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BLACK HOLE PHYSICS AND GENERAL RELATIVITY

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RESOURCE LETTER

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This is one of a series of Resource Letters on different topics intended to guide college physicists, astronomers, and other scientists to some of the literature and other teaching aids that may help improve course contents in specified fields. No Resource Letter is meant to be exhaustive and complete; in time there may be more than one letter on some of the main subjects of interest. Comments on these materials as well as suggestions for future topics will be welcomed. Please send such communications to Professor Roger H. Stuewer, Editor, AAPT Resource Letters, School of Physics and Astronomy, 116 Church Street SE, University of Minnesota, Minneapolis, MN 55455. Reprints: When ordering request Resource Letter BH-1. Enclose 35 cents per copy (not in stamps) together with a stamped and self-addressed envelope and send to: Executive Office, American Association of Physics Teachers, Graduate Physics Building, SUNY at Stony Brook, Stony Brook, NY 11794.

Resource letter BH-1: Black holes

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The author has selected resource material that focusses on black holes.

I. INTRODUCTION

A stationary black hole is pure vacuum endowed only with mass, charge, and angular momentum. It is the simplest object in general relativity.

Ironically, this simplest of objects is also the most interesting and has been at the center of much research in the last decade by both physicists and astronomers. Current understanding of stellar structure and evolution leads one to believe that a sufficiently massive star has a reasonable chance of ending its evolution as a black hole (item 2.10). And at least one such black hole candidate has been found (5.12). Moreover, supermassive black holes might form in the nuclei of some galaxies (5.16) creating quasars, radio galaxies, or perhaps a bright spot in M87 (5.14). With rather less acceptance black holes have also been invoked to account for a variety of other phenomena from the Tunguska meteorite event (3.10) to the lack of solar neutrinos (5.17).

Perhaps it is not surprising that black holes have also caught the attention of the general public. Black holes have been discussed in *Scientific American*, the *New York Times*, *Time* magazine, and *Rolling Stone* magazine, and they have appeared in comic strips and now a major motion picture.

Black holes conveniently fit into the physics and astronomy curricula at a variety of levels. The size of a black hole is estimated by setting the escape velocity of a mass equal to the speed of light (2.1). The introduction of an effective radial potential makes the problem of circular orbits an interesting variation on the Kepler problem (5.1). Certain aspects of geodesics around a rotating black hole present a rare and valuable example of Hamilton-Jacobi theory (5.3). Black holes, with their particle creation and

blackbody radiation, even fit into discussions of thermodynamics (5.5).

II. ARTICLES OF HISTORICAL INTEREST

The concept of a black hole began in 1783 with a British amateur astronomer, the Reverend John Michell. In a letter to Henry Cavendish (2.1) he discussed a calculation, based on Newtonian gravity, which showed that a sphere, of the same density as the sun, but 500 times larger, would have an escape velocity exceeding that of light. Fifteen years later, Laplace (2.2) published a similar calculation of his own and is usually given credit for the discovery. S. Schaffer (2.12) briefly reviews this interesting historical situation.

In the context of general relativity, the astronomer K. Schwarzschild in 1916 (2.3) found the celebrated solution representing the geometry outside of a point mass only months after Einstein's theory was published. At first the metric was thought to be singular at the Schwarzschild radius $R_s = 2GM/c^2$; but a coordinate transformation, found by Eddington (2.4), showed that the apparent singularity was only an artifact of Schwarzschild's coordinate system. Nonetheless in mks units G is so small and c so large that R_s seemed much smaller than any physical object and hence played no role in nature.

But in 1931 the astronomer S. Chandrasekhar (2.5 and 2.6) analyzed the sequence of equilibrium configurations of white dwarfs parametrized by the central density. And he showed clearly that as the mass of the star approaches a critical mass comparable to the mass of the sun "the completely relativistic model considered as the limit of the composite series is a point mass with $\rho_c = \infty$!" (2.6). Above the critical mass the equation of state of a relativistic de-

generate Fermi gas of electrons is too soft to counter the gravitational forces; hence when considering the end of evolution of such a star Chandrasekhar was "left speculating on other possibilities" (2.7).

Landau, in fact, did speculate—for masses greater than the critical mass "there exists in the whole quantum theory no cause preventing the system from collapsing to a point As in reality such masses exist quietly as stars and do not show any such ridiculous tendencies we must conclude that all stars heavier than $1.5 M_{\odot}$ certainly possess regions in which the laws of quantum mechanics are violated" (2.8). And Eddington's (2.9) response to Chandrasekhar's calculation was "the star has to go on radiating and radiating and contracting and contracting until, I suppose, it gets down to a few km radius, when gravity becomes strong enough to hold in the radiation, and the star can at last find peace. . . . I felt driven to the conclusion that this was almost a *reductio ad absurdum* of the relativistic degeneracy formula. Various accidents may intervene to save the star, but I want more protection than that. I think there should be a law of Nature to prevent a star from behaving in this absurd way!" With the unfair advantage of hindsight we can now look back and see Landau and Eddington rejecting the implications of relativistic quantum mechanics rather than allowing the formation of a black hole.

In 1939 Oppenheimer and Volkoff (2.10) discussed the possible existence of neutron stars as an end point of stellar evolution and used general relativity (rather than Newtonian gravity) to examine the equilibrium configurations. Their conclusion was similar to Chandrasekhar's: stars consisting of cold neutrons, supported by the Fermi pressure and more massive than some critical mass "will continue to contract indefinitely, never reaching equilibrium." Oppenheimer and Snyder (2.11) continued by examining the actual collapse of a pressureless star in the context of general relativity and concluded that no law of physics was likely to intervene and stop at least some stars from collapsing to form black holes.

