INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE CENTRATOM TRIESTE

SMR.762 - 54

IV

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

13 June - 29 July 1994

QUANTUM MECHANICS OF BLACK HOLES

STEVEN GIDDINGS
Dept. of Physics
University of California
Santa Barbara, CA
USA

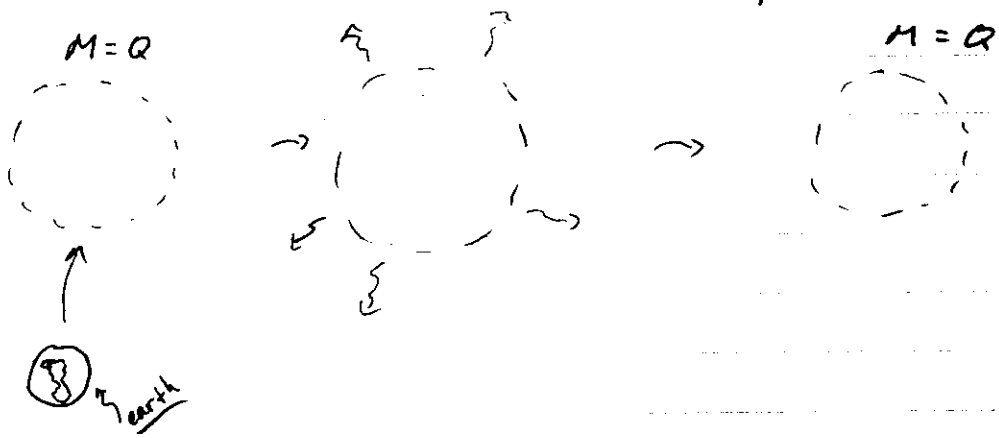
Please note: These are preliminary notes intended for internal distribution only.

Black hole pair production and remnants

- another interesting aspect of QM of Black holes
- relevant to remnant problem?

info loss	energy conservation
info return	locality / causality
remnants	naive crossing

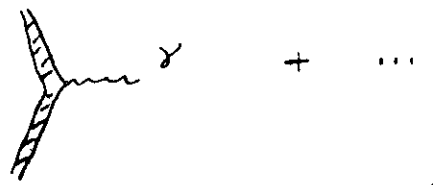
(extremal) RN bh : $\sim e^{\infty}$ states:



- finitely produced

$\Rightarrow$ model for remnants

Revisit ∞ prod: (magnetic field for convenience)

S_{eff} =  + ...

$$= \int d^4x \sum_{a\ \text{add}} - \left| D_\mu \; I_a \right|^2_{\text{add}} - m_A |I_A|^2_{\text{add}} + \text{higher dim}$$

$D_\mu = \partial_\mu + iqA_\mu$ just says has charge

$$\Gamma_{vac} \cdot V \cdot T = \text{Im ln} \int \underbrace{\prod_{a\ \text{add}} d\mathcal{O} I_a}\; e^{i S_{eff}}$$

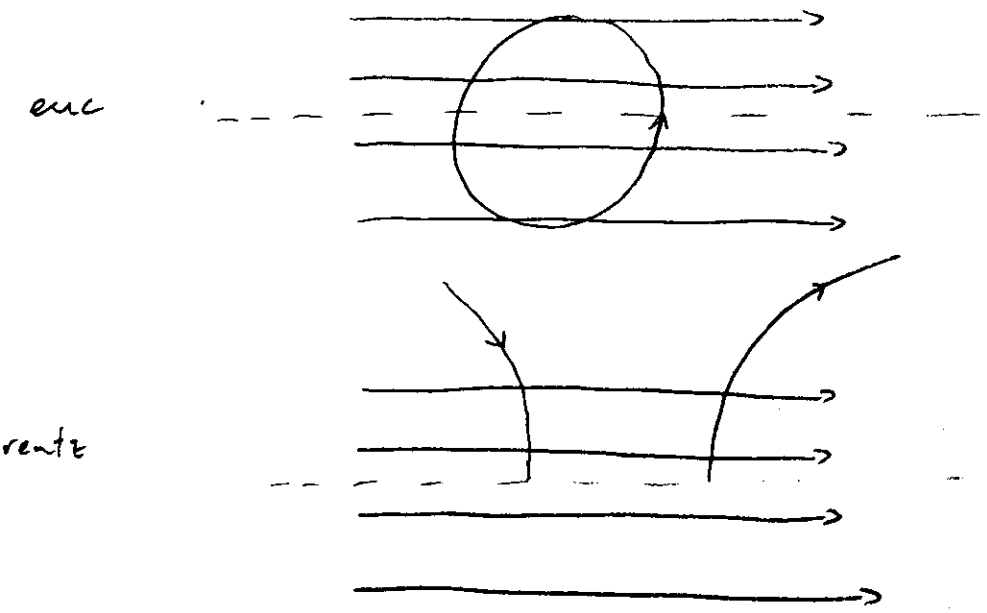
$$= \sum_{a\ \text{add}} \text{Im ln det}\left(-D^2 + m^2\right) + \ldots \quad \leftarrow \text{small corr.}$$

