INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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SMR.762 - 21

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

13 June - 29 July 1994

COMPUTATIONAL STUDIES OF INSTANTON INDUCED BARYON NUMBER VIOLATION

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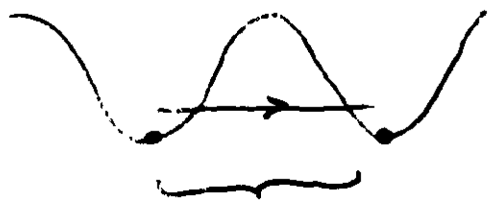
$$\Delta B = \Delta L = N_g \, \Delta Q$$

calculate the amplitude for processes with $\Delta Q \neq 0$

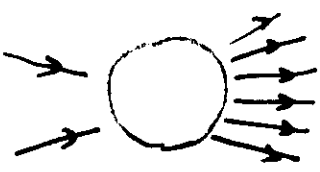
Tunnelling



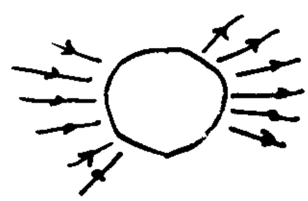
$\Delta Q = 1$

$$\sigma \propto e^{-\frac{16\pi^2}{g^2}}$$

but one must sum over final states (and initial st. and account for final (initial) state interactions



large E



large T

High energy processes

2 → anything (exclusive → inclusive)

very problematic

Semiclassical methods can be used to analyze

V. Rubakov
D. Son
P. Tinyakov

$N \to$ anything

with $N = \frac{\nu}{g^2}$ ν fixed

One finds:

$$\sigma \sim e^{-\frac{16\pi^2}{g^2} F\left(\frac{E}{E_0}, \nu\right) + O(g^0)}$$

F can be related to the action of a classical evolution along a complex time contour.

We will consider

A) the Abelian-Higgs model in 2D (1 space + 1 time)

B) the SU(2)-Higgs system in 4D, but with rotational symmetry

A) $\varphi(x,t)$ $\quad \bar{\varphi}(x,t)$ $\quad A_\mu(x,t)$ $\quad \mu = 0,1$

$$\mathcal{L} = (\overline{D_\mu \varphi})(D^\mu \varphi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\lambda}{4} (\bar{\varphi}\varphi - v^2)^2$$

$$D_\mu \varphi = \partial_\mu \varphi - i e A_\mu \varphi$$

vacuum states

$$|\varphi| = v$$

but phase is arbitrary

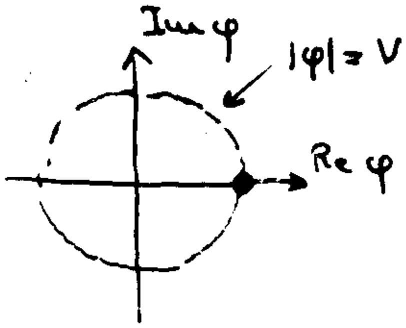
- because of local gauge invariance the phase is irrelevant, but phase rotations of multiples of 2π distinguish topologically inequivalent vacuum states

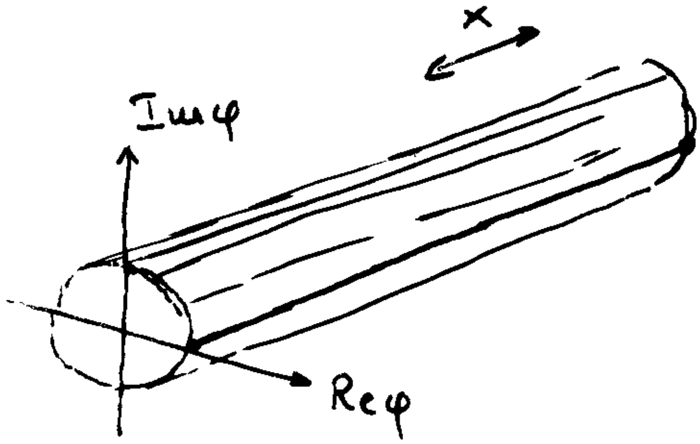
Consider $A_0 = 0$ gauge
Then the phase of φ at $x = \pm\infty$ has to be fixed, and we can take

$$\varphi\,(x = \pm\infty) = v \quad \text{real positive}$$

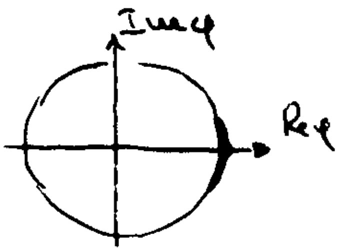
Vacuum states

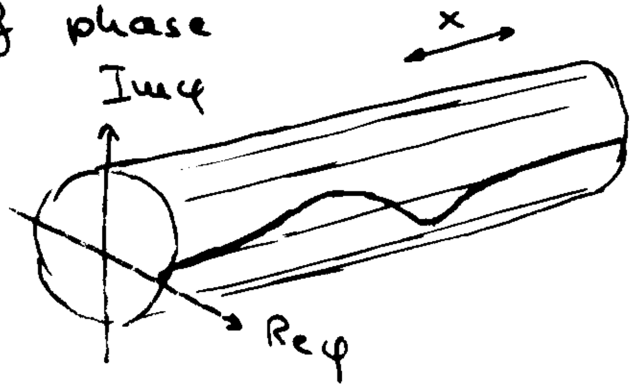
I:





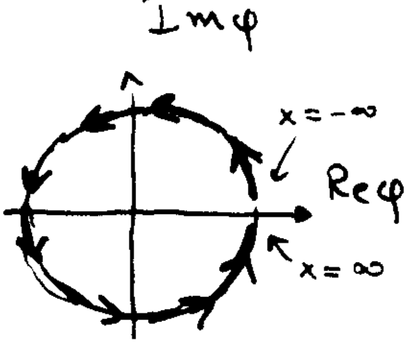
this is topologically equivalent to a state with a local variation of phase