- 2.1 "On the means of discovering the Distance, Magnitude, etc. of the Fixed Stars, in consequence of the Diminution of the velocity of their Light, in case such a Diminution should be found to take place in any of them, and such Data should be procured from Observations, as would be farther necessary for that purpose," J. Michell, *Philos. Trans.* 74, 35–57 (1783). First calculation of the necessary size of an object of a given density to have an escape velocity equal to the speed of light. Michell also points out that a black hole might be detected if it were an unseen companion in a binary system. (E)
- 2.2 *Exposition du système du monde*, P. S. Laplace (*Cercle-Social*, Paris, 1796), Vol. 2, p. 305. A rediscovery of Michell's result. A related translation appears in item 4.8. (E)
- 2.3 "On the gravitational field of a point mass according to the Einsteinian theory," K. Schwarzschild, *Sitzber. Preuss. Akad. Wiss., Phys. Math. Kl.* 189, 189–196 (1916). Discovery of the solution of Einstein's equations which represents the geometry of a nonrotating black hole. Partially translated and reprinted in item 2.14. (I)
- 2.4 "A comparison of Whitehead's and Einstein's Formulae," A. S. Eddington, *Nature* 113, 192 (1924). Expression of the Schwarzschild metric in a coordinate system which shows no singularity at the event horizon; although this fact is not mentioned by Eddington. In fact in *Space, Time and Gravitation* (Cambridge University, Cambridge, 1920) he implies that the event horizon ought to be thought of as the surface of a particle. (I)
- 2.5 "The Maximum Mass of Ideal White Dwarfs," S. Chandrasekhar, *Astrophys. J.* 74, 81–82 (1931). A calculation of the maximum mass which can be supported against gravity by relativistic, degenerate electrons. (I)

- 2.6 "The Highly Collapsed Configurations of a Stellar Mass," S. Chandrasekhar, *Mon. Not. R. Astron. Soc.* 91, 456 (1931). Quoted in the above discussion. (I)
- 2.7 "Stellar Configurations with degenerate Cores," S. Chandrasekhar, *Observatory* 57, 373–377 (1934). Quoted in the above discussion. (I)
- 2.8 "On the Theory of Stars," L. D. Landau, *Phys. Sow.* 1, 285–288 (1932). Quoted above; translated and reprinted in item 2.14. (I)
- 2.9 A. S. Eddington, *Observatory* 58, 37–39 (1935). Quoted in the above discussion. (E)
- 2.10 "On Massive Neutron Cores," J. R. Oppenheimer and G. M. Volkoff, *Phys. Rev.* 55, 374–381 (1939). Examine the equilibrium configurations of cold neutron stars in the context of general relativity. (A)
- 2.11 "On Continued Gravitational Contraction," J. R. Oppenheimer and M. Snyder, *Phys. Rev.* 56, 455–459 (1939). The first relativistic treatment of gravitational collapse to form a black hole. (A)
- 2.12 "John Michell and Black Holes," S. Schaffer, *J. Hist. Astron.* 10, 42–43 (1979). Reviews the history of the early concept of a black hole. (E)
- 2.13 "Why are the Stars as they are?," S. Chandrasekhar, in *Physics and Astrophysics of Neutron Stars and Black Holes*, edited by P. Giacconi and R. Ruffini, item 6.10. Historical development of the structure of compact objects. (I)
- 2.14 *A Source Book in Astronomy and Astrophysics 1900–1975*, edited by K. R. Lang and O. Gingerich (Harvard University, Cambridge, 1979). Includes reprints of some of the above papers along with short descriptive introductions. (E)

III. ELEMENTARY EXPOSITIONS

By now a plethora of elementary expositions of black holes have appeared. Nearly every recent book covering some aspect of physics or astronomy for the layman contains at least a paragraph or two about black holes. The books and articles listed below, for the most part, have a more thorough discussion—from at least a few pages up to nearly the entire book.

A. Books

These books seem not broad enough in scope to be considered suitable textbooks. They are, however, recommended for more than just their discussion of black holes.

- 3.1 *General Relativity from A to B*, R. Geroch (University of Chicago, Chicago, 1978). A lucid account of general relativity and of the structure of a black hole with discussions of the event horizon, singularity, and light cone structure. It contains no equations but is wonderfully clear. (E)
- 3.2 *Space, Time and Gravity*, R. M. Wald (University of Chicago, Chicago, 1977). A short general account of relativity, cosmology, and astrophysics. Discusses cosmic censorship, black hole uniqueness, energy extraction from black holes and particle creation near black holes. It also contains a simple, accurate, and reasonable description of the analytic extension of the Kerr geometry of a rotating black hole. (E)
- 3.3 *From the Black Hole to the Infinite Universe*, D. Goldsmith and D. Levy (Holden-Day, San Francisco, 1974). An elementary treatment of modern physics; it contains a little algebra, is very descriptive but a bit superficial. Despite the title, there is not much about black holes. (E)
- 3.4 *Spacetime in the Modern Universe*, P. C. W. Davies (Cambridge University, Cambridge, 1977). Covers some quite advanced concepts with just a little mathematics. There is not much of the usual topics of falling into a black hole or observing someone falling in, but rather it covers dragging of inertial frames and black hole evaporation. (E)

- 3.5 **Black Holes, Quasars and the Universe**, H. L. Shipman (Houghton-Mifflin, Boston, 1976). An excellent elementary exposition, a good bit of depth but with a minimum of mathematics. (E)
- 3.6 **Astronomy: The Structure of the Universe**, W. J. Kaufmann, III (Macmillan, New York, 1977). Approximately 20 pages on black holes, not much depth but descriptive and with excellent pictures. It also includes accretion disks and x-ray sources—topics not usually covered by authors with a background in physics. (E)
- 3.7 **The Cosmic Frontiers of General Relativity**, W. J. Kaufmann, III (Little and Brown, Boston, 1977). Purportedly a layman's guide to general relativity; in fact this book delves deeply into the specific subject of geodesic motion in and around black holes of all types. This is the only book for the layman that I know of which discusses Penrose-type conformal diagrams. But it puts far too much emphasis on the multiply connected asymptotically flat regions (i.e., "universes") in the analytically extended Kerr metric. (I)
- 3.8 **Principles of Cosmology and Gravitation**, M. Berry (Cambridge University, Cambridge, 1976). Attempts to fill the gap in books at the undergraduate level. It has detailed but often confusing discussions of falling in and observing someone falling into a black hole. This is slightly more sophisticated than *Scientific American*. (I)
- 3.9 **The Collapsing Universe**, I. Asimov (Walker, New York, 1977). This includes 20 pages of delightful reading about black holes and is particularly appropriate for the uninitiated. (E)
- 3.10 **Black Holes**, W. Sullivan (Anchor, New York, 1979). This is one of the better of the nontechnical books. With little mathematical sophistication it covers a great deal of the physics and astrophysics of black holes. However, the approach is essentially historical which is cumbersome for a subject with only ten years of detailed history. Too much emphasis is placed on the Tunguska meteorite event as possibly related to black holes. (E)

B. Journal articles

The following articles span a broad range of subjects and levels of sophistication. Perhaps some selection of these offer the best opportunities for an introduction to black holes to the nonspecialist. Their levels are best judged by the journals in which they appear.

