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SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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# PRECISION TESTS OF THE STANDARD MODEL

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Please note: These are preliminary notes intended for internal distribution only.

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# Precision electroweak physics within and beyond the Standard Model

mainly based on Nucl. Phys. B393 (1993) 3
Phys. Lett. B324 (1994) 173
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by J. Ellis, GLF and E. Lisi

1. Radiative corrections in the Standard Model
2. Model independent parametrization
3. Precision electroweak measurements
4. Comparing theory and experiment in the SM
   (before and after march-avril 1994)
5. Radiative corrections beyond the SM: Minimal Supersymmeytric Standard Model and Technicolour Theories
6. Conclusions and perspects

## 2. RENORMALIZATION OF THE SM AT 1-LOOP USING ONLY OBLIQUE CORRECTIONS

(...AND SOME TRICKS !)

2.1 **Propagators and vertices in the SM**

2.2 1-loop oblique corrections

2.3 A warm-up exercise: the charge renormalization in QED

2.4 Electric charge renormalization in the SM

2.5 Renormalization of the vector boson masses

2.6 The on-shell scheme and 1-loop calculation rules

2.7 The muon decay, $G_\mu$ and $\Delta r$

2.8 The universal corrections $\Delta r$, $\Delta\varrho$, $\Delta\kappa$

## 1. STANDARD MODEL AND BEYOND

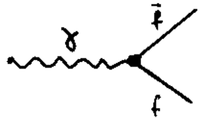




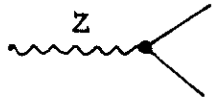
## 2.1 Propagators and vertices in the SM


$: \; -i\dfrac{g_{\mu\nu}}{q^2}$

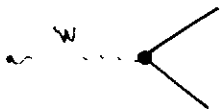

$: \; -i\dfrac{g_{\mu\nu}}{q^2 - M_V^2} \qquad (V = W, Z)$




$: \; -ieQ_f\gamma_\mu$




$: \; +\dfrac{ie}{2s_w c_w}\gamma_\mu(v_f - a_f\gamma_5) \qquad \begin{cases} v_f = I_3 - Qs_w^2 \\ a_f = I_3 \end{cases}$

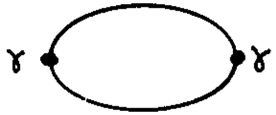



$: \; +\dfrac{ie}{2\sqrt{2}s_w}\gamma_\mu(1 - \gamma_5)$

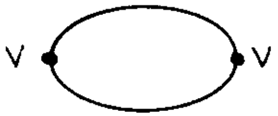
with $\quad s_w^2 = 1 - \dfrac{M_W^2}{M_Z^2}$ (at the tree level)

## 2.2 1–loop oblique corrections

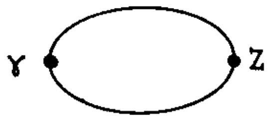
Let us introduce the $\boxed{\textit{unrenormalized self–energies}}$, 1–loop divergent integrals where all possible contributions from particles circulating in the loops must be summed up.




$: \quad -i\Sigma_\gamma(q^2)$




$: \quad -i\Sigma_V(q^2) \qquad (V = W, Z)$

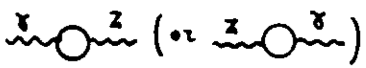



$: \quad -i\Sigma_{\gamma Z}(q^2)$

If we consider only the so–called "oblique" corrections, then the 1–loop vector boson self–energies enter as radiative corrections to the *vector boson propagators*. Since the integrals are divergent, a *renormalization procedure* is needed (if compatible with the theory).

$(V = W, Z)$

not present at tree level $\longrightarrow$





## 2.3 A warm–up exercise: the charge renormalization in QED

The *electric charge* is defined experimentally through the Thomson scattering of an electron off a proton (at rest) at $q^2 = 0$:


$\sim \quad e^2$

At 1–loop level for $q^2 \neq 0$:

i.e.

$$-i\frac{g_{\mu\nu}}{q^2} \longrightarrow -i\frac{g_{\mu\nu}}{q^2} + \left(-i\frac{g_{\mu\nu}}{q^2}\right)\left(-i\Sigma_\gamma(q^2)\right)\left(-i\frac{g_{\mu\nu}}{q^2}\right) =$$
$$-i\frac{g_{\mu\nu}}{q^2}\left(1 - \Pi_\gamma(q^2)\right)$$

where

$$\Pi_\gamma(q^2) = \frac{\Sigma_\gamma(q^2)}{q^2} \qquad (\textit{vacuum polarization})$$

So, in the Thomson scattering: $e^2 \to e^2(1 - \Pi_\gamma(0))$

*infinite!*

The electric charge receives an infinite contribution. It needs a

$\Rightarrow$ RENORMALIZATION PROCEDURE

Assume that the electric charge in the lagrangian is some unmeasurable "bare" charge $e_0$, so that:

$$\underset{\substack{\text{bare} \\ \text{(unmeasurable)}}}{e_0^2} \to e_0^2(1 - \Pi_\gamma(0)) = \underset{\substack{\text{physical} \\ \text{(measurable)}}}{e^2}$$

Call: $\delta e = e_0 - e$ *(charge counterterm)*

Then, neglecting 2nd order terms:

$$e^2 = e_0^2(1 - \Pi_\gamma(0)) \quad \Rightarrow \quad \frac{2\delta e}{e} = \Pi_\gamma(0)$$

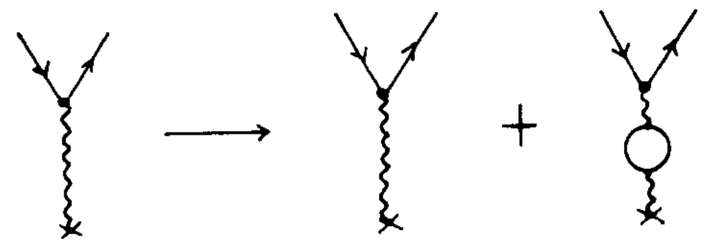
and the counterterm if fixed.

The *renormalization scheme* works (and it does in QED) if and only if this counterterm (or, in general, a finite number of counterterms) is sufficient to calculate:

- all possible processes (relatively easy to prove);
- all possible terms of the perturbative series for each process, removing the infinities (much more difficult to prove).



An example: RUTHEFORD SCATTERING (electron scattering by an external source at $q^2 \neq 0$)



$$\frac{e_0^2}{q^2} \to \frac{e_0^2}{q^2} - \frac{e_0^2}{q^2}\Pi_\gamma(q^2) =$$

$$\frac{(e+\delta e)^2}{q^2}(1 - \Pi_\gamma(q^2) + \Pi_\gamma(0) - \Pi_\gamma(0)) =$$

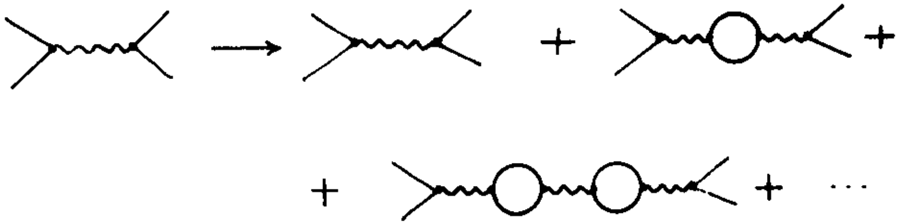
$$\frac{e^2}{q^2}(1 - \Pi_\gamma(q^2) + \Pi_\gamma(0)) = \frac{e^2}{q^2}(1 - \hat{\Pi}_\gamma(q^2))$$

where $\hat{\Pi}_\gamma(q^2) = \Pi_\gamma(q^2) - \Pi_\gamma(0)$ is *finite* due to the cancellation of infinities. So we get a finite result without introducing a new counterterm.

At this point, if we look at $\hat{\Pi}_\gamma(q^2)$ as a correction to the charge (instead to the propagator) we get the concept of *"running charge"*:

$$e^2(q^2) = e^2(1 - \hat{\Pi}_\gamma(q^2))$$

Resumming part of the higher order terms, we obtain an improved equation for the running charge:



$$e^2 \to e^2(1 - \hat{\Pi}_\gamma + \hat{\Pi}_\gamma^2 - \hat{\Pi}_\gamma^3 + \dots) = \frac{e^2}{(1 + \hat{\Pi}_\gamma(q^2))}$$

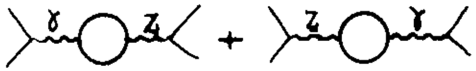
It can be shown that the leading $n$-loop contributions are included in such a geometric series. A more careful traitment leads to:

$$e^2(q^2) = \frac{e^2}{(1 + \mathrm{Re}\hat{\Pi}_\gamma(q^2))}$$



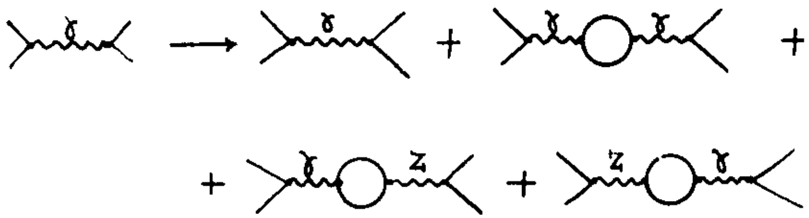
## 2.4 Electric charge renormalization in the SM

In the SM the charge renormalization is more tricky, since the $\gamma$ ***propagator*** gets contributions from the *vector* part of the $\gamma$–$Z$ exchange at the 1-loop level:




(the pure $Z$–exchange being suppressed at $q^2 = 0$)

This means (vector part alone)




i.e.

$$\frac{e_0^2}{q^2} \longrightarrow \frac{e_0^2}{q^2} - \frac{e_0^2}{q^2}\Pi_\gamma(0) - 2\frac{e_0^2}{q^2}\frac{s_w}{c_w}\frac{\Sigma_{\gamma Z}(q^2)}{q^2 - M_Z^2}$$

$$\xrightarrow{q^2=0} \frac{e_0^2}{q^2}\left(1 - \Pi_\gamma(0) + 2\frac{s_w}{c_w}\frac{\Sigma_{\gamma Z}(0)}{M_Z^2}\right)$$

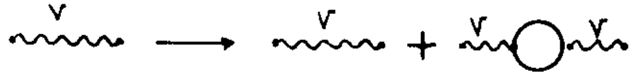
so that

$$\boxed{2\frac{\delta e}{e} = \Pi_\gamma(0) + 2\frac{s_w}{c_w}\frac{\Sigma_{\gamma Z}(0)}{M_Z^2}}$$



## 2.5 Renormalization of the vector boson masses

The correction to the vector boson propagator is given by:




$(V = W, Z)$

$$\frac{-ig_{\mu\nu}}{q^2 - M_{0V}^2} \longrightarrow \frac{-ig_{\mu\nu}}{q^2 - M_{0V}^2} +$$

bare mass

$$\left(\frac{-ig_{\mu\nu}}{q^2 - M_{0V}^2}\right)\left(-i\Sigma_V(q^2)\right)\left(\frac{-ig_{\mu\nu}}{q^2 - M_{0V}^2}\right) =$$

$$= \frac{-ig_{\mu\nu}}{q^2 - M_{0V}^2}\left(1 - \frac{\Sigma_V(q^2)}{q^2 - M_{0V}^2}\right)$$

after resummation

$$\frac{-ig_{\mu\nu}}{q^2 - M_{0V}^2} \longrightarrow \frac{-ig_{\mu\nu}}{q^2 - M_{0V}^2}\left(\frac{1}{1 + \frac{\Sigma_V(q^2)}{q^2 - M_{0V}^2}}\right) =$$

$$= \frac{-ig_{\mu\nu}}{q^2 - M_{0V}^2 + \Sigma_V(q^2)} =$$

$$= \frac{-ig_{\mu\nu}}{q^2 - M_{0V}^2 + \mathrm{Re}\Sigma_V(q^2) + i\mathrm{Im}\Sigma_V(q^2)}$$

$$\begin{cases} \mathrm{Re}\Sigma_V(q^2) \text{ is } \underline{\text{infinite}} \\ \mathrm{Im}\Sigma_V(q^2) \text{ is } \underline{\text{finite:}} \text{ it reflects the finite } \Gamma_V \text{ and removes the poles from the real axis.} \end{cases}$$