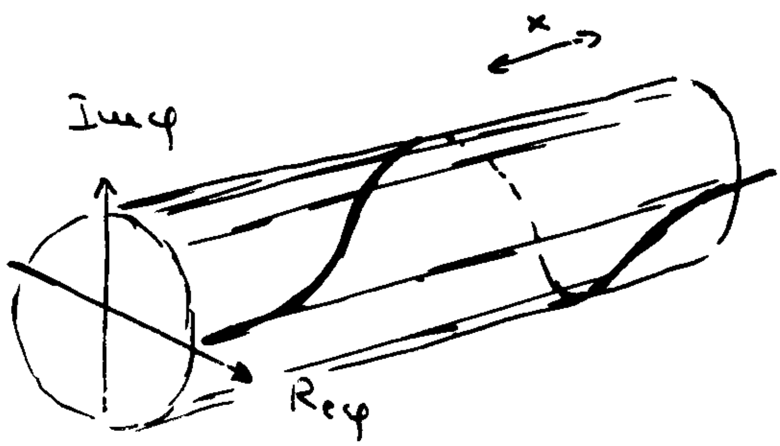




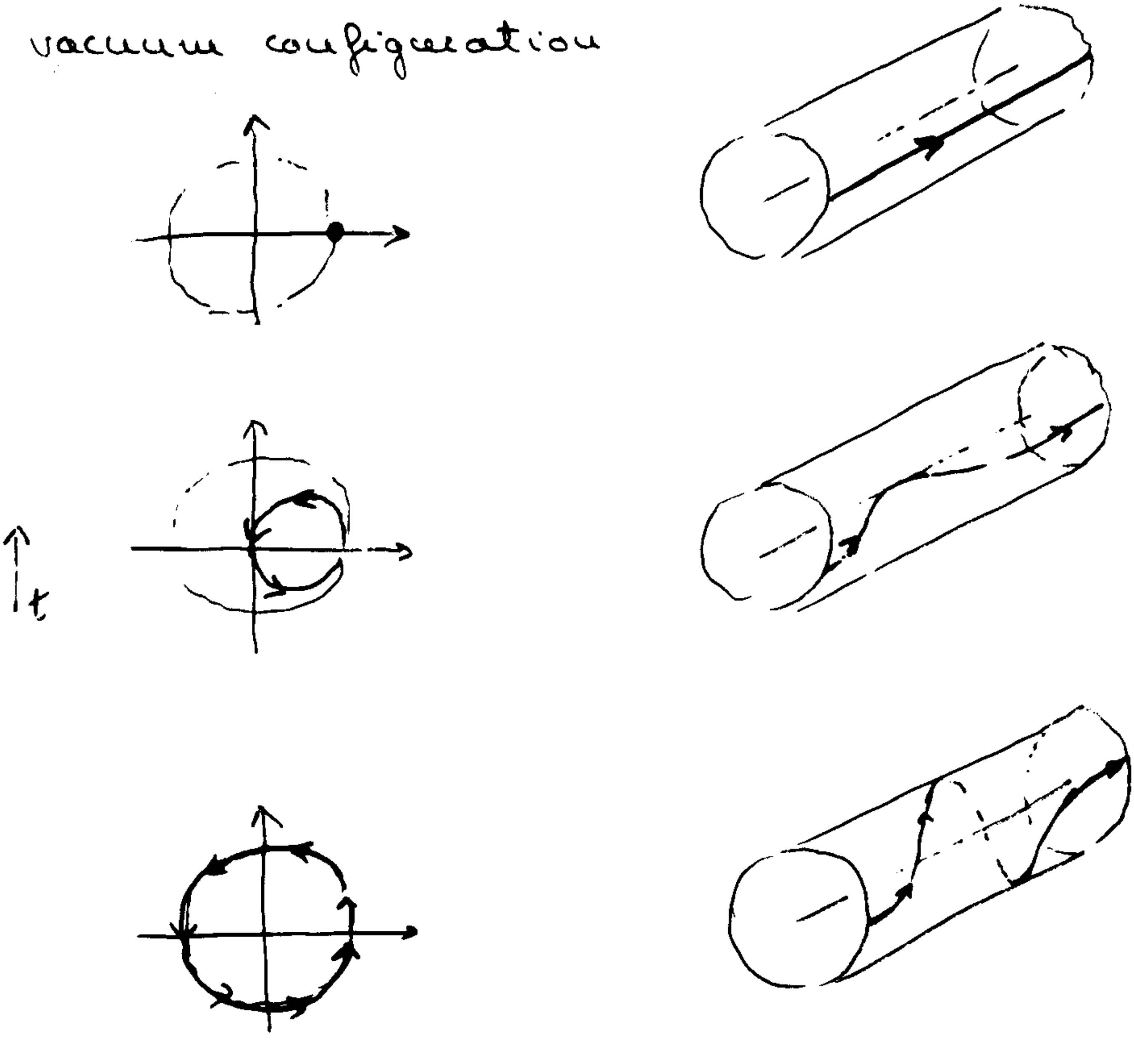
but inequivalent to the following

II:



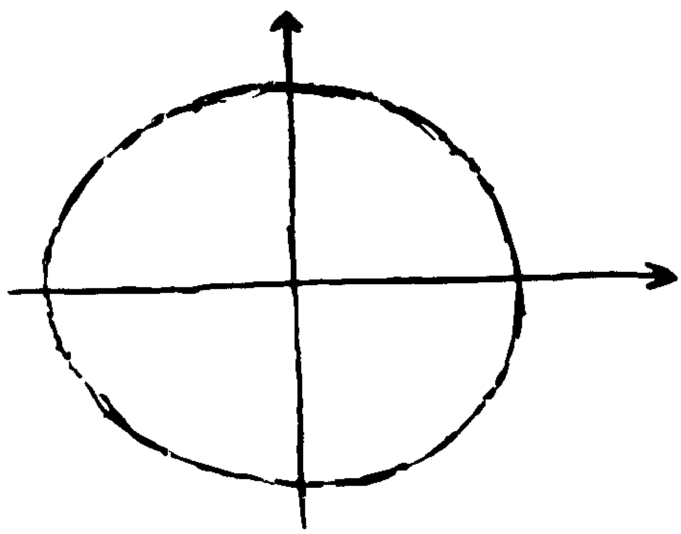


In a transition between the two topologically inequivalent vacuum state the field must necessarily leave its vacuum configuration



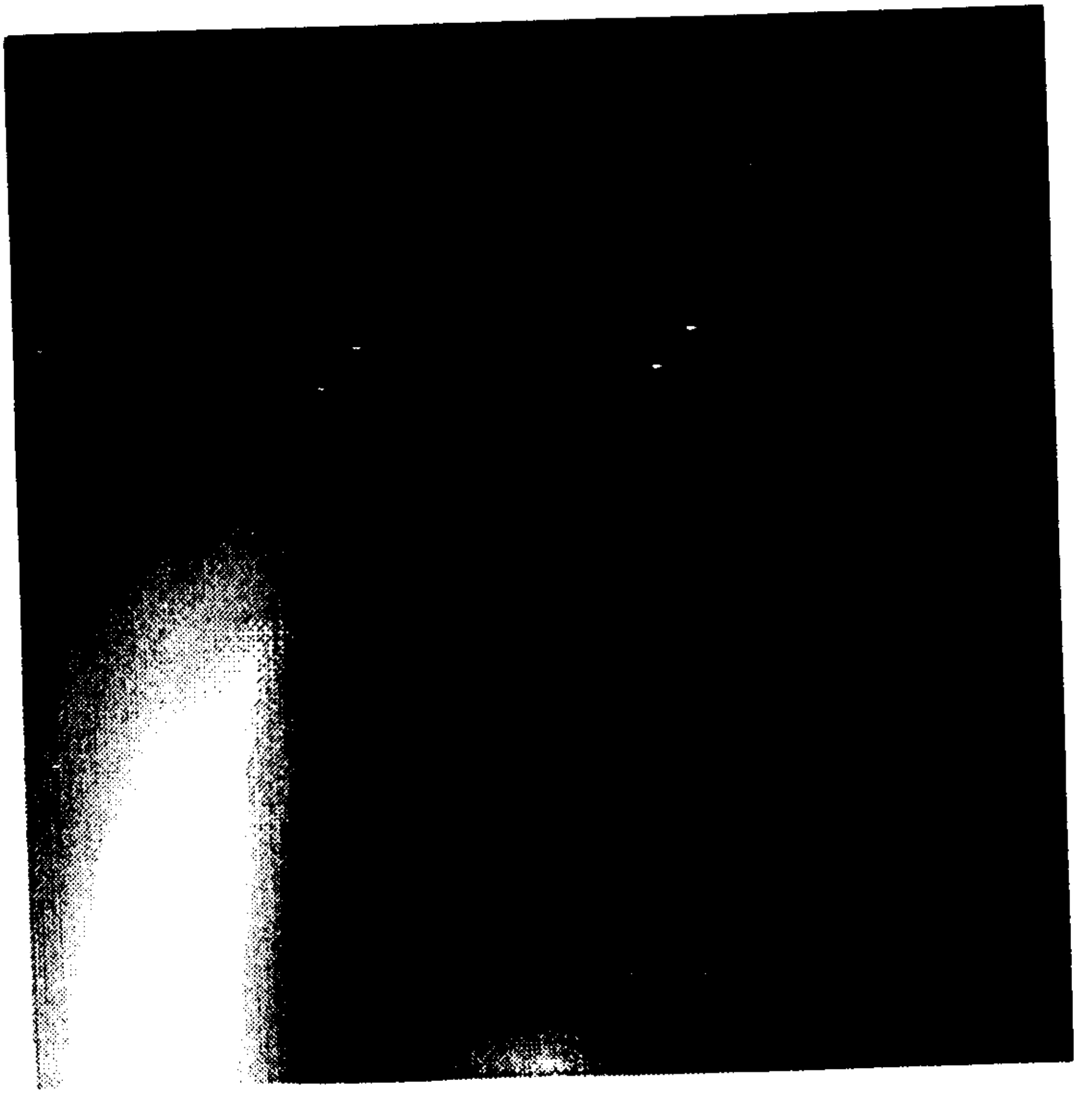
For a visualization of the space-time configurations (or evolutions - but not necessarily solutions of the eqs. of motion) of the fields it will be convenient to use color to code the complex phase.