- 3.11 **"Gravitational Collapse and the Death of a Star,"** K. S. Thorne, *Science* 150, 1671 (1965). An early review of the last stages of evolution of a star with an emphasis on general relativistic stellar structure and stability. (E)
- 3.12 **"Gravitational Collapse,"** K. S. Thorne, *Sci. Am.* 217, 88 (November 1967). An early discussion of gravitational collapse of a star and the formation of a black hole (although that phrase had not yet been coined, see the next item). It includes the appearance of a collapsing star and a treatment of the formation of the central singularity. (E)
- 3.13 **"Our Universe: The Known and the Unknown,"** J. A. Wheeler, *Am. Sci.* 56, 1-20 (1968). A general article on astrophysics and cosmology which uses the phrase "black hole" for the first time in print. (E)
- 3.14 **"Introducing the Black Hole,"** R. Ruffini and J. A. Wheeler, *Phys. Today* 24, 30 (January 1971). Elementary but descriptive and interesting; includes collapse and formation, the Kruskal diagram, and possibilities of observation. (E)
- 3.15 **"Black Holes,"** R. Penrose, *Sci. Am.* 226, 38-46 (May 1972). Excellent description of gravitational collapse, light cone structure, and rotation of black holes. (E)
- 3.16 **"The Search for Black Holes,"** K. S. Thorne, *Sci. Am.* 228, 32 (December 1974). X-ray observations and the early evidence that Cyg X-1 is a black hole. (E)
- 3.17 **"Black Holes: New Horizons in Gravitational Theory,"** P. C. Peters, *Am. Sci.* 62, 575-83 (1974). Introduction to the formation and physics of black holes and their effects on light and nearby matter. (E)
- 3.18 **"Black Holes and Their Astrophysical Implications,"** D. L. Block, *Sky and Telescope* 50, 20-23 and 87-90 (1975). Discusses gravi-

tational collapse and its optical appearance, accretion, a black hole in a binary system, and singularities. (E)

- 3.19 **"The Physics and Astronomy of Black Holes,"** D. W. Sciama, *Quart. J. R. Astron. Soc.* 16, 1-12 (1975). An interesting and readable account of black hole physics including discussions of singularities, energy generation, and the role of black holes in astronomy. Based on a lecture broadcast by the BBC. And **"Black Holes and their Thermodynamics,"** *Vistas Astron.* 19, 385-401 (1976). The four laws of black hole thermodynamics at an elementary level. (E)
- 3.20 **"Black Hole Physics Illustrated in Photon Orbits,"** H. Cohn, *Am. J. Phys.* 45, 239-241 (1977). Photon orbits of the Schwarzschild and Kerr metrics. (E)
- 3.21 **"Inside the Black Hole,"** R. W. Brehme, *Am. J. Phys.* 45, 423-428 (1977). The geometry inside as well as outside of a nonrotating black hole. (E)
- 3.22 **"Particle Creation near Black Holes,"** R. Wald, *Am. Sci.* 65, 585-589 (1977). Black hole evaporation and its thermodynamics. (E)
- 3.23 **"The Quantum Mechanics of Black Holes,"** S. W. Hawking, *Sci. Am.* 231, 34 (January 1977). Discusses gravitational collapse, intuitive interpretation of particle production, and the possibilities of primordial black holes. (E)
- 3.24 **"Black Holes,"** G. W. Gibbons, in *McGraw-Hill Yearbook of Science and Technology*, edited by Daniel N. Lapedes (McGraw-Hill, New York, 1977), pp. 92-102. A nice, concise, and technical account of the formation of a black hole, the event horizon and exterior geometry, the dynamics of black holes, and the observational evidence for their existence. (I)
- 3.25 **"Les trous noirs: maelstroms cosmique,"** B. Carter and J. P. Luminet, *Recherche* 94, 944-955 (November 1978, in French). An interesting discussion of black hole physics including a description of geodesics and observations of someone falling into a black hole. It also has a particularly illuminating comparison between whirlpools and the dragging of inertial frames outside of a rotating black hole. (E)
- 3.26 **"Our Elastic Spacetime: Black Holes and Gravitational Waves,"** L. L. Smarr and W. H. Press, *Am. Sci.* 66, 72-79 (1978). Colliding black holes as a source of gravitational waves. (E)
- 3.27 **"Galactic Nuclei and Quasars: Supermassive Black Holes,"** M. Rees, *New Sci.* 80, 188-191 (1978). The possibility that jumbo black holes are the ultimate energy source of quasars. (E)
- 3.28 **"Exploding Galaxies and Supermassive Black Holes,"** W. Kaufmann, *Mercury* 7, 97-105 (1978). Popular level introduction to black holes in active galactic nuclei. (E)
- 3.29 **"Possible Ultimate Fate of the Universe,"** J. N. Islam, *Quart. J. R. Astron. Soc.* 18, 3-8 (1977) and *Sky and Telescope* 57, 13-18 (1979). The role of black holes in the last evolutionary stages of the universe. Although this is highly speculative it makes particularly interesting reading. (E)
- 3.30 **"Black Hole Thermodynamics,"** J. D. Bekenstein, *Phys. Today* 33, 24-31 (1980). Discusses the conceptual development of the melding of general relativity, thermodynamics, and quantum mechanics in the study of black holes. It includes a useful list of references. (E)

IV. TEXTBOOKS

Black holes may be discussed at a variety of levels to stimulate student interest. However, a huge gap in the coverage by textbooks exists between elementary physics or astronomy for undergraduates and the advanced graduate courses in general relativity.

A. Elementary

At the elementary level most current textbooks devote at least a paragraph or two to black holes. In physics it is typical to have an example or problem which works out what size a mass must be to have an escape velocity of the

speed of light. In astronomy the elementary coverage often extends to include black holes in binary systems, accretion disks, and X-ray sources. But, at least in physics, black holes provide an interesting backdrop for examples and homework problems in a variety of contexts. Unfortunately there is no useful compilation of these examples and perhaps the best sources are those given in Sec. III.

