INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

SMR.762 - 36

I

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

13 June - 29 July 1994

TOPOLOGICAL N=2 STRING THEORY

M. BERSHADSKY
Dept. of Physics
Harvard University
Cambridge, MA
USA

Please note: These are preliminary notes intended for internal distribution only.

MAIN BUILDING STRADA COSTIERA, 11 TEL. 22401 TELEFAX 224163 TELEX 460392 ADRIATICO GUEST HOUSE VIA GRIGNANO, 9 TEL. 224241 TELEFAX 224531 TELEX 460449
MICROPROCESSOR LAB. VIA BEIRUT, 31 TEL. 224471 TELEFAX 224600 TELEX 460392 GALILEO GUEST HOUSE VIA BEIRUT, 7 TEL. 22401 TELEFAX 2240310 TELEX 460392

M. BERSHADSKY

Plan: 4 - Lectures

1) Topological Theories ⊕ Gravity

2) Intro in CY,
topological σ-models on CY

3) Anomaly Equation &
It's solution in perturbation series

4) KS & AKS

Lecture I

Brief Topological field theories

I) Metric independent

$\Rightarrow$ generally covariant on worldsheet.

1) spectrum $\{\phi_i\} = S$

2) $1 \in S$

3) Correlation functions

$$\langle \phi_{i_1} \dots \phi_{i_n} \rangle$$

are independent on the positions

4) metric $\eta_{ij} = \langle \phi_i | \phi_j \rangle$

5) Factoriz. $I = \sum_{ij} |\phi_i\rangle \eta^{ij} \langle \phi_i|$

Realization: $\exists$ nilpotent BRST symmetry Q

1) $Q^2 = 0$

2) $T_{ab} = [Q, G_{ab}]$

3) Corr. functions are invariant w/r BRST symmetry

$$\langle \dots [Q\, X] \rangle \equiv 0$$

4) Spectrum is given by Q- cohomologies

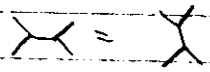
5) Yukawa: $C_{ijk} = \langle \phi_i \phi_j \phi_k \rangle$

6) metric $\eta_{ij} = C_{ij0} = \langle \phi_i \phi_j I \rangle$

(non degenerate)

All corr. functions are uniquely determined by factorization cond.

Symmetry condition $\left\{ C_{ijab} = C_{ijm} C_{abm} \eta^{mm} \right.$ ← symmetric w/r permutation (i,j,a,b)



Properties

1) Ex 1 $\langle \phi_{i_1}(x_1) \ldots \phi_{i_n}(x_n) \rangle$ is x independent

2) OPE: $\phi_i \phi_j = C_{ij}^{K} \phi_K + [Q, *]$

OPE algebra is associative

Coupling to gravity (Top. string theory).

$M_{g,n} \leftarrow$ moduli space of Riemann surfaces of genus g with n-punctures.

$\tau \in M_{g,n}$ 1) In Top. Theory $\langle \ldots \rangle$ is moduli independent!

$$\partial_\tau \langle .. \rangle \equiv 0$$

2) Coupling to gravity (by definition) means ~~to~~ first to construct a MEASURE on every $M_{g,n}$.
Correlation functions are given by INTEGRALS over moduli space $M_{g,n}$.
MEASURE is given by corr. functions in 2-d TFT.

Two possibilities

i) conformal theory,
indeed depends only on moduli, but not on metric (analog of critical theory)

ii) NON conformal $T_{z\bar{z}} \neq 0$ (still Q-exact)
depends on the metric REPRESENTATIVE

$$\cancel{(\partial_\kappa Z_0 + \partial_\sigma Z_0 - i\dot{Z}_0)(\partial_\kappa Z_\sigma + i\dot{Z}_\sigma)\, d\sigma +}$$
$$\cancel{+(\partial_\kappa Z_0 + \partial_\sigma Z_0 + i\dot{Z}_0)(\partial_\kappa Z_\sigma - i\dot{Z}_\sigma)\, d\sigma =}$$
$$\cancel{= \left[2(\partial_\kappa Z_0 + \partial_\sigma Z_0)\partial_\kappa Z_\sigma + \dot{Z}_0 \dot{Z}_\sigma \right] d\sigma}$$

Here we consider the first possibility (i)

1) $T_{z\bar{z}} = 0$ (conformal theory)

2) Nilpotent BRST Q : $Q^2 = 0$

3) Stress-energy is Q-exact

$$T_{ab} = [Q, G_{ab}]$$

Now with every moduli space one should associate a measure.

