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SMR.762 - 3

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

13 June - 29 July 1994

PERTURBATIVE QCD AND CHIRAL LANGRANGIANS

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PERTURBATIVE
QUANTUM
CHROMO
DYNAMICS

- •) INTRODUCTION
- •) ILLUSTRATION OF ASYMPTOTIC FREEDOM IN THE CONTEXT OF $e^+e^- \Rightarrow$ HADRONS
- •) JET PRODUCTION IN $e^+e^- \Rightarrow$ HADRON
- •) HIGH ENERGY COLLISIONS WITH ONE INCOMING HADRON: SCALING AND SCALING VIOLATIONS IN DEEP-INELASTIC SCATTERING
- •) PRODUCTION PHENOMENA IN HADRON-HADRON COLLISIONS

PROTONS, NEUTRONS AND MESONS INTERACT MAINLY VIA STRONG FORCES
STRONG FORCES APPEAR TO HAVE HIGHER SYMMETRY THAN WEAK AND ELECTROMAGNETIC INTERACTIONS. CONSERVE PARITY, AND HAVE (APPROXIMATE) ISOSPIN SYMMETRY

MAIN FEATURES:

- No small parameter is present (unlike QED, weak interactions)
- Characteristic scale $\approx 100 \div 300$ MeV
  - Typical cross sections $\approx 10 \text{ mb} \approx (300 \text{ MeV})^2$
  - Typical lifetime of excitations:
    $$\tau \approx 1/300 \text{ MeV}$$

VERY MUCH DIFFERENT (AND MUCH HARDER) THAN WEAK AND EM INTERACTIONS:
WE DON'T SEE (AT LEAST AT LOW ENERGY) THE "VERTICES" OF STRONG INTERACTIONS

MOTIVATIONS FOR QCD

1) SPECTRUM OF HADRONS

The whole hadron spectrum can be classified by assuming that

a) Hadrons are made up of quarks

$$\left.\begin{array}{l} u = \text{up} \quad (\text{charge } \tfrac{2}{3}) \\ d = \text{down} \quad (\text{charge} = -\tfrac{1}{3}) \\ s = \text{strange} \quad (\text{charge} = -\tfrac{1}{3}) \\ \ldots\ldots\ldots \end{array}\right\} \text{FLAVOURS}$$

b) Each quark "flavour" (u, d or s) comes in three different colours
We can represent quarks with the notation q_i^f, where f is the flavour index $(u, d, s, \ldots)$ and i is the colour index.

c) Observable hadrons are colour singlets under $SU(3)_{\text{COLOURS}}$

"Singlet under SU(3) colour" means that the wavefunction of a hadron must be invariant under $SU(3)_{\text{COLOUR}}$

Example 1:

A state formed by a quark of flavour f and an antiquark of flavour f', in the combination

$$\sum_i |q_i^f \; \bar{q}_i^{f'}\rangle \quad \text{is a singlet!}$$

transforms into

$$\sum_{i,l,m} U_{il} \; U^*_{im} \, |q_l^f \; \bar{q}_m^{f'}\rangle$$

for $U \in SU(3)$ ($U^\dagger U = \mathbb{1}$, $\det U = 1$)

But $\sum_i U_{il} \, U^*_{im} = \sum_i U^\dagger_{mi} \; U_{il} = \delta_{ml}$

Example 2:

$$\sum \varepsilon_{ijl} \, |q_i^f \; q_j^{f'} \; q_l^{f''}\rangle$$

is also a singlet; transforms into:

$$\sum_{kmn} \underbrace{\sum_{ijl} \varepsilon_{ijl} \, U_{ik} \, U_{jm} U_{ln}} \; |q_k^f \; q_m^{f'} \; q_n^{f''}\rangle$$

antisymmetric in k, m, n $\Rightarrow \overset{\updownarrow}{=} \; C \;\; \epsilon_{kmn}$

Only 1 antisymm tensor with 3 indices in 3 dimensions

Easy to show that $C = 1$ (From $\det U = 1$)

Enough to build mesons and barion.

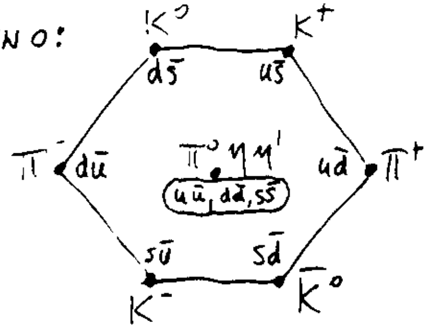
π, K, η : $\sum_i |q_i^f \; \bar{q}_i^{f'}\rangle$ in spin 0 state

$\rho, K^* \ldots$: " in spin 1 state

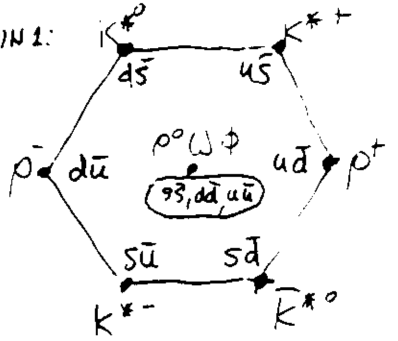
$p, \Lambda, n \ldots$: $\sum_{ijk} \epsilon_{ijk} \, |q_i^f \; q_j^{f'} \; q_k^{f''}\rangle$ spin $\frac{1}{2}$

$\Delta, \Omega \ldots$: " spin $\frac{3}{2}$

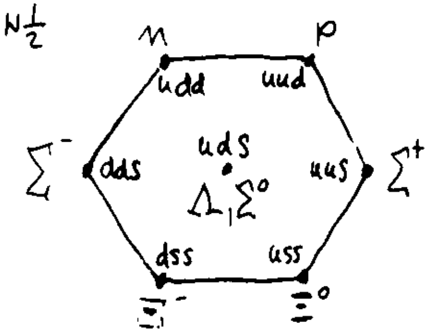
SPIN 0:



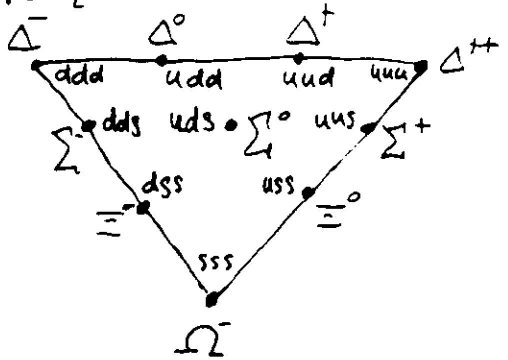
SPIN 1:



SPIN $\frac{1}{2}$



SPIN 3/2



Nice symmetry in each group; difficult to explain without colour

(Δ^{++}: 3 up with spin up; and the exclusion principle?)

ALSO:

Mass differences in each multiplet are well explained by assuming that they are due to quark mass differences (GELL-MANN; OKUBO). Therefore spatial wavefunctions must be the same for each member of the multiplet. Difficult to reconcile this with the fact that some of the members of the multiplet must have odd (antisymmetr wavefunctions.

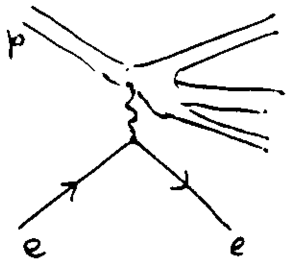
Further:

qq' states would give rise to fractionally charged hadrons. In $SU(3)_c$ we <u>CANNOT</u> make a singlet out of them.

In general, to make a singlet we need $n_q - n_{\bar{q}}$ multiple of 3. It follows that we cannot build fractionally charged combinations which are colour singlet.

2) HIGH ENERGY DYNAMICS

Scaling in Deep-Inelastic-Scattering (DIS) (SLAC-MIT, 1968)



$$p + e \Rightarrow e + X$$

For inclusive inelastic large angle scattering, the differential cross section (in terms of dimensionless variables) "SCALES" with energy:

Bjorke,
Feynm

$$\frac{d\sigma}{dx\,dy} \approx \frac{1}{s} \qquad (s = E_{CM}^2)$$

No dimensionful coefficient on the right hand side!!!

Suggests that strong interactions at high energy <u>RESEMBLES</u> a weakly interacting theory with dimensionless coupling!

(Same effect is seen in $e^+e^- \Rightarrow$ hadrons

3) THE THEORETICAL DISCOVERY OF ASYMPTOTICALLY FREE THEORIES
(GROSS, WILCZEK, POLITZER, 73-74)
('t Hooft (72))

Non abelian gauge theories are weakly coupled at high energies (short distances)
Good candidates to be the theory of strong interactions!

ALL THESE INGREDIENTS PUT TOGETHER POINT TO

A NON-ABELIAN GAUGE THEORY	THE ONLY ASYMPTOTICALLY FREE FIELD THEORIES
COUPLED TO $SU(3)_c$	IT BECOMES STRONG AT LOW ENERGY; IT MAY BIND HADRONS INTO COLOR NEUTRAL SYSTEMS (ASSUME CONFINEMEN (SINGLE SCALE AT LOW ENERGY)

OTHER ADVANTAGES:

•) (WEINBERG, 73) THE STRONG interaction group $SU(3)_c$ is completely independent (commutes) with the weak interaction group. Weinberg has proven that under this condition parity-violatin terms of order $\alpha_{weak} \alpha_s$ are not induced by the mixing of strong and weak interaction. Parity violating terms remain of order $\frac{\alpha_{weak}}{M_W^2}$, and no strangeness changing neutral currents are induced

•) Same type of theory for weak, strong, and electromagnetic interactions!
A step towards unification!

QCD LAGRANGIAN

flavours (u,d,s

$$\mathcal{L}_0 = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + \overset{\text{8 "gluons"}}{} \sum_f \bar{\Psi}_i^{(f)} \left((i\not\partial - m_f)\delta_{iJ} - g\, t^a_{iJ} \not A_a \right) \Psi_J^{(f)}$$

COLOUR

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f_{abc} A^b_\mu A^c_\nu$$

(Note the similarity with QED!)

The index i assumes 3 values ("Colors") 1 2 3

The matrices t^a_{iJ} form a complete basis of traceless $3 \otimes 3$ matrices, chosen to be hermitean

$$\left| \begin{array}{ccc} \text{Real} & \text{complex} & \text{complex} \\ * & \text{Real} & \text{complex} \\ * & * & \text{Fixed by trace=0} \end{array} \right| \Rightarrow 8 \text{ matrices}$$

Normalized by orthogonality:

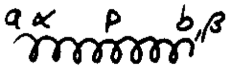
$$\mathrm{Tr}\left(t^a t^b\right) = \sum_{iJ} t^a_{iJ} \left(t^b_{iJ}\right)^* = \frac{1}{2}\delta^{ab}$$

COMPLETENESS

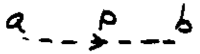
f defined by : $[t^a, t^b] = i f^{abc} t^c$

Property: $$\sum_a t^a_{iJ} t^a_{k\ell} = \frac{1}{2}\left(\delta_{i\ell}\delta_{kJ} - \frac{1}{3}\delta_{iJ}\delta_{k\ell}\right)$$

Feynman rules for QCD

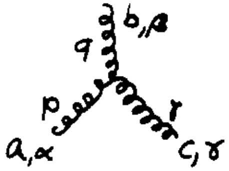


$$\delta^{ab}\left[-g^{\alpha\beta} + (1-\lambda)\frac{p^\alpha p^\beta}{p^2+i\varepsilon}\right]\frac{i}{p^2+i\varepsilon}$$



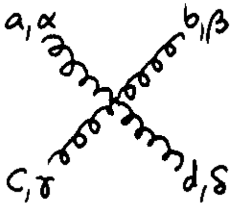
$$\delta^{ab}\frac{i}{p^2+i\varepsilon}$$

$$\delta^{ik}\frac{i}{(\not p - m + i\varepsilon)}_{mn}$$

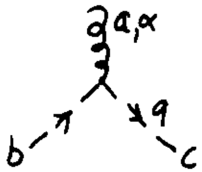


$$-g_s f^{abc}\left[g^{\alpha\beta}(p-q)^\gamma + g^{\beta\gamma}(q-r)^\alpha + g^{\gamma\alpha}(r-p)^\beta\right]$$

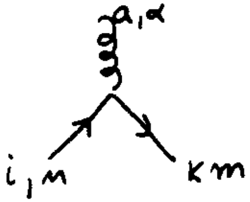
(all momenta incoming)



$$-i g_s^2 f^{xac} f^{xbd}\left(g_{\alpha\beta}g_{\gamma\delta} - g_{\alpha\delta}g_{\beta\gamma}\right)$$
$$-i g_s^2 f^{xad} f^{xbc}\left(g_{\alpha\beta}g_{\gamma\delta} - g_{\alpha\gamma}g_{\beta\delta}\right)$$
$$-i g_s^2 f^{xab} f^{xcd}\left(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}\right)$$



$$g_s f^{abc} q^\alpha$$

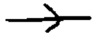


$$-i g_s t^a_{ki} \quad \gamma^\alpha_{mn}$$

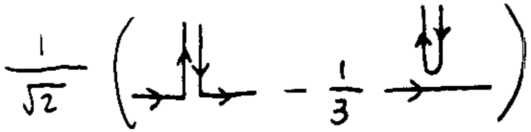
$$\alpha_s = \frac{g_s^2}{4\pi} \quad \left(\text{in analogy with QED}\right)$$

Colour Feynman rules

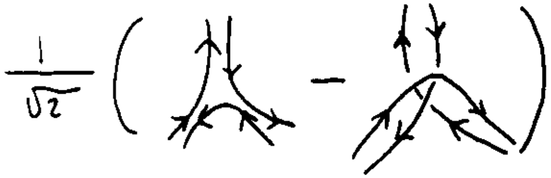
 = 3

 Fermion

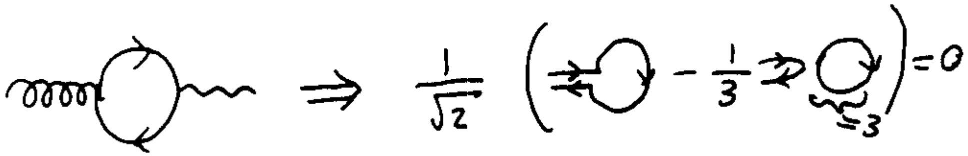
 Gluon

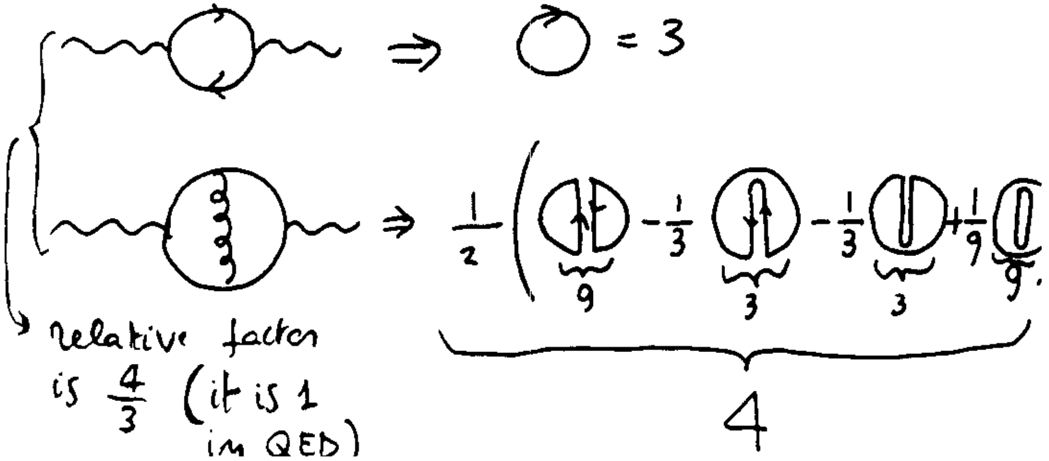


Fermion-Gluon Vertex (t^a)



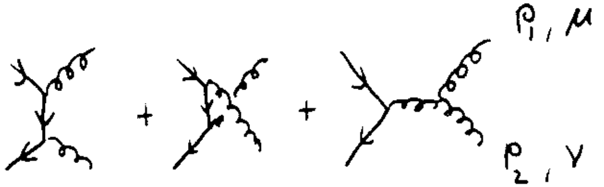
3-Gluon Vertex





relative factor is $\frac{4}{3}$ (it is 1 in QED)

GHOSTS!

