INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

SMR.762 - 7

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

13 June - 29 July 1994

INTRODUCTION TO GUTS AND SUPERSYMMETRY

A. MASIERO
INFN
University of Padova
Padova, Italy

ELW STANDARD MODEL

A (SHORT) APPRAISAL

- Renormalizable
  Spont. broken gauge theory

- Exceptional agreement with all experimental information so far available

- "Unification" of elw + weak inter.

- GIM mechanism $\Rightarrow$ FCNC suppression

- prediction of existence of CP $\neq$

- B and L automatic global symm.

- economical Higgs sector

* No true Unification

- $e, G_F \Rightarrow g, g'$
- g_s, g far apart
- gravity left out

* unpredictivity in the fermion spectrum

- intra-family level m_e/m_u?
- inter-family level m_e/m_μ?
- # generations
- parameters of the CKM matr

* arbitrariness of the Higgs sect

* G_F, M_{Pl} no relation

BASICS ON GRAND Unified Theories

- Physical (renorm.) coupling constants and fields (wave-function renorm. const.) are defined only when subtractions to render the theory finite are performed
- the subtraction scale μ is arbitrary $\Rightarrow$ change of μ is equivalent to a change in the scale of all momenta
- RGE $\Rightarrow$ correlation between Green's functions for one set of momenta and coupl. const. to Green's functions for a scaled set of momenta and different values of the coupl. const.

$$\frac{d\alpha_i}{d\ln q^2} = b_i \, \alpha_i^2 + O(\alpha_i^3)$$

$$b_i = -\frac{1}{4\pi}\left[\frac{11}{3}C_2(G_i) - \sum_f \frac{4}{3}T(R)_f + \text{scalar contrib.}\right]$$

$C_2(G_i)$ = eigenvalue of the Casimir operator of G_i

$$T(R) = \frac{d(R)\cdot C_2(R)}{r}$$

r = n° generators

$C_2(R)$ = Casimir for repres. R

$SU(N)$ $\quad C_2(SU(N)) = N$

$$T(N) = N.\ \frac{N^2-1}{2N}\cdot\frac{1}{N^2-1} = 1/2$$

$$b_3 = -\frac{1}{4\pi}\left[\frac{11}{3}\times \overset{C_2(3)}{3} - \frac{4}{3}\times\underset{T(3)}{\frac{1}{2}}\times\overset{2\text{ triplets}}{2}\times n_{gener} + \underset{0\text{ in SM}}{\overset{\text{scalars}}{\cdots}}\right]$$

$$b_2 = -\frac{1}{4\pi}\left[\frac{11}{3}\times 2 - \frac{4}{3}\times\frac{1}{2}\times 4\times\underset{\text{only left-handed}}{\frac{1}{2}}\times n_{gener} + \overset{1\text{ Higgs doublet}}{\cdots}\ \frac{1}{6}\right]$$

$$b_1 = -\frac{1}{4\pi}\left[0 - \frac{20}{9}n_{gener.}\right] \qquad Y = Q - T_3$$

redefinition of Y to obtain a correctly normalized generator

$$\mathrm{Tr}\, T_3^2 = \mathrm{Tr}\, Y'^2$$

$$\mathrm{Tr}\, T_3^2 \Big|_{\text{over 1 family}} = \left(\frac{1}{4} \times 2 \right) \times 4 = 2$$

$$\mathrm{Tr}\, Y^2 \Big|_{\text{over 1 family}} = \frac{1}{36} \times 6 + \frac{1}{4} \times 2 + \left(\frac{4}{9} + \frac{1}{9} \right) \times 3 + 1 = \frac{10}{3}$$

$$Y' \equiv \sqrt{\frac{3}{5}}\, Y \quad \text{then} \quad \mathrm{Tr}\, Y'^2 = \mathrm{Tr}\, T_3^2$$

$$\Downarrow$$

$$g_1' = \sqrt{\frac{5}{3}}\, g_1 \quad \Rightarrow \quad Q = T_3 + \sqrt{\frac{5}{3}}\, Y'$$

$$b_1' = -\frac{1}{4\pi} \left[0 - \frac{4}{3}\, n_{\text{gener}} + \underbrace{\cdots}_{\text{scalar}} \right]$$

Question :

given the above values for b_1', b_2, b_3

is there a value of q^2 $(q^2 = M_X^2)$

at which $\alpha_1' = \alpha_2 = \alpha_3$, i.e. such that

$$\alpha_1'(M_X^2) = \alpha_2(M_X^2) = \alpha_3(M_X^2) \equiv \alpha_G$$

$$\int_{q^2}^{M_X^2} \frac{d\alpha_i}{\alpha_i} = b_i \int_{q^2}^{M_X^2} d\ln q^2$$

$q^2 \to$ low value

$$\frac{1}{\alpha_i(q^2)} = \underbrace{\frac{1}{\alpha_i(M_X^2)}}_{\equiv \alpha_G} + b_i \ln \frac{M_X^2}{q^2}$$

inputs : α_s , α_{em} at low q^2 (for instance $q^2 = M_Z^2$)

unknown quantities to be determined :

M_X, α_G, $\sin^2\theta_W$

$$\begin{cases} \dfrac{1}{\alpha_3(q^2)} = \dfrac{1}{\alpha_G} + b_3 \ln \dfrac{M_X^2}{q^2} & (1) \\[2ex] \dfrac{1}{\alpha_2(q^2)} = \dfrac{1}{\alpha_G} + b_2 \ln \dfrac{M_X^2}{q^2} & (2) \\[2ex] \dfrac{1}{\alpha_1'(q^2)} = \dfrac{1}{\alpha_G} + b_1' \ln \dfrac{M_X^2}{q^2} & (3) \end{cases}$$

$e = g_1 \cos\theta_W = g_2 \sin\theta$

$$\alpha_{em} = \frac{3}{5} \alpha_1' \cos^2\theta_W = \alpha_2 \sin^2\theta_W$$

(1) − (2) $$\frac{1}{\alpha_3(q^2)} - \frac{1}{\alpha_2(q^2)} = (b_3 - b_2) \ln \frac{M_X^2}{q^2}$$

(1) − (3) $$\frac{1}{\alpha_3(q^2)} - \frac{1}{\alpha_1'(q^2)} = (b_3 - b_1') \ln \frac{M_X^2}{q^2}$$

$$\alpha_2(q^2) = \frac{\alpha_{em}(q^2)}{\sin^2\theta_W(q^2)} \qquad \alpha_1'(q^2) = \frac{\alpha_{em}(q^2)}{\cos^2\theta_W(q^2)} \frac{5}{3}$$

2 eqs with 2 unknowns: M_X^2, $\sin^2\theta_W$

M_X :

$$\frac{1}{\alpha_{em}(q^2)} = \frac{8}{3}\frac{1}{\alpha_3(q^2)} + \frac{11}{2\pi} \ln \frac{M_X^2}{q^2}$$

using previous b coeff.
without scalar contrib.

$$\boxed{M_X \sim 10^{15} \text{ GeV}}$$

!!

$\sin^2\theta_W$:

$$\sin^2\theta_W(q^2) = \alpha_{em}(q^2)\left[\frac{1}{\alpha_3(q^2)} + \frac{11}{12\pi} \ln \frac{M_X^2}{q^2}\right]$$

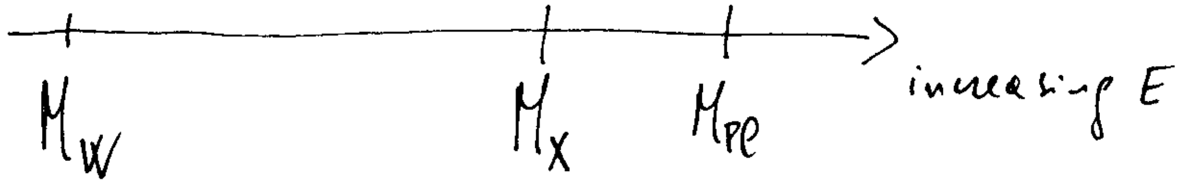
$$= \frac{5}{9}\frac{\alpha_{em}(q^2)}{\alpha_3(q^2)} + \frac{1}{6}$$

$$\boxed{\sin^2\theta_W(\text{low } q^2) \simeq 0.2}$$

10 - 100 GeV

!!

THE "BIG DESERT" PICTURE



$E > M_X \Rightarrow G \supset SU(3) \times SU(2) \times U(1)$

at $E \sim M_X \Rightarrow G \xrightarrow[\text{spont. break.}]{} SU(3) \times SU(2) \times U(1)$

vector (gauge) bosons of $G / SU(3) \times SU(2) \times U(1)$

get a mass $\sim M_X$

gauge bosons of $SU(3) \times SU(2) \times U(1)$ massless at this stage

ordinary fermions massless

$M_W < E < M_X$ Desert

$E \sim M_W$ $SU(3) \times SU(2) \times U(1) \xrightarrow[\text{spont. break.}]{} SU(3) \times U(1)_{em}$

W, Z mass fermion masses

1. WHY SUSY

- SOFTENING OF QUANTUM DIVERGENCES
  successes of quantum field theories associated with Lagrangians whose symmetries play a crucial role in the cancellation of divergences naively expected on dimensional grounds

- HELP IN SOLVING THE GAUGE HIERARCHY PROBLEM (absence of chiral protection for scalar mas

- THE LOCAL VERSION OF SUSY (SUPERGRAVIT IS A GOOD CANDIDATE FOR A POSSIBLE UNIFICATION OF ALL ELEMENTARY INTERACTION (in any case, convergence properties of quantum gravity highly improve in the SUSY version)

- SUSY IS THE MOST GENERAL SYMMETRY OF THE S-MATRIX (Haag, Lopuszanski, Sohnius)

THE GAUGE HIERARCHY PROBLEM

- CHIRAL PROTECTION OF FERMION MASSES

$\chi_L \; (1/2, 0)$ $\quad SU(2) \times SU(2)$ of Lorentz

$\chi_L^\alpha \chi_L^\beta \varepsilon_{\alpha\beta}$ $\quad \chi \in R$ of G (gauge group)

$\downarrow$ G invariant only if R real

$\chi_L \; (1/2, 0)$ $\quad \chi^c_L \; (1/2, 0)$

$\chi_L^\alpha \chi^{c\,\beta}_L \varepsilon_{\alpha\beta}$ $\quad \chi \in R, \; \chi^c \in R'$

if $R \times R'$ does not contain a G singlet also $\chi \chi^c$ mass term forbidden as long as G is unbroken

$\Rightarrow$ this is what happens in SM

$\Rightarrow$ fermion masses are constrained to be at most of the scale at which the elw symmetry breaks down

• GAUGE PROTECTION FOR THE

GAUGE BOSON MASSES

→ a gauge boson can acquire a mass only when its corresponding generator is "broken", i.e. for gauge bosons their masses are PROTECTED by a symmetry

• SCALAR MASSES DO NOT POSSESS AN ANALOGOUS CHIRAL OR GAUGE PROTECTION

Example: models with at least two widely different mass scales

$SU(2) \times U(1)$ breaking scale M_W ~

GUT breaking scale M_X ~

elw breaking $\rightarrow$ $\langle H \rangle \sim 10^{2}$ GeV

GUT breaking $\rightarrow$ $\langle \phi \rangle \sim 10^{15}$ GeV

$$V(H) = -\mu^2 |H|^2 + \lambda |H|^4 \quad ; \quad \langle H \rangle = \sqrt{\frac{\mu^2}{\lambda}}$$

$$M_W = \frac{g}{2}\sqrt{\frac{\mu}{\lambda}} \rightarrow \mu \sim 10^2 \text{ GeV}$$

