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II

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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QUANTUM MECHANICS OF BLACK HOLES

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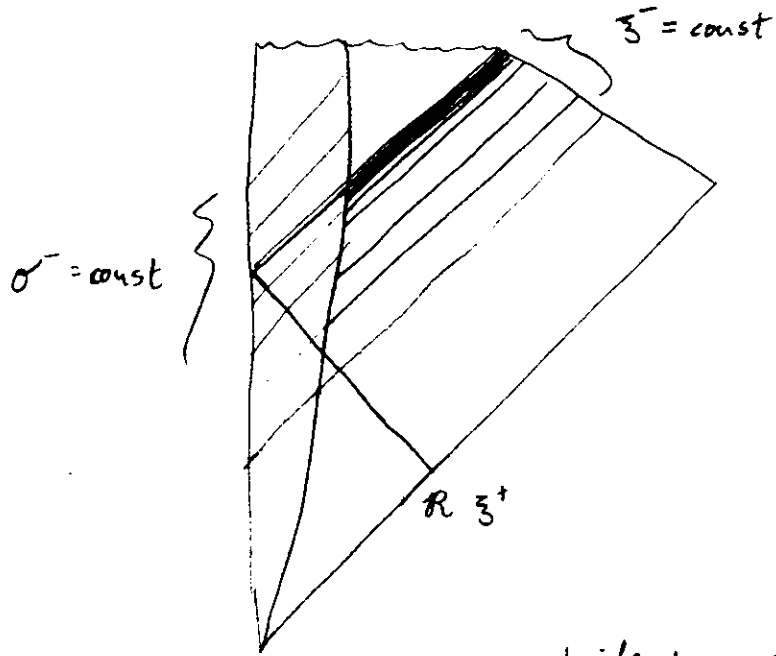
Please note: These are preliminary notes intended for internal distribution only.

Evap in 4d (spherically symm)



$$ds^2 = g_{tt}\,dt^2 + g_{rr}\,dr^2 + R^2(r,t)\,d\Omega_2^2$$

$$= -e^{2\rho}\left(-dx^{0\,2} + dx^{1\,2}\right) + R^2(x)\,d\Omega_2^2$$

outside : no t dep $\Rightarrow$

$$= -g_{tt}\left(-dt^2 + dr_*^2\right) + r^2 d\Omega_2^2$$

w/ $\dfrac{dr^*}{dr} = \sqrt{\dfrac{g_{rr}}{-g_{tt}}}$ "tortoise"

(Schw: $r^* = r + 2M\ln(r-2M)$)

$$S = -\frac{1}{2}\int d^4x\sqrt{-g}\,(\nabla f)^2 \quad ; \qquad f = \frac{u(r,t)}{R}\,Y_{lm}(\theta,\varphi)$$

2d solns

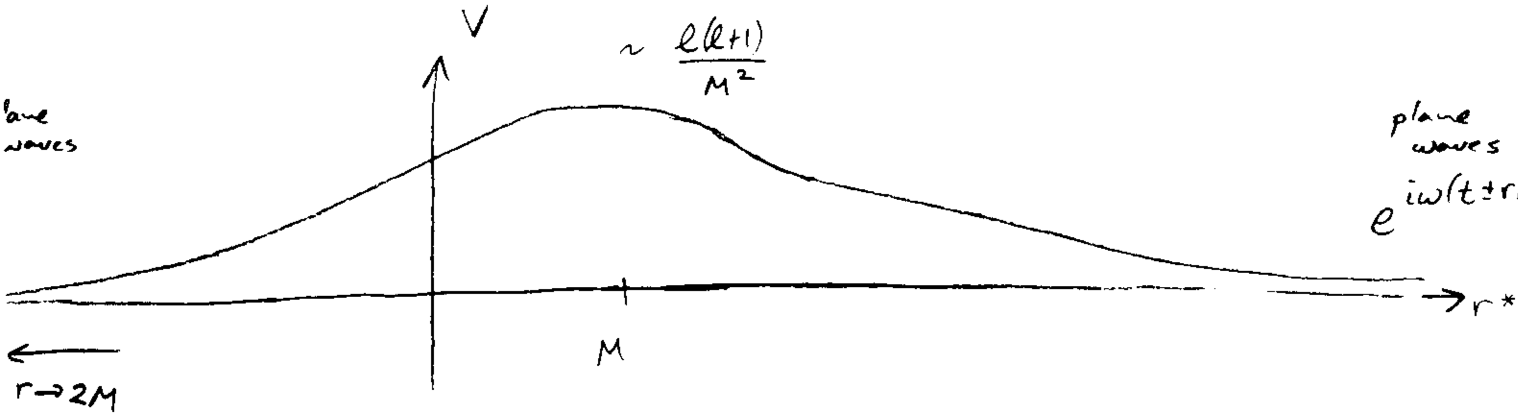
Comparison :