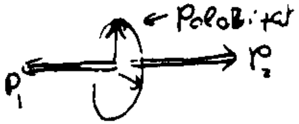


$= \mathcal{M}_{\mu\nu}$

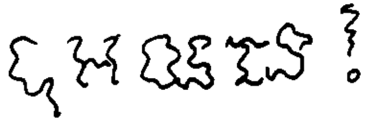
$$\sigma = \mathcal{M}_{\mu\nu} \mathcal{M}^*_{\mu'\nu'} \mathbb{P}^{\mu\mu'} \mathbb{P}^{\nu\nu'}$$

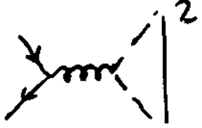
In the frame where $p_1^0 = p_2^0$, $\vec{p}_1 = -\vec{p}_2$, the polarization projector for μ and ν can be chosen as the projector onto the ⊥ plane

$$\mathbb{P}^{\mu\mu'} = -g^{\mu\mu'} + \frac{p_1^\mu p_2^{\mu'} + p_1^{\mu'} p_2^\mu}{p_1 \cdot p_2}$$



Feynman gauge projectors : $-g^{\mu\mu'}$ should give the same answer (as in QED, to preserve unitarity. It does not!

Unless you add  ?

$$\mathcal{M}_{\mu\nu} \mathcal{M}^*_{\mu'\nu'} \mathbb{P}^{\mu\mu'} \mathbb{P}^{\nu\nu'} = \mathcal{M}_{\mu\nu} \mathcal{M}^*_{\mu'\nu'} g^{\mu\mu'} g^{\nu\nu'} + \Big|$$  $$\Big|^2$$

2 COMMON POINTS OF VIEW

A) QCD is better established than EW theory ;

In fact: we have specified the full lagrangian, no room for modifications (In EW theories the Higgs sector is not well understood, lots of parameters, etc.)

B) EW theory are better established than QCD ;

In fact: we can compute everything we like to high accuracy, is accurately confirmed by experiment (In QCD we still don't understand hadron spectra dynamics, even perturbative applications rely on unproven assumptions.)

Point of view (A) relies on the assumption If we can't think of anything else, it must be true

Point of view (B) consider the possibility that our fanthasy may be limited

TESTING QCD IS IMPORTANT!

QCD and phenomenology

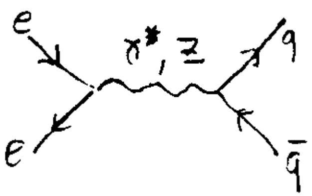
What can we calculate within QCD?

- •) Low energy:
  - i) consequences of the symmetry of the theory
  - ii) Lattice computations
- •) High energy: we can use perturbation theory. Only hadrons appear in initial and final states in reality, while perturbation theory deals with quarks and gluons. The predictions of perturbative QCD rely upon the assumption of the softness of the process of hadron formatio

Applications:

$e^+e^- \to$ hadrons :	Total cross section, Jets
$e\,p$ collisions :	deep inelastic scattering, Jet photoproduction, heavy quark photoproduction
hadron-hadron collisions :	Jet production, Drell-Yan pairs (or W and Z) production direct photon, heavy quark, etc

Illustration of perturbative QCD by its simplest application: $e^+e^- \to$ hadrons

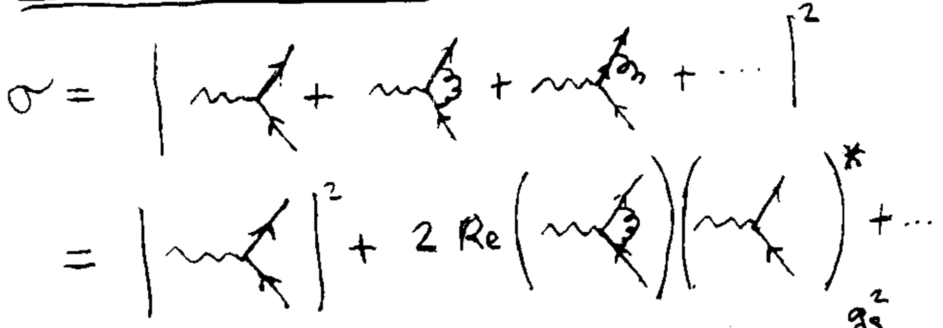


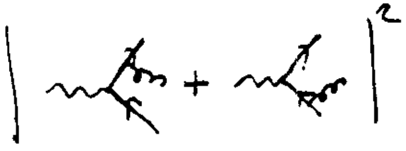
QCD at order α_s^0: (neglecting masses)

up down s c b

$$\frac{\sigma(\gamma^* \to \text{hadrons})}{\sigma(\gamma^* \to \mu^+\mu^-)} = 3 \cdot \left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9} + \frac{1}{9} \cdots\right.$$

QCD at order α_s:





$$= \left(1 + \frac{\alpha_s}{\pi}\right)\sigma_0 \qquad \alpha_s = \frac{g_s^2}{4\pi}$$

(Same as in QED, except for a colour factor $C_F =$

At order α_s^2

Divergent! M = UV cutoff!

$$\sigma = \left(1 + \frac{\alpha_s}{\pi} + \left(C + \pi b_0 \log\frac{M^2}{Q^2}\right)\frac{\alpha_s^2}{\pi^2}\right)\sigma_0$$

$$C = 1.986 - .115\, n_f \qquad b_0 = \frac{33 - 2n_f}{12\pi}$$

How do we fix this?

One can show that for any physical quantity G (FOR GENERIC) with the expansion:

$$G = G_0\,\alpha_s^n + (\;)\,\alpha_s^{n+1} + \cdots$$

the expansion has the form:

$$G = G_0\,\alpha_s^n + \left(G_1 + n\,G_0\,b_0 \log\frac{M^2}{K^2}\right)\alpha_s^{n+1} + \cdots$$

some scale in the process

In the case of σ, the quantity:

$$\frac{\sigma}{\sigma_0} - 1 = \frac{1}{\pi}\alpha_s + \left(\frac{C}{\pi^2} + \frac{1}{\pi} b_0 \log\frac{M^2}{Q^2}\right)\alpha_s$$

has precisely this form, with $n = 1$.

IF we now define:

$$\underline{\underline{\tilde{\alpha}_s(\mu) = \alpha_s + b_0 \log\left(\frac{M^2}{\mu^2}\right) \alpha_s^2}}$$

where μ is an arbitrary <u>finite</u> scale (while M is assumed to be <u>very</u> large) the physical quantity G expressed in terms of $\tilde{\alpha}_s(\mu)$ has the form:

$$G = G_0\, \tilde{\alpha}_s^n(\mu) + \left(G_1 + n G_0 b_0 \log\frac{\mu^2}{K^2}\right) \tilde{\alpha}_s^{n+}$$
$$+ \cdots \mathcal{O}\left(\alpha_s^{n+2}\right)$$

By a redefinition of the coupling constant, physical quantities are finite functions of the (redefined) coupling.

Patiently carry through the exercise:

$$G = G_0\left(\alpha_s + b_0 \log\frac{M^2}{\mu^2}\alpha_s^2\right)^n + \left(G_1 + n G_0 b_0 \log\frac{\mu^2}{K^2}\right)\alpha_s^{n+}$$

(<u>always neglecting α_s^{n+2} and higher</u>)

$$= G_0 \alpha_s^n + n G_0 b_0 \log\frac{M^2}{\mu^2}\alpha_s^{n+1}$$
$$+ G_1 \alpha_s^{n+1} + n G_0 b_0 \log\frac{\mu^2}{K^2}\alpha_s^{n+1}$$
$$= G_0 \alpha_s^n + G_1 \alpha_s^{n+1} + n G_0 b_0 \log\frac{M^2}{K^2}\alpha_s^{n+}$$
$$+ \mathcal{O}(\alpha_s^{n+2})$$

THIS IS <u>RENORMALIZATION</u>

Stated more carefully, if we compute physical quantities

$$G\big(\underset{\substack{\uparrow\\ \text{coupling}}}{\alpha_s},\ \underset{\substack{\uparrow\\ \text{cutoff UV}}}{M},\ \underset{\substack{\nwarrow\\ \text{physical variables}\\ \text{(energies, masses,}\\ \text{momenta, etc.)}}}{\ldots}\big)$$

I can always define a charge

$$\tilde{\alpha}_s(\mu, M, \alpha_s) = \alpha_s + C_1(\mu, M)\alpha_s^2 + \cdots$$

In such a way that:

$$G(\alpha_s, M, \ldots) = G'\left(\tilde{\alpha}_s(\mu, M, \alpha_s), \mu, \ldots\right.$$

so that the physical quantity has a finite expression in terms of $\tilde{\alpha}$, μ, and the physical variables

The <u>same</u> charge redefinition makes all physical quantities <u>finite</u>.

We can use one process to measure $\tilde{\alpha}_s(\mu, M,$ and then use it to predict other cross sections.

MANY IMPORTANT CONSEQUENCES:
TAKE DERIVATIVE OF BOTH SIDES WITH RESPECT TO $\log \mu^2$; left hand side is independent of μ, must give zero

$$0 = \frac{\partial G'(\tilde{\alpha}_s, \mu, \dots)}{\partial \tilde{\alpha}_s} \frac{\partial \tilde{\alpha}_s}{\partial \log \mu^2} + \frac{\partial G'(\tilde{\alpha}_s, \mu, \dots)}{\partial \log \mu^2}$$

$$\Rightarrow \quad \frac{\partial \tilde{\alpha}_s(\mu, M, \alpha_s)}{\partial \log \mu^2} = - \frac{\dfrac{\partial G'(\tilde{\alpha}_s, \mu, \dots)}{\partial \log \mu^2}}{\dfrac{\partial G'(\tilde{\alpha}_s, \mu, \dots)}{\partial \tilde{\alpha}_s}} = \beta(\tilde{\alpha}_s, \mu)$$

But β is dimensionless; if it does not depend upon M, it cannot depend upon μ: $\beta = \beta(\tilde{\alpha}_s)$

$$\frac{\partial \tilde{\alpha}_s(\mu, M, \alpha_s)}{\partial \log \mu^2} = \beta(\tilde{\alpha}_s)$$

In fact, from our initial 1-loop result: $\tilde{\alpha}_s(\mu) = \alpha_s + b_0 \log \frac{M^2}{\mu^2} \alpha_s^2$

$$\Rightarrow \quad \frac{\partial \tilde{\alpha}_s}{\partial \log \mu^2} = -b_0 \tilde{\alpha}_s^2 + \mathcal{O}(\tilde{\alpha}_s^3)$$

Up to now, no gain!

We have concluded that physical quantities can be given as an expansion in $\tilde{\alpha}$:

$$G'(\tilde{\alpha}, \mu, \dots) = \sum^{i} G_i(\mu, \dots) \, \tilde{\alpha}^i$$

and that if we change μ and $\tilde{\alpha}$ in such a way that:

$$\delta \tilde{\alpha} = \delta \log \mu^2 \, \beta(\tilde{\alpha}_s)$$

physical quantities don't change
(RENORMALIZATION GROUP TRANSFORMATION)

WHAT IS THE USE OF IT?

Look at our result as a function of the renormalized charge: (drop the ~ from now on)

$$\sigma = \left(1 + \frac{\alpha_s(\mu)}{\pi} + \left(c + \pi b_0 \log\left(\frac{\mu^2}{Q^2}\right)\right) \frac{\alpha_s^2(\mu)}{\pi^2} \right) \sigma_0$$

- One should always choose $\mu \approx Q$; in fact, if we choose $\mu \gg Q$ or $\ll Q$ the second order term may become larger then the first order one (and the third order even larger)

- Choosing μ and Q in a fixed ratio, for example $\mu = Q$, the energy dependence of the cross section is due to the running of α_s.

- Solving the evolution equation:

$$\frac{\partial \alpha(\mu^2)}{\partial \log \mu^2} = - b_0 \alpha_s^2(\mu) \Rightarrow \alpha_s = \frac{1}{b_0 \log\left(\frac{\mu^2}{\Lambda^2}\right)}$$

the integration constant Λ fixes the value of α_s at a given scale

$b_0 = \frac{33 - 2n_f}{12\pi} > 0$; in QED $b_0 = -\frac{4n_f}{12\pi} < 0$

$(\mu > \Lambda)$ $(\mu < \Lambda)$

$$\boxed{\begin{array}{llll} \text{QCD is only} & \text{calculable} & \text{for} & \mu \gg \Lambda \\ \text{QED} \ " & " & " & \mu \ll \Lambda \end{array}}$$

(The divergence comes mainly from graphs that correct the gluon propagator:

$b_0 = \frac{33 - 2n_f}{12\pi}$

In QED we only have the diagrams with the fermion loop.)

