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Lectures I and II

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SUPERSTRING PHENOMENOLOGY

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SUPERSTRING PHENOMENOLOGY

JAN LOUIS

TRIESTE, JULY 94

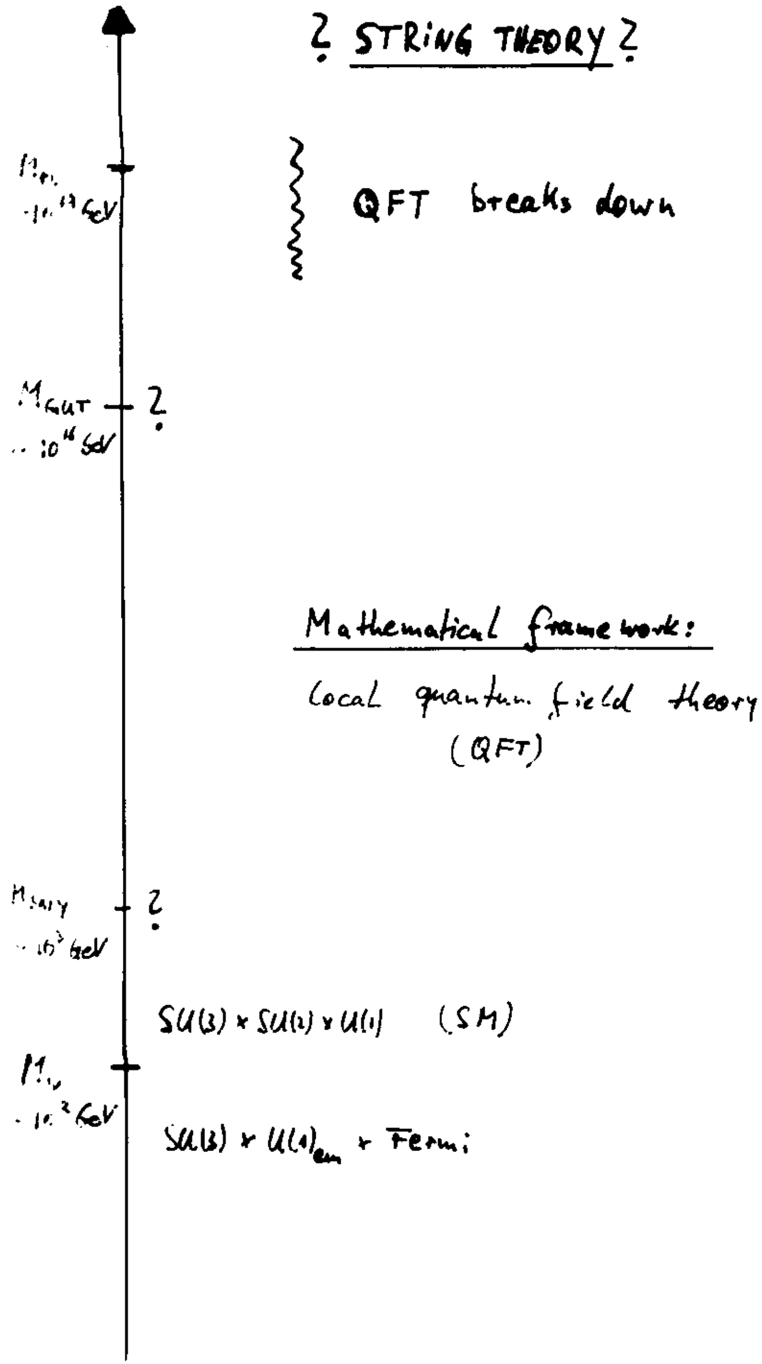
OUTLINE

1. INTRODUCTION — GOOD NEWS / BAD NEWS

2. GAUGE COUPLINGS IN STRING THEORY

3. NON-PERTURBATIVE EFFECTS I:
   - GAUGINO CONDENSATION -

4. NON-PERTURBATIVE EFFECTS II:
   - A phenomenological approach -



STRING THEORY

- gives up locality
  point-like particles $\longrightarrow$ extended object (string)

- particles appear as (excited) states of string

- finite number of massless states (L)
  infinite number of massive states (H)
  $M_H \sim O(M_{Pl})$

- among massless states: spin 2 graviton

- quantum corrections of graviton-graviton scattering can be computed

  $\Longrightarrow$ consistent ~~quantum~~ gravity

In addition:

the massless spectrum contains:

- spin 1 gauge bosons (in adjoint of G)

- families of massless chiral fermions (fundamental of G)

- Higgs-like states

and is anomaly free.

The low energy limit of string theory is an (effective) field theory

$\Rightarrow$ **Unification of all interactions?**

$\Rightarrow$ Is the SM the low energy limit?
(can we compute the top-quark mass!)

Problems

- only have a set of "rules" for computing (on-shell) scattering amplitudes in perturbation theory

- interactions governed by 2d field theory on the worldsheet

  consistency requires conformal field theory (CFT)

  $\Rightarrow$ 2d CFT $\hat{=}$ classical solution $\hat{=}$ string vacuum

  $\Downarrow$

  many exist $\Rightarrow$ vacuum is degenerate

- only one scale : M_{Pl} $\Rightarrow$ where does M_{weak} come from.

- the (dimensionless) coupling constant g_{string} is a free parameter

belief / hope: artefact of perturbation theory

What can we do ?

- study non-perturbative ST (String Field Theory, Matrix Models)

- study gravity in ST

  graviton scattering at high energies

  Black holes

  Dilaton gravity

  cosmological solutions

- study space of all classical string vacua ($\hat{=}$ string phenomen...)

  (assumes weak coupling)

  Look for vacuum $\supset$ SM as low energy limit

  (predictive power)

String phenomenology

select "promising" classical solutions
and study this "space" of string vacua

- this assumes weak coupling

Selection criteria: ("vacuum cleaning")

- 4-d flat Minkowski space ✓
- $G \supset SU(3) \times SU(2) \times U(1)$ ✓
- $n_g \geq 3$ ✓
- $N=1$ supersymmetry ?
  - stability of scales
  - preferred among consistent vacua with chiral fermions
- fermion masses, gauge couplings, proton decay, ...