However radiative corrections tend to induce a much larger effective μ^2:

$\rightsquigarrow m_H^2 \sim (10^{15}$ GeV

$\Rightarrow$ to keep $|\mu|$ small enough needs to calculate the radiative corrections to the scalar potential up to several orders in perturbation theory and then FINE-TUNE $|\mu|^2 \sim 10^2$ GeV to get $M_W \sim 10^2$ GeV and not 10^{15} GeV

$\underbrace{\text{FINE-TUNE}}_{\text{NATURALNESS PROBLEM}}$

This is a manifestation of [illegible]

the lack of protection for

scalar masses (if [illegible]

$m^2 \varphi^+ \varphi$ is G invariant [illegible]

Another aspect of the problem:

appearance of quadratic divergences

in the radiative corrections to scalar masses

$$\int^{\Lambda} d^4k \, \frac{1}{k^2-m^2} \sim \Lambda^2$$

$$\int^{\Lambda} d^4k \, \frac{m}{(k^2-m^2)} \ \text{[illegible]}$$

mass term
necessary to
realize the helicity flip

Effective theory: Λ? $\Lambda \sim$ GUT

$\Rightarrow m \ggg$ [illegible]

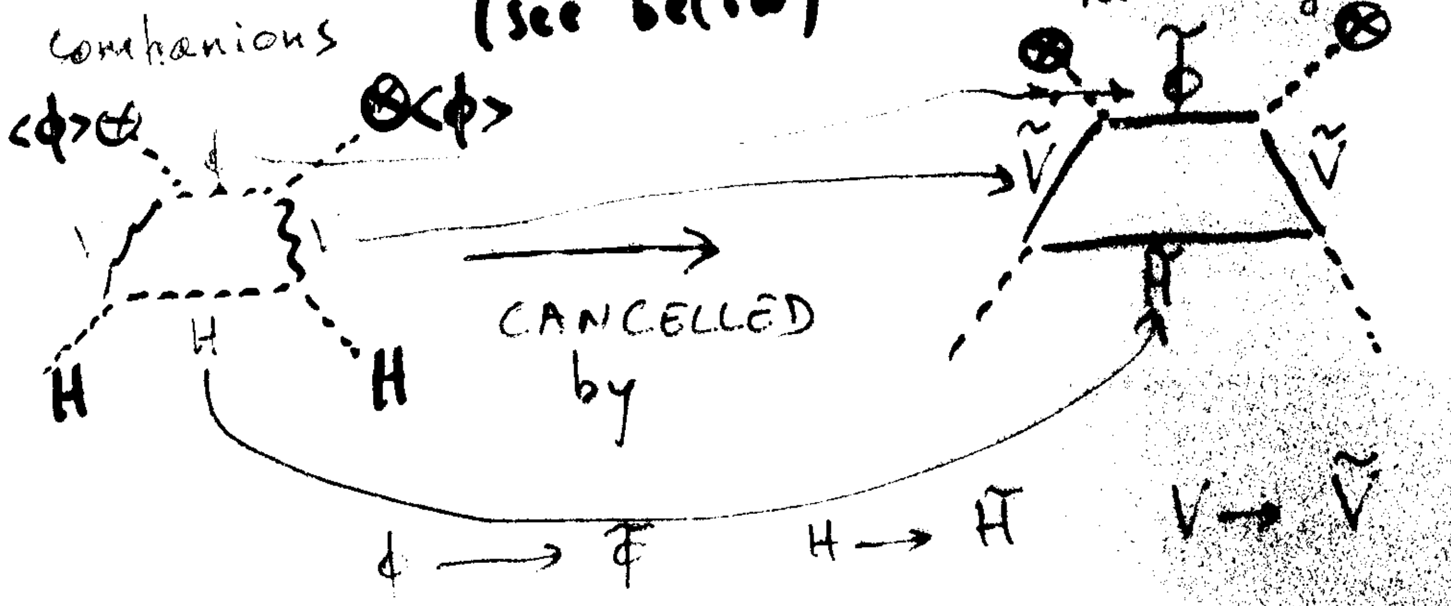
SUSY IS THE ONLY KNOWN EXCEPTION WHERE ALSO SCALAR MASSES ARE PROTECTED

→ it is an "induced" protection

scalars are put together with fermions of definite handedness → the chiral protection of fermion masses acts also on the masses of their scalar companions

(see below)

technically:



$\tilde{f}$, $\tilde{H}$, $\tilde{V}$ fermionic superpartners of ϕ, H, V

THE GAUGE HIERARCHY PROBLEM CONSISTS

OF TWO DIFFERENT PARTS

- WHY IS $M_X >>>> M_W$?

- ONCE WE HAVE FIXED $M_W <<< M_X$
AT TREE LEVEL IS IT POSSIBLE
TO GUARANTEE THIS HIERARCHY
AT ANY ORDER IN PERTURBATION THEORY?
(STABILITY OF THE GAUGE HIERARCHY)

SUSY HELPS FOR THE QUESTION

OF THE STABILITY

AS FOR THE FIRST QUESTION
ONLY A PARTICULAR CLASS OF SUPERGRAVITY
MODELS (NO-SCALE SUPERGRAVITY MODELS)
TRIES TO TACKLE IT.

Comment on the generality

SUSY as a symmetry of the S-matrix

- Coleman-Mandula theorem

the most general Lie algebra of symmetries of the S-matrix based on a local, relativistic quantum field theory in four-dimensional space-time contains

the en.-momentum operator P_m

the Lorentz rotation generator M_{mn}

a finite number of Lorentz scalar operators that belong to the Lie algebra of a compact Lie group

(ex.: Poincaré $\otimes$ $G_{\text{internal symmetries}}$)

no room for other operators which can transform a scalar into a fermion for instance

- Haag, Sohnius, Lopuszanski

catch in the previous theorem: Lie algebra

it is possible to avoid the constraints of the Coleman-Mandula theorem if one generalizes the notion of a Lie algebra to include algebras whose defining relations include ANTICOMMUTATORS AS WELL AS COMMUTATORS

(SUPERALGEBRAS or GRADED LIE ALGEBRA

$Q \to$ anticommuting operators

$X \to$ commuting operators

$$\{Q, Q'\} = X \qquad \underline{\underline{[X, X'] = X''}} \qquad [Q, X] = Q'$$

OF ALL THE GRADED LIE ALGEBRAS
ONLY THE SUPERSYMMETRY ALGEBRAS
GENERATE SYMMETRIES OF THE
S-MATRIX CONSISTENT WITH RELATIVISTI
QUANTUM FIELD THEORY !

In this sense SUSY
is the most general symmetry
of the S-matrix

Bagger - Wess

SPINOR REPRESENTATIONS OF THE LORENTZ GROUP

$$v^m \to v'^m = \Lambda^m{}_n v^n$$

$$\eta_{mn} = \Lambda^p{}_m \eta_{pq} \Lambda^q{}_n \qquad \eta_{mn} = \begin{pmatrix} -1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{pmatrix}$$

$$v = \sigma^m v_m \qquad \left(\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \; \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \; \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \; \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right)$$

$$\det v = -v^m v_m$$

M 2x2 matrix with $\det M = 1$, $M \in SL(2,\mathbb{C})$

$$v' = M v M^\dagger \qquad v' = \sigma_m v'_m \qquad \det v' = -v'^m v'_m$$

since $\det M = 1 \Rightarrow \det v' = \det v \Rightarrow v'^2 = v^2$

$\Rightarrow v'_m$ Lorentz transform of v_m

the Lorentz group can be represented by 2x2 complex matrices of det = 1, i.e. $SL(2,\mathbb{C})$

2-component Weyl spinors

$$\psi_\alpha \xrightarrow[\text{transf.}]{\text{Lor.}} \psi'_\alpha = M_\alpha{}^\beta \, \psi_\beta$$

$$\bar\psi_{\dot\alpha} \xrightarrow[\text{transf.}]{\text{Lor}} \bar\psi'_{\dot\alpha} = M^*_{\dot\alpha}{}^{\dot\beta} \, \bar\psi_{\dot\beta}$$

the dotted (undotted) spinors transform under the $(0, \frac{1}{2})$ $\left((\frac{1}{2}, 0)\right)$ representation of the Lorentz group

$$\psi^\alpha = \varepsilon^{\alpha\beta} \psi_\beta \qquad \psi_\alpha = \varepsilon_{\alpha\beta} \psi^\beta$$

$$\psi^\alpha \xrightarrow[\text{Lor.}]{} \psi'^\alpha = M^{-1}{}_\beta{}^\alpha \, \psi^\beta$$

$$\bar\psi^{\dot\alpha} \to \bar\psi'^{\dot\alpha} = (M^*)^{-1}{}_{\dot\beta}{}^{\dot\alpha} \, \bar\psi^{\dot\beta}$$

from $v' = M v M^\dagger \Rightarrow \sigma^m v_m = M \sigma^m v_m M^\dagger$

$\Rightarrow$ index structure of σ : $\sigma^m_{\alpha\dot{\alpha}}$

$\Longrightarrow$

$$\bar{\sigma}^{m\,\dot{\alpha}\alpha} \equiv \varepsilon^{\dot{\alpha}\dot{\beta}} \varepsilon^{\alpha\beta} \sigma^m_{\beta\dot{\beta}}$$

$$\sigma^m_{\alpha\dot{\alpha}} \Lambda_m{}^n = M_\alpha{}^\beta \sigma^n_{\beta\dot{\beta}} (M^*)_{\dot{\alpha}}$$

$$\sigma^m_{\alpha\dot{\alpha}} v_m \underset{Lor.}{\longrightarrow} \sigma^m_{\alpha\dot{\alpha}} \Lambda_m{}^n v_n = M_\alpha{}^\beta (M^*)_{\dot{\alpha}}{}^{\dot{\beta}} \sigma^n_{\beta\dot{\beta}} v_n$$

$\hookrightarrow$ transforms like a bispinor

Relationship of two-component spinors to four-component spinors

$$\gamma^m = \begin{pmatrix} 0 & \sigma^m \\ \bar{\sigma}^m & 0 \end{pmatrix}$$ (Weyl basis for the γ Dirac matrices)

$$\{\gamma^m, \gamma^n\} = -2\eta^{mn} \quad \left(\{\sigma^m, \bar{\sigma}^n\}_\alpha{}^\beta = -2\eta^{mn} \delta_\alpha{}^\beta \right.$$

Dirac spinors: $\psi_D = \begin{pmatrix} \chi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}$

Majorana spinors: $\psi_M = \begin{pmatrix} \chi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}$

2. WHAT IS SUSY

The simplest graded Lie algebra which is a symmetry of an S-matrix consistent with a relativistic quantum field theory involves **the generators of the Poincaré group** (even part of the algebra) as well as **two anticommuting spinor generators of supersymmetry**

$$[P^m, P^n] = 0$$

$$\{Q^\alpha, \bar{Q}_{\dot\alpha}\} = 2\,\sigma^m{}_{\alpha\dot\alpha}\,P_m$$

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0$$

$$[Q_\alpha, P^m] = [\bar{Q}_{\dot\alpha}, P^m] = 0$$

$$[M^{mn}, P^q] = -i\left(P^m \eta^{nq} - P^n \eta^{mq}\right)$$

$$[M^{mn}, M^{qr}] = i\left(\eta^{mq} M^{nr} - \eta^{mr} M^{nq} + \eta^{nr} M^{mq} - \eta^{nq} M^{mr}\right)$$

$$[M^{mn}, Q_\alpha] = -i\, \sigma^{mn\,\beta}_{\alpha}\, Q_\beta$$

$$[M^{mn}, \bar{Q}^{\dot\alpha}] = -i\, \bar\sigma^{mn\,\dot\alpha}{}_{\dot\beta}\, \bar{Q}^{\dot\beta}$$

$$\left(\sigma^{mn\,\beta}_{\alpha} = \frac{1}{4}\left[\sigma^m_{\alpha\dot\alpha}\, \bar\sigma^{n\dot\alpha\beta} - \sigma^n_{\alpha\dot\alpha}\, \bar\sigma^{m\dot\alpha\beta}\right]\right.$$

$$\bar\sigma^{mn\,\dot\alpha}{}_{\dot\beta} = \frac{1}{4}\left[\bar\sigma^{m\dot\alpha\alpha}\, \sigma^n_{\alpha\dot\beta} - \bar\sigma^{n\dot\alpha\alpha}\, \sigma^m_{\alpha\dot\beta}\right]$$

spinor representation of generators of the Lorentz group).