The ***physical** mass* $M_V$ is identified with the position of the pole:

$$q^2 - M_{0V}^2 + \mathrm{Re}\Sigma_V(q^2) = 0 \qquad \text{at} \quad q^2 = M_V^2 \quad \text{(on shell)}$$

$$\Downarrow$$

$$M_{0V}^2 = M_V^2 + \mathrm{Re}\Sigma_V(M_V^2)$$

Thus the counterterm $\delta M_V^2$ needed to render $M_{0V}^2$ finite ($M_{0V}^2 + \delta M_V^2 = M_V^2$) is identified with $\mathrm{Re}\Sigma_V(M_V^2)$:

$$\boxed{\begin{aligned} \delta M_W^2 &= \mathrm{Re}\Sigma_W(M_W^2) \\ \delta M_Z^2 &= \mathrm{Re}\Sigma_Z(M_Z^2) \end{aligned}}$$

The *dressed VB propagator* is then: $$\frac{-ig_{\mu\nu}}{q^2 - M_V^2 + i\mathrm{Im}\Sigma_V(q^2)}$$



Ignoring the $q^2$-dependence of $\Sigma_V$, one is led to identify *:

$$M_V^2 - i\mathrm{Im}\Sigma_V = M_V^2 - i\Gamma_V M_V \quad \Rightarrow \quad \mathrm{Im}\Sigma_V = \Gamma_V M_V$$

so that

$$\mathrm{Im}\Sigma_W = M_W\Gamma_W$$

$$\mathrm{Im}\Sigma_Z = M_Z\Gamma_Z$$

In particular at the $Z^0$ peak the dressed propagator is proportional to

$$\frac{1}{q^2 - M_Z^2 + iM_Z\Gamma_Z}$$

It can be shown that the $q^2$-dependence can be accounted for *(near the peak)* through:

$$\frac{1}{q^2 - M_Z^2 + i\frac{q^2}{M_Z^2}M_Z\Gamma_Z}$$

* A decaying particle satisfies

$$M \to M - i\frac{\Gamma}{2} \quad \text{i.e.} \quad M^2 \to M^2 - i\Gamma M$$

## 2.6 The on–shell scheme and 1–loop calculation rules

A very useful convention has been introduced *(Sirlin)*: the relation which connects VBM and *mixing angle* at the tree level

$$s_{0w}^2 = 1 - \frac{M_{0W}^2}{M_{0Z}^2}$$

is written in terms of the the physical masses $M_W$ and $M_Z$, and becomes the definition of $s_w^2$ *at any order*:

$$\boxed{s_w^2 = 1 - \frac{M_W^2}{M_Z^2}} \qquad \text{on shell scheme}$$

We are now ready to attack the 1–loop calculation of quantities as VB masses, scattering amplitudes and decay rates. The rules are:

- The classical lagrangian $\mathcal{L}(e, M_W, M_Z, \ldots)$ is sufficient for tree–level calculations. No distinction between bare and physical parameters is needed.

- For higher order calculations, use can be made of the same lagrangian, written now in terms of the bare parameters $\mathcal{L}(e_0, M_{0W}, M_{0Z}, \ldots)$, which are connected to the physical quantities $e$, $M_W$, $M_Z$ through the relations we have introduced above:

$$\begin{aligned} e_0 &= e + \delta e \\ M_{0W}^2 &= M_W^2 + \delta M_W^2 \\ M_{0Z}^2 &= M_Z^2 + \delta M_Z^2 \end{aligned}$$



- At the 1–loop level, including only *oblique* corrections,

$$\begin{aligned} 2\frac{\delta e}{e} &= \Pi_\gamma(0) + 2\frac{s_w}{c_w}\frac{\Sigma_{\gamma Z}(0)}{M_Z^2} \\ \delta M_{W,Z}^2 &= \mathrm{Re}\Sigma_{W,Z}(M_{W,Z}^2) \end{aligned}$$

- At any order

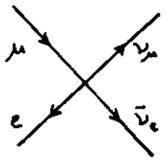
$$s_w^2 = 1 - \frac{M_W^2}{M_Z^2}$$

The approach works since the SM is *renormalizable*

## 2.7 The muon decay, $G_\mu$ and $\Delta r$

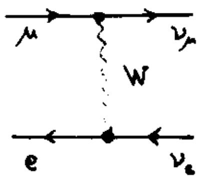
The set of constants we have chosen is $(e, M_W, M_Z)$. However, $M_W$ is not known with the same precision as $e$ and $M_Z$. It is then convenient to "switch" to a different choice, $(e, M_Z, G_\mu)$, $G_\mu$ being the *muon decay constant*.

This is simple at the tree level, by identifying the 4–fermion interaction




$$= i\frac{G_\mu}{\sqrt{2}} J_{cc}^{(\mu)} J_{cc}^{(e)} \quad \begin{cases} J_{cc}^{(\mu)} = \overline{u}_{\nu_\mu}\gamma_\mu(1-\gamma_5)u_\mu \\ J_{cc}^{(e)} = \overline{u}_e\gamma_\mu(1-\gamma_5)v_{\nu_e} \end{cases}$$

with the $q^2 \to 0$ limit of



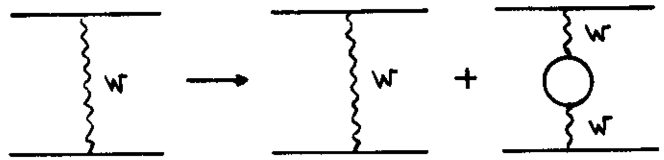

$$= i\left(\frac{e_0}{2\sqrt{2}s_{0w}}\right)^2 \frac{J_{cc}^{(\mu)} J_{cc}^{(e)}}{q^2 - M_{0W}^2}$$

*At the tree level:*

$$\frac{G_\mu}{\sqrt{2}} = \frac{e_0^2}{8s_{0w}^2 M_{0W}^2} \quad \Rightarrow \quad \begin{cases} M_{0W}^2 = \dfrac{\pi\alpha_0}{\sqrt{2}G_\mu}\dfrac{1}{s_{0w}^2} \\[2ex] M_{0Z}^2 = \dfrac{\pi\alpha_0}{\sqrt{2}G_\mu}\dfrac{1}{s_{0w}^2 c_{0w}^2} \end{cases}$$



*At the 1-loop level:*




that is:

$$\frac{e_0^2}{8s_{0w}^2 M_{0W}^2} \longrightarrow \frac{e_0^2}{8s_{0w}^2 M_{0W}^2}\left(1 - \frac{\Sigma_W(0)}{q^2 - M_{0W}^2}\right)_{q^2 \ll M_W^2} =$$

$$\frac{e_0^2}{8s_{0w}^2 M_{0W}^2}\left(1 + \frac{\Sigma_W(0)}{M_{0W}^2}\right)$$

By comparing

$$\frac{G_\mu}{\sqrt{2}} = \frac{e_0^2}{8s_{0w}^2 M_{0W}^2}\left(1 + \frac{\Sigma_W(0)}{M_{0W}^2}\right)$$

On the other hand

$$\begin{cases} e_0^2 = (e+\delta e)^2 = e^2\left(1 + \dfrac{2\delta e}{e}\right) \\[2ex] M_{0W}^2 = M_W^2\left(1 + \dfrac{\delta M_W^2}{M_W^2}\right) \\[2ex] s_{0w}^2 = 1 - \dfrac{M_W^2 + \delta M_W^2}{M_Z^2 + \delta M_Z^2} = s_w^2 + c_w^2\left(\dfrac{\delta M_Z^2}{M_Z^2} - \dfrac{\delta M_W^2}{M_W^2}\right) \end{cases}$$

so that

$$\frac{G_\mu}{\sqrt{2}} = \frac{e^2}{8s_w^2 M_W^2}\left[1 + \frac{2\delta e}{e} - \frac{c_w^2}{s_w^2}\left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right) + \frac{\Sigma_V(0) - \delta M_W^2}{M_W^2}\right]$$

or

$$\frac{G_\mu}{\sqrt{2}} = \frac{e^2}{8s_w^2 M_W^2}(1 + \Delta r)$$

!

where

$$\Delta r = \Pi_\gamma(0) - \frac{c_w^2}{s_w^2}\left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right) + \frac{\Sigma_V(0) - \delta M_W^2}{M_W^2} + 2\frac{c_w}{s_w}\frac{\Sigma_{\gamma Z}(0)}{M_Z^2}$$

is a ***finite*** correction.

It can be shown that the resummation–improved formula is:

$$\boxed{\frac{G_\mu}{\sqrt{2}} = \frac{e^2}{8s_w^2 M_W^2}\frac{1}{1 - \Delta r}}$$

so that we can forget, at the 1-loop level, about $M_W$, and make use of the quantities $(M_Z, e, G_\mu)$, measured with very high precision.

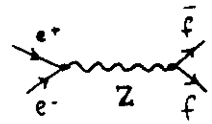
In particular, we can use

$$s_w^2 = 1 - \frac{M_W^2}{M_Z^2} = 1 - \frac{e^2\sqrt{2}}{8s_w^2 G_\mu}\frac{1}{1 - \Delta r}\cdot\frac{1}{M_Z^2} \quad \left\{\begin{array}{l}\text{as an implicit}\\ \text{equation in } s_w^2\end{array}\right.$$

Note that $\Delta r$ depends on virtual contributions, so that in the SM $\Delta r = \Delta r(m_t, M_H)$ and also $s_w^2 = s_w^2(m_t, M_H)$.



## 2.7 The process $e^+e^- \to f\bar{f}$ at the $Z^0$ peak

The tree level amplitude 

:

$$\frac{e_0^2}{4s_{0w}^2 c_{0w}^2}\frac{\left[\gamma_\mu(I_3^\ell - 2Q_\ell s_{0w}^2) - I_3^\ell\gamma_\mu\gamma_5\right] \otimes \left[\gamma_\mu(I_3^f - 2Q_f s_{0w}^2) - I_3^f\gamma_\mu\gamma_5\right]}{s - M_{0Z}^2}$$

becomes, at the 1-loop level (neglecting $\gamma$–exchange and $\gamma$–$Z$ mixing),

$$\frac{e_0^2}{4s_{0w}^2 c_{0w}^2}\frac{\left[\gamma_\mu(I_3^\ell - 2Q_\ell s_{0w}^2) - I_3^\ell\gamma_\mu\gamma_5\right] \otimes \left[\gamma_\mu(I_3^f - 2Q_f s_{0w}^2) - I_3^f\gamma_\mu\gamma_5\right]}{s - M_Z^2 + i\frac{s}{M_Z^2}M_Z\Gamma_Z}$$

On the other hand

$$\frac{e_0^2}{4s_{0w}^2 c_{0w}^2} = \frac{e^2}{4s_w^2 c_w^2}\left[1 + \frac{2\delta e}{e} - \frac{c_w^2 - s_w^2}{s_w^2}\left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right)\right] =$$

$$= \sqrt{2}G_\mu M_Z^2\left[1 + \frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} - \frac{\Sigma_V(0) - \delta M_W^2}{M_W^2}\right] =$$

$$= \sqrt{2}G_\mu M_Z^2\left[1 + \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2} - \frac{\Sigma_Z(M_Z^2) - \Sigma_Z(0)}{M_Z^2}\right] =$$

$$= \sqrt{2}G_\mu M_Z^2 \cdot \varrho$$

finite !

At the same time, $s_w^2$ in the neutral current trasforms according to

$$s_{0w}^2 = s_w^2 \left[ 1 + \frac{c_w^2}{s_w^2} \left( \frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right) \right] = \kappa \cdot s_w^2$$

finite!

