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Lectures III and IV

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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NON-PERTURBATIVE ELECTROWEAK EFFECTS IN BARYOGENESIS

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Please note: These are preliminary notes intended for internal distribution only.

Anomalous fermionic number non-conservation.

(1) Level crossing. 1+1

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \bar{\Psi}_1 D \Psi_1 + \bar{\Psi}_2 D \Psi_2 + \text{scalars}$$

$$D\Psi_1 = i\gamma_\mu(\partial_\mu - ie A_\mu(1+\gamma_5))\Psi_1$$

$$D\Psi_2 = i\gamma_\mu(\partial_\mu + ie A_\mu(1+\gamma_5))\Psi_2$$

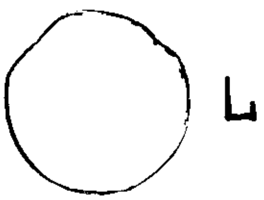
Conserved currents:

(i) electric current: $\bar{\Psi}_1 \gamma_\mu \Psi_1 - \bar{\Psi}_2 \gamma_\mu \Psi_2$

(ii) fermionic number $\bar{\Psi}_1 \gamma_\mu \Psi_1 + \bar{\Psi}_2 \gamma_\mu \Psi_2$

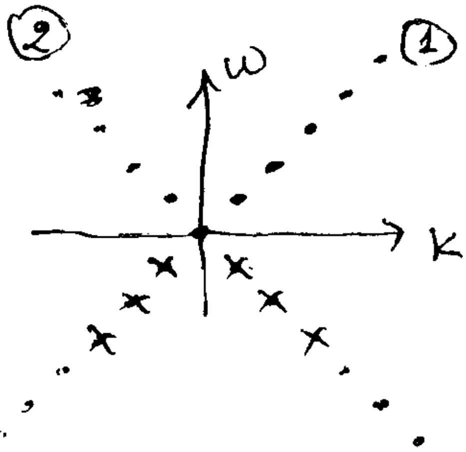
Are they conserved?

Fermionic level crossing.



$A = 0 \Rightarrow$ spectrum

$K = \frac{2\pi n}{L}$; $\omega = K$



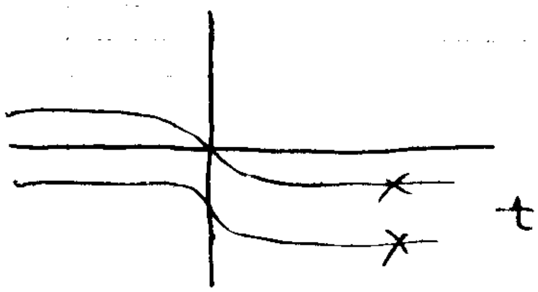
$A_1 \neq 0 \Rightarrow$ (constant, time indep.)

spectrum

$$\omega_1 = K - eA_1$$

$$\omega_2 = K + eA_1$$

if $A_1 = \frac{2\pi N}{eL}$ ← integer — spectrum is the sam



connect $A_1 = 0$ & $A_1 = \frac{2\pi}{eL}$

Creation of fermion!

Number of created fermions:

$$F = 2 \cdot \frac{eLA_1}{2\pi} = 2 \cdot \frac{e}{2\pi} \int dx\, A_1 =$$

$$= 2 \cdot \frac{e}{2\pi} \int dx \int dt\, \partial_t A_1 = 2 \cdot \frac{e}{2\pi} \int d^2x\, \epsilon_{\mu\nu} F^{\mu\nu}$$

$$\partial_\mu j_\mu^F = n_f \frac{e}{2\pi} \epsilon_{\mu\nu} F^{\mu\nu}$$ – anomaly equation

$$N_{cs} = \frac{e}{2\pi} \int dx\, A_1$$ – Chern-Simons number

$$\boxed{\Delta F = n_f\, \Delta N_{cs}}$$

vacua with $A = 0$ & $A = \frac{2\pi N}{eL}$ are related by non-trivial gauge transformation

$$\varphi \to e^{i\alpha(x)} \varphi \; ; \; A_\mu \to A_\mu + \frac{1}{e} e^{i\alpha} \partial_\mu e^{-i\alpha}$$

$$\alpha = \frac{x \cdot 2\pi N}{L}$$

Mapping: $S \to U(1)$; N - winding number

Electroweak theory: everything is the same: ③

Fermionic spectrum is identical in any of the fields

$\varphi = g\binom{0}{v}$; $A = g\partial g^{-1}$ g - gauge transformation.

