INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

SMR.762 - 43

I

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

13 June - 29 July 1994

(0,2) STRING COMPACTIFICATION

J. DISTLER
Dept. of Physics
Princeton University
Princeton, NJ
USA

Please note: These are preliminary notes intended for internal distribution only.

$N=1$ Spacetime SUSY $\Longleftrightarrow$ (0,2) SuperConf. sym on worldsheet
+ integer $U(1)_R$ charges
(so $\exists$ chiral GSO projection)

All our theories will have:

$(\widehat{U(1)} \ltimes \mathrm{Vir}\ ,\ N=2\ \mathrm{sVir})$

$$(c,\bar{c}) = (6+r, 9) \qquad \hat{U}(1)\ \text{level} = r \qquad J(z)\,J(w) = \frac{r}{(z-w)^2}$$

Add

X^μ $\quad \mu = 1,\dots,4$

ψ^μ $\quad$ right-moving Majorana-Weyl Spinors

λ^I $\quad I = 1,\dots,16-2r$ $\quad$ left-moving "

E_8 current algebra (left moving)

$\left.\begin{matrix}\lambda^I \\ E_8\end{matrix}\right\}$ gauge deg's

The $\hat{U}(1)$ plays a dual role:

- GSO projection for left-movers $\quad g = e^{-i\pi J_0}\,(-1)^{F_{\lambda^I}}$
  $\quad (J_0 + \frac{r}{2})$ in left-moving R-sector

- spinfield (left-moving spectral flow generator). Extends $SO(16-2r)$ $\to$ E_6 $\ r=3$; $\to$ $SO(10)$ $\ r=4$; $\to$ $SU(5)$ $\ r=5$
  – creates ground state of left-moving R-sector

Remind:

	E_6 rep	$SO(10)\times U(1)$
$r=3$	78	$\overline{16}_{-3/2} \oplus (45_0 \oplus 1_0) \oplus 16_{3/2}$
	27	$1_{-2} \oplus 16_{-1/2} \oplus 10_1$
	$\overline{27}$	$10_{-1} \oplus \overline{16}_{1/2} \oplus 1_2$

$r=4$	$SO(10)$ rep	$SO(8)\times U(1)$
	45	$8^{s'}_{-2} \oplus (28 \oplus 1_0) \oplus 8^{s'}_{2}$
	16	$8^{s}_{-1} \oplus 8^{v}_{1}$
	10	$1_{-2} \oplus 8^{s'}_{0} \oplus 1_2$

$r=5$	$SU(5)$ rep	$SO(6)\times U(1)$
	24	$\bar{4}_{-5/2} \oplus (5_0 \oplus 1_0) \oplus 4_{5/2}$
	10	$4_{-3/2} \oplus 6_1$
	$\bar{5}$	$\bar{4}_{-1/2} \oplus 1_2$

One realization is $(2,2)$ SUSY, $r=3$, since $(N=2 \text{ SVir})_{c=3r} \supset \hat{U}(1)_r$

Clearly, though, a special case.
Phenomenologically, it is also a somewhat unattractive one. $SO(10)$ or $SU(5)$ grand unified groups give much more attractive phenomenology.

$X : \Sigma \to M$ M Kähler, vanishing 1^{st} Chern class $c_1(T)=0$

(2,2) $$S = \frac{i}{2\pi}\int \tfrac{1}{2} g_{i\bar{j}}(\partial x^i \bar\partial x^{\bar j} + \partial x^{\bar j}\bar\partial x^i) + \tfrac{1}{2} b_{i\bar j}(x)(\partial x^i \bar\partial x^{\bar j} - \partial x^{\bar j}\bar\partial x^i)$$

$$+ i(\psi_{\bar i} D \psi^{\bar i} + \lambda_i \bar D \lambda^i) + R^k{}_{\ell}{}^{\bar i}{}_{\bar j}(x)\lambda_k \lambda^\ell \psi_{\bar i}\psi^{\bar j}$$

where

$$D\psi^{\bar i} = \partial \psi^{\bar i} + \partial x^{\bar j}\Gamma^{\bar i}_{\bar j \bar k}(x)\psi^{\bar k}$$

etc.

