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II

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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(0,2) STRING COMPACTIFICATION

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Where $t = r + i\theta$

Finally, a $(0,2)$ superpotential is

$$S_W = \int d^2z\, d\theta^- \,\Lambda F(\Phi) + \int d^2z\, d\theta^+ \,\overline{\bar{\Lambda}\ F(\Phi)}$$

$$= \int d^2z \left(l\, F(\phi) - \lambda \frac{\partial F}{\partial \phi} \psi \right) + h.c.$$

This is all that is required for $(0,2)$, but to describe $(2,2)$, need to "enlarge" the gauge multiplet. Introduce a complex fermionic superfield Σ & its conjugate $\bar{\Sigma}$ (no chiral constraint!)

New Gauge symmetry:

$$\Sigma \to \Sigma + i\Omega \qquad \bar{\Sigma} \to \bar{\Sigma} - i\bar{\Omega}$$

$$\Lambda \to \Lambda + 2iQ\Omega\Phi \qquad \bar{\Lambda} \to \bar{\Lambda} - 2iQ\bar{\Omega}\bar{\Phi}$$

all others invariant.

Σ has 4 indep components. The gauge sym allows us to gauge away 2 of them. The ones that remain ~~are~~ we'll call σ & β. The gauge-invariant

$$\bar{D}_+ \Sigma = \sigma + \theta^- \beta + \theta^- \theta^+ \partial_{\bar{z}} \sigma$$

The action S_Λ is not invariant under Ω-gauge transformations. Add:

$$S_\Sigma = \int d^2z\, d^2\theta\; 4Q^2 \,\tilde{\bar{\Phi}}\, \tilde{\Phi}\, \bar{\Sigma}\Sigma - Q\left(\tilde{\bar{\Lambda}}\, \tilde{\Phi}\, \Sigma - \tilde{\bar{\Phi}}\, \tilde{\Lambda}\, \bar{\Sigma} \right)$$ ← makes S_Λ invariant

$$+ \frac{1}{2e^2}\left(-\bar{D}_- \bar{\Sigma}\, \partial_{\bar{z}} \bar{D}_+ \Sigma + \partial_{\bar{z}} \bar{D}_- \bar{\Sigma}\, \bar{D}_+ \Sigma \right)$$ ← gives Σ a kinetic term

In "W.Z." Gauge,

$$S_\Sigma = \int d^2z\; 4Q^2 |\phi|^2 |\sigma|^2 + Q\left(\bar{\lambda}\psi\sigma - \lambda\bar{\psi}\bar{\sigma} \right) - Q\left(\beta\bar{\lambda}\phi - \bar{\beta}\lambda\bar{\phi} \right)$$

$$+ \frac{1}{2e^2}\left(\partial\bar{\sigma}\,\bar{\partial}\sigma + \partial\sigma\,\bar{\partial}\bar{\sigma} \right) + \frac{1}{e^2}\bar{\beta}\,\partial_z \beta$$

Example 1 $\mathbb{C}P^N$ (2,2) LoM

Φ^i, Λ^i $i=1,\dots,N+1$ all charge $Q=1$

After eliminating auxiliary field,

$$\mathcal{L} = (\text{kinetic terms for } \phi,\psi,\lambda,\alpha,\beta,\sigma) + (\bar\alpha\bar\psi^i\phi^i - \alpha\psi^i\bar\phi^i)$$
$$+(\bar\beta\lambda^i\bar\phi^i - \beta\bar\lambda^i\phi^i) + (\bar\lambda^i\psi^i\sigma - \lambda^i\bar\psi^i\bar\sigma)$$
$$+4\sum_i|\phi^i|^2|\sigma|^2 + \frac{e^2}{2}\Big(\sum_i|\phi^i|^2 - r\Big)^2$$
$$+\frac{1}{2e^2}f^2 + i\theta f$$

Semiclassical analysis:

$r \gg 0$ Scalar potential must vanish $\Rightarrow \begin{cases} \sum_i|\phi^i|^2 = r \\ \sigma = 0 \end{cases}$

sit on big sphere S^{2N+1}

but still have residual gauge inv. $\phi \to e^{i\theta}\phi$, etc, so after modding out, ϕ's live on $\mathbb{C}P^N$

$\left(\mathbb{C}P^N = S^{2N+1}/U(1)\right)$

Fermions: Since ϕ has a VEV, one lin. comb. of ψ's gets a mass with α. Which lin. comb. is it?