1) Effective pot

eg outside $\int(\nabla f)^2 \propto \int dr^* dt\left[(\partial_t u)^2 - (\partial_{r^*}u)^2 - V(r^*)\,u\right]$

w/ $V(r^*) = -g_{tt}\dfrac{l(l+1)}{r^2} + \dfrac{\partial_{r^*}^2 r}{r}$

$\mapsto 0$
$\sim 1/r^3$



reflection $\rightsquigarrow$ "gray body" factors
(negligable for $\omega^2 \gg \frac{\ell(\ell+1)}{M^2}$)

2) Relation betw modes :

out: $ds^2 = -g_{tt}\, d\xi^+ d\xi^- + \text{ang.}$ $\qquad \xi^\pm = t \pm r^*$

inside $= -\Omega^2(\sigma)\, d\sigma^+ d\sigma^- + \text{ang}$ $\qquad \sigma^\pm = \tau \pm r$

have chosen σ to be well behaved through horizon
[see picture]

relate ξ^+ (in) & ξ^- (out)

1^{st} $\sigma^- \to \xi^-$: (then $\cdot\ \xi^+ \to \sigma^-$)

$$\frac{d\sigma^-}{d\xi^-} = \frac{1}{2}\left(\frac{\partial\sigma^-}{\partial t} - \frac{\partial\sigma^-}{\partial r^*}\right) = \frac{1}{2}\left(\frac{\partial\tau}{\partial t} - \frac{\partial\tau}{\partial r^*} + \frac{\partial r}{\partial r^*}\right)$$

$$0 = \frac{d\sigma^+}{d\xi^-} \propto \frac{\partial\sigma^+}{\partial t} - \frac{\partial\sigma^+}{\partial r^*} = \frac{\partial\tau}{\partial t} - \frac{\partial\tau}{\partial r^*} - \frac{\partial r}{\partial r^*}$$

(near horizon)

$$\Rightarrow \frac{d\sigma^-}{d\xi^-} = \frac{\partial r}{\partial r^*} = \frac{\partial r}{\partial r^*}\left(2m - \frac{\sigma^-}{2}\right) = \underbrace{-\frac{1}{2}\frac{\partial}{\partial r}\left(\frac{\partial r}{\partial r^*}\right)\Big|_{\text{horizon}}}_{\kappa \ \text{surface gr.}} \sigma^- + \cdots$$

so $\sigma^- = -A e^{-\kappa\xi^-}$ ... increasing redshift

$\quad\hookrightarrow$ const.

bounce; then

near $\mathcal{R}$: $\sigma^+ \sim \frac{d\sigma^+}{d\xi^+}\Big|_{\mathcal{R}} \xi^+$

$\quad\downarrow$ const blueshift; modifies A

Remaining argument goes through, w/ $\omega \to \omega/\kappa$

$$\Rightarrow \boxed{T = \frac{\kappa}{2\pi}}$$

Schw: $T = \frac{1}{8\pi M}$ (ex)

State similar; entropy $dS = \frac{dE}{T} = 8\pi M\,dM \Rightarrow S = 4\pi M^2 =$