$\Rightarrow$ new problem: break down of supersymmetry at low energies

$$\begin{cases} L, & M_L = 0 \\ H, & M_H = O(M_{Pl}) \end{cases}$$

effective theory $\mathcal{L}_{eff}(L)$

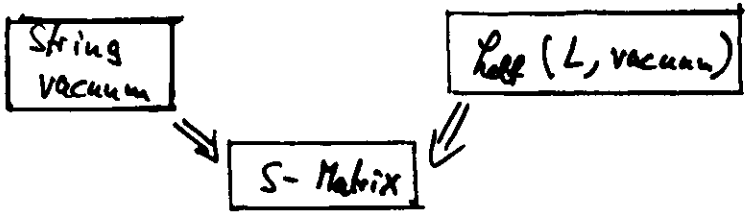
$$\begin{cases} L = \text{Leptons: } e, \mu, \tau, \nu \\ H = \text{vector bosons: } W^{\pm}, Z^0 \\ \quad M_{Z^0} \sim M_{W^{\pm}} \sim O(M_W) \end{cases}$$

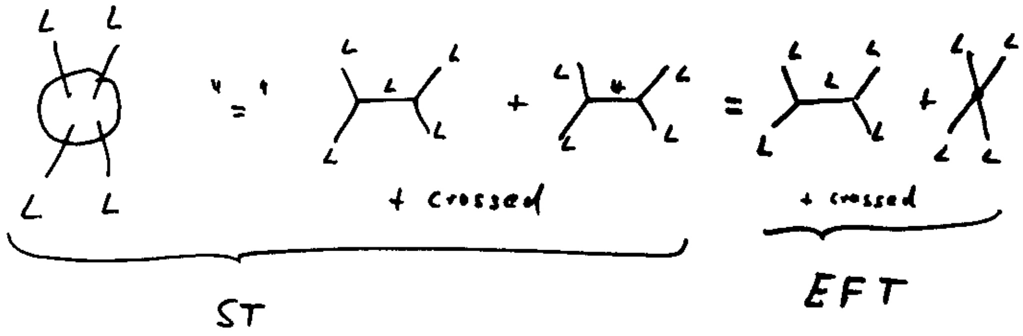
effective theory $\mathcal{L}_{Fermi}(L)$

Low energy limit:

external momentum $K \ll M_{pe}$

and "integrate out" H





$\Rightarrow$ expand $\mathcal{L}_{eff}$ in powers of K

$\Rightarrow$ $\mathcal{L}_{eff}^{(2)}(K^2, L, \text{vacuum})$

$\Rightarrow$ different $\mathcal{L}_{eff}$ for different vacua

$\Rightarrow$ string vacuum $\cong$ "model"

(in the sense of model building)

$\mathcal{L}_{eff}$ is constrained by the symmetries of the string vacuum

$N=1$ supergravity in $d=4$:

ALL couplings [at $O(K^2)$] are encoded in 3 functions

Kähler potential $K(\phi, \bar\phi)$ (real)

SUPER potential $W(\phi)$ (holomorphic)

gauge kinetic function $f(\phi)$ (holomorphic)

[ϕ are chiral superfields $\bar{D}_{\dot\alpha} \phi = 0$]

$$\mathcal{L}_{eff} = -3 \int d^4\Theta E\, e^{-\frac{1}{3}K(\phi,\bar\phi)} + \int d^2\Theta \mathcal{E}\, W(\phi) + \text{h.c.}$$

$$+ \int d^2\Theta \mathcal{E} f(\phi) \operatorname{tr} W^\alpha W_\alpha \quad + \text{h.c.}$$

$$W_\alpha = -\frac{1}{4} (\bar{D}^2 - 8R)\, e^{-V} D_\alpha e^{V}$$

supersymmetric field strength, V = vector superfield

Symmetries of $\mathcal{L}_{eff}$

1) Kähler invariance

$g_{i\bar{j}} = \partial_i \partial_{\bar{j}} K$ is metric on a Kähler manifold

and invariant under

$$K \to K(\phi,\bar{\phi}) + F(\phi) + \bar{F}(\bar{\phi})$$

$\mathcal{L}_{eff}$ is invariant if also

$$W(\phi) \to W(\phi)\, e^{-F(\phi)} \qquad \text{(superpotential)}$$

$$\chi^i \to \chi^i\, e^{-\frac{i}{2}\mathrm{Im}\,F} \qquad \text{(chiral fermions)}$$

$$\lambda^a \to \lambda^a\, e^{\frac{i}{2}\mathrm{Im}\,F} \qquad \text{(gauginos)}$$

$$\psi \to \psi\, e^{-\frac{i}{2}\mathrm{Im}\,F} \qquad \text{(gravitino)}$$

(local chiral rotation on all fermions of $\mathcal{L}_{eff}$)

2) coordinate transformations

$$\phi^i \to \phi'^i(\phi) \qquad \text{(holomorphic change)}$$

$$\chi^i \to \frac{\partial \phi'^i}{\partial \phi^j}\,\chi^j$$

$$g_{i\bar{j}} \to \frac{\partial \phi^l}{\partial \phi'^i}\,\frac{\partial \bar{\phi}^{\bar{m}}}{\partial \bar{\phi}'^{\bar{j}}}\, g_{l\bar{m}}$$

N=1 Supergravity

at $O(\kappa^2)$:

$$\mathcal{L}_{eff}\big(K(\phi,\bar{\phi}),\, W(\phi),\, f(\phi)\big) = e\Big\{ -\frac{1}{2} R + \frac{1}{2}\bar{\psi}\not{D}\psi$$
$$- \frac{1}{4}\frac{1}{g^2} F_{\mu\nu}^2 - \frac{1}{4}\Theta F\tilde{F} - \frac{i}{g^2}\bar{\lambda}\not{D}\lambda$$
$$- g_{i\bar{j}}\, D_\mu \phi^i D_\mu \bar{\phi}^{\bar{j}} - i\, g_{i\bar{j}}\, \bar{\chi}^{\bar{j}}\not{D}\chi^i$$
$$- \frac{1}{2} e^{K/2} D_i D_j W\, \chi^i \chi^j + h.c.$$
$$+ i\sqrt{2}\, g\, g_{i\bar{j}} \frac{\partial K}{\partial \phi^i}\chi^i \lambda + h.c.$$
$$- V(\phi,\bar{\phi}) + \cdots \Big\}$$

$$\frac{1}{g^2} = \mathrm{Re}\, f(\phi) \qquad \text{(gauge coupling)}$$

$$\Theta = \mathrm{Im}\, f(\phi) \qquad \text{(}\Theta\text{-angle)}$$

$$g_{i\bar{j}} = \frac{\partial}{\partial \phi^i}\frac{\partial}{\partial \bar{\phi}^{\bar{j}}} K \qquad \text{(Kähler metric)}$$

$$V = \frac{1}{2} D^2 + e^K\left[|DW|^2 - 3|W|^2\right] \qquad \text{(scalar potentia}$$

$$D_i W = \frac{\partial}{\partial \phi^i} K + \frac{\partial K}{\partial \phi^i} W \qquad \text{(Kähler covariant derivati}$$

$$D^a = g\, \frac{\partial K}{\partial \phi^i}\, T^a \phi^i \qquad \text{(D-term)}$$

