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QUANTUM MECHANICS ACCORDING TO FEYNMAN-SCHWINGER

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Please note: These are preliminary notes intended for internal distribution only.

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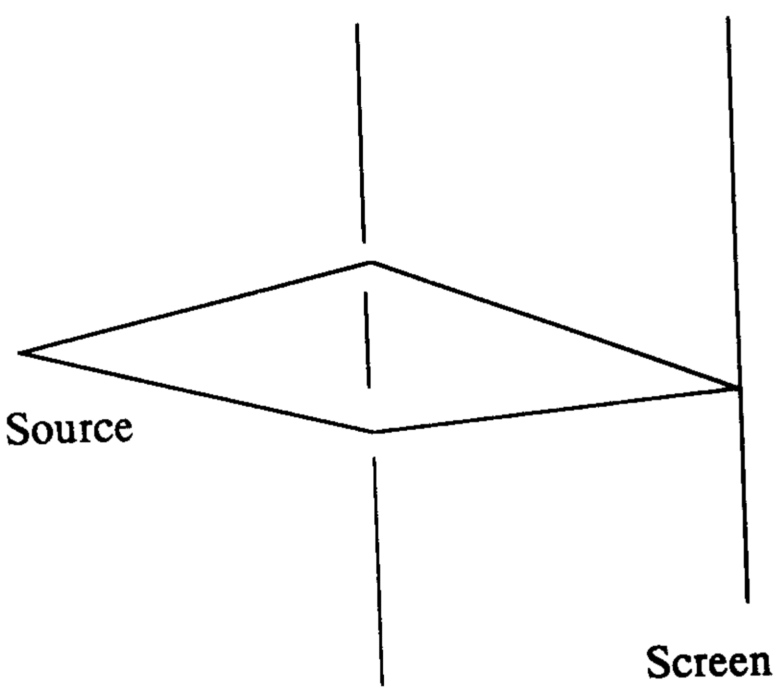
QUANTUM MECHANICS ACCORDING TO FEYNMAN-SCHWINGER

$$K(x,y;T) = \sum_{\text{paths}} e^{iS}$$



$$\langle T(\phi(x_1)\ldots\phi(x_n)\rangle = \int\prod_x \delta\phi(x)\exp iS\ \phi(x_1)\ldots\phi(x_n)$$

IN THE GREEN'S FUNCTION IS CODED ALL PHYSICAL INFORMATION:

PROPAGATOR POLES $\Rightarrow$ MASS-SPECTRUM

REDUCTION FORMULAS (LSZ) $\Rightarrow$ S-MATRIX

PROBLEMS

a) OSCILLATING ESPONENTIAL

b) ULTRAVIOLET DIVERGENCES

a) IS ALSO A PROBLEM FOR QUANTUM MECHANICS

SOLUTION:

$$\Downarrow$$

EUCLIDEAN CONTINUATION

(NOT INNOCUOUS)

$$\exp\left[-iH\,t_M\right] \Rightarrow \exp\left[-H\,t_E\right]$$

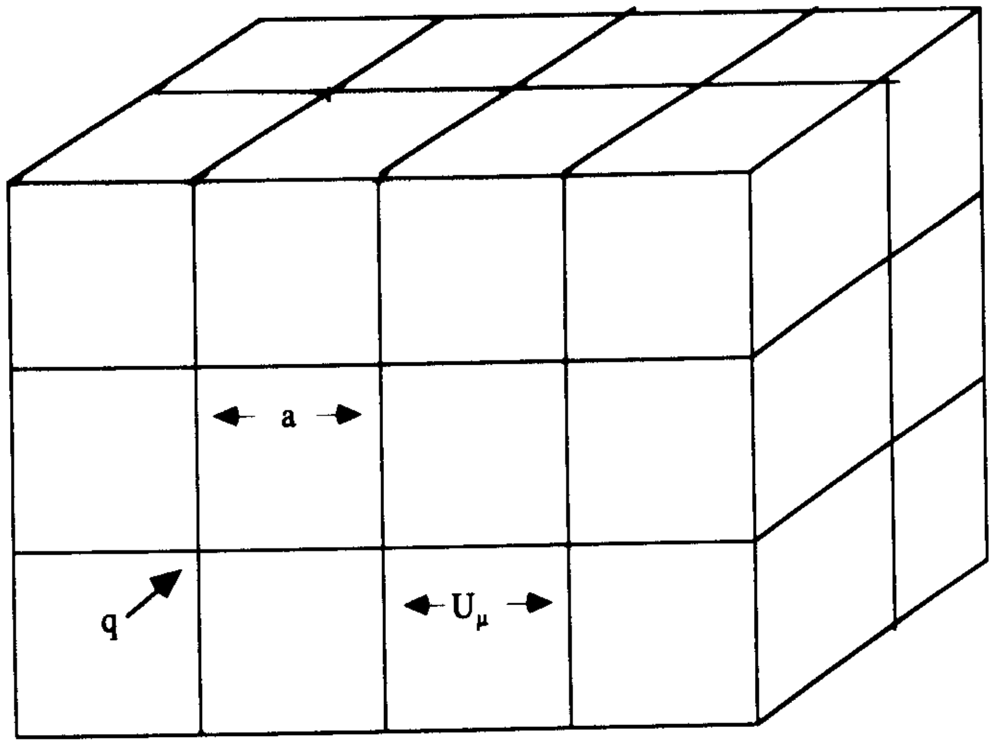
NOW WE CODE THE PHASE INFORMATION IN A COMPLICATED WAY (OSTERWALDER-SCHRADER FORMULATION)