In QED, for $\mu = m_e \Rightarrow \alpha(\mu) = \alpha_{QED}$

So:

$$\alpha_{QED} = \frac{1}{\left(-\frac{4n_f}{12\pi}\right) \log\left(\frac{m_e^2}{\Lambda_{QED}^2}\right)}$$

$$\Lambda_{QED}^2 \simeq m_e^2 e^{\frac{3\pi}{n_f \alpha_{QED}}} \Leftarrow \text{Astronomic Scale!}$$

That's why we never talk about Λ_{QED}

In QCD, we must have Λ of the order of typical hadronic scale (~ 500 MeV) so that the hadronic systems become strongly coupled at ~~high~~ low energy.

QCD : SINCE strong interactions have characteristic scales of few hundred

PREDICTION : $\alpha_s(Q^2) = \frac{1}{b_0 \log\left(\frac{Q^2}{\Lambda^2}\right)}$

WITH $\Lambda \simeq 100 - 500$ MeV

$$\Longrightarrow \alpha(M_Z) = 0.1 \div 0.13 \qquad (\pm 13\%$$

$$\alpha_s(10^7 \text{GeV}) = 0.040 \div 0.044 \qquad (\pm 5$$

Present Status of $e^+e^- \Rightarrow$ hadrons

total cross section

GORISHNY, KATAEV, LARIN

$$\frac{\sigma}{\sigma^0} = 1 + \frac{\alpha_s}{\pi}\left(1 + .448\,\alpha_s - 1.30\,\alpha_s^2\right) + \ldots$$

DINE, SAPIRSTEIN
CELMASTER, GONSALVES

(Coefficients are for $n_f = 5$; general expression very complicate

$$\alpha_s = \alpha_s^{\overline{MS}}(Q)$$

Corrections are well behaved; with $\alpha \approx .12$ $\mathcal{O}(\alpha_s^2)$ term is 5%, $\mathcal{O}(\alpha_s^3)$ is 2% of total QCD correction (on Z peak)

In principle, we could determine $\alpha_s(M_Z)$ with 1% accuracy

β function:

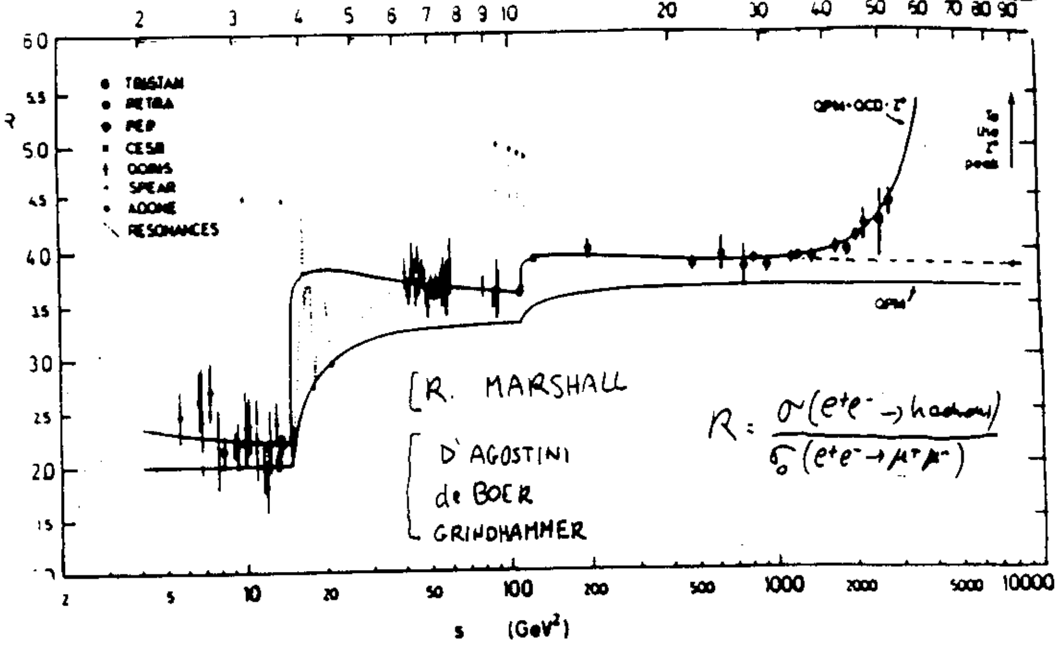
$$\frac{\partial \alpha_s}{\partial \log \mu^2} = -b_0\,\alpha_s^2 - b_1\,\alpha_s^3 - b_2\,\alpha_s^4$$

TARASOV
VLADIMIROV
ZHARKOV

$$b_0 = \frac{33 - 2n_f}{12\pi}, \quad b_1 = \frac{153 - 19n_f}{24\pi^2}, \quad b_2 = \frac{2857 - \frac{5033}{18}n_f + \frac{325}{54}n_f^2}{(4\pi)^3}$$

Commonly used 2-loop formula:

$$\boxed{\alpha_s(\mu) = \frac{1}{b_0 \log\frac{\mu^2}{\Lambda^2}}\left(1 - \frac{b_1 \log\log\frac{\mu^2}{\Lambda^2}}{b_0^2 \log\frac{\mu^2}{\Lambda^2}}\right)}$$



[R. MARSHALL

D'AGOSTINI
de BOER
GRINDHAMMER

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma_0(e^+e^- \to \mu^+\mu^-)}$$

$e^+e^- \Rightarrow$ hadrons

below the Z peak
Large errors in each experiment, must combine them to get smaller error.

Recent analysis of D. Haidt: (MARSEILLE '93)

$$20 < \sqrt{s} < 65 \text{ GeV}$$

$$\alpha_s(35\,\text{GeV}) = 0.146 \pm 0.030 \xrightarrow{\text{NLO evolution}} \alpha_s(M_Z) = 0.124 \pm 0.021$$

But: Branchina, Castorina, Consoli, Zappalà (1991)

$$22 < \sqrt{s} < 61.4 \text{ GeV} \Rightarrow \alpha_s = .155 \pm .02$$

AT LEP:

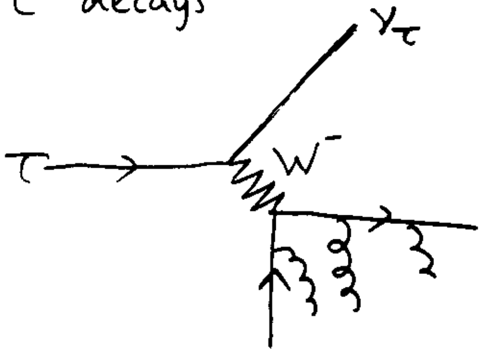
$$\Gamma(Z^0 \to \text{had}) \Rightarrow \alpha_s(M_Z) = .122 \pm .009$$

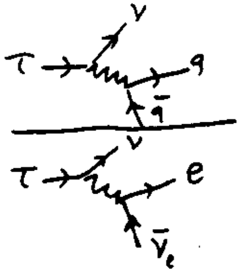
0.007 ↙ ↘ 0.005

↑ EXP. ERROR   ↑ TH. ERR

(Marseille, EPS 93)

τ decays



0^{th} order QCD :  = 3

QCD correction desplaces value from 3

$$\alpha_s(M_\tau) = 0.36 \pm 0.05 \Longrightarrow \alpha_s(M_Z) = 0.122 \pm 0.005$$

(Dangerous game)   (Marseille, EPS 93)

Back to theory ...

3 questions:

1) Our cross section at order α_s for producing two quarks alone, and two quarks + 1 gluon. However, we never see neither the two quarks, nor the gluon in the final state. How comes that we get the right answer?

2) The leading quark antiquark pair flies away pulling away colour quantum numbers in the opposite emispheres of the events. How comes that in reality, for any pair of emispheres we get neutrality in colour?

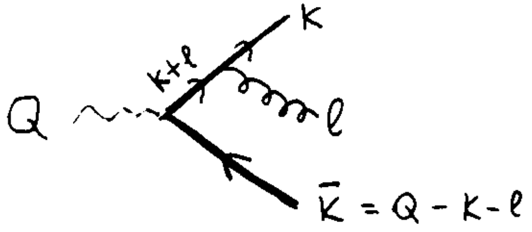
3) Following 1 and 2 it seems that our final state is not described by the feynman graphs we computed. Is there any feature of the final state that we can compute?

Let us proceed to show that questions 1 and 2 imply no contradiction, and 3 (under some assumptions) has a positive answer.

Using some approximations (just for illustration) we will look at details of the $\mathcal{O}(\alpha_s)$ conection to $e^+e^- \Rightarrow$ hadrons, trying to understand how the final state looks like

Look at $\gamma^* \to q\bar{q}g$;

Make for simplicity the assumption that the energy of the gluon is small compared to Q



Assume $\ell \ll K, \bar{K}$

the amplitude $\mathcal{M}$ is given by:

$$\bar{u}(K)\gamma^\alpha \frac{\not{K}+\not{\ell}}{(K+\ell)^2}\gamma^\nu v(\bar{K}) \approx \bar{u}(K)\frac{\gamma^\alpha \not{K}}{2K\cdot\ell}\gamma^\nu v($$

using $\gamma^\alpha \not{K} + \not{K}\gamma^\alpha = 2K^\alpha$, and $\bar{u}(K)\not{K} = 0$

$$= \bar{u}(K)\frac{\gamma^\alpha \not{K} + \not{K}\gamma^\alpha}{2K\cdot\ell}\gamma^\nu v(\bar{K}) = \frac{K^\alpha}{K\cdot\ell}\bar{u}(K)\gamma^\nu v(\bar{K}$$

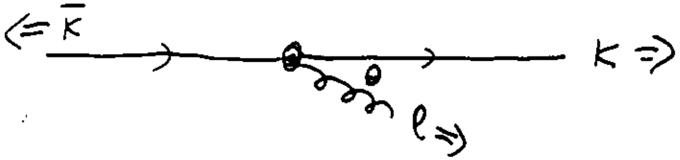
$$= \frac{K^\alpha}{K\cdot\ell}\mathcal{M}_0$$

Including the conection on the antiquark line we get:

$$\mathcal{M} = \mathcal{M}_0\left(\frac{K^\alpha}{K\cdot\ell} - \frac{\bar{K}^\alpha}{\bar{K}\cdot\ell}\right)$$

Squaring:

$$|\mathcal{M}|^2 = -|\mathcal{M}_0|^2 \left(\frac{k^\alpha}{k\cdot l} - \frac{\bar{k}^\alpha}{\bar{k}\cdot l}\right)^2 = |\mathcal{M}_0|^2 \frac{2k^\alpha \cdot \bar{k}^\alpha}{k\cdot l \;\; \bar{k}\cdot l}$$



$$k\cdot l = k^0 l^0 (1-\cos\theta)$$
$$\bar{k}\cdot l = k^0 l^0 (1+\cos\theta)$$

$$|\mathcal{M}|^2 = |\mathcal{M}_0|^2 \frac{k^\alpha \cdot \bar{k}^\alpha}{k^{0^2} l^{0^2} (1-\cos^2\theta)}$$

Phase space: $\frac{d^3 l}{2l^0 (2\pi)^3} = \frac{l\, dl^0\, d\cos\theta}{2(2\pi)^2}$

Integrating we get: (include $g_s^2\, C_F$, $C_F = 4/3$)

$$\frac{d^3 l}{2l^0(2\pi)^3} |\mathcal{M}|^2 = |\mathcal{M}_0|^2 \frac{\alpha_s C_F}{2\pi} \int \frac{dl^0}{l^0} \int \frac{d\cos\theta}{1-\cos^2\theta}$$

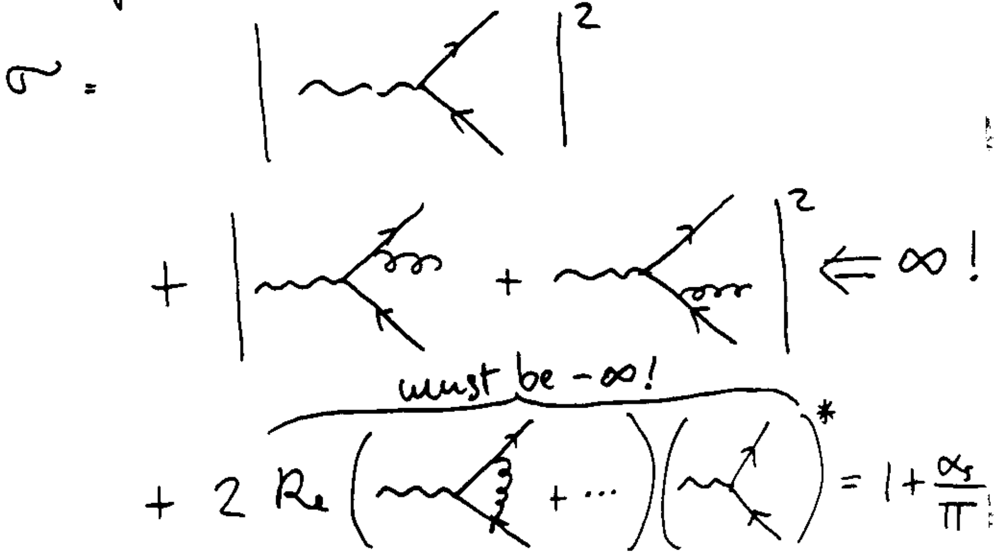
Two divergent integrals.

$\int \frac{dl^0}{l^0}$ ⇒ "SOFT" divergence

$\int \frac{d\cos\theta}{1-\cos^2\theta}$ ⇒ "COLLINEAR" divergence

Remember: $\mathcal{O}(\alpha_s)$ correction is of order $(1+\frac{\alpha_s}{\pi})$; finite!

Therefore:



Only possibility: virtual corrections $= -\infty$ to exactly cancel real correction.

