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SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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# MONOPOLE STORY

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# MONOPOLE STORY

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THE MOST POWERFUL METHOD OF ADVANCE THAT CAN BE SUGGESTED AT PRESENT IS TO EMPLOY ALL THE RESOURCES OF PURE MATHEMATICS IN ATTEMPTS TO PERFECT AND GENERALISE THE MATHEMATICAL FORMALISM THAT FORMS THE EXISTING BASIS OF THEORETICAL PHYSICS, AND AFTER EACH SUCCESS IN THIS DIRECTION, TO TRY TO INTERPRET THE NEW MATHEMATICAL FEATURES IN TERMS OF PHYSICAL ENTITIES.

DIRAC, 1931

## PLAN OF MY TALK

I. DIRAC'S CLASSICAL MONOPOLE; MOTIVATION FOR IT; SAHA'S CONTRIBUTION

II. NONABELIAN MONOPOLES; 't HOOFT-POLYAKOV MONOPOLES

III. MONOPOLES IN GRAND UNIFIED THEORIES (GUT MONOPOLES)

IV. MONOPOLE-PRODUCTION IN THE GUT PHASE OF THE EARLY UNIVERSE. THE MONOPOLE PROBLEM

V. MONOPOLES IN CURVED SPACE-TIME

VI. CONCLUSIONS

# I. DIRAC MONOPOLE

## MAXWELL'S ELECTROMAGNETIC FIELD EQNS

$$\vec{\nabla}\cdot\vec{E} = \rho$$

$$\vec{\nabla}\times\vec{B} - \frac{\partial\vec{E}}{\partial t} = \vec{j}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0$$

**COVARIANT FORM**

$$\partial_\nu F^{\mu\nu} = -j^\mu$$

$$\partial_\nu \tilde{F}^{\mu\nu} = 0 ,$$

WHERE

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}F_{\lambda\rho}$$

IN EMPTY SPACE, THE ABOVE EQNS HAVE THE SYMMETRY, $\vec{E}\to\vec{B}$ AND $\vec{B}\to -E$, REMAINING UNCHANGED. WHEN THE CURRENTS DUE TO MATTER FIELDS ARE PRESENT, THIS SYMMETRY IS LOST.

THERE IS A "MONO CHARGE," BUT NO "MONOPOLE"

THIS ASSYMETRY BOTHERED DIRAC. HE WONDERED IF THIS SYMMETRY CAN BE MAINTAINED EVEN IN THE PRESENSE OF MATTER FIELDS. SUPPOSE WE CONSIDER

$$\partial_\nu F^{\mu\nu} = -j^\mu \qquad \partial_\nu \tilde{F}^{\mu\nu} = -k^\mu$$

$$j^\mu(x) = \sum_i q_i \int dx_i^\mu \delta^4(x-x_i); \qquad k^\mu = \sum_i g_i \int dx_i^\mu \delta(x_i-$$

IS SUCH A SYSTEM CONSISTENT WITH QUANUM MECHANICS ?

DIRAC'S STARTNG POINT : PHYSICAL INTERPRETATION OF A PATH-DEPENDENT OR NON-ITEGRABLE PHASE OF A QUANTUM MECHANICAL WAVEFUNCTION.

PATH-DEPENDENT PHASE ⟶ PARTICLE MOVING IN AN EM POTENTIA
NOT NEW,
(EQUIVALENT TO "WEYL'S GAUGE INVARIANCE IN MODERN FORM")

HOWEVER, PHASE AROUND A CLOSED CURVE IS ARBITRARY TO THE EXTENT OF $2\pi n$, WHERE $n$ IS AN INTEGER.

THEREFORE, IF ONE CONSIDERS A WAVE FUNCTIO THAT VANISHES ALONG A LINE (NODAL LINE), IT LEADS TO A SINGULAR POTENTIAL AND THE POSSIBILITY OF A MAGNETIC MONOPOLE CONSISTENT QUANTUM MECHANICS IF AND IF ONLY

$$\frac{qg}{4\pi\hbar} = \frac{1}{2} n, \quad n \text{ AN INTEGER}$$

THE CELEBRATED DIRAC'S QUANTIZATION CONDITION

- A SUBTLE AND PROFOUND INTERPRETATION OF THE PHASE OF A QUANTUM-MECHANICAL WAVE FUNCTION

- GENERALIZATION OF MAXWELL'S ELECTRODYNAMICS WITH A MAGNETIC MONOPOLE AND HENCE SYMMETRY BETWEEN CHARGES AND MONOPOLES

MOST IMPORTANT, FOR DIRAC,

- THE EXPLAINATION FOR THE QUANTIZATIC OF ELECTRIC CHARGE. SMALLEST UNIT OF ELECTRIC CHARGE

$$\boxed{q = \frac{(2\pi\hbar)}{g} n}, \quad n \text{ AN INTEGER}$$

" ONE WOULD BE SURPRISED IF NATURE HAD NOT MADE USE ... " OF THE MONOPOLE CONCEPT.