E.g.



The modulus of φ can then be represented by color intensity (in a 2-D diagram) or by elevation along the z-axis (in a 3D plot)

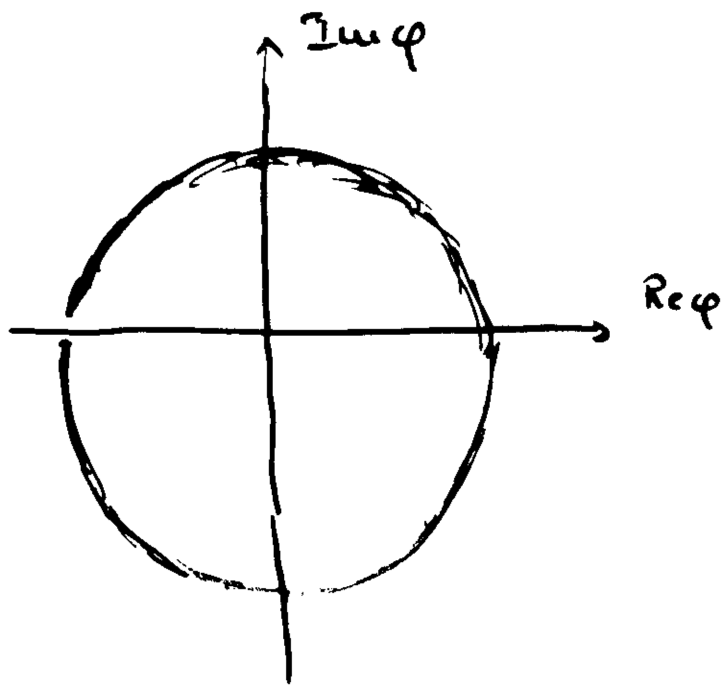
~~The~~ A transition between two inequivalent vacua would then look as follows ---



Instanton transition in the 2–d Abelian Higgs model

C. Rebbi and R. Singleton. June 1994

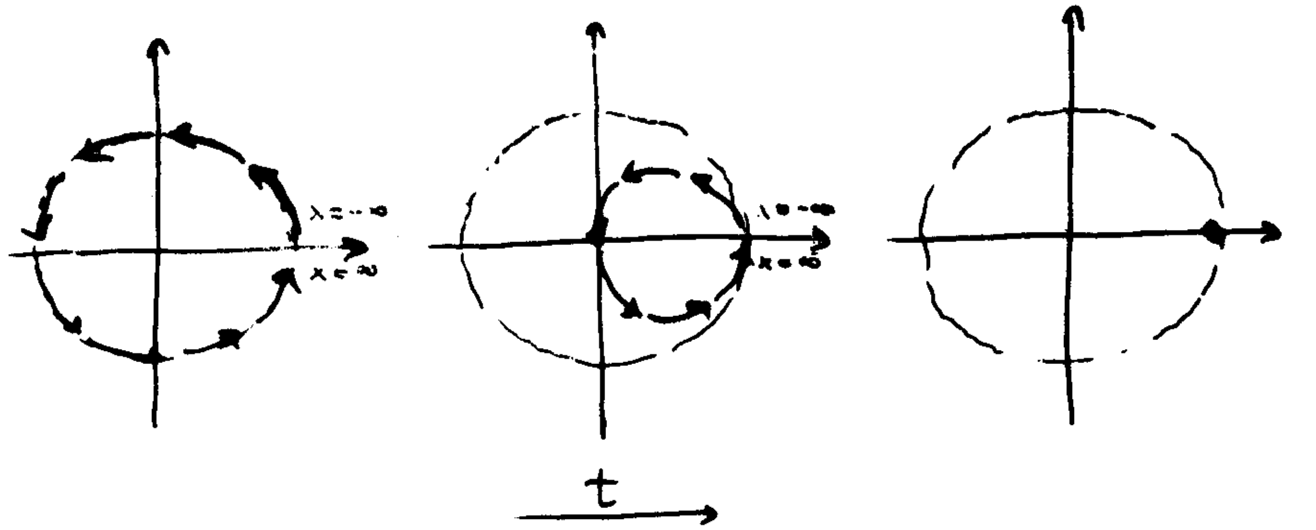
Note: we will not always use the same color coding for the phase. E.g., in some future illustrations the following will be used



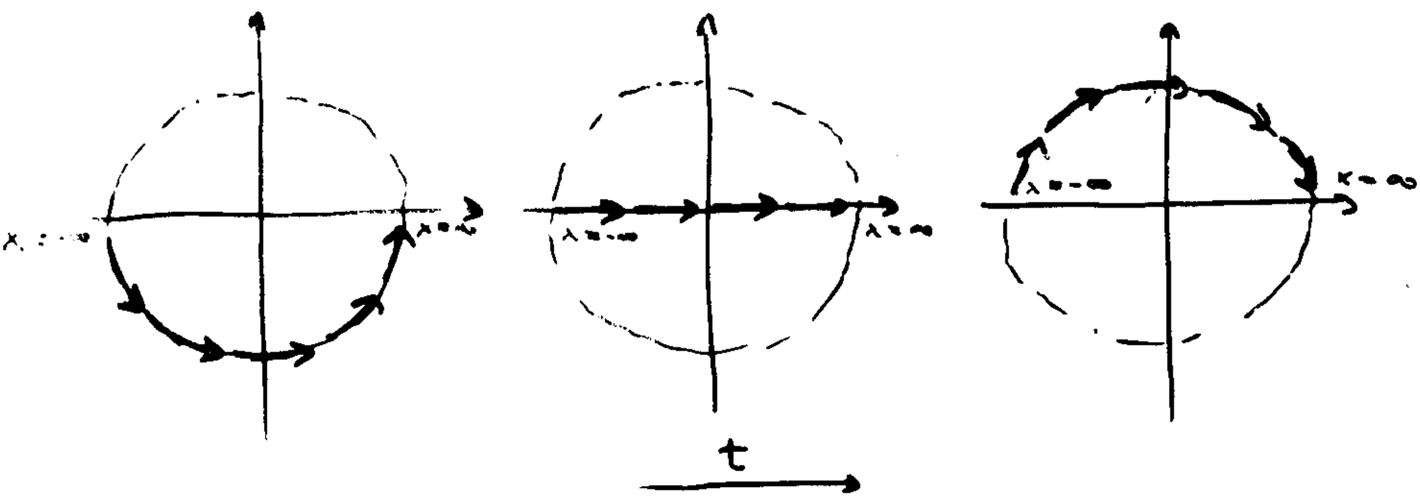
Also, more important, the condition $\varphi(\pm\infty) = v$, real positive, is only a convention.