Q_α, $\bar{Q}^{\dot\alpha}$ are generators of SUSY trans. and form the grading representation of the Poincaré algebra.

Simple examples of Lagrangians exhibit the conservation of a spinorial charge Q_α of supersymmetry

(1) $\mathcal{L}_0 = \partial_\mu \varphi^* \partial^\mu \varphi + i \bar{\psi} \not{\partial} \psi$

cons. curr. $J^\mu_\alpha = (\not{\partial} \varphi \sigma^\mu \psi)_\alpha$; $\partial_\mu J^\mu_\alpha = 0$

cons. charge $Q_\alpha \equiv \int d^3x \, J^0_\alpha$

what makes SUSY interesting is that such a symmetry can be defined for interacting Lagrangians, too.

(2) $\mathcal{L} = \mathcal{L}_0 - h^2 |\varphi|^4 - h (\varphi \psi_\alpha \psi^\alpha + h.c.)$

cons. curr. : $J^{\mu\prime}_\alpha = J^\mu_\alpha + h \, \sigma^\mu_{\alpha\dot{\beta}} (\varphi^*)^2 \bar{\psi}^{\dot{\beta}}$

This is the so-called massless Wess-Zumino model

crucial that the Yukawa coupling is $\sqrt{\ }$ of the $|\varphi|^4$ coefficient $\rightarrow$ examples of new relations among coupling constants in SUSY

Q_α conserved charge

$\bar{Q}^{\dot\alpha}$ also cons. charge $\Rightarrow$ $\{Q_\alpha, \bar{Q}_{\dot\alpha}\}$ also conserved

$\rightarrow$ the only conserved quantity is the 4-momentum P^μ contracted with σ^μ to have the right spinor structure

$$\Rightarrow \quad \{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2 \not{P}_{\alpha\dot\alpha}$$

Notice that

$P^2 = P^m P_m$ commutes with <u>all</u> the generators, but

$W^2 = W^m W_m$ $\left(W^m = \frac{i}{4} \varepsilon^{mnqr} P_n M_{qr} \right.$

Pauli-Lubanski spin vector$\Big)$

does <u>NOT</u> commute with the SUSY generators Q_α, $\bar{Q}_{\dot\alpha}$

$\Rightarrow$ the SUSY multiplets (SUPERMULTIPLETS) upon which the SUSY symmetry is realized contain states with EQUAL MASS but DIFFERENT SPIN (indeed Q generators [illegible]

SUPERMULTIPLETS

Poincaré symmetry is realized on fields of definite spin and mass (scalars, fermions, gauge bosons)

analogously one realizes supersymmetry on SUSY-multiplets (chiral or scalar multiplets, vector multiplet, ...)

simplest case: CHIRAL MULTIPLET

$$\Phi \equiv (\varphi, \psi_\alpha, F) : \begin{cases} \varphi & \text{complex scalar} \\ \psi_\alpha & \text{2-comp. Weyl spinor (bispinor)} \\ F & \text{complex scalar (auxiliary field)} \end{cases}$$

does not propagate - it can be eliminated using the eqs. of motion

the SUSY transformation rotates the components φ, ψ_α, F into each other the infinitesimal parameter of such a SUSY "rotation" must be a fermionic (anticommuting) parameter since it transforms fermions into bosons and viceversa

the construction of the above smallest component susy multiplet (chiral multiplet) is realized starting from the first (lowest) component, the scalar field φ in such a way that the susy transformation is linear in the multiplet fields

$$\varphi \xrightarrow[\text{SUSY}]{} \psi \qquad (\varphi \text{ dim } 1 \quad \psi \text{ dim } 3/2)$$

try $\delta_\xi \varphi = \sqrt{2}\, \xi \psi$
$\hookrightarrow$ infinites. param. of the SUSY rotation

this SUSY rotation must satisfy

$$\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2\, \sigma^m_{\alpha\dot\alpha} P_m \quad \text{or}$$

$$[\xi Q, \bar\eta \bar{Q}] = 2\, \xi \sigma^m \bar\eta P_m$$

(ξ^α, η^α anticomm. (Grassmann) parameters the infinit. SUSY transf. δ_ξ on a field $f(x)$ defined by:

$$\delta_\xi f(x) \equiv (\xi Q + \bar\xi \bar Q) \times f(x)$$

Q has mass dim. 1/2

$\Rightarrow$ Susy transf. maps fields of dim. ℓ into fields of dim $\ell + 1/2$ or derivatives of fields of dim. $\ell + 1/2 - n$, n positive integer

if we try $\delta_\xi \varphi = \sqrt{2}\, \xi \psi$

then $[\delta_\eta, \delta_\xi] \varphi = \sqrt{2} (\xi \delta_\eta - \eta \delta_\xi) \psi$

so that to satisfy $[\xi Q, \bar\eta \bar Q] = 2 \xi \sigma^m \bar\eta P_m$ (*)

one can ask for ψ to transform

$$\delta_\xi \psi = \sqrt{2}\, i \sigma^m \bar\xi \partial_m \varphi$$

however to close the algebra ψ must also satisfy the fundamental relation (*)

$$[\delta_\eta, \delta_\xi] \psi = \sqrt{2}\, i \sigma^m \bar\xi \partial_m \delta_\eta \varphi - \sqrt{2}\, i \sigma^m \bar\eta \partial_m \delta_\xi \varphi$$

$$= 2 i \sigma^m \bar\xi (\eta \partial_m \psi) - 2 i \sigma^m \bar\eta (\xi \partial_m \psi)$$

$$\neq -2i (\eta \sigma^m \bar\xi - \xi \sigma^m \bar\eta) \partial_m \psi$$

⇒ we <u>cannot</u> close the algebra with just φ and ψ

⇒ we alter the transformation law for ψ by introducing an additional bosonic field F, so that

$$\delta_\xi \psi = \sqrt{2}\, i\, \sigma^m \bar{\xi}\, \partial_m \varphi + \sqrt{2}\, \xi F$$

choosing $\delta_\xi F = \sqrt{2}\, i\, \bar{\xi}\, \bar{\sigma}^m \partial_m \psi$

then

$$[\delta_\eta, \delta_\xi] \psi = -2i \left(\eta \sigma^m \bar{\xi} - \xi \sigma^m \bar{\eta} \right) \partial_m \psi$$

for φ still $[\delta_\eta, \delta_\xi] \varphi = -2i \left(\eta \sigma^m \bar{\xi} - \xi \sigma^m \bar{\eta} \right) \partial_m \varphi$

moreover

$$[\delta_\eta, \delta_\xi] F = -2i \left(\eta \sigma^m \bar{\xi} - \xi \sigma^m \bar{\eta} \right) \partial_m F$$

⇒ THE ALGEBRA DOES INDEED CLOSE !

$\Rightarrow$ chiral multiplet $\{\varphi, \psi, F\}$ with

$$\begin{cases} \delta_\xi \varphi = \sqrt{2}\, \xi \psi \\ \delta_\xi \psi = \sqrt{2}\, i\, \sigma^m \bar{\xi}\, \partial_m \varphi + \sqrt{2}\, \xi F \\ \delta_\xi F = \sqrt{2}\, i\, \bar{\xi} \bar{\sigma}^m \underline{\underline{\partial_m}} \psi \end{cases}$$

forms a linear representation of the SUSY algebra

→ EQUAL NUMBER OF BOSONIC (φ, F) and FERMIONIC (ψ_1, ψ_2) fields

→ THE COMPONENT F TRANSFORMS UNDER SUSY AS A TOTAL DIVERGENCE

THIS IS ALWAYS TRUE FOR THE COMPONENT OF HIGHEST DIMENSION OF ANY MULTIPLET to construct an action out of this multiplet which is invariant under SUSY transformations, it is sufficient to construct a Lagrangian which transforms as a TOTAL DIVERGENCE

when dealing with components only, this procedure is one of trial and error.

When SUPERFIELDS are introduced we shall have a systematic way of constructing invariant actions

Before doing that we make an important observation once again derived from the fundamental relation

$$\{\bar{Q}_{\dot{\alpha}}, Q_{\alpha}\} = 2\,\sigma^{m}_{\alpha\dot{\alpha}}\, P_m$$

$$\Downarrow$$

En-momentum tensor

$$\{\bar{Q}_{\dot{\alpha}}, J^{m}_{\alpha}\} = 2\,\sigma^{m}_{\alpha\dot{\alpha}}\, T_{m}{}^{n} + \text{Schwinger terms}$$

$\langle 0 | \text{Schwinger terms} | 0 \rangle = 0$

$$\Rightarrow \langle 0 | T_m{}^n | 0 \rangle = 0$$

if $Q_\alpha | 0 \rangle = 0$

i.e. if SUSY is unbroken

$\Rightarrow$ this might (?) account

for the vanishing of

the COSMOLOGICAL CONSTANT

(i.e. the most severe fine-tuning

or naturalness problem in physics)

Other important consequence of

$$\{ \bar{Q}_{\dot{\alpha}}, Q_\alpha \} = 2 \sigma^m_{\alpha\dot{\alpha}} P_m$$

$$P_0 \equiv H = 1/4 \left(\bar{Q}_1 Q_1 + Q_1 \bar{Q}_1 + \bar{Q}_2 Q_2 + Q_2 \bar{Q}_2 \right)$$

$$\Rightarrow \langle 0 | H | 0 \rangle \geq 0$$

$\langle 0|H|0\rangle = 0 \Rightarrow Q_\alpha|0\rangle = 0$ SUSY conserved

$\langle 0|H|0\rangle > 0 \Rightarrow Q_\alpha|0\rangle \neq 0$ SUSY broken

$\Rightarrow$ the vacuum energy is the order parameter of supersymmetry

(all what is said above is true in global supersymmetry - In local SUSY, i.e. supergravity, major differences occur)

SUSY is unbroken if and only if a state exists, to be identified with a ground state of the theory, which is annihilated by the SUSY charges

$Q|0\rangle = \langle 0|\bar{Q} = 0$

this property is the basis of difficulties encountered in trying to spont. break SUSY

SUPERFIELDS and SUPERSPACE

Poincaré symmetry $\rightarrow$ transf. in the 4-dim. Minkowski space (x_0, x_1, x_2, x_3)

SUSY symmetry $\rightarrow$ transf. defined by the ferm. anticommuting param. θ_α, $\bar{\theta}_{\dot{\alpha}}$

$$\Downarrow$$

natural parameter space for

SUSY + POINCARÉ

("graded" Poincaré transf.)