Analog: descent Equation

ϕ ← physical state

$$[Q, \phi] = 0$$
$$[Q, \phi^{(1)}] = d\phi$$
$$[Q, \phi^{(2)}] = d\phi^{(1)}$$

5

$\phi \leftarrow$ 0-form on worldsheet

$\phi^{(1)} \leftarrow$ 1-form on worldsheet

$\phi^{(2)} \leftarrow$ 2-form on worldsheet

Then $\langle \phi^{(2)}(1) \dots \phi^{(2)}(n) \rangle \leftarrow$ measure on the moduli space of the sphere with n-punctures.

With every moduli space $M_{g,n}$ we associate set of forms $\omega_{g,n}^{(K)}$

$$\omega_{g,n}^{(K)} \in \Omega_K (M_{g,n}, \otimes^n \mathcal{H})$$

where $\omega^{(K)}$ are related by descent equatio

$$[Q, \omega_{g,n}^{(K)}] = d\,\omega_{g,n}^{(K-1)}$$

$d \leftarrow$ differential on $M_{g,n}$

and $\omega_{g,n}^{(0)} = \phi(1) \otimes \dots \otimes \phi(n)$

Let $\mu = d\tau^i \mu_i$ be 1-form on $M_{g,n}$ with coefficients in Beltrami differentials

$$\mu_i = \mu_{i\bar{z}}{}^{z}\, d\bar{z}\, \partial_z$$

The descent equation can be solved

$$\omega_{g,n}^{(k)} = \langle \phi_{i_1} \dots \phi_{i_n} (\mu G)^k \rangle_{\Sigma_{g,n}}$$

$$\left(\text{indeed} \quad [Q, \omega_{g,n}^{(k)}] = d\omega_{g,n}^{(k-1)} \right)$$

The measure on $M_{g,n}$ is given by top form on $M_{g,n}$: $k = \dim M_{g,n}$

In fact, we are going to discuss topological theories that can be obtained by TWISTING $N=2$ superconformal system

Some WORDS about Twisting

After Twisting

$T_{zz}, \; G^-_{zz}, \; G^+_z, \; H_z$

$Q = \int G^+_z \quad ; \quad F = \int H_z$

$T_{zz} = [Q, G^-_{zz}] \qquad G^+_z = [Q, H_z]$

$Q \leftarrow$ BRST charge

$F \leftarrow U(1)$ charge (or fermion number, or ghost number)

In order to get a measure of $M_{g,n}$

one should satisfy a charge conservation

$$\langle \phi_{i_1} \ldots \phi_{i_n} (\mu G)^{2(3g-3+n)} \rangle \qquad -3(g-1)$$

$$\sum_{j=1}^{n} (q_j - 1) + 3(1-g) = \hat{c}\,(1-g)$$

$$\sum^{n} (q_j - 1) = (3 - \hat{c})(g-1)$$

1) $\hat{c} = 3$ and $q = 1$ (marginal operators)

For example: σ-models on Calabi–Yau

2) $\hat{c} = 2$ For Example σ-models on K_3

$$\sum (q_j - 1) = g - 1$$

$g > 1$ Let Ω be spectral flow operator

$$q_\Omega = 2 \qquad \#_\Omega = g - 1$$

⊕ marginal

Torus $g = 1$ marginal.

Sphere $g = 0$ one puncture

3) For $\hat{c} \leq 1$ gravitational descendant are important

In what follows we consider the following problem.

Lecture II

Take $N=2$ Twisted system

T_{zz} ; H_z ; G^+_z ; G^-_{zz}

Physical spectrum $\{\phi_a\}$ is given by BRST cohomologies

~~[illegible]~~ $Q_{tot} = Q + \bar{Q}$

$\phi \in H_{Q^{tot}} \iff [Q,\phi]=0$ & $\phi \neq [Q,*]$ ...

