INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

SMR.762 - 19

Lectures I and II

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

13 June - 29 July 1994

NON-PERTURBATIVE ELECTROWEAK EFFECTS IN BARYOGENESIS

M. SHAPOSHNIKOV
CERN
TH DIVISION
GENEVA, SWITZERLAND

Please note: These are preliminary notes intended for internal distribution only.

Non-perturbative EW effects & baryogenesis

M. Shaposhnikov

3 TOPICS:

* Phase transitions at high T

** B- violation in SM

*** Electroweak baryogenesis mechanisms

Why this is interesting?

1. 1st order phase transitions: large deviations from th. equilibrium

   * Monopole production
   ** cosmic string production
   *** baryogenesis
   **** density perturbations
   - magnetic fields in the universe
   - axions

   etc

2. B- violation: may explain baryonic asymmetry

   Might be tested experimentally???

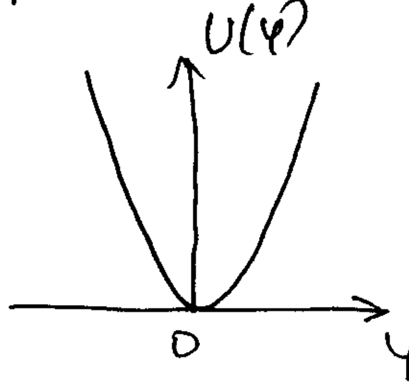
3. fix the spectrum of EW theory ??!

Symmetry restoration in $\lambda\varphi^4$ theory.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\varphi)^2 - \frac{m^2}{2}\varphi^2 - \frac{\lambda}{4}\varphi^4 \qquad (\lambda > 0)$$

$$V(\varphi) = \frac{m^2}{2}\varphi^2 + \frac{\lambda}{4}\varphi^4$$

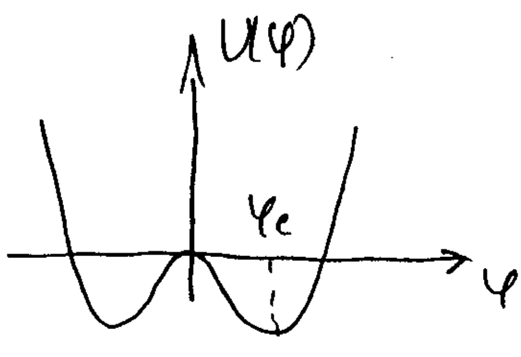
(i) $m^2 > 0$:



$$\varphi = \frac{1}{(2\pi)^3}\int \frac{d^3k}{2k^0}\left[e^{ikx}a^+(k) + e^{-ikx}a^-(k)\right]$$

$$[a, a^+] = (2\pi)^3 2k^0 \delta(k-k')$$
$$k^0 = \sqrt{\vec{k}^2 + m^2}$$

(ii) $m^2 < 0$: spontaneous breaking of the symmetry $\varphi \to -\varphi$



φ_c :

$$\frac{dU}{d\varphi} = m^2\varphi + \lambda\varphi^3 \Rightarrow$$

$$\varphi_c^2 = -\frac{m^2}{\lambda}$$

Quantization:

$$\varphi = \varphi_c + \chi \, ; \qquad \varphi_c = +\sqrt{\frac{m^2}{\lambda}}$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \chi)^2 - V(\chi);$$

$$V(\chi) = m^2\chi^2 + \frac{\lambda}{4}\chi^4 + \lambda\varphi_c\chi^3$$

$$\text{mass of } \chi = 2m^2 = m_\chi^2$$

Finite temperatures

density matrix $\rho = \frac{1}{Z}\exp(-H/T)$

H is the Hamiltonian

tree approximation for H:

$$H = \underbrace{\int \frac{d^3K}{(2\pi)^3 2K^0}}_{\text{Lorentz-invariant measure}} \cdot \underbrace{K^0}_{\text{energy of excitation}} a^\dagger(K)\,a(K)$$