In the $A_0 = 0$ gauge $\varphi(-\infty)$ and $\varphi(+\infty)$ must remain fixed (with respect to t), but one can always perform a time independent gauge transformation.

Thus the transition



can be gauge transformed to

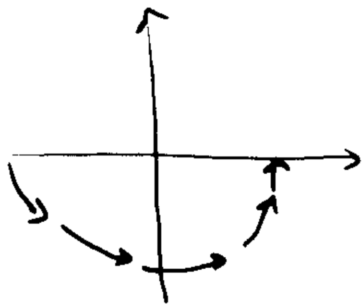


(more explicitly symmetric with respect to $t \to -t$)

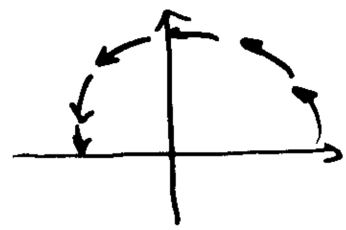
Now $\quad \varphi(-\infty) = -1 \qquad \varphi(\infty) = 1$

These are all space-time field configurations with a change of topology ($\Delta Q = 1$), not necessarily solutions to any equation of motion.

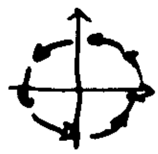
The "instanton" is a solution to the Euclidean equations of motion, where the field goes from



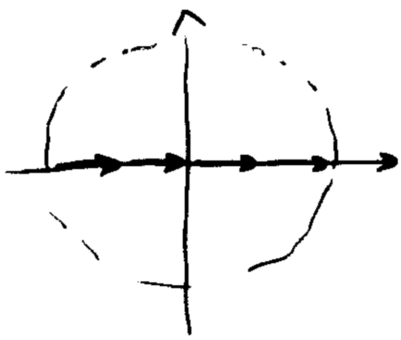
to



in ∞ time.

[Or from  to ]

The mid-point configuration



where

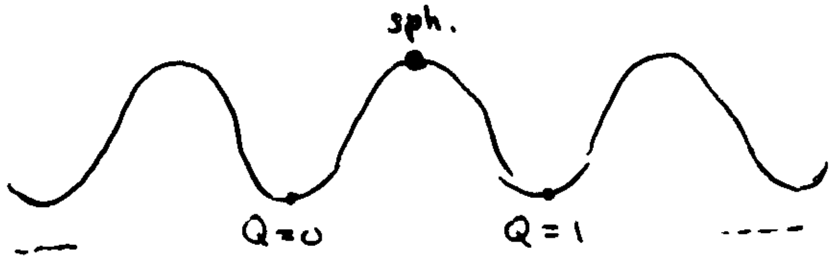
$$\varphi = \text{th}\left[\tfrac{1}{2}\sqrt{\lambda}\,(x - x_0)\right]$$

$(v = 1)$

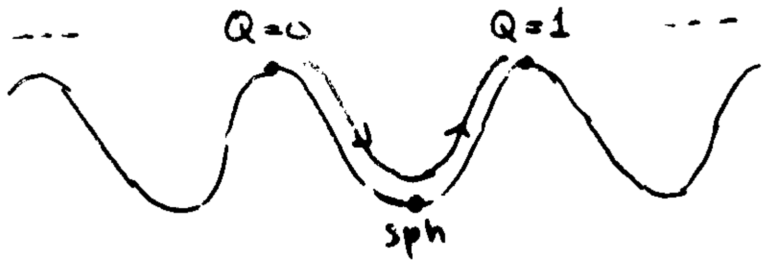
$$A_\mu = 0$$

is also a static (time-independent) solution to the Minkowskian eqs. of motion, and is called the "sphaleron".

In Minkowski space-time the sphaleron sits on top of the energy barrier:



In Euclidean space-time, with $V \to -V$, the sphaleron lies at the bottom of the well



$\longrightarrow$ instanton

$V \to -V$ is however an oversimplification, because $|\vec{\nabla}\varphi|^2 \to -|\vec{\nabla}\varphi|^2$ and the hyperbolic evolution eqs. of Minkowski space-time become elliptic in Euclidean space-time

In Minkowski space-time the sphaleron is unstable and, if perturbed, it will decay, as illustrated in the following graphs ..

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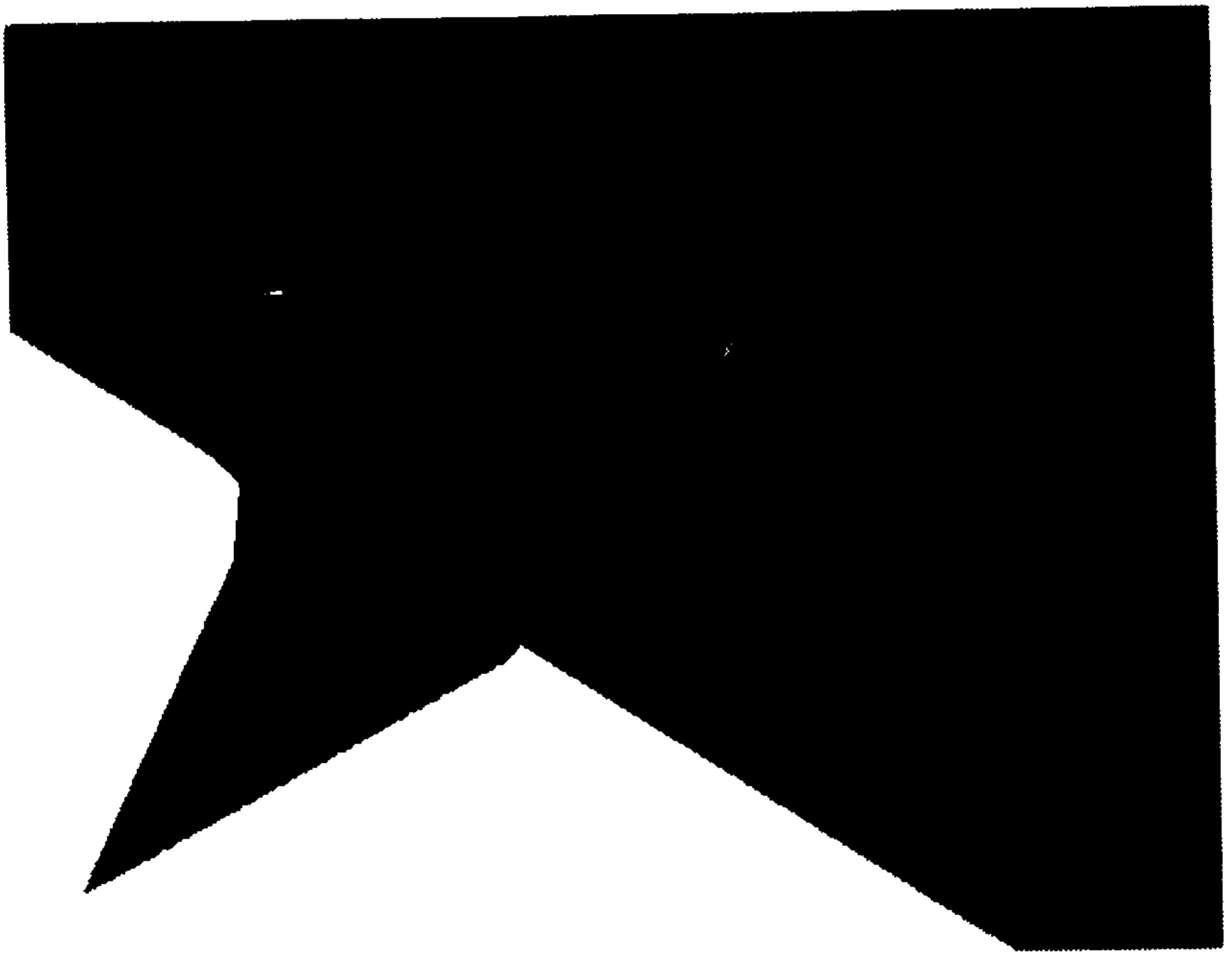
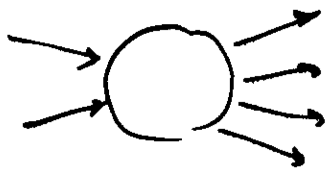