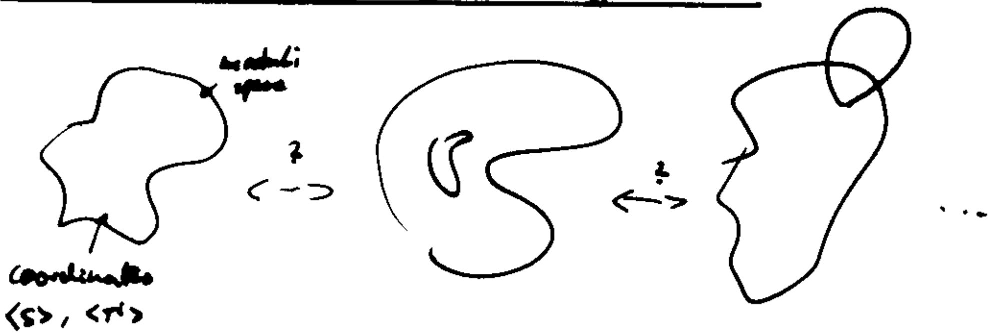
Massless spectrum (in $N=1$ super multiplets)

- graviton - ino
- gauge bosons - ino [in adjoint of G]
- "matter" fields Q^I (charged under G)
  - ↑ flavour index
- dilaton/axion S (~ gauge coupling) } neutral under G
- moduli T^i

} $\sim \phi$

S, T^i are flat directions of $V(S,T,Q)$
($\hat{=}$ exactly marginal perturbation of CFT)

Space of (promising) string vacua:



classification ?

What do we know about K, W, f as a function of Q, S, T

(i) expand around $\langle Q^I\rangle = 0$

(ii) dilaton $(S+\bar S) \sim g_{string}^{-2}$ organizes the string perturbation theory

Kähler potential:

$$K = \hat K + Z_{I\bar J}\, Q^I \bar Q^{\bar J} + \tfrac{1}{2}\left(H_{IJ}\, Q^I Q^J + \text{h.c.}\right) + O(Q^3)$$

K is renormalized at all orders in string perturbation theory

$$\hat K = \underbrace{-\ln[S+\bar S] + \hat K^{(0)}(T,\bar T)}_{\text{tree level}} + \underset{\text{1 loop}}{\frac{\hat K^{(1)}(T,\bar T)}{S+\bar S}} + \underset{\text{2 loop}}{\frac{\hat K^{(2)}(T,\bar T)}{(S+\bar S)^2}} + \dots$$

$$Z_{I\bar J} = Z^{(0)}_{I\bar J}(T,\bar T) + \frac{Z^{(1)}_{I\bar J}(T,\bar T)}{S+\bar S} + \frac{Z^{(2)}_{I\bar J}(T,\bar T)}{(S+\bar S)^2} + \dots$$

$$H_{IJ} = H^{(0)}_{IJ}(T,\bar T) + \frac{H^{(1)}_{IJ}(T,\bar T)}{(S+\bar S)} + \frac{H^{(2)}_{IJ}(T,\bar T)}{(S+\bar S)^2} + \dots$$

gauge kinetic function: (holomorphic)

$$f = f(S,T) + O(Q^2)$$

$$f(S,T) = S + f^{(1)}(T) + \text{" } \varrho \text{ "}$$

S: tree level; $f^{(1)}(T)$: 1 loop

no renormalization beyond 1 loop

Shifman
Vainshtein
Nilles
Antoniadis
Narain
Taylor

superpotential:

$$W = \tfrac{1}{2} M_{IJ}(T)\, Q^I Q^J + \tfrac{1}{3} Y_{IJK}(T)\, Q^I Q^J Q^K + O(Q^4)$$

Y_{IJK}: Yukawa couplings

- no dilaton dependence
- not renormalized at any order in string perturbation theory

Dine
Seiberg
Martinec
⋮

$\Rightarrow$ model (string vacuum) dependent couplings are:

tree level: $\hat{K}^{(0)}(T,\bar{T})$, $Z^{(0)}_{IJ}(T,\bar{T})$, $H^{(0)}_{IJ}(T,\bar{T})$, $M_{IJ}(T)$, $Y_{IJK}(T)$

1 loop: $\hat{K}^{(1)}$, $Z^{(1)}$, $H^{(1)}$, $f^{(1)}(T)$

⋮ ⋮ ⋮

Computation of the (tree level) couplings

recall: scattering of strings governed by 2d CFT of central charge $(c,\bar{c}) = (26,15)$ for heterotic string

selection criteria imposed imply

(i) 4-d flat Minkowski space $\Rightarrow$

4 space-time coordinates	$X^\mu(\sigma,\tau)$	$(4,4)$
4 right-moving fermions	$\psi^\mu(\sigma^+)$	$(0,2)$

$$\Rightarrow \quad (26,15) = \underbrace{(4,6)}_{\text{space time}} \oplus \underbrace{(22,9)}_{\text{internal CFT}}$$

(ii) $N=1$ space-time supersymmetry

$\Rightarrow$ $(0,9)$ CFT has $N=2$ worldsheet supersymmetry and quantized $U(1)$ charges

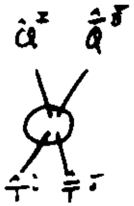
left over $(22,0)$ makes up gauge degrees of freedom

Computation of the couplings

recall:

- scattering of strings is governed by 2d CFT
- states in space-time $\hat{=}$ operators in CFT
- couplings in $\mathcal{L}_{eff}$ $\hat{=}$ correlation functions of CF

e.g.:



