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SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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TOPOLOGICAL N=2 STRING THEORY

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Please note: These are preliminary notes intended for internal distribution only.

Anomaly Equation (σ-models on CY-3-folds)

Partition function F_g $(g>1)$ & $\langle \phi_a \rangle_{g=1}$

$g>1$
$$F_g = \int_{M_g} \langle (\mu G)^{3g-3} (\bar{\mu} \bar{G})^{3g-3} \rangle$$

$g=1$
$$F_{a,1} = \langle \phi_a \rangle = \int_{F} \langle (\mu G)(\bar{\mu} \bar{G})\, \phi_a \rangle$$

For Unitary theories

F_g is finite

as $\tau \to \infty$ only finite number of ground states (chiral primaries) contribute.

~~Partition~~

Anomaly $\bar{\partial}_t F_g \neq 0$

Simple Example $\bar{\partial}_{\bar{j}}\, C_{abc} \equiv 0$ (easy to check)

But $\bar{\partial}_{\bar{j}}\, C_{abcd} = \bar{\partial}_{\bar{j}}\, D_a C_{bcd} = [\bar{\partial}_{\bar{j}}\, D_a]\, C_{bcd} =$
$= \hat{R}_{\bar{j}a} \cdot C \neq 0$

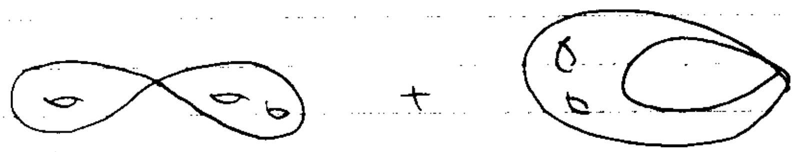
Anomaly eq. for $g > 1$

$$\bar\partial_j F_g = \int_{\mathcal{M}_g} \langle (\mu G)^{3g-3} (\bar\mu \bar G)^{3g-3} \int [Q[\bar Q \phi_a^+]] \rangle =$$

$$= (3g-3)^2 \int_{\mathcal{M}_g} \langle (\mu T)(\bar\mu \bar T)(\mu G)^{3g-4} (\bar\mu \bar G)^{3g-4} \int \phi_a^+ \rangle =$$

$$= \int_{\mathcal{M}_g} \partial_\tau \bar\partial_\tau \langle (\mu G)^{3g-4} (\bar\mu \bar G)^{3g-4} \int \phi_a^+ \rangle =$$

boundary contribution $= \sum$ (different degenerations)



k $\quad$ $g-k$ $\quad + \quad$ $g-1$

Coordinated around degeneration divisor

$$\mathcal{M}_g \longrightarrow (\mathcal{M}_k, \mathcal{M}_{g-k}, z, w, \tau = T + i\theta)$$

punctures $\quad$ Tube

or

$$\mathcal{M}_g \longrightarrow (\mathcal{M}_{g-1}, z, w, \tau = T + i\theta)$$

Consider an example of degeneration

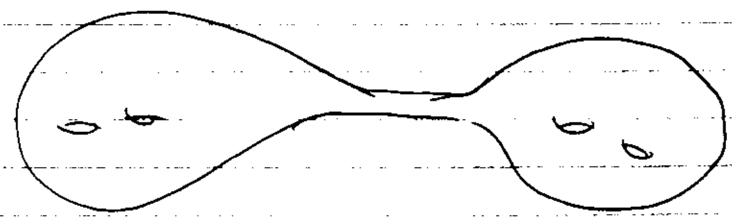
$$\mathcal{M}_g \to \mathcal{M}_K \times \mathcal{M}_e.$$

$$\Rightarrow \int_{\mathcal{M}_K} dm_K \int_{\mathcal{M}_{g-K}} dm_{g-K} \, dz \, dw \, dT \wedge d\theta \times$$

$$\times (\partial_T - i\partial_\theta)(\partial_T + i\partial_\theta) \langle (\mu G)^{3g-4} (\bar{\mu}\bar{G})^{3g-4} \int \phi_a^+ \rangle_{\Sigma_K \times \Sigma_{g-K} \times T} =$$

(only ∂_T^2 is relevant)

$$= \int dm_K \int dm_{g-K} \, dz \, dw \, d\theta \, \frac{\partial}{\partial T}\Big|_{T\to\infty} \langle (\mu G)^{3g-4} (\bar{\mu}\bar{G})^{3g-4} \int \phi_a^+ \rangle$$

$=$ 

1). Only ground states propagates along the tube (mass supression $\sim e^{-\Delta T}$)

2) We have to pick contribution to corr. functions which behaves as $\sim T$. In this case one will get a finite contribution

That means the operator should be inserted on the tube

$$= \int_{\mathcal{M}_h} dm_h\, dz \,\langle (\mu G)^{3h-3} (\bar\mu \bar G)^{3h-3} \int \phi_i^{(2)} \rangle \times \quad D_i$$

$$\times \frac{\partial}{\partial T} \langle \phi_i^{i} | \int \phi_{\bar a}^{+} \; | \phi_j^{j} \rangle_{T\to\infty} \times$$

$$\times \int_{\mathcal{M}_\ell} dm_\ell\, dw \,\langle (\mu G)^{3\ell-3} (\bar\mu \bar G)^{3\ell-3} \int \phi_{\bar a}^{(2)} \rangle$$

$$\frac{\partial}{\partial T} \langle \phi_i | \int \phi_{\bar a}^{+} | \phi_j \rangle_{T\to\infty} = \frac{\partial}{\partial T} T \langle \phi_i^{j} | \phi^{\bar b} \rangle \langle \phi_{\bar b}^{+} | \oint \phi_{\bar a}^{+} \; | \phi_{\bar c}^{+} \rangle \times$$