Sphaleron decay in the Abelian Higgs model (detail)

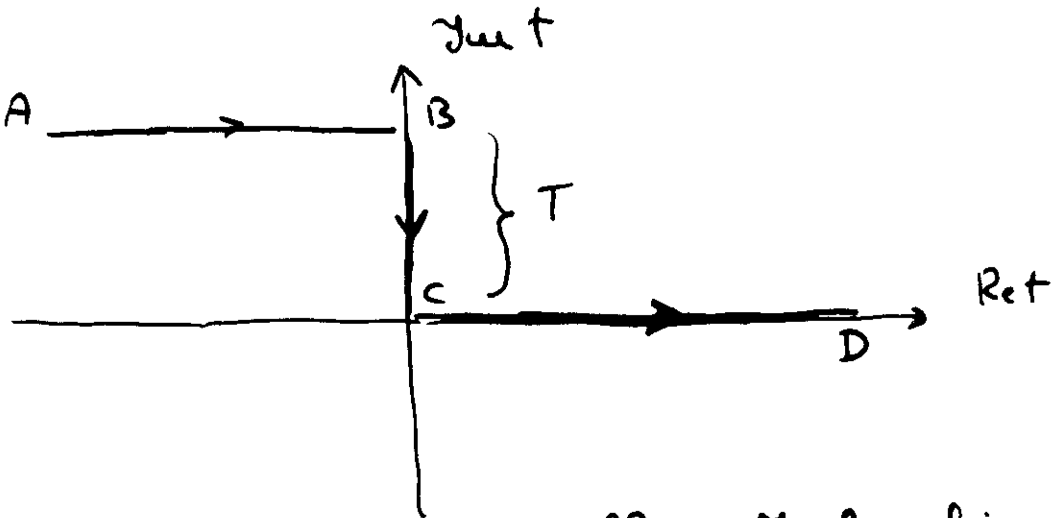
[illegible] June 1994

Semiclassical calculation of



$N = \cancel{8} \frac{\nu}{g^2} \Rightarrow$ anything
(Rubakov, Son, Tinyakov)

Evolve along complex-time path



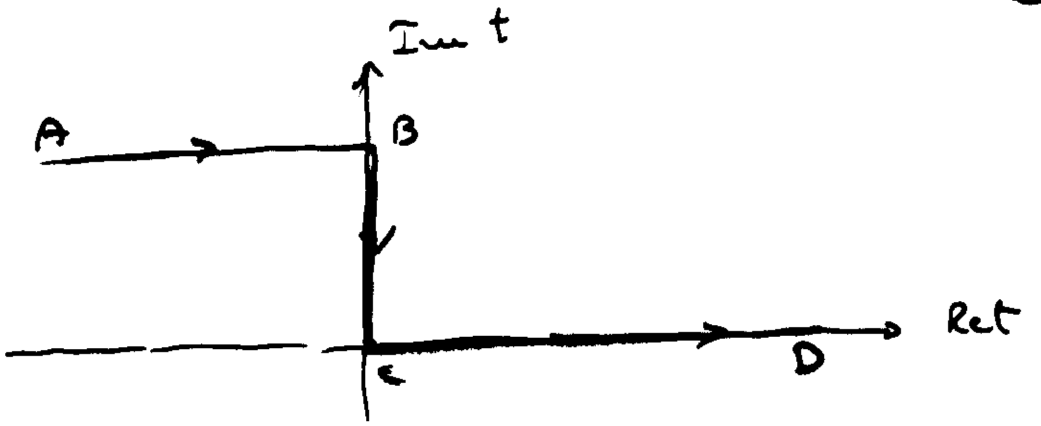
AB : Minkowskian eq.o.m.
BC : Euclidean " - -
CD : Minkowskian - - -

for Re t << (A)
and Re t >> (D) the fields will evolve like free fields, thus

$$\varphi(x,t) = \sum_\lambda \quad a_\lambda \varphi^{(\lambda)}(x) e^{-i\omega_\lambda t} + \bar{a}_\lambda \varphi^{(\lambda)*}(x) e^{i\omega_\lambda t}$$

Demand
$$\bar{a}_\lambda = e^{\vartheta} a_\lambda^*$$

The real parameter ϑ acts as a chemical potential and controls the mean number of particles ν



A: $\quad \vartheta \neq 0 \qquad \bar{a}_\lambda = e^{\vartheta} a_\lambda^*$

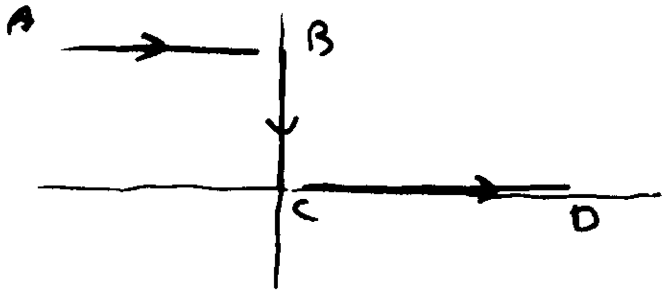
D: $\quad \vartheta = 0 \qquad \bar{a}_\lambda = a_\lambda^*$, φ is real

[NB: with charged, complex fields the
[condition is slightly different
[φ is expanded into amplitudes b , $\bar{c}$
[$\bar{\varphi}$ " " " " $\bar{b}$, c
[and the b.c. read $\bar{c} = e^{\vartheta} c^*$ $\bar{b} = e^{\vartheta} b^*$,

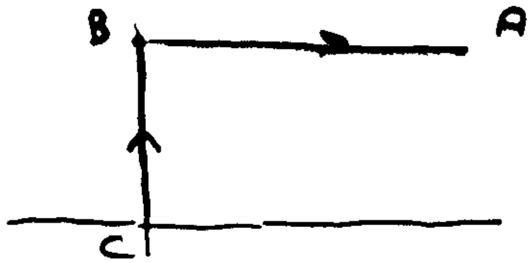
[for $\vartheta = 0$ $\qquad \bar{\varphi} = \varphi^*$]

Thus the fields are 'real' on CD,
but, with $\vartheta \neq 0$ at A, will be complex
along BC and AB

Use a time inversion, for convenience



$\Rightarrow$



The computational problem:

•) at C, initial point for the Euclidean evolution along $C \to B$

φ is real $\qquad \pi = \dot{\varphi}$ is pure imaginary

(2N real degrees of freedom, after discretization)

•) evolve along $C \to B$ and along $B \to A$

•) identify the normal mode amplitudes a_λ and $\bar{a}_\lambda$ of the "complex" field φ

a_λ: (N complex) = 2N real $\qquad \bar{a}_\lambda$: (N complex = 2N real)

•) impose the N complex = 2N real eqs:

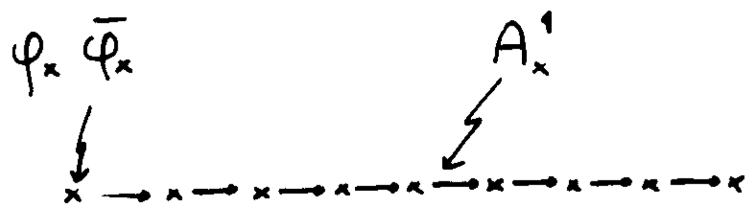
$$\bar{a}_\lambda = e^{\theta} a_\lambda^*$$

Computational solution of the equations of motion.