$$= \sum_{a\ \text{add}} \text{Im Tr ln} \left(\quad \right) + \ldots$$

$$= \sum_{a\ \text{add}} \text{Im} \int \frac{dT}{T} \int dx \; \underbrace{\langle x | \, e^{-H_{\text{1 particle}} T} | x \rangle}_{x} + \ldots$$

$$\int \mathcal{D}x \; e^{-\int_0 d\tau \left[\frac{1}{2}\dot{x}^2 + iqA_\mu^0 \dot{x}^\mu + \frac{m^2}{2}\right]}$$

Instanton



$$\Rightarrow \Gamma \sim \sum_a e^{-\frac{\pi m^2}{qB}} + \ldots = \infty$$

↑ add ↓ add.

RN, Dilaton } Black holes : analagous solutions !

RN : Ernst

Dilaton : DGKT

Refs for this talk : Garfinkle, S.G., and Strominger gr-qc/9306023

Dowker, Gauntlett, Giddings & Horowitz hep-th/931217

SG - Comments on Info ... hep-th/9310101

Metric: $ds^2 = \frac{\Lambda^2}{A^2(x-y)^2}\left[+G(y)\,dt^2 - G^{-1}(y)\,dy^2 + G^{-1}(x)\,dx^2\right] + \frac{G(x)}{\Lambda^2 A^2 (x-y)^2} d$

$A_\varphi = \dots$

$G(\xi) = (1-\xi^2 - r_+ A \xi^3)(1 + r_- A \xi)$

↑ zeroes ξ_2, ξ_3, ξ_4 ← ξ_1

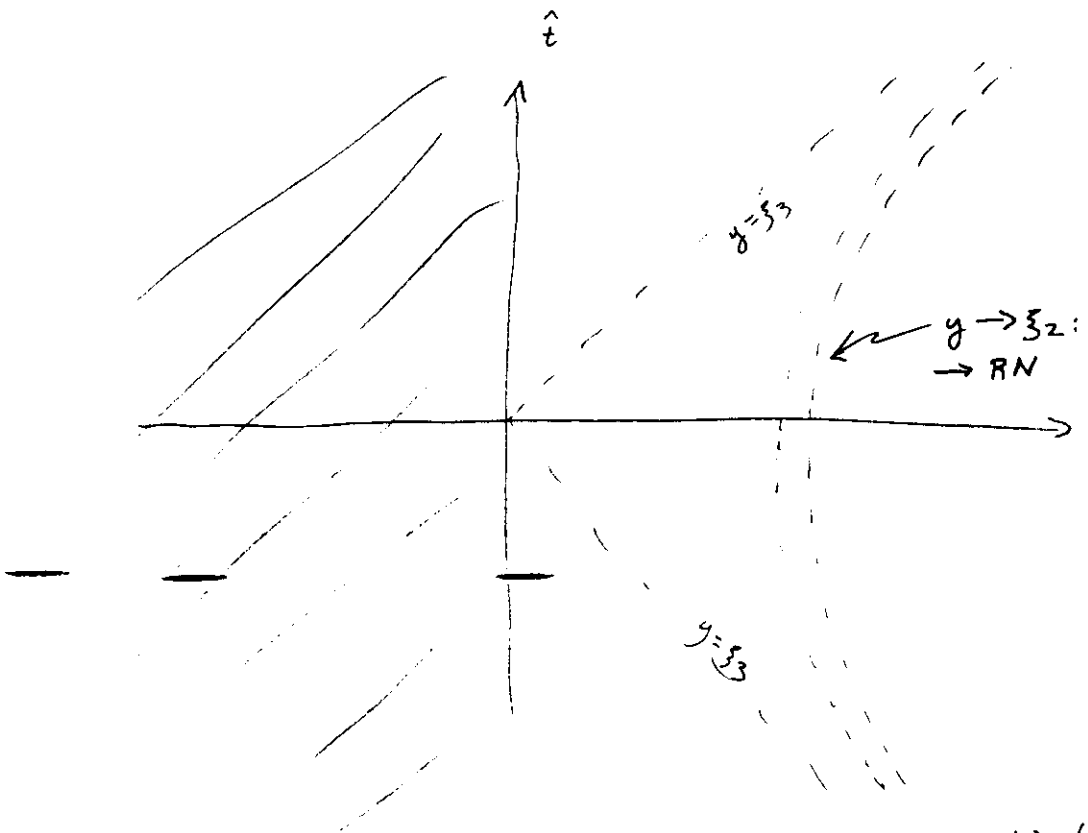
$\Lambda = (1 + qBx)^2 + \frac{B^2 G(x)}{4A^2(x-y)^2}$