SEVENTEEN YEARS LATER, DIRAC ELABORATED ON THIS AND SAID,

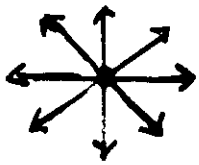
-- THE MERE EXISTENCE OF ONE POLE OF STRENGTH $g$ WOULD REQUIRE ALL ELECTRIC CHARGES TO BE QUANTIZED IN UNITS OF $\left(\frac{2\pi\hbar}{g}\right)$ AND, SIMILARLY, THE EXISTENCE OF ONE CHARGE WOULD REQUIRE ALL POLES TO BE QUANTIZED. THE QUANTIZATION OF ELECTRICITY IS ONE OF THE MOST FUNDAMENTA AND STRIKING FEATURES OF ATOMIC PHYSICS, AND THERE SEEMS TO BE NO EXPLANATION FOR IT APART FROM THE THEORY OF POLES. THIS PROVIDES SOME GROUNDS FOR BELIEVING IN ~~POLES~~ THE EXISTENCE OF THESE POLES

(DIRAC 1948)

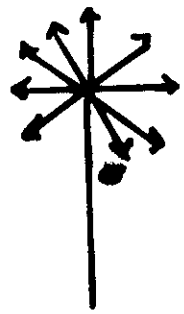
1931 - <u>AN INCOMPLETE THEORY</u>

MONOPOLE - NOT A POINT CHARGE, BUT A POINT CHARGE WITH A STRING

<u>PHYSICAL PICTURE</u> : AN INFINITELY THIN AND LONG SOLENOID, ONE END OF WHICH IS AT INFINITY



ELECTRIC CHARGE



MAGNETIC MONOPOLE

SYMMETRY IN MAXWELL'S EQUATIONS BUT CHARGE AND POLE NOT QUITE SYMMETRIC

A RECIPROCAL RELATIONSHIP:

UNIT OF ELECTRIC CHARGE IS SMALL IF THE UNIT OF MAGNETIC CHARGE IS LARGE AND VICE VERSA

$$e^2 = \frac{1}{137} \quad , \quad g^2 = \frac{137}{4}$$

STRONG BINDING BETWEEN TWO OPPOSITE POLES
DIRAC ESTIMATED THE BINDING ENERGY OF 500 MeV.
A POLE WOULD BE A HEAVILY IONIZING PARTICLE
SUGGESTED THAT ONE SHOULD LOOK FOR SUCH PARTICLES IN ATOMIC PROCESSES.

IN 1948, DIRAC PROVIDED A COMPLETE DYNAMICAL THEORY BASED ON ACTION PRINCIPLE. TREATMENT OF STRING COORDINATES ETC.

# SAHA'S INTERPRETATION OF THE QUANTIZATION CONDITION

MOTION OF A CHARGED PARTICLE IN THE MAGNETIC FIELD OF A FIXED MONOPOLE

FIELD

$$\vec{B} = \frac{g}{4\pi r^3}\hat{r}$$

EQUATION OF MOTION

$$m\ddot{\vec{r}} = q(\vec{r} \times \vec{B})$$

$$\frac{d}{dt}(\vec{r} \times m\dot{\vec{r}}) = \frac{d}{dt}\left(\frac{qg}{4\pi}\hat{r}\right)$$

WHAT IS CONSERVED IS NOT THE USUAL ORBITAL ANGULAR MOMENTUM $\vec{r} \times m\dot{\vec{r}}$, BUT

$$\boxed{\vec{J} = \vec{r} \times m\dot{\vec{r}} - \frac{qg}{4\pi}\hat{r}}$$

ADDITIONAL TERM POINTING ALONG THE LINE JOINING THE POLE AND THE CHARGE WITH

$$\boxed{\vec{J} \cdot \vec{r} = -\frac{qg}{4\pi}}$$

(DERIVED EARLIER (AROUND 1895) BY POINCARE' AND J. J. THOMSON)