Mapping: $S_3 \Rightarrow SU(2)$; winding number:

$$\underbrace{W(g)} = \frac{1}{24\pi^2} \mathrm{Tr}\, \epsilon_{ijk} \int d^3x (g\partial_i g^{-1})(g\partial_j g^{-1})(g\partial_k g^{-1})$$

integer

$g = \exp\left[i \frac{2\pi \tau^a x^a}{\sqrt{x^2+\rho^2}}\right]$

vacuum with $W=0$ $\Rightarrow$ vacuum with $W=1$ $\rightarrow$ fermionic level crossing.

selection rules: (9 quark doublets + 3 leptonic doublets) $\Rightarrow$

creation of 9 quark + 3 leptons

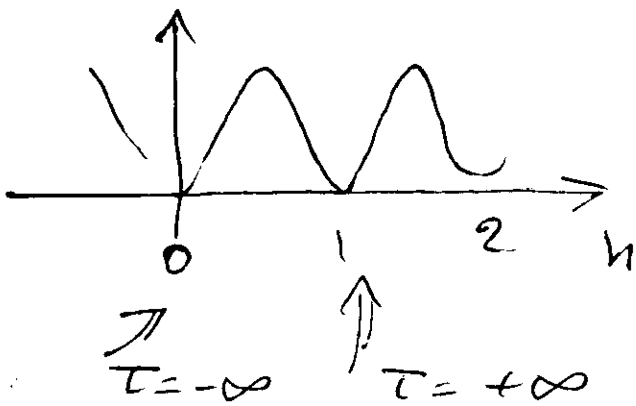
$$\partial_\mu j_\mu^B = n_f \frac{g^2}{32\pi^2} F_{\mu\nu} \tilde{F}_{\mu\nu} ; \quad \tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F_{\rho\sigma}$$

$$\partial_\mu j_\mu^L = \ldots$$

$$\frac{g^2}{32\pi^2} F\tilde{F} = \partial_\mu K_\mu ;$$ ⟸ Chern-Simons current.

$$K^\mu = \frac{g^2}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \left\{ F_{\nu\lambda}^a A_\sigma^a - \frac{2}{3} g \epsilon^{abc} A_\nu^a A_\lambda^b A_\sigma^c \right\}$$

Energy barriers



$$E_B = \min_{\{paths\}} \max_{\{\tau\}} \underbrace{E(\tau)}_{\text{static energy}}$$

Configuration saturating the minimax procedure: unstable solution to the classical eq. of motion.

sphaleron:

~~$A_i^a = \frac{2}{g} \epsilon^a_{i\,j}$~~

Barrier in 3+1 case:

sphaleron:

$$A_i^a = \frac{1}{g}\frac{\varepsilon_{aij}x_j}{r^2} \cdot f(\xi) \qquad \xi = gv\sqrt{2}\,|\vec{x}|$$

$$\varphi = i\,\frac{\tau^a x^a}{r}\,h(\xi)\begin{pmatrix}0\\ v\end{pmatrix} \qquad f(0)=h(0)=0$$

$$f(\infty)=h(\infty)=1$$

$$E_{sph} = \frac{2M_W}{\alpha_W}\,B\left(\frac{\lambda}{\alpha_W}\right) = \begin{cases} 8\,\text{TeV}, & m_H \ll M_W \\ 14\,\text{TeV}, & m_H \gg M_W \end{cases}$$

What is the origin of the scale $\frac{M_W}{\alpha_W}$?

$$E = \int d^3x \left[\frac{1}{4}(F_{ij})^2 + (D_i\varphi)^2 + V(\varphi)\right]$$

dimensionless varyables

$$x = \frac{1}{m_W}\xi, \quad A = \frac{m_W}{g}\tilde{A}, \quad \varphi = \frac{v}{g}\tilde{\varphi}$$

$$E = \frac{M_W}{g^2}\int d^3\xi\left[\frac{1}{4}(\tilde{F}_{ij})^2 + (D_i\tilde{\varphi})^2 + \frac{\lambda}{g^2}(\tilde{\varphi}^2-1)^2\right]$$