- 4.1 **University Astronomy**, J. M. Pasachoff and M. L. Kutner (Saunders, Philadelphia, 1977) and **Contemporary Astronomy**, J. M. Pasachoff (Saunders, Philadelphia, 1977). The sections on black holes in these two books are virtually identical. They are about ten pages long, have no mathematics, are very descriptive and make good reading. (E)
- 4.2 **Physics**, P. A. Tipler (Worth, New York, 1978). A calculus-based introductory physics text which has a two-page essay by A. P. Lightman on black holes. (E)

B. Advanced

These books are suitable for a graduate-level course in general relativity. For the most part the mathematics of general relativity is too advanced for undergraduates, but in some cases these books might be useful at that level.

- 4.3 **Principles of Relativity Physics**, J. L. Anderson (Academic, New York, 1967). Discusses the Schwarzschild solution, its analytic extension and the Eddington form for the metric. The Kerr solution is given but its interpretation is "still obscure." (A)
- 4.4 **Introduction to General Relativity**, R. Adler, M. Bazin, and M. Schiffer (McGraw-Hill, New York, 1st ed. 1965, 2nd ed. 1975). This text is suitable for a graduate or undergraduate course. The first edition derives the Schwarzschild solution but then discusses it only in the context of experimental tests of general relativity in the weak-field limit. The second edition has a much expanded section on black holes including the Kruskal diagram, a verification that the Kerr metric satisfies Einstein's equations (which is rare anywhere, let alone in a textbook!), trajectories around a rotating black hole, and a discussion of the ergosphere. (A)
- 4.5 **Gravitation and Spacetime**, Hans C. Ohanian (Norton, New York, 1976). One of the few textbooks which includes general relativity and is intended for undergraduates. It contains an excellent discussion of the Schwarzschild metric including its derivation and its use in the analysis of tests of general relativity. This book also includes a chapter on gravitational collapse and the formation of black holes. (I)
- 4.6 **General Relativity and Cosmology**, G. C. McVittie (University of Illinois, Urbana, 1962). Includes a straightforward discussion of the Schwarzschild metric and its geodesics. (A)
- 4.7 **Relativistic Astrophysics, Vol. I: Stars and Relativity**, Ya. B. Zel'dovich and I. D. Novikov (University of Chicago, Chicago, 1971). Includes a thorough discussion of the astrophysical implications of black holes with the formation in collapse at the end of stellar evolution, accretion, the presence in the nuclei of galaxies, and also the formal aspects of geodesics, and the analytic extension of the Schwarzschild, and Reissner-Nordstrom metrics. The Russian edition was published in 1967 so the treatment of rotating black holes is not up to date. (A)
- 4.8 **The Large Scale Structure of Space-Time**, S. W. Hawking and G. F. R. Ellis (Cambridge University, Cambridge, 1973). Includes a modern mathematical discussion of *all* the black hole solutions including maximal analytic extensions, topology, nature of singularities, a variety of coordinates, and the causal structure but just a scant description of either astrophysical applications or geodesics. (A)
- 4.9 **Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity**, S. Weinberg (Wiley, New York, 1972). Includes a standard treatment of the Schwarzschild geometry along with its geodesics, but for an excellent modern text on general relativity this book contains rather little discussion of black holes. (A)
- 4.10 **Gravitation**, C. W. Misner, K. S. Thorne, and J. A. Wheeler (Freeman, San Francisco, 1973). Over 160 pages on black holes and gravitational collapse (but that's only 13% of this tome!). All aspects are covered including geodesics, the event horizon, analytic extensions, and different useful coordinate systems. (A)
- 4.11 **The Classical Theory of Fields**, L. D. Landau and E. M. Lifshitz, 3rd ed. (Addison-Wesley, Reading, MA, 1971); 4th ed. (Pergamon, New York, 1975). This has long been the classic textbook for general relativity. There is a considerable change in the treatment of black holes between the third and fourth editions. The earlier edition concisely derives the Schwarzschild metric and discusses its geodesics. The later edition is expanded to include gravitational collapse and a short discussion of the Kerr metric. (A)

V. RESEARCH ARTICLES

Research on the physics and astrophysics of black holes continues to move apace in a variety of directions. New black hole candidates are found occasionally if somewhat irregularly. Some X-ray sources are believed to result from the interaction between a black hole and in-falling matter with the subsequent formation of an accretion disk. Numerical computations are being used to examine supernovas, gravitational collapse, and even the collision of two black holes. On a formal level the role of event horizons and trapped surfaces in the creation of singularities in space-time appears likely to lead to some version of a cosmic censorship theorem. Finally, the creation of particles by black holes provides an example of thermodynamics, quantum mechanics, and general relativity all interacting in a consistent manner and gives some hope to the possibilities of quantizing gravity. For black holes the last decade has been exciting; the next is likely to be more so.

Nearly all articles on black holes have been published in the last decade. With so little hindsight available it is difficult to decide which were the seminal articles and which topics of research will have the greatest long-term impact on physics. Hence what follows is not intended to be complete or exhaustive but rather representative of the state of black hole physics from my perspective.

A. Analytic structure of black holes

Solutions of Einstein's equations have now been found which represent black holes endowed with various amounts of mass, angular momentum, and electric charge. And there are indications that these are the *unique* stationary solutions (see item 5.4). But the exterior geometry, the analytic extension to the interior of the event horizon and the geodesics vary considerably for the different types of black holes.