Superspace with coordinates

$$\text{SUPERPOINT} = (x_0, x_1, x_2, x_3, \underbrace{\theta_\alpha, \bar{\theta}_{\dot{\alpha}}}_{\text{anticommuting c-numbers}})$$

$$\theta_1^2 = \theta_2^2 = 0 \qquad \theta_1\theta_2 = -\theta_2\theta_1$$

fields in superspace $\Longrightarrow$ superfields

they may or may not have Lorentz indices

the superfields $F(x,\theta,\bar\theta)$ are understood in terms of their Taylor expansion in θ and $\bar\theta$

(given that $\theta, \bar\theta$ are Grassmann anticommuting variables, the series terminates after a finite number of terms

general expression for $F(x,\theta,\bar\theta)$

$$
\begin{aligned}
F(x,\theta,\bar\theta) = {} & \underline{f(x)} + \theta\, \underline{\eta(x)} + \bar\theta\, \underline{\bar\chi(x)} \\
& + \theta\theta\, \underline{m(x)} + \bar\theta\bar\theta\, \underline{n(x)} \\
& + \theta\sigma^{m}\bar\theta\, \underline{v_m(x)} + \theta\theta\bar\theta\, \underline{\bar\lambda(x)} + \bar\theta\bar\theta\theta\, \underline{\psi(x)} \\
& + \theta\theta\bar\theta\bar\theta\, \underline{d(x)}
\end{aligned}
$$

16 component fields, each a function of x

8 carry tensor indices and 8 carry spinor indices

A superfield is a function acting on superspace which transforms under the SUSY transformation

$$\delta_\xi F = (\xi Q + \bar\xi \bar Q) F,$$

where Q^α, $\bar Q^{\dot\alpha}$ are the linear differential operators

$$Q_\alpha = \frac{\partial}{\partial \theta^\alpha} + i\, \sigma^m_{\alpha\dot\alpha}\, \bar\theta^{\dot\alpha}\, \partial_m \equiv D_\alpha$$

$$\bar Q^{\dot\alpha} = \frac{\partial}{\partial \bar\theta_{\dot\alpha}} - i\, \theta^\alpha\, \sigma^m_{\alpha\dot\beta}\, \varepsilon^{\dot\beta\dot\alpha}\, \partial_m \equiv \bar D^{\dot\alpha}$$

which represent Q^α and $\bar Q^{\dot\alpha}$ on superspace

(i.e. they induce the translation in the parameter space of superspace

like $T^m \to i\partial^m$ for ordinary space)

This general 16-component superfield $F(x,\theta,\bar\theta)$ carries a linear representation of the SUSY algebra. However it is REDUCIBLE

(it could be because previously with only $\underline{4}$ components in the case of the scalar multiplet it was possible to obtain a linear representation of the SUSY algebra)

to obtain irreducible representations of the SUSY algebra one can impose appropriate constraints on $F(x,\theta,\bar\theta)$ which are covariant under SUSY and eliminate the extra components. We study two such constraints leading to

SCALAR and VECTOR superfields $\rightarrow$ building blocks for the construction of susy lagrangians

3. CONSTRUCTIONS OF SUSY LAGRANGIANS

SCALAR or CHIRAL SUPERFIELDS

They are defined imposing the constraint on F:

$$\bar{D}_{\dot{\alpha}} F = 0$$

this constraint leads to a substantial reduction of the field components

$$\Phi = \varphi(x) + i\,\theta\sigma^m\bar{\theta}\,\partial_m\varphi(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\,\partial^2\varphi(x)$$

$$+ \sqrt{2}\,\theta\,\psi(x) - \frac{i}{\sqrt{2}}\,\theta\theta\,\partial_m\psi(x)\,\sigma^m\bar{\theta}$$

$$+ \theta\theta\,F(x)$$

neglecting components involving total divergences, the superfield is independent of $\bar{\theta}$

ANTI-CHIRAL SUPERFIELD Φ^+ satisfies

$$D_\alpha \Phi^+ = 0$$

$$\Phi^+ = \varphi^*(x) - i\,\theta\sigma^m\bar{\theta}\,\partial_m\varphi^*(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\,\partial^2\varphi^*(x)$$

Since D_α is a LINEAR differential operator

$\Rightarrow$ $\Phi_i \cdot \Phi_j \cdot \ldots \cdot \Phi_k$ IS A SCALAR SUPERF.

$\Rightarrow$ the $\theta\theta$ (highest) component of a product of scalar superfields trasforms as a TOTAL DIVERGENCE under SUSY transformations

On the other hand

$\underline{\underline{\Phi_i^\dagger \Phi_i}}$ is $\underline{\text{NOT}}$ a SCALAR SUPERF.

however it is still a superfield

$\Rightarrow$ its $\theta\theta\bar\theta\bar\theta$ component transforms under SUSY as a total divergence

$\Rightarrow$ we have now a recipe to construct the MOST GENERAL RENORM. SUSY ACTION COMPOSED OF ONLY SCALAR SUPERFIELDS

$$\mathcal{L} = \Phi_i^\dagger \Phi_i \Big|_{\theta\theta\bar\theta\bar\theta \text{ component}} +$$

$$+ \left\{ \left[\frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} g_{ijk} \Phi_i \Phi_j \Phi_k + \lambda_i \Phi_i \right] \Big|_{\theta\theta \text{ component}} + \text{h.c.} \right\}$$

where m_{ij} and g_{ijk} are totally symmetric

terms in $\mathcal{L}$ with products of more than 3 superfields destroy renormalizability as can be seen by simple power counting

mass dimension: $[\theta] = -1/2$

$\Rightarrow [\varphi] = 1, \quad [\psi] = 3/2, \quad [F] = 2$

the product of 4 superfield would have a $\theta\theta$ component containing $\varphi_i \varphi_j \varphi_k F_l \Rightarrow$ dim 5

not allowed if we are to preserve renormalizability

SUPERPOTENTIAL $\quad W = \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} g_{ijk} \Phi_i \Phi_j \Phi_k + \lambda_i \Phi_i$

Let us write $\mathcal{L}$ in terms of the component fields (dropping total divergences)

$$\Phi_i \Phi_i^{\dagger}\Big|_{\theta\theta\bar\theta\bar\theta} = i\,\partial_m \bar\psi_i \bar\sigma^m \psi_i - \partial_m \varphi_i^* \partial_m \varphi_i + F_i^* F_i$$

$$\Phi_i \Phi_j\Big|_{\theta\theta} = \varphi_i F_j + F_i \varphi_j - \psi_i \psi_j$$

$$\Phi_i \Phi_j \Phi_k\Big|_{\theta\theta} = \varphi_i \varphi_j F_k + \varphi_i F_j \varphi_k + F_i \varphi_j \varphi_k$$
$$- \varphi_i \psi_j \psi_k - \psi_i \varphi_j \psi_k - \psi_i \psi_j \varphi_k$$

$$\mathcal{L} = i\,\partial_m \bar\psi_i \bar\sigma^m \psi_i - \partial_m \varphi_i^* \partial^m \varphi_i - F_i^* F_i$$
$$+ \Big[m_{ij} \left(\varphi_i F_j - \tfrac{1}{2} \psi_i \psi_j \right) +$$
$$+ g_{ijk} \left(\varphi_i \varphi_j F_k - \psi_i \psi_j \varphi_k \right) + \lambda_i F_i + h.c. \Big]$$

the auxiliary fields F_i, F_i^* can be eliminated using their field equations

$$F_k^* = - \lambda_k - m_{ik} \varphi_i - g_{ijk} \varphi_i \varphi_j$$

$\Rightarrow$ $\mathcal{L}$ solely in terms of the dynamical fields φ, ψ:

$$\mathcal{L} = i\partial_m \bar{\psi}_i \bar{\sigma}^m \psi_i - \partial_m \varphi_i^* \partial^m \varphi_i$$
$$- \frac{1}{2} m_{ij} \psi_i \psi_j - \frac{1}{2} m_{ij}^* \bar{\psi}_i \bar{\psi}_j$$
$$- g_{ijk} \varphi_k \psi_i \psi_j - g_{ijk}^* \varphi_k^* \bar{\psi}_i \bar{\psi}_j$$
$$- V(\varphi_i, \varphi_i^*)$$

superpotential

$$V(\varphi_i, \varphi_i^*) = F_k^* F_k = \sum_i \left| \frac{\partial W}{\partial \varphi_i} \right|^2$$

since $V(\varphi_i, \varphi_i^*) \geq 0$ its absolute minimum corresponds to points where $F_k = F_k^* = 0$

[Coming back to the previous ex. of the massless Wess-Zumino model, we see that now we can write its action very simply with superfields

$$S = \int d^4x \, d\theta^2 d\bar{\theta}^2 \phi^\dagger \phi + h \int d^4x \, d^2\theta \, \phi^3 + \dots$$

VECTOR SUPERFIELDS

Vector superfields are defined by the conditio

$$F(x,\theta,\bar{\theta}) = F^{+}(x,\theta,\bar{\theta})$$

$$\Rightarrow f^{*}=f, \quad \eta = \chi, \quad m^{*}=n, \quad \lambda = \psi, \quad d^{*}=d$$

one can introduce a generalization of the usual concept of gauge transformations of fields and gauge bosons to chiral and vector multiplets → these SUSY gauge transf. allow for the possibility of gauging away the unphysical fields. In this physical gauge the Wess-Zumino gauge the vector multiplet V is the power series in θ_α and $\bar{\theta}_{\dot{\alpha}}$:

$$V(x,\theta,\bar{\theta}) = \theta\sigma^{m}\bar{\theta}\, v_m(x) - i\,\theta^2\bar{\theta}_{\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}}(x) + i\,\bar{\theta}^2\theta^{\alpha}\lambda_\alpha(x) - \frac{1}{2}\theta^2\bar{\theta}^2 D(x)$$

$\dim v_m = 1 \;\rightarrow\; \dim V = 0 \;\rightarrow\; \dim D = 2$

D is a non-propagating auxiliary field as it was F in the chiral superfield

only physical fields: gauge boson $v_m(x)$

its fermionic partner gaugino $\lambda_\alpha(x)$

in the W-Z gauge $V^n = 0$ for $n > 2$

say generalization of the field strength (F_{mn})

$$\left.\begin{aligned} W_\alpha &= -\frac{1}{4}\,\bar{D}\bar{D}D_\alpha V \\ \bar{W}_{\dot\alpha} &= -\frac{1}{4}\,DD\bar{D}_{\dot\alpha} V \end{aligned}\right\} \text{scalar superfields, gauge invariant}$$

$\Rightarrow W^\alpha W_\alpha$ is a scalar superf. and it is gauge invar.

