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SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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AN INTRODUCTION TO QUANTUM GEOMETRY

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Please note: These are preliminary notes intended for internal distribution only.

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N=2 SUPERCONFORMAL SYMMETRY AND QUANTUM GEOMETRY

OUTLINE:

I) INTRODUCTION & MOTIVATION

II) THE N=2 SUPERCONFORMAL ALGEBRA
- GENERAL PROPERTIES
- EXAMPLES OF REALIZATIONS

III) FAMILIES OF N=2 THEORIES
- TRULY MARGINAL OPERATORS
- MODULI SPACES I: CFT & NLσM

IV) INTERRELATIONS BETWEEN REALIZATIONS OF N=2 SCA
- LG/CY
- LG/MIN. MODELS
- MODULI SPACES II

V) MIRROR MANIFOLDS
- MOTIVATION, CONSTRUCTION, IMPLICATIONS
- MODULI SPACES III

VI) SPACETIME TOPOLOGY CHANGE

I) Introduction & Motivation

A) String Theory

- Initially: $M_{10} \to M_4 \times K_6$

  K_6: Ricci flat, complex, Kähler (to lowest order): CY

- More generally:

$$M_{10} \to M_4 \times \left\{ \begin{matrix} N=2 \text{ SCFT} \\ c = 9 \end{matrix} \right\}$$

B) Statistical Mechanics

C) Mathematical Physics

MAIN RESULTS TO BE COVERED

i) MIRROR MANIFOLDS:

- $M_4 \times K_6$
- $M_4 \times \tilde{K}_6$

$\Big\}\rightarrow$ IDENTICAL STRING PHYSICS

- NLσM ON K_6
- NLσM ON $\tilde{K}_6$

$\Big\}\rightarrow$ IDENTICAL N=2 SCFT's

"QUANTUM GEOMETRY"

ii) SPACETIME TOPOLOGY CHANGE:

$$M_4 \times K_6(\lambda) \begin{array}{l} \nearrow K_6(0) \\ \searrow K_6(1) \end{array} \Bigg\}$$

DIFFERENT TOPOLOGY YET PHYSICALLY SMOOTH TRANSITION

TRULY "STRINGY" PHENOMENA.

3.5
3.[illegible]
3.52
3.53

- WHY CONFORMAL FIELD THEORY?

  - IN ORDINARY QFT:

WITTEN

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\rangle \sim \text{YUKAWA COUPLINGS}$$

$$\langle \phi(x_1)\phi(x_2)\rangle \sim \text{MASS SHIFTS}$$

$$\langle \phi(x_1)\rangle \sim \,?$$

$$\langle \phi(x_1)\rangle \neq 0 \Rightarrow \text{MISIDENTIFIED VACUUM}$$

$\Rightarrow$ SHIFT: $\phi \to \phi - \langle\phi\rangle = \phi'$

$\Rightarrow \langle \phi' \rangle = 0$

$\Rightarrow$ 1 PT FCN $\longleftrightarrow$ EQNS OF MOTION

  - IN STRING THEORY:

$\langle V(z,\bar{z})\rangle = 0$ ARE EQNS OF MOTION

IF CFT: $\langle V(z,\bar{z})\rangle = \langle V(0,0)\rangle$

$$= \lambda^{-1}\bar{\lambda}^{-1}\langle V(0,0)\rangle \Rightarrow \langle V\rangle = 0 \checkmark$$

CFT's SOLVE STRING EQNS OF MOTION.

- WHY N=2 CFT?

  - N=2 SUSY ON WORLDSHEET ⇒
    N=1 SUSY IN SPACETIME

  - NONRENORMALIZATION THEOREMS
    ⇒ SOL'N TO ALL ORDERS IN STRING PERTURBATION THEORY
    &
    ⇒ PHENOMENOLOGICAL RELEVANCE

II) N=2 SUPERCONFORMAL ALGEBRA

II.1) THE ALGEBRA

- USUAL (N=0) CONFORMAL ALGEBRA: GENERATED BY $T(z)$

① $$T(z)\,T(\omega) \sim \frac{c/2}{(z-\omega)^4} + \frac{2}{(z-\omega)^2}\,T(\omega) + \frac{1}{(z-\omega)}\,\partial_\omega T(\omega) + \cdots$$

($\cdots$ = NON-SINGULAR TERM $\sim \sum_n T(\omega)$ = NEW FIELDS)
$= \oint [dz]\,(z-\omega)^{n+1}\,T(z)\,T(\omega)$

①′ $$[L_n, L_m] = (n-m)\,L_{n+m} + c/12\,(n^3-n)\,\delta_{n+m,0}$$

$$T(z) = \sum_{n\in\mathbb{Z}} z^{-n-2} L_n \rightarrow L_n = \oint [dz]\, z^{n+1}\, T(z)$$

- N=1 SCA: ADD $G(z)$, dim = 3/2

① AS ABOVE

② $$T(z)\,G(\omega) \sim \frac{3}{2}\,\frac{G(\omega)}{(z-\omega)^2} + \frac{\partial G(\omega)}{(z-\omega)} + \cdots$$