$\sim \langle \hat{Q}^I \hat{\bar{Q}}^{\bar{J}} \hat{T}^i \hat{\bar{T}}^{\bar{j}} \rangle_{CFT} \rightarrow \frac{\partial}{\partial T^i}\frac{\partial}{\partial \bar{T}^{\bar{j}}} Z_{I\bar{J}}\Big|_{T=\langle T\rangle}$ is computed

$\sim \langle Q^I Q^J Q^K T^i \rangle_{CFT} \rightarrow \frac{\partial}{\partial T^i} Y_{IJK}{}^{(T)}\Big|_{T=\langle T\rangle}$ is computed

(both functions are needed for m_{top})

problems:

- $\langle \dots \rangle_{CFT}$ are different for different string vacuum
- $\langle \dots \rangle_{CFT}$ only known in a few "fortunate" cases (e.g. orbifolds, Gepner constructions, free fermions, ...)

detailed phenomenological investigations
(Yukawa coupling, fermion masses, CP-violation, ...)
have been done for particularly promising vacua:

(i) 3 generation Calabi-Yau compactifications — Ross

(ii) orbifolds with $G = SU(3) \times SU(2) \times U(1) \times G'$ — Ibanez & [illegible]

(iii) 3 generation Gepner-models — Gepner

(iv) "flipped" $SU(5) \times U(1)$ — Antoniadis, ...

problem:

- all of these models sit at particular point in moduli space (where we can compute) but why should nature sit at this point?
- many low energy properties depend on mechanism for supersymmetry breaking
- almost no new insight into problem of fermion mass hierarchy, origin of CP-violation, number of light generation

(2,2) string vacua

split $\quad (22,9) = (13,0) \oplus (9,9)$

and impose $\quad N=(2,2)$ w.s. supersymmetry on $(9,9)$

$\Rightarrow$

- $G = E_8 \oplus E_6 \oplus G'$
- $h^{(1,1)}$ families of $\underset{\sim}{27}$ of E_6 and moduli T^i
- $h^{(2,1)}$ families of $\overline{\underset{\sim}{27}}$ of E_6 and moduli U^i
- $\hat{K}^{(0)} = K_1(T,\bar{T}) + K_2(U,\bar{U})$
- $\frac{\partial}{\partial U} Y^{(27)} = \frac{\partial}{\partial T} Y^{(\overline{27})} = 0$
- $Y_{ijk} = \partial_i \partial_j \partial_k \mathcal{F} \leftarrow$ holomorphic prepotential

special Kähler geometry

- $R_{i\bar{j}k\bar{l}} = g^{(1)}_{i\bar{j}} g^{(1)}_{k\bar{l}} + g^{(1)}_{i\bar{l}} g^{(1)}_{k\bar{j}} - e^{2K_1} Y^{(27)}_{ikm} \, g^{m\bar{n}} \, \bar{Y}^{(27)}_{\bar{j}\bar{l}\bar{n}}$

  where $g^{(1,2)}_{i\bar{j}} = \partial_i \partial_{\bar{j}} K_{(1,2)}$

- $Z^{(27)}_{i\bar{j}} = e^{\frac{1}{3}(K_2 - K_1)} g^{(1)}_{i\bar{j}}$

  $Z^{(\overline{27})}_{i\bar{j}} = e^{\frac{1}{3}(K_1 - K_2)} g^{(2)}_{i\bar{j}}$

Recent excitement: Mirror Symmetry   Greene's lecture

$$\begin{array}{c} \boxed{\begin{array}{c}(c,\bar{c}) = (9,\bar{9}) \\ N=(2,2)\end{array}} \begin{array}{c} \nearrow \mathcal{M}_1 \\ \quad\updownarrow \\ \searrow \mathcal{M}_2 \end{array} \end{array}$$

topological distinct C-Y manifolds

use techniques from algebraic geometry to compute $Y^{(27)}(U)$ on $\mathcal{M}_1$

$Y^{(\overline{27})}(U)$ on $\mathcal{M}_2$

mirror symmetry: $\quad Y^{(27)}(T)\big|_{\mathcal{M}_1} \cong Y^{(\overline{27})}(U)\big|_{\mathcal{M}_2}$

In simple (low dimensional) examples $Y^{(27)}(T)$, $K^{(0)}(T,\bar{T})$, $Z(T,\bar{T})$ have been computed on the entire moduli space <u>without</u> solving any CFT correlation function

Candelas
de la Ossa
Green
Parkes
Morrison
Font
Klemm
Theisen
Yau
⋮

What did we learn?

- improved computational techniques (extension to more interesting $(0,2)$ vacua?)
- flavors of generic couplings $K(T,\bar{T})$, $Z(T,\bar{T})$, $Y(T)$
- "connected" moduli spaces of topologically different C-Y spaces

<u>However:</u>

$\langle T \rangle$ is still a free parameter

$\Rightarrow$ $Y(\langle T \rangle)$ is a free parameter

$\Rightarrow$ need to find correlations or zero's among $Y_{IJK}(T)$ to obtain information or constraints about fermion masses

<u>Note</u>: Fermion masses are almost independent of mechanism for supersymmetry breaking

Discrete symmetries

Other possible constraints on the couplings arise from discrete symmetries which exist in almost all string vacua.

Example: $SL(2,\mathbb{Z})$-duality (T-duality)

$$T \to \frac{aT - ib}{icT + d} \qquad \begin{array}{l} a,b,c,d \in \mathbb{Z} \\ ad - bc = 1 \end{array}$$

$\Rightarrow$ $Y_{IJK}(T)$ have to be modular forms of $SL(2,\mathbb{Z})$

- generalization is not understood
- many other discrete symmetries exist origin is still unclear, some are remnants of broken gauge symmetries anomaly free?