$$\sim O\left(\frac{M_W}{g^2}\right)$$

Important properties of the sphaleron

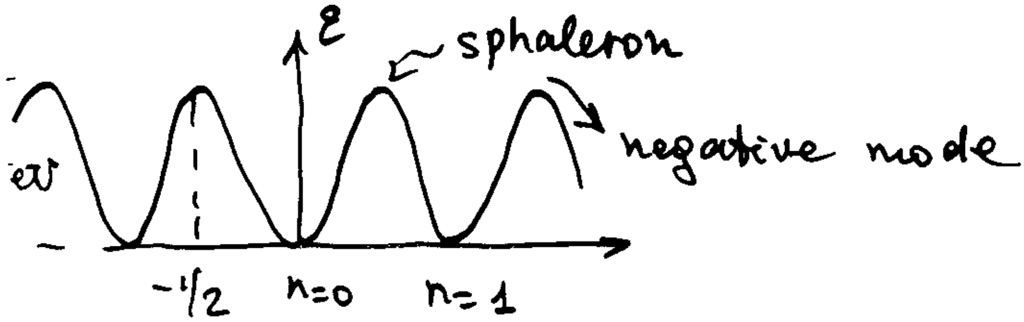
(i) topological charge = 1/2

$$\frac{g^2}{32\pi^2}\int_0^{t_0} F_{\mu\nu}\tilde{F}_{\mu\nu} = \frac{1}{2}$$

$t = 0$: $\varphi = \binom{0}{v}$, $A = 0$

$t = t_0$: $\varphi = \varphi_{sph}$, $A = A_{sph}$

(ii) it has exactly one negative mode



(iii) fermionic zero modes is sphaleron background

⇒ non-conservation under usual conditions:

tunneling through the barrier

Amplitude : $\exp\left(-\frac{2\pi}{\alpha_W}\right) \sim \exp(-180)$

too small

HIGH TEMPERATURES!

Rate of sphaleron transitions at high T:

sphaleron : analogy with critical bubble

Rate : $T < T_c$:

$$\Gamma \sim \exp\left(-\frac{M_{sph}}{T}\right) \simeq$$

$$\cong T^4 \left(\frac{\alpha_W}{4\pi}\right)^4 N_{tr} N_{rot} \left(\frac{2M_W}{\alpha_W T}\right)^7 \exp\left(-\frac{M_{sph}}{T}\right) \kappa$$

↗ temperature dependent sphaleron mass.

$T > T_c$:

no barrier

$$\Gamma \simeq (\alpha_W T)^4 \qquad [\text{from } m_W \simeq \alpha_W T]$$

Cosmology :

compare rate with the rate of Universe expansion

$$t = \frac{M_{Pl}}{T^2} \Rightarrow$$

equilibrium for $\Delta B \neq 0$

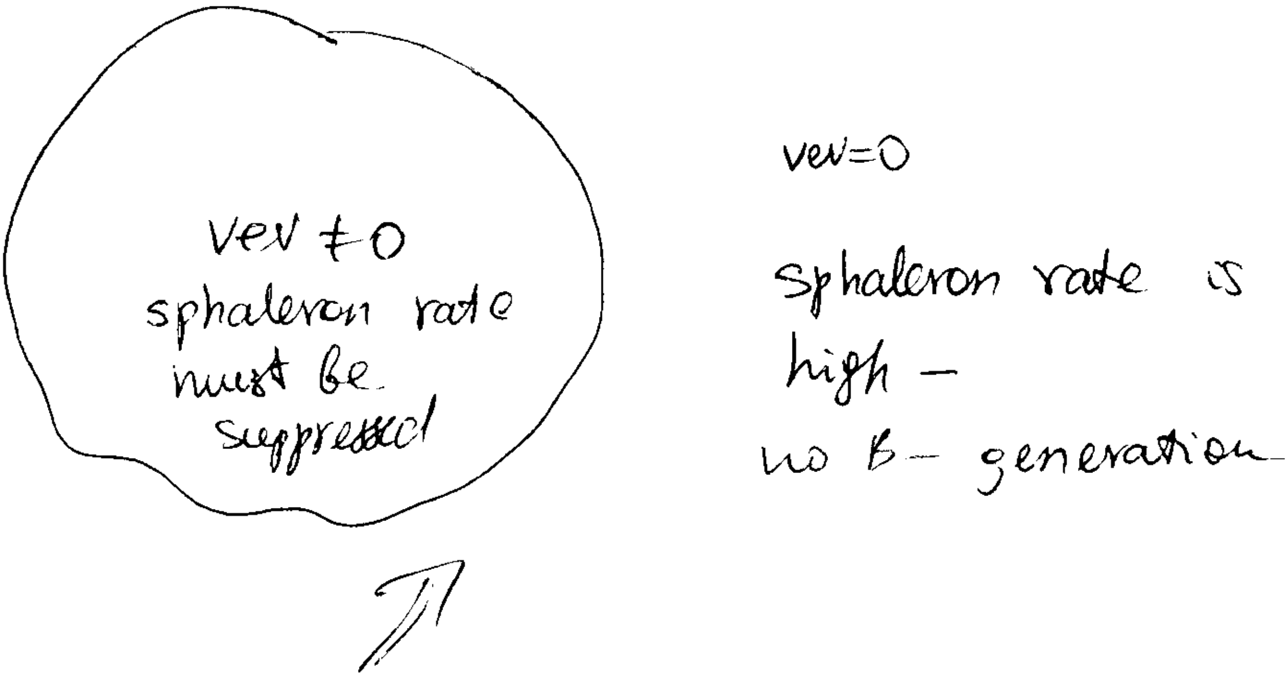
$100\,\text{GeV} \lesssim T \lesssim 10^{12}\,\text{GeV}$