So, we get

- An *overall multiplicative factor $\varrho$ (finite)* for $Z$–exchange amplitudes
- A *multiplicative factor $\kappa$ (finite)* for $s_w^2$ in the expression of the neutral current (NC)

It can be shown that this is true for any NC amplitude $A$. We have then the following rules to obtain a 1–loop corrected amplitude:

- Write a NC amplitude at the tree level *in terms of* $G_\mu$
- Multiply the amplitude by $\varrho$ and $s_w^2$ in the NC by $\kappa$
- Dress the $Z$ propagator (at the $Z$ peak)

This is the so–called "improved Born approximation"



## 2.8 The universal corrections $\Delta r$, $\Delta\varrho$, $\Delta\kappa$

A more careful traitment of the electroweak processes at the $Z^0$ peak requires also the inclusion of the $\gamma$–$Z$ exchange contribution. This leads to the following expressions (assuming $\varrho = 1 + \Delta\varrho$ and $\kappa = 1 + \Delta\kappa$):

$$\begin{cases} \Delta\varrho = \dfrac{\Sigma_Z(0)}{M_Z^2} - \dfrac{\Sigma_W(0)}{M_W^2} - 2\dfrac{s_w}{c_w}\dfrac{\Sigma_{\gamma Z}(0)}{M_Z^2} & \text{(finite)} \\ \Delta\kappa = \dfrac{c_w^2}{s_w^2}\left( \dfrac{\delta M_Z^2}{M_Z^2} - \dfrac{\delta M_W^2}{M_W^2} - \dfrac{s_w}{c_w}\dfrac{\Sigma_{\gamma Z}(M_Z^2) - \Sigma_{\gamma Z}(0)}{M_Z^2} \right) & \text{(finite)} \end{cases}$$

The improved Born approximation can be used also in the *Z decay widths* and in the low–energy processes (if small $q^2$–dependent ~~terks~~ terms are ignored).

Note that the *asymmetries* at the $Z^0$ peak, being cross–section ratios, do not get contributions from $\Delta\varrho$ but only from $\Delta\kappa$.

The general procedure to calculate theoretically the measured quantities at the 1–loop level in terms of the electroweak parameters is essentially the following:

1) Fix $m_t$, $M_H$
2) Calculate $\Delta r \rightarrow M_W \rightarrow s_w^2$
3) Calculate $\Delta\varrho$ and $\Delta\kappa$
4) Use the improved Born approximation for the neutral current processes

# 1. Radiative corrections in the SM

## 1.1 General approach

Gauge group

$$SU(2)_L \otimes U(1)_Y \xrightarrow{SSB} U(1)_{em}$$

$g \qquad g' \qquad v = \langle H_0 \rangle$ three parameters at the tree level

$$\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2} = \frac{1}{4\pi\alpha}$$

fixes the interaction scale $\boxed{\alpha = 1/137.0359895(61)}$

from

$tg\theta_W = g'/g$ mixing angle

$$\begin{cases} M_W^2 = (gv/2)^2 \\ M_Z^2 = M_W^2 / c_W^2 \end{cases}$$ gauge boson masses

it follows

$v = (\sqrt{2} G_F)^{-\frac{1}{2}} = 250$ GeV typical mass restoring the e.w. unification, derived from the low-energy 4-fermion coupling

$\boxed{G_F}$ well measured from μ-decay (see later)

the third parameter can be

$\boxed{M_Z}$ well measured at LEP (see later)

In conclusion...

$\boxed{\alpha}$ $\boxed{G_F}$ $\boxed{M_Z}$ three parameters at the tree level



It follows that...

at zeroth order (Born approximation) each physical observable is given by

$$R_i^{(0)} = R_i^{(0)}(g, g', v) = R_i^{(0)}(\alpha, G_F, M_Z)$$

fundamental constants

Including $\boxed{\text{radiative corrections}}$ (ew + QCD)

$$R_i^{(0)} = R_i^{(0)} + \Delta R_i(\alpha, G_F, M_Z; \boxed{\alpha_s, m_t, M_H})$$

new parameters

with the well-known functional dependence

$\Delta R(m_t) \sim m_t^2 / M_W^2$ + subdominant terms

$\Delta R(M_H) \sim \ln(M_H / M_W)$ asymptotically (large $M_H$)

"screening theorem" due to the "custodial SU(2)"

$\boxed{\text{Radiative corrections}}$ depend on $\boxed{\alpha_s}$, $\boxed{m_t}$, $\boxed{M_H}$

Radiative corrections will include

- all one-loop contributions
- $O(\alpha\alpha_s)$ contributions
- $O(\alpha_s^2)$ and $O(\alpha_s^3)$ in $\Gamma_h$ (hadronic width)
- resummation of higher order terms (where possible)

In particular "oblique corrections", the corrections to the gauge boson propagators,are universal, i.e. the same in all processes: the bare propagators $D^0_{\gamma,Z,W}$

$$\begin{pmatrix} D^0_{\gamma\gamma}(s) & & \\ & D^0_{ZZ}(s) & \\ & & D^0_{WW}(s) \end{pmatrix} = \begin{pmatrix} s & & \\ & s - M^2_{Z0} & \\ & & s - M^2_{W0} \end{pmatrix}^{-1}$$

after insertion of the self-energies $\Sigma_{\gamma,Z,W}$ and the usual renormalization procedure, transform in the renormalized propagators $\hat{D}_{\gamma,Z,W}$

$$\begin{pmatrix} \hat{D}_{\gamma\gamma}(s) & \hat{D}_{\gamma Z}(s) & \\ \hat{D}_{\gamma Z}(s) & \hat{D}_{ZZ}(s) & \\ & & \hat{D}_{WW}(s) \end{pmatrix} = \begin{pmatrix} s + \hat{\Sigma}_{\gamma\gamma}(s) & \hat{\Sigma}_{\gamma Z}(s) & \\ \hat{\Sigma}_{\gamma Z}(s) & s - M_Z^2 + \hat{\Sigma}_{ZZ}(s) & \\ & & s - M_W^2 + \hat{\Sigma}_{WW}(s) \end{pmatrix}^{-1}$$



the right behaviour at poles being guaranteed by

$$\mathrm{Re}\,\hat{\Sigma}_{WW}(M_W^2) = \mathrm{Re}\,\hat{\Sigma}_{ZZ}(M_Z^2) = \hat{\Sigma}_{\gamma\gamma}(0) = \hat{\Sigma}_{\gamma Z}(0) = 0$$

The renormalized self-energies are expressed in terms of the non-renormalized ones through the following - finite - relations

$$\hat{\Sigma}_{\gamma\gamma}(s) = \Sigma'_{\gamma\gamma}(0)$$

$$\hat{\Sigma}_{\gamma Z}(s) = \Sigma_{\gamma Z}(s) - \Sigma_{\gamma Z}(0) + s\left[2\frac{\Sigma_{\gamma Z}(0)}{M_Z^2} - \frac{c_W}{s_W}\left(\frac{\Sigma_{ZZ}(M_Z^2)}{M_Z^2} - \frac{\Sigma_{WW}(M_W^2)}{M_W^2}\right)\right]$$

$$\hat{\Sigma}_{ZZ}(s) = \Sigma_{ZZ}(s) - \Sigma_{ZZ}(0) + (s - M_Z^2)\left[-\Sigma'_{\gamma\gamma}(0) - 2\frac{c_W^2 - s_W^2}{s_W c_W}\frac{\Sigma_{\gamma Z}(0)}{M_Z^2} + \right.$$
$$\left. + \frac{c_W^2 - s_W^2}{s_W^2}\left(\frac{\Sigma_{ZZ}(M_Z^2)}{M_Z^2} - \frac{\Sigma_{WW}(M_W^2)}{M_W^2}\right)\right]$$

$$\hat{\Sigma}_{WW}(s) = \Sigma_{WW}(s) - \Sigma_{WW}(0) + (s - M_Z^2)\left[-\Sigma'_{\gamma\gamma}(0) - 2\frac{c_W}{s_W}\frac{\Sigma_{\gamma Z}(0)}{M_Z^2} + \right.$$
$$\left. + \frac{c_W^2}{s_W^2}\left(\frac{\Sigma_{ZZ}(M_Z^2)}{M_Z^2} - \frac{\Sigma_{WW}(M_W^2)}{M_W^2}\right)\right]$$

Let us define the vacuum polarization functions

$$\hat{\Pi}_{\gamma\gamma}(s) = \mathrm{Re}\,\frac{\hat{\Sigma}_{\gamma\gamma}(s)}{s}$$

$$\hat{\Pi}_{\gamma Z}(s) = \mathrm{Re}\,\frac{\hat{\Sigma}_{\gamma Z}(s)}{s}$$

$$\Pi_{ZZ}(s) = \mathrm{Re}\,\frac{\hat{\Sigma}_{ZZ}(s)}{s - M_Z^2}$$

$$\hat{\Pi}_{WW}(s) = \mathrm{Re}\,\frac{\hat{\Sigma}_{WW}(s)}{s - M_W^2}$$

The relevant (finite) corrections are then given by

$$\Delta\alpha(s) = -\hat{\Pi}_{\gamma\gamma}(s)$$

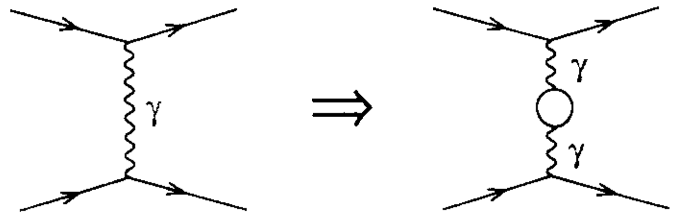
$$\Delta\rho(s) = \hat{\Pi}_{WW}(s) - \hat{\Pi}_{ZZ}(s)$$

$$\Delta\kappa(s) = -c_W/s_W\,\hat{\Pi}_{\gamma Z}(s)$$

$$\Delta r = -\hat{\Pi}_{WW}(0)$$



## 1.1 The electromagnetic vacuum polarization and $\Delta\alpha$




corresponds to

$$\alpha\,\frac{J_\mu^Q \otimes J_\mu^Q}{q^2} \;\Longrightarrow\; \alpha\,\frac{J_\mu^Q \otimes J_\mu^Q}{q^2 + \hat{\Sigma}_{\gamma\gamma}(q^2)} = \alpha(q^2)\,\frac{J_\mu^Q \otimes J_\mu^Q}{q^2}$$

with

$$\boxed{\alpha(q^2) = \frac{\alpha}{1 - \Delta\alpha(q^2)}} \qquad \left(\text{LEP physics: } \Delta\alpha(M_Z^2)\right)$$

$\Delta\alpha(q^2)$ is a QED correction:

- contributions of heavy particles decouple
- contributions of light quarks are important, and affect the precise estimate of $\alpha(M_Z^2)$

Approach:
estimate of the hadronic contribution $\Delta\alpha(M_Z^2)_h$ through a dispersive representation, independent of quark masses,

$$\boxed{R(s) = \frac{3s}{4\pi\alpha^2}\,\sigma(s)}$$

$$\Delta\alpha(M_Z^2)_h = \frac{\alpha}{3\pi} M_Z^2 \int_{4m_\pi^2}^{\infty} ds \, \frac{R(s)}{s(s - M_Z^2)}$$

hadronic cross-section of the normalized process
$e^+e^- \longrightarrow \gamma \longrightarrow$ hadrons

an interpolation of the experimental measurements of $\sigma(s)$ gives

Jegerlhener (1991)

$$\boxed{\alpha^{-1}(M_Z^2) = 128.7 \pm 0.12}$$

main source of theoretical uncertainties in the RC estimates

Note that Born approximation with $\alpha \longrightarrow \alpha(M_Z^2)$ was able, until 1993, to reproduce the experimental quantities within $\pm 1\sigma$

$\Downarrow$

**What about "genuine" weak corrections?**

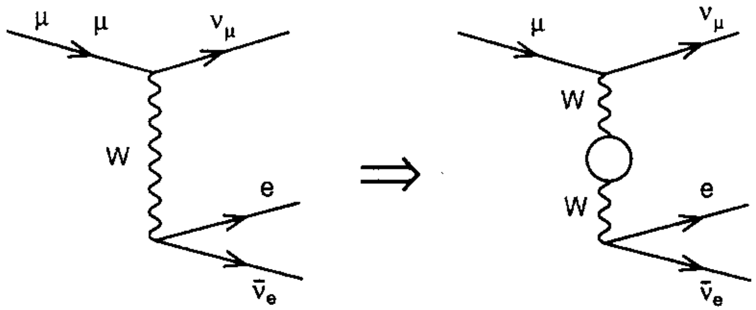


## 1.3 Charged currents, W mass and $\Delta r$

At the tree level the W mass is related to the 3 fundamental constants

$$M_W^2\left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_F}$$

The correction to the W propagator induces the correction to the charged current coupling constant $G_F$