The system must be discretized

$$x = 0 \quad a \quad 2a \quad \dots \quad na \quad \dots \quad Na$$

Discretize preserving gauge invariance



- the matter field (Higgs field) is defined over the sites of the lattice $\varphi_x \quad \bar\varphi_x$
- the space component (A^1) of the gauge field is defined over the oriented links joining neighbouring sites $A^1_x : \quad x \to x+a$
- we will use the $A^0 = 0$ gauge for the time evolution, but, in a Lagrangian formulation, A^0 is defined over the sites A^0_x, like $\varphi\,\bar\varphi$

Consider the 4-D SU(2)-Higgs system

Vacuum structure (in the $A_0 = 0$ gauge)

$$A_i(\vec{x}) = \frac{1}{g} U^{-1}(\vec{x}) \frac{\partial}{\partial x^i} U(\vec{x})$$

$$U \in SU(2)$$

$$|\vec{x}| \to \infty \qquad U(\vec{x}) \to \mathbb{I}$$

The mapping $\vec{x} \to SU(2)$ defined above is then, topologically, a mapping $S_3 \to S_3$

There is a discrete infinity of topologically inequivalent vacuum states.

The BPST instanton is a solution to the Euclidean equations of motion interpolating, in ∞ time, between two neighboring vacua.

Adding a Higgs field in the fundamental representation does not alter the vacuum structure, just changes the detailed form of the solutions.

Spherically symmetric Ansatz
combine L (dependence of gauge fields A^0, A^i and of Higgs field Φ on $\vec{x}$)
S (dependence of A^i on vector index i)
and T (dependence of all fields on the SU(2) gauge indices) into $L+S+T=0$
(cfr B. Ratra + G. Yaffe '88)

$$A_0(x) = \frac{1}{g}\frac{1}{2i}\, a_0(r,t)\, \vec{\tau}\cdot\hat{x}$$

$$\vec{A}(x) = \frac{1}{g}\frac{1}{2i}\Big[a_1(r,t)\,(\vec{\tau}\cdot\hat{x})\hat{x} + \frac{\alpha(r,t)}{r}\,(\vec{\tau}-(\vec{\tau}\cdot\hat{x})\hat{x}) + \frac{1+\beta(r,t)}{r}\, i((\vec{\tau}\cdot\hat{x})\vec{\tau}-\hat{x})\Big]$$

$$\Phi(x) = \frac{1}{g}\,[\mu(r,t) + i\nu(r,t)\,\vec{\tau}\cdot\hat{x}]\}$$

The 4 fields $a_0(r,t)$ $a_1(r,t)$

$$\chi(r,t) = \alpha(r,t) + i\beta(r,t)$$

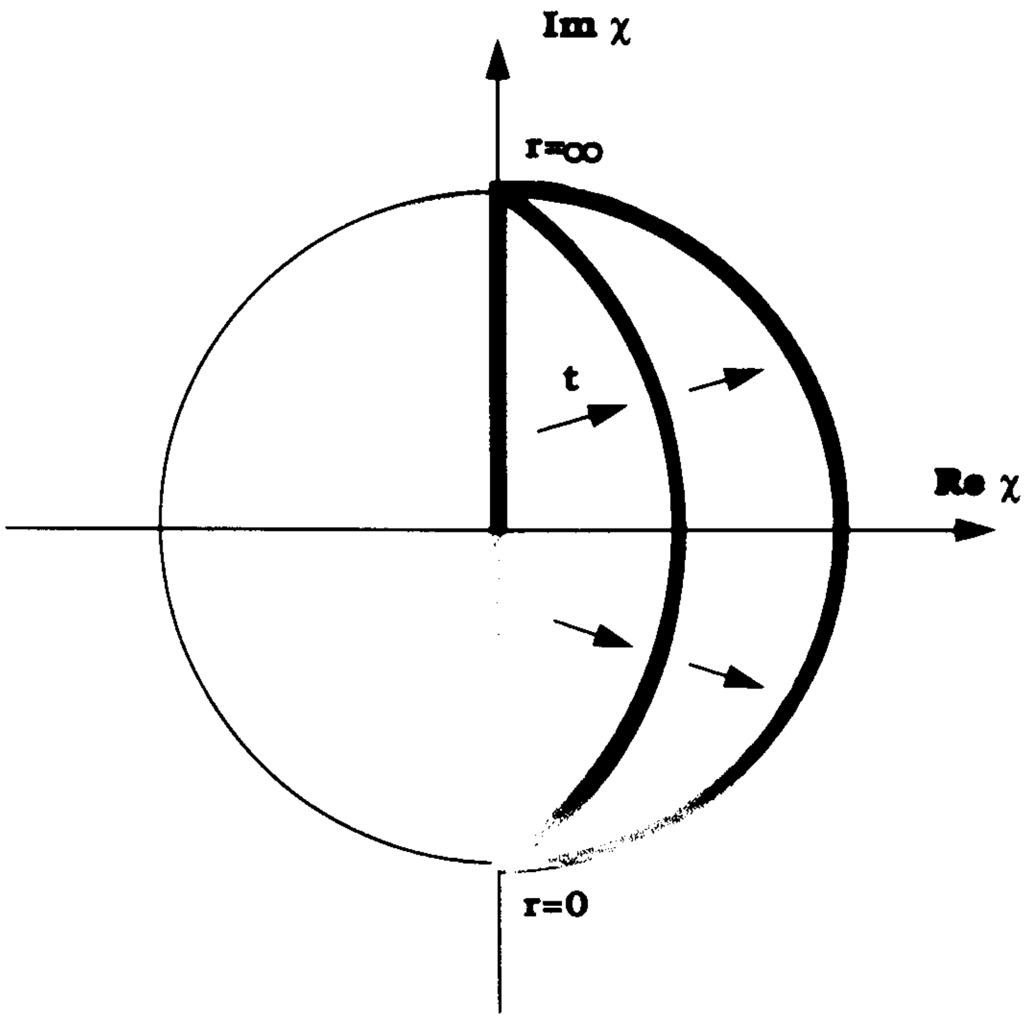
and

$$\varphi(r,t) = \mu(r,t) + i\nu(r,t)$$

transform like one Abelian gauge field $(a_0\, a_1)$ and two charged Higgs fields of charge