③ $$G(z)\,G(\omega) \sim \frac{2c}{3(z-\omega)^3} + \frac{2\,T(\omega)}{(z-\omega)} + \cdots$$

IN TERMS OF MODES: $G_n = \oint [dz]\, z^{n+1/2}\, G(z)$

(1') AS ABOVE

(2') $[L_n, G_m] = \left(\frac{m}{2} - n\right) G_{n+m}$

(3') $\{G_m, G_n\} = 2L_{m+n} + c/3\,(m^2 - 1/4)\,\delta_{m+n,0}$

- $N=2$ SCA: $G^1(z), G^2(z)$

(1) AS ABOVE

(2) AS ABOVE: $G \to G^1, G^2$

(3) $G^\alpha(z) G^\beta(\omega) \sim \left(\frac{2c}{3(z-\omega)^3} + \frac{2T(\omega)}{(z-\omega)}\right)\delta^{\alpha\beta} + i\left(\frac{2J(\omega)}{(z-\omega)^2} + \frac{\partial J(\omega)}{(z-\omega)}\right)\varepsilon^{\alpha\beta} + \cdots$

$J(\omega)$: DIMENSION 1 CURRENT

(4) $J(z) G^\alpha(\omega) \sim i\,\varepsilon^{\alpha\beta} G^\beta(\omega) / (z-\omega) + \cdots$

(5) $T(z) J(\omega) \sim J(\omega)/(z-\omega)^2 + \partial J(\omega)/(z-\omega) + \cdots$

(6) $J(z) J(\omega) \sim c/3\,/(z-\omega)^2 + \cdots$

• USEFUL TO INTRODUCE : $G^{\pm}(z) = \frac{1}{\sqrt{2}}\left(G^1(z) \pm i\, G^2(z)\right)$

③ $\rightarrow\quad G^+(z)G^-(\omega) \sim \frac{2c}{3(z-\omega)^3} + \frac{2J(\omega)}{(z-\omega)^2} + \frac{2T(\omega)+\partial J(\omega)}{(z-\omega)} + \ldots$

$$G^+(z)G^+(\omega) \sim G^-(z)G^-(\omega) \sim O(1)$$

④ $\rightarrow\quad J(z)\,G^{\pm}(\omega) \sim \pm\, G^{\pm}(\omega)/(z-\omega)$

• IN TERMS OF MODES :

$$T(z) = \sum L_n z^{-n-2} \qquad J(z) = \sum J_n z^{-n-1}$$

$$G^{\pm}(z) = \sum G^{\pm}_{n\pm a}\, z^{-(n\pm a)-3/2} \qquad 0 \le a < 1$$

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}\, m(m^2-1)\,\delta_{m+n,0}$$
$$[J_m, J_n] = \frac{c}{3}\, m\, \delta_{m+n,0}$$
$$[L_n, J_m] = -m\, J_{m+n}$$
$$[L_n, G^{\pm}_{m\pm a}] = \left(\frac{n}{2} - (m\pm a)\right) G^{\pm}_{m+n\pm a}$$
$$[J_n, G^{\pm}_{m\pm a}] = \pm\, G^{\pm}_{m+n\pm a}$$
$$\{G^+_{n+a}, G^-_{m-a}\} = 2L_{m+n} + (n+m+2a)J_{n+m} + \frac{c}{3}\left[(n+a)^2 - \frac{1}{4}\right]\delta_{m+n}$$

RAMOND : $a \in \mathbb{Z}$ (0) $\Rightarrow G^{\pm}(z)$ PERIODIC IN $z \to e^{2\pi i} z$

NS : $a \in \mathbb{Z}+1/2$ (1/2) $\Rightarrow G^{\pm}(z)$ ANTIPERIODIC "

II.2) HIGHEST WEIGHT REPRESENTATIONS

• IN N=0 CASE:

PRIMARY FIELD ϕ: $T(z)\phi(\omega) \sim \frac{h_\phi}{(z-\omega)^2}\phi(\omega) + \frac{1}{(z-\omega)}\partial_\omega \phi(\omega) + \cdots$

HWS: $|\phi\rangle = \phi(0)|0\rangle$ SATISFIES

$L_0|\phi\rangle = h_\phi|\phi\rangle$; $L_n|\phi\rangle = 0 \quad \forall\, n>0$

STATES IN REP: $(\prod L_{-n_i})|\phi\rangle$

• IN N=2 CASE:

SUPERCONFORMAL PRIMARY FIELD ϕ

$T(z)\phi(\omega) \sim$ AS ABOVE
$J(z)\phi(\omega) \sim (Q_\phi/(z-\omega))\phi(\omega) + \cdots$
$G^\pm(z)\phi(\omega) \sim (1/(z-\omega))\,\hat{\phi}^\pm(\omega) + \cdots = (1/(z-\omega))(G^\pm_{-1/2}\phi)(\omega)$
↳ SUPERPARTNERS OF ϕ