ϕ are in one to one correspondence with chiral primary fields for untwisted system (when $N=2$ is unitary)

The fields $\phi \in H_{Q^{tot}}$ are labeled by left & right charges

$$[F, \phi_{p,q}] = p\,\phi_{p,q} \qquad [\bar{F}, \phi_{p,q}] = q\,\phi_{p,q}$$

$$[L_0, \phi_{p,q}] = [\bar{L}_0, \phi_{p,q}] = 0$$

2 forms of chiral primaries (lowest components).

$$\phi^{(2)} = \oint G_{zz} \oint \bar{G}_{\bar{z}\bar{z}} \, \phi$$

~~this~~ defines exactly marginal perturbations

$$\therefore \quad S_0 \to S_0 + t^a \int \phi_a^{(2)}$$

It is clear that $\phi_a^{(2)}$ are

1) chargeless
2) dimension $(1, \bar{1})$

The original $N=2$ system was hermitian:
~~which means~~ Hermitian conjugation
ϕ (chiral) $\to$ ϕ^+ (antichiral)

The lowest component of antichiral field is

$$\tilde{\phi}^+ = \oint G_z^+ \int \bar{G}_{\bar{z}}^+ \, \phi^+ \equiv [Q[\bar{Q}, \phi^+]]$$

and is BRST exact in ~~the~~ twisted model.
Namely the perturbations $\tilde{\phi}^+$ decouple.

Therefore we may consider

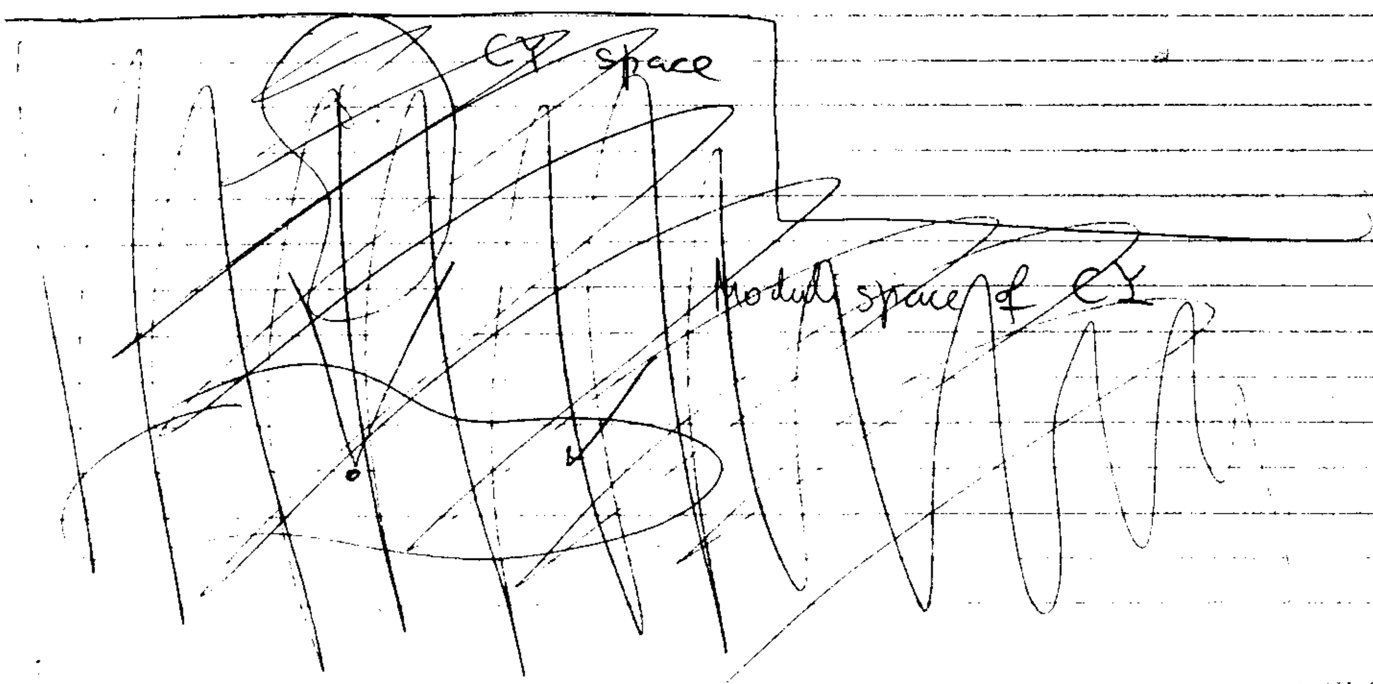
$$S_0 \to S_0 + t^a \int \phi_a^{(2)} + \bar{t}^a \int \{Q[\bar{O}\,\phi^+]\}$$

Naively one expects that nothing depends on $\bar{t}$.

There is an ANOMALY

Insertion of BRST trivial operators produce the total derivative on the moduli space which gets a contribution from the boundary.