Lorentz-invariant measure

energy of excitation

For bosons:

$$\langle a^+_k a_{k'} \rangle = \frac{1}{Z} \mathrm{Tr}\, a^+_k a_{k'} \exp(-H/T) =$$

$$= (2\pi)^3 2k^0 n_B(\varepsilon_k) \delta(k-k') \quad \varepsilon_k = k^0 = \sqrt{k^2+m^2},$$

$$n_B = \frac{1}{e^{\varepsilon_k/T}-1} \Leftarrow \text{Bose distribution}$$

For fermions:

$$\langle a^+_k a_{k'} \rangle = (2\pi)^3 2k^0 n_F(\varepsilon_k) \delta(k-k')$$

$$n_F(\varepsilon_k) = \frac{1}{e^{\varepsilon_k/T}+1} \Leftarrow \text{Fermi distributio}$$

—//—

equation of motion

$$(\Box + m^2)\varphi \equiv \ddot{\varphi} - \Delta\varphi + m^2\varphi = -\lambda\varphi^3$$

Mean field:

$$\tilde{\varphi} = \frac{1}{Z} \mathrm{Tr}\, e^{-H/T} \varphi \Rightarrow$$

$$(\Box + m^2)\tilde{\varphi} = -\lambda \langle \varphi^3 \rangle$$

(1c)

$$\langle \varphi^3 \rangle \approx 3\langle \varphi^2\rangle\langle\varphi\rangle + \langle\varphi\rangle^3 \approx 3\langle\varphi^2\rangle\tilde{\varphi} + \tilde{\varphi}^3$$

symmetry factor:

$\underbrace{\varphi\ \varphi}\ \varphi$: 3

$\langle\varphi^2\rangle =$ 

$$\langle\varphi^2\rangle = \left\langle \int \frac{d^3k}{(2\pi)^3 2k^0}\left[a_k^+ + a_k\right] \int \frac{d^3k'}{(2\pi)^3 2k^0}\left[a_{k'}^+ + a_k'\right]\right\rangle$$

$$= 2\int \frac{d^3k}{(2\pi)^3 2k^0} n_B(k) + \text{infinities} \Rightarrow$$

$$\langle\varphi^2\rangle = \frac{T^2}{12} \quad (\text{for } m \ll T) \Rightarrow$$

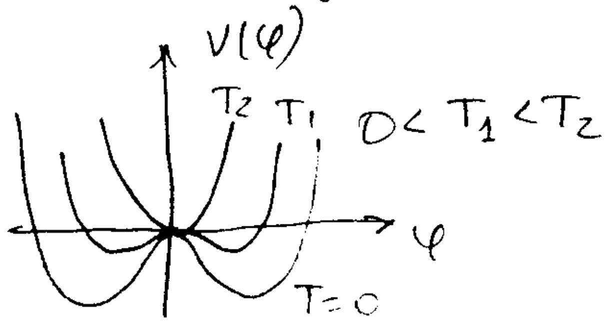
$$\left[\Box\tilde{\varphi} + m^2\tilde{\varphi}\right] = -\frac{T^2}{4}\tilde{\varphi} - \lambda\tilde{\varphi}^3$$

or,

$$\left[\Box\tilde{\varphi} + m_{eff}^2\tilde{\varphi}\right] = -\lambda\tilde{\varphi}^3$$

$$m_{eff}^2 = m^2 + \frac{T^2}{4}\lambda$$

Evolution of the potential:

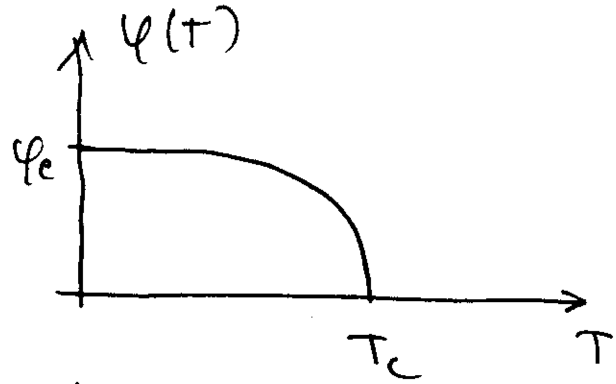


Critical temperature

$$T_c^2 = -\frac{4m^2}{\lambda}$$

Temperature dependence of the expectation value:

$$\langle \varphi(T) \rangle = \sqrt{\varphi_c^2 - \frac{T^2}{4}}$$



Second order phase transition with restoration of the symmetry $\varphi \to -\varphi$

We want to extend this analysis to gauge theories, study higher corrections, ets:

we should have a gauge-invariant formalism.