$\Longleftrightarrow$ $|\phi\rangle = \phi(0)|0\rangle$ SATISFIES:

$L_0|\phi\rangle = h_\phi|\phi\rangle$; $J_0|\phi\rangle = Q_\phi|\phi\rangle$

$L_n|\phi\rangle = J_n|\phi\rangle = G^\pm_n|\phi\rangle = 0 \quad n>0$

STATES IN REP: "$\prod L_{-n_i} J_{-m_j} G^\pm_{-l_k}$" $|\phi\rangle$

II.3) Chiral Primary Fields

W. ARR 262

- Special subset of NS primary fields "chiral primary":

$$G^{+}_{-1/2}|\phi\rangle = 0, \text{ i.e. } G^{+}(z)\phi(w) \sim \mathcal{O}(1)$$

- Also: "antichiral primary":

$$G^{-}_{-1/2}|\phi\rangle = 0, \text{ i.e. } G^{-}(z)\phi(w) \sim \mathcal{O}(1)$$

Either of these additional constraints eliminates half of the superpartners of $|\phi\rangle$.

- Recall: Holomorphic & Antiholomorphic algebras $\{T(z), J(z), G^{\pm}(z)\} \times \{\bar{T}(\bar{z}), \bar{J}(\bar{z}), \bar{G}^{\pm}(\bar{z})\} \Rightarrow$

(chiral, chiral) $\quad G^{+}_{-1/2}|\phi\rangle = \bar{G}^{+}_{-1/2}|\phi\rangle = 0$

(antichiral, chiral) $\quad G^{-}_{-1/2}|\phi\rangle = \bar{G}^{+}_{-1/2}|\phi\rangle = 0$

(chiral, antichiral) $\quad G^{+}_{-1/2}|\phi\rangle = \bar{G}^{-}_{-1/2}|\phi\rangle = 0$

(antichiral, antichiral) $\quad G^{-}_{-1/2}|\phi\rangle = \bar{G}^{-}_{-1/2}|\phi\rangle = 0$

- SPECIAL FEATURE OF SUCH CHIRAL (ANTI) FIELDS :
  - THERE ARE A FINITE NUMBER OF THEM
  - THEIR OPERATOR ALGEBRA IS CLOSED & NONSINGULAR

- LETS SEE HOW THIS GOES:

i) CONSIDER $\{G^-_{1/2}, G^+_{-1/2}\} = 2L_0 - J_0$ ACTING ON CHIRAL PRIMARY $|\phi\rangle$:

$$0 = \{G^-_{1/2}, G^+_{-1/2}\}|\phi\rangle = (2L_0 - J_0)|\phi\rangle = 2h_\phi - Q_\phi$$

$$\Rightarrow h_\phi = Q_\phi/2$$

- IN FACT : USE $(G^+_{-1/2})^\dagger = G^-_{1/2} \Rightarrow \forall |\psi\rangle$:

$$\langle\psi| \{G^-_{1/2}, G^+_{-1/2}\} |\psi\rangle \geq 0 \Rightarrow h_\psi \geq Q_\psi/2$$

ii) ϕ, χ CHIRAL PRIMARY

$$\phi(z)\chi(\omega) \sim \sum_i (z-\omega)^{h_{\psi_i} - h_\phi - h_\chi} \psi_i(\omega)$$

$$Q_{\psi_i} = Q_\phi + Q_\chi \,;\; h_{\psi_i} \geq \tfrac{1}{2} Q_{\psi_i} = \tfrac{1}{2} Q_\phi + \tfrac{1}{2} Q_\chi = h_\phi + h_\chi$$

$$\Rightarrow \lim_{z \to \omega} \phi(z)\chi(\omega) = (\phi\chi)(\omega) = \sum_{\text{CHIRAL PRIMARY}} \psi_i$$

$\Rightarrow$ CHIRAL PRIMARY FIELDS FORM A (NON-SINGULAR) RING.

iii) FINALLY:

CONSIDER $\{G^-_{3/2}, G^+_{-3/2}\} = 2L_0 - 3J_0 + 2c/3$.
LET ϕ BE CHIRAL PRIMARY:

$$\langle\phi|\{G^-_{3/2}, G^+_{-3/2}\}|\phi\rangle = \langle\phi|2L_0 - 3J_0 + 2c/3|\phi\rangle \geq 0$$

$$\Rightarrow \quad 2h_\phi - 3Q_\phi + 2c/3 \geq 0$$

AND $h_\phi = Q_\phi/2$

$$\Rightarrow \quad -4h_\phi + 2c/3 \geq 0 \Rightarrow \underline{h_\phi \leq c/6}$$

$\Rightarrow$ FINITE # OF CHIRAL PRIMARY FIELDS

- $\Rightarrow$ CHIRAL PRIMARY FIELDS FORM A FINITE NON-SINGULAR RING
- MORE PRECISELY: A GENERAL $N=2$ SCFT HAS 4 SUCH RINGS

1) (c,c) 1*) (a,a)