Back to question #1

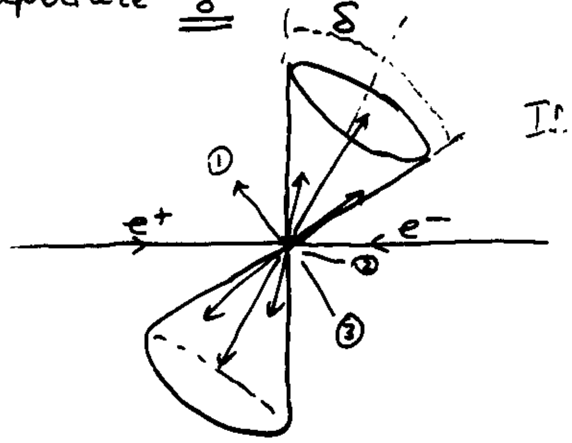
Already at order α_s, the cross section for producing a $q\bar{q}g$ state has a (+) infinite coefficient, and the cross section for producing ONLY a $q\bar{q}$ pair has a (−) infinite coefficient.

It is easy to guess that these infinities will arise to all order in the perturbative expansion – To answer #1 we m. resum the "large" terms in the expansion.

We don't know how to do this in QCD; but it may well be that the cross section for producing any finite # of quarks/gluons in the final state is zero

STERMAN & WEINBERG (1977) answer to #3

Define the cross section for the production of "Sterman-Weinberg Jets" as the cross section for the production of a hadronic event in which all the produced energy, except for a fraction ε of the total, almost contained in two opposite cones of aperture δ



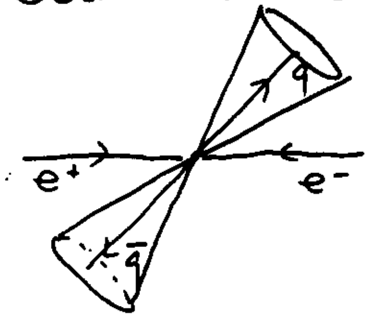
If: $E_{①} + E_{②} + E_{③} < \epsilon E$,

E = total energy

An event contributes to the Sterman Weinberg cross section if we can find a cone that satisfies the above condition.

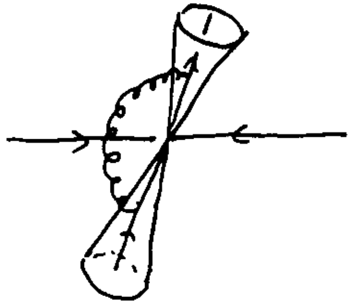
Computation of the Sterman-Weinberg cross section

"Born" contribution



<u>ALL</u> BORN cross section contribute, independentl of δ and ϵ !

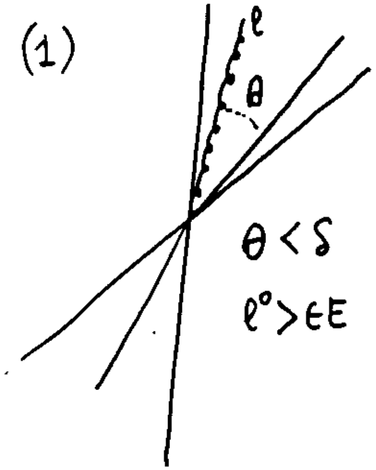
Virtual contribution $\mathcal{O}(\alpha_s)$



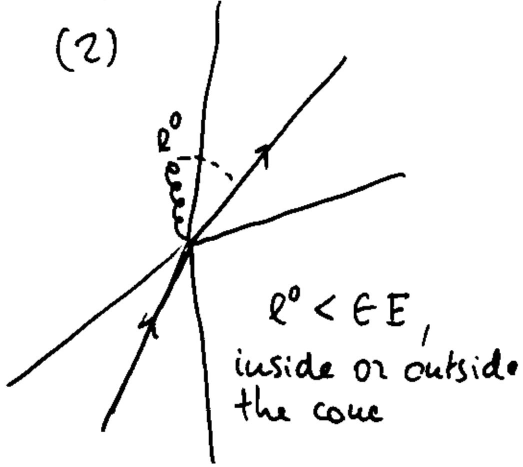
All Virtual term, as in the Born case

Real contributions, 2 types

(1)



(2)



<u>Divergent</u> parts in the various contributions (Neglect finite, $\mathcal{O}(\alpha_s)$ pieces)

Real (1)

$$\sigma_0 \frac{\alpha_s C_F}{2\pi} \int_{\epsilon E}^{E} \frac{d\ell^0}{\ell^0} \left[\int_{\theta=0}^{\delta} d\cos\theta \frac{1}{1-\cos^2\theta} + \int_{\theta=\pi-\delta}^{\pi} d\cos\theta \frac{1}{1-\cos^2\theta} \right]$$

Real (2)

$$\sigma_0 \frac{\alpha_s C_F}{2\pi} \int_0^{\epsilon E} \frac{d\ell^0}{\ell^0} \int_{\theta=0}^{\pi} \frac{d\cos\theta}{1-\cos^2\theta}$$

Virtual

$$-\sigma_0 \frac{\alpha_s C_F}{2\pi} \int_0^{E} \frac{d\ell^0}{\ell^0} \int_{\theta=0}^{\pi} \frac{d\cos\theta}{1-\cos^2\theta}$$

Because it must cancel the total ∞ part of the REAL cross section

Sum them up :

$$\text{Real}\Big((1) + (2)\Big) + \text{Virtual} = -\sigma_0 \frac{\alpha_s C_F}{2\pi} \int_{\epsilon E}^{E} \frac{d\ell^0}{\ell^0} \int_{\theta=\delta}^{\pi-\delta} \frac{d\cos\theta}{1-\cos^2\theta}$$

$$= -\sigma_0 \frac{\alpha_s C_F}{2\pi} \log\epsilon \log\delta \left(+ \underset{\text{when } \delta\to 0 \text{ or } \epsilon\to 0}{\text{other terms which are finite}} \right)$$

$+$ Born :

$$\sigma_0 \left(1 - \sigma_0 \frac{\alpha_s C_F}{2\pi} \log\epsilon \log\delta + \mathcal{O}(\alpha_s, \text{no logs}) \right)$$

FINITE !!!

NOW, JUST BE BRAVE!

TAKE THE STERMAN-WEINBERG RESULT SERIOUSLY, AND SEE WHAT IT PREDICTS FOR THE HADRONIC FINAL STATE IN e^+e^- ANNIHILATION!

The cross section is (approximately)

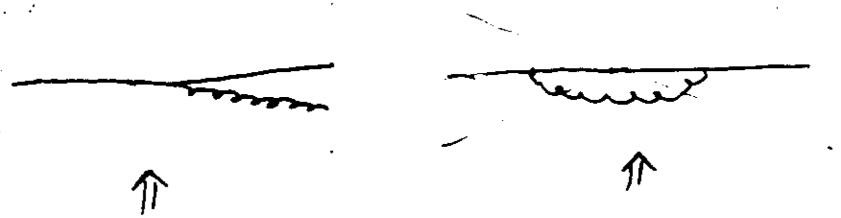
$$\sigma_0 \left(1 - \frac{\alpha_s C_F}{2\pi} \log\epsilon \log\delta + \mathcal{O}(\alpha_s, \mathrm{mo}\right.$$

if ϵ and δ are small, but not TOO small ($\log\epsilon$ and $\log\delta$ are not too large) the cross section for producing two Sterman-Weinberg jets is close to 1: almost all events have most of their energy contained in opposite cones! It predicts therefore that most events will be 2 Jets events, and that their angular distributi will be that of σ_0. This has indeed been observed since long time.

Why is Sterman-Weinberg cross Section finite?

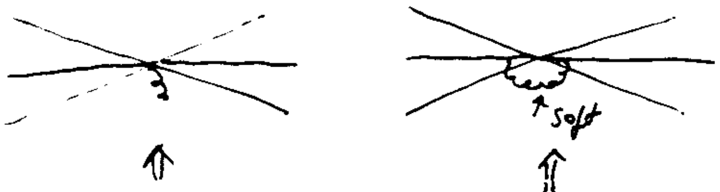
2 reasons:

a) It is not too sensitive to collinear splitting (because of $\delta > 0$)



Both contribute to SW cross sections

b) It is not too sensitive to soft emission (because of $\epsilon > 0$)



Both contribute

After SW, a whole Zoo of soft and collinear safe variables have been invented: Thrust, Oblateness, C-parameter, Jet clusters, Heavy Jet mass, etc.

Assume that THE SAME DEFINITION IS APPLIED TO HADRONS (WHEN THEY ARE MEASURED)

Some examples

Thrust:

$$T = \mathop{\mathrm{Max}}_{\vec{v}} \frac{\sum_i |\vec{p}_i \cdot \vec{v}|}{\sum_i |\vec{p}_i|}$$

$\vec{v}$ is a vector of unit length, and the maximum is taken over all possible orientations of $\vec{v}$.

For two partons (hadrons) in the final state $T = 1$

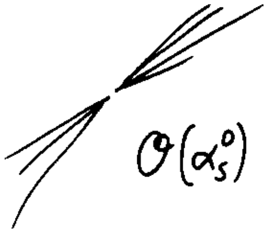
T starts being non trivial with three or more partons.

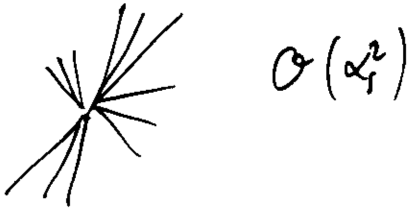
Jet clusters:

Form the invariant mass of all pairs of hadrons (partons) in the final state. The pair with minimum invariant mass is merged into a single pseudoparticle. The procedure is continued untill the minimum invariant mass obtained is larger than a given cut: $\frac{(p_i + p_j)^2}{s} < Y$

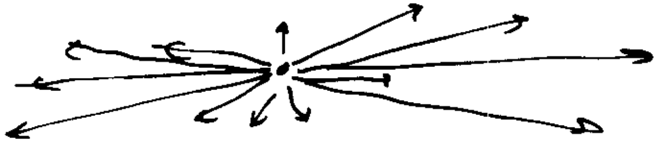
With the jet clusters definition, events are classified into two, three, four clusters events, according to how many clusters we have at the end

Clearly, quantities to which two parton configurations contribute (two jets) prevail, since they are of order 0 in α_s. Planar type of events are a small fraction of the total, and highly aplanar events ($\mathcal{O}(\alpha_s^2)$) are even less. The distinction between two, three, four or more jet events is not, however, a unique one. The nature of perturbative QCD forbids us to ask too detailed (too exclusive) questions about the final state. There will always be a jet resolution parameter, like a Y cut, or the ε-δ of Sterman-Weinberg, to decide wether two nearby jets should be merged into a single one.



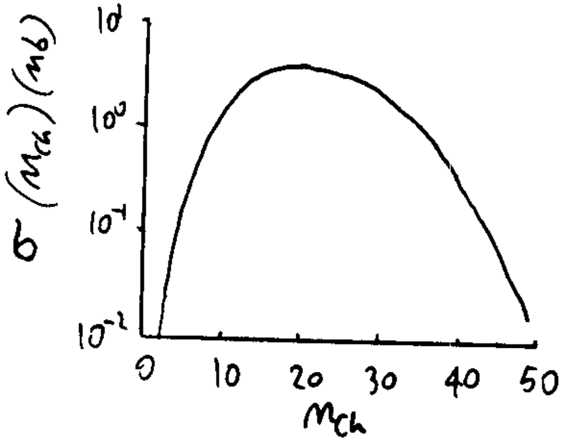


Typical structure of a hadronic event at LEP



LARGE multiplicity : $\langle n_{ch} \rangle = 20$

with large fluctuations



Average mass of the fattest Jet: ≈ 20 GeV

QCD predicts: $\left\langle \frac{M_H^2}{s} \right\rangle = \frac{\alpha}{\pi} 1.05 + O(\alpha^2)$

THEORETICAL STATUS OF JET CROSS SECTIONS AND SHAPE VARIABLE DISTRIBUTION

JET CROSS SECTIONS: STERMAN - WEINBERG

\# OF CLUSTERS

A SOFT AND COLLINEAR INSENSITIVE DEFINITION OF JETS

EACH EVENT THAT CONTRIBUTES IS ADDED TO THE CROSS SECTION

SHAPE VARIABLES DISTRIBUTIONS: THRUST, OBLATENESS C - PARAMETER, ETC.

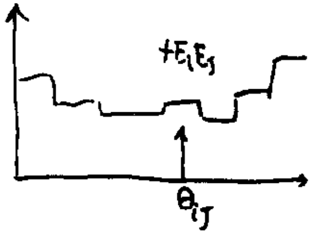
A VARIABLE CHARACTERIZING THE FINAL STATE (I.E. A FUNCTION OF FINAL STATE MOMENTA AND ENERGIES)

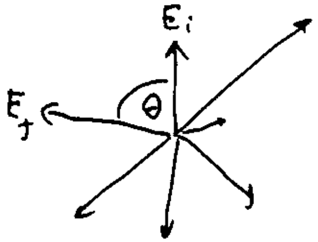
SOFT AND COLLINEAR INSENSITIV

EACH EVENT CONTRIBUTES TO 1 BIN OF A DISTRIBUTION

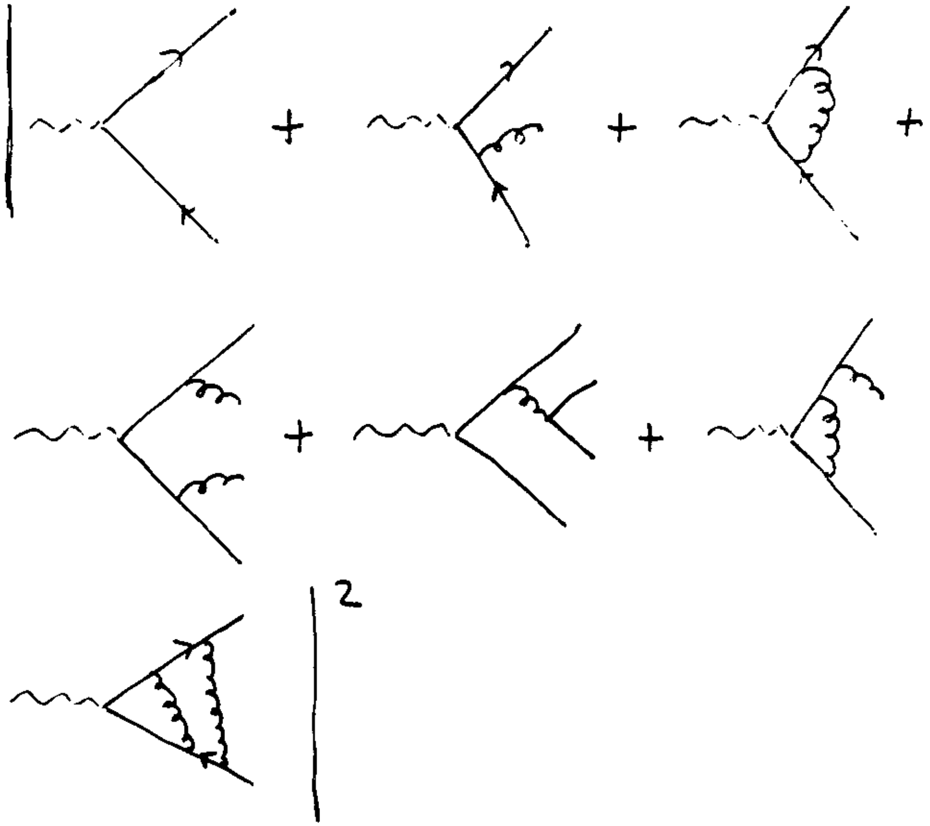
OTHERS: ENERGY - ENERGY CORRELATION

EACH EVENT CONTRIBUTES TO SEVERAL BINS OF A DISTRIBUTION





ELLIS - ROSS - TERRANO have computed for us the NEXT-TO-LEADING CROSS SECTION FOR $e^+e^- \to 3,4$ partons (1981)



WE CAN USE THEIR RESULT TO COMPUTE ANY SHAPE VARIABLE DISTRIBUTION, OR JET CROSS SECTION, OR OTHER TYPE OF VARIABLE CHARACTERIZING THE FINAL STATE.