Phase transitions at high T – formalism

Equilibrium state:

$$\rho = \frac{1}{Z} \exp\left(-\frac{H}{T}\right)$$

H – Hamiltonian ; T : temperature

Real time Green functions:

$$\langle \varphi(x_1, t_1) \ldots \varphi(x_n, t_n) \rangle =$$

$$= \frac{1}{Z} \operatorname{Tr} \varphi(x_1, t_1) \ldots \varphi(x_n, t_n) e^{-H/T}$$

φ : Heizenberg operators

Imaginary time Green functions:

$$\varphi(x, \tau) = e^{H\tau} \varphi(x, 0) e^{-H\tau}$$

Correspondence:

Minkowski space-time

density matrix

$$U = e^{-iHt} \qquad \rho = \frac{1}{Z} e^{-H/T}$$

$$\underline{\underline{it = \tau}} \; ; \quad T \equiv 1/\beta$$

Bosonic fields:

$$G(x_1,\tau_1;x_2,\tau_2)=$$

$$=\theta(\tau_1-\tau_2)\,\langle\varphi(x_1,\tau_1)\,\varphi(x_2,\tau_2)\rangle$$

$$+\,\theta(\tau_2-\tau_1)\,\langle\varphi(x_2,\tau_2)\,\varphi(x_1,\tau_1)\rangle$$

Boundary conditions:

let $\tau_1<\tau_2 \Rightarrow$

$$G=\theta(\tau_2-\tau_1)\,\frac{1}{Z}\,\mathrm{Tr}\Big[e^{H\tau_2}\varphi(x_2,0)\,e^{-H(\tau_2-\tau_1)}\varphi(x_1,0)\,e^{-H\tau_1-\beta H}\Big]=$$

$$=\cancel{\frac{1}{Z}\,\mathrm{Tr}\,e}$$

$$G(\tau_1+\beta,\tau_2)=\frac{1}{Z}\,\mathrm{Tr}\Big[e^{H(\tau_1+\beta)}\varphi(x_1)\,e^{-H(\tau_1+\beta-\tau_2)}\varphi(x_2,0)\,e^{-H\tau_2-\beta H}\Big]=G(\tau_1,\tau_2).$$

periodicity

Bosonic ~~fr~~ Green function is periodic

Fermionic fields

$$G(x_1, \tau_1; x_2, \tau_2) =$$

$$= \theta(\tau_1 - \tau_2) \langle \psi(x_1, \tau_1) \bar{\psi}(x_2, \tau_2) \rangle$$

$$- \theta(\tau_2 - \tau_1) \langle \bar{\psi}(x_2, \tau_2) \psi(x_1, \tau_1) \rangle$$

The same consideration as before:

$$G(\tau_1 + \beta, \tau_2) = - G(\tau_1, \tau_2)$$

Fermionic Green function is anti-periodic.

— // —

Functional integral representation

$$Z = \mathrm{Tr}\, e^{-H/T} = \int dA, d\varphi \langle A, \varphi | e^{-H/T} | A, \varphi \rangle =$$

$$= \int dA\, d\varphi \exp\left[- \int_0^\beta \mathcal{L}_{euclid}\, d^4x \right]$$

Boundary conditions
(due to trace!)

Bosonic fields: $A(0,\vec{x}) = A(\beta,\vec{x})$ periodic

fermionic fields: $\psi(0,\vec{x}) = -\psi(\beta,\vec{x})$ antiperiodic

Hosts $\quad c(0,\vec{x}) = c(\beta,\vec{x})$

periodic boundary conditions!!

Equilibrium statistical mechanics is equivalent to Euclidean field theory on a final time interval $0 \le \tau \le \beta \equiv 1/T$.