- 5.1 "The Particle Problem in General Relativity," A. Einstein and N. Rosen, *Phys. Rev.* **48**, 73 (1935); "Past-Future Asymmetry of the Gravitational Fields of a Point Particle," D. Finkelstein *Phys. Rev.* **110**, 965 (1958); "Maximal Extension of the Schwarzschild Metric," M. D. Kruskal, *Phys. Rev.* **119**, 1743 (1960); "The Gravity Field of a Particle," C. Darwin, *Proc. R. Soc. London A* **249**, 180 (1959). Original studies of the analytic structure and geodesics of nonrotating black holes. (A)
- 5.2 "Über die Eigengravitation des elektrischen Feldes nach der Einsteinschen Theorie," H. Reissner, *Ann. Phys.* **50**, 106 (1916); "On the Energy of the Gravitational Field in Einstein's Theory," G. Nordström, *Proc. Kon. Ned. Akad. Wet.* **20**, 1238-1245 (1918). The first solutions of the coupled Einstein-Maxwell equations for a charged, nonrotating black hole. "Oscillatory Character of Reissner-Nordstrom Metric for an Ideal Charged Wormhole," J. C. Graves and D. R. Brill, *Phys. Rev.* **120**, 1507 (1960). The maximal analytic extension of a charged black hole. (A)

- 5.3 "Gravitational Field of Spinning Mass as an Example of Algebraically Special Metrics," R. P. Kerr, *Phys. Rev. Lett.* 11, 237 (1963); "Metric of a Rotating, Charged Mass," E. T. Newman, E. Couch, K. Chinnapared, A. Exton, A. Prakash, and R. Torrence, *J. Math. Phys.* 6, 918 (1965); "Maximal Analytic Extension of the Kerr Metric," R. H. Boyer and R. W. Lindquist, *J. Math. Phys.* 8, 265 (1967); "Global Structure of the Kerr Family of Gravitational Fields," B. Carter, *Phys. Rev.* 174, 1559 (1968); "Black Holes in General Relativity" S. W. Hawking, *Comm. Math. Phys.* 25, 152 (1972). The original research on the geometry of rotating black holes (both charged and uncharged) along with the analytic and topological structure and the geodesics. (A)
- 5.4 "Event Horizons in Static Vacuum Spacetimes," W. Israel, *Phys. Rev.* 164, 1776 (1967); "Axisymmetric Black Hole Has Only Two Degrees of Freedom," B. Carter, *Phys. Rev. Lett.* 26, 331 (1971); "Black Holes in Static Vacuum Space-Times," H. Müller zum Hagen, D. C. Robinson, and H. J. Seifert, *Gen. Rel. Grav.* 4, 53 (1973); "Uniqueness of the Kerr Black Hole," D. C. Robinson, *Phys. Rev. Lett.* 34, 905 (1975); "The Vacuum Black Hole Uniqueness Theorem and Its Conceivable Generalization" B. Carter, in *Proceedings of the First Marcel Grossman Meeting on General Relativity*, edited by R. Ruffini (North-Holland, Amsterdam, 1977), pp. 243-254. An important series of papers culminating in the Israel-Robinson theorem which states that the only stationary, uncharged black hole geometries are those of the Kerr family. (A)

B. Black hole dynamics

The dynamics and interactions of black holes have been examined with a variety of tools in a variety of contexts. The possibility of particle creation by black holes has caused great excitement in physics as a first step in the melding of thermodynamics, quantum mechanics, and general relativity. Fully relativistic black hole collisions have been studied with large scale numerical computations. And perturbation methods have been used to analyze black hole stability and interactions with gravitational waves.

An outstanding research problem today is the lack of direct evidence of the existence of a black hole (see part C below). In fact as the black hole bicentennial in 1983 (see item 2.1) approaches it is difficult to imagine any observation in the near future which would convince even a mild skeptic. Perhaps the most likely possibility for positively identifying a black hole is via the gravitational waves emitted in some violent black hole event. But until gravitational wave astronomy becomes a reality the direct confirmation of black holes will be lacking.

- 5.5 "Black Holes and the Second Law," J. D. Bekenstein, *Nuovo Cimento Lett.* 4, 737-740 (1972) and "Black Holes and Entropy," *Phys. Rev. D* 7, 2333-2346 (1973); "The Four Laws of Black Hole Mechanics," J. M. Bardeen, B. Carter, and S. W. Hawking, *Comm. Math. Phys.* 31, 161-170 (1973); "Black Hole Explosions," S. W. Hawking, *Nature* 248, 30-31 (1974); "Particle Creation by Black Holes," *Comm. Math. Phys.* 43, 199-220 (1975); "Black Holes and Thermodynamics," *Phys. Rev. D* 13, 191-197 (1976); "On Particle Creation by Black Holes," R. M. Wald, *Comm. Math. Phys.* 45, 9-34 (1975); "Origin of the Particles in Black Hole Evaporation," W. G. Unruh, *Phys. Rev. D* 15, 365-369 (1977); "Thermodynamics of Black Holes," P. C. W. Davies, *Rep. Prog. Phys.* 41, 1313-1355 (1978); see also item 5.17. These papers discuss the intertwining between thermodynamics, quantum mechanics, and black hole physics. (A)
- 5.6 "Gravitational Radiation from Distant Encounters and from Head-on Collisions of Black Holes: The Zero Frequency Limit," L. Smarr, *Phys. Rev. D* 15, 2069-77 (1977); "Gauge Conditions, Radiation Formulae and the Two Black Hole Collision," in item 6.2; "Collision of Two Black Holes: Theoretical Framework," L. Smarr, A. Cadez, B. DeWitt, and K. Eppley, *Phys. Rev. D* 14, 2443-2452 (1976); "Ultrarelativistic Black Hole Encounters," G. E. Curtis, *Gen. Rel.*

Grav. 9, 999-1008 (1978); "Gravitational Radiation from Hyperbolic Encounters," P. D. D'Eath, in item 6.2; and "Interaction of Two Black Holes in the Slow-Motion Limit," *Phys. Rev. D* 12, 2183-2199 (1975); "Some Remarks on the Two-Body Problem in Geometrodynamics," A. Cadez, *Ann. Phys.* 91, 58-74 (1975). An interesting series of papers which discusses the nature of the gravitational radiation when two black holes collide. (A)