(0,2) $$S = \frac{i}{2\pi}\int \ldots i(\ldots + \lambda_a \bar D \lambda^a) + F^a{}_b{}^{\bar i}{}_{\bar j}(x)\lambda_a\lambda^b\psi_{\bar i}\psi^{\bar j}$$

- λ^a transform as sections of a holomorphic vector bundle $V \to M$ with

$$c_1(V) = 0$$
$$c_2(V) = c_2(T)$$

Data: $\underset{\text{Kähler metric}}{g_{i\bar j}}$, $\underset{\text{closed 2-form}}{b_{i\bar j}}$, $\&\ A^a{}_{bi}$

For string theory, interested in conf-inv theory

1-loop β-f$\underline{\text{cn}} = 0 \Rightarrow g_{i\bar j}$ Ricci-flat

corrected at higher orders in pert. th$\underline{y}$

<u>Philosophy</u>

1) Imagine we could construct exact fixed pt. th$\underline{y}$ order by order. But for weak coupling, assume approximate th$y$ given by 1-loop $\beta=0$ is good enough.

2) Accept σ-model (at least what we can write down) is not conf-inv.
RG flow $\rightsquigarrow$ in IR, flow to desired fixed pt.

2^{nd} pt of view is useful. Suggest several helpful ways of thinking:

- RG flow is dissipative $g, b, (\& A)$ represent ∞ # couplings of 2-d th$y$. All but a finite # are marginally-irrelevant.
Fixed pt charaterized by a finite # params, which are RG-invariant

a) complex structure of M

b) holomorphic str. of V

c) cohomology class of (complex) Kahler form

$$J = \frac{i}{2} g_{i\bar{j}}\, dx^i \wedge dx^{\bar{j}}$$
$$B = \frac{1}{2} b_{i\bar{j}}\, dx^i \wedge dx^{\bar{j}}$$
$$\mathcal{J} = B + iJ$$

The first two are automatic, assured by the ~~[illegible]~~ chiral $U(1)_L \times U(1)_R$ symmetry of the model:

$$\psi^{\bar{i}} \to e^{i\theta_R} \psi^{\bar{i}} \qquad \lambda^a \to e^{i\theta_L} \lambda^a$$
$$\psi_{\bar{i}} \to e^{-i\theta_R} \psi_{\bar{i}} \qquad \lambda_a \to e^{-i\theta_L} \lambda_a$$

J.6: Nonanomalous since $c_1(T) = c_1(V) = 0$

The third is quite nontrivial. Proven to all orders in perturbation theory by Alvarez-Gaumé, Ginsparg & Coleman (who showed that all perturbative corrections to $\mathcal{J}$ are exact 2-forms)

Beyond perturbation theory, need to worry about σ-model instantons (topologically-nontrivial maps: $\Sigma \to M$). Hard to see, since corrections to $g_{i\bar{j}}$ generally expected to be instanton-antiinstanton effects.

Use basic principle:

Worldsheet β-f$\underline{\text{cn}}$ eq$\underline{\text{n}}$s $\iff$ spacetime eq$\underline{\text{n}}$s of motion

$\Downarrow$

spacetime SUSY $\Rightarrow$ satisified if $W = \partial W = 0$

(a tadpole for certain vertex operator (BRST cohomology class) $\Rightarrow$ nonzero β-f$\underline{\text{cn}}$ for corresponding operator in worldsheet lagrangian)

Simpler to calculate superpotential, W, an instanton effect.

For (2,2) th$\underline{\text{y}}$s, one can argue $\exists$ no superpotential for the Kahler moduli.

$\Rightarrow$ nonperbatively, $\mathcal{J}$ is RG-INV.

[SKIP discussion]

upshot is ~~that~~ neither argument applies to (0,2).

Since NLσMs are so hard, envoke another great principle of RG:

Universality: Many QFT's renorm to same IR fixed pt.

- Find a simpler class of QFT, in same universality class!

LσMs

Ultimately, we will be interested in (0,2) LσMs, so first discuss (0,2) superfields.

(0,2) superspace has coordinates $(z, \bar{z}, \theta^+, \theta^-)$

Spinor derivatives:

$$\bar{D}_\pm = \frac{\partial}{\partial \theta^\pm} + \theta^\mp \partial_{\bar{z}}$$

Chiral (scalar) superfield Φ satisfies

$$\bar{D}_+ \Phi = 0$$

In components

$$\Phi = \phi + \theta^- \psi + \theta^- \theta^+ \partial_{\bar{z}} \phi$$

A (chiral) Fermi superfield, Λ, satisfies $\bar{D}_+ \Lambda = 0$. Components:

$$\Lambda = \lambda + \theta^- l + \theta^- \theta^+ \partial_{\bar{z}} \lambda$$

where λ is a left-moving fermion & l is auxiliary

(0,2) gauge multiplet

(in Minkowski space, where $\partial_z, \partial_{\bar{z}}$ are really ∂_+, ∂_-)