ONLY TIME IS CHANGED, NOT HAMILTONIAN

NOT EVERYTHING CAN BE OBTAINED EASILY FROM THE EUCLIDEAN REGION

b) IS SOLVED BY REGULARIZATION

(E.G. DISCRETIZING ON A LATTICE)



THE LATTICES AVAILABLE TODAY (THE LIMITATIONS COME FROM CPU AND MEMORY) HAVE A LATTICE SPACING $1/a \approx 3.9$ GeV AND A SPATIAL EXTENSION $(32)^3 \times 48$

THE FUNCTIONAL INTEGRAL BECOMES AN "ORDINARY" (FERMIONS) FINITE DIMENSIONAL INTEGRAL
(IN A FINITE SPACE-TIME VOLUME)

$$\langle \phi(x_1)\ldots\phi(x_k)\rangle = \int \prod_n d\phi_n \exp\left[-S_a\right] \phi(x_1)\ldots\phi(x_k)$$

$$x = an$$

$$n \equiv \{n_1, n_2, n_3, n_4\}$$

FOR QUANTUM CHROMODYNAMICS (QCD) WE HAVE THE WILSON ACTION:

$$S_a = S_U + S_F$$

$$S_F = \sum_x \left\{ -\frac{1}{2a} \sum_\mu \left[\overline{q}(x)(r-\gamma_\mu)U_\mu(x)q(x+\hat{\mu}) + \right.\right.$$

$$\left. + \overline{q}(x+\hat{\mu})U_\mu^+(x)(r+\gamma_\mu)q(x) \right] +$$

$$\left. + \overline{q}(x)\left(M_0 + \frac{4r}{a} \right) q(x) \right\}$$

TERMS PROPORTIONAL TO r VIOLATE CHIRALITY (IN ORDER TO AVOID FERMION DOUBLING)

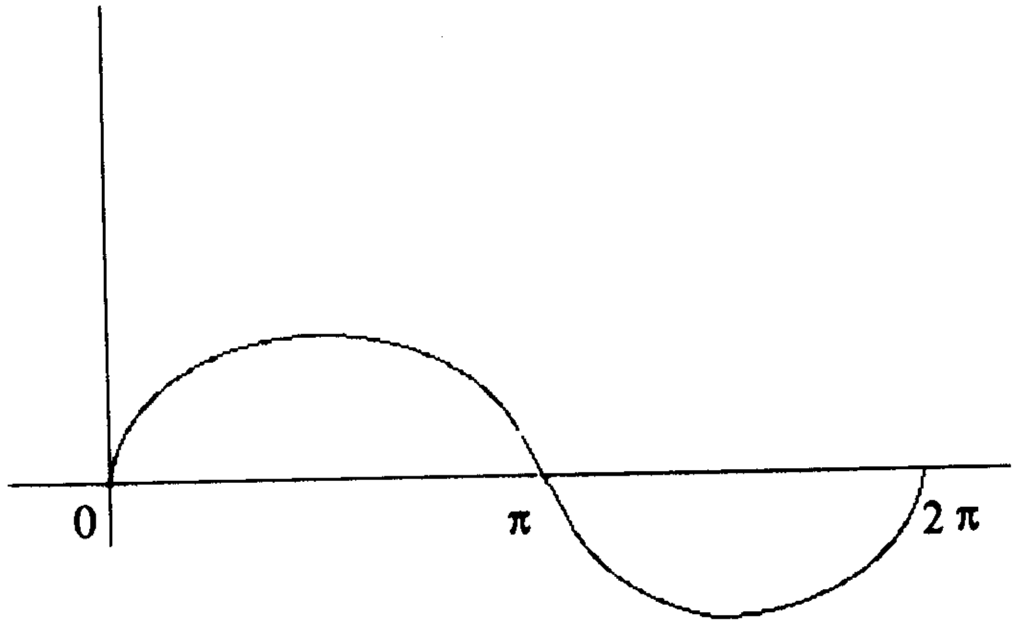
- **DOUBLING PROBLEM**:

THE NAIVE DISCRETIZATION OF THE DIRAC ACTION

$$\overline{\psi}\left(\partial_\mu \gamma_\mu\right)\psi$$

LEADS TO THE FERMION PROPAGATOR

$$S(p) = \frac{1}{\frac{1}{a}\sum_{\mu=1}^{4} \gamma_\mu \sin(ap_\mu)}$$



- **NIELSEN-NINOMIYA THEOREM**

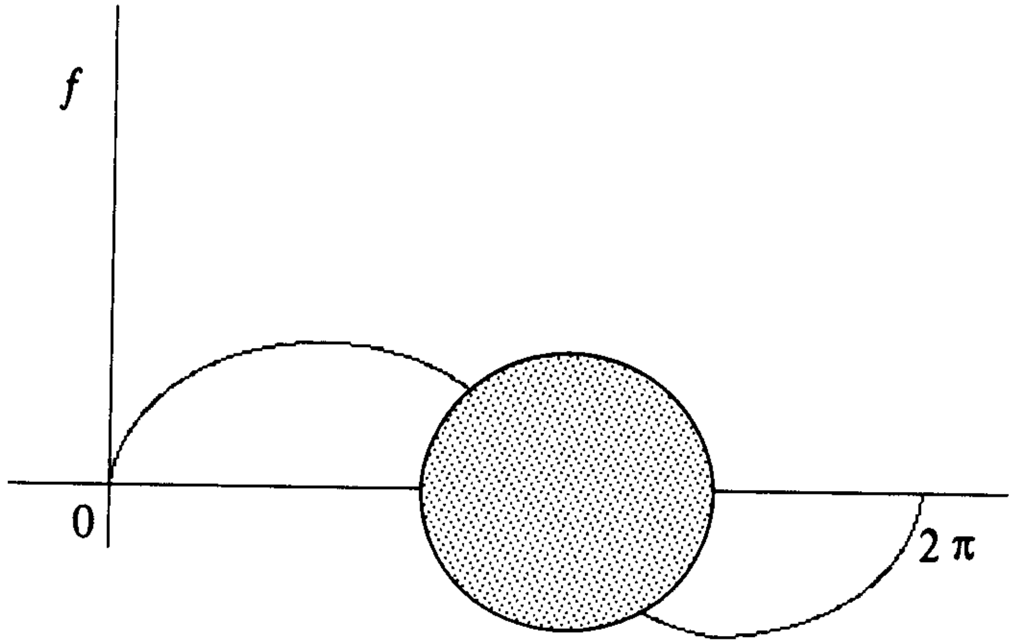
(TOPOLOGICAL IN THE FOURIER TRANSFORM)

ANY LOCAL, CHIRAL SYMMETRIC, BILINEAR ACTION IMPLIES SPECTRUM DOUBLING.

IN FACT THE PROPAGATOR IS OF THE FORM:

$$S^{-1} = \sum \gamma_\mu f^\mu(p)$$

WITH



ADDING THE WILSON TERM (CHIRAL VIOLATING):

$$L_W \approx r\overline{\psi}_L a\partial^2 \psi_R + h.c.$$

GIVES:

$$S^{-1} = \frac{1}{a}\sum_{\mu=1}^{4} \gamma_\mu \sin(ap_\mu) + \frac{r}{a}\sum_{\mu=1}^{4} \sin^2(\frac{ap_\mu}{2})$$

AND WE AVOID DOUBLING

THIS IS NOT A MERE TECHNICAL POINT, BUT A DEEP PHYSICAL ONE. IT IS RELATED TO THE (MATHEMATICAL) FACT THAT NO REGULATOR PRESERVES CHIRALITY.

LATTICE IS NOT AN EXCEPTION.