$\Downarrow$

$$\mathcal{L} = \frac{1}{4}\left[W^\alpha W_\alpha \big|_{\theta\theta \text{ compon}} + \bar{W}_{\dot\alpha}\bar{W}^{\dot\alpha}\big|_{\bar\theta\bar\theta \text{ compon}} \right]$$

$\Rightarrow$ neglecting total div

$$\mathcal{L} = \frac{1}{2}D^2 - \frac{1}{4}v^{mn}v_{mn} - i\lambda\sigma^m\partial_m\bar\lambda$$

SUSY and LOCAL ...

$$\Phi_i \xrightarrow[U(1)]{\text{local}} \Phi_i' = e^{-i t_i \Lambda(x)} \Phi_i$$

parameter of the transf. now depends on x, and it is a superfield

Φ' is a scalar superfield only if Λ is a scalar superfield

$$\Phi_i^+ \Phi_i \xrightarrow[U(1)]{\text{local}} \Phi_i'^+ \Phi_i' = \Phi_i^+ \Phi_i \, e^{i t_i (\Lambda^+ - \Lambda)}$$

$\Rightarrow$ $\Phi_i^+ \Phi_i$ is NOT by itself locally $U(1)$ invariant

$\rightarrow$ standard remedy: add a "compensating" vector superfield V such that

$$V \longrightarrow V' = V + i(\Lambda - \Lambda^+)$$

$$\Rightarrow \Phi_i^+ e^{t_i V} \Phi_i \xrightarrow[U(1)]{\text{local}} \Phi_i'^+ e^{t_i V'} \Phi_i' = \Phi_i^+ e^{t_i V} \Phi_i$$

locally $U(1)$ invariant (like for $\bar{\psi} \partial \psi \rightarrow \bar{\psi} D \psi$)

$$\mathcal{L}^{\text{SUSY and local } U(1) \text{ invar}} = \frac{1}{4}\left[W^\alpha W_\alpha \Big|_{\theta\theta} + \bar{W}_{\dot\alpha} \bar{W}^{\dot\alpha} \Big|_{\bar\theta\bar\theta} \right.$$

$$+ \Phi_i^+ e^{t_i V} \Phi_i \Big|_{\theta\theta\bar\theta\bar\theta} +$$

$$\left. + \left[\left(\frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} g_{ijk} \Phi_i \Phi_j \Phi_k \right)\Big|_{\theta\theta} + h.c. \right] \right]$$

$m_{ij} = 0$ if $t_i + t_j \neq 0$

$g_{ijk} = 0$ if $t_i + t_j + t_k \neq 0$

Since $\Phi_i^+ e^{t_i V} \Phi_i$ is gauge invariant, we can calculate it in the W-Z gauge, where $V^n = 0$ $n > 2$

$$\Phi_i^+ e^{t_i V} \Phi_i \Big|_{\theta\theta\bar\theta\bar\theta} = F_i^* F_i - \partial_m \varphi_i^* \partial^m \varphi_i + i \partial_m \bar\psi_i \bar\sigma^m \psi_i$$

$$+ \frac{t_i}{2} v_m \bar\psi_i \bar\sigma^m \psi_i$$

$$+ i \frac{t_i}{2} v^m \left(\varphi_i^* \partial_m \varphi_i - \partial_m \varphi_i^* \varphi_i \right)$$

$$- \frac{i}{\sqrt{2}} t_i \left(\varphi \bar\lambda \bar\psi - \varphi^* \lambda \psi \right)$$

$$+ \frac{1}{2} \left(t_i D - \frac{1}{2} t_i^2 v_m v^m \right) \varphi_i^* \varphi_i$$

$\Rightarrow$ all terms are of dim $= 4$ no problem for renormalizability

SCALAR POTENTIAL FOR A GENERAL SUSY THEORY

WITH BOTH CHIRAL AND GAUGE INTERACTIONS

$$V(\varphi_i) = \sum_i |F_i|^2 + \frac{1}{2} |D^a|^2$$

(a index of the gauge group)

$$= \sum_i \left|\frac{\partial W}{\partial \varphi_i}\right|^2 + \frac{g^2}{2} \left|\varphi_i^* T^a_{ij} \varphi_j\right|^2$$

$$F_i^* = - \frac{\partial W}{\partial \varphi_i} \quad ; \quad D^a = - g\, \varphi_i^* T^a_{ij} \varphi_j$$

generators of the gauge group

* F_i, D^a auxiliary fields → unphysical, indeed from the above expressions one sees that they can be written in terms of the physical fields

* $V(\varphi)$ is NON-NEGATIVE

SUPERSYMMETRY BREAKING

) SPONT. $\mathcal{L}_{SUSY}$ but vacuum non SUSY invar.

$\langle 0|H|0\rangle > 0 \Rightarrow Q_\alpha |0\rangle \neq 0$ susy broken

$\langle 0|H|0\rangle = 0 \Rightarrow Q_\alpha |0\rangle = 0$ susy unbroken

$$H = \frac{1}{4}\left(\bar{Q}_1 Q_1 + Q_1 \bar{Q}_1 + \bar{Q}_2 Q_2 + Q_2 \bar{Q}_2\right)$$

$$\langle 0|H|0\rangle = \langle 0|V|0\rangle = \left(\sum_i |F_i|^2 + \frac{1}{2}|D^a|^2\right)\Big|_{\text{at the minimum}} \geq 0$$

to break SUSY spontaneously

$\langle 0|F_i|0\rangle \neq 0$ Fayet-O'Raifertaigh mechanism

or

$\langle 0|D^a|0\rangle \neq 0$ Fayet-Iliopoulos mechanism

Since the broken generator Q_α is fermionic spin $\frac{1}{2}$
$\Rightarrow$ the associated Goldstone particle is a spin $= \frac{1}{2}$ fermion GOLDSTINO

PHENOMEN. PROBLEM: NO REALISTIC ELW. SUSY MODEL WITH SPONT. BREAK. OF SUSY HAS EVER BEEN CONSTRUCTED

b) EXPLICIT BREAKING OF SUSY

→ add to $\mathcal{L}_{SUSY}$ new terms which explicitly violate SUSY

how about the reappearance of the dangerous quadratic divergences? ( $\int \frac{d^4k}{k^2-m^2} \sim \Lambda^2$)

If the SUSY violating terms enter a certain class of terms then it is possible to avoid the quadr. divergences.

This "protected" class of explicitly SUSY violating terms is called SOFT BREAKING TERMS

i) any dimension 2 operator $\begin{cases} m^2 \varphi\varphi^* \\ m^2(\varphi\varphi + h.c.) \end{cases}$

ii) gaugino mass terms $\quad m(\lambda\lambda) + h.c.$

iii) trilinear scalar couplings $\quad \lambda\varphi^3 + h.c.$

notice that other dim 3 terms like explicit chiral fermion masses $\bar{\psi}\psi$ are NOT soft

NO-RENORMALIZATION THEOREMS

W superpotential

Theorem: the parameters in W (λ's, m's, f's)

* do not suffer any renormalization

* (finite or infinite) from loop corrections

in particular this is important for

the Higgs doublet of the G-W-S model

if it starts with a small mass <<<< 10^{15} GeV,

radiative corrections will not push its mass

up to 10^{15} GeV

this is true as long as SUSY is exact

$\Rightarrow$ to protect the Higgs mass say at the TeV

scale

SUSY must be exact down to a

scale of O(TeV) !!!

"LOW ENERGY SUSY MODELS

4. THE SUPERSYMMETRIC STANDARD MODEL

Particle content

VECTOR MULTIPLETS

$J=1$		$J=1/2$	
g (gluon)	$\longrightarrow$	$\tilde{g}$	(gluinos)
$W^{\pm}, W^{3}$	$\longrightarrow$	$\tilde{W}^{\pm}, \tilde{W}^{3}$	(winos)
B	$\longrightarrow$	$\tilde{B}$	(bino)

CHIRAL MULTIPLETS

q_L, q_R (quarks)	$\longrightarrow$	$\tilde{q}_L, \tilde{q}_R$	(squarks)
ℓ_L, ℓ_R (leptons)	$\longrightarrow$	$\tilde{\ell}_L, \tilde{\ell}_R$	(sleptons)
(Higgsinos) $\tilde{H}_1, \tilde{H}_2$	$\longrightarrow$	H_1, H_2	(higgses)

$\hookrightarrow$ 2 higgs doublets

In SM $\mathcal{L}_{Yuk} = Q H d^c + Q H^* u^c + \dots$

H superfield $\begin{pmatrix} H \\ \tilde{H} \end{pmatrix}$ $\quad H^+$ antichiral superfield

but $Q H^+ u^c$ (Q, u^c chiral superf. H^+ antichiral superf.)

is NOT a chiral superf. $\Rightarrow$ its $\theta\theta$ component is not a total divergence

$\Rightarrow$ in the superpotential W we cannot put a term $Q H^+ u^c$

$\Rightarrow$ we have to add a NEW chiral superfield H' which transforms under $SU(2) \times U(1)$ as H^+

$$W \ni Q H_1 d^c + Q H_2 u^c$$

with the previous superfields and imposing the $SU(3) \times SU(2) \times U(1)$ invariance, the most general superpotential is:

$$
\begin{aligned}
W = {} & h_{U_{ij}} Q_i H_2 u_j^c + h_{D_{ij}} Q_i H_1 d_j^c \\
& + h_{L_{ij}} L_i H_1 e_j^c + \mu H_1 H_2 \\
& + \lambda_{ijk} L_i L_j e_k^c + \lambda'_{ijk} L_i Q_j d_k^c \\
& + \lambda''_{ijk} u_i^c d_j^c d_k^c + \lambda_i L_i H_2
\end{aligned}
$$

... quantum numbers

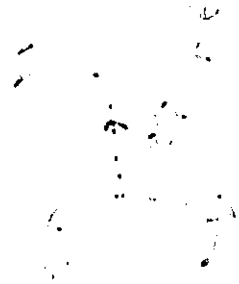
(... but it is not possible to use L eliminating H_1, i.e. L and H_2 only, because the $\tilde{H}_1$ contribution to the $U(1)$ ABJ anomalies would not be cancelled)

$\Rightarrow$ The above red terms in W violate Lepton and Baryon numbers - danger for from $LQd^c|_{\theta\theta} \to \overset{\theta}{L}\ \overset{\theta}{Q}\ \tilde{d}^c$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\ \downarrow\ \ \ \downarrow\ \ \ \downarrow$

ferm. ferm. scalar

from $u^c d^c d^c|_{\theta\theta} \to u^c d^c \tilde{d}^c$



since SUSY is unbroken down to $O(1\,\mathrm{TeV})$

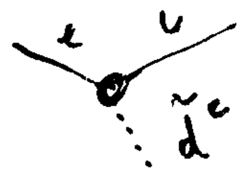
$m_{\tilde{d}^c} \lesssim O(1\,\mathrm{TeV})$

$\Rightarrow$ too fast p - decay

add to $SU(3) \times SU(2) \times U(1) \times$ SUSY $\times$ Lorentz

a NEW symmetry R-parity which distinguishes between ordinary and super particles

$R(q) = +1 \quad R(\tilde{q}) = -1 \quad R(\ell) = +1, \quad R(\tilde{\ell}) = -1.$