LET US FOCUS UPON THRUST AS AN EXAMPLE (PAG. 37). THE THRUST DISTRIBUTION HAS THE PERT. EXPANSION:

$$\frac{1}{\sigma_0}\frac{d\sigma}{dt} = \overset{\text{BORN}}{\delta(1-t)} + \underbrace{\frac{\alpha_s(\mu)}{2\pi}A(t)}_{\text{LEADING}} +$$

$$\text{NEXT-TO-LEADING} \Rightarrow \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2\left[A(t)\, 2\pi b_0 \boxed{\log\frac{\mu^2}{Q^2}} + B(t)\right]$$

$$+\ \mathcal{O}(\alpha_s^3)$$

AS IN THE TOTAL CROSS SECTION CASE THE NEXT-TO-LEADING TERM HAS A μ DEPENDENCE WHIC IS FIXED BY THE LEADING TERM (Pag 17)

REMEMBER: THE LEFT HAND SIDE IS μ INDEPENDENT. ON THE RIGH HAND SIDE WE ARE NEGLECTING TERMS OF ORDER α_s^3; THEREFORE WE MAY HAVE A RESIDUAL SCALE DEPENDENCE OF ORDER α_s^3. CHECK THIS AS AN EXERCISE! (USE $\partial\alpha/\partial\log\mu^2 = -b_0\alpha_s^2$)

RADIATIVE CORRECTIONS ARE GENERALLY QUITE LARGE. SOME EXAMPLES:

$$\langle 1-t\rangle = \frac{1.05}{\pi}\alpha_s(Q)\left(1+\underline{3\alpha_s}\right)$$

$$\langle Q\rangle = 1.29\ \alpha_s(Q)\left(1-\underline{4.3\,\alpha_s}\right)$$

$$\left\langle\frac{M^2_{D,t}}{S}\right\rangle = \frac{1.05}{\pi}\alpha_s(Q)\left(1-\underline{0.025\ \alpha_s}\right)$$

<u>UP TO 40% AT LEP ENERGIES!</u>

$\mathcal{O}(\alpha_s^3)$ EFFECTS MUST ALSO BE IMPORTANT. IF WE WANT TO COMPARE THEORY WITH DATA WE MUST FIND A WAY TO ESTIMATE HIGHER ORDER (α_s^3 and higher) EFFECTS.

A COMMON METHOD IS THE FOLLOWING: KEEP $\mu \neq Q$:

$$\langle 1-t\rangle = \frac{1.05}{\pi}\alpha_s(\mu)\left(1+\alpha_s\left(b_0\log\frac{\mu^2}{Q^2}+3\right)\right)+\mathcal{O}($$

WE KNOW THAT FOR THE EXPANSION TO WORK WE MUST HAVE $\underline{\mu \simeq Q}$. CHOOSE A RANGE:

$$\boxed{\frac{Q}{2} < \mu < 2Q}$$

LEFT HAND SIDE IS μ INDEPENDENT; THE RESIDUAL μ VARIATON OF RIGHT HAND SIDE SHOULD BE COMPENSATED BY HIGHER ORDER TERMS. THEREFORE, HIGHER ORDER TERMS ARE AT LEAST AS LARGE AS THE μ VARIATION!

HADRONIZATION EFFECTS

IN PERTURBATIVE QCD ONE USUALLY <u>ASSUMES</u> THAT HADRON FORMATION IS A <u>SOFT</u> PHENOMENON (IT GIVES CORRECTIONS OF ORDER $\frac{\Lambda}{Q}$)

LOOK AT THRUST: SIMPLE EXAMPLE:

pion with few hundred MEV transverse momentum (with respect to thrust axis)

(From simple reasoning, this should have probability of <u>order 1</u>, because involves α_s (few hundred MeV).)

EFFECT ON THRUST:

$$\delta t = \frac{\sqrt{m_\pi^2+p_\perp^2}}{Q} \simeq \frac{.5}{100} \quad \text{AT LEP}$$

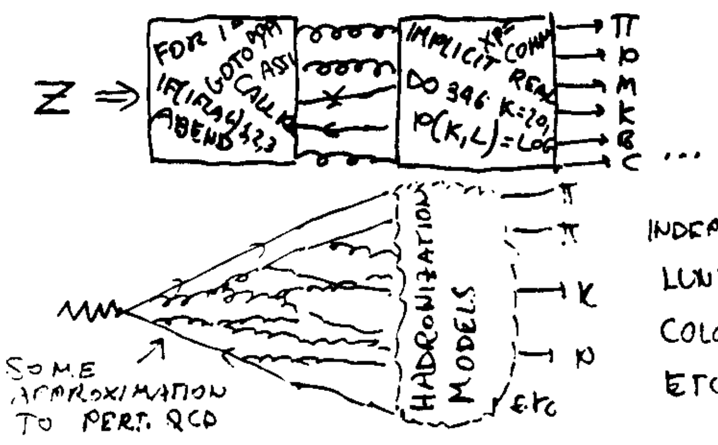
To be compared with the perturbative value:

$$\langle 1-t\rangle \simeq \frac{\alpha}{\pi} \simeq .06 \quad \text{AT LEP}$$

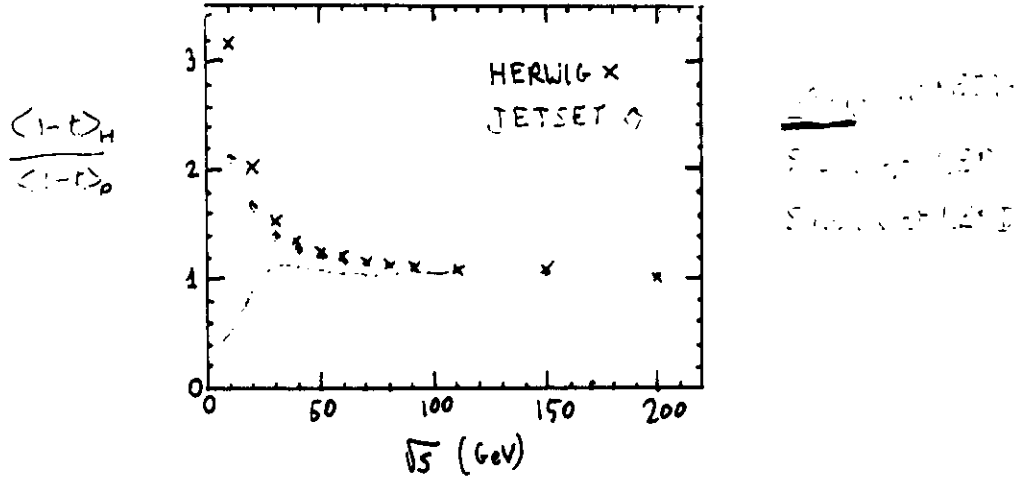
$$\text{So}:\ \frac{\delta t}{\langle 1-t\rangle} = \frac{\frac{.5}{100}}{.06} = \boxed{8\ \%}$$

This will affect directly α_s determinations!

MORE SOFISTICATED WAYS TO ESTIMATE HADRONIZATIO EFFECTS: USE MONTECARLO MODELS OF HADRON PRODUCTION (JETSET, HERWIG, ARIADNE



INDEPENDENT FRAGMENTATI
LUND STRINGS
COLOUR CLUSTERS
ETC.



Estimated by switching on and off the hadronization stage in a shower montecarlo (W. Giele)

(Montecarlo estimates of hadronization effects are always data-biased)

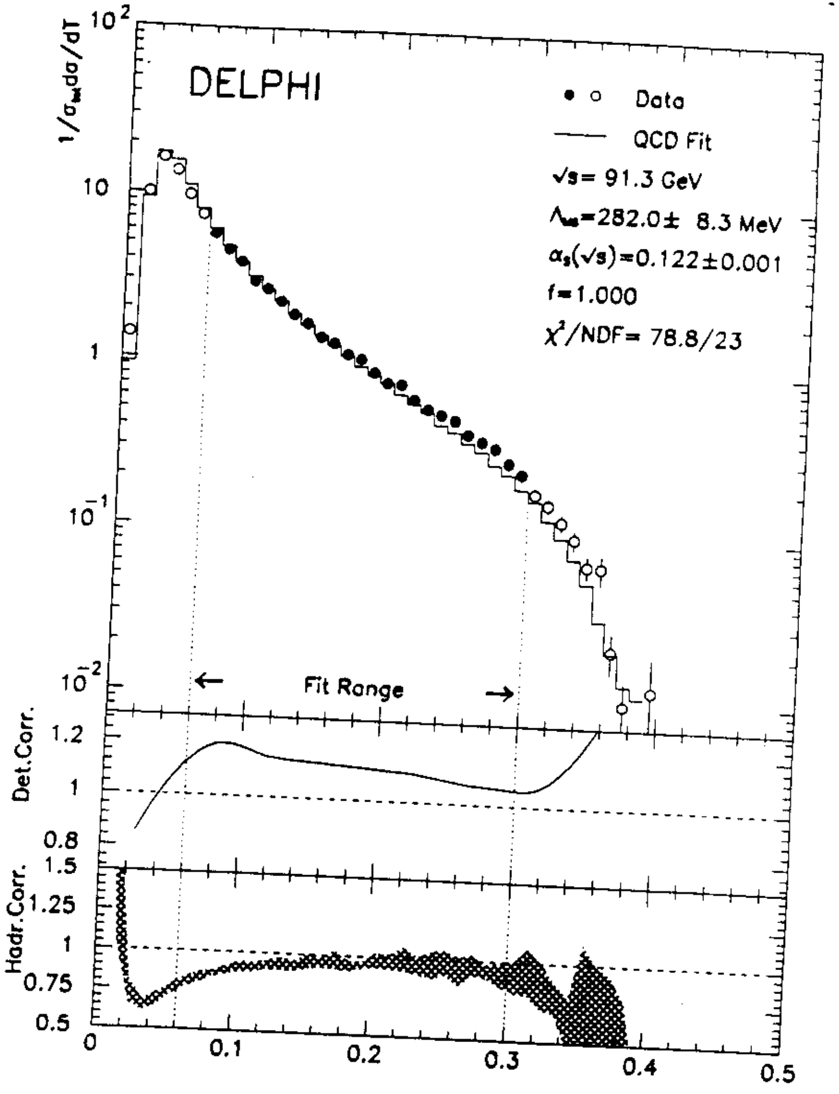


Figure 3:

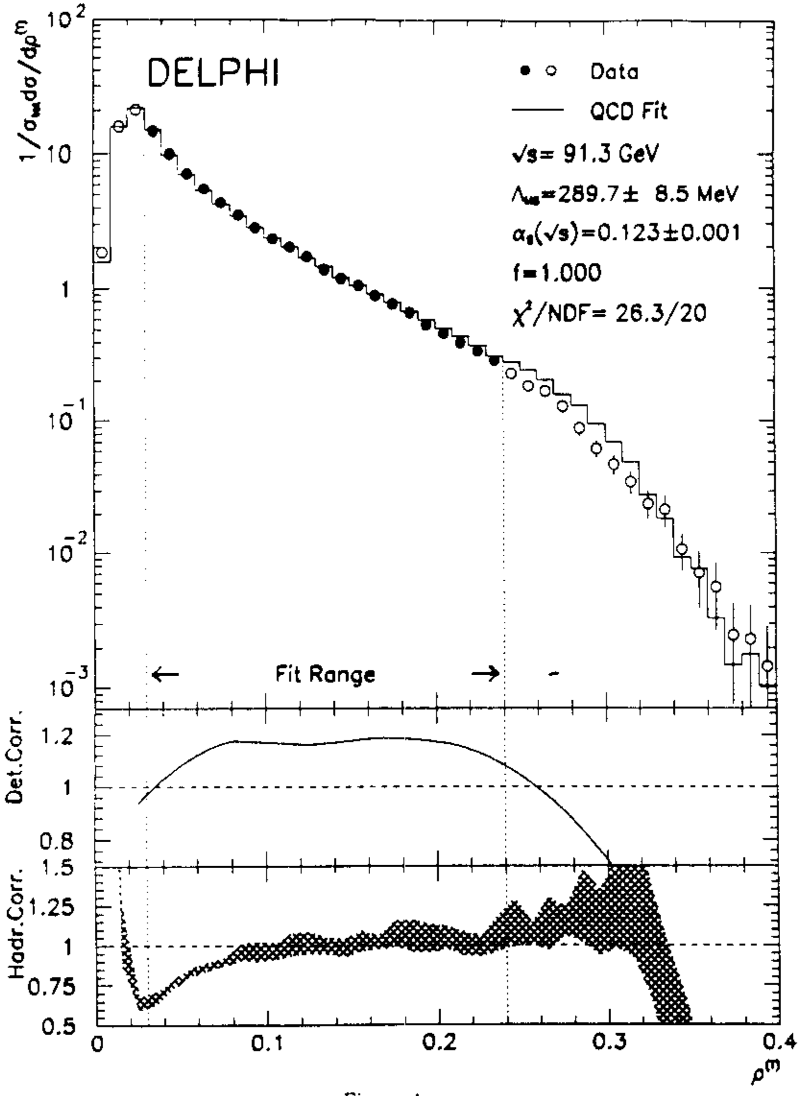


Figure 4:

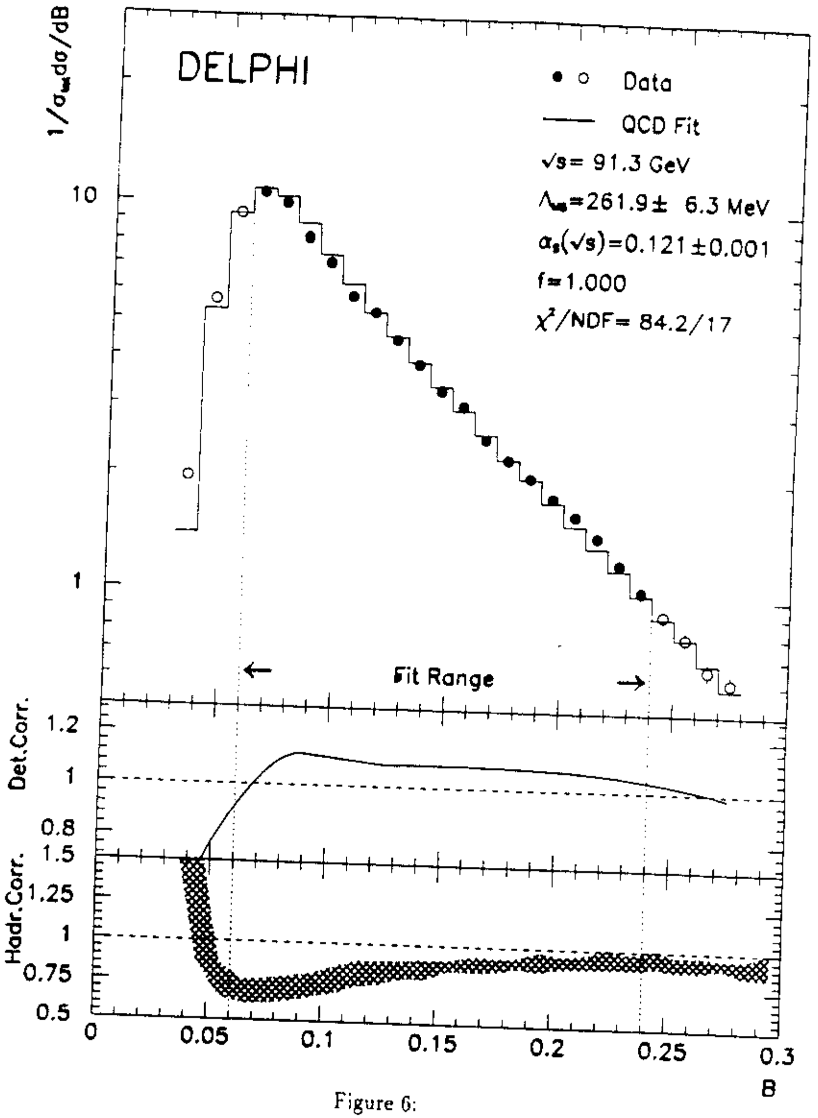


Figure 6:

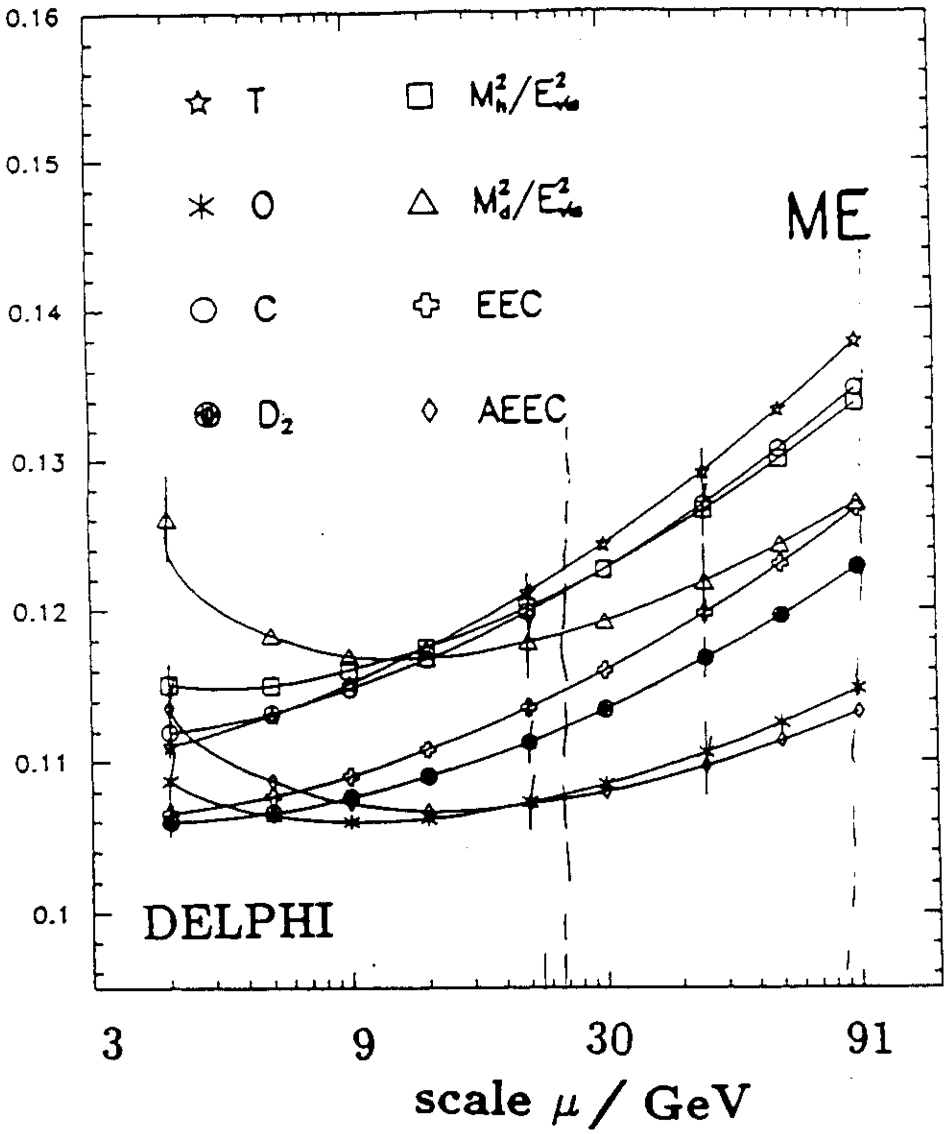


Figure 4.9: α_s values determined by DELPHI USING VARIOUS SHAPE VARIABLES

S. CATANI

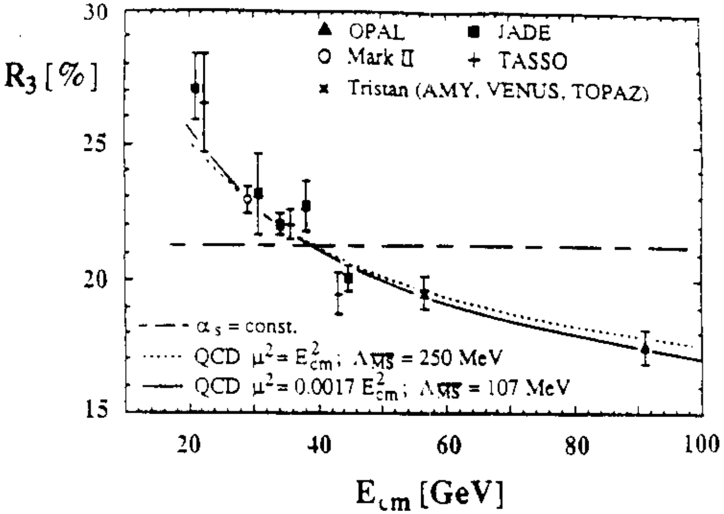
	Process	Q [GeV]	$\alpha_s(Q)$	$\alpha_s(M_{Z^0})$	$\Delta\alpha_s(M_{Z^0})$ exp.	theor.	order of perturb.
*	GLS [CCFR]	1.73	0.32 ± 0.05	0.115 ± 0.006	0.005	0.003	NNLO
*	R_τ [world]	1.73	0.36 ± 0.05	0.122 ± 0.005	0.002	0.004	NNLO
	DIS [ν]	5.0	$0.193 {}^{+0.019}_{-0.018}$	0.111 ± 0.006	0.004	0.004	NLO
	DIS [μ]	7.1	0.180 ± 0.014	0.113 ± 0.005	0.003	0.004	NLO
* *	$c\bar{c}$ mass splitting	5.0	0.174 ± 0.012	0.105 ± 0.004	0.000	0.004	LGT
	$J/\Psi, \Upsilon$	10.0	$0.167 {}^{+0.015}_{-0.011}$	$0.113 {}^{+0.007}_{-0.005}$	0.001	${}^{+0.007}_{-0.005}$	NLO
*	e^+e^- [σ_{had}]	35.0	0.146 ± 0.030	0.124 ± 0.021	-	-	NLO
	e^+e^- [ev. shapes]	35.0	0.14 ± 0.02	0.119 ± 0.014	-	-	NLO
	e^+e^- [ev. shapes]	58.0	0.130 ± 0.008	0.122 ± 0.007	0.003	0.007	NLO
	$p\bar{p} \to b\bar{b}X$	20.0	$0.138 {}^{+0.028}_{-0.019}$	$0.109 {}^{+0.016}_{-0.012}$	${}^{+0.012}_{-0.007}$	${}^{+0.011}_{-0.010}$	NLO
	$p\bar{p} \to W$ jets	80.6	0.123 ± 0.025	0.121 ± 0.024	0.017	0.016	NLO
	e^+e^- [scal. viol.]	91.2	0.118 ± 0.005	0.118 ± 0.005	0.001	0.005	NLO
*	$\Gamma(Z^0 \to had.)$	91.2	0.12~~7~~2 ± 0.0~~10~~09	0.12~~7~~2 ± 0.0~~10~~09	0.00~~9~~7	${}^{+0.005}_{-0.004}$	NNLO
	Z^0 [ev. shapes]	91.2	0.119 ± 0.006	0.119 ± 0.006	0.001	0.006	NLO
	Z^0 [ev. shapes]						
*	SLD	91.2	0.126 ± 0.007	0.126 ± 0.007	0.004	0.006	resum.
	ALEPH	91.2	0.125 ± 0.005				resum.
*	DELPHI	91.2	0.123 ± 0.006				resum.
*	L3	91.2	0.124 ± 0.009				resum.
*	OPAL	91.2	0.120 ± 0.006				resum.
	LEP *Average*	91.2		0.123 ± 0.006	0.002	0.005	resum.

Table 1: Summary of measurements of α_s (EPS Conference, Marseille, July '93). For details see text.

* new w.r.t. Dallas Conf. '92

* * theoretical uncertainty ?) [→ Sachrajda talk]

"RESUM" HAS TO DO WITH RESUMMING TO ALL ORDER CERTAIN EFFECTS (ANALOG OF $(\alpha_s \log \epsilon \log \delta)^n$ in STERMAN-WEINBERG TEST)



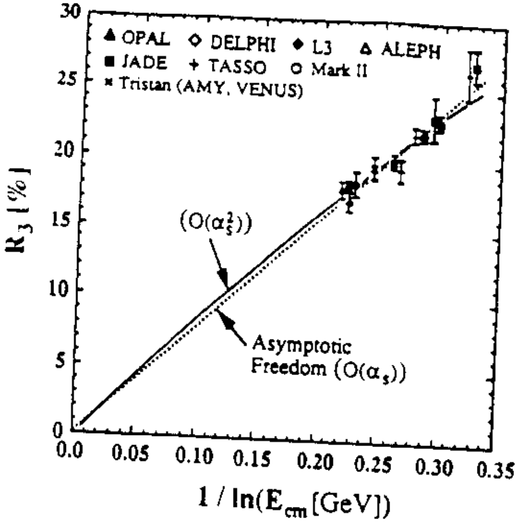


Fig. 10. The same data as shown in Fig. 9, as a function of $1/\ln(E_{cm})$, compared to the prediction of Aysmptotic Freedom.

- RUNNING OF α_s

Based upon R_3. Each event is classified according to the number of clusters.
Look at event: from all particle pairs i,j
find the pair with minimum $y_{ij} = \frac{M_{ij}^2}{s}$
If $y_{ij} < y_{cut}$, combine i,j into a single (pseudo)particle
repeat untill no pair has $y_{ij} < y_{cut}$

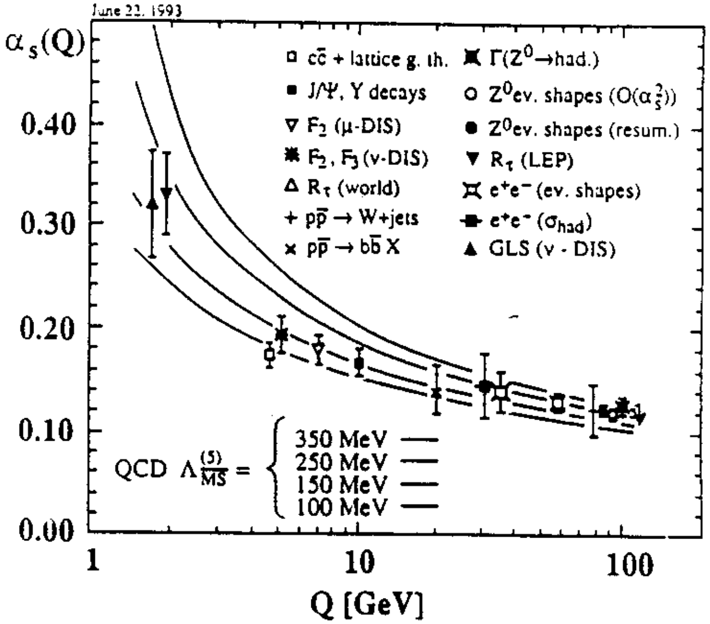
$$R_3 = \frac{\text{number of events with 3 clusters}}{\text{total}}$$

'Jade' variation: $y_{ij} = E_i E_j (1 - \cos\theta_{ij}) / s$

- Based upon montecarlo estimates of hadronization effects, the 'Jade' cluster algorithm shows little sensitivity to hadronization. A good start, but it would be nice to see the running also with other variables.