V lowest component is a scalar. V is Hermitian (where Hermitian conjugation: $\theta^+ \leftrightarrow \theta^-$)

A lowest component is the left-moving component of the gauge field, a. A is anti-Hermi[tian]

Super gauge-inv.

$$V \to V - i(\chi - \bar{\chi}) \quad , A \to A - i(\chi + \bar{\chi})$$

where χ is a chiral superfield, $\bar{D}_+ \chi = \bar{D}_- \bar{\chi} = 0$.

In W.Z. gauge, the [illegible] [illegible] [illegible] ... [illegible] $\partial_{\bar z}\rho$, with ρ real)

$$V = \theta^- \theta^+ \bar{a}$$

$$A = a + \theta^+ \kappa - \theta^- \bar{\kappa} + \theta^- \theta^+ D$$

(so $a, \bar{a}$ are anti-hermitian, $\kappa \leftrightarrow \bar{\kappa}$ under h.c. & D is hermitian) (again, these make sense in Minkowski space)

left & right-moving components of gauge field — left-moving gauginos — auxiliary field

Under a super-gauge transf'n:

$$\Phi \to e^{2iQ\chi}\,\Phi \quad , \quad \bar{\Phi} \to e^{-2iQ\bar{\chi}}\,\bar{\Phi}$$

& similarly for Λ.

• Let $\tilde{\Phi} = e^{QV}\,\Phi \quad , \quad \tilde{\bar{\Phi}} = e^{QV}\,\bar{\Phi}$ (& similarly for Λ)

Gauge-inv. kinetic term for Φ:

$$S_\Phi = \frac{1}{2}\int d^2z\, d^2\theta \;\; (\partial_z - QA)\tilde{\bar{\Phi}}\,\tilde{\Phi} - \tilde{\bar{\Phi}}\,(\partial_z + QA)\tilde{\Phi}$$

$$= \int d^2z\;\; (\partial_z - Qa)\bar{\phi}\,(\partial_{\bar z} - Q\bar{a})\phi + (\partial_{\bar z} - Q\bar{a})\bar{\phi}(\partial_z + Qa)\phi + \bar{\psi}(\partial_z + Qa)\psi$$

$$+ Q\,(\bar{\kappa}\bar{\psi}\phi - \kappa\psi\bar{\phi}) - Q\bar{\phi}\phi D$$

Inv. kinetic term for Λ:

$$S_\Lambda = \frac{1}{2}\int d^2z\, d^2\theta\;\; \tilde{\bar{\Lambda}}\,\tilde{\Lambda} \quad = \int d^2z\;\bar{\lambda}\,(\partial_{\bar z} + Q\bar{a})\lambda - \frac{1}{2}\bar{\ell}\ell$$

Define gauge-covariant spinor derivatives

$$\bar{\mathcal{D}}_\pm = \pm e^{\pm V}\,\bar{D}_\pm\, e^{\mp V} \qquad \mathcal{D} = \partial_z + A$$

Can define gauge field strengths (N.b. $\bar{D}_+ \Upsilon = 0$)

$$\Upsilon = [\mathcal{D}, \bar{\mathcal{D}}_+] = -\kappa + \theta^-(D + f) - \theta^-\theta^+\partial_{\bar z}\kappa$$

$$\bar{\Upsilon} = [\mathcal{D}, \bar{\mathcal{D}}_-] = -\bar{\kappa} + \theta^+(D - f) + \theta^-\theta^+\partial_{\bar z}\bar{\kappa}$$

Gauge Action is

$$S_\Upsilon = -\frac{1}{2e^2}\int d^2z\, d^2\theta\;\; \Upsilon\bar{\Upsilon} \quad = \frac{1}{e^2}\int d^2z\left(\frac{1}{2}f^2 - \frac{1}{2}D^2 + \kappa\,\partial_{\bar z}\bar{\kappa}\right)$$

Fayet-Iliopoulos D-term:

$$S_D = \frac{t}{2}\int d^2z\, d\theta^-\,\Upsilon + \frac{\bar{t}}{2}\int d^2z\, d\theta^+\,\bar{\Upsilon} \quad = r\int d^2z\, D + i\theta\int d^2z\, f$$