



INTERNATIONAL ATOMIC ENERGY AGENCY  
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

**INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS**

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SMR762 - 49



III

**SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY**

**13 June - 29 July 1994**

**(0.2) STRING COMPACTIFICATION**

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Please note: These are preliminary notes intended for internal distribution only.

But now, since we have a theory with chiral fermions coupled to gauge field, need to worry about gauge anomaly

$$\text{bubble} = \underbrace{\sum n_a^2 + d^2}_{\text{left-movers}} - \underbrace{\sum w_i^2 - m^2}_{\text{right-movers}} \equiv 0$$

rearrange

$$\frac{1}{2}(-m^2 + \sum n_a^2) = \frac{1}{2}(-d^2 + \sum w_i^2)$$

or, in c.v. phase,

$$c_2(V) = c_2(T) \quad ! \quad \text{yow!}$$

General principle: The condition  $c_2(V) = c_2(T) \iff$  cancellation of worldsheet gauge anomalies.

Ex 6

No need to restrict to hypersurfaces. Complete intersection Calabi-Yau's are just as easy  $\Rightarrow$  several  $\Gamma^k$

$$S_W = \int d^2z d\theta^- \Gamma^k W_k(\Phi) + \Lambda^a P F_a(\Phi) + \text{h.c.}$$

field	charge
$\Phi^i$	$w_i$
$P$	$-m$
$\Lambda^a$	$n_a$
$\Gamma^k$	$-d_k$

As before

$$-\sum d_k + \sum w_i = 0 \iff c_1(T) = 0$$

$$-m + \sum n_a = 0 \iff c_1(V) = 0$$

absence of gauge anomaly

$$\sum d_k^2 - \sum w_i^2 = m^2 - \sum n_a^2 \iff c_2(T) = c_2(V)$$

N.b., in this, and Ex 5,  $\sum$  scalar charges  $\neq 0$  in general.

Easy to fix: Add:

field	charge
$S$	$m - \sum d_k$
$\Xi$	$-m + \sum d_k$

$$S_W = \int d^2z d\theta^- \dots + \Xi S + \text{h.c.}$$

Setting  $\mathcal{U} = 0 \Rightarrow S = 0$   $\&$  all deg's of freedom associated to  $\Xi$   $\&$   $S$  are massive. They serve only to cancel the short-distance divergence in the one-loop correction to  $\Gamma$ .

Could go on  $\&$  discuss more examples, but you get the idea...

## Remarks on Renormalization of LOM's

- Theory is almost super-renormalizable. The only renormalizable coupling is  $t$ .  
But, unlike in NLOMs,  $t$  is coefficient of an F-term,  $\frac{t}{2} \int d^2z d\theta^2 \mathcal{F}$   
So its renorm is tightly-constrained
- Beyond 1-loop (which we have already discussed) there are no further corrections to  $t$  to all orders in perturbation theory!
- Nonperturbatively,  $t$  should only be corrected by instantons (holomorphic) and in favourable circumstances, one can show this vanishes.

A real discussion would get kinda technical. Instead, let's discuss...

## Extracting some Physics

Calculate RG-invariant quantities

Rich class come considering "twisted" model.

Compute on cylinder with all right-movers periodic

Unbroken SUSY  $\bar{Q}^+$ ,  $\{\bar{Q}^+, \bar{Q}^-\} = \bar{L}_0$  ← in Minkowski

If we are interested in states with  $\bar{L}_0 = 0$ , we can represent these as elements of the  $\bar{Q}^+$ -cohomology.

This is eminently computable, so we obtain

- spectrum of states in right-moving R-sector (spacetime fermions) with  $\bar{L}_0 = 0$

Similarly, we can compute matrix elements of  $\bar{Q}^+$ -inv operators between these fermion states:

- Yukawa couplings (spacetime superpotential!)

## Landau-Ginzburg

As we saw, semiclassical analysis becomes exact for  $r \rightarrow \pm\infty$ .

$r \rightarrow +\infty$  weakly-coupled  $\sigma$ -model

$r \rightarrow -\infty$  L-G.

In both cases, instanton corrections to LOM suppressed. We will look at latter.

To describe Low Energy th, need, in particular, two unbroken nonanomalous  $U(1)$  sym. One will be the  $U(1)_R$  (right-moving  $N=2$ ). The other will become the left-moving  $U(1)$ .