μ−decay

i.e.

$$G_F \Longrightarrow G_F\left[\frac{q^2 - M_W^2 - \hat{\Sigma}_{WW}(q^2)}{q^2 - M_W^2}\right]_{q^2=0} = G_F(1 - \Delta r)$$

$G_F$ is defined in terms of $\tau_\mu$: 

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^2}{192\pi^3}\left(1 - 8\frac{m_e^2}{m_\mu^2}\right)\left[1 - \left(\frac{\alpha}{\pi}\right)\left(\frac{25}{4} - \pi^2\right)\right]$$

$G_F$ includes explicitly QED corrections

$$\Downarrow$$

$$\boxed{G_F = 1.16639(2) \times 10^{-5}\ \mathrm{GeV}^{-2}}$$

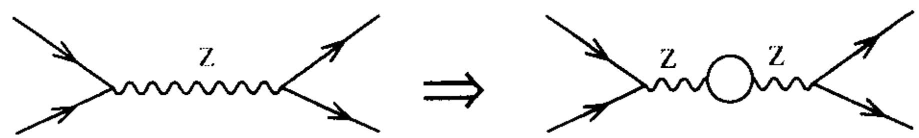
As a consequence, the relation between the W mass and the other fundamental constants is modified into (Sirlin)

$$M_W^2\left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_F(1 - \Delta r)} = \frac{\pi\alpha(M_Z^2)}{\sqrt{2}G_F(1 - \Delta r_w)}$$

"genuine" ew correction
(QED contribution extracted)

The mass spectrum, which contributes to $\Delta r_w$, contains $M_W$ $\Longrightarrow$ The equation is then implicit in $M_W$ and must be solved by recursion

## 1.4 Neutral currents, Z mass and $\Delta\rho$ 




corresponds to

$$G_F\,\frac{J_\mu^{NC}\otimes J_\mu^{NC}}{s - M_Z^2} \Longrightarrow G_F(1 - \Delta r)\,\frac{J_\mu^{NC}\otimes J_\mu^{NC}}{s - M_Z^2 + \hat{\Sigma}_{ZZ}(s)} = G_F\,\rho(s)\,\frac{J_\mu^{NC}\otimes J_\mu^{NC}}{s - M_Z^2}$$

i.e.

$$\boxed{G_F \Longrightarrow G_F\rho(s) = G_F\left(1 + \Delta\rho(s)\right)}$$ (Veltman)

Dominant contribution to $\Delta\rho(s)$ from isospin-breaking effects: asymptotic one-loop contribution from the top quark mass

$$\Delta\rho\Big|_{top} = \frac{3\alpha}{16\pi s_w^2 c_w^2}\,\frac{m_t^2}{M_Z^2}$$

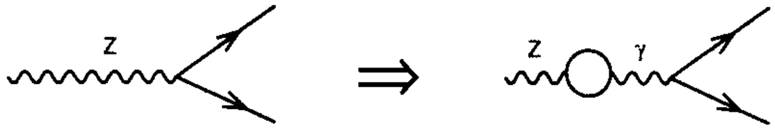
Non-minimal Higgs content induces further contributions (non-minimal Standard Model): in general

$$\rho_{Born} = \frac{\Sigma_h\ v_h^2\ [T_h\ (T_h + 1) - T_{3h}^2]}{2\ \Sigma_h\ v_h^2\ T_{3h}^2}$$

- $\Sigma_h$ sums over the Higgs representations
- $v_h$ Higgs v.e.v.'s
- $T_h$ $SU(2)_L$ quantum numbers

## 1.5 The weak mixing angle and $\Delta\kappa$

A further contribution to the $J^Q_\mu$ component of $J^{NC}_\mu$ comes from the $\gamma$-Z correction




i.e.

$$J^{NC}_\mu = J^3_\mu - s_W^2 J^Q_\mu \Longrightarrow J^3_\mu - s_W^2 J^Q_\mu \left(1 - \frac{c_W}{s_W}\frac{\hat{\Sigma}_{\gamma Z}(s)}{s}\right) = J^3_\mu - s_W^2 \kappa(q^2) J^Q_\mu$$

The radiative correction is reabsorbed into an "effective mixing angle"

$$\boxed{s_W^2 \Longrightarrow s_W^2(q^2) = s_W^2 \kappa(q^2) = s_W^2[1 + \Delta\kappa(q^2)]}$$

An equivalent parametrization can be used, by introducing $\kappa'(q^2) = 1 + \Delta\kappa'(q^2)$ through the relation

$$s_W^2 \kappa(q^2) = s_{W0}^2 \kappa'(q^2) \qquad \text{with} \qquad s_{W0}^2 = s_W^2\big|_{\Delta r_w = 0}$$

It follows

$$\Delta\kappa'(q^2) = \Delta\kappa(q^2) + \frac{c_W^2}{c_W^2 - s_W^2}\Delta\alpha(M_Z^2)$$

which isolates the "genuine electroweak correction" $\boxed{\Delta\kappa'(q^2)}$



## 1.6 Higher order corrections to $\Delta\rho$ and $\Delta r$

We can extract the "1 Particle Irreducible" terms out from $\Delta\rho(0)$:

$$\Delta\rho(0) = \Delta\rho_{dom} + \Delta\rho_{sub}$$

with

$$\Delta\rho_{dom} = 3\,x_t\left[1 - \left(\frac{\pi^2}{2} - 19\right)x_t - \frac{2\alpha_s}{3\pi}\left(\frac{\pi^2}{3} + 1\right)\right] \qquad \left(x_t = \frac{G_F m_t^2}{8\sqrt{2}\pi^2}\right)$$

and similarly for $\Delta r$

$$\Delta r = \Delta r_{dom} + \Delta r_{sub} = \Delta\alpha(M_Z^2) - \frac{c_W^2}{s_W^2}\Delta\rho_{dom} + \Delta r_{sub}$$

Then a geometrical resummation is performed

$$\left\{\begin{array}{lcl} 1 + \Delta\rho_{dom} & \longrightarrow & \dfrac{1}{1 - \Delta\rho_{dom}} \\ 1 + \Delta\alpha(M_Z^2) & \longrightarrow & \dfrac{1}{1 - \Delta\alpha(M_Z^2)} \end{array}\right.$$

The W mass is now expressed through the recursion relation

$$M_W^2 = \frac{1}{1 - \Delta\rho_d}\frac{M_Z^2}{2}\left[1 + \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2}G_F M_Z^2}\left(1 - \Delta\rho_d\right)\left(\frac{1}{1 - \Delta\alpha(M_Z^2)} + \Delta r_s\right)}\,\right]$$

(Halzen & Kniehl)

with non-negligible effects for $m_t$ > 100-150 GeV.

## 2. Model independent parametrization



At zeroth order (but with QED corrections included) all physical observables, as chiral couplings of quarks and leptons ($q^2_{L,R}$, $g_{L,R}$, $C_{1,2}$), W mass, total and partial Z widths and asymmetries, can be expressed in terms of

$\boxed{G_F}$ $\boxed{M_Z}$ $\boxed{\alpha(M_Z^2)}$ using $\boxed{s^2_{w0}} = s^2_w\big|_{\Delta r_w = 0}$

which satisfies

$$c^2_{w0}\, s^2_{w0} = \frac{\pi\alpha(M_Z^2)}{\sqrt{2} G_F M_Z^2}$$

At this point the "genuine" weak corrections are contained in

$\boxed{\Delta r_w}$ $\boxed{\Delta\rho(s)}$ $\boxed{\Delta\kappa'(s)}$

- neglecting the s-dependence of $\Delta\rho$ and $\Delta\kappa'$
- performing the substitution $s^2_{w0} \longrightarrow \kappa' s^2_{w0}$
- introducing $\Delta r_w$ in the expression of $M_W$
- multiplying by $\Delta\rho$ where necessary

we obtain a model-independent parametrization of the radiative corrections affecting each observable

$$R_i^{(1)} \cong R_i^{(0)} + \frac{\partial R_i}{\partial\rho}\Delta\rho + \frac{\partial R_i}{\partial s^2_{w0}}\Delta\kappa' + \frac{\partial R_i}{\partial\Delta r}\Delta r_w$$



Equivalent parametrizations in terms of linear combinations of $\Delta\rho$, $\Delta\kappa'$, $\Delta r_w$:

$\boxed{S, T, U}$ (Peskin & Takeuchi) $\boxed{\varepsilon_1, \varepsilon_2, \varepsilon_3}$ (Altarelli & Barbieri)

related by

$$\alpha S = 4s^2_w\varepsilon_3 = 4s^2_w c^2_w\Delta\rho + 4s^2_w(c^2_w - s^2_w)\Delta\kappa'$$

$$\alpha T = \varepsilon_1 = \Delta\rho$$

$$\alpha U = -4s^2_w\varepsilon_2 = -4\frac{s^4_w}{c^2_w - s^2_w}\Delta r_w - 4s^2_w c^2_w\Delta\rho + 8s^4_w\Delta\kappa'$$

In the model independent approach $\Delta\rho$, $\Delta\kappa'$, $\Delta r_w$ are unknown. In the Standard Model they depend on $m_t$ and $M_H$: retaining only the dominant terms

$$\varepsilon_1 \simeq \frac{3G_F m_t^2}{8\pi^2\sqrt{2}} - \frac{3G_F M_Z^2}{4\pi^2\sqrt{2}}\, s^2_w \ln\left(\frac{M_H}{M_Z}\right)$$

$$\varepsilon_2 \simeq -\frac{G_F M_W^2}{2\pi^2\sqrt{2}}\ln\left(\frac{m_t}{M_Z}\right)$$

$$\varepsilon_3 \simeq \frac{G_F M_W^2}{12\pi^2\sqrt{2}}\ln\left(\frac{M_H}{M_Z}\right) + \frac{G_F M_W^2}{6\pi^2\sqrt{2}}\ln\left(\frac{m_t}{M_Z}\right)$$

The model-independent approach, however, contains several approximations, when compared with the approach in the Standard Model: 16

- vertex and box corrections are not included
- the s-dependence from low ($s \simeq 0$) to high ($s = M_Z^2$) energies is neglected
- all contributions beyond one-loop cannot be included (for example, the resummation discussed before)

In order to reduce these effects, two approaches have been followed:

**I** Expansion of the $R_i$'s performed around values $R_i(m_t^*, M_H^*)$ estimated in the SM at the "reference point" $(m_t^*, M_H^*)$

$\downarrow$

precise estimate of the "non-oblique" corrections at the reference point, but approximation less reliable far from it

(Ellis, GLF, Lisi)

**II** Box and vertex corrections added to the $\varepsilon_i$'s ("hybrid" $\varepsilon_i$) estimated for three variables. A fourth $\varepsilon$, $\varepsilon_b$, added in order to account for the important vertex [redacted]

(Altarelli, Barbieri, Jadach)

(Altarelli, Barbieri, Caravaglios)

# 3. Precision electroweak measurements 17

## 3.1 The fundamental parameters

After the inclusion of ew+QCD radiative corrections, estimated in the SM, each physical observable is given by

$$C^{(rc)} = C^{(0)}(\alpha, G_F, M_Z) + \Delta C(\alpha, G_F, M_Z; \alpha_s, m_t, M_H)$$

**$\alpha$** fundamental constant: $\boxed{\alpha = 1/137.0359895(61)}$

but the inclusion of QCD corrections at the $M_Z^2$ scale gives (Jegerlhener 1991)

$\boxed{\alpha^{-1}(M_Z^2) = 128.7 \pm 0.12}$ main source of theoretical uncertainties in the RC estimates

**$G_F$** measured with great accuracy from $\mu$-decay, including QCD corrections

$\boxed{G_F = 1.16639(2) \times 10^{-5} \ \mathrm{GeV}^{-2}}$

**$M_Z$** measured with high precision at LEP

$\boxed{M_Z = 91.190 \pm 0.0044 \ \mathrm{GeV}}$

The error, however, is not neglected. It is included in the theoretical uncertainty affecting the parametrization

$\alpha_s$ in principle, a free parameter

(18)

but ...

independently estimated from a QCD analysis of the hadronic jets at LEP. A recent estimate gives

$$\boxed{\alpha_s(M_Z^2) = 0.123 \pm 0.006}$$ (Catani, Marseille, 1993)

error mainly of theoretical origin: hadronization, higher orders in perturbative QCD...)