Bekenstein-Hawking

Flux estimate:

transmission

$$\frac{dM}{dt} = \sum_{\ell} (2\ell+1) \int \frac{d\omega}{2\pi}\,\omega \frac{\Gamma_{\omega,\ell}}{e^{8\pi M\omega} - 1}$$

energy density

$$\Gamma_{\omega,\ell} \sim \Theta(a\,\omega M - \ell)$$

scale out $M \rightarrow \quad \frac{dM}{dt} \propto \frac{1}{M^2}$

Lifetime $\quad \tau \sim \left(\frac{M}{m_{P\ell}}\right)^3 t_{P\ell}$

$\tau \sim T_{universe}$ for $M \sim 10^{-18} M_\odot$

Correlations

1) Missing info due to corr. between modes inside & outside horizon

2) Calc. refers to ultrahigh frequencies, but just relies on properties of vacuum.

Backreaction (semiclassical)

attempts at full quantum - see Hermann

Want to understand effect of HR on BH.

Recall:

$$ds^2 = \quad = e^{2\rho} d\sigma^+ d\sigma^- = d\xi^+ d\xi^-$$

w/ $$e^{-2\rho} = \frac{d\sigma^+}{d\xi^+}\frac{d\sigma^-}{d\xi^-}$$

$$\leadsto \quad :T_{--}:_\xi = :T_{--}:_\sigma - \frac{1}{24}\left(\frac{\sigma^{-'''}}{\sigma^{-'}} - \frac{3}{2}\frac{\sigma^{-''}}{\sigma^{-'}}\right)^2$$

$$= :T_{--}:_\sigma + \underbrace{\frac{1}{12}\left[\partial_-^2\rho - (\partial_-\rho)^2\right]}_{t_{--}}$$

True, general ρ.

Suppose $:T_{+-}:_\sigma = 0$;

$$0 = \nabla_+ T_{--} + \nabla_- T_{+-} \quad \leadsto$$

$$0 = \frac{1}{12}\partial_+\left[\partial_-^2\rho - (\partial_-\rho)^2\right] + \partial_- t_{+-} - \Gamma^-_{--} t_{+-}$$

$$\Gamma^-_{--} = g^{+-}\Gamma_{--,+} = g^{+-} g_{+-,-} = \partial_- \log g_{+-} = 2\partial_-\rho$$

$$\Rightarrow \qquad t_{+-} = -\frac{1}{12}\partial_+\partial_-\rho$$

$$\boxed{\begin{array}{l} :T_{+-}:_\xi = -\frac{1}{12}\partial_+\partial_-\rho \quad \propto R \\ \text{conformal anomaly} \end{array}}$$

$$-\frac{1}{12}\partial_+\partial_-\rho = \langle :T_{+-}: \rangle = -\frac{1}{i}\frac{2\pi}{\sqrt{g}}\frac{\delta}{\delta g^{+-}} \ln\left[\int \mathcal{D}f \quad e^{-\frac{i}{4\pi}\int(\nabla f)^2}\right]$$

$$= +\frac{\pi}{4i}\frac{\delta}{\delta\rho}\, i S_{eff}$$

$$\Rightarrow \qquad S_{eff} = -\frac{1}{6\pi}\int d^2\sigma\, \rho\,\partial_+\partial_-\rho$$

$$= -\frac{1}{24\pi}\int d^2x\,\sqrt{-g}\;\rho\,\Box\rho \quad = -\frac{1}{96\pi}\int d^2x\sqrt{-g}\,d^2x'\sqrt{-g'}\; R(x)\Box^{-1}R$$

$$R = -2\Box\rho \qquad\qquad \equiv S_{PL}$$

... general metric

To quantize: (reinstate $\hbar$)

$$\int \mathcal{D}g\,\mathcal{D}\phi\; e^{\frac{i}{\hbar}S_{grav}} \int \mathcal{D}f\; e^{\frac{i}{\hbar}S_f} \quad (\;\cdots\;)$$