Electroweak baryogenesis

Sakharov conditions

(i) B - non-conservation
(ii) CP- violation
(iii) thermal non-equilibrium

How it can happen?



domain wall baryogenesis.

Out of equilibrium condition ;
upper limit on the Higgs mass

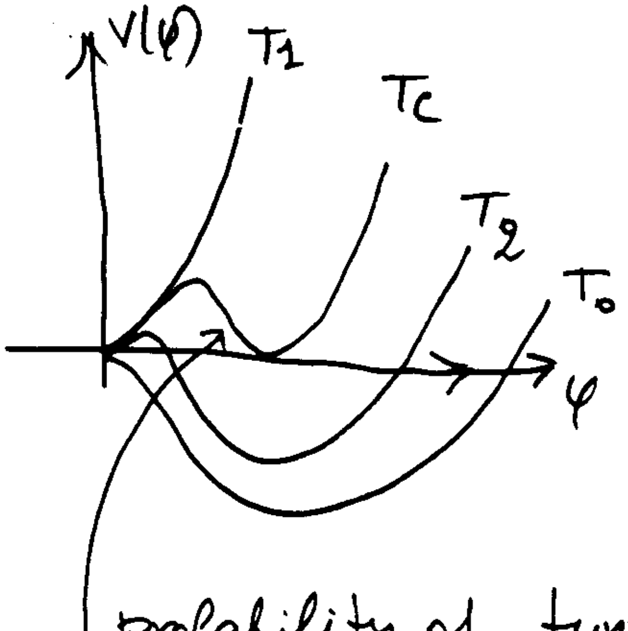
(2)

Scalar potential:

$$V(\varphi) = \mu^2(T)\varphi^2 - \delta T \varphi^3 + \lambda \varphi^4$$

$$\mu^2(T) = \frac{5}{16} g^2 (T^2 - T_0^2), \quad \delta = \frac{3}{32\pi} g_w^3$$

Phase transition occures near $T = T_0$



$T_1 > T_c > T_2 > T_0$

probability of tunneling is small, at $T \simeq T_0$ it is of order of 1.

vev just after the phase transition:

$$\left.\frac{dV}{d\varphi}\right|_{T=T_0} = 0 \Rightarrow \varphi = \frac{3\delta T}{\lambda}$$

Sphaleron mass after Phase transition

$$M_{sph} \approx \frac{3M_W}{\alpha_W} = \frac{3}{\alpha_W} \cdot \frac{1}{2} g_W \cdot \frac{3\delta T}{\lambda}$$

Out of equilibrium condition:

$$\frac{M_{sph}}{T} \geq 45 \Rightarrow \underbrace{\lambda \leq \frac{3}{80} g_W^2}$$

upper bound on the Higgs mass

$$m_H \lesssim 45\,\text{GeV} \Leftarrow \text{one-loop result.}$$

Theory with two Higgs doublets:

$$m_H \lesssim 120\,\text{GeV}.$$

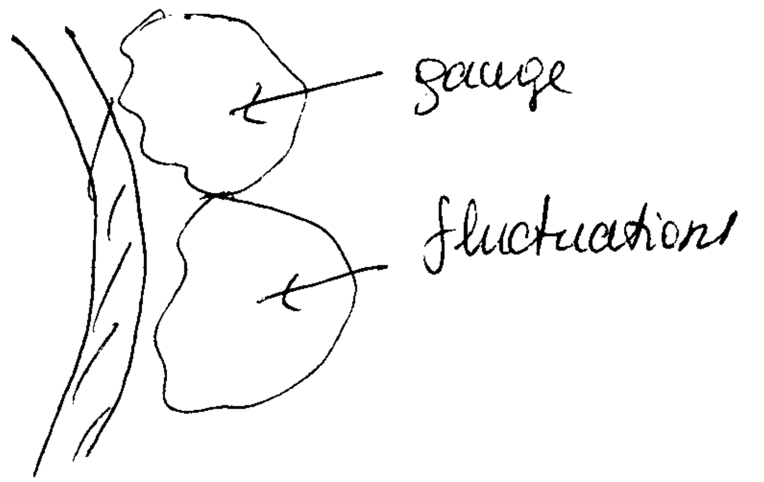
One-loop result should not be trusted: infrared divergences