THIS FACT LEADS TO DIRECT EXPERIMENTAL OBSERVATION IN THE $\pi_0 \rightarrow \gamma\gamma$ DECAY

ALSO THE GLOBAL STRUCTURE OF THE STANDARD MODEL IS DEEPLY AFFECTED BY THE NON EXISTENCE OF A GAUGE INVARIANT REGULARIZATION

IT MAKES THE CHIRAL CLASSIFICATION OF LOCAL OBSERVABLES (CURRENT ALGEBRA) A COMPLICATED PROBLEM ALREADY IN QCD.

IT REPRESENTS A SERIOUS OBSTRUCTION TO THE PROGRAM OF PUTTING THE STANDARD MODEL (OR ANY G.U.T.) ON THE LATTICE

FERMION FIELDS ARE GRASSMANN VARIABLES AND ALWAYS APPEAR IN BILINEAR FORM (GAUSSIAN). THEY CAN BE,

THEREFORE, ESPLICITLY INTEGRATED AWAY, LEAVING AN EFFECTIVE ACTION FOR YANG-MILLS VARIABILES:

$$S_{EFF} = S_U + Tr \log(\not{D})$$

$$S_{EFF} = S_U +$$ 

IN THE "QUENCHED APPROXIMATION" WE OMIT FERMION LOOPS

"QUENCHING" IS A PRACTICAL APPROXIMATION (COMPUTER CPU&MEMORY) NOT UNDER COMPLETE CONTROL (EXPECIALLY FOR SMALL QUARK MASSES). THERE ARE INTERESTING RECENT ATTEMPTS TO EVALUATE ITS EFFECTS BASED ON CHIRAL PERTURBATION THEORY

NOW WE CAN COMPUTE NUMERICALLY

(MONTECARLO)

WHICH KIND OF PHYSICAL INFORMATION CAN BE EXTRACTED?

THE BASIC IDEA IS TO KEEP A FINITE CUTOFF a, PROVIDED $1/a >> m$

(FOR ACTUAL COMPUTERS $1/a \approx 3.9$ GeV BUT.....SEE LATER)

THE PHYSICAL LIMIT IS ATTAINED WHEN $a \to 0$ ADJUSTING THE ACTION PARAMETERS IN SUCH A WAY THAT A CORRESPONDING NUMBER OF PHYSICAL QUANTITIES STAY FINITE
(NON PERTURBATIVE RENORMALIZATION)

THE QCD PARAMETERS ARE: g_0 AND M_0
(ONE MASS PARAMETER FOR EACH QUARK TYPE)
IF WE ARE INTERESTED IN THE COMPUTATION OF PHYSICAL QUANTITIES ONLY, NO WAVE FUNCTION RENORMALIZATION IS REQUIRED

PHYSICAL INPUT (E.G.):

PROTON AND PION MASSES

ASYMPTOTIC FREEDOM ALLOWS A CONTROL OF THE FINITE CUTOFF FOR EXAMPLE EVERY MASS (IN THE CHIRAL LIMIT) MUST BE PROPORTIONAL TO:

$$m = c\frac{\exp\left(-\frac{1}{2\beta g_0^2}\right)}{a}$$

SO THAT WE KNOW WHEN WE CAN CONSIDER THE U.V. CUTOFF AS INFINITE

MASS SPECTRUM

IN THIS CASE THE EUCLIDEAN CONTINUATION IS ADVANTAGEOUS!!!

WE CHOOSE A "GOOD LOCAL INTERPOLATING FIELD"

$$O(x)$$

SUCH THAT

$$\langle 0|O(x)|p\rangle \neq 0$$

FROM THE (EXACT) SPECTRAL REPRESENTATION WE HAVE:

$$F(x_4) \equiv \int d^3x \, e^{ipx} \langle O(x) O(0) \rangle$$

$$F(x_4) \xrightarrow[x_4 \to \infty]{} |\langle 0|O(0)|p\rangle|^2 \exp\left[-\left(\sqrt{m^2 + p^2}\right) x_4\right]$$

THE HADRON MASS SPECTRUM IN QCD TURNS OUT TO BE REASONABLE

(APPROXIMATIONS: QUENCHING, FINITE ULTRAVIOLET AND INFRARED CUTOFF (VOLUME))

A BIG SUCCESS IS THE FACT THAT CHIRAL *SU(3)⊗SU(3)* TURNS OUT TO BE REALIZED IN THE SPONTANEOUSLY BROKEN PHASE, WHILE THE VECTOR SYMMETRY IS REALIZED A LA WIGNER