- 5.7 "Stability of a Schwarzschild Singularity," T. Regge and J. A. Wheeler, *Phys. Rev.* 108, 1063-1069 (1957); "Stability of the Schwarzschild Metric," V. C. Vishveshwara, *Phys. Rev. D* 1, 2870-2879 (1970); "Stability of a Reissner-Nordstrom Black Hole," V. Moncrief, *Phys. Rev. D* 10, 1057-1063 (1974); "Perturbations of a Rotating Black Hole. II. Dynamical Stability of the Kerr Metric," W. H. Press and S. A. Teukolsky, *Astrophys. J.* 185, 649-673 (1973). These important papers demonstrate the stability of the geometry of a black hole to small perturbations. If black holes were unstable, then we might not expect to find them in the real world. (A)
- 5.8 "Effective Potential for Even-Parity Regge-Wheeler Gravitational Perturbation Equations," F. J. Zerilli, *Phys. Rev. Lett.* 24, 737-738 (1970); "Odd-Parity Stability of a Reissner-Nordstrom Black Hole," V. Moncrief, *Phys. Rev. D* 9, 2707-2709 (1974); "Perturbations of a Rotating Black Hole. I. Fundamental Equations for Gravitational, Electromagnetic, and Neutrino-Field Perturbations," S. A. Teukolsky, *Astrophys. J.* 185, 635-647 (1973); "Perturbations of a Rotating Black Hole. III. Interaction of the Hole with Gravitational and Electromagnetic Radiation," S. A. Teukolsky and W. H. Press, *Astrophys. J.* 193, 443-461 (1974); "The Solution of Dirac's Equation in the Kerr Geometry," S. Chandrasekhar, *Proc. R. Soc. London A*, 349, 571-575 (1976); and "An Introduction to the Theory of the Kerr Metric and Its Perturbations," in item 6.12; "Gravitational Radiation from a Particle Falling Radially into a Schwarzschild Black Hole," M. Davis, R. Ruffini, W. H. Press, and R. H. Price, *Phys. Rev. Lett.* 27, 1466-1469 (1971); "Gravitational Perturbation Theory and Synchrotron Radiation," R. A. Breuer (Springer-Verlag, Berlin, 1975); "Black Holes and Gravitational Waves: Perturbation Analysis," S. Detweiler, in item 6.2. These papers, along with those in item 5.7, apply perturbation methods to problems involving the interaction of black holes with waves and sources of gravitational and other fields. (A)
- 5.9 "Gravitational Collapse: The Role of General Relativity," R. Penrose, *Riv. Nuovo Cimento* 1 (special number), 252-276 (1969); "Reversible and Irreversible Transformation in Black Hole Physics," D. Christodoulou, *Phys. Rev. Lett.* 25, 1596 (1970); "Rotating Black Holes: Locally Nonrotating Frames, Energy Extraction, and Scalar Synchrotron Radiation," J. M. Bardeen, W. H. Press, and S. A. Teukolsky, *Astrophys. J.* 178, 347-369 (1972); "Energy Limits on the Penrose Process," R. M. Wald, *Astrophys. J.* 191, 231-233 (1974); "Upper Bounds on Collisional Penrose Processes Near Rotating Black-Hole Horizons," T. Piran and J. Shaham, *Phys. Rev. D* 16, 1615-1635 (1977). These papers, along with some of those mentioned in 5.8 consider the extraction of the rotational energy of a black hole via various, possibly observable, mechanisms. (A)
- 5.10 "Nonspherical Perturbations of Relativistic Gravitational Collapse. I. Scalar and Gravitational Perturbations," R. Price, *Phys. Rev. D* 5, 2419-2438 (1972); "Radiation from Collapsing Relativistic Stars. I. Linearized Odd-Parity Radiation," C. T. Cunningham, R. H. Price, and V. Moncrief, *Astrophys. J.* 224, 643-667 (1978); and "II. Linearized Even-Parity Radiation," *ibid.* 230, 870-892 (1979); "Collapse to a Rotating Black Hole," L. S. Kegeles, *Phys. Rev. D* 18, 1020-1029 (1978). Examine the emission of gravitational radiation from collapse forming a black hole. (A)
- 5.11 "Instability of Black Hole Inner Horizons," J. M. McNamara, *Proc. R. Soc. London A* 358, 499-517 (1978); and "Behavior of Scalar Perturbations of a Reissner-Nordstrom Black Hole Inside the Event Horizon," *Proc. R. Soc. London A* 364, 121-134 (1978); "Evolution of Scalar Perturbations near the Cauchy Horizon of a Charged Black Hole," Y. Gursel, V. D. Sandberg, A. A. Starobinsky, and I. D. Novikov, *Phys. Rev. D* 19, 413-420 (1979); "Instability of the Cauchy Horizon of Reissner-Nordstrom Black Holes" R. A.

Matzner, N. Zamorano, and V. D. Sandberg, *Phys. Rev. D* **19**, 2821-2826 (1979). Discuss the possible geometry inside of a black hole. While this subject poses no observational questions, it is important in principle. (A)

C. Astrophysical processes

Any search for the effects of black holes must come through astronomy and astrophysics. Only there can we envisage gravitational fields strong enough and masses great enough to overcome the pressure of matter resisting the formation of a black hole.

There is no shortage of theoretical proposals of where black holes might arise. Mini-black holes might have been created in the very early stage of the universe shortly after the big bang. Medium-sized black holes might have been created in supernovae explosions. And jumbo-sized black holes might reside in the centers of many galaxies or provide the power source for quasars.

On the other hand, observational evidence is in short supply and in every case is only indirect. For example, in Cygnus X-1 and in the nucleus of the nearby galaxy M87 there appear to be objects which are massive enough and small enough that most likely they are black holes. Cygnus X-1 appears to be a collapsed star with a mass substantially above the Chandrasekhar limit; hence it cannot be a neutron star or white dwarf. And in M87 there seems to be a central mass concentration of about $3 \times 10^9 M_\odot$ and only 100 pc in radius. In both these cases the black hole hypothesis is most likely but it wins only by default. No relativistic effects of black holes have yet been observed. And in fact every phenomenon possibly associated with a black hole can equally well be modeled by just a point mass and Newtonian gravity. As a result the most compelling reasons for believing in black holes are both indirect and theoretical.