Non-perturbative effects, lattice simulations $\Rightarrow$ m_H^{crit} may be as large as

$$m_H^{crit} \simeq 80\text{-}100\,\text{GeV}$$

Two general types of mechanisms
(i) Bosonic fields are important
(ii) fermions are important.

(i) "Bosonic" mechanism (a)



domain wall (thin)

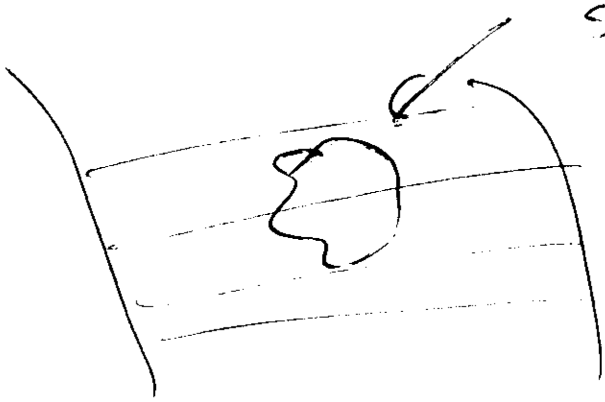
CP- violation :

fluctuations decay and produce non-zero $F\tilde{F}$

$$\left\{ \begin{array}{l} F\tilde{F} > 0 \\ \tilde{F}F < 0 \end{array} \right. \neq \text{ due to CP } \Rightarrow$$

fermions are created.

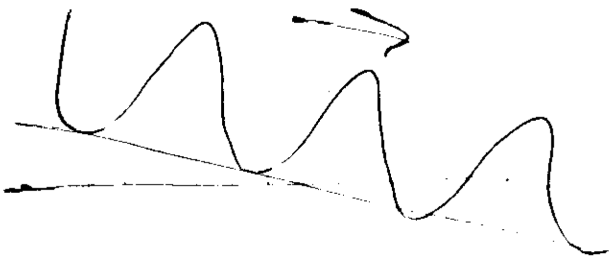
Bosonic mechanism (B)



sphaleron transitions inside domain wall

wall (thick)

time varying background :

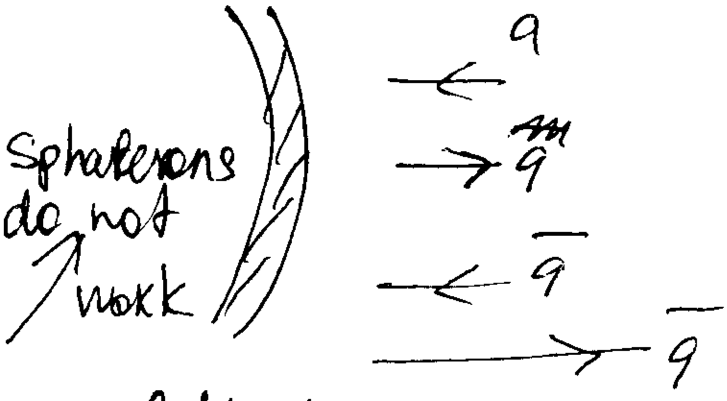


$\Delta B \neq 0$

(ii) fermionic mechanism

charge transport:

(i) thin wall



reflection of quarks is different from that of $\bar{q}$

wall thickness $\ell \ll \lambda$ mean free path

sphalerons are in equilibrium

if $T_q > T_{\bar{q}}$: quarks inside survive

antiquarks outside dissapear $\Rightarrow$

$\Delta B \neq 0$

(ii) thick walls $\rightarrow$

interaction of quarks with gluons inside the domain walls should be taken into account

Results :

(1) MSM :

Farrar, MS

$\frac{n_B}{n_S} \simeq 10^{-10} - 10^{-18}$ –

may be OK.

(2) Extended EW theories

(i) two Higgs doublets - OK

(ii) SUSY SM - maybe OK

Phase transitions:

Effective potential:

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