Let $\psi^i = \psi\phi^i$

$$-\alpha\psi^i\bar\phi^i = -\alpha\psi|\phi^i|^2 = -r\alpha\psi$$

so it is the particular lin. comb. represented by ψ which becomes massive. The remaining ψ's transform as sections of the tangent bundle to $\mathbb{C}P^N$.

Mathematically,

$$0 \to \mathcal{O} \xrightarrow{\otimes\phi^i} \mathcal{O}(1)^{\oplus N+1} \to T_{\mathbb{C}P^N} \to 0$$

Digression. Line bundles on $\mathbb{C}P^N$

Recall $\mathbb{C}P^N \simeq \mathbb{C}^{N+1}/\sim$, where $(z_0, z_1, \dots, z_N) \simeq (\lambda z_0, \lambda z_1, \dots, \lambda z_N)$, $\lambda \in \mathbb{C}^*$

$\mathcal{O}(-1)$, the "tautological line bundle" has, as fiber of the point $[z_0, \dots, z_N] \in \mathbb{C}P^N$, the complex line through the origin in $\mathbb{C}^{N+1}$ which passes through $(z_0, z_1, \dots, z_N)$

$\mathcal{O}(-n) \simeq \mathcal{O}(-1)^{\otimes n}$, negative powers mean positive powers of the dual line bundle.

. Fly in the ointment:

r is renormalized in this theory. There's a 1-loop log-divergent diagram 
with ϕ's running around the loop. The coefficient is $\sum Q_i$, where Q_i are the charges of the scalars in the model. The sign is such that $r(\mu)$ <u>decreases</u> in the IR. So, even if we start out at large r, where semiclassical is valid, we don't stay there.

But this is exactly the behaviour we <u>expect</u>. The $\mathbb{C}P^N$ NLσM also has a nonzero β-f<u>cn</u>, and flows to strong coupling in the IR.
It develops a mass gap, and low-energy th^y is a $c=0$ CFT (topological field theory)

<u>Ex 2</u> Calabi-Yau Hypersurface in $W\mathbb{P}^4$

Φ^i, Λ^i charges w_i $i = 1, \dots, 5$

P, Γ charge $-d$, where $d = \sum w_i$

(under Ω-gauge transf'ns, $\Gamma \to \Gamma - 2id\Omega P$)

Action as before, but now add gauge-inv. superpotential

$$S_W = \int d^2z\, d\theta^- \left(\Gamma W(\Phi) + \Lambda^i P \frac{\partial W}{\partial \Phi^i} \right) + h.c.$$

Where $W(\Phi)$ is a homogeneous polynomial of (weighted) degree d in the Φ^i.

This is obviously invariant under χ-gauge transformations. Inv. under Ω holds by virtue of

$$\sum_i w_i \Phi^i \frac{\partial W}{\partial \Phi^i} = d\, W(\Phi)$$

Adding the superpotential introduces new terms in the scalar potential, from integrating out auxiliary fields in Λ^i, Γ:

$$\mathcal{U} = \frac{e^2}{2}\left(\sum w_i|\phi_i|^2 - d|p|^2 - r\right)^2 + |W|^2 + |p|^2\left|\frac{\partial W}{\partial \phi_i}\right|^2 + 4|\sigma|^2\left(\sum_i w_i|\phi_i|^2 + d^2|p|^2\right)$$

Also, get some new Yukawa couplings:

$$\mathcal{L} = \cdots - (\gamma\psi^i + \lambda^i\pi)\frac{\partial W}{\partial \phi_i} - \lambda^i\psi^j p \frac{\partial^2 W}{\partial\phi_i\partial\phi_j} + h.c.$$

We assume W such that $W = \frac{\partial W}{\partial \phi_i} = 0 \Rightarrow \forall \phi_i = 0$

Semiclassical analysis

$r \gg 0$ $\quad \sum w_i|\phi^i|^2 = r$, $\quad p = 0$, $\quad \sigma = 0$ $\quad$ and $W(\phi) = 0$ $\Rightarrow$ after modding out by $U(1)$ live on hypersurface $W(\phi) = 0$ in $W\mathbb{P}^4$

fermions: As before, one lin. comb. of ψ^i gets a mass with the gauge fermion. Another lin comb gets a mass from $-\gamma\psi^i\frac{\partial W}{\partial\phi^i}$