We will think about Topological Twisted models as being realized as Topological σ-models on Calabi-Yau Manifolds.



Brief Intro in CY manifolds

1) Kähler n-dim manifold
2) Trivial Ricci Tensor

$$R = R_{i\bar{j}}\, dz^i \wedge d\bar{z}^{\bar{j}}$$

$$[R] = 0$$

or in other words $c_1(CY) = 0$

This condition is equivalent to either

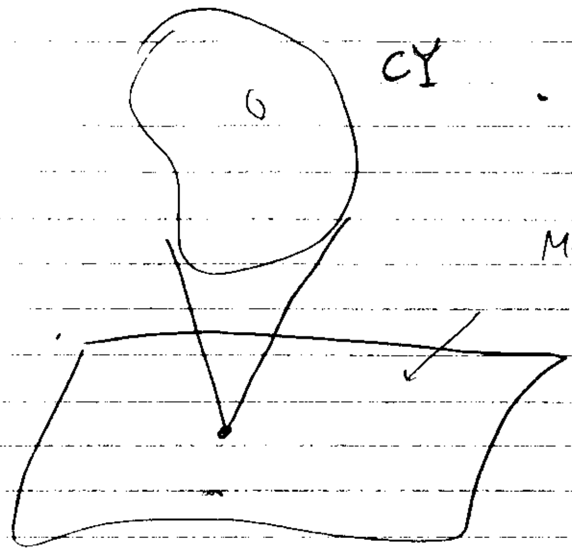
1) $GL(n) \longrightarrow SU(n)$ holonomy
2) or $\exists!$ holomorphic n-form $\Omega (dz)^n$

Metric $g_{ab} = \omega_{ac} J_b^{\ c}$

$[\omega] \leftarrow$ defines Kähler structure $\}$ $[\omega] \in H_d$

$[J] \leftarrow$ defines Complex structure $\}$ $J^2 = -I$ & some integrability condition

CY manifolds always comes in families



Moduli space of CY manifolds

Locally $\mathcal{M} = \mathcal{M}_c \times \mathcal{M}_K$

$\mathcal{M}_c \leftarrow$ deformations of Complex structures

$\mathcal{M}_K \leftarrow$ deformations of Kahler structure

Both manifolds $\mathcal{M}_c$ & $\mathcal{M}_K$ are special geometry Manifolds.

For example $\mathcal{M}_c$

1) $\Omega (dz)^n$ is a section of Line bundle $\mathcal{L}$ over $\mathcal{M}_c$

2) Kahler potential on $\mathcal{M}_c$

$$e^{-K} = \int_{CY} \Omega \wedge \bar{\Omega} \quad ; \quad e^{-K} \in \mathcal{L} \times \bar{\mathcal{L}}$$

3) For CY 3-folds

$$\Omega \in H_{3,0} \; , \quad \partial_a \Omega = [\;]_{3,0} + [\;]_{2,1}$$

$$\partial_a \partial_b \Omega = [\ ]_{3,0} + [\ ]_{2,\bar{1}} + [\ ]_{1,\bar{2}} \quad {}_{0,\bar{3}}$$

$$\partial_a \partial_b \partial_c \Omega = [\ ]_{3,\bar{0}} + [\ ]_{2,\bar{1}} + [\ ]_{1,\bar{2}} + [\ ]_{0,\bar{3}}$$

Indeed, deforming complex structure $\bar{\partial} \to \bar{\partial} + A \cdot \partial$

$$dz \to dz - A\, d\bar{z}$$

which implies the form mixing

Yukawa Coupling:

$$C_{ijk} = \int \Omega \wedge \partial_i \partial_j \partial_k \Omega = - \int \partial_i \Omega \wedge \partial_j \partial_k \Omega$$

4) Special geometry relation

$K \leftarrow$ Kahler potential

$G_{i\bar{j}} = \partial_i \bar{\partial}_j K \leftarrow$ Kahler metric

$R_{i\bar{j}k\bar{e}} \leftarrow$ Curvature

$$\boxed{R_{i\bar{j}k\bar{e}} = G_{i\bar{j}} G_{k\bar{e}} + G_{i\bar{e}} G_{\bar{j}k} - C_{ika} C_{\bar{j}\bar{e}\bar{a}}\, e^{2K}\, G^{a\bar{a}}}$$

(**) The same relation is valid for $\mathcal{M}_K$

!!! C_{ijk} and $G_{i\bar{j}}$ are given by correlation functions in FCFT

Back to Correlation functions.