SAHA SUGGESTED THAT THIS TERM IS DUE TO THE ELECROMAGNETIC FIELD AND APPLYING THE USUAL QUANTIZATION CONDITION FOR ANG. MOMENT OBTAINED

$$\boxed{\frac{qg}{4\pi\hbar} = \frac{1}{2}n}, \quad n \text{ AN INTEGER}$$

SAHA ALSO CONTEMPLATED COMPOSITES OF A CHARGE AND A MONOPOLE – DYONS

SPINLESS CHARGE AND SPINLESS POLE FORMING AN OBJECT OF HALF-INTEGRAL SPIN !!

STATISTICS ? ARE THEY REAL FERMIONS ?

ARE ALL ELEMENTARY PARTICLES MADE OF DYONS?
(SCHWINGER, BARUT)

EXPERIMENTAL SEARCH

PROPERTIES OF AN ELEMENTARY POLE

$g = \frac{137}{2} e = 3.29 \times 10^{-8}$ CGS UNITS ;

$g^2/\hbar c = 34.24 \qquad (e^2/\hbar c = \frac{1}{137})$

ENERGY ACQUIRED IN A MAGNETIC FIELD $\qquad W(eV) = 2.04 \times 10^4 \, B(\text{gauss}) \, 1(\text{cm})$

COULD ACQUIRE LARGE ENERGIES IN MODEST MAGNETIC FIELDS

IONIZATION ENERGY LOSS $\qquad \left(\frac{dE}{dx}\right)_g = \left(\frac{dE}{dx}\right)_q \left(\frac{g}{e}\right)^2 \beta^2$

$\downarrow$

4,700

FOR RELATIVISTIC VELOCITIES, IONIZATION IS 4,700 TIMES THE VALUE OF THE MINIMUM IONIZING CHARGED PARTICLE.

THE TWO CAN BE EQUAL FOR $\beta \sim 1.4 \times 10^{-3}$

SEARCHES FOR OBJECTS WITH THE ABOVE FEATURES CONDUCTED

HIGH ENERGY ACCELERATORS ( $p+p \to p+p+q+\bar{q}$ )

COSMIC RAYS (EMULSIONS)

BULK MATTER (TRAPPED MONOPOLES, TRAPPED BY AN IMAGE FORCE)

(ASSUMED MASS $m_g = \left(\frac{g}{e}\right)^2 m_e \Rightarrow 2{\cdot}4$ GeV for $n=1$

QUARKS $\frac{1}{3}e$ $m_g = \left(\frac{137}{2} \times 3\right)^2 m_e \simeq 22$ GeV

NO MONOPOLES !! EXPTS NULL RESULTS

TRIESTE CONF. 1981 TO CELEBRATE THE 50TH "BIRTH ANNIVERSARY OF ~~THE "BIRTH" THE~~ MONOPOLE

RESPONDING TO THE INVITATION, EXPRESSING HIS INABILITY TO DO SO, DIRAC SAID

> I AM INCLINED NOW TO BELIEVE THAT MONOPOLES DO NOT EXIST. SO MANY YEARS HAVE GONE BY WITHOUT ANY ENCOURAGEMENT FROM THE EXPERIMENTAL SIDE. IT WILL BE INTERESTING TO SEE IF YOUR CONFERENCE CAN PRODUCE ANY NEW ANGLE OF ATTACK ON THE PROBLEM

## II 't HOOFT POLYAKOV MONOPOLES; THE NEW ANGLES

RISE OF THE STANDARD MODEL IN THE SEVENTIE

ELECTRIC CHARGE WAS A "QUANTUM NUMBER" ASSOCIATED WITH A GENERATOR OF A GROUP. NO NEED OF A MONOPOLE.

QUANTIZATION OF ELECTRIC CHARGE DID NOT NEED A MAGNETIC MONOPOLE

YET, A NEW KIND OF MONOPOLE WAS UNAVOIDABLE IN NONABELIAN GAUGE THEORIES WITH SPONTANEOUS SYMMETRY BREAKING.