e.g. classical sources

$$= \underbrace{\int \mathcal{D}g\,\mathcal{D}\phi \quad e^{\frac{i}{\hbar}S_{grav} + \frac{i}{\hbar}S^f_{cl}} \quad \underbrace{\int \mathcal{D}f\; e^{\frac{i}{\hbar}S_f}}_{e^{\,i S_{PL}}}}_{\mathcal{O}(1)\text{ effects.}}$$

backreaction

f_i ... N matter fields $\rightsquigarrow$

$$\int \mathcal{D}g\,\mathcal{D}\phi \quad e^{\frac{i}{\hbar} S_{grav} + \frac{i}{\hbar} S_f^{cl} + \frac{i}{\hbar}(N\hbar) S_{PL}}$$

Large N, semiclassical : $\hbar \to 0$, $N\hbar$ fixed :

[in practice : $S_{cl} \to$ large]

$$G_{++} = T_{++}^{cl} + \frac{N}{12}\left[\partial_+^2\rho - (\partial_+\rho)^2\right]$$

$$G_{--} = \quad \cdots$$

$$-e^{-2\phi}\left(2\partial_+\partial_-\phi - 4\partial_+\phi\partial_-\phi - \lambda^2 e^{2\rho}\right) \equiv G_{+-} = -\frac{N}{12}\partial_+\rho\partial_-\rho$$

numerical solns ...

RST model

Must add counterterms to quantize.

$\exists$ choices that make semiclass eqns <u>soluble</u>

(also Bilal & Callan, de Alwis)

RST : keep $\partial_\mu(\rho - \varphi)$ conserved :

$g = e^{2\rho}\eta \quad \rightsquigarrow$

$$S_{sc} = \frac{1}{2\pi}\int d^2x \left\{ e^{-2\phi}\left[-2\Box\rho + 4(\nabla\phi)^2 + 4e^{2\rho} \right] - \frac{N\hbar}{12}\,\rho\,\Box\rho \right\}$$

$$= \frac{1}{2\pi}\int d^2x \left\{ 2\,\nabla(\rho-\varphi)\cdot\nabla e^{-2\phi} + 4e^{2(\rho-\varphi)} + \frac{N\hbar}{12}\nabla\rho\cdot\nabla\rho \right\}$$

$$-\frac{N\hbar}{12}\nabla\varphi\cdot\nabla\rho$$

add $\hbar S_{AST} = -\frac{\hbar N}{48\pi}\int d^2x\,\sqrt{-g}\,\phi R$ $\longrightarrow$

$$= \frac{1}{2\pi}\int d^2x \left\{ 2\,\nabla(\rho-\varphi)\,\nabla\Big(e^{-2\phi} + \underbrace{\frac{N\hbar}{24}}_{\kappa/2}\rho\Big) + 4e^{2(\rho-\varphi)} \right\}$$

Soln:

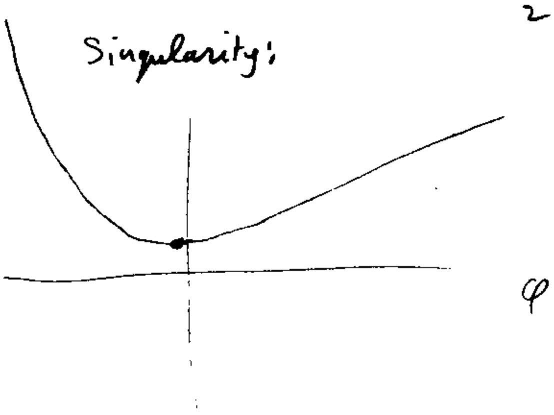
$$\rho-\varphi = \tfrac{1}{2}(\sigma^+-\sigma^-)$$

$$e^{-2\phi} + \frac{\kappa}{2}\rho = M + e^{\sigma^+}(e^{-\sigma^-}-\Delta)$$

i.e. explicit soln for evaporating black hole.