$\Rightarrow$ 

forbidden

... BARYON VIOLATING

... IF R IS IMPOSED

alternative discrete symmetries forbidding either

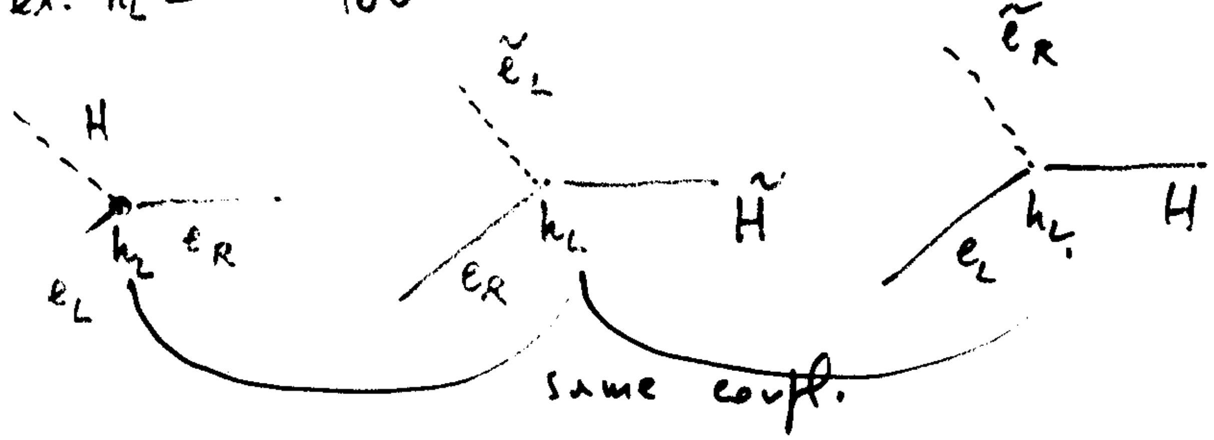
Imposing R-parity implies that

i) super-particles can be produced or annihilated only in pairs

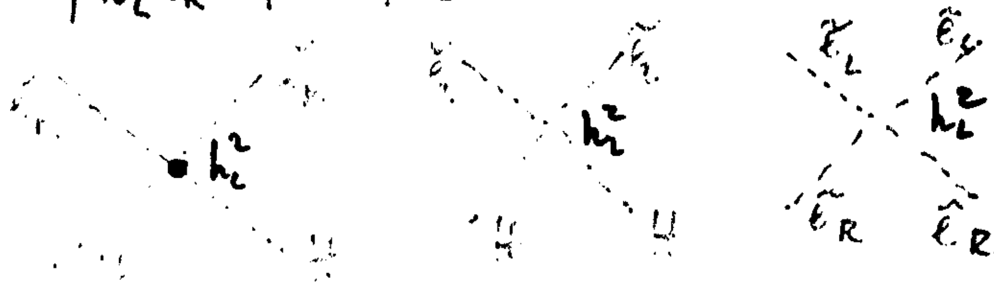
ii) the lightest super-particle (i.e. the lightest particle with $R=-1$) is ABSOLUTELY STABLE

NEW INTERACTIONS IN MSSM

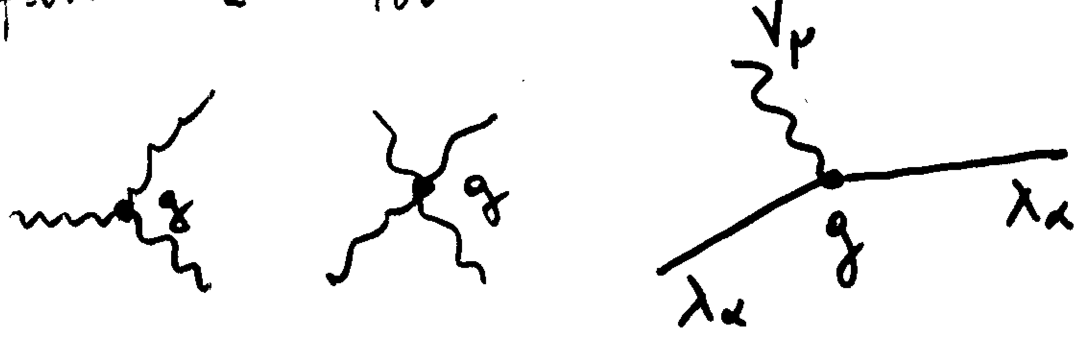
from W ex. $h_L L H e^c|_{\theta\theta}$



in $\mathcal{L}$ from $\left|\frac{\partial W}{\partial \phi_i}\right|^2 \Rightarrow |h_L \tilde{e}_R H|^2 + |h_L \tilde{e}_L H|^2 + |h_L \tilde{e}_L \tilde{e}_R|^2$



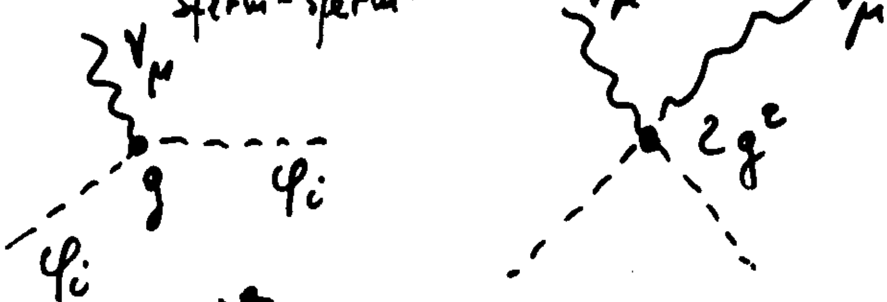
from $W_\alpha W^\alpha|_{\theta\theta}$

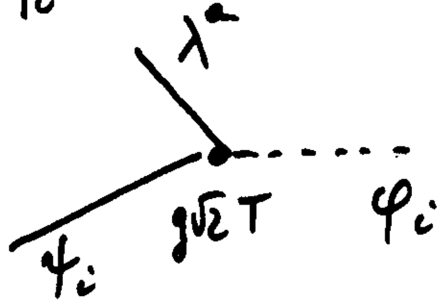


gaugino - gaugino - vector boson

from $\Phi^\dagger e^V \Phi|_{\theta\theta\bar\theta\bar\theta}$

sferm - sferm - vector boson(s)





ferm - sferm - gaugino

from $W_\alpha W^\alpha|_{\theta\theta} \rightarrow D^2$ $\quad$ D aux. field

$$\Rightarrow V_g(\varphi_i) = \frac{g^2}{2} \left| \varphi_i^* T^a_{ij} \varphi_j \right|^2$$

"gauge contribution to the scalar potential"

So far we have an exact $N=1$ Susy version of SM, i.e. a theory with

with superpotential W coming from the supersymmetrization of $\mathcal{L}_{Yuk}$ of SM

SPON. BREAK. OF $N=1$ SUPERGRAVITY IN A HIDDEN SECTOR

SOFT BREAKING TERMS OF SUSY

i) equal mass $\tilde{m}^2$ for all the scalars

ii) new scalar interactions: $\sim \tilde{m}\, W(\varphi_i)$; $\tilde{m}\, \varphi_i \frac{\partial W}{\partial \varphi_i}$

iii) gaugino masses (equal λ masses M at GUT?)

$$\mathcal{L}_{\substack{\text{soft breaking}\\ \text{of } N=1 \text{ global susy}}} = \tilde{m}^2 \sum_{\substack{i=\tilde{q},\tilde{\ell},\\ H_1,H_2}} |\varphi_i|^2 +$$

$$+ A\tilde{m}\, W_{YUK}(\tilde{q},\tilde{\ell},H_1,H_2)$$

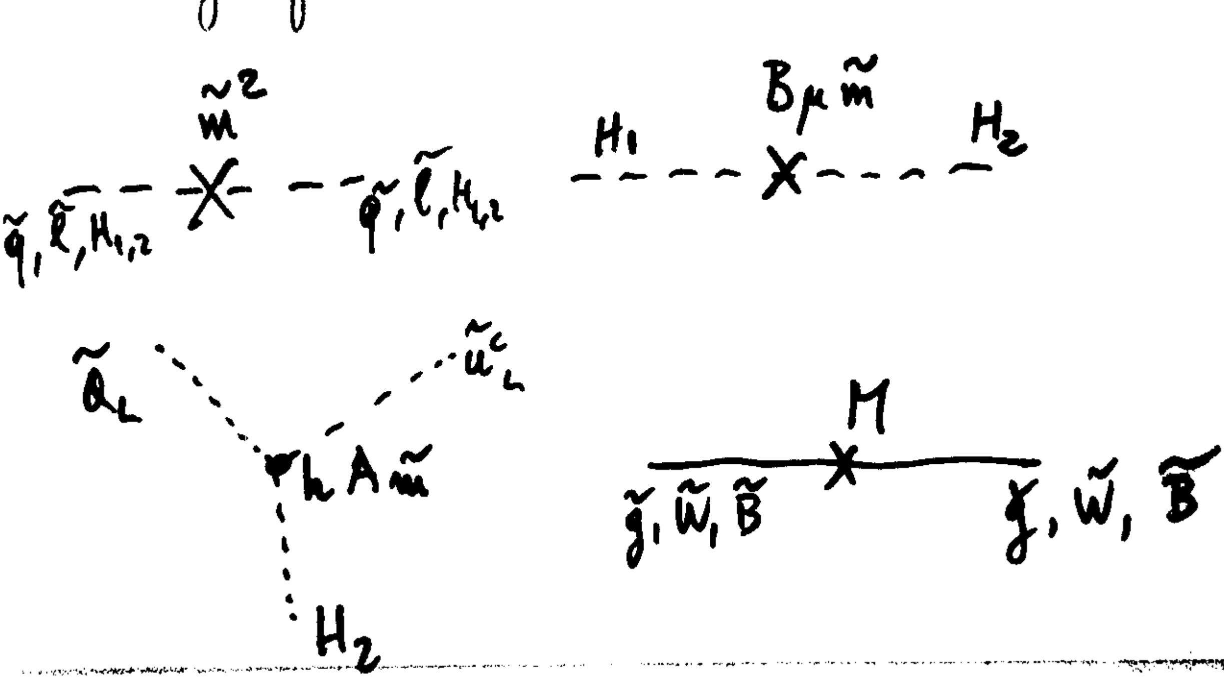
$$+ B\,\tilde{m}\,\mu\, H_1 H_2$$

$$+ M\,\underbrace{\lambda_i\,\lambda_i}_{\to\ \text{gaugino fields}}$$

$W_{Yuk} \to$ Yukawa part of W

A, B c-numbers (of $O(1)$)

M common mass of the $SU(3), SU(2), U(1)$ gauginos at GUT



RENORM. OR FINITE INITIAL THEORY

T.O.E. (THEORY OF EVERYTHING) ?
Superstrings ?

$\downarrow$

N=1 Supergravity (non-renorm.)
"observable" sector (all the known part. and their SUSY partner)

commun. only through grav. effects

+ "hidden" sector $\rightarrow$ breaks N=1 supergr. spontan. scale $\sim 10^{10}-10^{11}$ GeV

$\rightarrow$ goldstino swallowed by gravitino
$\rightarrow$ massive gravitino $m_{3/2} \sim 10^2 - 10^3$ G[illegible]

the communication that SUSY is broken reaches the observable sector only through gravitation interact. $\Rightarrow$ in spite of the superlarge breaking of local N=1 SUSY in the observable sector the SUSY breaking terms are $\propto m_{3/2}$

$\downarrow$

RENORM. THEORY $\rightarrow$ $SU(3) \times SU(2) \times U(1) \times$ global N=1 susy + soft breaking terms $\tilde{m} \sim m_{3/2} \sim 10^2 - 10^3$ GeV range