- 5.12 "Cygnus X-1: A Spectroscopic Binary with a Heavy Companion?," B. L. Webster and P. Murdin, *Nature* **235**, 37-38 (1972); "Dimensions of the Binary System HDE 226868: Cygnus X-1," C. T. Bolton, *Nature: Phys. Sci.* **240**, 124-127 (1972); and "Orbital Elements and an Analysis of Models for HDE 226868: Cygnus X-1," *Astrophys. J.* **200**, 269-277 (1975); "Scattering Model for X-ray Bursts: Massive Black Holes in Globular Clusters," J. Grindlay and H. Gursky, *Astrophys. J. Lett.* **205**, L131-L133 (1976); "A Status Report on Cygnus X-1," D. M. Eardley, A. P. Lightman, N. I. Shakura, S. L. Shapiro, and R. A. Sunyaev, *Comments Astrophys.* **7**, 151-160 (1978); "Two More Globular Cluster X-ray Sources?," J. E. Grindlay, *Astrophys. J. Lett.* **224**, L107-L111 (1978); "Masses of Neutron Stars and Black Holes in X-ray Binaries," J. N. Bahcall, *Ann. Rev. Astron. Astrophys.* **16**, 241-264 (1978). Examine the evidence that some X-ray sources, particularly Cygnus X-1, might be black holes. (A)
- 5.13 "Astrophysics of Black Holes," I. D. Novikov and K. S. Thorne, in item 6.1; "Accretion onto Massive Black Holes," J. E. Pringle, M. J. Rees, and A. G. Pacholczyk, *Astron. Astrophys.* **29**, 179-184 (1973); "Accretion Disk Models for Compact X-ray Sources," J. E. Pringle and M. J. Rees, *Astron. Astrophys.* **21**, 1-9 (1972); "Black Holes in Binary Systems. Observational Appearance," N. I. Shakura and R. A. Sunyaev, *Astron. Astrophys.* **24**, 337-355 (1973); "Black Holes in Binary Systems: Instability of Disk Accretion," A. P. Lightman and D. M. Eardley, *Astrophys. J. Lett.* **187**, L1 (1974); "A Theory of the Instability of Disk Accretion onto Black Holes and the Variability of Binary X-ray Sources, Galactic Nuclei and Quasars," N. I. Shakura and R. A. Sunyaev, *Mon. Not. R. Astron. Soc.* **175**, 613-632 (1976); "A Model of a Self-gravitating Accretion Disk with a Hot Corona," B. Paczynski, *Acta Astron.* **28**, 241-251 (1978). These papers discuss the accretion of matter onto compact objects and the formation of an accretion disk. This is done with an eye toward matching theoretical predictions with X-ray observations. (A)
- 5.14 "Evidence for a Supermassive Object in the Nucleus of the Galaxy M87 from SIT and CCD Area Photometry," P. J. Young, J. A. Westphal, J. Kristian, C. P. Wilson, and F. P. Landauer, *Astrophys. J.* **221**, 721-730 (1978); "Dynamical Evidence for a Central Mass Concentration in the Galaxy M87," W. L. W. Sargent, P. J. Young, A. Boksenberg, K. Shortridge, C. R. Lynds, and F. D. A. Hartwick, *Astrophys. J.* **221**, 731-744 (1978); "Luminosity Distribution in the Central Regions of Messier 87: Isothermal Core, Point Source or Black Hole," G. de Vaucouleurs and J. L. Nieto, *Astrophys. J.* **230**, 697-712 (1979). Examines the evidence, based on optical observations, for the existence of a black hole in M87. (A)
- 5.15 "Star Distribution Near a Collapsed Object," P. J. E. Peebles, *Astrophys. J.* **178**, 371-375 (1972); "Star Distribution Around a Massive Black Hole in a Globular Cluster," J. N. Bahcall and R. A. Wolf, *Astrophys. J.* **209**, 214-232 (1976); "The Dissolution of Globular Clusters Containing Massive Black Holes," S. L. Shapiro, *Astrophys. J.* **217**, 281-286 (1977); "The Distribution of Stars Around a Massive Central Black Hole in a Spherical Stellar System. I. Results for Test Stars with a Unique Mass and Radius," J. R. Ipser, *Astrophys. J.* **222**, 976-990 (1978); "Star Clusters Containing Massive Central Black Holes: Monte Carlo Simulations in Two Dimensional Phase Space," S. L. Shapiro and A. B. Marchant, *Astrophys. J.* **225**, 603-624 (1978); see also the following item. These papers discuss the stellar dynamical effects, which are possibly observable, of a massive black hole in the center of a spherical star cluster. (A)
- 5.16 "Galactic Nuclei as Collapsed Old Quasars," D. Lynden-Bell, *Nature* **223**, 690-694 (1969); "On Quasars, Dust and the Galactic Centre," D. Lynden-Bell and M. J. Rees, *Mon. Not. R. Astron. Soc.* **152**, 461 (1971); "Death of White Holes in the Early Universe," D. M. Eardley, *Phys. Rev. Lett.* **33**, 442-444 (1974); "Massive Black Holes in Globular Clusters?," J. N. Bahcall and J. P. Ostriker, *Nature* **256**, 23-24 (1975); "Effects of Massive Central Black Holes on Dense Stellar Systems," J. Frank and M. J. Rees, *Mon. Not. R. Astron. Soc.* **176**, 633-647 (1976); "The Distribution and Consumption Rate of Stars Around a Massive Collapsed Object," A. P. Lightman and S. L. Shapiro, *Astrophys. J.* **211**, 244-262 (1977); "On the Nature of Quasars and Active Galactic Nuclei," V. L. Ginzburg and L. M. Ozernoi, *Astrophys. Space Sci.* **48**, 401-420 (1977); "Sustenance of a Black Hole in a Galactic Nucleus," G. A. Shields and J. C. Wheeler, *Astrophys. J.* **222**, 667-674 (1978); "Relativistic Jets and Beams in Radio Galaxies," M. J. Rees, *Nature* **275**, 516-517 (1978); "Massive Black Holes in Nuclei of Galaxies and Quasars," L. M. Ozernoi and M. Reinhardt, *Astrophys. Space Sci.* **59**, 171-192 (1978); "Gravity Power," D. Lynden-Bell, *Phys. Scr.* **17**, 183-191 (1978); see also the preceding item. As suggested by Lynden-Bell these papers discuss the possible role of black holes as the energy source of active galactic nuclei, radio sources, and quasars. (A)
- 5.17 "The Hypothesis of Cores Retarded During Expansion and the Hot Cosmological Model," Ya. B. Zel'dovich and I. D. Novikov, *Sov. Astron. A. J.* **10**, 602-603 (1967); "Gravitationally Collapsed Objects of Very Low Mass," S. W. Hawking, *Mon. Not. R. Astron. Soc.* **152**, 75 (1971); "The Primordial Black Hole Mass Spectrum," B. J. Carr, *Astrophys. J.* **201**, 1-19 (1975); and "On the Cosmological Density of Black Holes," *Comments Astrophys.* **7**, 161-173 (1978); "The Hydrodynamics of Black Hole Formation," D. K. Nadezhin, I. D. Novikov, and A. G. Polnarev, *Sov. Astron.* **22**, 129-138 (1978); "Gamma Rays from Primordial Black Holes," D. Page and S. W. Hawking, *Astrophys. J.* **106**, 1-7 (1976); "Upper Limits for Gamma Ray Bursts from Primordial Black Holes," N. A. Porter and T. C. Weekes, *Nature* **277**, 199 (1979); "Particle Emission Rates from a Black Hole: Massless Particles from an Uncharged, Nonrotating Hole," D. N. Page, *Phys. Rev. D* **13**, 198-206 (1976); "A Better Way of Searching for Black Hole Explosions," M. J. Rees, *Nature* **266**, 333-334 (1977); "Spectrum of a Radio Pulse from an Exploding Black Hole," R. D. Blandford, *Mon. Not. R. Astron. Soc.* **181**, 489-498 (1977); "Solar Model of Low Neutrino-counting Rate: The Central Black Hole," D. D. Clayton, M. J. Newman, and R. T. Talbot, Jr., *Astrophys. J.* **201**, 489 (1975). Discuss the possibilities for both formation and ob-