SUMMARY

- classical vacua of heterotic string resemble some extension of SM
- string (gauge) coupling constant is a free parameter $\sim \langle \mathrm{Re}\, S \rangle$
- Yukawa couplings are free parameters $Y_{IJK}^{(T)}$
- only one scale M_{Pl}
- only perturbative information available
- $\mathcal{L}_{eff}(L)$ can be computed in perturbation theory for a steadily rising class of string vacua
- origin of fermion mass hierarchy, CP violation, number of light generation

  is still mysterious
- missing mechanism for supersymmetry breaking is stumbling block for low energy phenomenology

GAUGE COUPLINGS

gauge group: $G = \bigotimes_a G_a$ (e.g. $E_8 \otimes E_8$)

[cannot be arbitrarily big: $\frac{d(G)\cdot K}{K + C_2(G)} \leq 22$]

gauge couplings: $\frac{1}{g_a^2} = K_a \; \mathrm{Re}\, \langle S \rangle$ $\forall$ vacua at tree level

Dilaton

level of Kac-Moody algebra (integer for G_a non-abelian)

$\Rightarrow$ g_a^{-2} is universal (up to K_a)

$\Rightarrow$ no need for covering GUT-group !?

however:

- $\langle S \rangle$ is a free parameter in perturbation theory
- gauge coupling unification at

$$M_{String} = g_{String} \cdot 5 \cdot 10^{17}\, \mathrm{GeV}$$

Gauge coupling unification in GUTs

Georgi - Quinn - Weinberg:

low energy couplings $\rightarrow \dfrac{1}{g_a^2(\mu)} = \dfrac{k_a}{g_{GUT}^2} + \dfrac{b_a}{8\pi^2} \ln \dfrac{M_{GUT}}{\mu} + \dfrac{\Delta_a}{16\pi^2}, \quad \mu \ll M_{GUT}$

($\dfrac{b_a}{8\pi^2}$: 1 loop β-function; $\dfrac{\Delta_a}{16\pi^2}$: threshold corrections)

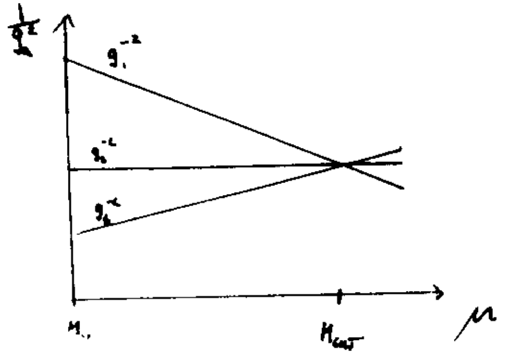
for $SU(3) \times SU(2) \times U(1)$ (embedded in $SU(5)$)

$k_2 = k_3 = 1, \quad k_1 = 5/3$

3 equations and 2 unknown: g_{GUT}, M_{GUT}

$\Rightarrow$ 1 prediction: α_s or $\sin^2\theta_W$

for supersymmetric $SU(5)$:



Gauge couplings in String Theory

Low energy running is determined by same GQW:

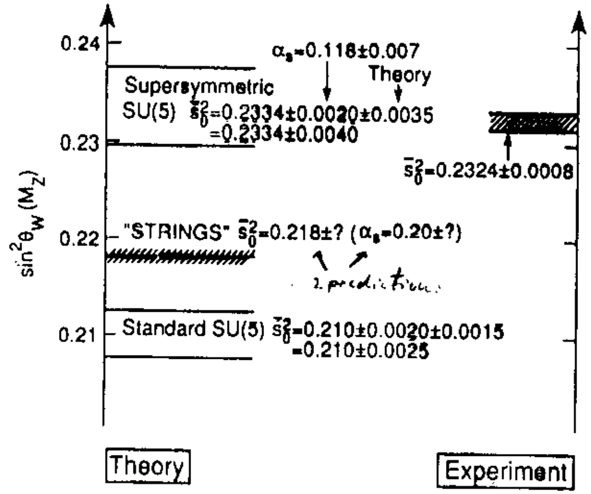
$$g_a^{-2}(\mu) = k_a \,\mathrm{Re}\langle S\rangle + \frac{b_a}{8\pi^2} \cdot \ln \frac{M_{String}}{\mu} + \frac{\Delta_a(T)}{16\pi^2}$$

but:

- $M_{String} \sim 5 \cdot 10^{17}$ GeV is fixed
- $k_1 = 5/3$, $k_2 = k_3 = 1$ not necessary
- $\Delta_a(T)$ is T-dependent and can be large

from: R. Barbieri, Moriond 93

Coupling unification; summary



$K_1 = 5/3$

$M_{GUT} = M_{string} \cdot 6 \cdot 10^{?}$

1989: before LEP

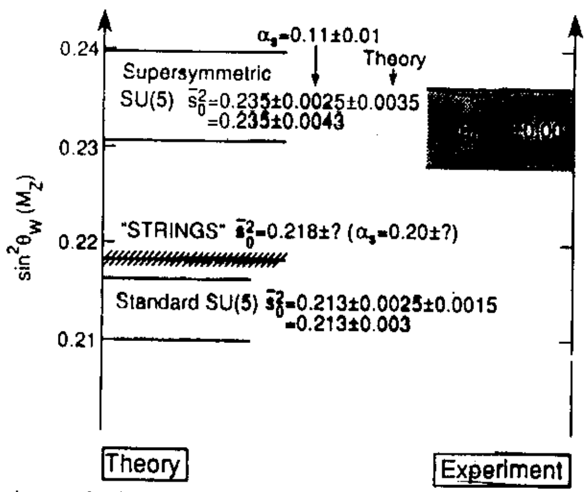


Fig.5. Comparison of theory and experiment in the determination of the weak mixing angle from the unification hypothesis now and before LEP.

... I'banez Phys. Lett. B 318

form the doublet–triplet splitting is just hoping for a miracle. Thus the situation concerning SUSY-GUTs is quite puzzling: they are theoretically in quite a bad shape but give the correct result required for unification!

In the present note I remark that there is another class of well motivated theories which have comparable success concerning gauge coupling unification but are free from the theoretical problems of the SUSY-GUTs mentioned above. This class of theories correspond to the assumption that the SUSY standard model is directly unified into a string theory close to the Planck mass, without any GUT intermediate step. In fact gauge coupling unification within this type of scheme has been considered in the recent past reaching apparently a different conclusion [5,6]. The origin of this difference is a matter of appropriately identifying what are actually the free parameters in string unification. This will be clarified below.