$SU(2) \times U(1)$ BREAKING MSSM

$$V(H_1, H_2) = \left.\frac{1}{8}\left[\left(H_1^* \tau H_1 + H_2^* \tau H_2\right)^2 g^2 + \left(|H_2|^2 - |H_1|^2\right)^2 g'^2 \right] \right\} \leftarrow |D|^2$$

$$+ \mu_1^2 |H_1|^2 + \mu_2^2 |H_2|^2 - \mu_3^2 (H_1 H_2 + h.c.)$$

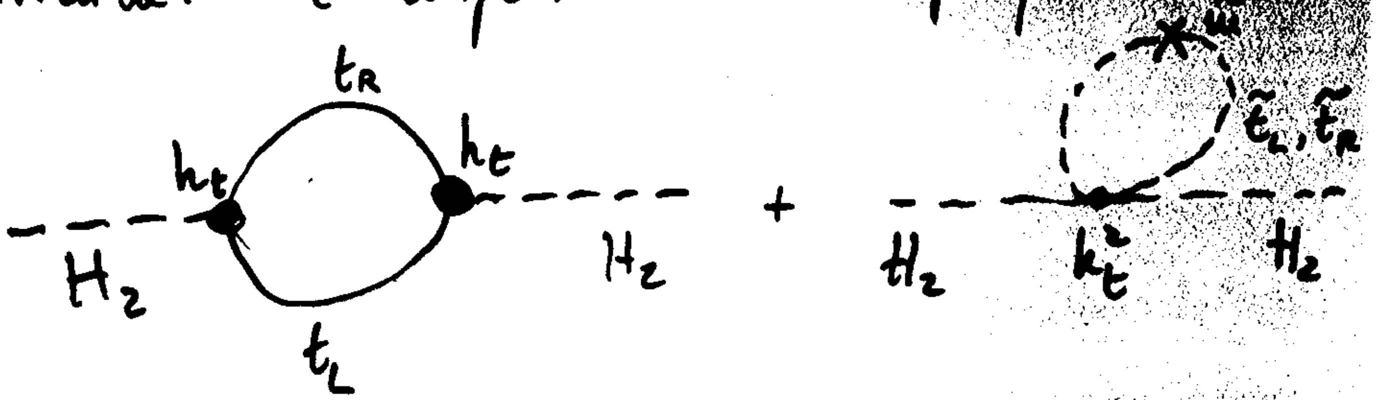
$\mu_1^2 = \tilde{m}^2 + \mu^2$; $\mu_2^2 = \tilde{m}^2 + \mu^2$; $\mu_3^2 = B\tilde{m}\mu$

in this potential $\mu_1^2 = \mu_2^2 > 0$ and a minimum with $\langle H_{1,2} \rangle \neq 0$ may only be obtained if there is a negative $(\text{mass})^2$ eigenvalue in the Higgs mass matrix, i.e. if $\mu_1^2 \mu_2^2 - \mu_3^4 < 0$

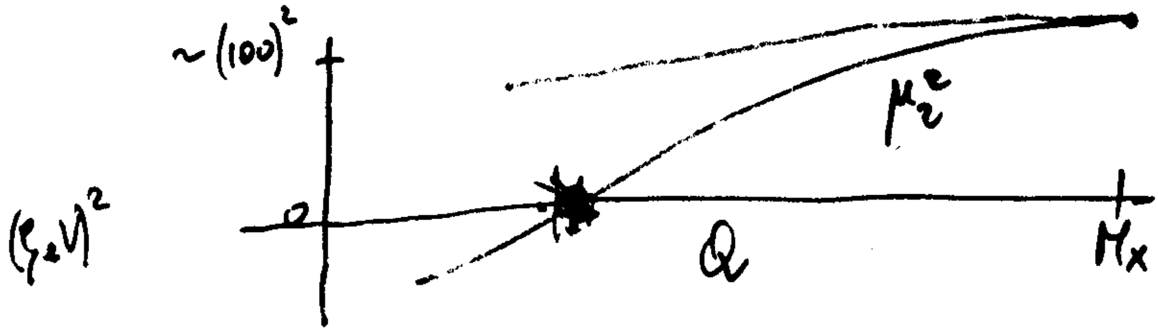
however if $\mu_1^4 = \mu_2^4 < \mu_3^4$ the potential is unbounded from below in the direction $\langle H_1 \rangle = \langle H_2 \rangle \to \infty$

<u>SOLUTION</u> : μ_1^2 and μ_2^2 are equal only at $\sim M_X$ but below that point they will get renormalized differently if H_1 and H_2 couple with different strength to quarks and leptons

in particular H_2 couples to the top quark



$$\delta\mu_2^2 \simeq -\frac{3}{8\pi^2} h_t^2 \tilde{m}^2 \log\left(M_X/Q\right)$$



INITIAL PARAM. (new SUSY param.)

$$\tilde{m},\ M,\ A,\ B,\ \mu$$

+

m_t or h_t

the condition that the breaking of $SU(2)\times U(1)$ occurs at the correct scale implies a relation between h_t and the SUSY param.

$\Rightarrow$ only 4 indep. new SUSY param.

moreover in the simplest cases (i.e. simple "hidden" sectors) $\underline{B = A - 1}$

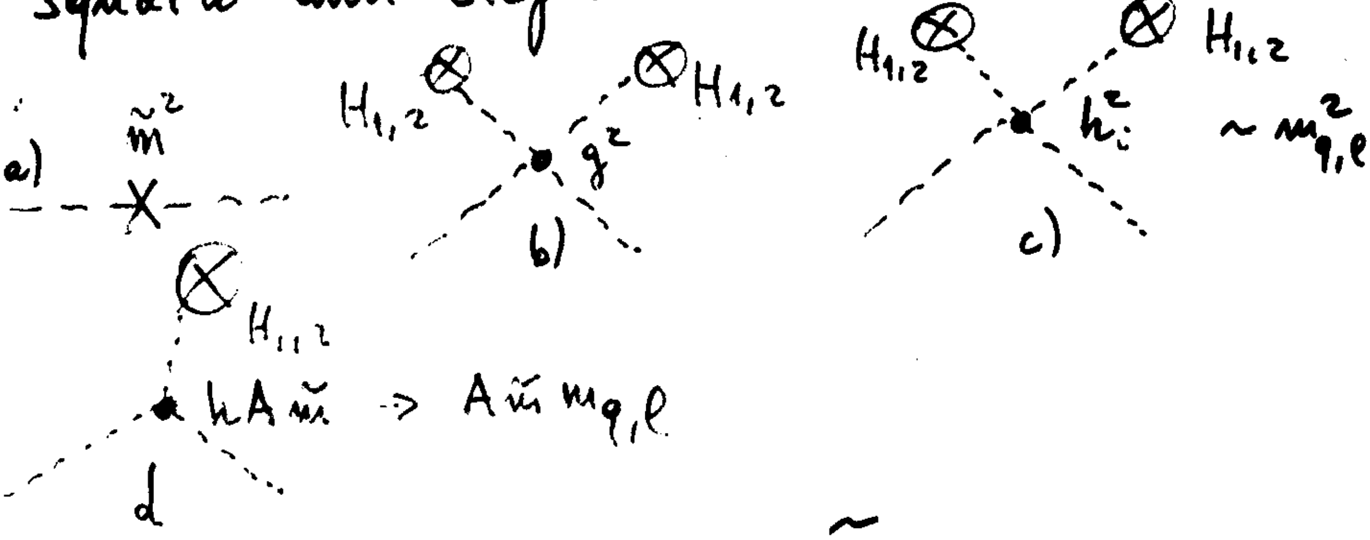
$\Downarrow$

IN THE MINIMAL SUSY STANDARD MODEL WITH RADIATIVE BREAKING OF $SU(2) \times U(1)$ THERE ARE ONLY <u>THREE</u> NEW INDEPENDENT SUSY PARAM

fixing one of the three and for fixed values of the param. of SM (in particular m_t) one can characterize the SUSY phenomenology in the PLANE of the SUSY param. space

5. SOME PHENOMENOLOGICAL IMPLICATIONS OF MSSM

a) squark and slepton masses



$$\begin{array}{ccc} & \tilde{f}_L & \tilde{f}_R \\ \begin{array}{c}\tilde{f}_L \\ \\ \tilde{f}_R\end{array} & \multicolumn{2}{c}{\begin{pmatrix} m_a^{L^2} + m_b^{L^2} + m_c^{L^2} & m_d^2 \\ & \\ m_d^2 & m_a^{R^2} + m_b^{R^2} + m_c^{R^2} \end{pmatrix}} \end{array}$$

for sleptons

$$(\tilde{\ell}_L, \tilde{\ell}_R) \begin{pmatrix} L^2 \tilde{m}^2 + m_e^2 & A\tilde{m}\, m_e \\ A\tilde{m}\, m_e & R^2 \tilde{m}^2 + m_e^2 \end{pmatrix} \begin{pmatrix} \tilde{\ell}_L \\ \tilde{\ell}_R \end{pmatrix}$$

L, R terms of $O(1)$

mass eigenstates

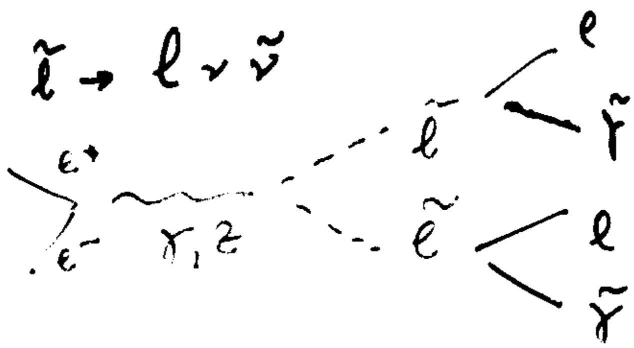
$\tilde{l}_1 = \tilde{l}_L \cos\theta + \tilde{l}_R \sin\theta$; $l_2 = -\tilde{l}_L \sin\theta + \tilde{l}_R \cos\theta$

$$tg\, 2\theta = (2 A m_e) / ((L^2 - R^2)\, \tilde{m})$$

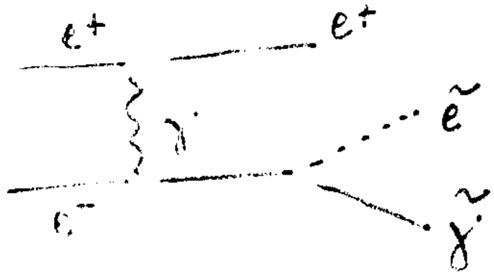
$\tilde{l}$ decay: $\tilde{l}^\pm \to l^\pm \tilde{\gamma}$

if $\tilde{\gamma}$ too heavy $\tilde{l} \to l \nu \tilde{\nu}$

$\tilde{e}$ production:



acolinear $l^+ l^-$ pairs with lots of missing energy



$$\frac{\Gamma(Z^0 \to \tilde{e}_L^+ \tilde{e}_L^- + \tilde{e}_R^+ \tilde{e}_R^-)}{\Gamma(Z^0 \to e^+ e^-)} = \frac{1}{2} \left(1 - 4 \frac{\tilde{m}_e^2}{m_Z^2}\right)^{3/2}$$

LEP $m_{\tilde{e}} > M_Z/2$

squarks mass matrix similar to $\tilde{\ell}$ mass matrix

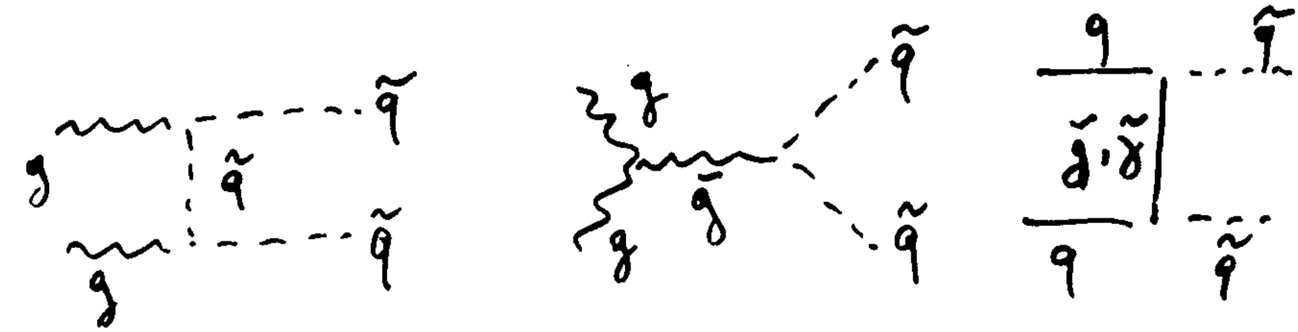
only exception $\tilde{t}$ $\begin{pmatrix} \tilde{m}^2 + m_t^2 & A \tilde{m} m_t \\ A \tilde{m} m_t & \tilde{m}^2 + m_t^2 \end{pmatrix}$

$\Rightarrow$ it is possible to have one of the two eigenv.