$q = \sqrt{r_+ r_-}$

A, B params ~ acceleration, B field

smooth ~ $A = A(B, r_+, r_-)$

mass & charge

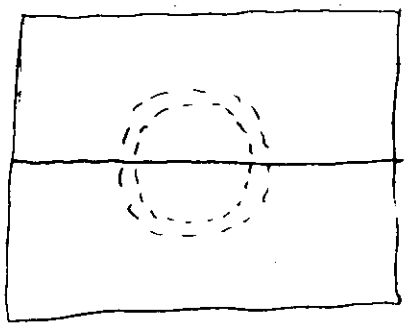


asymptotically Melvin

$ds^2 = \left(1+\frac{B^2\rho^2}{4}\right)^2\left[-dt^2 + dz^2 + d\rho^2\right.$

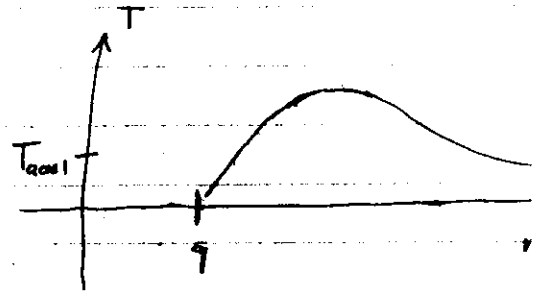
$\left. + \left(1+\frac{B^2\rho^2}{4}\right)^{-1}\rho^2 d\varphi^2\right.$

$t = i\tau$



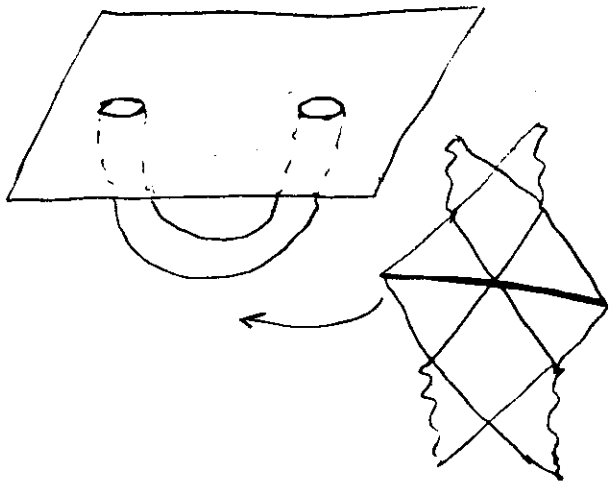
smoothness $\longleftrightarrow$ $T_H = T_{accel} \sim A$

$A \ll 1$:



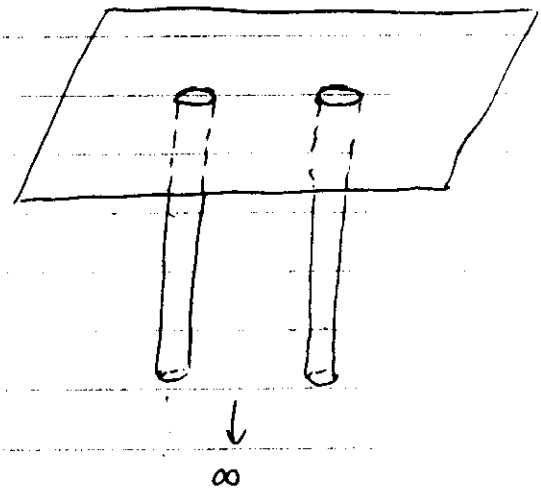
1) $m = q$ + small

"Wheeler wormhole"

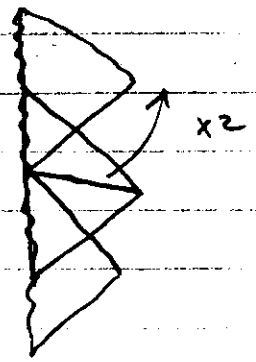


2) $m = q$; T ambiguous

"extremal pair"



$S_{inst} = \frac{\pi q}{\beta} + O(q^2)$



$\Gamma \propto e^{-S_{inst}}$

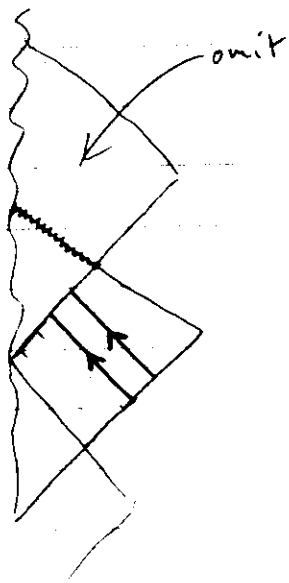
?

Naive expectation: $N \to \infty$

Source : fluctuations about soln. (nearby configurations)

∞ states : can feed ∞ info into BH. :

eg. (neglect BR) :



recall modes :

$$\sim \frac{e^{-i\omega(t \pm r^*)}}{R} Y_{lm}$$

u

horizon

∞ states $\leftrightarrow$ ∞ volume in t $\leftrightarrow$ ∞ volume in r_*

[coordinates bunch @ horizon]

qm evolution : $\langle B | e^{-iHt} | A \rangle \sim \int \mathcal{D}g \, \mathcal{D}_*f \, e^{iS[f,g]}$

$$(\delta\phi, \delta\phi)_* = \int d^2x \sqrt{g_*} \, \delta u \, \delta u$$

w/ $ds_*^2 = -dt^2 + dr_*^2 = \frac{ds^2}{-g_{tt}}$

$\therefore$ corresponds to $\omega^2 < \Lambda^2$ (cf, eg., Fumita)

w/ $g_* \to g$: corresponds to $\frac{\omega^2}{-g_{tt}} < \Lambda^2$

"evolve past cutoff"

⇒ To include ∞ states in functional integral:

$$\int_{\text{instanton}} \mathcal{D}_x f \; e^{iS[f]}$$

eg. 2d: $\int_{\text{inst}} \mathcal{D}_x f \; e^{iS[f]} = e^{-S_L(-\ln(-g_{tt}))} \int \mathcal{D} f \; e^{iS[f]}$ (δS_{PL})

effect on calculation? $\int d^2x \left[(\nabla \ln g_{tt})^2 + R \ln g_{tt} \right]$

$T_{\mu\nu}$:

Wheeler: $T^{(L)}_{\mu\nu} = \infty$ regular metric ⇒ $\langle T_{\mu\nu} \rangle < \infty$

Extremal: $\langle T_{\mu\nu} \rangle = \infty$ (either)

T diverges ⇒ strong coupling

SC approx fails

∴

A) $\Gamma \neq \mathcal{N} \underset{\infty}{} e^{-S_{inst}}$ - failure of naive crossing