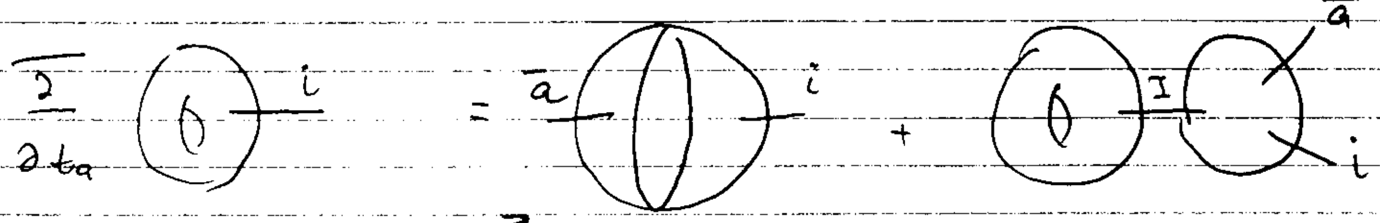
$$\times \langle \phi^{\bar c} | \phi^{j} \rangle =$$

$$= \bar C_{\bar a \bar b \bar c}\, G^{j\bar b} G^{i\bar c} e^{2K} = \bar C_{\bar a}^{\;ij}$$

As the Result

$$\bar\partial_{\bar a} F_g = \frac{1}{2} \bar C_{\bar a}^{\;ij} \left(\sum_{k=1}^{g-1} D_i F_k D_j F_{g-k} + D_i D_j F_{g-1} \right)$$

Similar Equation can be derived for 1-point corr. function on a torus.



Familia degeneration

$$\sum C_{inm} \times \bar{C}_{\bar{a}}^{nm}$$

Another degeneration comes from the contribution when ϕ_a^+ & ϕ_i get close to each other

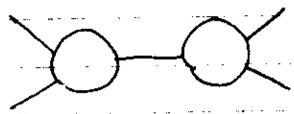
In this case the contribution is proportional to $G_{i\bar{a}} \times \frac{\chi}{12}$

$$\partial_{\bar{a}} \partial_i F_1 = \sum_{(n,m)} C_{inm} \bar{C}_{\bar{a}}^{nm} + \left(\frac{\chi}{12} - 2\right) G_{\bar{a}i}$$

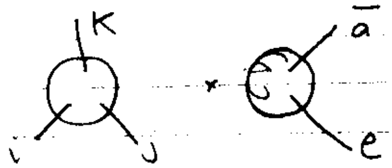
marginal !!

The computation of Special geometry relation is pretty straightforward.

$$\bar{\partial}_a C_{ijke} = \sum_{3\text{ channels}} C_{ijr}\, C_{kes}\, \bar{C}_{\bar{a}}{}^{rs} \; +$$



$$+ \sum_{4\text{ terms}} C_{ijk}\, G_{\bar{a}e}$$



It is easy to check that

$$\bar{\partial}_a C_{ijke} = \sum_{\substack{\text{perm}\\(j,k,e)}} R_{\bar{a} i \bar{n} j}\, G^{n\bar{n}}\, C_{nke}$$

where R = special geometry.

Anomaly Master Equation

$$\mathcal{F} = \sum \lambda^{2g-2} F_g \qquad D_a = \partial_a + \Gamma_a + 2(\partial_a K)\lambda\,\partial_\lambda$$

Then $\left(\bar\partial_{\bar a} - \bar\partial_{\bar a} F_1 - \frac{1}{2}\lambda^2 \bar C_{\bar a}{}^{bc} D_b D_c\right) e^{\mathcal{F}} = 0$

is equivalent to anomaly equations for $g > 1$

Introduce $\bar{\mathcal{D}}_{\bar a} = \bar\partial_{\bar a} - \bar\partial_{\bar a} F_1 - \frac{1}{2}\lambda \bar C_{\bar a}{}^{bc} D_b D_c$

Integrability condition

$$[\bar{\mathcal{D}}_a, \bar{\mathcal{D}}_b] \sim \left(\begin{matrix}\text{equation}\\ \text{of } F_1\end{matrix}\right) \equiv 0$$

Anomaly Equation for corr. function (check this).

$$\bar\partial_{\bar a}\, e^{W(x,t,\bar t)} = \frac{1}{2}\lambda\Big[\bar C_{\bar a}{}^{bc}\frac{\partial}{\partial x^b}\frac{\partial}{\partial x^c} + G_{\bar a b}\, x^b x^c \frac{\partial}{\partial x^c} + G_{\bar a b}\, x^b\left(\lambda\partial_\lambda + x^k\frac{\partial}{\partial x^k}\right)\Big] e^{W}$$