$q$        $\bar{q}$  ← the R-symmetry

$\Phi^i$	$q_i$	$q_i$	$i=1, \dots, 3+n$	$q_i = \frac{w_i}{m}$	$S_W = \int d^2z d\theta^- (F^{\alpha\beta} W_\alpha(\Phi) + \Lambda^q F_a(\bar{\Phi}))$
$\Lambda^a$	$q_a - 1$	$q_a$	$a=1, \dots, r+1$	$q_a = \frac{n_a}{m}$	
$\Gamma^\alpha$	$q_\alpha - 1$	$q_\alpha$	$\alpha=1, \dots, n$	$q_\alpha = 1 - \frac{d_\alpha}{m}$	

We need to check

- 1) These are nonanomalous under the gauge sym. of the underlying theory 
- 2)  $J \cdot \bar{J}$  anomaly diagram  vanishes, so these symmetries can be candidates for commuting L & R currents in conformal limit.

Both of these follow from the linear & quadratic conditions which we previously imposed on the gauge charges. Translated into our current notation, these are

$$\sum (q_a - 1) = -\sum q_i$$

$$(*) \quad \sum q_a = 1$$

$$\sum (q_a - 1)^2 + \sum q_a^2 = 1 + \sum q_i^2$$

Now we can calculate the central charge of the IR  $N=2$  superconformal algebra

Recall the OPE

$$\bar{J}(z) \bar{J}(w) = \frac{\bar{c}/3}{(\bar{z}-\bar{w})^2}$$

in the  $N=2$  algebra measures the central charge. But this is also simply a measure of the  $\bar{J} \cdot \bar{J}$  anomaly, which we can again compute with a 1-loop diagram 

$$\frac{\bar{c}}{3} = \sum (q_i - 1)^2 - \sum q_a^2 - \sum q_\alpha^2 = 3$$

Exactly as expected! Yow!

Similarly, the  $J \cdot J$  anomaly computes the level of the left-moving  $U(1)$  current algebra

$$r = \sum (q_a - 1)^2 + \sum (q_\alpha - 1)^2 - \sum q_i^2$$

But things are even better than this. The operators

$$T' = -\sum (\partial\phi_i \partial\bar{\phi}_i + \frac{q_i}{2} \partial(\phi_i \partial\bar{\phi}_i)) + \sum (\lambda_a \partial\bar{\lambda}_a - \frac{1-q_a}{2} \partial(\lambda_a \bar{\lambda}_a)) + \sum (\gamma_\alpha \partial\bar{\gamma}_\alpha - \frac{1-q_\alpha}{2} \partial(\gamma_\alpha \bar{\gamma}_\alpha))$$

$$J' = -\sum q_i \phi_i \partial\bar{\phi}_i + \sum (1-q_a) \lambda_a \bar{\lambda}_a + \sum (1-q_\alpha) \gamma_\alpha \bar{\gamma}_\alpha$$

commute with  $\bar{Q}_+$  & generate a  $\widehat{U(1)}$  & Virasoro algebra on the  $\bar{Q}_+$  cohomology.

To see this, note that rescaling the superpotential can be absorbed in a redefinition of the kinetic terms, a  $\bar{Q}_+$ -trivial operation. Thus, while working on the level of the  $\bar{Q}_+$ -cohomology, one can use free-field OPE's. And it is easy to verify that these operators satisfy the correct algebra & commute with  $\bar{Q}_+$  using free-field OPE's.

The central charge &  $\widehat{U(1)}$  level are, as expected,  $6+r$  &  $r$ , respectively.

Also,  $J_0$  differs from the previously-defined  $q$  by  $\bar{Q}_+$ -trivial terms, as does  $L'_0$  from the canonical  $L_0$ .

These operators are actually specializations of operators which exist in the linear  $\sigma$ -model.

Simply replace  $\tilde{\tau} \rightarrow \mathbb{D}$  & add a term involving the gauge fields.

Silverstein & Witten verified that the operators in the L-M obey the correct algebra (modulo  $\bar{Q}_+$ -trivial terms) provided (\*) is satisfied.

### GSO projection

$$g = e^{-i\pi\tilde{g}} (-1)^{F_L}$$

Since we are projecting, we must as in usual orbifold constructions add twisted sectors.

As all charges are multiples of  $1/m$ , this is a  $\mathbb{Z}_{2m}$  orbifold.