Definition

$$A_{j_1 \ldots j_n} = \lim_{\varepsilon\to 0} \int_{|z_i - z_j| \geq \varepsilon} d^2 z_1 \ldots \int d^2 z_n \left\langle \phi^{(2)}_{j_1}(z_1) \ldots \phi^{(2)}_{j_n}(z_n) \right\rangle$$

$A_{j_1 \ldots j_n}(t, \bar{t})$ ← depends on the Moduli of CY

Variation $t \to t + \delta t$

$\bar{t} \to \bar{t} + \delta \bar{t}$

NB ~~$A_{j_1 \ldots j_n}(t, \bar{t})$ well defined~~ ~~TENSORS on the moduli~~ space.

$$S_{t,\bar{t}} \to S_{t,\bar{t}} + \delta t^a \int_\Sigma \phi_a^{(2)} d^2 z + \delta \bar{t}^a \int [Q[\bar{Q} \phi_a^{+}]]$$

$$\partial_a A_{j_1 \ldots j_n} = \lim_{\varepsilon \to 0} \int d^2 x \int d^2 z_1 \ldots d^2 z_n \left\langle \phi_a^{(2)}(x)\, \phi^{(2)}_{j_1}(z_1) \ldots \phi^{(2)}_{j_n}(z_n) \right\rangle$$

It is clear that $\partial_a A_{j_1 \ldots j_n}$ differs from $A_{a j_1 \ldots j_n}$ by contact term.

Contact terms have to identified with connection (for marginal perturbations of CFT)

$$\phi_a(z)\, \phi_b(w) = \ldots\ \delta(z-w)\, \Gamma^n_{ab}\, \phi_n(w) + \ldots$$

(D. Kutasov).

Therefore $\partial_a A_{j_1 \dots j_n} - \begin{pmatrix}\text{contact}\\ \text{terms}\end{pmatrix} =$

$$= \partial_a A_{j_1 \dots j_n} - \sum_k \Gamma^b_{a j_k} A_{j_1 \dots b \dots j_n} = D_a A_{j_1 \dots j_n}$$

$\boxed{\text{Lesson: Covariant derivatives make field insertion}}$

It turns out that $A^{(g)}_{i_1 \dots i_n}$ is not just n-th rank tensor on $\mathcal{M}_c$, but also a section of Line Bundle. The reason for this is that one should consistently normalize integral over fermion zero modes.

B-model $\quad \#(\psi^{i}_{z}) = \underbrace{d}_{\dim(CY)} \cdot g \qquad \#(\rho^{\bar{i}}) = n$

$$\underbrace{d\alpha_{i_1} \quad d\alpha_{i_{d(g-1)}}}_{\text{contracted with } \Omega} \; \underbrace{d\alpha_{i_{d(g-1)+1}} \quad d\alpha_{i_{dg}} \; d\beta_{j_1}' \quad d\beta_{j_g}}_{\text{contracted with metric}}$$

$$A^{(g)}_{i_1 \dots i_n} \in \mathcal{L}^{2(1-g)} \otimes T^{\otimes n}$$

That means, we should slightly modify our prescription

$$D_a = D_a^{(z)} + 2(g-1)\,\partial_a K$$

connection in the line bundle

$$A^{(g)}_{i_1 \cdots i_n} = \int_{\mathcal{M}_g} d\mu \int d^2 z_1 \cdots \int_{z_i \neq z_j} d^2 z_n \left\langle \phi^{(2)}(z_1) \cdots \phi^{(2)}(z_n) \right\rangle_{\Sigma_g}$$

$$A^{(g)}_{i_1 \cdots i_n} \in \mathcal{L}^{2(1-g)} \otimes T^{\otimes n}$$

$$D_a A^{(g)}_{i_1 \cdots i_n} = A^{(g)}_{a i_1 \cdots i_n}$$

BRIEF Intro in Topological σ-models on CY_3

A model

$$\mathcal{L} = G_{i\bar j}\left(\partial x^i \bar\partial x^{\bar j} + \bar\partial x^i \partial x^{\bar j} + \psi^i_{\bar z} D_z \bar\rho^{\bar j} + \underline{\psi^{\bar i}_z \bar D_{\bar z} \rho^{j}}\right) +$$
$$+ R_{i\bar j n\bar m}\, \psi^i_{\bar z}\psi^{\bar j}_z\, \rho^n \bar\rho^{\bar m} + F^2$$

to be compared

Spectrum $\quad x^i \; x^{\bar i} \qquad F^{\bar i}_z \; F^i_{\bar z}$

$\qquad\qquad \psi^i_{\bar z} \; \psi^{\bar i}_z \qquad \bar\rho^{\bar i} \; \rho^i$