Two possible attitudes:

- take $\alpha_s(M_Z^2)$ from jets, including the error in the theoretical uncertainties
- derive $\alpha_s(M_Z^2)$ from the precision ew data, and compare with the value coming from jets

$m_t$ The main goal of the precision electroweak physics for several years. Now [redacted] at the $p\bar{p}$ collider:

$$\boxed{m_t = 174 \pm 10^{+13}_{-12} \ \text{GeV}}$$ (CDF @ FERMILAB)

$M_H$ Only remaining totally unknown mass. Rather elusive dependence on the e.w. parameters ("screening theorem" due to the "custodial SU(2)"). Present experimental lower limit

$$\boxed{M_H > 63 \ \text{GeV}}$$ (LEP)

## 3.2 Low-energy experiments

(19)

**NC couplings**

- $\nu q$ interaction from deep-inelastic scattering $\nu N$
- $\nu e$ interaction at accelerators and reactors
- $\ell q$ interaction from parity violation effects

**$\nu q$ couplings**

from an overall analysis of the NC/CC data of all the deep-inelastic scattering experiments

$R_\nu^I$, $R_{\bar\nu}^I$, $R_\nu^p$, $R_{\bar\nu}^p$ — Quark Parton Model with QCD effects $\longrightarrow$ $u_L^2$, $d_L^2$, $u_R^2$, $d_R^2$

model-independent parametrization of the $\nu q$ interaction

(correlation effects and theoretical uncertainties taken into account)

**$\nu e$ couplings**

From all the data concerning

$\nu_\mu e$ (accelerator)

$\nu_e e$ (reactors) $\longrightarrow$ $g_V$, $g_A$

model-independent parametrization of the $\nu e$ interaction

**$\ell q$ couplings**

parity violation in atoms (Boulder, Paris) — $C_{1u}$

asymmetry in $\mu$C scattering (polarized $\mu$ at CERN) $\longrightarrow$ $C_{1d}$

asymmetry in eD scattering (polarized e at SLAC) — $C_{2u} - \frac{1}{2}C_{2d}$

model-independent parametrization of the q interaction

## 3.3 High-energy experiments

[redacted]

$$\begin{cases} e^+e^- & \text{(LEP, SLD)} \\ p\bar{p} & \text{(UA2, CDF)} \end{cases}$$

**$e^+e^-$ measurements at LEP**

The largest amount of precision data. A short discussion required here. We distinguish

**lineshape measurements**: partial widths and cross-section in the process

$$e^+e^- \longrightarrow Z \longrightarrow f\bar{f} \qquad \text{at the Z peak}$$

given by

$$\Gamma_{f\bar{f}} = N_c \frac{G_F M_Z^2}{12\pi\sqrt{2}}\left[1 - 4|Q_f| s_w^2 + 8 Q_f s_w^4\right]$$

$$\sigma_f^0 = \frac{12\pi\, \Gamma_{e^+e^-} \Gamma_{f\bar{f}}}{M_Z^2 \Gamma_Z^2}$$

Including $\gamma$-exchange and $\gamma Z$ interference

$$\sigma_f^0(s) = \frac{s}{(s - M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2}\left[\frac{12\pi\, \Gamma_{e^+e^-}\Gamma_{f\bar{f}}}{M_Z^2} + \frac{\boxed{I_f} N_c (s - M_Z^2)}{s}\right] + \frac{4}{3}\pi Q_f^2 \frac{\alpha^2(M_Z^2) N_c}{s}$$

resonant Z exchange

$\gamma Z$ interference

$\gamma$ exchange

Cannot be reliably estimated from the fit. Estimated in the SM, but essentially independent of the model in the extensions of the SM

Experimental estimates start from the above formula, with a fit to the different cross-sections, after deconvolution of the bremsstrahlung effects and correction for experimental efficiencies and acceptances.



The quantities measured at LEP are then (assuming lepton universality)

$$M_Z \qquad , \qquad \Gamma_Z$$

and

$$\begin{cases} \Gamma_h = \Gamma_{u\bar{u}} + \Gamma_{d\bar{d}} + \Gamma_{s\bar{s}} + \Gamma_{c\bar{c}} + \Gamma_{b\bar{b}} \\ \Gamma_\ell = \Gamma_{e^+e^-} + \Gamma_{\mu^+\mu^-} + \Gamma_{\tau^+\tau^-} \end{cases} \quad \underset{\text{through}}{\overset{\text{appearing}}{\Longrightarrow}} \quad \begin{cases} \sigma_h^0 = 12\pi \dfrac{\Gamma_{e^+e^-}\Gamma_h}{M_Z^2\Gamma_Z^2} \\ R_h = \dfrac{\Gamma_h}{\Gamma_{e^+e^-}} \end{cases}$$

As far as hadronic partial widths are concerned, LEP measures

$$\begin{cases} \Gamma_{b\bar{b}} & \text{with a vertex contribution} \sim m_t^2\ ! \\ \Gamma_{c\bar{c}} \end{cases}$$

**asymmetries**: At LEP, with unpolarized beams, one measures asymmetries in the final states (exclusive cross-sections)

$$\begin{cases} A_{FB}(e^+e^- \longrightarrow f\bar{f}) = \dfrac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} \quad \Longrightarrow \quad A_{FB}^\ell\ ,\ A_{FB}^{b\bar{b}}\ ,\ A_{FB}^{c\bar{c}} \\ A_{pol}(e^+e^- \longrightarrow f\bar{f}) = \dfrac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \quad \Longrightarrow \quad A_{pol}^\tau \end{cases}$$

## $e^+e^-$ measurements at SLD



With polarized beams, SLD measures

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$$

initial state polarization, then "production" asymmetry with inclusive measurement

$\Downarrow$

- probes e-coupling independently of final states
- does not require the assumption of universality
- highest precision on $s_w^2$ from a single measure

In the first analysis the value is used $\boxed{A_{LR} = 0.100 \pm 0.044}$

but the last value exhibited by SLD is

$$\boxed{A_{LR} = 0.1637 \pm 0.0075}$$

$\Downarrow$ which implies

$$\boxed{s_w^2\big|_{LEP} - s_w^2\big|_{SLD} \cong 2.6\,\sigma}$$

discrepancy !

## measurements at the $p\bar{p}$ collider

Estimate of the W mass, using the measurement of $M_Z$ at LEP to avoid the error due to the energy scale calibration.
Present estimate

$M_W = 80.22 \pm 0.26$ GeV (UA2 + CDF)



(before march-avril 1994)

| | | | | |
|---|---|---|---|---|
| CCFR + ..... | $u_L^2 = 0.1205 \pm 0.0117$<br>$d_L^2 = 0.1796 \pm 0.0119$<br>$u_R^2 = 0.0261 \pm 0.0082$<br>$d_R^2 = 0.0051 \pm 0.0078$ | vq | LE | ALL |
| CHARM II + ..... | $g_L = -0.2714 \pm 0.0180$<br>$g_R = 0.2277 \pm 0.0217$<br>$s_w^2\big\|_{CH\,II} = 0.232 \pm 0.009$ | ve | | |
| Boulder + ..... | $C_{1u} = -0.2052 \pm 0.0180$<br>$C_{1d} = 0.3577 \pm 0.0217$<br>$C_{2ud} = 0.0104 \pm 0.1879$ | eq | | |
| CDF + UA2 | $M_W = 80.22 \pm 0.26$ GeV | $M_W$ | HE | |
| LEP | $\Gamma_h/\Gamma_\ell = 20.763 \pm 0.049$<br>$\sigma_h^0 = 41.56 \pm 0.14$ nb<br>$\Gamma_Z = 2.489 \pm 0.007$ GeV<br>$\Gamma_{b\bar{b}}/\Gamma_h = 0.2200 \pm 0.0027$<br>$\Gamma_{c\bar{c}}/\Gamma_h = 0.1710 \pm 0.0140$ | WID | | |
| | $A_{FB}^b = 0.0990 \pm 0.0060$<br>$A_{FB}^\mu = 0.0158 \pm 0.0018$<br>$A_{pol}^\tau = -0.1390 \pm 0.0140$ | ASY | | |
| SLD | $A_{LR} = 0.100 \pm 0.044$ | | | |

# 4. Comparing theory and experiment in the SM

(before march-avril 1994)

All radiative corrections included: 1loop + higher order estimated contributions ($\alpha\alpha_s$, $\alpha_s^2$, $\alpha_s^3$, resummation, ...)

goals
- best determinations of [redacted]
- compatibility between different sets of data
- discrepancies $\longrightarrow$ "new physics" ?

**$m_t$** To derive informations on the top quark mass quite usual. We give a "historical" survey of the subject:

- LE data are able to put an upper limit on $m_t$ — Fig. 1
- ALL data put lower and upper limits on $m_t$ for fixed $M_H$ — Fig. 2

$$\boxed{m_t = 144 ^{+15}_{-17} \text{ GeV}} \quad \text{for} \quad M_H = M_Z$$

- ALL data constrain $m_t$ independently of $M_H$ — Fig. 3

$$\boxed{m_t = 136 ^{+18}_{-22} \text{ GeV}}$$

- What about the most recent value of $A_{LR}$ measured by SLD ? — [redacted]



**$M_H$** The determination of $M_H$ from precision data is rather controversial. A brief "historical" survey is reported:

- $M_H$ can be costrained at fixed $m_t$ — Fig. 5

(note the ambiguity in the behaviour of WID+$M_Z$)

$$\boxed{M_H = 70 ^{+180}_{-57} \text{ GeV}} \quad \text{for} \quad m_t = 145 \text{ GeV}$$

- Central value depending in a critical way on the value of $m_t$ .Low values slightly preferred — Fig. 6

$$\boxed{M_H = 35 ^{+300}_{-27} \text{ GeV}}$$

**$m_t$, $M_H$** With $\alpha_s(M_Z^2)$ fixed at the jet value (LEP) we can derive the allowed regions in the plane ($M_H$, $m_t$)

- Limits on both parameters simultaneously (note the correlation induced by the LEP measurements) — Fig. 7

**$m_t$, $M_H$, $\alpha_s(M_Z^2)$** Assuming also $\alpha_s(M_Z^2)$ as free parameter, complete analysis in the ($M_H$, $m_t$, $\alpha_s$) space.

- $\Delta\chi^2 = 1$ ellipsoid in the ($M_H$, $m_t$, $\alpha_s$) space Fig.8

- Best values of the three parameters:

$$m_t = 140\ ^{+21}_{-22}\ \text{GeV}$$

$$M_H = 35\ ^{+205}_{-26}\ \text{GeV}$$

$$\alpha_s = 0.116\ ^{+0.007}_{-0.006}$$

- $\alpha_s(M_Z^2)$ is essentially independent of $m_t$, but positively correlated to $M_H$: assuming "typically" $M_H = M_Z$

$$m_t = 146\ ^{+16}_{-18}\ \text{GeV}$$

$$\alpha_s = 0.117 \pm 0.006 \pm 0.001\ (m_t) \pm 0.002\ (M_H)$$

- In the past discrepancy between $\alpha_s|_{jets}$ and $\alpha_s|_{ew}$. Now agreement.

**In conclusion ...**

Impressive agreement between the different sets of data.

Impressive agreement of the data with the SM.



