$

Mass Fluct.

Mass Perturb. in a sphere of radius R "comoving"

$$\frac{\delta M}{M}(R) = \frac{\int_V d^3x \, \rho \, \delta(\bar{x},\eta)}{\int_V d^3x \, \rho} = \frac{\int_V d^3x \, \rho \int d^3k \, \delta(\bar{k},\eta) \, e^{i\underline{k}\cdot x}}{\int_V d^3x \, \rho}$$

$$= \int d^3k \, \delta_{\underline{k}}(\eta) \, W(kR)$$

Window fn: $W(kR) = \int_V \frac{d^3x}{V} \, e^{i\underline{k}\cdot x}$

$$= \frac{3}{k^3R^3}\left(\sin(kR) - kR\cos(kR)\right)$$

Appr. $W(kR) = \begin{cases} 0 & kR > 1 \\ 1 & kR \leqslant 1 \end{cases}$

mean square mass fluct $\Rightarrow$

$$\left\langle \frac{\delta M^*}{M}(R) \, \frac{\delta M}{M}(R) \right\rangle = \int d^3k \, d^3k' \langle \delta^*_{\underline{k}}(\eta) \, \delta_{\underline{k}'}(\eta) \rangle W(kR) \, W(k'R)$$

$$= \int \frac{d^3k}{k^3} \, \epsilon_H^2 \left(\frac{k\eta}{2}\right)^4 W^2(kR)$$

$$\simeq k^3 \frac{1}{k^3} \, \epsilon_H^2 \left(\frac{k\eta}{2}\right)^4 \Bigg|_{k\sim R^{-1}} \simeq \epsilon_H^2$$

$\Downarrow$

Scale-inv.

at horizon crossing ($k^{-1} \sim R = \eta/2$)

TEMPERATURE FLUCTUATIONS

Existence of density inhomogeneities $\longrightarrow$ temperature fluct. of CBR

For $\theta \geq 2^\circ$, the dominate effect is the "Sachs-Wolfe effect"

dens. fluct. on the "last-scattering surface" $\Rightarrow$ "scalar potential fluct. Φ" $\rightarrow$ temp fluct. (regions with a deep grav. potential will cause γ to lose energy as they climb up the well $\rightarrow$ appear cooler)

For $\theta \leq 2^\circ$ two other phys effects dom.

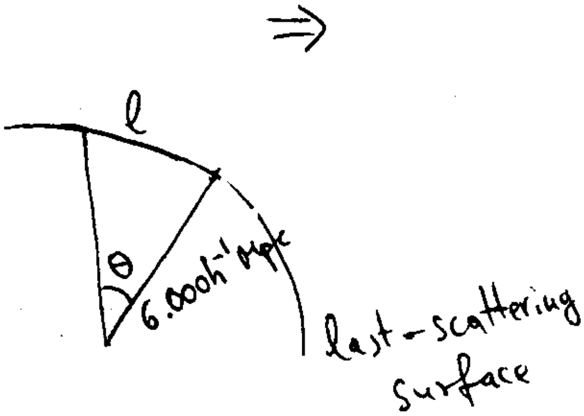
$\rightarrow$ (i) motion of last-scattering surface $\rightarrow$ Doppler shifts

(ii) intrinsic fluct. of photon temp.

More difficult to calculate (microphysics: ionization history, photon streaming, ...)