A MODEL

$$\boxed{\mathcal{L} = -\frac{1}{4} \underline{F}_{\mu\nu} \underline{F}^{\mu\nu} + \frac{1}{2} \underline{D}_\mu \underline{\Phi} \, \underline{D}^\mu \underline{\Phi} - V(\underline{\Phi})}$$

$G \simeq SO(3)$. GAUGE AND HIGGS FIELD BELONGING TO THE ADJOINT TRIPLET REPRESENTATION

$$\left[ \begin{array}{l} \underline{F}_{\mu\nu} = \partial_\mu \underline{W}_\nu - \partial_\nu \underline{W}_\mu - e\,[\underline{W}_\mu, \underline{W}_\nu] \\ \underline{D}_\mu \underline{\Phi} = \partial_\mu \underline{\Phi} - i e \, \underline{W}_\mu \, \underline{\Phi} \\ V(\Phi) = -\frac{\lambda a^2}{2} \Phi^2 + \frac{\lambda}{4} \Phi^4 \end{array} \right]$$

## SYMMETRIC VACUUM (FALSE)

ENERGY DENSITY $\theta_{00} = 0$ EVERYWHERE

$$\Phi^a = 0 \quad , \quad W_a^\mu = 0 \, , \quad V(\Phi) = 0$$

## HIGGS VACUUM (TRUE)

$$\Phi^2 = a^2 \, , \qquad W_a^\mu \neq 0$$

$$V(\Phi) = -\frac{\lambda}{4} a^2$$

$$SO(3) \xrightarrow[\text{BROKEN}]{\text{SPONTANEOUSLY}} SO(2) \simeq U(1)$$

LEFT OVER $U(1)$ PROVIDES A BASIS FOR AN ELECTROMAGNETIC GAUGE THEORY. STATIC SPHERICALLY SYMMETRIC SOLNS WITH

$$\Phi^a = \frac{r^a}{er^2} H(aer) \, , \qquad W_\mu^a = -\epsilon_{aij} \frac{r^j}{er^2} [1 - K(aer)]$$

COUPLED DIFF. EQNS FOR H AND K. A WHOLE BODY OF LITERATURE AROUND THESE EQUATIONS. EXACT SOLUTIONS IN THE LIMIT $\lambda/e^2 \to 0$, THE WELLKNOWN PRASAD-SOMMERFIELD SOLUTIONS, BOGOMOLN'YI BOUND ....

## PARTICLES AT CLASSICAL LEVEL

| | MASS |
|---|---|
| HIGGS PARTICLE | $\mu = a(2\lambda)^{1/2}\hbar$ |
| MASSIVE GAUGE PARTICLES (CHARGED) | $M = ae\hbar$ |

## AND A MONOPOLE !!

DESCRIBED BY

- FINITE ENERGY CLASSICAL CONFIG. OF GAUGE AND HIGGS FIELDS. FINITE RADIUS DETERMINED BY THE COMPTON WAVELENGTHS $\hbar/M$, $\hbar/\mu$.

- ASYMPTOTICALLY, SURVIVING EM FIELD OF A MAGNETIC MONOPOLE (IN AN APPROPRIATE GAUGE)

- ITS CHARGE CONNECTED WITH TOPOLOGY AND LEADS TO THE QUANTIZATION CONDITION

$$\frac{gq}{4\pi\hbar} = \frac{1}{2} N, \quad N \text{ AN INTEGER}$$

BEING THE NUMBER OF TIMES THE HIGGS FIELD $\Phi$ COVERS $M_0$ (VACUUM MANIFOLD) AS $r$ COVERS THE SPHERE $\Sigma$ SURROUNDING THE MONOPOLE

- MASS OF THE MONOPOLE A MONOTONICALLY INCREASING FUNCTION OF $\lambda/e^2 = \frac{M^2}{M_2}$

$$\frac{M}{\alpha} \longrightarrow 1.8\,\frac{M}{\alpha}$$

$$\lambda/e^2 \to 0 \qquad \frac{\lambda}{e^2} \to \infty$$

## SAME QUANTIZATION CONDITION FROM A DIFFERENT ANGLE !

### DIRAC

QUANTUM MECHANICAL
↓
EM WITH MONOPOLE WITH STRING AND A POINT SINGULARITY

### 't HOOFT-POLYAKOV

CLASSICAL EXTENDED OBJECT WITH QUANTUM-PARTICLE LIKE CHARACTERISTICS

NO DIRAC STRING
REGULAR AT ORIGIN.

NEW THEORETICAL OBJECTS TO PLAY WITH.