B) Need planck physics

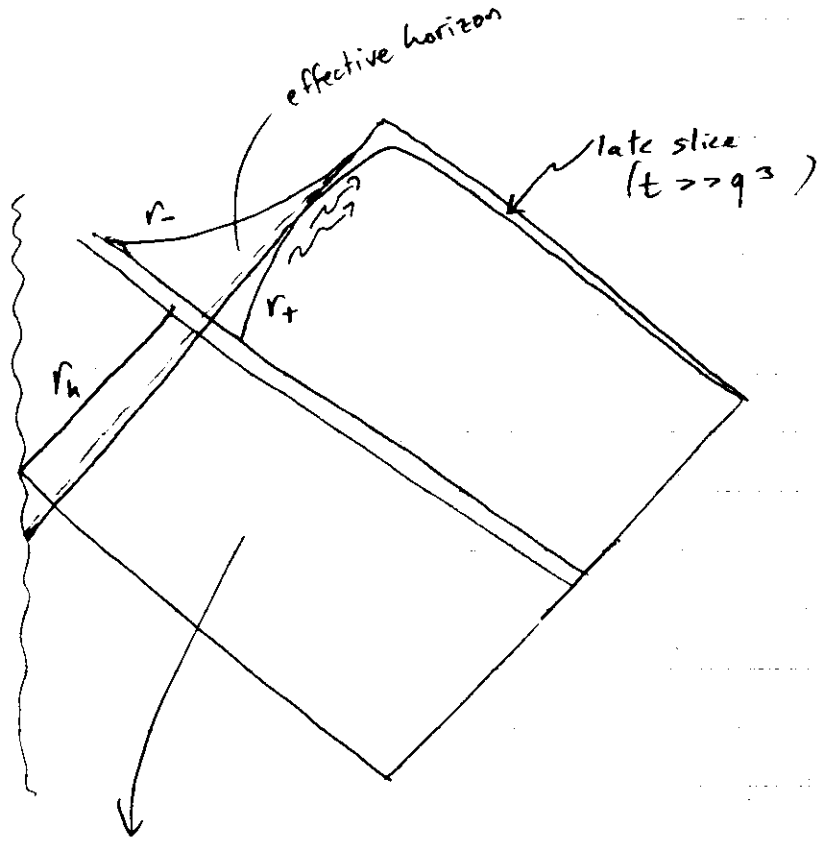
— — C) — finite ??? —

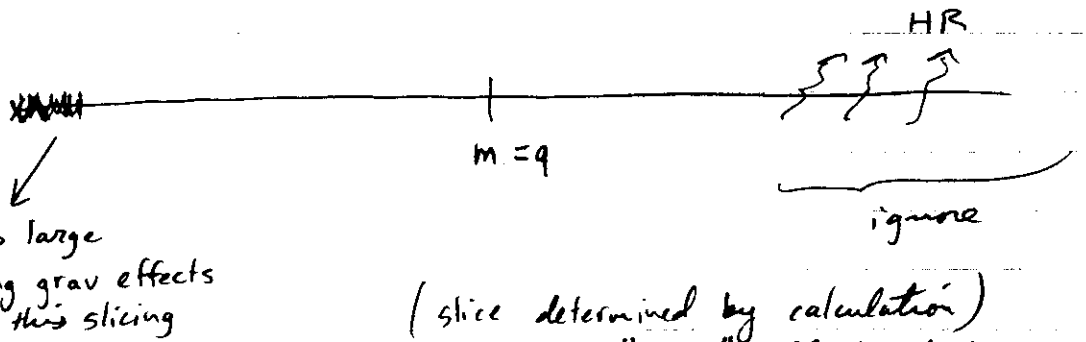
Questions

1) why does naive crossing fail?
   no min coupling ??

2) why planck physics relevant?

RN "remnant" w/ BR: (cf. Strominger-Trivedi's analysis)





T → large
strong grav effects
w/ this slicing

(slice determined by calculation)
reminiscient of "return" efforts, but no nonlocality

Conclusions:

- BH evaporation = window on planck physics
- sharp conflict of GR & QM : info problem
- diversity of opinion
- BH pair production = rich & interesting { Hawking rad, Accel rad, remnants }
- A theory of remnants might exploit tricks of RN!