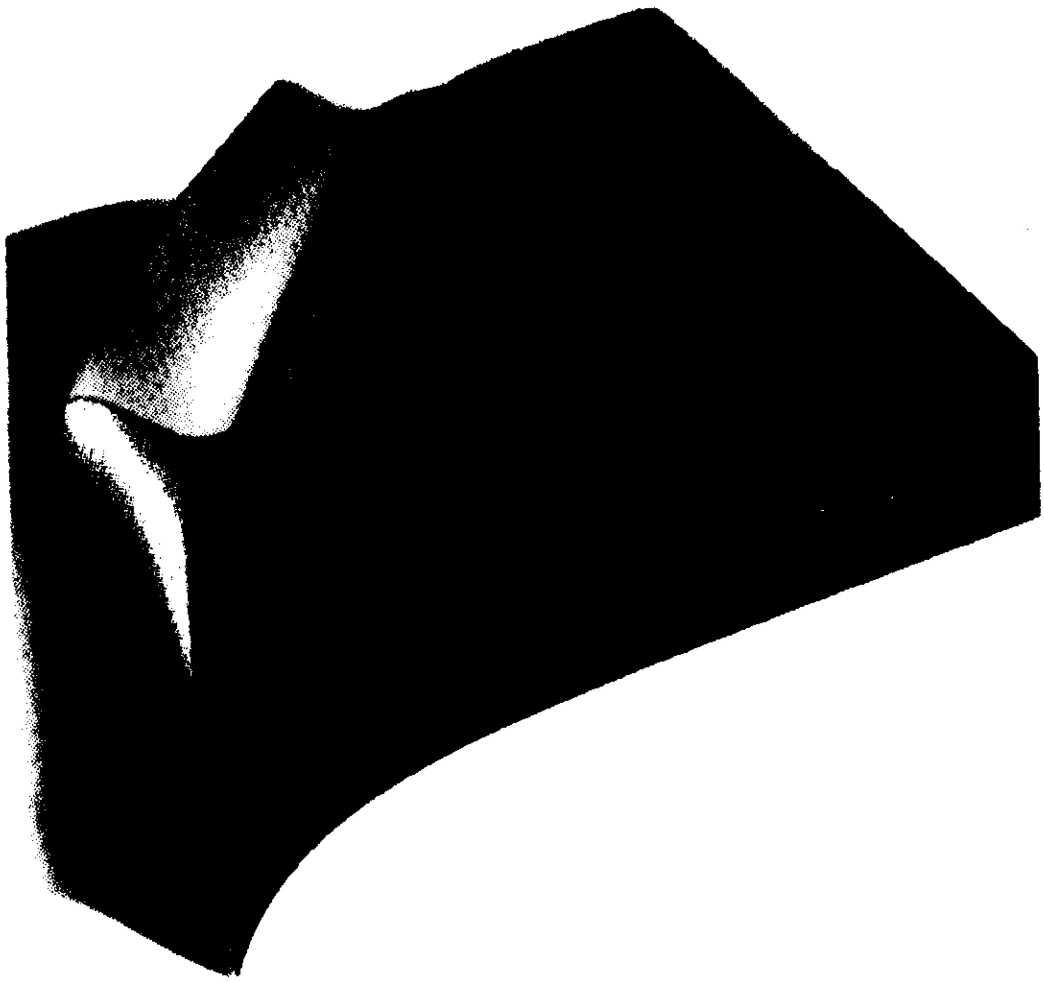
The situation is rather similar to the Abelian-Higgs model in so far as the relation between the phase variation of χ and the topological charge of the vacuum is concerned (with the present conventions $\chi(r=0) = -i$ $\chi(r=\infty) = i$; the vacuum states are always characterized by $|\chi| = 1$ $|\varphi| = 1$. In the sphaleron configuration χ is purely imaginary).
But the various metric factors in the equations of motion which reflect the reduction from 3 space dimensions to the radial coordinate give origin to a much faster dispersion and quicker approach to the linear regime.

Identifying the normal mode amplitudes is non-trivial. It requires performing a gauge transformation (straightforward) and finding numerically the eigenvectors of the equations of motion.



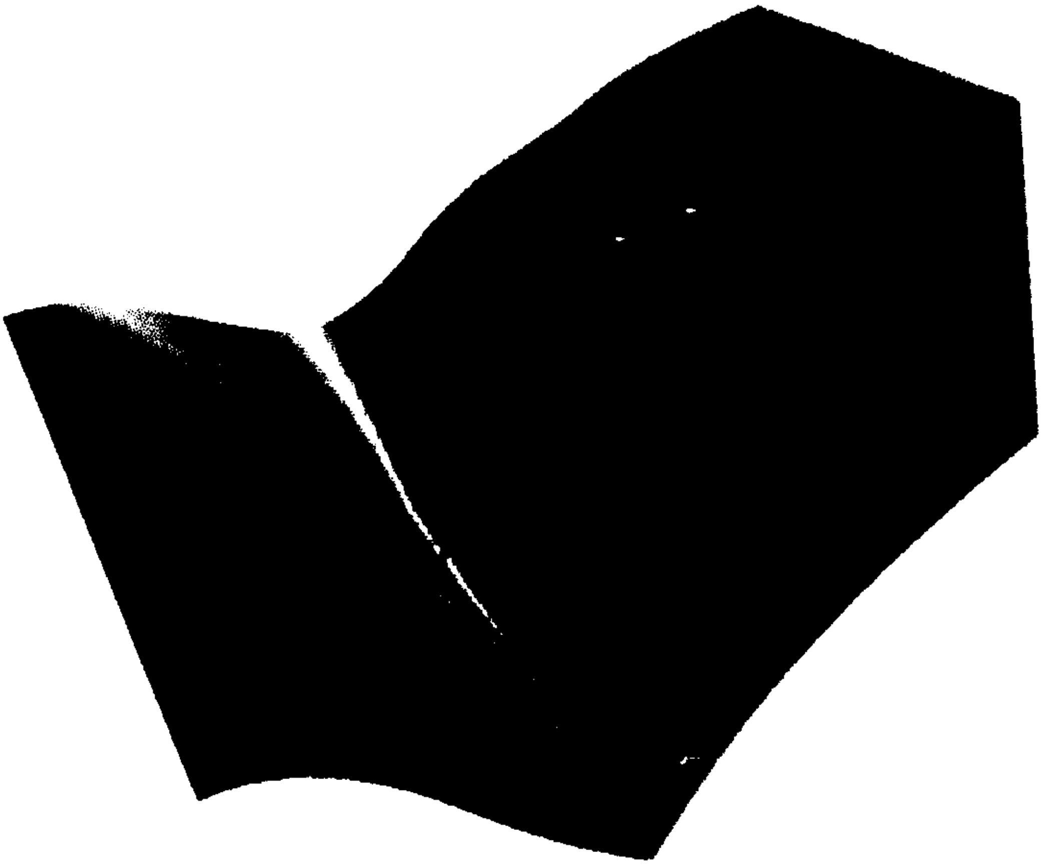
Behavior of the χ field in the course of the decay of the sphaleron. The color represents the phase of the complex field.

C. Rebbi and R. Singleton, June 1994



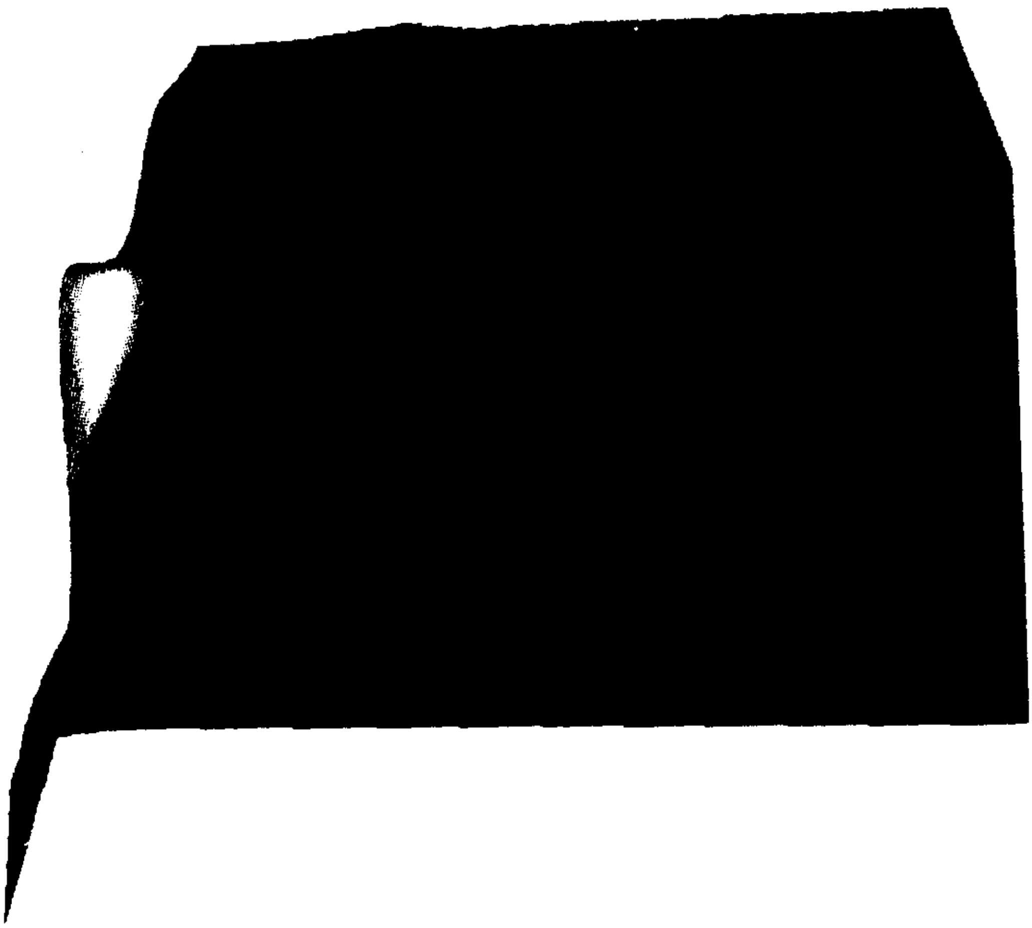
Sphaleron decay in the 4D SU(2) Higgs model: the gauge field