WHAT ARE THEIR INTERACTIONS ?

MULTI-MONOPOLE SOLNS

ETC.

# III GUT MONOPOLES

SO(3) MONOPOLES GENERIC TO ALL NONABELIAN GAUGE THEORIES WITH SSB.

IN ANY GRAND UNIFIED THEORY,

$$G \xrightarrow[SSB]{M_G} SU(3) \otimes SU(2) \otimes U(1) \xrightarrow[SSB]{M_W} SU(3) \times U(1)_{EM}$$

ARGUMENTS BASED ON HOMOTOPY AND TOPOLOGY SHOW THAT MAGNETIC MONOPOLE SOLUTIONS EXIST. BRIEFLY STATED,

IF THE VACUUM MANIFOLD $\mathcal{M} = G/H$ ASSOCIATED WITH THE SYMMETRY BREAKING $G \to H$ CONTAINS ~~NON-TRIVIAL~~ NON-SHRINKABLE SURFACES, i.e., IF THE MAPPING OF $\mathcal{M}$ ONTO THE TWO-SPHERE IS NON-TRIVIAL, MONOPOLES MUST EXIST.

IF THE GROUP $G$ IS SIMPLY CONNECTED,

$$\pi_2(G/H) = \pi_1(H)$$

AND IF $H$ CONTAINS A $U(1)$ FACTOR, SINCE $\pi_1(U(1)) = Z$ (GROUP OF INTEGERS) IS NON-TRIVIAL

GUT GROUPS SUCH AS $SU(5)$, $SO(10)$ WILL HAVE MONOPOLES

$M_M$ = MASS OF THE MONOPOLE

$= \frac{M_G}{g^2} \simeq 10^{16}$ GeV IF $M_G = 10^{14} - 10^{15}$ GeV AND $g = .1$

ENORMOUS MASS FOR AN ELEMENTARY MONOPOLE !

VALENTINE DAY PRESENT TO CABRERA !

CABRERA SAW ON FEB 14, 1982 A GOLDPLATED EVENT THAT COULD BE NOTHING OTHER THAN A MONOPOLE !

CREATED A GREAT DEAL OF EXCITEMENT, MANY PAPERS.

COULD NOT BE A GUT MONOPOLE, BECAUSE OF THE FLUX PROBLEM.

WAS CABRERA VICTIM OF A HOAX ?

# IV MONOPOLES IN PHASE TRANSITIONS OF THE EARLY UNIVERSE

## THE KIBBLE MECHANISM

- COMBINE GUTS WITH COSMOLOGY

- THE VERY EARLY HOT UNIVERSE WENT THROUGH A SEQUENCE OF PHASE TRANSITIONS

- DURING SUCH PHASE TRANSITIONS, VARIOUS KINDS OF DEFECTS ARE TO BE EXPECTED

DOMAIN WALLS, STRINGS, MONOPOLES

- ALTHOUGH SUCH DEFECTS ARE NOT MINIMUM ENERGY CONFIGURATIONS, THEY REMAIN TRAPPED AND SHOULD BE RELICS OF THE BYGONE ERA

(FORMAL ARGUMENTS OF KIBBLE SUBSTANTIATED BY LABORATORY EXPERIMENTS !!!)

## ESTIMATE MONOPOLE PRODUCTION

- CORRELATION LENGTH LIMITED BY THE PARTICLE HORIZON. STANDARD BIG BANG COSMOLOGY $d_H \sim M_{Pl}/T^2$. ONE MONOPOLE PER HORIZON VOLUME, $n_M \sim d_H^{-3} \sim T_c^6/M_{Pl}^3$; $T_c \sim 10^{14}$ GeV, $n_M/s \sim 10^{-13} \rightarrow$ PRESENT MONOPOLE MASS DENSITY $\sim 10^{11} \rho_c$ !

NOT ACCEPTABLE.

# V. NONABELIAN MONOPOLES COUPLED TO GRAVITY

- RECENT FLURRY OF PAPERS ON THE STUDY OF EINSTEIN-YANG-MILLS-HIGGS SYSTEM

- t HOOFT-POLYAKOV MONOPOLES — QUANTUM PARTICLES LIKE OBJECTS DESCRIBED BY CLASSICAL, FINITE ENERGY, CONFIGURATIONS HAVE AN EXTENDED STRUCTURE COULOMB INTERACTIONS ARE ALTERED NO INTERACTION THEOREM ( O'RAIFEARTAIGH, SOO YONG PARK AND K.C.W). SIMPLE INTERPRETATIO

WHAT HAPPENS IF ONE INCLUDES GRAVITY ?