(A STRIKING EMPIRICAL FACT)

ASYMPTOTIC FREEDOM

↓

PERTURBATION THEORY

AT THE LATTICE LEVEL CHIRALITY IS BROKEN BY THE WILSON TERM (RESPONSIBLE FOR THE ANOMALIES IN THE LATTICE FORMULATION) AND THE CHIRAL LIMIT IS NOT RECOVERED FOR $M_0=0$, BUT, RATHER, AT $M_0=M_{cr}$ WITH:

$$M_{cr} = \frac{f(g_0)}{a}$$

$$f(g_0) = \sum f_k g_0^k + \exp(-\frac{c}{g_0^2})$$

M_{cr} CONTAINS, THEREFORE, PERTURBATIVE CONTRIBUTIONS AND NON PERTURBATIVE ONES
(DIVERGENCE OF THE PERTURBATIVE SERIES AND "RENORMALONS"):

$$\frac{\exp(-\frac{1}{2\beta g_0^2})}{a} = \Lambda_{QCD}$$

NUMERICALLY, FOR $g_0=1$ WE HAVE:

$$M_{cr} a = 0.8051$$

WHILE PERTURBATION THEORY GIVES:

$$M_{cr} a = 0.6386$$

MATRIX ELEMENTS OF LOCAL OPERATORS

SEVERAL KINDS OF MATRIX ELEMENTS CAN BE (**AND ARE**) EVALUATED FROM LATTICE NUMERICAL COMPUTATIONS:

a) MESON DECAY COSTANTS:

$$G(t) = \sum_{x} \langle A_4(\boldsymbol{x},t) A_4(\boldsymbol{0},0) \rangle \underset{t \to \infty}{\approx} \frac{f_P^2 M_P}{2} e^{-M_P t}$$

b) FORM FACTORS

(AT SPACELIKE MOMENTUM TRANSFERS)

$$G(t_1 t_2; \boldsymbol{q}_1 \boldsymbol{q}_2) = \left\langle \phi_{\boldsymbol{q}_1}(t_1) \phi_{\boldsymbol{q}_2}(t_2) J(0) \right\rangle$$

$$\lim_{\substack{t_1 \to +\infty \\ t_2 \to -\infty}} G \approx \frac{Z_\pi}{\sqrt{2E_{q_1} 2E_{q_2}}} \langle q_1 | J(0) | q_2 \rangle \exp(-E_{q_1} t_1) \exp(E_{q_2} t_2)$$

COMPOSITE LOCAL OPERATORS WHICH DESCRIBE PHYSICAL QUANTITIES REQUIRE AN ACCURATE DEFINITION DUE TO ULTRAVIOLET PROBLEMS (RENORMALIZATION):
IN FACT WE HAVE, FOR EXAMPLE:

$$\lim_{x \to y} \bar{q}(x) q(y) = \text{divergent quantity}$$

WE NEED TO REGULARIZE AND CONSTRUCT SUITABLE LINEAR COMBINATIONS OF OPERATORS WITH DIMENSIONS ≤ OF THE FORMAL OPERATOR WE ARE CONSIDERING:

$$\hat{O} = Z_O O + \sum c_n O_n$$

THE COEFFICIENTS ARE DETERMINED FROM RENORMALIZATION CONDITIONS AS IS DONE IN PERTURBATION TEORY. THE RATIOS OF THE COEFFICIENTS CAN BE FIXED FROM (E.G.) THE REQUIREMENT OF COVARIANCE UNDER CHIRAL TRANSFORMATIONS (CURRENT ALGEBRA) (THIS IS A NON TRIVIAL REQUIREMENT BECAUSE THE WILSON TERM VIOLATES CHIRAL SYMMETRY)

SOME OPERATORS (PARTIALLY CONSERVED CURRENTS) MAY NEED FINITE RENORMALIZATIONS ONLY.
A PARTICULARLY RELEVANT CASE IS PROVIDED BY THE AXIAL CURRENT (NEEDED IN THE COMPUTATION OF $f\pi$)
IN THIS CASE IT IS POSSIBLE TO COMPUTE THE RENORMALIZATION CONSTANT BOTH PERTURBATIVELY AND NON PERTURBATIVELY (AGAIN THROUGH CURRENT ALGEBRA).
SINCE THERE ARE NO POWER DIVERGENCES INVOLVED (CURRENTS ARE MULTIPLICATIVELY RENORMALIZABLE)

$$Z = \sum Z_k g_0^k + \exp \textit{small terms}$$

THE DISCREPANCY BETWEEN THE TWO EVALUATIONS GIVES AN ESTIMATE OF THE CORRECTIONS DUE TO A FINITE LATTICE SPACING (OF THE ORDER OF *20-30 %*)