C. Rebbi and R. Singleton, June 1994



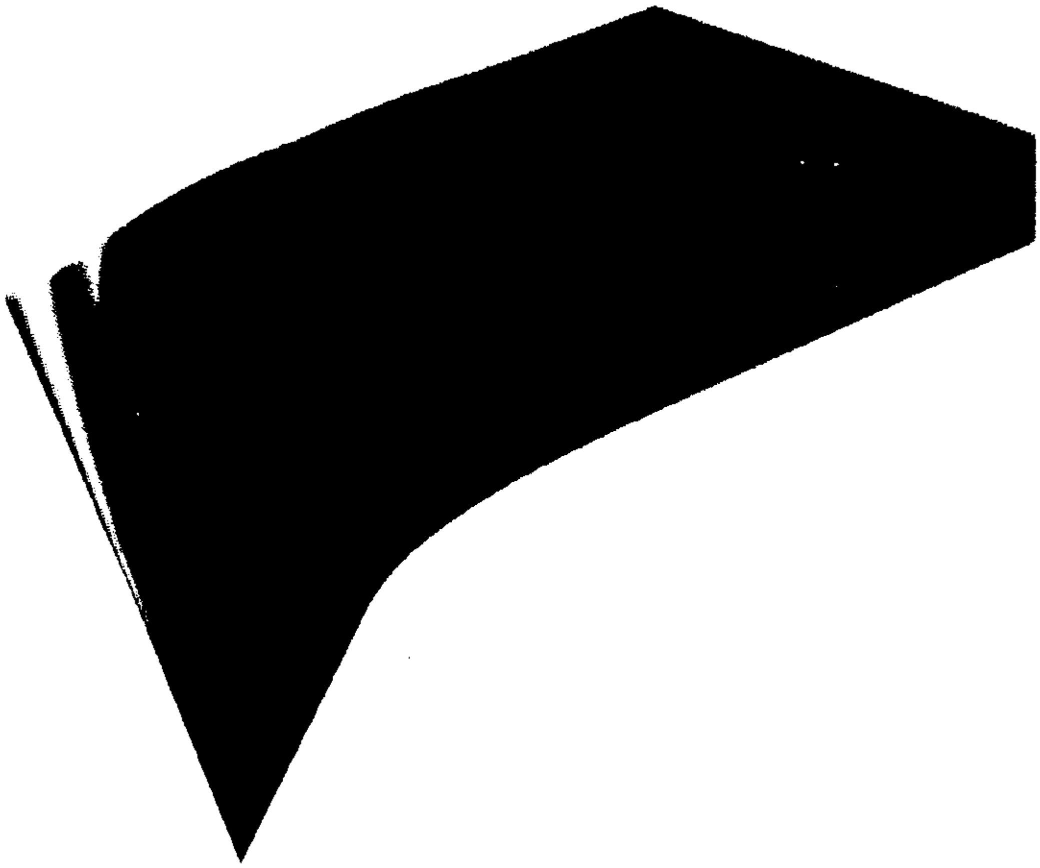
Sphaleron decay in the 4D SU(2) Higgs model:
the gauge field (blow up)

C. Rebbi and R. Singleton, June 1994



**Sphaleron decay in the 4D SU(2) Higgs model:
the Higgs field**

Rebbi and R. Singleton June 1994



Sphaleron decay in the 4D SU(2) Higgs model: the Higgs field (blow up)

Rebbi and R. Singleton, June 1994

V

Another interesting semiclassical study.

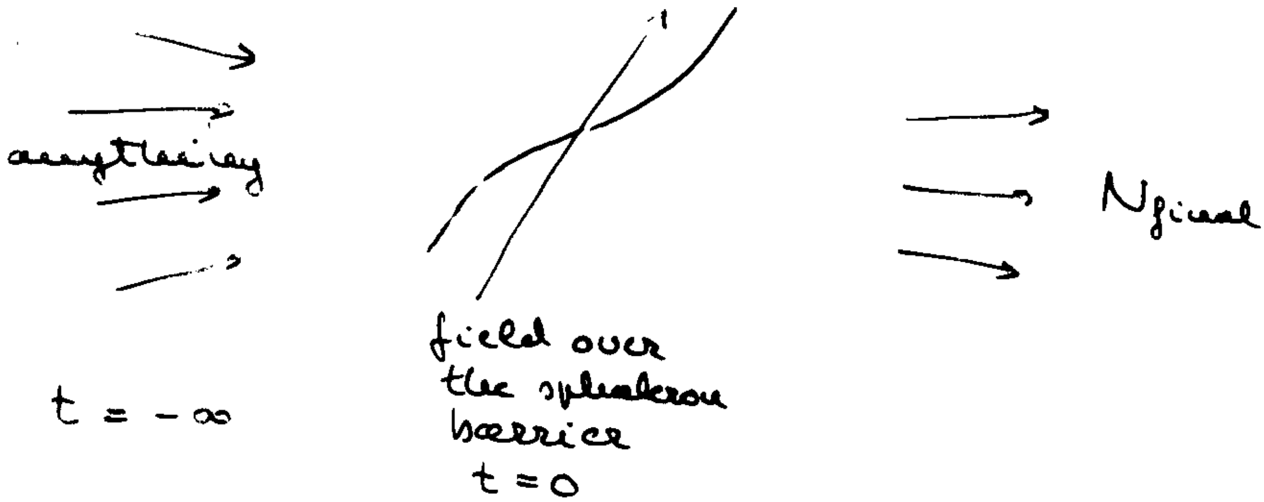
Consider processes with $E > E_{sph.}$ with a change of topology (and therefore baryon number violation). Can these be induced by collisions where the number of particles in the initial state is small?

$$N_{init} = \frac{\nu}{g^2} \Longrightarrow \Delta Q \rightarrow \text{anything}$$

Now the process is over the energy barrier and can be described, in the saddle point approx., by an entirely Minkowskian evolution.

The question is whether, with $\Delta Q = 1$ and a given E, there is a lower bound on ν (and, if so, how does this depend on E)

As usual, it is computationally more convenient to consider the time reversed process



Computationally:

consider the most general parametrization of the field at the sphaleron transition,

take this as initial field configuration,

- calculate E
- evolve and calculate N_f

vary the parameters to minimize N_f

The calculation is in progress ...