## NEWTONIAN MECHANICS

STATIC CONFIGURATION OF CHARGED PARTICLES

## GENERAL RELATIVITY

COUPLED EINSTEIN-MAXWELL EQNS AND MAJUMDAR, PAPAPETROU SOLUTION
HARTLE-HAWKING INTERPRETATION

DOES AN ANALOGOUS SITUATION EXISTS IN THE CASE OF NONABELIAN MONOPOLES COUPLED TO GRAVITY ?

NUMERICAL SOLNS TO COUPLED EYMH EQNS STUDIED BY SEVERAL AUTHORS HAVE REVEALED MANY INTERESTING FEATURES

MONOPOLES WITHIN BLACKHOLES
BLACK HOLES WITHIN MONOPOLES

UNCONVENTIONAL COUPLING. REDUCTION TO FIRST ORDER EQNS USING BOGOMOLN'YI CONDITION MOSTLY ANALYTICAL.

AN EXTREME REISNER-NORDSTROM BLACK HOLE WITH A MAGNETIC CHARGE WITHIN EYMH SYSTEM.

INTERPLAY BETWEEN GRAVITATIONAL FORCES AND GAUGE INTERACTIONS WITH HIGGS MECHANISM PROVIDES A FERTILE GROUND FOR FURTHER STUDIES INTO EXTENDED CONFIGURATIONS THAT CONSTITUTE MONOPOLES.

A SPECULATIVE REMARK.

DO MASSIVE MONOPOLES BECOME BLACK HOLES ?

BALAKRISHNA, WALI ; NGUYEN AI VIET, WALI

$$S = \int d^4x \sqrt{-g}\left[-\frac{1}{16\pi G v^2} R\phi^2 + \mathcal{L}_{MATTER}\right]$$

$$\mathcal{L}_{MATTER} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \frac{1}{2}(D_\mu\Phi)^2 - \frac{\lambda}{4}(\phi^2 - v^2)^2$$

IN THE PRASAD-SOMMERFIELD LIMIT, ONE CAN SHOW THAT THE SYSTEM HAS STATIC SOLNS GIVEN BY FIRST ORDER DIFF. EQNS (BOGOMOLN'YI TECHNIQUE)

FINITE ENERGY. SOLNS DETERMINED BY SOME WELL-KNOWN CLASSICAL EQNS (NON-LINEAR)

$$\mathcal{E} = \frac{1}{2}\int d^3x \left[(E_i^a \mp B_i^a)(E_i^a \mp B_i^a)\right]$$

$$+ \int d^3x \left[\pm E_i^a B_i^a + \frac{1}{2}\partial_i\left(\sqrt{-g}\, \partial^i \sqrt{g_{00}}\, \phi^2\right)\right]$$

$$\geq \int d^3x \left[\pm E_i^a B_i^a + \frac{1}{2}\partial_i\left(\sqrt{-g}\, \partial^i \sqrt{g_{00}}\, \phi^2\right)\right]$$

IF $E_i = \pm B_i$, $\mathcal{E}$ is saturated -

ISOTROPIC COORDINATE SYSTEM

$$r^4 F'' = e^{-2F}, \quad \frac{g}{A} = \frac{1}{r} + F' \; ; \; W = e^{-F}/r$$

STANDARD COORDINATE SYSTEM

$$y' = -\frac{2xy}{y - 1 + x}$$

# VI CONCLUSION

THE CONCEPT OF A MAGNETIC MONOPOLE INVENTED BY DIRAC, MOTIVATED PRIMARILY BY ASKING THE PHYSICAL MEANING OF THE ARBITRARINESS OF QUANTUM MECHANICAL PHASE AND RELATING IT TO THE SYMMETRIES IN MAXWELL'S THEORIES, HAS PLAYED A SIGNIFICANT AND DIVERSE ROLE IN ELEMENTARY PARTICLE PHYSICS. IT MAY NEVER BE FOUND EXPERIMENTALLY. BUT, IT APPEARS THAT IT WILL CONTINUE TO EXIST IN THE THEORISTS' LABORATORY AND THEORISTS WILL CONTINUE TO EXPERIMENT WITH IT.