[redacted]

(after march-avril 1994)

| [redacted] | [redacted] | [redacted] | | |
|---|---|---|---|---|
| CCFR + ..... | same values | νq | LE | ALL |
| CHARM II + ... | same values | νe | LE | ALL |
| Boulder + ... | same values | eq | LE | ALL |
| CDF + UA2 | $M_W = 80.22 \pm 0.16$ GeV | $M_W$ | HE | ALL |
| LEP | $\Gamma_h/\Gamma_\ell = 20.79 \pm 0.040$<br>$\sigma_h^0 = 41.51 \pm 0.12$ nb<br>$\Gamma_Z = 2.497 \pm 0.0038$ GeV<br>$\Gamma_{b\bar{b}}/\Gamma_h = 0.2208 \pm 0.0026$<br>$\Gamma_{c\bar{c}}/\Gamma_h = 0.172 \pm 0.013$ | WID | HE | ALL |
| LEP | $A_{FB}^b = 0.0961 \pm 0.0043$<br>$A_{FB}^\mu = 0.0170 \pm 0.0016$<br>$A_{pol}^\tau = -0.15 \pm 0.01$<br>$A_{FB}^c = 0.070 \pm 0.011$ | ASY | HE | ALL |
| SLD | $A_{LR} = 0.1637 \pm 0.0075$ | ASY | HE | ALL |

Moreover ...

| | | |
|---|---|---|
| CDF | $m_t = 174 \pm 10\ ^{+13}_{-12}$ GeV | direct measurement |

and

| | |
|---|---|
| $\alpha_s(M_Z^2) = 0.118 \pm 0.007$ | world average! |

# 4'. Comparing theory and experiment in the SM

(after march-avril 1994)

Now the top quark mass has been measured by CDF at Fermilab

Fig. 9

- Contours in the ($M_H$, $m_t$) plane of all the e.w. data (ALL) including (+) or not including (-)

  the SLD measurement of $A_{LR}$ (±ALR)

  the CDF kinematic fit to $m_t$ (±CDF)

Fig. 10

- Indications on $M_H$ from different combinations of precision electroweak data

Fig. 11

Higgs boson hunting is open !



# 5. radiative corrections beyond the SM

## 5.1 Why beyond the SM ?

Several reasons justify an analysis beyond the SM, even if the large effect related to the top quark mass in the MSM tends to mask other possible minor effects of new physics.

- Impossible to predict fermion masses, quark mixings, Higgs mass, number of generations.
- Large, unnatural, and unjustified, mass difference between light and heavy quark
- The three couplings of $SU(3)_c$ , $SU(2)_L$ , $U(1)_Y$ do not converge to the same value when evolution equations are applied starting from the known MSM mass spectrum.
- There is no place for gravity.

↓

Minimal Standard Model as low energy limit of a GUT ?

This would answer to the two last points.

But...

If the SM is considered as the limit of a GUT theory characterized by a new scale $\Lambda$, with $\boxed{\Lambda >> v}$

$$\Downarrow$$

the lower scale v is destabilized, since the new scale $\Lambda$ contributes through radiative corrections to the masses of the scalars of the theory:

$$M_H^2 \approx O(\Lambda^2) >> v^2 \qquad \boxed{\text{hierarchy problem}}$$

$$\Downarrow \text{ requires}$$

"fine tuning" at the level of radiative corrections (cancellation between $M^2_{H0}$ and corrections $O(\Lambda^2)$ with precision of the order $v^2/\Lambda^2$) $\boxed{\text{naturalness problem}}$

$$\Downarrow$$

$\boxed{\text{either}}$ a new symmetry ensuring the cancellation of divergencies

$\boxed{\text{or}}$ scalar (Higgs) fields no more fundamental: compositeness

If naturalness is required, only two possible wayouts $\Longrightarrow$ [redacted]



## 5.2 The supersymmetric option

Only the supersymmetric extensions of the MSM (SUSY theories) do not present quadratically divergent radiative corrections.

$$\Downarrow$$

Stability of the breaking scale.

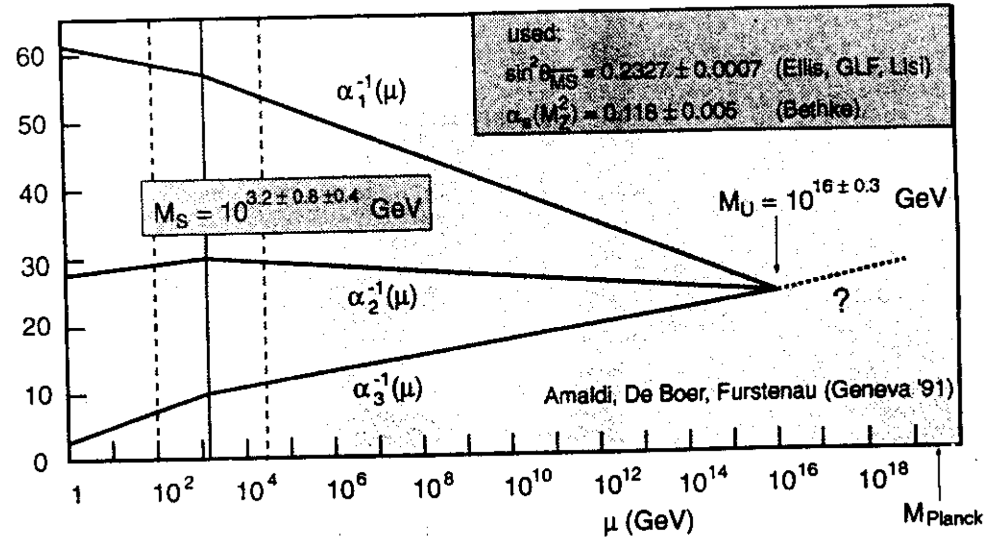
A soft breaking of SUSY must be introduced, with

$$\boxed{M_{SUSY} \approx 1 - 10 \text{ TeV}}$$

The hierarchy

$$\boxed{v < M_{SUSY} << M_{GUT}}$$

leads to a satisfactory unification of the three gauge couplings $\alpha_{em}, \alpha_W, \alpha_S$ at a scale $M_{GUT} \approx 10^{16}$ GeV, compatible with the present limits on the proton lifetime



## 5.3 The Minimal Supersymmetric Standard Model (MSSM)

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The simplest version of SUSY theories. It satisfies

- Agreement with the present phenomenology
- Minimal number of parameters

Only one supersymmetric generator, which introduces the SUSY partners of the usual particles. Two kinds of particle-sparticle pairs:

spin $(\frac{1}{2}, 0)$ : chiral supermultiplets as $(f, \tilde{f})$ and $(\tilde{H}, H)$

spin $(1, \frac{1}{2})$ : vector supermultiplets as $(V, \tilde{V})$

MSSM requires:

**R-parity invariance** (to preserve L and B invariance)

$\Downarrow$

- sparticles produced in pairs
- lightest SUSY particle stable

After a soft breaking of supersymmetry, two parameters must be introduced

$m_0$ universal mass of scalars

$m_{\frac{1}{2}}$ universal mass of gauginos $\left(m_{\tilde{g}} = \frac{\alpha_s}{\alpha_{GUT}} m_{\frac{1}{2}}\right)$

32

**Higgs sector** of the MSSM:

Two doublets, with v.e.v.'s $v_1$, $v_2$ and coupling parameter $\mu$ (the coefficient of the term $\mu H_1 H_2$ in the superpotential)

After spontaneous symmetry breaking:

2 neutral Higgs with CP = +1 and masses related to $v_1$, $v_2$ (h, h')

1 neutral Higgs with CP = -1 and mass $m_A$ (A)

2 charged Higgs with masses $m^2_{H^\pm} = m^2_A + M^2_W$

from the constraint $v_1^2 + v_2^2 = v^2$

$\Downarrow$

only 2 independent parameters $\Longrightarrow$ $\tan\beta = v_2/v_1$, $m_A$

After diagonalization of the neutral Higgs mass matrix

$$m^2_{h,h'} = \frac{1}{2}\left[ m_A^2 + M_Z^2 \mp \sqrt{\left(m_A^2 + M_Z^2\right)^2 - 4m_A^2 M_Z^2 c^2_{2\beta}} \right]$$

$\Downarrow$

$m_h < M_Z < m_{h'}$ (at the tree level)

In conclusion, 5 parameters

$m_0$, $m_{\tilde{g}}$, $m_A$, $\tan\beta$, $\mu$ to be added to $m_t$

## 5.4 Radiative corrections in the MSSM

When $m_0$, $m_{\tilde{g}}$, $m_A$, $\mu$ grow, sparticle masses become larger and larger. It follows

| "light" SUSY spectrum $\Longrightarrow$ MSM spectrum |
|---|

the only "signature" of the MSSM being $m_h < M_Z$.

Still valid at 1-loop: in fact

radiative corrections from a soft breaking $\Longrightarrow$ "decoupling" of the sparticle spectrum

In particular, $m_h \underset{m_A \to \infty}{\Longrightarrow} M_H$

but with the limit $\boxed{m_h < M_Z \pm 30 \text{ GeV}}$ uncertainty strongly dependent on the top quark mass

Radiative corrections in the MSSM include:

- all supersymmetric contributions to "oblique" corrections
- contributions to $\Gamma_Z$ of sparticle decay channels, if open
- supersymmetric corrections to the $Z \to b\bar{b}$ vertex
- threshold effect for chargino and neutralino masses slightly over the kinematical limit $M_Z/2$



Properties of the MSSM radiative corrections:

- Purely supersymmetric corrections ($m_t$-independent) are in general smaller than the corresponding MSM corrections
- corrections induced by sfermions (in particular the doublet stop-sbottom) are of the same sign as those induced by $m_t$
- corrections induced by charginos on the decay widths for $m_{\chi^\pm} \to M_Z/2$ can be of opposite sign with respect to those induced by $m_t$
- corrections induced by charged Higgs to the $Z \to b\bar{b}$ vertex have the same sign of those induced by $m_t$
- corrections due to the Higgs sector before the decoupling are typical of the MSSM (they involve two doublets, with effects on their masses and mixing)

Further constraint: $\boxed{\text{unification of the couplings at } M_{GUT}}$

$$g(M_{GUT}^2) = g'(M_{GUT}^2) = g_s(M_{GUT}^2)$$

$$\Downarrow$$

if estimated in some SUSY GUT (for example, the SUSY version of SU(5))

- a prediction for $M_{GUT}$ ($M_{GUT} \sim 10^{16}$ GeV)
- a rather complicate expression for

$$\sin^2\theta_w(M_Z^2)\big|_{GUT}$$

in terms of the couplings and of the MSSM spectrum, to be compared with $s_w^2$ extracted from the precision ew data

## 5.5 Comparing theory and experiment in the MSSM (35)

Within the previous scheme, all radiative corrections in the MSSM are taken into account. It is remarkable that, in spite of the large number of parameters, only in a few, well specified cases one finds an improvement with respect to the MSM. This allows to derive limits on the MSSM parameters.

Only a few results reported:

- Comparison between SM and MSSM as far as the prediction of the Higgs boson mass is concerned, for several values of the top quark mass — Fig. 12

- Contours in the $(m_h, m_t)$ plane of all the e.w. data (ALL) including (+) or not including (-)
  - the SLD measurement of $A_{LR}$ (±ALR)
  - the CDF kinematic fit to $m_t$ (±CDF)

  Fig. 13

- Exclusion plot in the plane $(m_0, m_{\tilde{g}})$. Comparison of the MSSM limits from radiative corrections (90 % C.L.), the limits on slepton and chargino masses from LEP and the negative results of CDF searches for gluinos and squarks — Fig. 14

## 5.6 Electroweak data and model-independent parametrization (36)

Precision electroweak data reported in the space $(\varepsilon_1, \varepsilon_2, \varepsilon_3)$: [redacted]

In the same space reported the [redacted], for plausible values of $(m_t, M_H)$:

90 GeV < $m_t$ < 250 GeV ; 50 GeV < $M_H$ < 1 TeV — Fig. 18

The figure refers to old (1991) data. With the recent data, the agreement with the SM is considerably improved. At present

[redacted]

The model-independent approach, however, contains several approximations, when compared with the approach in the Standard Model:

- vertex and box corrections are not included

- the s-dependence from low ($s \simeq 0$) to high ($s = M_Z^2$) energies is neglected

- all contributions beyond one-loop cannot be included

## 5.7 Radiative corrections in technicolour theories



[redacted] cannot be ascribed to $\begin{cases} \text{pions} \\ <t\bar{t}> \text{ condensates} \end{cases}$

- pions cannot be the longitudinal components of the gauge vector bosons ($M_W = gf_\pi/2$ required)
- $<t\bar{t}>$ condensates require large $m_t$ : $m_t$ = 200-250 GeV

$\boxed{\text{then}}$

New gauge (techni)interactions at a scale $\Lambda_{TC} >> \Lambda_{QCD}$ required for dynamical breaking.
Correct decay constant for technipions if

$$F_\pi = v \quad \Rightarrow \quad \Lambda_{TC} \approx \frac{F_\pi}{f_\pi} \Lambda_{QCD} \approx O(1\ \text{TeV})$$

It is assumed that the gauge group is $SU(N_{TC})$ with

number of technicolours

- $N_{TF}$ families of technifermions in the fundamental IR
- (bound) physical states $\Rightarrow$ singlets of $SU(N_{TC})$

Radiative corrections depend on $N_{TC}$ and $N_{TF}$ and are essentially independent of the details of the spectrum.