Feinmann rules:

Minkowski		finite T
p_0	$\rightarrow$	$i\omega$
d^4p	$\rightarrow$	$2\pi i T \int d^3p \sum_\omega$
$\delta^4(p)$	$\rightarrow$	$(2\pi i T)^{-1} \delta_{\omega,0}\, \delta^3(\vec{p})$

$\omega = 2\pi n T$ – bosons

$\omega = (2n+1)\pi T$ – fermions

Symmetry properties: effective potential

<u>zero temperatures</u>:

$$V(\varphi) = \frac{1}{V} \langle \varphi_c | H | \varphi_c \rangle \Big|_{\text{min on } |\varphi_c\rangle};$$

$$\langle \varphi_c | \hat{\varphi} | \varphi_c \rangle = \varphi_c$$

[redacted]

Functional integral representation:

$$\exp(-V(\varphi)\cdot V) = \int d[\text{fields}]\, e^{-S}\, \delta\Big(\varphi_c - \frac{1}{V}\int d^4x\, \varphi\Big)$$

Physical meaning: $\frac{dV}{d\varphi_c} = 0$ at $\varphi_c \neq 0 \Rightarrow$ $\langle \text{vac} | \hat{\varphi} | \text{vac} \rangle = \varphi_c \neq 0$ – symmetry is broken.

Non-zero temperatures: the same functional integral representation, but new boundary conditions:

$$\int_{-\infty}^{+\infty} dt \to \int_0^{\beta} d\tau, \quad \varphi(x,0) = \varphi(x,\tau)$$

$$\psi(x,0) = -\psi(x,\tau)$$

One - loop at $T = 0$:

$$V_{1\text{-loop}}(\varphi) = \sum \int \frac{d^4 p}{(2\pi)^4} \log\left[p^2 + \underbrace{m^2(\varphi)}_{}\right] =$$

sum over all degrees of freedom

particle mass in φ background

$$= \frac{1}{64\pi^2}\left(\sum_{\text{Bose}} - \sum_{\text{ferm}}\right) m_i^4 \log \frac{m_i^2}{\mu^2}$$

Coleman - Weinberg

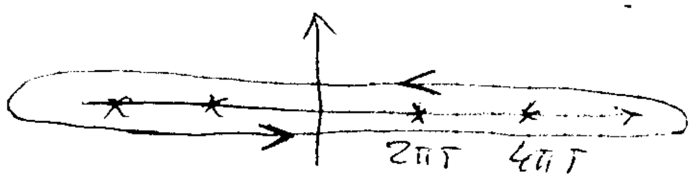
$T \neq 0$:

$$\int \frac{d^4 p}{(2\pi)^4} \to T \sum_n \int \frac{d^3 k}{(2\pi)^3}$$ – thats it!

How to compute finite temperature sums?

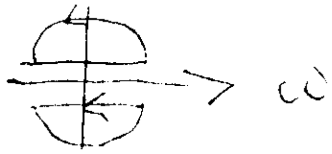
(1)

$$\sum_{n,\,\text{Bose}} f(\omega_n) \sim \frac{1}{2\pi i} \int \frac{d\omega}{2T} f(\omega)\, \mathrm{ctg}\left(\frac{\omega}{2T}\right)$$



$$\sum_{n,\,\text{fermi}} f(\omega_n) = \frac{1}{2\pi i} \int d\omega\, f(\omega)\, \mathrm{tg}\, \frac{\omega}{2T}$$

(2) Change the countour of integration



(4c)

$$V = V_{tree} + \underset{\text{1-loop}}{V_{T=0}} + \Delta V_T$$

$$\Delta V_T = \frac{T^4}{2\pi^2}\left(\sum_{\text{bose}} I_B(m) + \sum_{\text{fermi}} I_f(m)\right)$$

$$I_b = \int_0^\infty p^2 dp \log\left[1 - \exp\left(-\frac{\sqrt{p^2+m^2}}{T}\right)\right]$$

$$I_f = -\int_0^\infty p^2 dp \log\left[1 + \exp\left(-\frac{\sqrt{p^2+m^2}}{T}\right)\right]$$

$$T \to 0 \Rightarrow I_b, I_f \to 0 \text{ as } \exp\left(-\frac{m}{T}\right)$$

$$\Delta V_T \approx -\frac{\pi^2}{90} T^4 \left(\sum_b + \frac{7}{8}\sum_f\right)$$

for $m \to 0$

$$+ \frac{1}{24} T^2 \left(\sum m_b^2 + \frac{1}{2}\sum m_f^2\right) + \ldots O(m^3)$$