(Alternatively, we can think of first projecting onto integer  $U(1)$  charges using  $e^{2\pi i g}$  & then projecting with  $g$ . Either way, the result is the same.)

$\mathbb{Z}_{2m}$  orbifold: sectors labelled by  $k = 0, 1, \dots, 2m-1$

$k$  even  $\Leftrightarrow$  left-moving R sector

$k$  odd  $\Leftrightarrow$  left-moving N's sector.

We will see how to compute spectrum of states with  $L_0 = 0$  in right-moving R sector (spacetime fermions)

### Modular Invariance

Level-matching (which is certainly satisfied in our construction) is not obviously sufficient here.  
(unlike usual toroidal orbifolds)

Principle: worldsheet anomalies  $\Leftrightarrow$  spacetime anomalies

Can check whether the  $\mathbb{Z}_{2m}$  quantum symmetry of the L-G orbifold (a discrete R-symmetry in spacetime) is anomalous in spacetime.

The quantum symmetry had better be anomaly-free!

In fact, this holds for all  $(2,2)$  theories and for most  $(0,2)$  theories, but is violated for some  $(0,2)$  theories which should be rejected (presumably violate modular invariance)

Details are in hep-th/9406091.

We define the generator of the quantum symmetry by composing it with a gauge transformation (in the  $U(1) \subset G$ , where  $G = E_6$  ( $r=3$ ),  $SO(10)$  ( $r=4$ ) or  $SU(5)$  ( $r=5$ )) to make it commute with the left-spectral flow (so that reps of  $G$  transform homogeneously)

$$A_1 = \text{[Diagram: Triangle with wavy lines and a vertex labeled } r_k=2g \text{]} \quad G\text{-gauge bosons} \quad \text{mod } 2\pi r$$

$$A_2 = \text{[Diagram: Triangle with wavy lines]} \quad E_8 \text{ gauge bosons} \quad \text{mod } 2\pi r$$

$$A_3 = \text{[Diagram: Triangle with wavy lines]} \quad \text{gravitons} \quad \text{mod } 2\pi r$$

Each of these has expressions in terms of traces of degrees of freedom in right-moving R-sector.

You might think we should demand that each of these anomalies vanish separately. But this is false, and would fail even for well-known  $(2,2)$  theories (eg, the 3-generation Gepner model). All we should demand is that these should vanish for combinations of gauge & gravitational instantons which satisfy  $\text{tr } R^2 = \text{tr } F_1^2 + \text{tr } F_2^2$

Or

$$\left. \begin{aligned} A_1 - A_2 &= 0 \\ A_3 - 24A_2 &= 0 \\ A_3 - 24A_1 &= 0 \end{aligned} \right\} \text{mod } 2\pi r$$

A more direct worldsheet interpretation of this anomaly would be desirable (hint, hint).

### Constructing the spectrum

Basic idea: rescaling L.G. superpotential is a  $\bar{Q}_+$ -trivial operation (since can be compensated by rescaling D-terms), so we can scale its coefficient to be as small as we want & use free-field OPE's

So we have a fairly standard orbifold construction: we have free fields with twisted b.c.'s.  
 We build up Fock spaces using the (fractionally-moded) oscillators acting on the twisted sector ground states.  
 Then we need to calculate the  $\bar{Q}_+$  -cohomology on this Fock space.

$$\bar{Q}_+ = \bar{Q}_{+R} + \bar{Q}_{+L}$$

$$\bar{Q}_{+R} = \oint (\psi_i \bar{\psi}_i - \bar{\psi}_i \psi_i)$$

$$\bar{Q}_{+L} = \oint (\gamma^* W_\alpha(\phi) + \lambda^* F_\alpha(\phi))$$

Spectral sequence (or above perturbation argument)

$$\Rightarrow H_{\bar{Q}_+}^* = H_{\bar{Q}_{+L}}^* (H_{\bar{Q}_{+R}}^*)$$

- But, since  $\bar{Q}_{+R}$  is quadratic, its cohomology is easy to compute its cohomology:

$$H_{\bar{Q}_{+R}}^* = \text{indep of } \psi_i, \bar{\psi}_i \text{ oscillators}$$

‡ (in untwisted sector) holomorphic in zero mode of  $\phi_i$

So one just needs to compute  $H_{\bar{Q}_{+L}}^*$  on this smaller Fock space.

Straightforward...