In technicolour theories RC cannot be estimated perturbatively, the Higgs sector being strongly interacting.

But RC can be estimated through dispersive representations of the contributions to the self-energies, added to some further "ad hoc" assumptions

By rescaling hadronic data and spectra of QCD to the technicolour scale (Peskin & Takeuchi, 1992)

$$\Delta S_{TC} \simeq 0.05\ N_{TF}\, N_{TC} + 0.127$$

number of technifermions ($\geq 2$)

number of technicolours ($\geq 2$)

contribution coming from $M_H = M_Z$

Similar corrections to S are obtained also assuming that TC dynamics differs from that of QCD (Appelquist & Triantaphillou, 1992).

Assuming asymmetry in the hypercharge interaction of techniquark doublets, it follows (Peskin & Takeuchi, 1992)

$$\Delta T_{TC} \simeq 0.150\ N_C\, N_{TC} \frac{m_t^2}{M_Z^2}$$

number of colours of technifermions

Technicolour theories are clearly disfavoured by experimental data, in particular for large $N_{TC}$ , $N_{TF}$ .

Fig. 19

# 6. Conclusions and prospects

Now that the first indications about $m_t$ are available ...

precision electroweak physics exhibits an extraordinary agreement with the SM, with

- an impressive consistency on the top quark mass $m_t$
- first indications about the Higgs mass $M_H$ (supersymmetric Higgs !?)

conversely ...

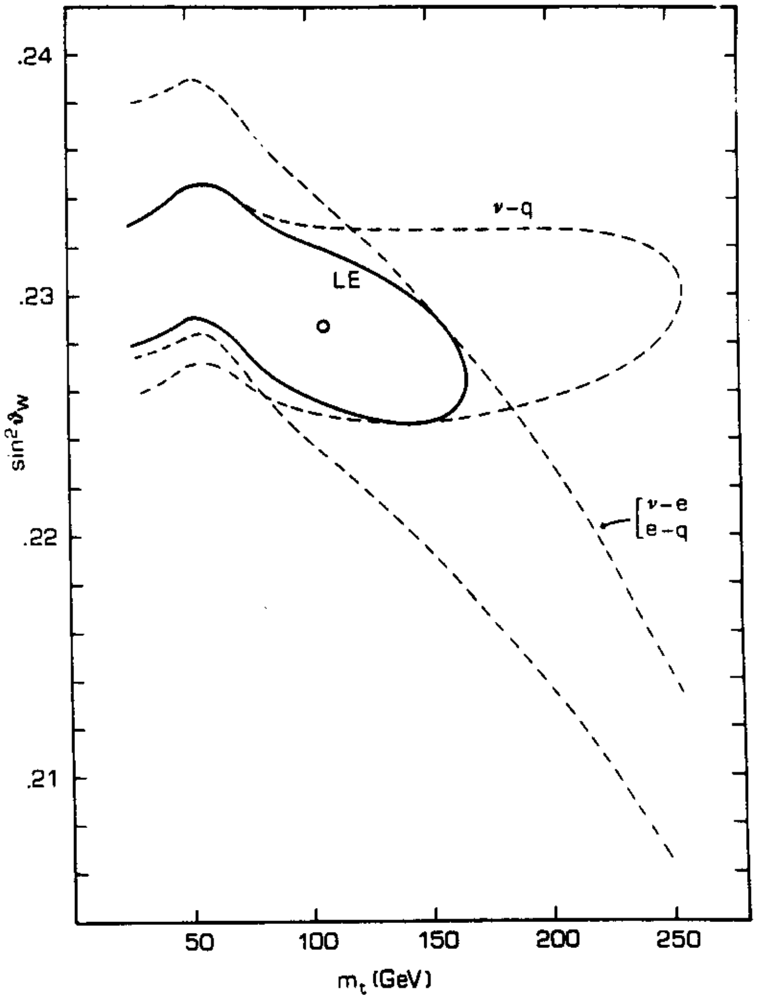
very difficult to extract indications of "new" physics. At present

- Higgs mass consistent with a light supersymmetric Higgs (too optimistic ?!)
- technicolour theories poorly compatible with the data (in particular for large $N_{TC}$, $N_{TF}$)

however ...

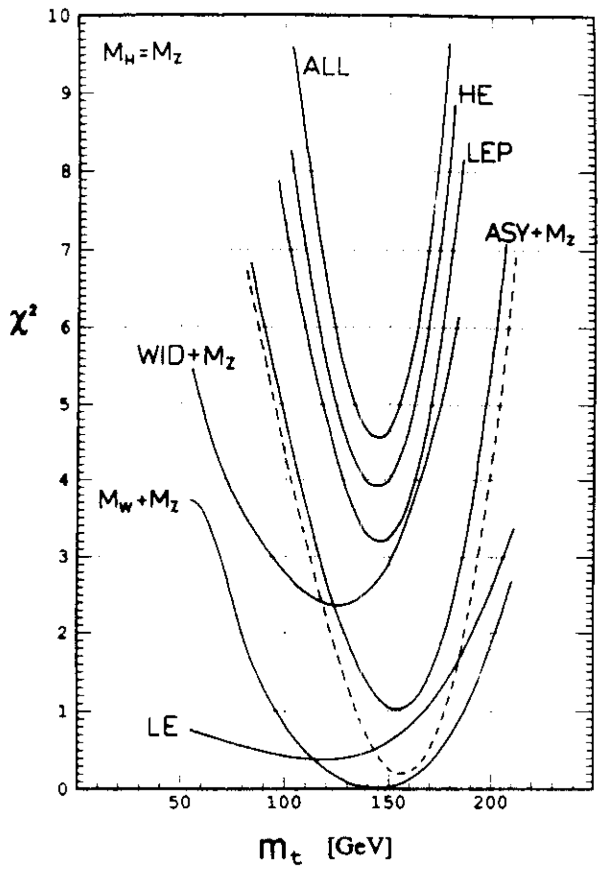
- some discrepancies still remain:
  - ⟹ what about $A_{LR}$ measured by SLD?
  - ⟹ why a rather slow $m_t$ when extracted by $\Gamma_{bb}$ ?

Further work is needed !



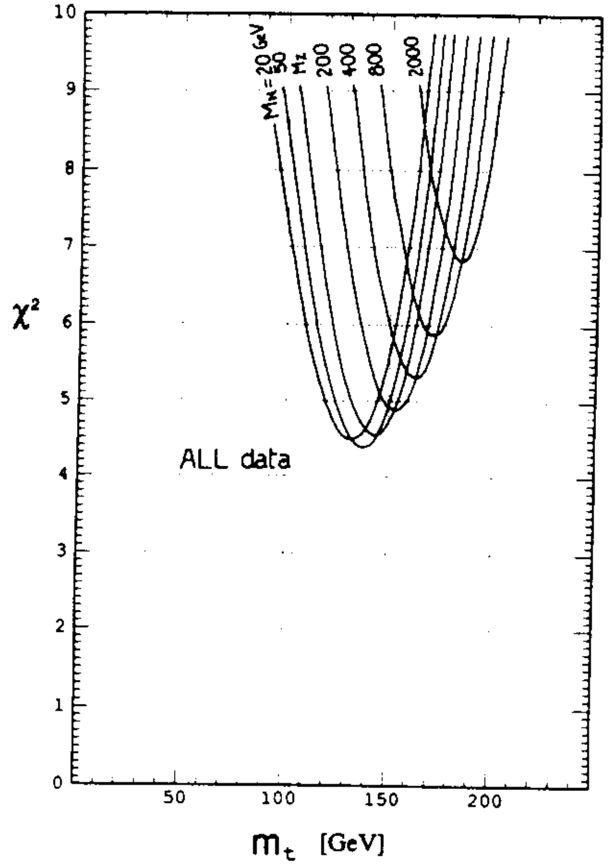

$M_H = M_Z$

Fig. 2



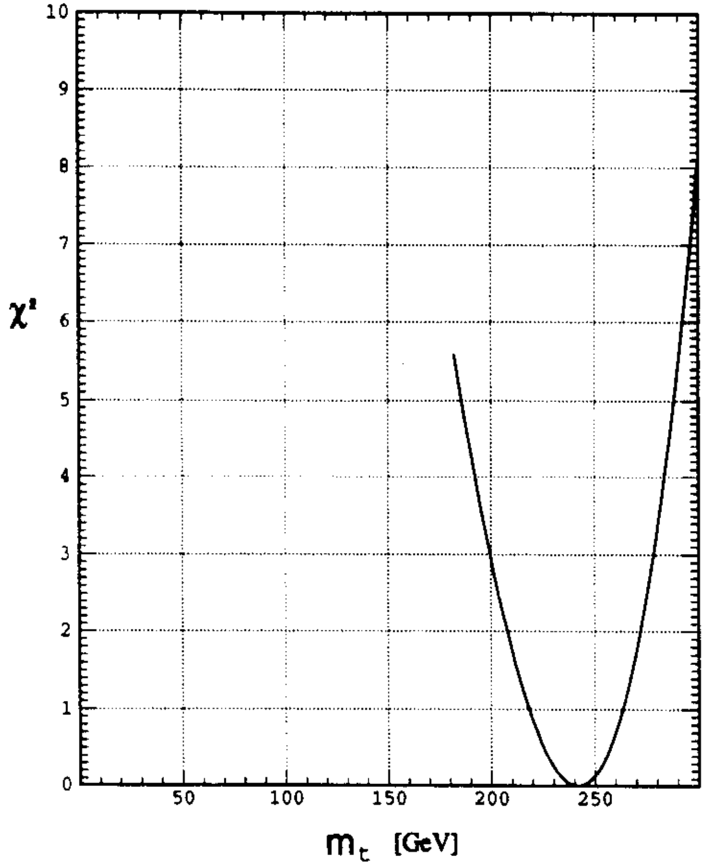
$M_H = M_Z$
ALL
HE
LEP
ASY+$M_Z$
WID+$M_Z$
$M_W$+$M_Z$
LE
$\chi^2$
0
1
2
3
4
5
6
7
8
9
10
50
100
150
200
$m_t$ [GeV]