Running of $\alpha_s(Q)$

[up-to-date version of S. Bethke, Dallas Conf. '92]

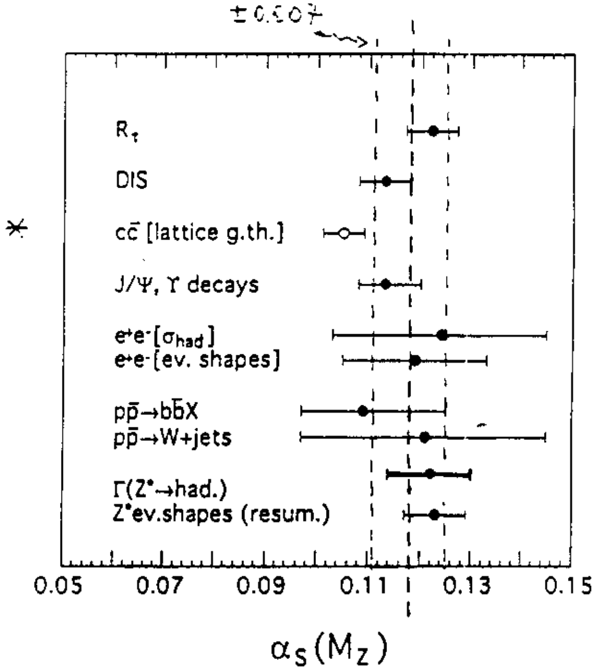


$\alpha_s(Q)$ from jet physics consistent with QCD running

Summary of $\alpha_s(M_Z)$

weighted average $\alpha_s(M_Z) = 0.118$

error** $\Delta\alpha_s(M_Z) = \begin{cases} \pm 0.005 & \text{(optimistic)} \\ \pm 0.007 & \text{(conservative} \\ ? & \text{(pessimistic}; \end{cases}$



ONE SAMPLE ANALYSIS

(RATTAZZI
MAGNOLI
P. N., 1990)

-) FOR A GIVEN SET OF SHAPE VARIABLES DETERMINE $\alpha_s(M_Z)$ WITH 3 Methods

  (a) LEADING ORDER,
  (b) NEXT-TO-LEADING ORDER
  (c) NEXT-TO-LEADING ORDER + ESTIMATE OF HADRONIZATION EFFECTS

-) ERRORS FOR (UNKNOWN) HIGHER ORDER EFFECTS ARE ESTIMATED AS FOLLOWS:

  1) USE PERTURBATIVE FORMULA WITH $\mu = M_Z$, determine $\alpha(M_Z)$

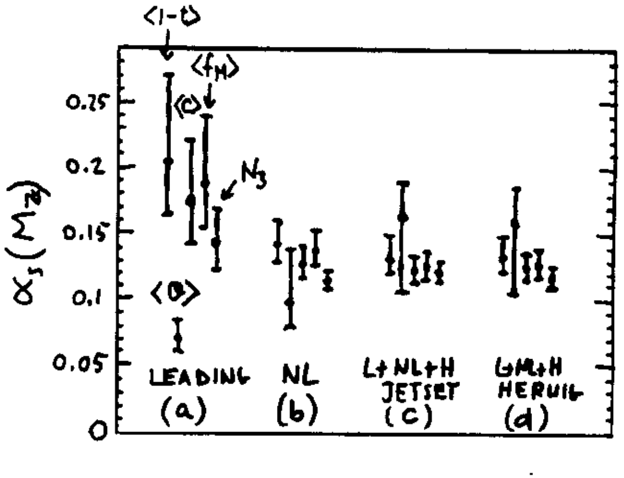
  2) Use PERTURBATIVE FORMULA FOR $\mu = \frac{M_Z}{2}$; determine $\alpha\left(\frac{M_Z}{2}\right)$, evolve it to $\alpha(M_Z)$ USING EVOLUTION EQUATION

  3) AS IN 2), WITH $\mu = \frac{M_Z}{4}$

  THE CHOICE 2) WILL GIVE OUR CENTRAL VALUE, 1) AND 2) WILL GIVE THE EXTREMES OF THE ERROR BAR (THEORETICAL)

-) EXPERIMENTAL ERRORS ARE ADDED IN QUADRATURE TO THE THEORETICAL ONES.

EARLY DATA FROM OPAL ON THRUST, C-PARAMETER, f_{MAJOR}, OBLATENESS N_3 (FRACTION OF EVENTS WITH 3 CLUSTER! FOR $Y = M^2_{CLUS}/M_Z^2 < 0.1$)



-) REMARKABLE IMPROVEMENT IN CONSISTENCY WHEN RADIATIVE CORRECTIONS ARE INCLUDED

-) HADRONIZATION CORRECTIONS BRING ABOUT TOO MUCH CONSISTENCY. DATA BIAS ?

-) LARGE UNCERTAINTIES (≈ 12 % OR SO...)

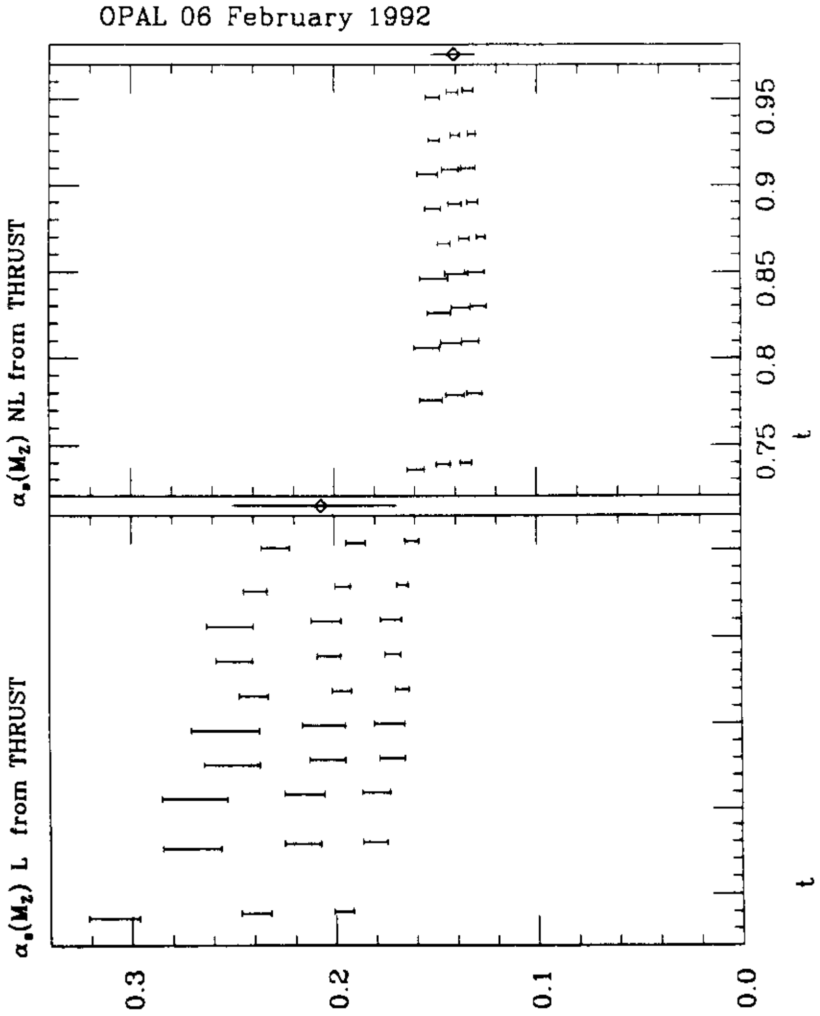
TODAY THE DATA IS SO GOOD THAT WE CAN REPEAT THIS PROCEDURE FOR EACH BIN OF MANY DISTRIBUTIONS (NO NEED TO TAKE AVERAGE VALUE). ONLY, STAY NOT TO CLOSE TO EXTREME VALUE OF SHAPE VARIABLES (REMEMBER: STERMAN-WEINBERG CALCULATION DOES NOT WORK WHEN δ, ϵ ARE TOO SMALL)

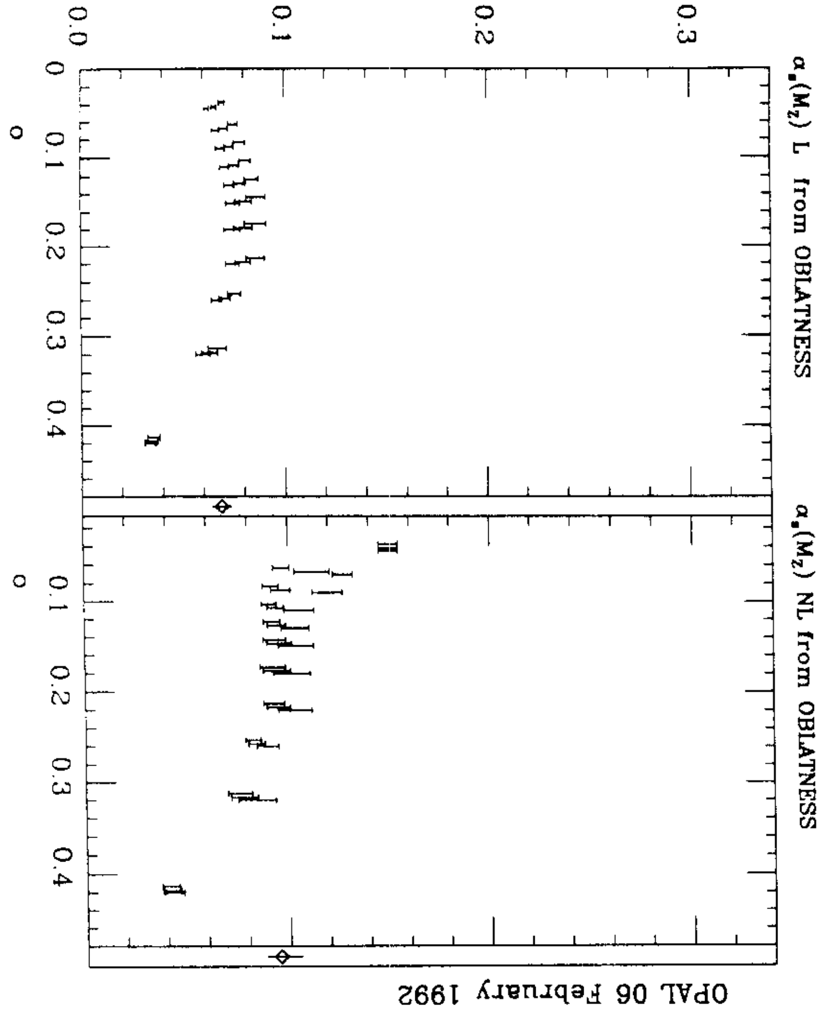
•) USE OPAL AND DELPHI PUBLISHED DATA

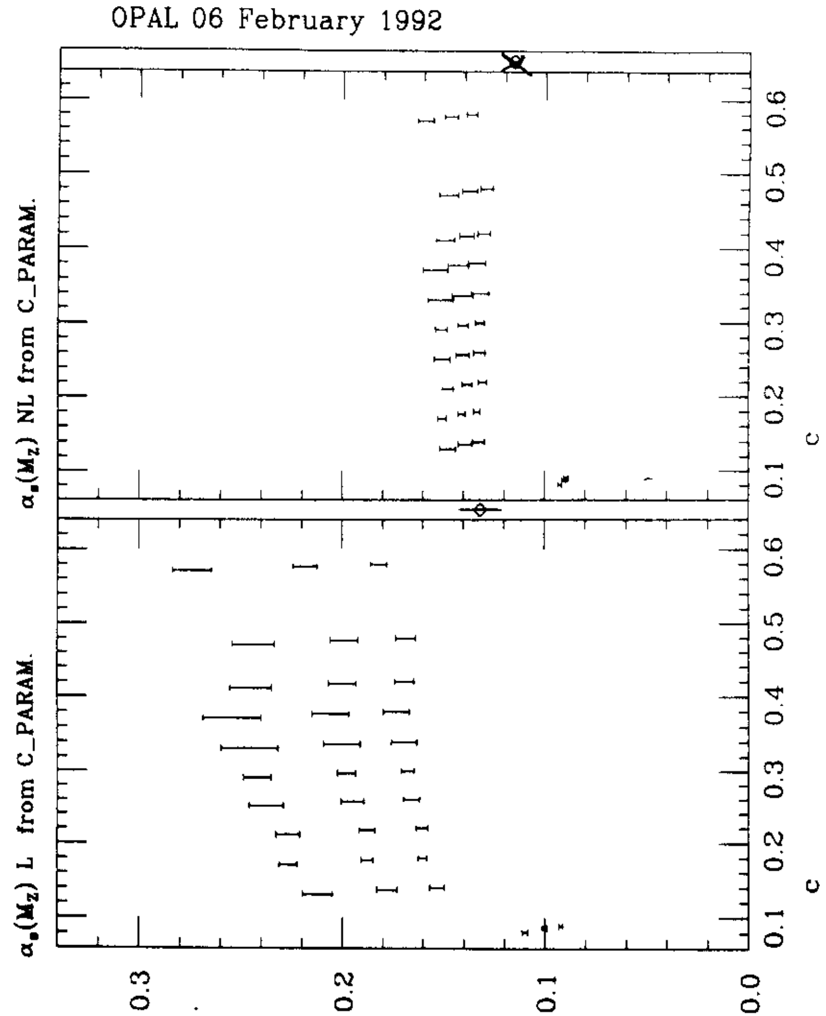
AS IN PREVIOUS (PAG. 52), BUT FOR EACH BIN OF DISTRIBUTION;
DO LEADING AND LEADING + NL.