For $\lambda\varphi^4$ theory: $m^2(\varphi) = m^2 + 3\lambda\varphi^2$

$$\Delta V_T = \ldots + \frac{1}{24} 3\lambda T^2 \varphi^2 ;$$

$$T_c^2 = \frac{4m^2}{\lambda}$$

Physical meaning:

Mass operator for φ:

$$\simeq \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2+m^2} \to$$

$$\to T\sum \int \frac{d^3k}{(2\pi)^3} \frac{1}{\vec{k}^2+m^2+\omega_n^2} \to$$

$$\to \frac{T}{2\pi i} \int d\omega \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2+m^2+\omega^2} \, \mathrm{ctg} \frac{\omega}{2T} \to$$

poles: $\omega = \pm i\sqrt{\vec{k}^2+m^2} \equiv \epsilon_k$

$$\to \frac{T}{(2\pi)^3} \int \frac{d^3k}{(2k_0)} \, \mathrm{cth} \frac{\epsilon_k}{2T}$$

$$\mathrm{cth} \frac{\epsilon_k}{2T} \approx 1 + 2n_B(\epsilon_k) \to$$

$$\to \int \frac{d^3k}{(2\pi)^3 2k^0} n_B(\epsilon_k) \sim T^2$$

effectively the same calculation as on sheet #

To estimate T_c: just compute

$m^2(T)$.

Electroweak theory: (for simplicity no $U(1)$)

$$m^2(T) = -\frac{1}{2} m_H^2 + \left(\frac{3}{16} g^2 + \frac{1}{2}\lambda + g^2 \frac{m_t^2}{8 m_W^2}\right) T^2$$

Second order p.t.?

Approximations:
(i) 1-loop
(ii) high T expansion

We can easily go beyond (ii).

$$I_b = \ldots + \frac{1}{24} T^2 \sum m_b^2$$

$$- \frac{1}{12\pi} T \sum [m_b^2]^{3/2}$$

Sign $(-)$!

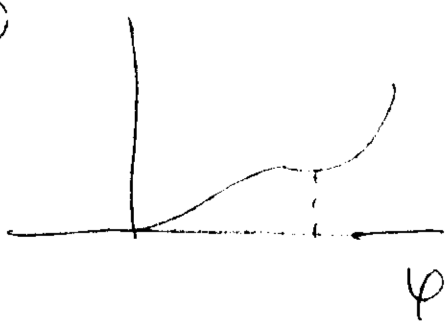
How $[m_b^2]^{3/2}$ can appear?

n = 0 sector: $T g^4 \varphi^4 \int \underbrace{\frac{d^3k}{[k^2+g^2\varphi^2]^2}}_{\sim \frac{1}{g\varphi}} \sim$

$$\sim T(g\varphi)^3 \Rightarrow$$

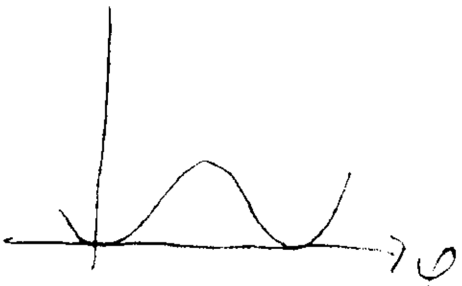
First order p.t. 3 important T:

(1)



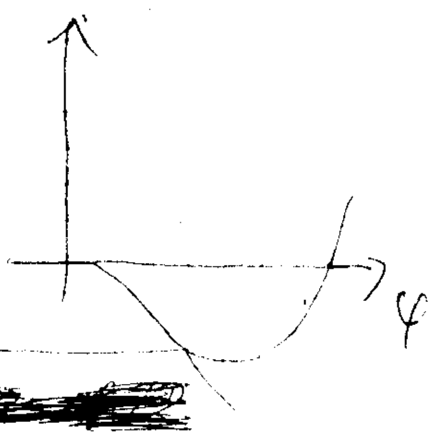
$T=T_f$: additional min at $\langle\varphi\rangle \neq 0$ appears

(2)



T_c: $V(\varphi) = V(\varphi_c)$

(3)



$T=T_0$: unbroken phase is absolutely unstable

[redacted]

Convergence of perturbation theory.