servation of miniblack holes from the early universe. See also item 5.5. (A)

VI. RESOURCES, REVIEWS, AND CONFERENCE PROCEEDINGS

The physics and astrophysics of black holes have evolved so rapidly that often reviews and conference proceedings are outdated by the time they are published. Nonetheless these provide the quickest means of becoming familiar with both the literature and current topics of interest.

- 6.1 **Black Holes**, edited by B. S. DeWitt and C. DeWitt (Gordon and Breach, New York, 1973) includes "The Event Horizon," S. W. Hawking; "Black Hole Equilibrium States," B. Carter; "Timelike and Null Geodesics in the Kerr Metric," J. M. Bardeen; and "Rapidly Rotating Stars, Disks, and Black Holes"; "Observations of Galactic X-ray Sources," H. Gursky; "Black Hole Astrophysics," I. D. Novikov and K. S. Thorne; and "On the Energetics of Black Holes," R. Ruffini. This grand volume is an excellent source for reviews of the current understanding of the physics of black holes. The authors are first rate and together they cover all aspects of black hole physics. (A)
- 6.2 **Sources of Gravitational Radiation**, edited by L. Smarr (Cambridge University, Cambridge, 1979). Contains a variety of articles on black holes as sources of gravitational radiation—one by Smarr on the collision of two black holes, Novikov and Polnarev on primordial black holes, Blandford on massive black holes, Detweiler on perturbations, Moncrief, Cunningham, and Price on gravitational collapse of dust, and D'Eath on hyperbolic encounters of two black holes. (A)
- 6.3 "Black Holes: the Outside Story," J. Stewart and M. Walker, Springer Tracts Mod. Phys., 69-115 (1973). A technical description of the geometry of black holes along with discussions of tidal forces and perturbations. (A)
- 6.4 "Astrophysical Processes Near Black Holes," D. M. Eardley and W. H. Press, Ann. Rev. Astron. Astrophys. 13, 381-422 (1975). Reviews the then current ideas concerning gravitational collapse, accretion, X-ray binaries, and black hole dynamics. (A)
- 6.5 **Astrophysics and Gravitation: Proceedings of the Sixteenth Solvay Conference on Physics**, edited by R. Debever (University of Brussels, Brussels, 1974). Includes a general article by Wheeler, a review of observational evidence by Novikov, and a lengthy article by Ruffini on a variety of aspects of black holes. (A)
- 6.6 **Neutron Stars Black Holes and Binary X-ray Sources**, edited by H. Gursky and R. Ruffini (Reidel, Dordrecht, Holland, 1975). Includes reviews by Ruffini, Gursky, and Kraft with an emphasis on astrophysical aspects of black holes. There are also reprints of some "classic" papers along with some recent interesting ones. (A)
- 6.7 **Gravitational Radiation and Gravitational Collapse**, edited by C. DeWitt, I. A. U. Symposium No. 64 (Reidel, Dordrecht, Holland, 1973). Includes articles by Markov, Bardeen, and Rees on black hole dynamics and accretion and a review of binary X-ray sources by Giacconi. (A)
- 6.8 **Black Holes, Gravitational Waves and Cosmology: An Introduction to Current Research**, M. Rees, R. Ruffini, and J. A. Wheeler (Gordon and Breach, New York, 1974). This volume aims at the beginning graduate student—one who has had an introductory course in general relativity. It reviews a variety of topics from the collapse of a dust cloud to the ergosphere of the Kerr metric. But it has not quite enough depth to bring one up to the level of current research. (A)
- 6.9 **Proceedings of the Seventh International Conference on General Relativity and Gravitation**, edited by G. Shaviv and J. Rosen (Wiley, New York, 1975). Includes articles by Press and by Rees on black hole processes both exotic and astrophysically reasonable. (A)
- 6.10 **Physics and Astrophysics of Neutron Stars and Black Holes**, edited by P. Giacconi and R. Ruffini, *Proceedings of the Enrico Fermi International School of Physics, Course LXV* (North-Holland, Amsterdam, 1978). (A)
- 6.11 **Proceedings of the First Marcel Grossman Meeting on General Relativity**, edited by R. Ruffini (North-Holland, Amsterdam, 1977). Includes a large number of short articles describing various technical research problems. (A)
- 6.12 **General Relativity: An Einstein Centenary Survey**, edited by S. W. Hawking and W. Israel (Cambridge University, Cambridge, 1979). Includes "The General Theory of the Mechanical, Electromagnetic and Thermodynamic Properties of Black Holes," B. Carter; "An Introduction to the Theory of the Kerr Metric and Its Perturbations," S. Chandrasekhar; and "Black Hole Astrophysics" R. D. Blandford and K. S. Thorne, among other fine articles not so closely related to black holes. This volume constitutes a unique compendium of general relativity as of 1980. Details of the results of all aspects of black hole physics are presented unencumbered by the actual calculations and proofs. (A)
- 6.13 **Black Holes 1969-1974**, Institution of Electrical Engineers (McGraw-Hill, London, 1974). This is a complete bibliography of all research papers written on black holes from 1969 to 1974. This was a particularly fruitful time for black hole physics which makes this a helpful resource. (E)