Mathematically:

$$0 \rightarrow \mathcal{O} \xrightarrow[\otimes w_i\phi^i]{f} \bigoplus_i \mathcal{O}(w_i) \xrightarrow[\otimes \frac{\partial W}{\partial \phi^i}]{g} \mathcal{O}(d) \rightarrow 0$$

$$T_M = \frac{\ker(g)}{\mathrm{im}(f)}$$

$r \ll 0$ $\quad |p|^2 = \frac{|r|}{d}$, $\phi = 0$, $\sigma = 0$

γ gets a mass with the gauge fermion $\bar{\beta}$ (from $\mathcal{L} = \dots - d\bar{\beta}\gamma\bar{p} + \dots$)
π gets a mass with the gauge fermion α (from $\mathcal{L} = \dots + d\alpha\pi\bar{p} + \dots$)

Low energy theory described by a superpotential

$$\int d^2z\, d\theta^- \quad \overset{c}{\text{const}} \quad \Lambda^i \frac{\partial W}{\partial \Phi^i} \quad + \text{h.c.}$$

This should be recognizable as the superpotential for (2,2) Landau-Ginzburg theory.

Actually, it is an L.G. <u>orbifold</u>. Since p had charge $-d$, giving it a VEV doesn't completely break the gauge symmetry. Gauge transf'<u>n</u>s by d^{th} roots of unity still unbroken: $U(1) \to \mathbb{Z}_d$.

Should still mod out our L.G. thy by $\mathbb{Z}_d$.

<u>Ex 3</u> The superpotential of <u>Ex</u> 2 was not the most general one compatible with Ω-gauge transformations.

More generally,

$$S_W = \int d^2z\, d\theta^- \left(\Gamma\, W(\Phi) + \Lambda^i P\, F_i(\Phi) \right) \quad + \text{h.c.}$$

where

$$(*) \qquad \sum_i w_i \Phi^i F_i(\Phi) = d\, W(\Phi)$$

is invariant under Ω. Generically, this breaks (2,2) SUSY, preserv. (0,2).

E_8, for $W(\Phi)$ quintic polynomial in $\mathbb{C}P^4$, there is a 224 dimensional space of F_i satisfying $(*)$

But rank of V remains $r=3$. So E_6 is unbroken & (one can show) # 27 s & # $\overline{27}$ s is unchanged.

$r \gg 0$: The right-moving fermions ψ^i which remain massless, again, transform as sections of T. But the left-moving fermions transform as sections of V (a holomorphic deformation of T), where

$$0 \longrightarrow \mathcal{O} \xrightarrow[f]{\otimes w_i \phi_i} \bigoplus_i \mathcal{O}(w_i) \xrightarrow[g]{\otimes F_i(\phi)} \mathcal{O}(d) \to 0 \qquad V = \frac{\ker(g)}{\operatorname{im}(f)}$$

Ex 4 Why do we need Σ multiplet & corresponding Ω gauge-inv.?

Say we dispensed with them and considered

$$S_W = \int d^2z\, d\theta^- \left(\Gamma W(\Phi) + \Lambda^i P F_i(\Phi) \right) + h.c.$$

Where now, since we are freed of the Ω-gauge invariance conditions, we can take the $F_i(\phi)$ to be <u>arbitrary</u>.

Recall that previously (for C.Y. phase, $r \gg 0$) there were two mass terms for the left-moving fermions

$$\mathcal{L} = \cdots + w_i \bar{\beta} \lambda^i \bar{\phi}^i - \lambda^i \pi F_i(\phi) + h.c.$$

Now there's only one (since $\bar{\beta}$ is absent from the theory)

So V' is defined by

$$0 \to V' \to \bigoplus_i \mathcal{O}(w_i) \xrightarrow{\otimes F_i(\phi)} \mathcal{O}(d) \to 0$$

V' now has rank $r = 4$.

It turns out that this theory is ill-behaved; we'll see why later.

Ex 5 Since the Λ^i's are now supposed to be unrelated to the Φ's, we shouldn't give them the same label & assume they have the same gauge charges.

So

$$S_W = \int d^2z\, d\theta^- \left(\Gamma W(\Phi) + \Lambda^a P F_a(\Phi) \right)$$

Where now

field	charge
Φ^i	w_i
P	$-m$
Λ^a	n_a
Γ	$-d$

We still require:

$$\left.\begin{array}{l} \sum_i w_i = d \\ \sum_a n_a = m \end{array}\right\} \underset{\text{in semiclassical } r \gg 0 \text{ analysis}}{\Longleftrightarrow} \left\{\begin{array}{l} c_1(T) = 0 \\ c_1(V) = 0 \end{array}\right.$$