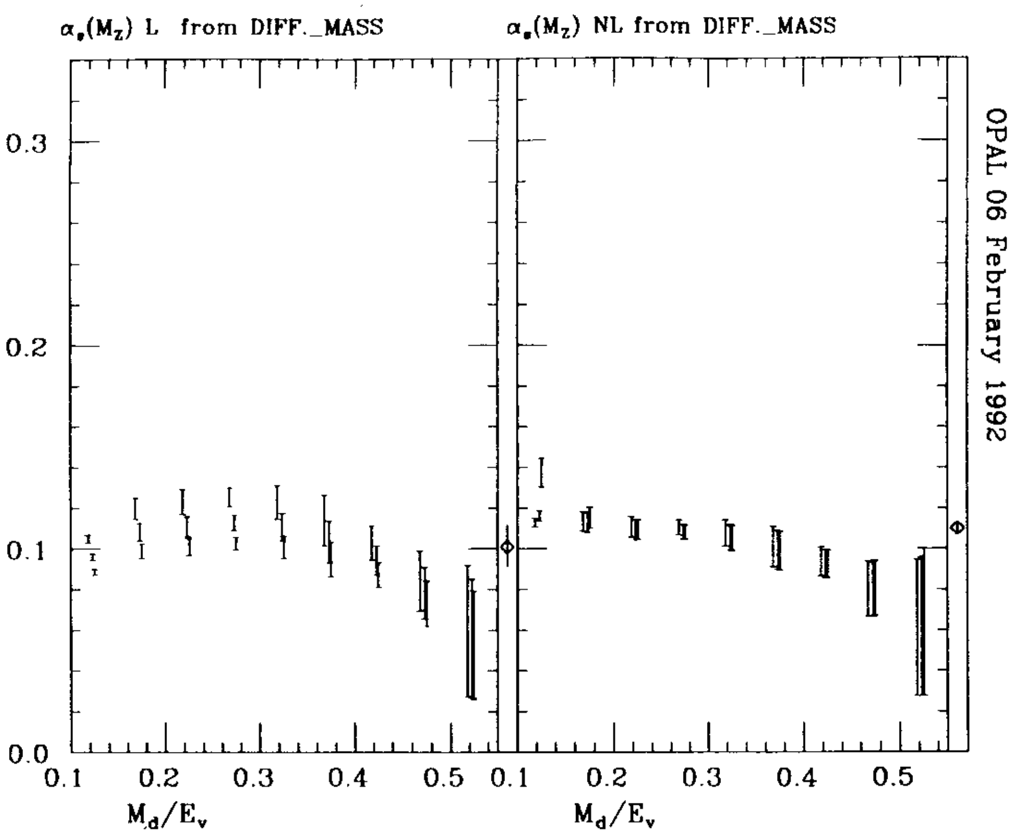
LIVE OUT HADRONIZATION (IT MAY BE BIASED).

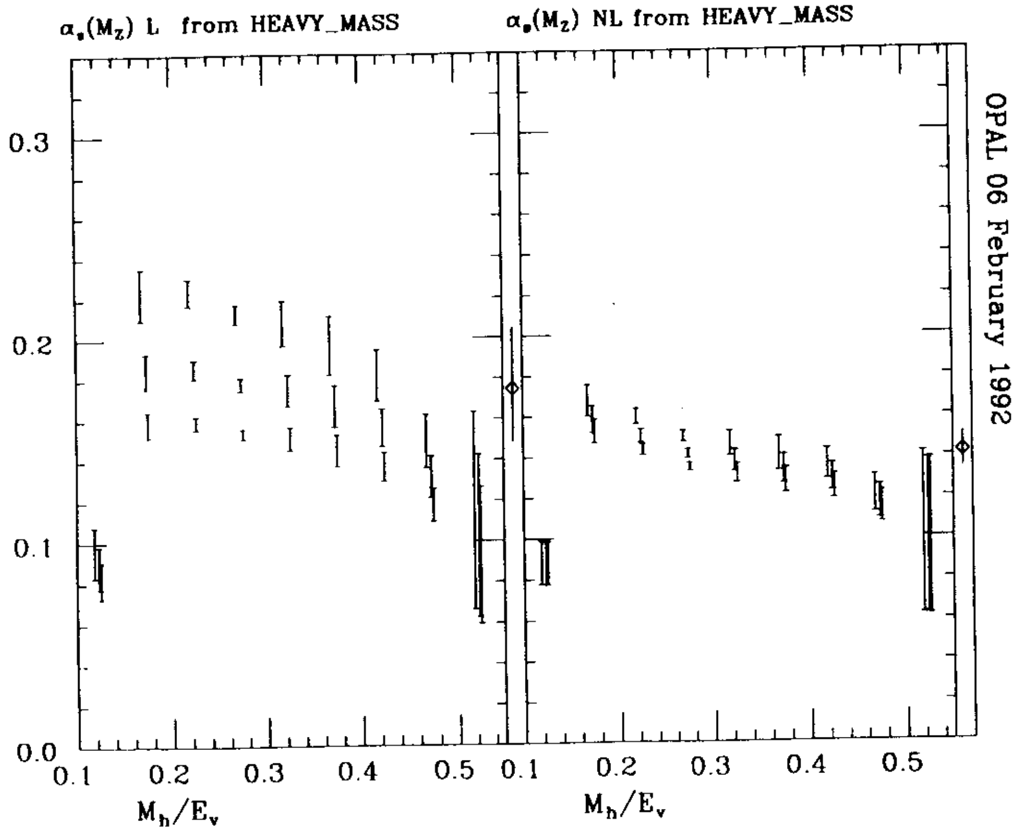
(N. BLUNDA, R. RATTAZZI, P.N., UNPUBLISHED)



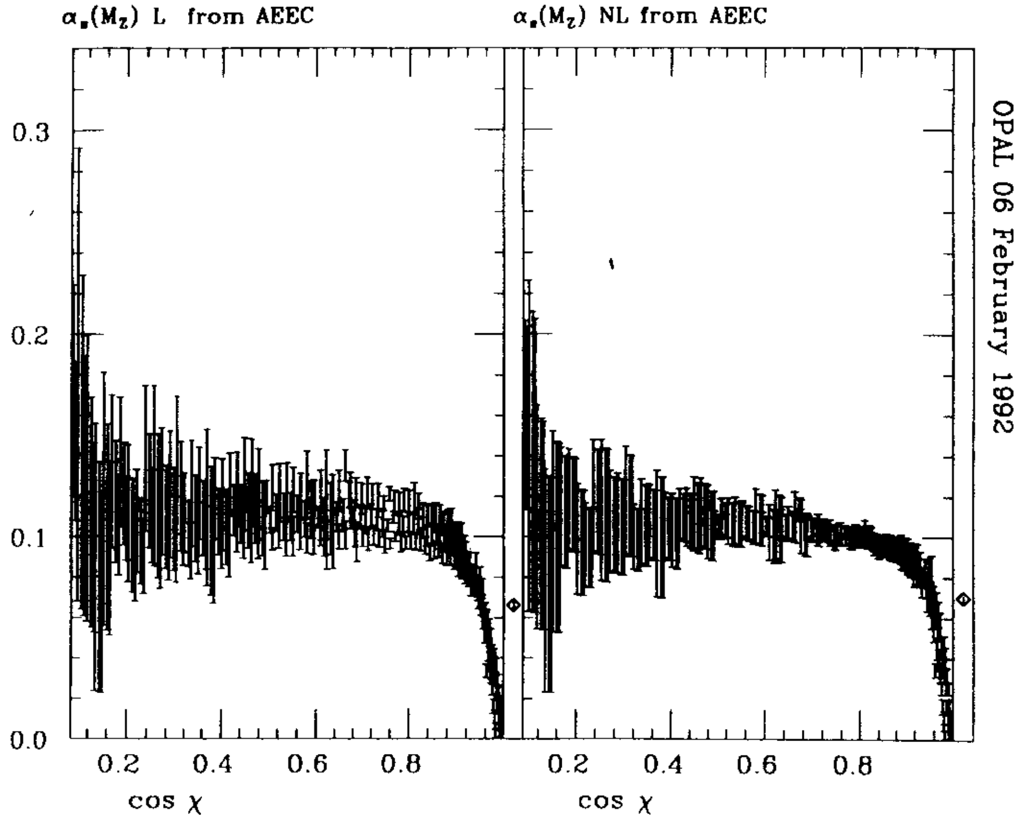

α_s(M_Z) L from OBLATNESS
α_s(M_Z) NL from OBLATNESS
o
o
0.0
0.1
0.2
0.3
0
0.1
0.2
0.3
0.4
0.1
0.2
0.3
0.4
OPAL 06 February 1992


OPAL 06 February 1992
α_s(M_Z) NL from C_PARAM.
α_s(M_Z) L from C_PARAM.
c
c
0.1
0.2
0.3
0.4
0.5
0.6
0.1
0.2
0.3
0.4
0.5
0.6
0.3
0.2
0.1
0.0

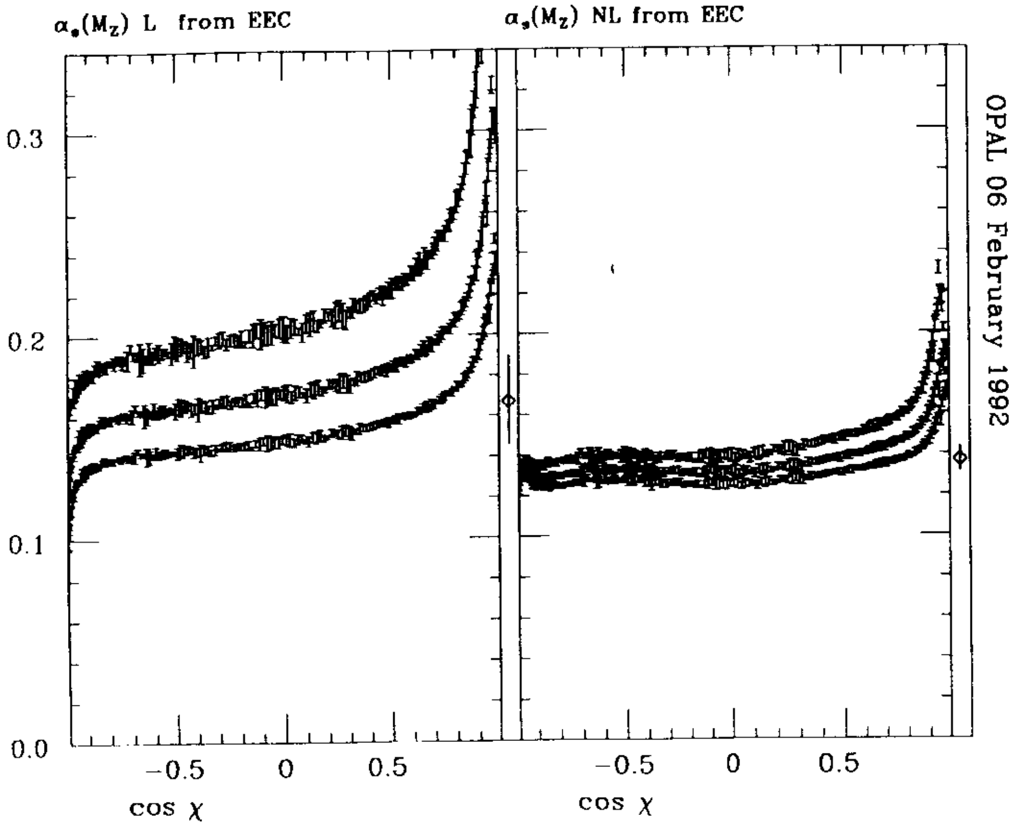

$\alpha_s(M_Z)$ L from DIFF._MASS
$\alpha_s(M_Z)$ NL from DIFF._MASS
OPAL 06 February 1992
0.3
0.2
0.1
0.0
0.1
0.2
0.3
0.4
0.5
0.1
0.2
0.3
0.4
0.5
M_d/E_v
M_d/E_v


$\alpha_s(M_Z)$ L from HEAVY_MASS
$\alpha_s(M_Z)$ NL from HEAVY_MASS
OPAL 06 February 1992
0.3
0.2
0.1
0.0
0.1
0.2
0.3
0.4
0.5
0.1
0.2
0.3
0.4
0.5
M_h/E_v
M_h/E_v

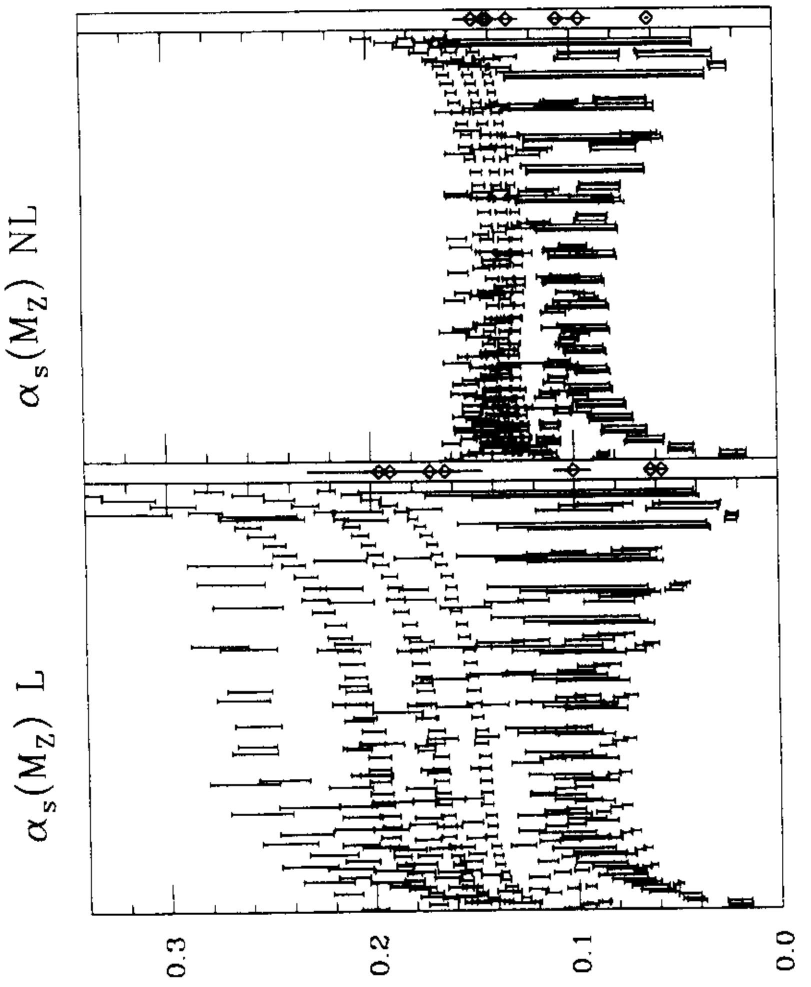
(61)


$\alpha_s(M_Z)$ L from AEEC
$\alpha_s(M_Z)$ NL from AEEC
0.0
0.1
0.2
0.3
0.2
0.4
0.6
0.8
cos χ
0.2
0.4
0.6
0.8
cos χ
OPAL 06 February 1992

(60)


$\alpha_s(M_Z)$ L from EEC
$\alpha_s(M_Z)$ NL from EEC
0.0
0.1
0.2
0.3
−0.5
0
0.5
cos χ
−0.5
0
0.5
cos χ
OPAL 06 February 1992

DELPHI 23 July 1991



OPAL 06 February 1992