Much more to say. The best I can do is refer you to:

J. D & S. Kachru, Nucl. Phys. B413 (1994) 213, hep-th/9309110

J. D & S. Kachru, hep-th/9406091

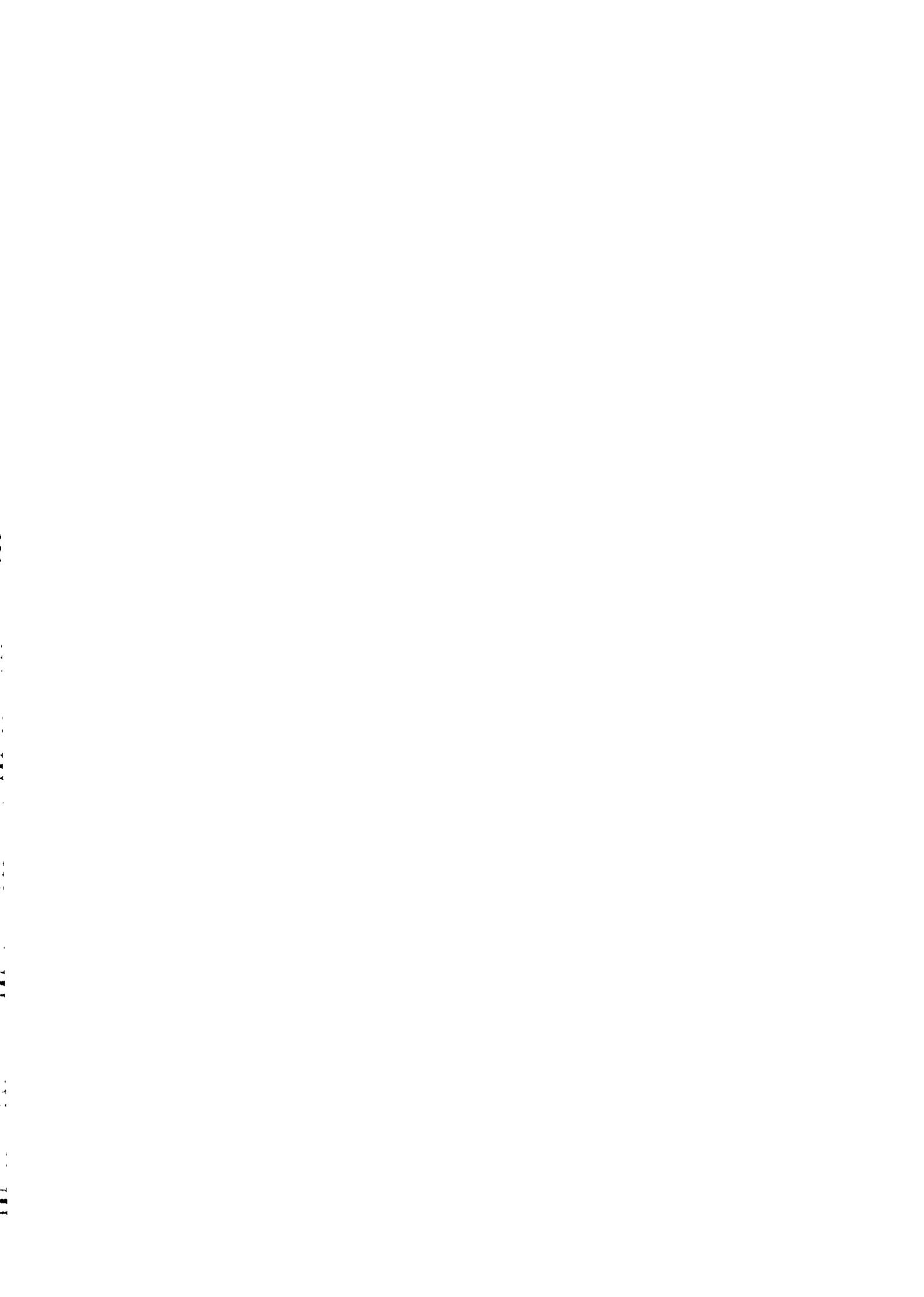
J. D & S. Kachru, hep-th/9406090

S. Kachru & E. Witten, Nucl. Phys. B407 (1993) 637, hep-th/9307038

E. Silverstein & E. Witten, hep-th/9403054

E. Witten, Nucl. Phys. B403 (1993) 159, hep-th/9301042

and references therein.