2) (a,c) 2*) (c,a)

SPECTRAL FLOW

$$G^{\pm}(z) = \sum G^{\pm}_{n\pm a} z^{-(n\pm a)-3/2} \qquad 0 \le a < 1$$

CLAIM: DIFFERENT CHOICES FOR "a" (DIFF BOUNDARY CONDITIONS) YIELD *ISOMORPHIC ALGEBRAS*

PROOF: (DIRECT CALCULATION) LET $\eta \equiv a - 1/2$

WANT TO SHOW: ALGEBRA GENERATED BY $\{L_n, J_n, G^{\pm}_{n\pm(\eta+\frac{1}{2})}\}$ IS ISOMORPHIC TO THAT GENERATED BY $\{L_n, J_n, G^{\pm}_{n\pm 1/2}\}$.

DEFINE:
$$L_n' = L_n + \eta J_n + 1/6\, \eta^2 c\, \delta_{n,0}$$
$$J_n' = J_n + \tfrac{1}{3} \eta c\, \delta_{n,0}$$
$$G^{\pm\prime}_r = G^{\pm}_{r\pm\eta}$$

NB: ALG $\{L_n', J_n', G^{\pm\prime}_r\}$ IS TRIVIALLY ALG $\{L_n, J_n, G^{\pm}_{r\pm\eta}\}$

NOW: $[L_n', L_m'] = [L_n + \eta J_n + \frac{1}{6}\eta^2 c\, \delta_{n,0},\ L_m + \eta J_m + \frac{1}{6}\eta^2 c\, \delta_{m,0}]$

$$= (n-m) L_{n+m} + c/12\, m(m^2-1)\,\delta_{m+n,0}$$
$$+ \eta(-m) J_{m+n} - \eta(-n) J_{m+n}$$
$$+ \eta^2 \frac{c}{3} n\, \delta_{m+n,0}$$
$$= (n-m)\left(L_{n+m} + \eta J_{n+m} + \frac{\eta^2}{6} c\, \delta_{m+n,0}\right) + \frac{c}{12} m(m^2-1)\,\delta_{m+n,0}$$

$$[L_n', L_m'] = (n-m) L'_{n+m} + c/12\, m(m^2-1)\,\delta_{m+n,0} \ \checkmark$$

- SIMILARLY FOR ALL OTHER PARTS OF THE ALGEBRA.

- WE CAN REALIZE THIS ISOMORPHISM BY A UNITARY OPERATOR:
  - $L_n' = \mathcal{U}_\eta L_n \mathcal{U}_\eta^{-1}$ • $J_n' = \mathcal{U}_\eta J_n \mathcal{U}_\eta^{-1}$ • $G_r^{\pm\prime} = \mathcal{U}_\eta G_r^{\pm} \mathcal{U}_\eta^{-1}$

AND GIVEN A REPRESENTATION $\{|S_j\rangle\}$ ON WHICH $\{L_n, J_n, G_r^{\pm}\}$ ACT, GET REP. $\{\mathcal{U}_\eta |S_j\rangle\}$ ON WHICH $\{L_n', J_n', G_r^{\prime\pm}\}$ ACT.

- $\eta = 0$: SPACETIME BOSONS (NS)
  $\eta = 1/2$: SPACETIME FERMIONS (R)

"SPECTRAL FLOW OPERATOR" $\mathcal{U}_{1/2}$ $\leftrightarrow$ BOSONS $\leftrightarrow$ FERMIONS
$\leftrightarrow$ SUSY IN SPACETIME

EXPLICITLY: $J_n' = J_n + \frac{1}{3}\eta c\, \delta_{n,0} \Rightarrow J_0' = J_0 + \frac{1}{3}\eta c$

$\Rightarrow$ EXPECT $\mathcal{U}_\eta$ SHIFTS $\mathcal{U}(1)$ CHARGE OF STATES BY $\frac{1}{3}\eta c$.

- GENERALLY: $\mathcal{U}(1)$ CURRENT $= i\sqrt{\frac{c}{3}}\, \partial_z \phi$
- FIELD OF CHARGE q: $f = \underset{\text{(NEUTRAL)}}{\hat{f}}\, e^{iq\sqrt{3/c}\,\phi}$
- $\mathcal{U}_\eta f = \hat{f}\, e^{i\phi\left(q\sqrt{\frac{3}{c}} + \eta\sqrt{\frac{c}{3}}\right)}$

(CHECK: L_n' WORKS OUT)

$\Rightarrow \mathcal{U}_\eta = e^{i\sqrt{\frac{c}{3}}\eta\phi} \Rightarrow \mathcal{U}_{1/2} = e^{i\sqrt{\frac{c}{3}}\frac{1}{2}\phi}$ (cf GAVA LECTURE)

(Left SIDE ONLY).

$N=2 \rightarrow$ SPECTRAL FLOW $\rightarrow$ SPACETIME SUSY.