B- model

$$\mathcal{L} = G_{i\bar j}\left(\partial x^i \bar\partial x^{\bar j} + \bar\partial x^i \partial x^{\bar j} + \psi^i_{\bar z} D_z \bar\rho^{\bar j} + \underline{\psi^{i}_z \bar D_{\bar z} \rho^{\bar j}}\right) +$$
$$+ R_{i\bar j n\bar m}\, \psi^i_z \psi^n_{\bar z}\, \rho^{\bar j} \bar\rho^{\bar m}$$

Spectrum $\quad x^i \; x^{\bar i} \qquad F^i_{z\bar z} \;;\; F^{\bar i}$

$\qquad\qquad \psi^i_{\bar z} \; \psi^i_z \qquad \rho^{\bar j} \; \bar\rho^{\bar j}$

Tiny difference $\quad \psi^{\bar i}_z \; \rho^i \quad \leftarrow$ A-model

$\qquad\qquad\qquad \psi^i_z \; \rho^{\bar i} \quad \leftarrow$ B-model

Both theories are topological theories.

A-model

$$\delta X^i = \rho^i \qquad \bar\delta X^{\bar i} = \bar\rho^{\bar i}$$

$$\delta \psi_z^{\bar i} = \partial_z X^{\bar i} \qquad \bar\delta \psi_{\bar z}^{i} = \bar\partial_z X^{i}$$

$$\delta \psi_{\bar z}^{i} = F_{\bar z}^{i} + \Gamma^{i}_{nm} \rho^n \psi^m_{\bar z} \qquad \delta \psi_z^{\bar i} = F_z^{\bar i} + \Gamma^{\bar i}_{\bar n \bar m} \rho^{\bar n} \psi_z^{\bar m}$$

$$\delta F_z^{\bar i} = -\partial_z \bar\rho^{\bar i} - \delta\left(\Gamma^{\bar i}_{\bar n \bar m} \psi_z^{\bar n} \bar\rho^{\bar m}\right) \qquad \dots$$

BRST invariant configurations

$$\partial_z X^{\bar i} = \bar\partial_z X^i = 0$$

$$(\text{fermions}) = 0$$

BRST inv configurations are given by holomorphic maps!

In the large volume limit one can identify

$$Q \Leftrightarrow \partial ; \quad \bar Q \Leftrightarrow \bar\partial$$

and Q_{tot} cohomologies with d-cohomologies

$$\left\{\begin{matrix}\text{physical} \\ \text{spectrum}\end{matrix}\right\} \equiv H_d .$$

$$\phi = A_{i_1 \dots i_n \bar{j}_1 \dots \bar{j}_m}(x,\bar{x})\, \rho^{i_1} \dots \rho^{i_n} \bar{\rho}^{\bar{j}_1} \dots \bar{\rho}^{\bar{j}_m}$$

Special fields $\quad \phi \in H_2$

$$\phi = A_{i\bar{j}}(x,\bar{x})\, \rho^i \bar{\rho}^{\bar{j}}$$

2-form : $\phi^{(2)} = A_{i\bar{j}}\, \partial x^i \bar{\partial} x^{\bar{j}}$ + fermions

Antichiral field (Non physical!!)

$$\phi^{+} = A_{i\bar{j}}(x,\bar{x})\, \psi^i_{\bar{z}} \psi^{\bar{j}}_{z}$$

$$[Q[\bar{Q}\phi^{+}]] = A_{i\bar{j}}\, \bar{\partial} x^i \partial x^{\bar{j}} + \text{fermions}$$

$$S_0 = \int G_{i\bar{j}} \left(\partial x^i \bar{\partial} x^{\bar{j}} + \bar{\partial} x^i \partial x^{\bar{j}} \right) + \dots$$

$$S_0 + \delta t \int \phi^{(2)} + \delta\bar{t} \int [Q[\bar{Q}\phi^{+}]] =$$

$$= \int (G_{i\bar{j}} + \delta t\, A_{i\bar{j}})\, \partial x^i \bar{\partial} x^{\bar{j}} +$$

defines variation of Kähler structure

$$+ \int (G_{i\bar{j}} + \delta\bar{t}\, \tilde{A}_{i\bar{j}})\, \bar{\partial} x^i \partial x^{\bar{j}} + \dots$$