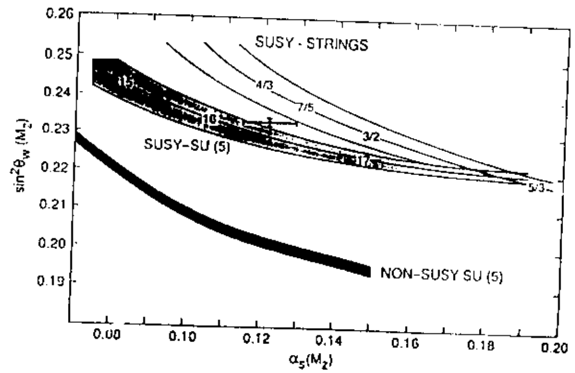
Let us first briefly recall the situation in SUSY-GUTs. Here the normalization of the U(1) factor is known ($k_1 = \frac{5}{3}$) and the one-loop expressions for the weak angle and α_s yield

$$\sin^2\theta_w(M_Z) = \frac{3}{8}\left[1 + \frac{5\alpha(M_Z)}{6\pi}\,(b_2 - \tfrac{3}{5}b_1)\log\left(\frac{M_X}{M_Z}\right)\right], \qquad (1)$$

$$\frac{1}{\alpha_s(M_Z)} = \frac{3}{8}\left[\frac{1}{\alpha(M_Z)} - \frac{1}{2\pi}\,(b_1 + b_2 - \tfrac{8}{3}b_3)\log\left(\frac{M_X}{M_Z}\right)\right], \qquad (1\text{ cont'd})$$

where one has $b_1 = 11$, $b_2 = 1$ and $b_3 = -3$ in the SUSY case. In principle M_X is unknown and the formulae (1) give us a constraint between the values of $\sin\theta_w$ and α_s consistent with unification. This constraint is shown numerically in fig. 1 in which it is represented as a line in the $\sin\theta_w$–α_s plane. Different points in the line correspond to different values for M_X ($\log_{10}M_X$ is shown at various points on the line). We will not attempt here to include a detailed treatment of the errors. We have included an error band corresponding to an uncertainty of ± 0.01 in the resulting value for α_s. There are different sources of errors coming from the uncertainty in the low-energy and superheavy thresholds, two-loop effects etc. (see e.g., ref. [7] for a detailed discussion of these points). The success of the SUSY-GUT predictions correspond to this line going through the experimental results also depicted in the figure (α_s is taken from jet event shape analysis). On the other hand the lower curve in the figure corresponds to the non-supersymmetric GUT result. It is clear that this latter case is ruled out.

Let us go now to the supersymmetric string case.



.. I'banez

Fig. 1. Constraints in the $\sin^2\theta_w(M_Z)$–$\alpha_s(M_Z)$ plane coming from unification of gauge coupling constants.

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$\frac{1}{\alpha_s(M_Z)} =$

$\times\left[\frac{1}{\alpha(M}\right.$

$\times\log\left(\frac{M}{}\right.$

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possible explanations of discrepancy

- "desert" does not exist, more layers of intermediate symmetry breaking

- In ST at M_{GUT}: $SU(5) \to SU(3) \times SU(2) \times U(1)$ — Lewellen

  $\Rightarrow K_{SU(5)} > 1$ necessary for standard Higgs mechanism

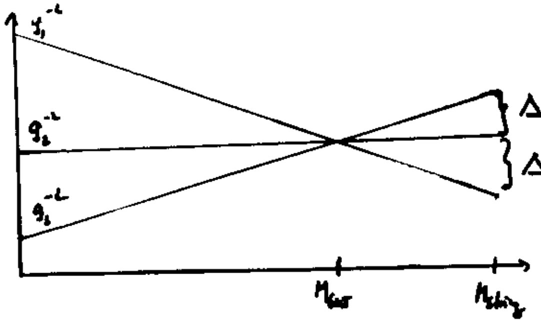
  or "flipped" $SU(5) \times U(1)$ — Antoniadis, Hagelin, Nanopoulos, Ellis

- $K_1 \neq 5/3$ but smaller; — Ibanez

  $\Rightarrow$ "exotic" representations of $SU(3)/SU(2)$ — Schellekens

  and/or fractionally charged particles

- Large threshold corrections at M_{string} — Ibanez, Lüst, Ross



constraint on size and sign

Computation of $\Delta_a^{(1)}$

in F.T.:

Weinberg

in S.T.:

Kaplunovsky

Heavy & light modes run in loop $\Rightarrow$ need to know string partition function

example: $N=1$ orbifolds

T^i = "untwisted" moduli

can evaluate string loop diagram and find

$$\Delta_a(T^i) = -\sum_i B_a^i \ln\left[|\eta(iT^i)|^4 (T^i + \bar{T}^i)\right]$$

Dixon, Kaplunovsky, J.L., Antoniadis, Gava, Narain, Taylor

B_a^i = const., computable from massless spectrum

$$\eta(iT) = q^{\frac{1}{24}} \prod_{n=1}^{\infty} (1 - q^n) \, ; \quad q = e^{-2\pi T}$$

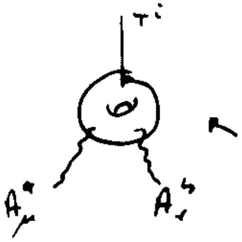
Puzzle: $\boxed{\partial_i \partial_{\bar{j}} \Delta_a \neq 0}$ — Kähler (holomorphic) anomaly

$\boxed{\Delta_a \neq \mathrm{Re}\, f^{1\,loop}}$

contradiction with $N=1$ supersymmetry?

No

technically simpler is the computation of the CP-odd part of

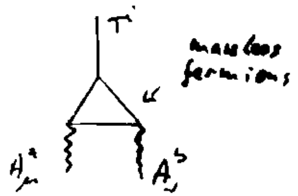


which computes $\left.\frac{\partial\theta}{\partial T^i}\right|_{T^i=\langle T^i\rangle}$

related to g_a^{-2} via the supersymmetric relation

$$\left.\frac{\partial g_a^{-2}}{\partial T^i}\right|_{T^i=\langle T^i\rangle} = i \left.\frac{\partial\theta}{\partial T^i}\right|_{T=\langle T^i\rangle}$$

The string diagram includes the graph



coupling of T^i to fermions

$$\mathcal{D}_\mu\chi^I = \partial_\mu\chi^I + \Gamma^I_{iK}(\partial_\mu T^i)\chi^K - K_\mu\chi^I + \dots \quad \text{matter fermions}$$

$$\mathcal{D}_\mu\lambda^a = \partial_\mu\lambda^a + \qquad + K_\mu\lambda^a + \dots \quad \text{gauginos}$$

$$\Gamma^I_{iK} = Z^{I\bar{J}}\,\partial_i Z_{\bar{J}K} \qquad \text{Christoffel connection}$$

$$K_\mu = \tfrac{1}{4}\left[\partial_j \hat{K}\,\partial T^j - \bar{\partial}_{\bar{j}}\hat{K}\,\partial\bar{T}^{\bar{j}}\right] \quad \text{Kähler connection}$$