II.4) Four Examples

A) One Complex Boson $X(z) = X^1(z) + i X^2(z)$
One Complex Fermion $\psi(z) = \psi^1(z) + i \psi^2(z)$ $\Big\}\; c = 3$

$$S = \int d^2 z \left(\partial X \bar{\partial} X^* + i\, \psi^* \bar{\partial} \psi + \psi \bar{\partial} \psi^* \right)$$

$$T(z) = -\partial X \partial X^* + \tfrac{1}{2} \psi^* \partial \psi + \tfrac{1}{2} \psi \partial \psi^*$$
$$G^+(z) = \psi^* \partial X$$
$$G^-(z) = \psi \partial X^*$$
$$J(z) = \psi^* \psi$$

Excercise: Using $\langle X(z) X^*(w) \rangle \sim -\log(z-w)$
$\langle \psi(z) \psi^*(w) \rangle \sim -1/(z-w)$
show that the above generate an $N=2$ SCA.

- By direct calculation

$\psi^*(\omega)$ $\quad h = 1/2 \quad Q = -1$ antichiral primary
$\psi(\omega)$ $\quad h = 1/2 \quad Q = +1$ chiral primary

$$\Rightarrow (c,c) = \{1, \psi, \bar{\psi}, \psi\bar{\psi}\} \qquad h \le c/6 \,?\ \text{Yes: } \tfrac{1}{2} = \tfrac{3}{6}.$$
$$(a,c) = \{1, \psi^*, \bar{\psi}, \psi^* \bar{\psi}\}$$
$$(c,a) = \{1, \psi, \bar{\psi}^*, \psi \bar{\psi}^*\}$$
$$(a,a) = \{1, \psi^*, \bar{\psi}^*, \psi^* \bar{\psi}^*\}$$

<u>NOTE</u>: FOR STRING-THEORY IN LIGHT CONE GAUGE

$$\mathbb{M}_4 \qquad \times \qquad \text{“ INTERNAL ”}$$

$$\left\{\begin{array}{l} \text{2 REAL BOSONS} \\ \text{+ 2 REAL FERMIONS} \begin{pmatrix} \text{ON LEFT} \\ \text{\& RIGHT} \end{pmatrix} = \begin{array}{l} \text{1 COMPLEX BOSON} \\ \text{1 COMPLEX FERMION} \end{array} \end{array}\right\} \times \left\{\begin{array}{c} \text{“ INTERNAL ”} \\ c = 9 \quad \mathcal{N} = 2 \\ \text{SCFT} \end{array}\right\}$$

$\downarrow$ EXAMPLE A

$\downarrow$ OTHER EXAMPLES

B) Nonlinear Sigma Model: (NLσM)

- More bosonic/fermionic fields
- Require bosonic fields span (are coords of) some Riemannian manifold (fermions are sections of pullback of tangent bundle times spin bundle)

- Fields: $X^i, X^{\bar{i}}$ complex bosonic fields $i = 1, \ldots, \dim_{\mathbb{C}} M$ (Kähler)

  $\psi^i \; \psi^{\bar{i}}$ RMoving complex fermions

  $\lambda^i, \lambda^{\bar{i}}$ LMoving complex fermions

- Action:

Antisymmetric tensor

$$S = \frac{i}{4\pi\alpha'}\int\Big\{g^M_{i\bar{j}}\big(\partial_z X^i \bar{\partial}_{\bar z} X^{\bar{j}} + \bar{\partial}_{\bar z} X^i \partial_z X^{\bar{j}}\big) - i B_{i\bar{j}}\big(\partial X^i \bar{\partial} X^{\bar{j}} - \bar{\partial} X^i \partial X^{\bar{j}}\big)$$
$$+ i\, \lambda^{\bar{i}} \mathcal{D}_z \lambda^{j} g_{\bar{i}j} + i\, \psi^{\bar{i}} \mathcal{D}_{\bar{z}} \psi^{j} g_{\bar{i}j}$$
$$+ R_{i\bar{i}j\bar{j}}\, \psi^i \psi^{\bar{i}} \lambda^j \lambda^{\bar{j}} \Big\}\, d^2 z$$

$$\left(\mathcal{D}_{\bar{z}} \psi^i = \partial/\partial\bar{z}\, \psi^i + \frac{\partial X^j}{\partial \bar{z}} \Gamma^i_{jk} \psi^k \right)$$

$$\left(S = \int d^2z\, d^4\theta\, K(\Phi^i, \Phi^{\bar{j}}) + B \;\; ; \;\; \Phi^i = X^i + \theta^- \psi^i + \bar{\theta}^- \lambda^i + \theta^- \bar{\theta}^- F^i \right)$$