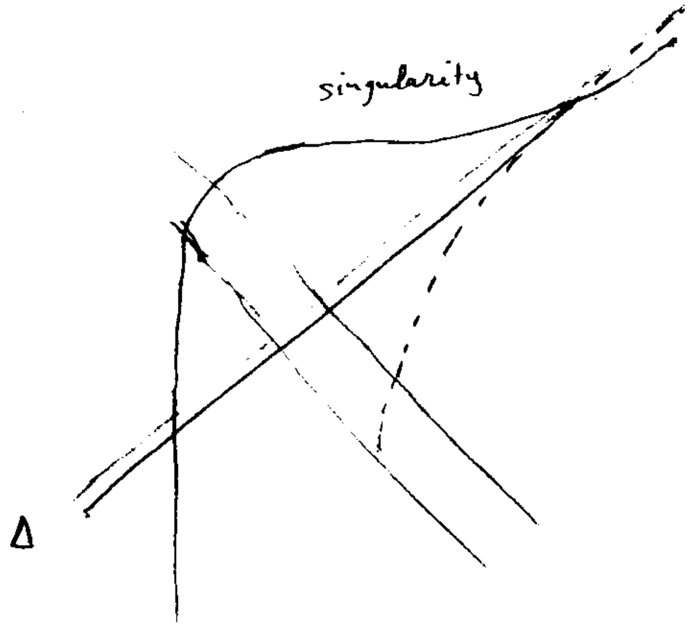
$$\rightsquigarrow \quad e^{-2\phi} + \frac{\kappa}{2}\varphi = M + e^{\sigma^+}(e^{-\sigma^-}-\Delta) - \kappa(\sigma^+-\sigma^-)$$

Singularity:



$$\rightsquigarrow \quad \phi_{cr} = -\tfrac{1}{2}\ln(\kappa/4)$$

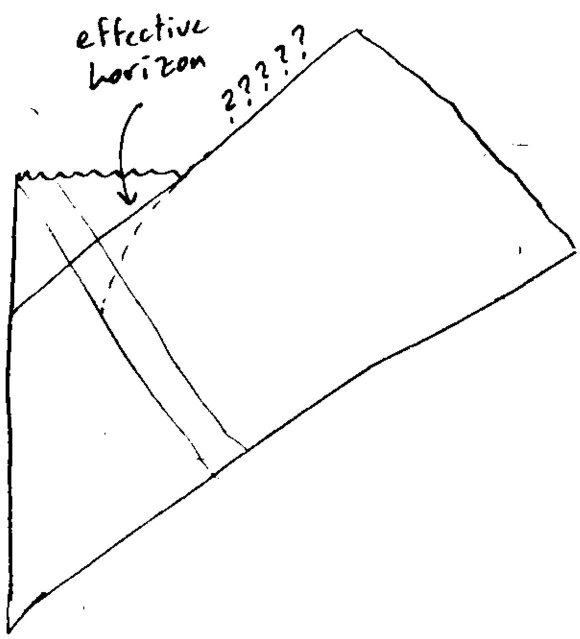
(Quantum effects important)



Apparent horizon: $\partial_+ \phi = 0$ (past such line inevitably go to stronger coupling):

$$e^{-\sigma^-} = \Delta + \kappa e^{-\sigma^+}$$

Crossing: $$e^{-\sigma^-_{AS}} = \frac{+\Delta}{1 - e^{-4M/\kappa}}$$



semiclassical breakdown

HR: $\sigma^+ \to \infty$, $ds^2 \to \dfrac{-d\sigma^+ d\sigma^-}{1-\Delta e^{\sigma^-}}$ as before

$\langle :T_{--}: \rangle$ as before, $\xrightarrow[\xi^- \to \text{large}]{} \dfrac{1}{48}$

check: (ex) $\displaystyle\int_{-\infty}^{\xi^-_{NS}} d\xi^- \langle :T_{--}: \rangle = M - \underbrace{\frac{\text{const}}{\kappa}}_{\sim M_{Pl}}$

Missing corr., thermal; $dE = T\,dS \rightsquigarrow S \propto M$,