$\Rightarrow$ In any locally supersymmetric field theory ($d=4$) the low energy gauge couplings obey

$$\boxed{\begin{aligned} g_a^{-2}(\mu) = {} & \mathrm{Re}\,f_a + \frac{b_a}{8\pi^2}\ln\frac{M_{Pl}}{\mu} + \frac{c_a}{16\pi^2}\hat{K} \\ & - \sum_R \frac{T_a(R)}{8\pi^2}\ln\det Z_{(R)}(\mu) + \frac{T(G)}{8\pi^2}\ln g_a^{-2}(\mu) \end{aligned}} \quad (*)$$

$\mathrm{Tr}_R(T^aT^b) = T_a(R)\,\delta^{ab}$, $\quad T_a(G) \equiv T_a(\text{adjoint})$

$c_a = \sum_R n_R T_a(R) - T(G)$, $\quad b_a = \sum_R n_R T_a(R) - 3T_a(G)$

<u>Remarks:</u>

- $\partial_i\bar{\partial}_{\bar{j}}\, g_a^{-2}$ (at 1 loop) is determined by the spectrum and tree level couplings of the massless modes
- $f_a = f_a^{tree} + \frac{f_a^{1\,loop}}{8\pi^2}$, the threshold contribution of the massive modes is summarized in $f_a^{1\,loop}$

  $\Rightarrow$ f_a can be viewed as the Wilsonian coupling of the theory, $g_a^{-2}(\mu)$ as the momentum dependent (running) effective coupling
- Shifman & Vainshtein: $(*)$ is exact to all orders in perturbation theory

Anomalous symmetries:

① Kähler invariance

$$K \to K + F(T) + \bar{F}(\bar{T}) \quad \Rightarrow$$

$$g_a^{-2}(\mu) \to g_a^{-2}(\mu) + \frac{c_a}{16\pi^2}\,(F + \bar{F}) \qquad \text{Kähler anomaly}$$

② coordinate transformations

$$\left.\begin{array}{l} Q^I \to \gamma^I{}_J(T)\, Q^J \\ Z_{I\bar{J}} \to \gamma^{-1} Z \bar{\gamma}^{-1} \end{array}\right\} \Rightarrow$$

$$g_a^{-2} \to g_a^{-2} + \frac{2}{16\pi^2}\sum_R T_a(R)\left[\ln\det\gamma^{(R)} + \ln\det\bar{\gamma}^{(R)}\right]$$

Both anomalies can be cancelled by assigning a (holomorphic) 1 loop transformation

$$\boxed{f_a^{1loop} \to f_a^{1loop} - 2\,c_a F(T) - 4\sum_R T_a(R)\ln\det\gamma(T)}$$

(f^{tree} is invariant under both symmetries)

$\Rightarrow$ f_a^{1loop} acts as a local WZ-counterterm

Compare to string theory

ⓐ Dilaton dependence

use universal tree level couplings of dilaton

$$f_a = k_a S + \frac{1}{16\pi^2} f^{1loop}(T)$$

$$\hat{K} = -\ln(S+\bar{S}) + \hat{K}(T,\bar{T})$$

$$Z_{I\bar{J}} = Z_{I\bar{J}}(T,\bar{T})$$

$$\Rightarrow \frac{1}{g_a^2(\mu)} = k_a \operatorname{Re} S + \frac{b_a}{8\pi^2}\left[\ln\frac{M_{pl}}{\mu} - \frac{1}{2}\ln(S+\bar{S})\right]$$

$$+ \frac{1}{16\pi^2}\left[\operatorname{Re} f_a^{1loop}(T) + c_a \hat{K}(T,\bar{T}) - 2\sum_R T_a(R)\ln\det Z^{(R)}(T,\bar{T})\right]$$

use: $M_{string}^2 \sim \frac{1}{\alpha'} \sim M_{pl}^2\, g_{string}^2 \sim \frac{M_{pl}^2}{S+\bar{S}}$

$$\Rightarrow \boxed{\begin{array}{l} g_a^{-2}(\mu) = k_a \operatorname{Re} S + \frac{b_a}{8\pi^2}\ln\frac{M_{string}}{\mu} + \frac{\Delta_a(T,\bar{T})}{16\pi^2} \\[2ex] \Delta_a(T,\bar{T}) = \operatorname{Re} f_a^{1loop}(T) + c_a \hat{K}(T,\bar{T}) - 2\sum_R T_a(R)\ln\det Z^{(R)}(T,\bar{T}) \end{array}}$$

(b) $N=1$ orbifolds, I = untwisted moduli

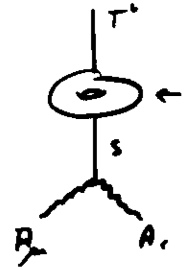
$$\hat K = -\sum_i \ln(T^i+\bar T^i) \;, \qquad Z_{I\bar J} = \frac{\delta_{I\bar J}}{\prod_i (T^i+\bar T^i)^{q_I^i}}$$

cont. computed

$$\Rightarrow \Delta_a = \mathrm{Re}\, f_a^{1loop} + \sum_i \alpha_a^i \ln(T^i+\bar T^i)$$

$$\alpha_a^i = T_a(G) + \sum_I T_a(R_I)\,(2 q_I^i - 1)$$

String amplitude also contains a universal (gauge group independent) 1PR diagram:



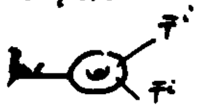
← 1 loop "mixing" of S, T^i

given by $\dfrac{K^{(1)}(T,\bar T)}{(S+\bar S)}$

GREEN-SCHWARZ-TERM

Derendinger, Ferrara, Kounnas, Zwirner; Ovrut + Cardoso

$K^{(1)}(T,\bar T)$ can be determined from

- consistency of F.T and S.T. analysis
- direct string loop computation  Antoniadis, Gava, Narain, Taylor

find $K^{(1)}(T,\bar T) = \sum_i \frac{\delta_{GS}^i}{8\pi^2} \ln\left[|\eta(iT^i)|^4 (T^i+\bar T^i)\right]$

$\Rightarrow$ altogether F.T and S.T. are consistent if

$-B_a^i = \alpha_a^i + k_a \delta_{GS}^i$ has been checked for

Constraints on $f_a^{1loop}(T)$ (in orbifolds)

Contribution of massive string modes sits in $f_a^{1loop}(T)$. this is the "stringy" part.

f^{1loop} can be constraint from anomaly equation.