• $N=2$ SUSY : FORMULATED IN $N=2$ SUPERSPACE

IN COMPONENTS e.g.: $\delta X^i = i\bar{\alpha}_+ \psi^i + i\alpha_+ \lambda^i$

$$\delta \psi_+^i = -\bar{\alpha}_- \partial_z X^i - i\alpha_+ \lambda^j \Gamma^i_{jm} \psi^m$$

$$\vdots$$

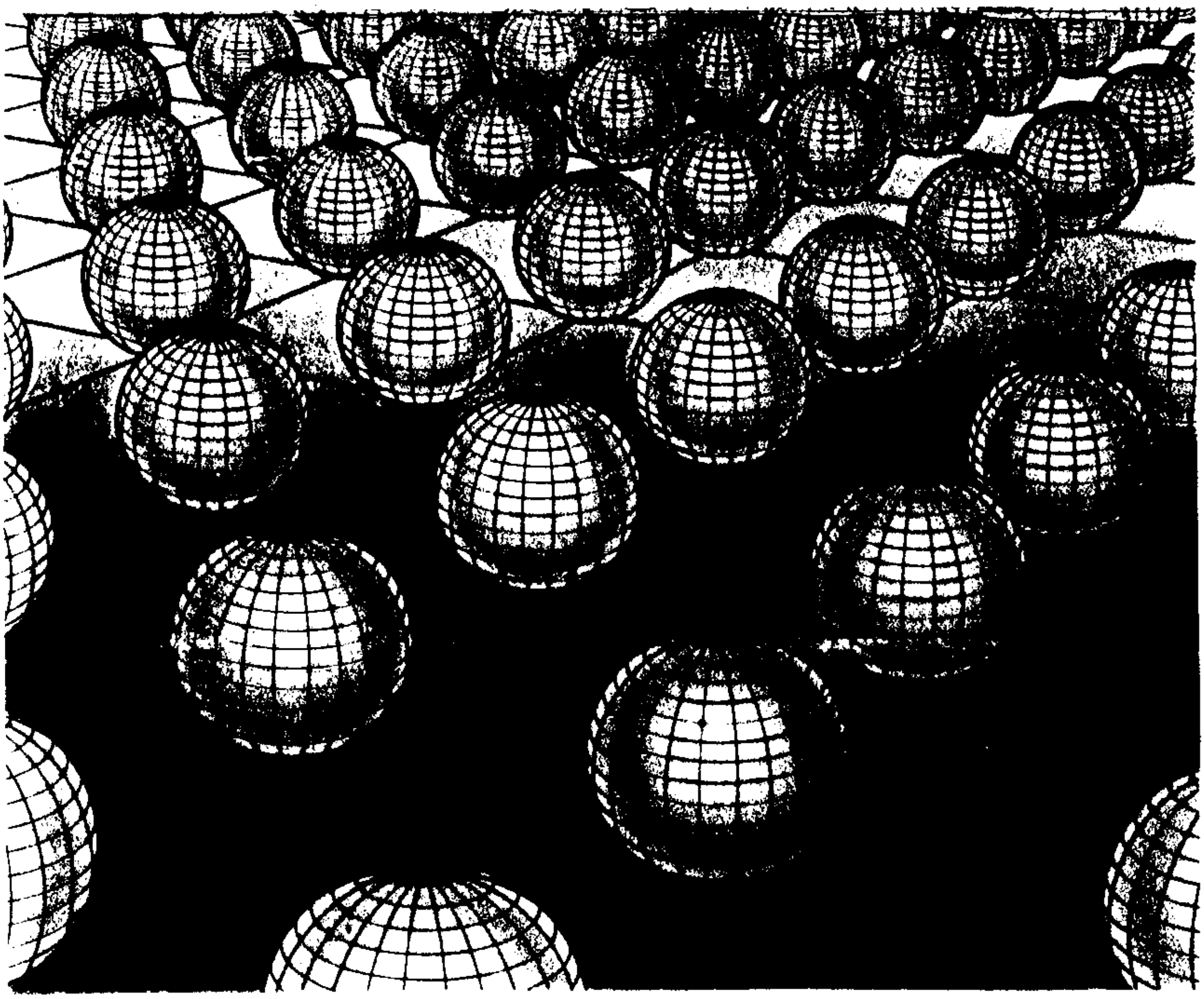
• HOW ABOUT CONFORMAL SYMMETRY?