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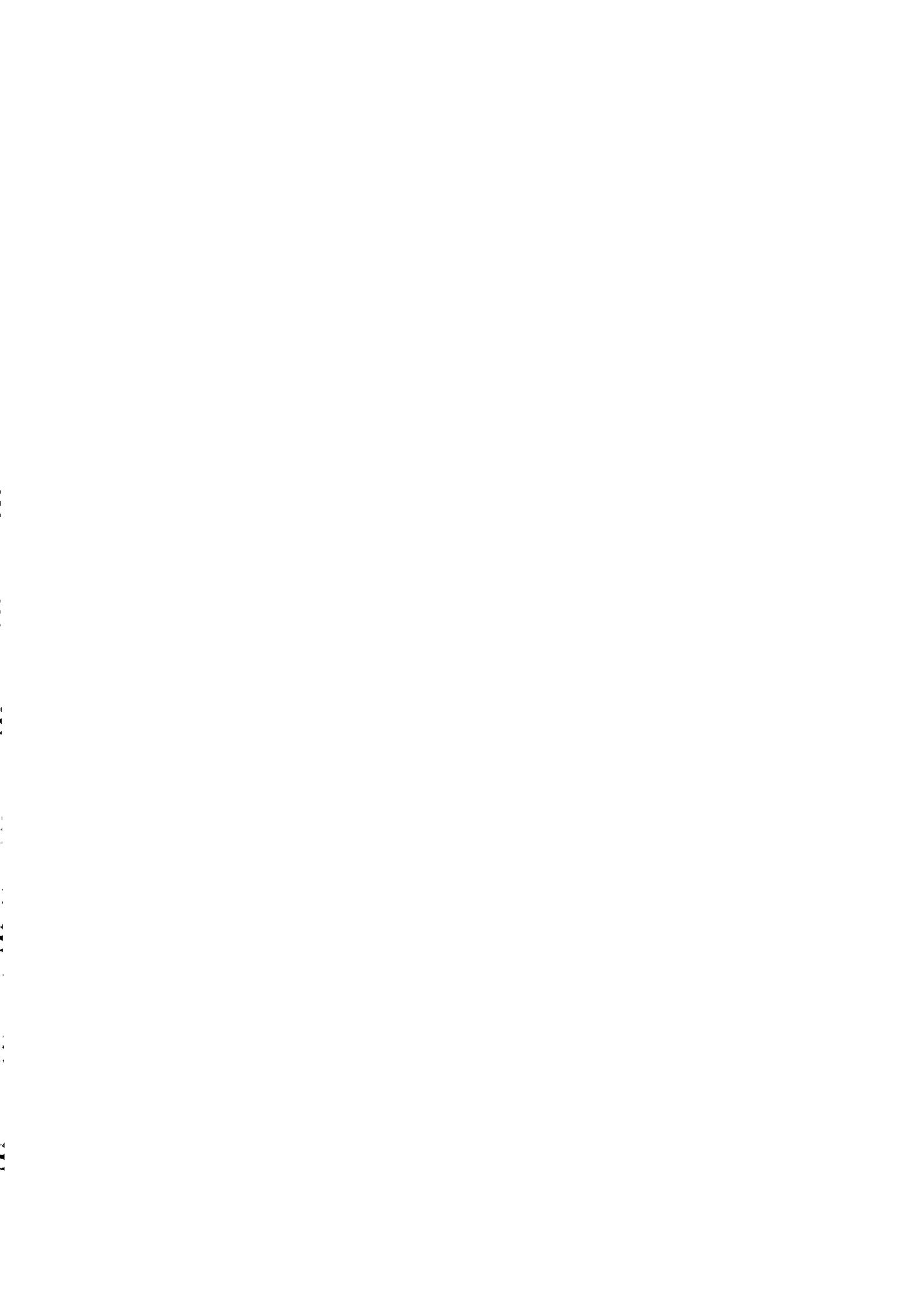
**SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY**

**13 June - 29 July 1994**

QUANTUM MECHANICS OF BLACK HOLES

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Please note: These are preliminary notes intended for internal distribution only.



Info problem

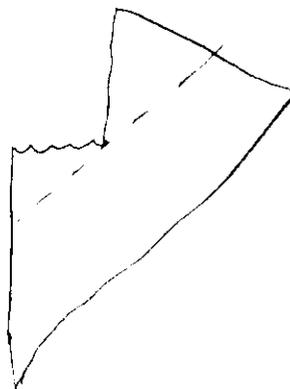
what happens to info?