For simplicity: only one T $\Rightarrow$ $SL(2,\mathbb{Z})$ is a symmetry of the string vacuum (T-duality)

$\Rightarrow$ physical gauge couplings g_a^{-2} should be invariant

$SL(2,\mathbb{Z})$: $T \to \dfrac{aT-ib}{icT+d}$, $ad-bc=1$, $a,b,c,d\in\mathbb{Z}$

$\Rightarrow K = -\ln(T+\bar T) \to K + \ln|icT+d|^2$

$\ln\det Z \to \ln\det Z - \sum_I q_I \ln|icT+d|^2$

$\Rightarrow$ Kähler transformation with $F(T) = \ln(icT+d)$

coordinate – – $\gamma_{\bar J}^{I}(T) = \delta_J^I (icT+d)^{q_I}$

$\Rightarrow g_a^{-2}$ invariant if $\boxed{f_a^{1loop} \to f_a^{1loop} + 2\alpha_a \ln(icT+d)}$

$\Rightarrow \boxed{f_a^{1loop} = 2\alpha_a \ln \eta(iT) + H[j(T)]}$

$SL(2,\mathbb{Z})$ invariant j-function

additional physical requirements

(i) g_a^{-2} should not be singular at finite T

(ii) at large T g_a^{-2} should diverge at most like T^3

reason: $\frac{g_4^{-2}}{R^6} = g_{10}^{-2}$

4d-gauge coupling

radius of M_6

10-d gauge coupling

In the decompactification limit $R \to \infty$, g_4^{-2} should 'approach' g_{10}^{-2} or in other words $\frac{g_4^{-2}}{R^6}$ should stay fixed.

$\Rightarrow$ f_a^{1loop} should diverge at most as $T^3 (\sim R^6)$

(i) + (ii) $\Rightarrow$ $H[j(T)] = 0$

$\Rightarrow$ $f_a^{1loop} = 2\alpha_a \ln \eta^2(iT)$ $\quad \left(\to -\frac{\pi}{12}\alpha_a(T+\bar T)\right)$

$\Rightarrow$ uniquely reconstructed f_a^{1loop}!

Mayr

Relation with topological index

Bershadsky
Cecotti
Ooguri
Vafa

• (2,2) vacua recall

$K = K_1(T,\bar T) + K_2(U,\bar U)$, $\quad T \hat= (1,1)$-moduli, $U \hat= (2,1)$-moduli

$Z^{(11)}_{i\bar j} = e^{\frac{1}{3}(K_2-K_1)} G_{1\,i\bar j}$ $\qquad G_{1\,i\bar j} = \partial_{T^i}\partial_{\bar T^{\bar j}} K_1$

$Z^{(21)}_{i\bar j} = e^{\frac{1}{3}(K_1-K_2)} G_{2\,i\bar j}$ $\qquad G_{2\,i\bar j} = \partial_{U^i}\partial_{\bar U^{\bar j}} K_2$

$$\Rightarrow \quad \Delta_{E_8} = \mathrm{Re}\, f_8^{1loop} - 30(K_1+K_2)$$

$$\Delta_{E_6} = \mathrm{Re}\, f_6^{1loop} + (5h^{(1,1)} + h^{(1,2)} - 12)K_1 - 6\ln\det G_1 + (5h^{(1,2)} + h^{(1,1)} - 12)K_2 - 6\ln\det G_2$$

• observe mirror symmetry $K_1 \leftrightarrow K_2$, $h^{(1,1)} \leftrightarrow h^{(1,2)}$

$$\Delta_{E_6} - \Delta_{E_8} = \mathrm{Re}(f_8 - f_6) + 6\left[(3 + h^{(1,1)} - \tfrac{\chi}{12})K_1 - \ln\det G_1\right] + 6\left[(3 + h^{(2,1)} + \tfrac{\chi}{12})K_2 - \ln\det G_2\right]$$

$\partial_T\partial_{\bar T}(\Delta_{E_6} - \Delta_{E_8}) = 12\,\partial_T\partial_{\bar T} F_1$ $\leftarrow$ top. index defined in BCOV

can show for all (2,2) vacua:

$$\boxed{\Delta_{E_6} - \Delta_{E_8} = 12\, F_1}$$

Example: Quintic three fold $W = X_1^5 + X_2^5 + X_3^5 + X_4^5 + X_5^5 - 5\psi\, x_1 x_2 x_3 x_4 x_5$

$h_{1,1} = 1, \quad h_{1,2} = 101$

CDGP: compute $K_1(\psi)$,

$\psi = 0$ $\hat{=}$ Gepner point

$\psi = 1$ $\hat{=}$ conifold singularity

modular group $\mathbb{Z}_5$: $\psi \to e^{\frac{2\pi i}{5}} \psi$, K_1 chosen inv

$$\Delta_6 - \Delta_8 = \mathrm{Re}(f_6^1 - f_8^1) - 124\, K_1(\psi,\bar\psi) - 6 \ln G_1(\psi,\bar\psi)$$

should be invariant and regular everywhere except at $\psi = 1$

$\Rightarrow$ Ansatz: $f_6 - f_8 = \ln\left[\psi^{5a}(1-\psi^5)^b\right]$

$\underline{\psi = 0}$: $K \to -\ln|\psi|^2$; $G \to$ const.

$\Rightarrow \boxed{5a = 248}$

$\underline{\psi = \infty}$: $\Delta_6 - \Delta_8 \longrightarrow 25(\ln\psi^5 + \ln\bar\psi^5)$ BCOV

$G \to \frac{1}{\psi\bar\psi}$

$\Rightarrow \boxed{b = -2}$

$\Rightarrow f_6 - f_8$ reconstructed uniquely ! Can be generalized

Summary

- gauge coupling unification in string theory
- $M_{GUT} \approx 3 \cdot 10^{16}$ GeV is close to but not identical with $M_{string} \approx 5 \cdot 10^{17}$ GeV
- gauge couplings are moduli dependent at 1 loop
- $\Delta_a(T,\bar T)$ can be computed explicitly or constrained via anomaly equation
- $f^{1\,loop}(T)$ can be viewed as local WZ-counterterm.