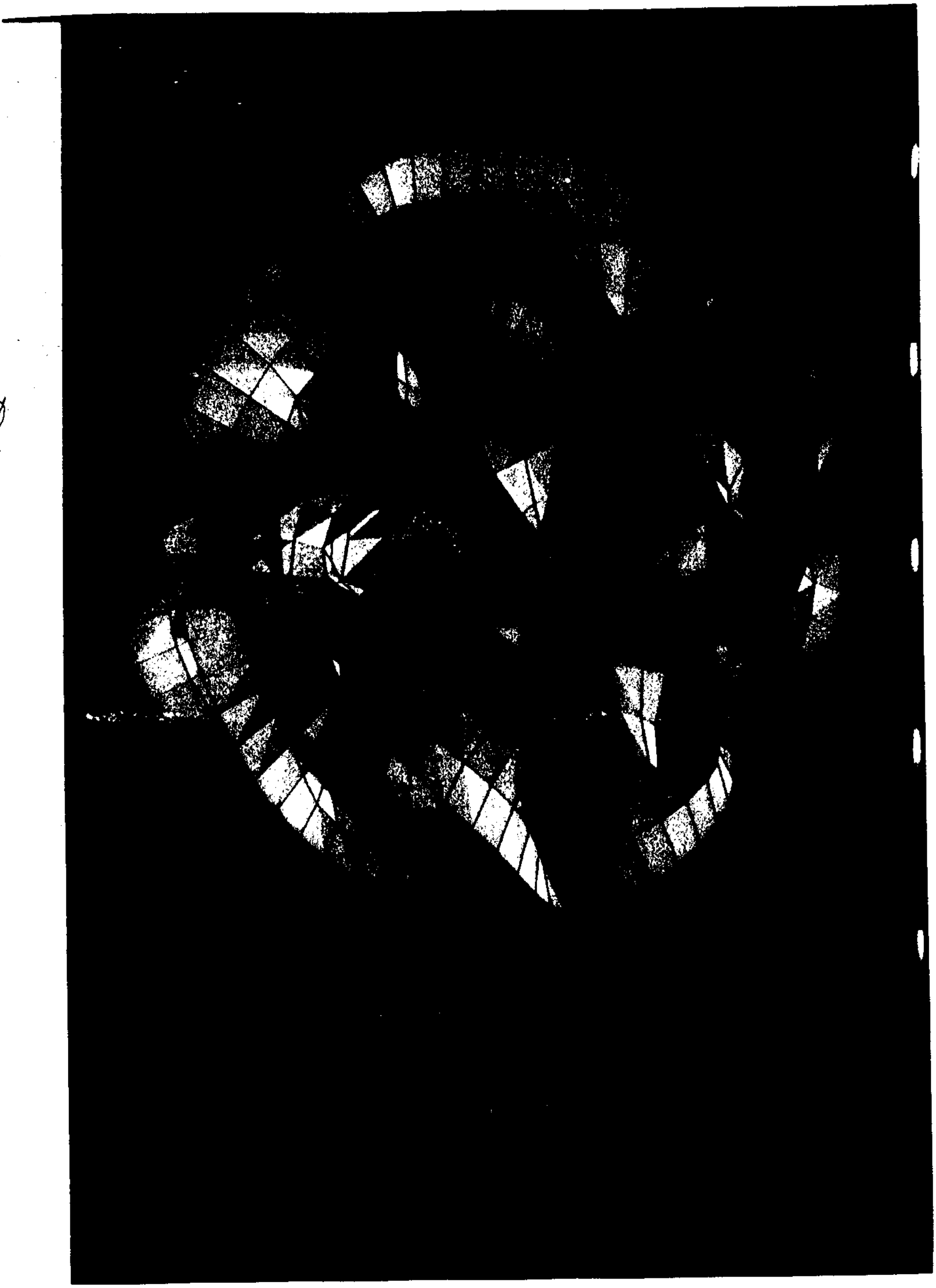
• $\beta(g_{i\bar{j}}) = R_{i\bar{j}} \Rightarrow M$ MUST ADMIT RICCI FLAT METRIC

$\Rightarrow M$ IS COMPLEX, KÄHLER, ADMITTING METRIC WITH $R_{i\bar{j}}=0$

$\Rightarrow M$ IS A CALABI-YAU MANIFOLD.

(B_{ij} MUST BE CLOSED 2-FORM)

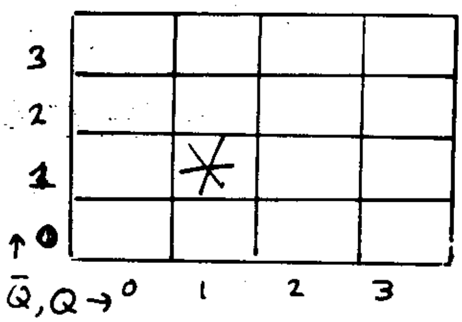




• Chiral (Anti-Chiral) Rings?

• **(c,c)**: Central charge $= 9 \Rightarrow h \leq 9/6 = 3/2$

$$\Rightarrow Q = 2h \leq 3$$
$$(\bar{Q} \leq 3)$$



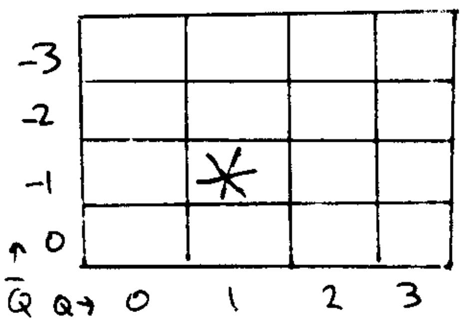
$*$: $Q = \bar{Q} = 1 \quad h = \bar{h} = 1/2$

Geometrically: $\rightarrow$ harmonic (2,1) forms $h_{ij,\bar{k}}\, dx^i dx^j dx^{\bar{k}}$

via $\quad \phi \sim (h_{ij,\bar{k}}\, \underbrace{\Omega^{ijk}}_{(3,0)})\, \psi^{\bar{k}}\, \underbrace{\lambda_k}_{g_{k\bar{n}}\lambda^{\bar{n}}}$

Why harmonic? # depends on M $=$ "$h^{2,1}$"

• **(a,c)**:



Geometrically: Harmonic (1,1) forms $h_{i\bar{j}}\, dx^i \wedge dx^{\bar{j}}$

$$\phi \sim h_{i\bar{j}}\, \psi^i \lambda^{\bar{j}}$$

Ring structure?