1. Lost (Hawking ...)

conservative

but radical;

- QM breaks down



2. Returned (in HR) ('t Hooft, Verlinde, Susskind)

- mistake? (unlikely?)

- violate locality/causality?

Return after  $M \sim m_{pl}$ :

$$4d: \quad S \sim M^2, \quad E \sim m_{pl}$$

state @  $\infty$ :

$$\text{soft } \gamma: \quad 1 \text{ bit / photon} \Rightarrow M^2 \text{ photons}$$

$$E_\gamma \sim \frac{1}{M^2}$$

$$\text{decay time: } 1 \text{ photon } \Gamma_\gamma \sim \frac{1}{E_\gamma} \sim M^2$$

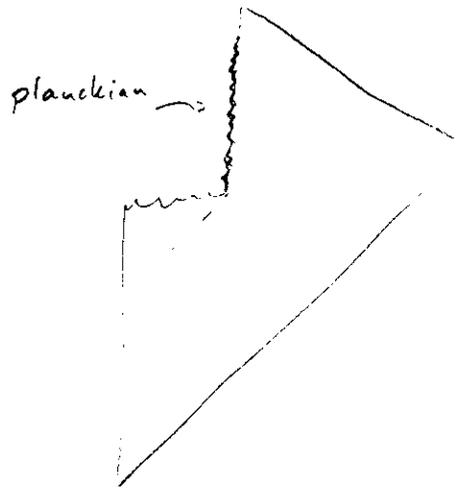
$$\Rightarrow \tau \sim \left( \frac{M}{m_{pl}} \right)^4 t_{pl}$$

$$\tau = T_{\text{univ}} \\ \left[ \rightarrow (10\text{m})^3 \text{ of } \rho = 10\text{H}_2\text{O} \right]$$

$$\sim T_{\text{univ}} \text{ for } M = M_{\text{building}}$$

$\Rightarrow$

3. Remnants — stable or long lived (Banks?, SG, ...)

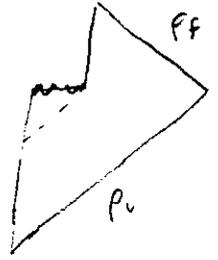


(in Revsach blot !)

Each stridently advocated

Each seems to violate sacred low energy principles

1. Info loss - violates qm  
 also energy conservation  
 (we're very attached to this!)  
 information  $\leftrightarrow$  energy



want Effective description of BH evap:  $\rho_i \rightarrow \rho_f$

useful model: Two Hilbert spaces,  $\mathcal{H}_o$ ,  $\mathcal{H}_h$  - exchange info

$$H = H_o + H_i + H_h$$

$\downarrow$   
 interaction

BH:  $\mathcal{H}_o = \text{exterior}$   
 $\mathcal{H}_i = \text{interior}$

(other types of info loss possible, but this = good enough for BH?)

Note: info loss repeatable:

$$n \text{ experiments, } \Delta t \geq \tau_{\text{evap}} \Rightarrow \Delta S_n = n \Delta S_o$$

$\leadsto$  energy violations, expect  $\Delta E \sim 1/\tau_{\text{evap}}$ .

reason: assume not, ie  $H_h = 0$

$$\text{eg } \mathcal{H}_h = \{ |x\rangle \}$$

$$H_i = \sigma \hat{x}$$

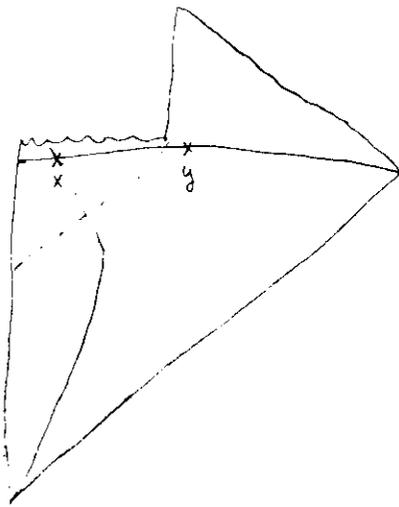
$\downarrow$   
 ext.



Indeed: Hawking:

$$\rho \rightarrow \mathcal{S}\rho\mathcal{S}^\dagger$$

$\downarrow$   
 linear,  
 ...