OTHER INTERESTING QUANTITIES:

- σ-TERM (AFFECTED BY QUENCHING)

- B-PARAMETER

PROBLEMS REQUIRING PERTURBATION TEORY AS A FUNDAMENTAL INGREDIENT

DUE TO THE FACT THAT IT IS VERY DIFFICULT TO PUT THE STANDARD MODEL ON THE LATTICE (FOR TECHNICAL ($M_{W,Z} >> 1/a$) & **THEORETICAL** REASONS (WILSON TERM)) WE MUST "INTEGRATE" ANALITICALLY THE HEAVY DEGREES OF FREEDOM (W'S, Z'S)

↓

EFFECTIVE LOW ENERGY ACTIONS FOR NON-LEPTONIC DECAYS ($K\text{-->}\pi\pi$ AND (THE LONG (AND STILL) STANDING PROBLEM) $\Delta I=1/2$):

STATISTIC PROBLEMS ($\infty-\infty$?) FOR NON PERTURBATIVE SUBTRACTIONS DUE TO THE MIXING WITH LOWER DIMENSION OPERATORS)

- STATIC QUARKS $M_Q >> 1/a$

RENORMALIZATION AND SYMANZIK "IMPROVEMENT"

$$S_a \approx S_{cont} + a \sum_{\dim 5} O$$

DRASTICALLY REDUCES DISCREPANCES WITH PERTURBAZION TEORY, WHEN APPLICABLE

FORM FACTORS IN THE TIME-LIKE REGION AND SCATTERING AMPLITUDES

HERE EUCLIDEAN CONTINUATION IS DEVASTATING. IN FACT THE MINKOWSKI EVOLUTION OPERATOR:

$$e^{-iHt_M} = \sum_n e^{-iE_n t_M} |n\rangle\langle n|$$

BECOMES:

$$e^{-Ht_E} = \sum_n e^{-E_n t_E} |n\rangle\langle n|$$

FOR NON DEGENERATE STATES (VACUUM AND SINGLE PARTICLE) THERE IS NO AMBIGUITY.

FOR MANY PARTICLE STATES, WE SHOULD IDENTIFY THE CORRECT COMBINATIONS OF ASYMPTOTIC STATES (IN E OUT).
IT IS, THEREFORE, NECESSARY TO CONTINUE BACK TO THE MINKOWSKIAN REGION (A NUMERICALLY IMPOSSIBLE TASK).

SOME INFORMATION ON SCATTERING LENGHTS IS STILL ACCESSIBILE FROM EUCLIDEAN THROUGH VOLUME DEPENDENCE, OR THROUGH THE RELATION:

$$G(t,t,\mathbf{0},\mathbf{0})_{(t>>t>>0)} = \frac{Z_\pi}{(2M_\pi)} \exp(-M_\pi t_1)\exp(-M_\pi t_2) f(4M_\pi) * \left[1 - a\sqrt{\frac{M_\pi}{4\pi t_2}} + \ldots\right]$$

$$G(t_1 t_2; \mathbf{q}_1 \mathbf{q}_2) = \left\langle \phi_{\mathbf{q}_1}(t_1) \phi_{\mathbf{q}_2}(t_2) J(0) \right\rangle$$

NUMERICALLY HARD

CONCLUSIONS

LATTICE QCD IS **ESSENTIAL** TO OUR PHENOMENOLOGICAL UNDERSTANDING OF THE STANDARD MODEL

LATTICE QCD IS **NOT** JUST NUMERICAL COMPUTATION

COMPUTING POWER IS, OF COURSE, WELCOME, BUT IS NOT ENOUGH:

SOPHISTICATED FIELD THEORY TECHNIQUES ARE ESSENTIAL IN ORDER TO CONNECT NUMERICAL SIMULATIONS WITH PHYSICS

THERE ARE STILL MANY OUTSTANDING OPEN PROBLEMS

IN PARTICULAR IT IS NOT YET COMPLETELY CLEAR HOW CHIRAL THEORIES CAN BE TREATED IN A NON PERTURBATIVE WAY

UNSUSPECTED UTILITY OF PERTURBATION THEORY AS A (SEMI)-FUNDAMENTAL COMPUTATION INSTRUMENT