C) N=2 Landau-Ginsburg Theories

TOR, MARTINEC
VCAR;
?, WARNER

LET $\Phi(z,\bar{z},\theta^+,\theta^-,\bar{\theta}^+,\bar{\theta}^-)$ BE A COMPLEX $N=2$ SCALAR SUPERFIELD WHICH IS CHIRAL:

$$D_+\Phi = \bar{D}_+\Phi = 0 \qquad (D_\pm = \partial/\partial\theta^\pm + \theta^\mp \partial/\partial z)$$
$$(\bar{D}_\pm = \partial/\partial\bar{\theta}^\pm + \bar{\theta}^\mp \partial/\partial\bar{z})$$

$$S_{LG} = \int K(\Phi^i,\bar{\Phi}^{\bar{i}})\,d^2z\,d^4\theta + \int W(\Phi^i)\,d^2\theta^-\,d^2z + \int \bar{W}(\bar{\Phi}^{\bar{i}})\,d^2\theta^+\,d^2z$$

$(d^2\theta^\pm = d\theta^\pm d\bar{\theta}^\pm)$

- K : e.g. $K = \sum \bar{\Phi}^i \Phi^i$
- W HOLOMORPHIC POLYNOMIAL
  - $V(\phi) \sim |\nabla W(\phi)|^2$; TAKE W: $\partial W/\partial\Phi^i|_{\Phi^k=0} = 0$
  - $\det(\partial^2 W/\partial\Phi^i\partial\Phi^j)|_0 = 0$ (DEGENERATE CRIT. PTS)
  - $\partial W/\partial\Phi^i = 0 \Rightarrow \Phi^i = 0$

- ALLOW RG TO FLOW THEORY TO IR - ASSUME $\exists$ FIXED POINT $\Rightarrow$ SUPERCONFORMAL THEORY

- KEY FACT: $K_{\text{FIXED POINT}}$ IN GENERAL $\neq K$
  $W_{\text{FIXED POINT}} = W$

• A LITTLE MORE PRECISELY:

NONRENORMALIZATION THEOREM $\Rightarrow$

- F TERM NOT RENORMALIZED
- $W(\Phi^i)$ INVARIANT UP TO WAVE FCN RENORMALIZATION

NOW: UNDER CHANGE OF SCALE: $z \to \lambda^{-1} z$, $\theta \to \lambda^{-1/2}\theta$

$\Rightarrow$

$$\int d^2z\, d^2\theta \;\longrightarrow\; \lambda^{-1} \int d^2z\, d^2\theta$$

$\Rightarrow$ $\quad W(\Phi^i)$ MUST $\to$ $\lambda W(\Phi^i)$ UNDER

WAVE FCN RENORMALIZATION $\Phi^i \to \lambda^{\omega_i} \Phi^i$.

- ω_i = ANOMALOUS DIM OF Φ^i ; Φ^i SCALAR $\Rightarrow h_i = \bar{h}_i = \omega_i/2$
- AT FIXED POINT: $W(\Phi^i)$:

$$W(\lambda^{\omega_i}\Phi^i) = \lambda W(\Phi^i) \quad \text{QUASIHOMOGENEOUS}$$

HEURISTICALLY:

LG THEORY AT IR FIXED POINT: UNIQUE CLASSICAL VACUUM STATE, DEGENERATE CRITICAL POINT, (MASSLESS FLUCTUATIONS).

D) Minimal Models

- $N=0$ CFT : Unitary $c<1$ theories

$$C = 1 - 6/m(m+1)\ ;\ h_{p,q}(m) = \frac{[(m+1)p - mq]^2 - 1}{4m(m+1)}$$

$$1 \leq p \leq m-1\ ;\ 1 \leq q \leq p$$

- $N=1$, $N=2$ Analogous minimal models
  - $N=1$ $\quad c = \frac{3}{2}\left(1 - 8/m(m+2)\right) \leq 3/2 \quad m=3,\ldots$
  - $N=2$ $\quad c = 3\left(1 - 2/m+2\right) \leq 3 \quad m=1,2,\ldots$

$$= 3m/m+2 = \underline{3p/p+2}$$

As in $N=0$ case, finite # superconformal primary fields.

$$h_{\ell,m} = \frac{\ell(\ell+2)}{4(p+2)} - \frac{m^2}{4(p+2)}\ ,\ Q_{\ell,m} = m/p+2$$

$(\ell = 0,1,\ldots,p\ ;\quad m = -\ell, -\ell+2, \ldots, \ell)$
(NS sector)