BPS: violates  $E$  conserv.these arguments:  $\sim$  thermal bath,  $T = \infty!$ 2. Info return - violates locality/causality

mistake?

$$\frac{dE}{dt} \sim N$$

also want  $\frac{dI}{dt} \sim N$  after  $u \sim \frac{M}{2}$ 

(Page)

no obvious evidence?

also if state not Hawking state,  
trace back, difficult to see something  
that looks like vac.

$$1) [\theta(x), \theta(y)] = 0 \Rightarrow \psi = \sum_{\alpha} |\psi_{\alpha}\rangle_{in} |\psi_{\alpha}\rangle_{out}$$

2) Infalling guy carries info, no trouble @ horizon  
 $\Rightarrow$  non-trivial correlations

$$\Rightarrow \Delta I \neq 0, \sim M^2$$

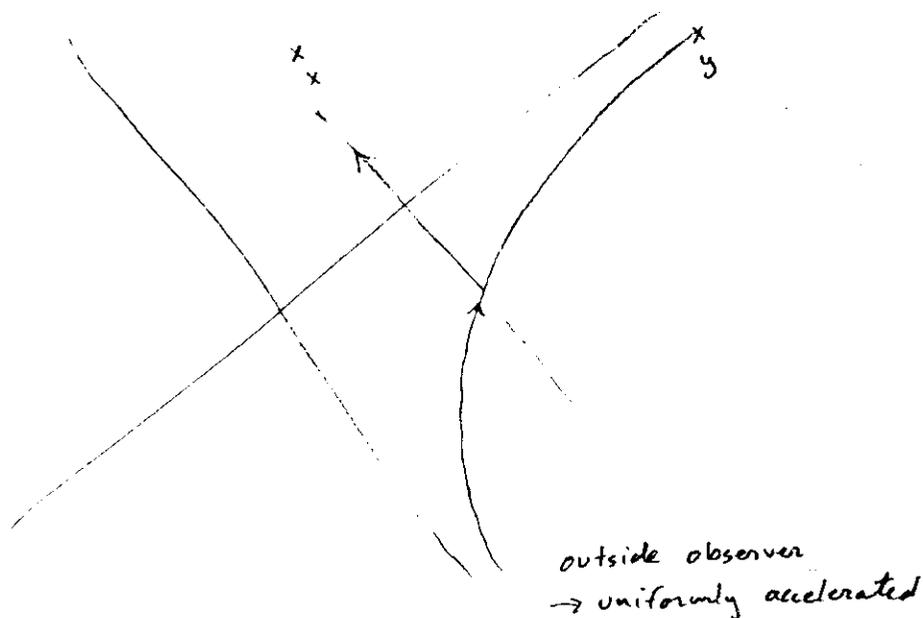
defining observables notoriously difficult in QG

Info: 6

(cf Hermann)

maybe nonlocal?

extreme case:  $M \rightarrow \infty$  : Rindler space



all info in left half of universe is actually on or to R of horizon;  $[O(x), O(y)] \neq 0$ .

't Hooft }  
Susskind } new physics responsible

't Hooft : cellular automata (can't assess)

Susskind : strings - are nonlocal  
(non-perturbative)

String theory:

$R_1$

$R_2$

Info. 7

$$[\theta(R_1), \theta(R_2)] \sim e^{-|R_1 - R_2|^2 / l_{st}^2}$$

ordinary measurement

$$\sim 1$$

high energy measurement  
(Lowe, Susskind, Uglum)

⇒ maybe all info is in state on R half of world!

Problem: transfer to HR? (no demonstrated mechanism)

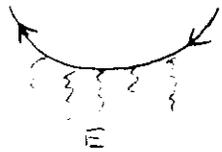
(indeed, could stay @ horizon → remnant!)

### 3. Remnants violate naive crossing

Eg. if charged



→



: Schwinger production

$$\Gamma_{vac} \propto N e^{-\frac{\pi m^2}{qE}} = \infty$$

$\downarrow$   
 $\infty$

likewise:

Hawking:

$$\frac{dM}{dt} \propto N \cdot \frac{1}{M^2} = \infty$$

$\downarrow$   
 $\infty$

(is self consistent)

Thermal:

$$n \sim N \cdot e^{-m/T} = \infty$$

(can't really make!)

Summary:

proposal	violates
Info loss	energy conservation
Info return	locality/causality
Remnants	naïve crossing

each objection phrased in terms of low E physics:

~> Information "paradox"

Relevance of planck scale ~> problem

(Hermann;  
next time)