"Sachs-Wolfe effect"

$$\Rightarrow \qquad \left(\frac{\delta T}{T}\right)_\theta = -\left(\frac{\Phi}{3}\right)_\ell$$



ℓ = "comoving" scale which subtends the angle θ = $100h^{-1}$ Mpc $(\theta/\deg)$

We know $\Phi = -\frac{3}{2}\epsilon_H \hat{s}(\bar{k})$ "always"

$$\Rightarrow \left(\frac{\delta T}{T}\right)_\theta = \frac{1}{2}\epsilon_H \hat{s}(\bar{k})$$

We also know $\left(\frac{\delta\rho}{\rho}\right)_{\ell = k^{-1}} \equiv \delta_{\bar{k}}(\eta_H) = \epsilon_H \hat{s}(\bar{k})$

$$\Rightarrow \left(\frac{\delta T}{T}\right)_\theta = \frac{1}{2}\left(\frac{\delta\rho}{\rho}\right)_\ell$$

If $\theta \simeq 60^\circ \longrightarrow \ell =$ COBE scale (our present horizon)

$$\left(\frac{\delta\rho}{\rho}\right)_\ell \propto \frac{V^{3/2}(\phi_\ell)}{M_p^3 V'(\phi_\ell)} \propto N_\ell^{\frac{\nu+2}{4}}$$

$$\left(\frac{\delta T}{T}\right)_S^2 \propto \frac{V^3(\phi_\ell)}{M_p^6 V'(\phi_\ell)} \propto N_\ell^{\frac{\nu+2}{2}}$$

Analysis of temperature fluct. in spherical harmonics

$\Rightarrow$ Quadrupole unisotropy due to Sachs–Wolfe effect ("scalar")

$$: \left(\frac{\delta T}{T}\right)_{Q-S}^2 = \frac{32\pi}{45}\frac{V^3}{V'^2 M_{Pl}^6}$$

$$(V(\phi) = \lambda\phi^\nu) \qquad = \frac{32\pi}{45\nu^2 M_p^6}\lambda\,\phi^{\nu+2} \propto \lambda N^{\frac{\nu+2}{2}}$$

Comparing this with COBE

$$\frac{\delta T}{T} = 6 \times 10^{-6}$$

$$\longrightarrow \quad \lambda \sim 10^{-14} - 10^{-15} \quad (\text{for } \nu = 4)$$

$$\text{COBE} \Rightarrow \quad n = 1.1 \pm 0.5$$

$$\epsilon_H = (5.3 \pm 1.3) \times 10^{-6}$$

Gravitational waves:

Graviton is massless and has two polarizations

"scalar fields" $\longrightarrow$ ϕ^i $(i=1,2)$ related to the dimensionless tensor

metric perturbations $\longrightarrow$ $h^i = \sqrt{16\pi G}\,\phi^i$

They obey massless Klein-Gordon eq.

$$\rightarrow \quad \ddot{h}^i_{\underline{k}} + 3H\dot{h}^i_{\underline{k}} + \frac{k^2}{a^2} h^i_{\underline{k}} = 0$$

"superhorizon modes" $\quad k^{-1} \gg \frac{1}{aH} \quad \Longrightarrow \quad h^i_k = \text{const.}$

→ like dens. pert. they become classical metric pert. as they cross outside the horizon with const. amplitude till postinflation hor. crossing!

The amplitude in "de Sitter space" is $h^i_{\underline{k}} = \sqrt{16\pi G}\,(H/2\pi)$
$\sim 2H/M_p\sqrt{\pi}$

These gravitational waves have ampl. at horizon crossing

$$h \simeq 4(2/3)^{1/2} \frac{V^{1/2}(\phi)}{M_p^2}$$

and they also produce temp. fluct.

$$\left(\frac{\delta T}{T}\right)_\theta \simeq h_\ell$$

$\longrightarrow \left(\frac{\delta T}{T}\right)^2_{Q-T} \simeq 0.6 \frac{V(\phi)}{M_{P\ell}^4}$ is the quadrupole tensor anisotropic of CBR to be added to the "scalar"

$$\Rightarrow r \equiv \frac{T}{S} \equiv \frac{(\delta T/T)^2_{Q-T}}{(\delta T/T)^2_{Q-S}} \cong 0.28 \frac{M_{P\ell}^2 V'^2(\phi)}{V^2(\phi)} \simeq \frac{3.4\,\nu}{N_H}$$