Most dangerous: sector with $n=0$: static modes.

$$\int_0^\beta \mathcal{L}\, d^4x = \frac{1}{T}\int \mathcal{L}\, d^3x$$

$\Uparrow$ only static fields.

Effective 3d theory $\Rightarrow$ back to canonical dimensions

$$\mathcal{L}_{3d} = \frac{1}{4}F_3^2 + (D\varphi_3)^2 + m^2\varphi_3^2 + \lambda_3(\varphi^\dagger\varphi_3)^4$$

$\varphi_3 : [GeV]^{1/2} \quad A_3 : [GeV]^{1/2}$

$\lambda_3 : \lambda T \;;\; g_3^2 = g^2 T \quad [GeV]$: dimension full.

Dimensionless expansion parameter:

$$\frac{(g^2T)^N}{(g\varphi)^N} = \left(\frac{gT}{\varphi}\right)^N \Rightarrow$$

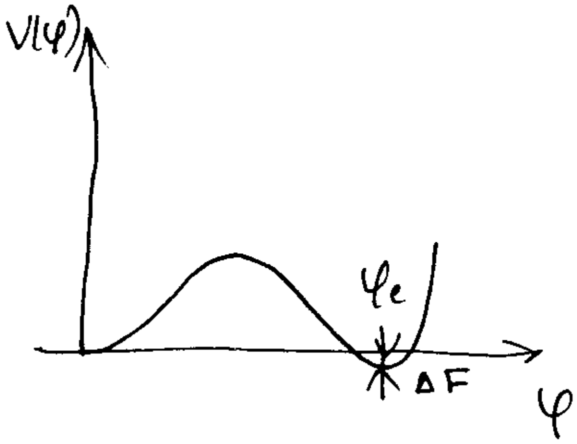
perturbation theory works only for $\varphi \gg gT$! $\Rightarrow$ Pt cannot be used for determination of T_c, etc. Lattice, etc

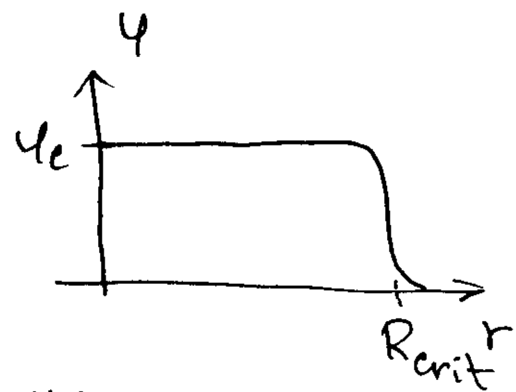
Dynamies of the first order phase transitions at $T \neq 0$.

theory with Lagrangian:

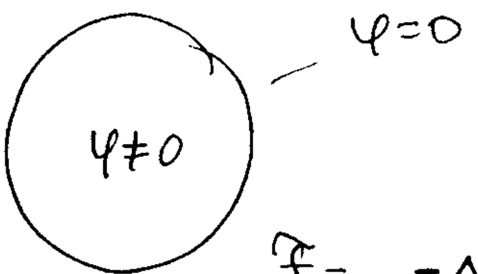
$$\mathcal{L}(\varphi) = \frac{1}{2}(\partial_\mu \varphi)^2 - V(\varphi)$$

Thin wall approximation:





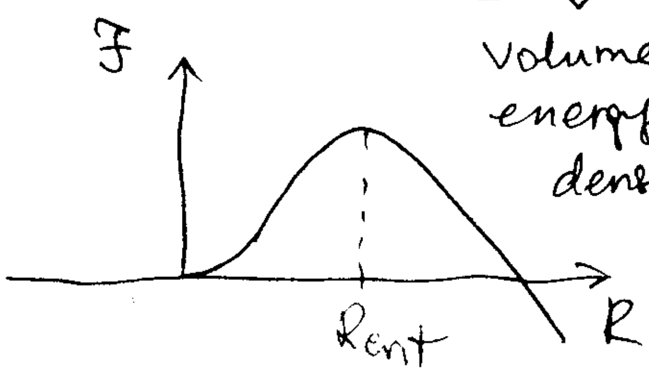
Bubble of the new phase:



Free energy of the bubble:

$$\mathcal{F} = \underbrace{-\Delta F \cdot \frac{4}{3}\pi R^3}_{\text{volume energy density}} + 4\pi R^2 S$$

surface energy density



if $R < R_{crit} \Rightarrow$ bubble collapces

$R > R_{crit} \Rightarrow$ bubble expands

R_{crit}:

$$\frac{\partial F}{\partial R} = -4\pi \Delta F R^2 + 8\pi R S = 0 \Rightarrow$$

$$R_{crit} = \frac{2S}{\Delta F};$$

$$\mathcal{F}_{crit} = \frac{16\pi S_1^3}{3(\Delta F)^2}$$

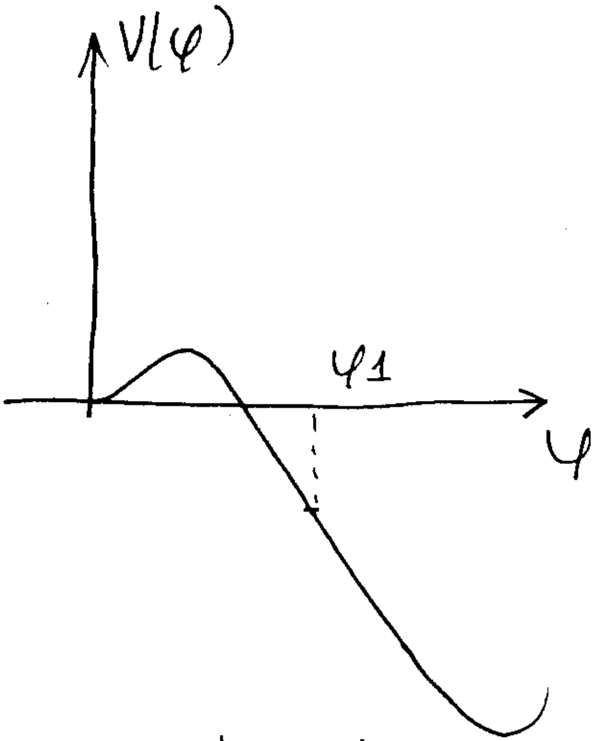
Probability of the bubble nucleation:

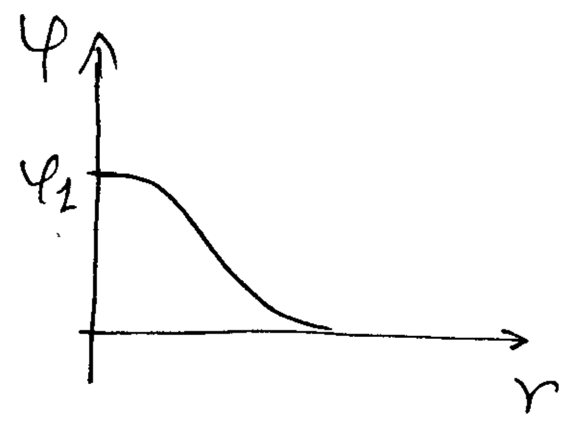
$$P \sim \exp\left(-\frac{F_{crit}}{T}\right) * (\text{prefactor})$$