$< \tilde{m}, m_t$

LEP $m_{\tilde{q}} > M_Z/2$

CDF



jet + missing energy

$m_{\tilde{q}} > 100$ GeV (roughly)

(however a light $\tilde{t}$ still possible

the above limit is obtained assuming

all $\tilde{q}$ degenerate - if only one light

$\Rightarrow$ the bound goes down LEP lim. 45 GeV still

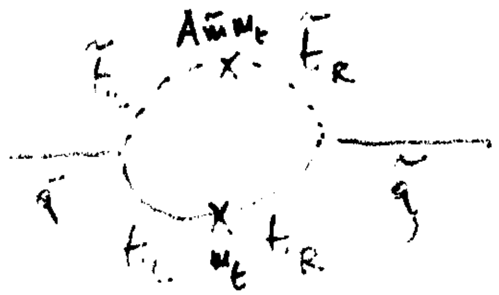
valid)

- GLUINOS

mass: soft breaking term $M\lambda\lambda$

the existence of this term depends on the initial $N=1$ supergravity lagrangian

if $M=0$ → radiative mass for the $\tilde g$



$\frac{\alpha_s}{2\pi}\frac{m_t^2}{\tilde m}$ if $\tilde m > m_t$

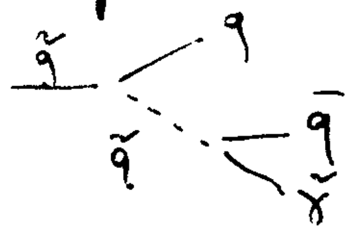
$\frac{4\alpha_s}{3\pi}\tilde m$ if $\tilde m < m_t$

at most $\sim 1 \div 2$ GeV

from CDF $m_{\tilde g} > 100$ GeV (roughly)

however there still exists a window for $\tilde g$ of few GeV's (can the $\tilde g$ be the lightest SUSY particle? Unlikely, but not completely excluded → strong bounds from searches of exotic nuclei)

typical decay



jets + missing en.

- CHARGINOS

$$W^{\pm}, \; H_1^+, \; H_2^- \longrightarrow \tilde{W}^{\pm}, \; \tilde{H}_1^+, \; \tilde{H}_2^-$$

$$\begin{pmatrix} \tilde{W}^- & \tilde{H}_1^- \end{pmatrix} \begin{pmatrix} M & \frac{g v_2}{\sqrt{2}} \\ \frac{g v_1}{\sqrt{2}} & \mu \end{pmatrix} \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_2^+ \end{pmatrix}$$

if $M = \mu = 0 \rightarrow$ 2 Dirac spinors $\tilde{\chi}_1^+ = \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_1^{-*} \end{pmatrix}$

with masses $m_{\chi_1} = \frac{g v_1}{\sqrt{2}}$ $\tilde{\chi}_2^+ = \begin{pmatrix} \tilde{H}_2^+ \\ \tilde{W}_R^{-*} \end{pmatrix}$

$$m_{\chi_2} = \frac{g v_2}{\sqrt{2}}$$

if M or $\mu = 0$, or $M \cdot \mu \ll g^2 \frac{v_1 v_2}{2}$

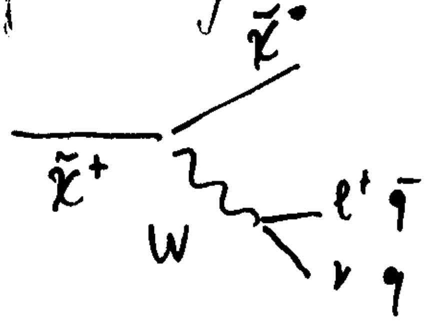
then $\tilde{M}_1 \cdot \tilde{M}_2 \simeq g^2 \frac{v_1 v_2}{2}$

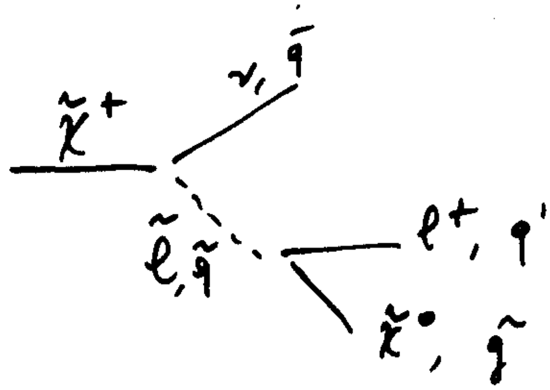
$$\rightarrow m_W^2 - \tilde{M}_1 \cdot \tilde{M}_2 = g^2 \frac{(v_1 - v_2)^2}{4} \geq 0$$

$\Rightarrow$ one the two chargino is lighter than the W !

however for M and μ large there is no need for one chargino to be lighter than m_W

chargino decay

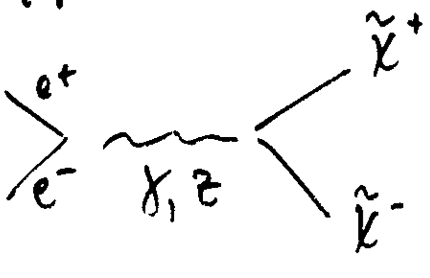


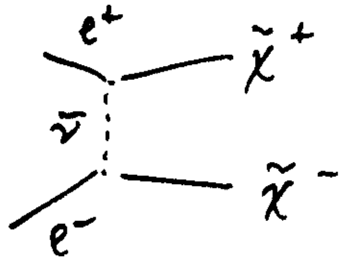


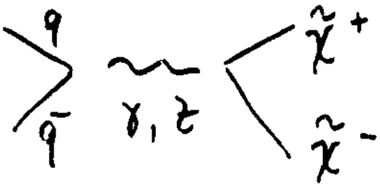
χ^0 neutralino

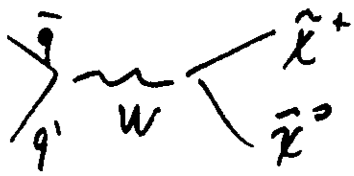
$$\chi^+ \to \begin{cases} q\bar{q}\; q\bar{q} \\ q\bar{q} \\ l^+ \\ l^+ l^+ l^- \\ l^+ q\bar{q} \end{cases}$$

production









present bound LEP $m_{\chi^+} > M_Z/2$

- NEUTRALINOS

$$W_3, B, H_1^\circ, H_2^\circ \longrightarrow \tilde{W}_3, \tilde{B}^\circ, \tilde{H}_1^\circ, \tilde{H}_2^\circ$$

$$(\tilde{B}, \tilde{W}_3, \tilde{H}_1^\circ, \tilde{H}_2^\circ) \begin{pmatrix} M' & 0 & -g'\frac{v_1}{2} & g'\frac{v_2}{2} \\ 0 & M_2 & g\frac{v_1}{2} & -g\frac{v_2}{2} \\ -g'\frac{v_1}{2} & g\frac{v_1}{2} & 0 & -\mu \\ g'\frac{v_2}{2} & -g\frac{v_2}{2} & -\mu & 0 \end{pmatrix} \begin{pmatrix} \tilde{B} \\ \tilde{W}_3^\circ \\ \tilde{H}_1^\circ \\ \tilde{H}_2^\circ \end{pmatrix}$$

in GUT $M' = \frac{5}{3} \frac{\alpha_1}{\alpha_2} M_2$

photino $\tilde{\gamma} \simeq g_1 \tilde{W}^\circ + g_2 \tilde{B}^\circ \qquad m_{\tilde{\gamma}} \simeq \frac{8}{3} \frac{g_1^2}{g_1^2+g_2^2} M_2$

in general, however, it is not one of the eigenvectors of the neutralino mass matrix

if GUT $M_2 = M_3 = M$ then $m_{\tilde{g}} = \frac{\alpha_3}{\alpha_{GUT}} M$

→ from the CDF bound on $m_{\tilde{g}}$ it is possible to infer a bound on M' and M_2 in the above matrix

if that is taken $m_{\text{lightest } \chi^0} > (10-20)$ GeV for LE

THE LIGHTEST $\tilde{\chi}^0$ is a good candidate for COLD DARK M

- Higgs sector

3 physical neutral scalars

1 physical charged scalar

at the tree level

one of the three neutral scalar

has a mass < m_Z

however radiative corrections spoil

this result → sensitivity to m_t

in any case it remains a fact

that in MSSM there exists a

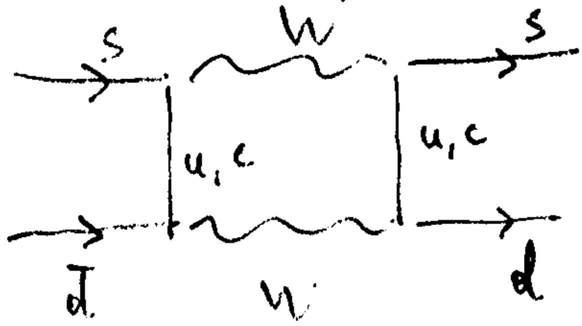
light neutral Higgs (say, of mass ≲ 120 GeV

given that m_t < 200 GeV from rad. corr.)

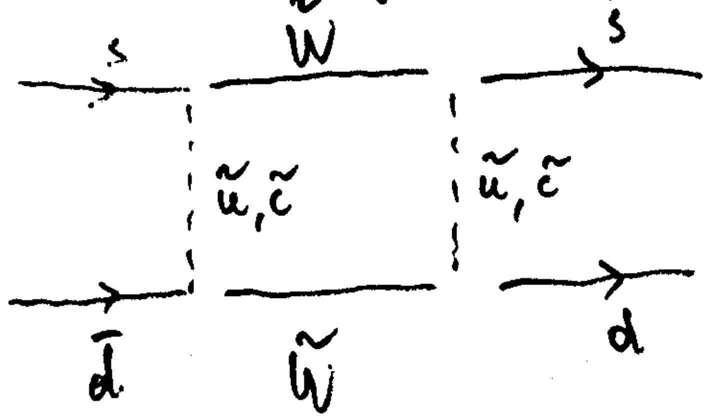
→ challenge for LEP 2

200 ?

SUSY avoids the usual FCNC trap



$$\sim \frac{(m_c - m_u)^2}{M_W^4}$$



$$\sim \frac{m_{\tilde{c}}^2 - m_{\tilde{u}}^2}{m_{\tilde{q}}^2} \, \frac{1}{m_\chi^2}$$

$\tilde{c}$ $\tilde{u}$ highly degenerate $\begin{pmatrix} \tilde{m}^2 + m_q^2 & A\tilde{m} m_q \\ A\tilde{m} m_q & \tilde{m}^2 + m_q^2 \end{pmatrix}$

$\Rightarrow$ MSSM passes unscathed the famous FCNC tests !

1961 Gauge model of W.I.

1964 Higgs mech.

1967-8 W-S model

1971 proof of renorm ('t Hooft)

1972 searches for NC

1973 discovery of NC in Gargam.

1974 charm discovery

1975 tau lepton "

1977 bottom quark "

1983 W "

1977 first phenom. SUSY models

1981 SUSY for gauge hierarchy problem

1982-3 N=1 supergrav.

1985-6 Superstrings

⋮ ← EXP. CONFIRMATIONS ????

Fayet - Ferrara, Phys. Rep. 32 (1977) 249

Salam - Strathdee, Fortschr. Phys. 26 (1978) 57

Bagger - Wess Supersymmetry and Supergravity, Princeton Univ. Press 1983

P. Nilles, Phys. Rep. 1984