BRST trivial

$$\omega = G_{i\bar{j}}\, dX^i \wedge dX^{\bar{j}} \quad \to \quad \omega + \delta\omega = (G_{i\bar{j}} + \delta t A_{i\bar{j}})\, dX^i \wedge dX^{\bar{j}}$$

Marginal perturbations for A-model
$\Leftrightarrow$ deformations of Kahler structure

$\delta S = \int \phi^{(2)} d^2z$, where $\phi \in H_2$

A-model does not depend of Complex structure

~~Topol~~ B-model

$$\delta X^{\bar{i}} = \rho^{\bar{i}}$$
$$\delta \psi^i_z = \partial_z X^i$$
$$\delta \rho^{\bar{i}} = F^{\bar{i}} + \Gamma^{\bar{i}}_{\bar{n}\bar{m}} \rho^n \bar{\rho}^{\bar{m}}$$
$$\delta F^i_{z\bar{z}} = -\partial_z \psi^i_{\bar{z}} - \delta(\Gamma^i_{nm} \psi^n_z \psi^m_{\bar{z}})$$

$$\bar{\delta} X^{\bar{i}} = \bar{\rho}^{\bar{i}}$$
$$\bar{\delta} \psi^i_{\bar{z}} = \bar{\partial}_z X^i$$
$$\bar{\delta} \bar{\rho}^{\bar{i}} = -F^{\bar{i}} - \Gamma^{\bar{i}}_{\bar{n}\bar{m}} \rho^{\bar{n}} \bar{\rho}^{\bar{m}}$$
$$\bar{\delta} F^i_{z\bar{z}} = \ldots$$

BRST invariant configurations

$$\begin{cases} \partial_z X^i = \bar{\partial}_z X^i = 0 \\ (\text{fermions}) = 0 \end{cases}$$

Constant maps !!

It is easy to see that

1) $\theta^{\bar{i}} = \rho^{\bar{i}} + \bar{\rho}^{\bar{i}}$ is BRST inv.

2) on the other hand

$$\delta_{tot}\left(\rho^{\bar{i}} - \bar{\rho}^{\bar{i}}\right) = 2\Gamma^{\bar{i}}_{\bar{n}\bar{m}}\, \rho^{\bar{n}} \bar{\rho}^{\bar{m}}$$

Ex $\boxed{\eta_i = g_{i\bar{i}}\left(\rho^{\bar{i}} - \bar{\rho}^{\bar{i}}\right)}$

η_i is BRST inv. (take into account $\delta g_{i\bar{j}}$)

Again, in the large volume limit

$Q_{tot} \Longleftrightarrow \bar{\partial}$ operator

Physical observables are given by $\bar{\partial}$-cohomologis

$$\phi = A^{i_1 \ldots i_n}_{\bar{j}_1 \ldots \bar{j}_m}\, \theta^{\bar{j}_1} \ldots \theta^{\bar{j}_m}\, \eta_{i_1 \ldots i_n}$$

$$\phi \in H\left(\Lambda^* T_M\right)$$

(forms with coefficient with vector fields

Again, there are some special fields

$$\phi \in H_1(\Lambda^1 T_n)$$

$$\phi = A^j_{\bar{i}}\, \theta^{\bar{i}} \eta_j \quad \Leftrightarrow \quad A^j_{\bar{i}}\, dx^{\bar{i}} \frac{\partial}{\partial x^j}$$

Perturbation by $\int \phi^{(2)}$ defines the deformation of complex structure

$$\bar{\partial} \to \bar{\partial} + A^j \cdot \partial_j$$

Similarly: B-model is insensitive to deformation of complex structure.

Yukawas for B-model } $C_{ABC} = \langle \phi_A \phi_B \phi_C \rangle = \int_{CY} A^{j_1} \wedge B^{j_2} \wedge C^{j_3}\, \Omega_{j_1 j_2 j_3} \wedge \Omega$

$$C_{ABC} \in \mathcal{L}^2$$

Yukawas for A-model $\quad C_{ABC} = \langle \phi_A \phi_B \phi_C \rangle = \int A \wedge B \wedge C + \{\text{Instantons Correction}\}$