$$\text{prefactor} \sim \left(\frac{1}{R_{crit}}\right)^4$$

$$S = \int_0^{\varphi_0} d\varphi \sqrt{2V(\varphi)} \Big|_{\Delta F \to 0}.$$

Thick walls





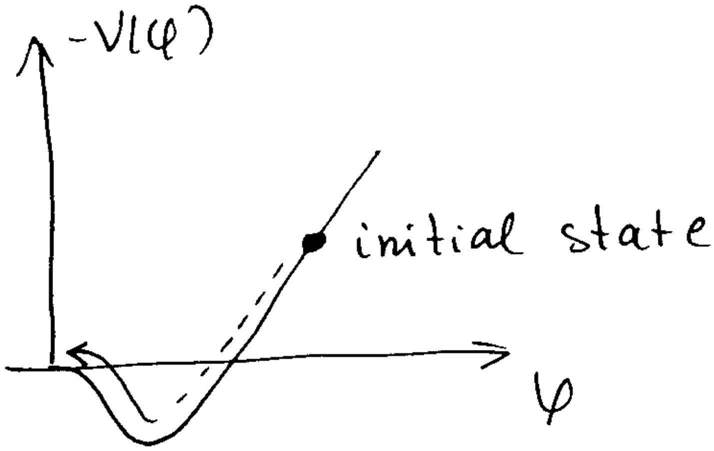
One has to find a saddle point of the free energy:

$$\mathcal{F} \sim \int d^3x \left[\frac{1}{2} (\partial_i \varphi)^2 + V(\varphi) \right]:$$

$\frac{\delta F}{\delta \varphi} = 0 \Rightarrow$ for spherical symmetry

$$\frac{d^2\varphi}{dr^2} + \frac{2}{r} \frac{d\varphi}{dr} = \frac{dV(\varphi)}{d\varphi} \qquad (*)$$

if r is time, then $(*)$ describes a motion of the particle in upside-down potential $V(\varphi)$ with friction $\frac{2}{r}$



$$\mathcal{F}_{crit} = \int d^3x \left[\frac{1}{2}(\partial_i \varphi)^2 + V(\varphi) \right] ;$$

$$\text{Probability} \sim \exp\left(-\mathcal{F}_{crit}/T\right)$$

Elements of the universe evolution

Radiation dominated universe

$$t = \frac{M_0}{T^2} \quad ; \quad M_0 = \frac{M_{Pl}}{1.66\, N_{eff}^{1/2}} \simeq 10^{18}\,\text{GeV}$$

$$t\,[\text{sec}] = \frac{1}{T^2\,[\text{MeV}]}$$

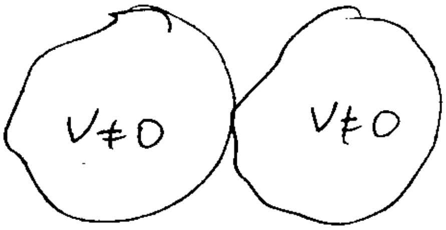
density of entrophy

Constant entrophy : $S \sim R^3 T^3$

$R \sim t^{1/2}$

scale factor

Percolation temperature T^*



(End of the phase transition)
bubbles of a new phase fill out the Universe

$$\mathcal{P}(t) \sim \frac{1}{R^4} \exp\left(-\frac{\mathcal{F}(T)}{T}\right) dt\, dV :$$

probability to create a bubble

Volume of the bubble created at $t = t_1$ at the moment t:

$$\frac{4}{3}\pi v^3 (t-t_1)^3$$

Part of the space occupied by bubbles:

$$\int_{t_c}^{t} \frac{4}{3}\pi v^3 (t-t_1)^3 \frac{1}{R^4} \exp\left(-\frac{\mathcal{F}(T_1)}{T_1}\right) dt_1 \simeq$$

$$\simeq \int \frac{4}{3}\pi v^3 \frac{M_0^4}{(RT_1)^4 T_1^4} \exp\left(-\frac{\mathcal{F}(T_1)}{T_1}\right) \frac{dT}{T} \sim$$

$$\sim \int \frac{dT}{T} \exp\left[-\frac{\mathcal{F}(T_1)}{T_1} + \log \frac{4}{3}\pi v^3 \frac{M_0^4}{(RT)^4 T^4}\right]$$

Phase transition is over if

$$\frac{\mathcal{F}(T^*)}{T^*} \simeq 4 \log \frac{M_0}{T^*}$$

For grand unified theories

$$T \sim 10^{15}\ \text{GeV} \Rightarrow \frac{\mathcal{F}(T^*)}{T^*} \sim 30$$

for electroweak theory:

$$T \sim 100\ \text{GeV} \Rightarrow \frac{\mathcal{F}(T^*)}{T^*} \sim 150$$

Generally, $T_0 < \overset{*}{T} < T_c$

$\uparrow$ critical temperature

$\uparrow$ temperature of absolute instability of the unbroken phase.