Fig. 3



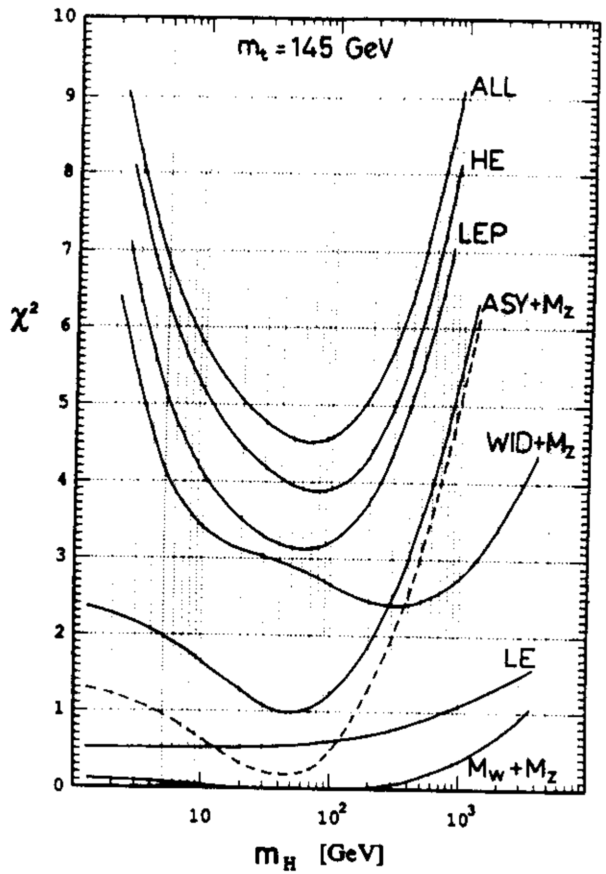
$M_H$=20 GeV
50
$M_Z$
200
400
800
2000
ALL data
$\chi^2$
0
1
2
3
4
5
6
7
8
9
10
50
100
150
200
$m_t$ [GeV]

Fig. 4



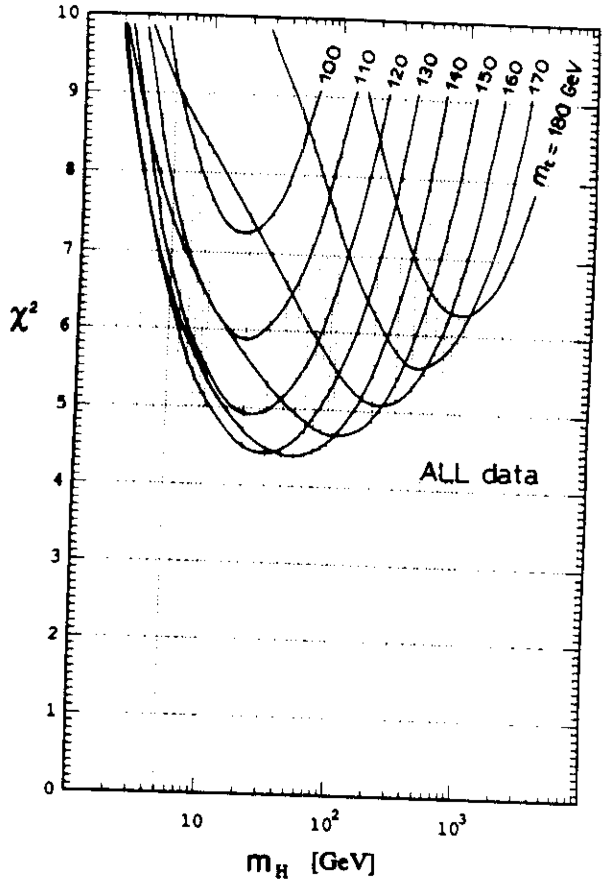
$\chi^2$
$m_t$ [GeV]


Fig. 5



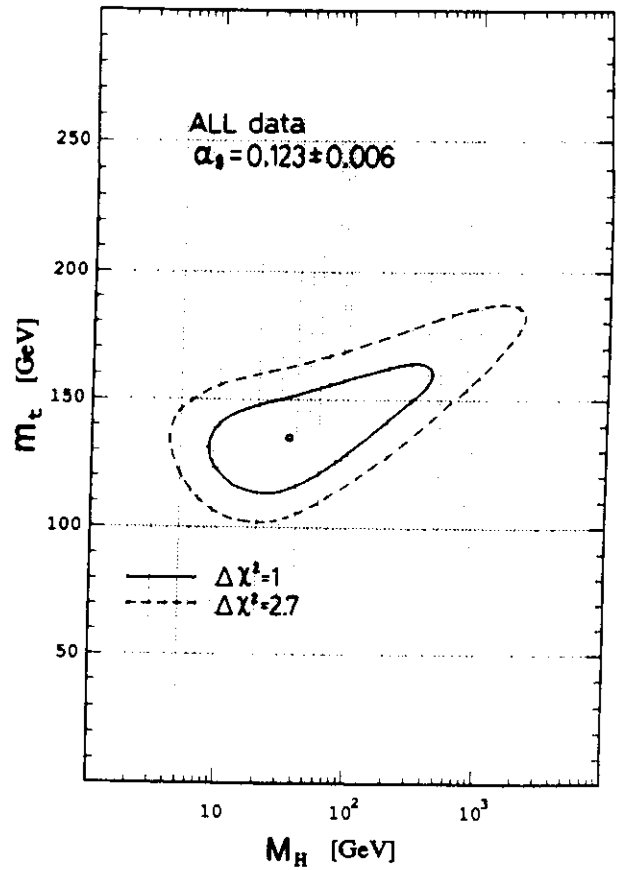
$m_t = 145$ GeV
ALL
HE
LEP
ASY+$M_Z$
WID+$M_Z$
LE
$M_W + M_Z$
$\chi^2$
$m_H$ [GeV]

Fig. 6



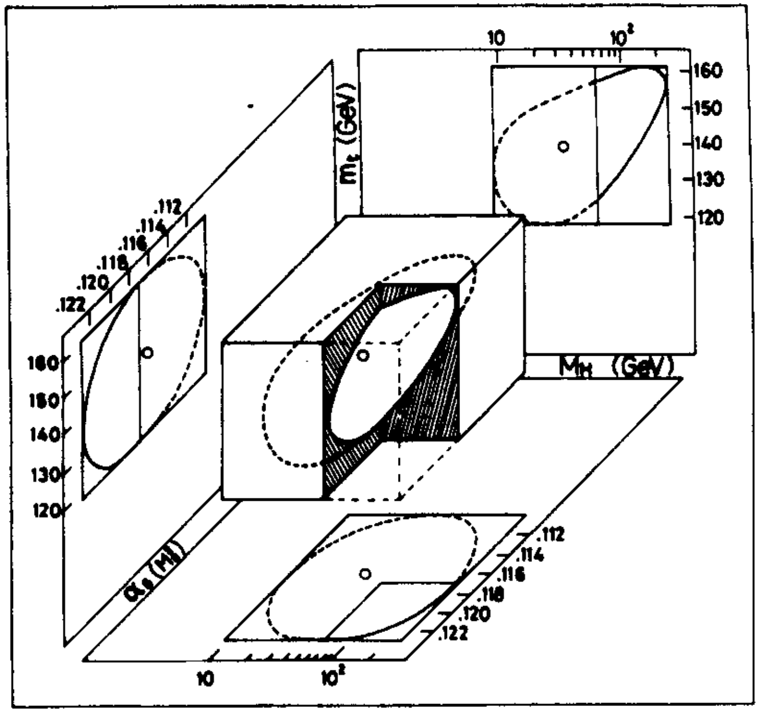
$m_t$ = 180 GeV
100
110
120
130
140
150
160
170
ALL data
$\chi^2$
$m_H$ [GeV]


Fig. 7



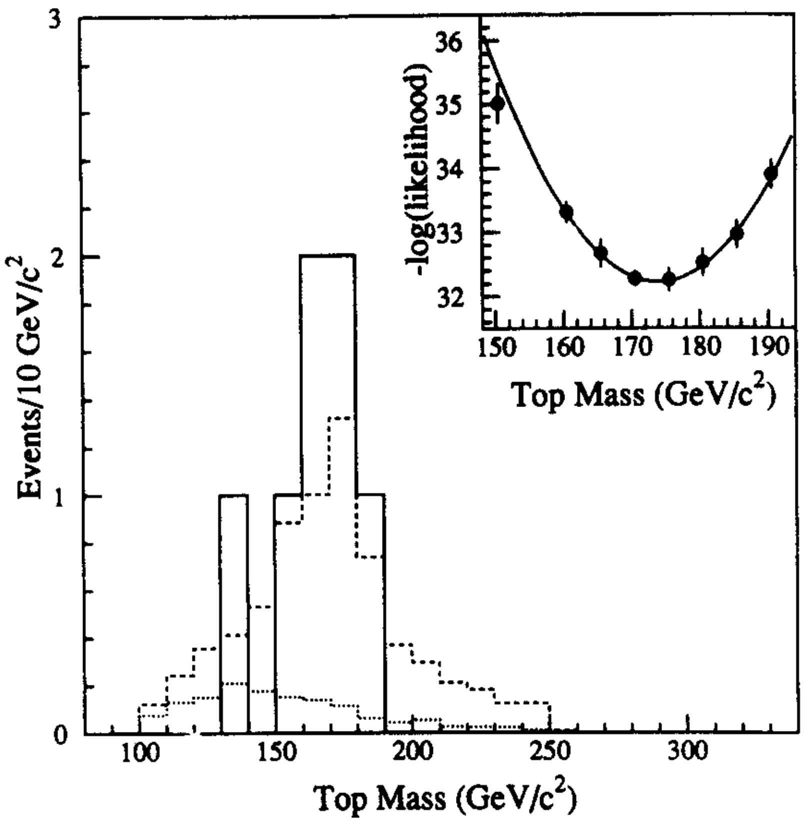
ALL data
$\alpha_s = 0.123 \pm 0.006$
$\Delta\chi^2$=1
$\Delta\chi^2$=2.7
$m_t$ [GeV]
$M_H$ [GeV]

Fig. 2

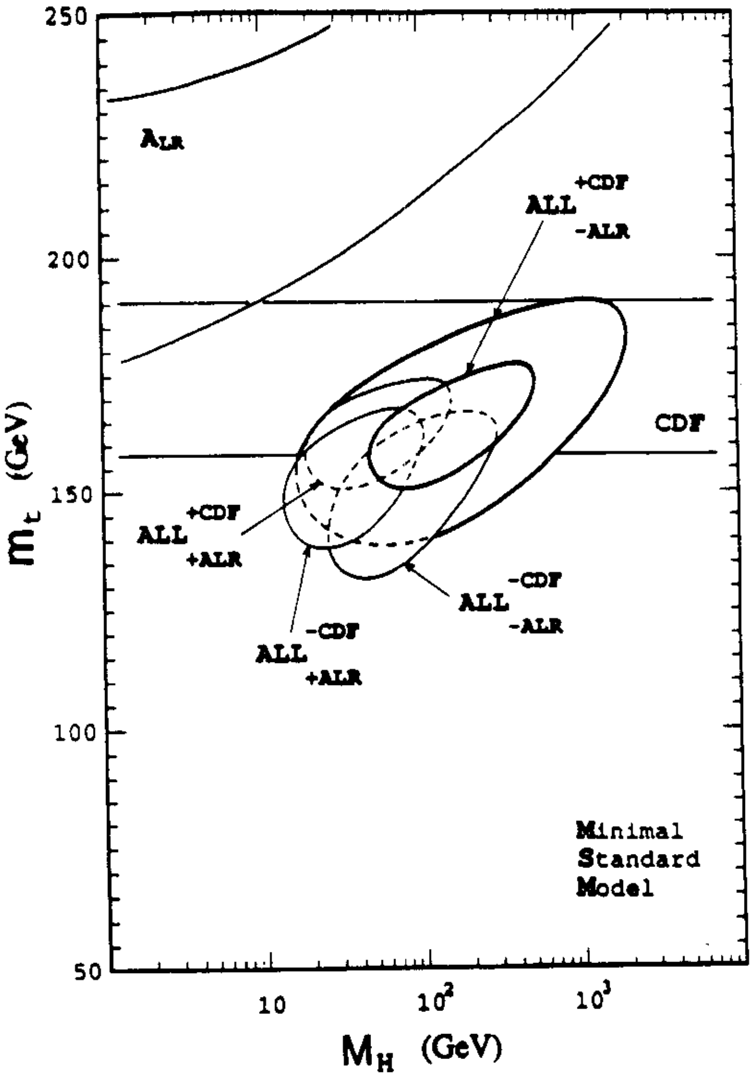


10
$10^2$
$m_t$ (GeV)
160
150
140
130
120
$M_H$ (GeV)
.112
.114
.116
.118
.120
.122
160
150
140
130
120
$\alpha_s$ ($M_Z$)
.112
.114
.116
.118
.120
.122
10
$10^2$

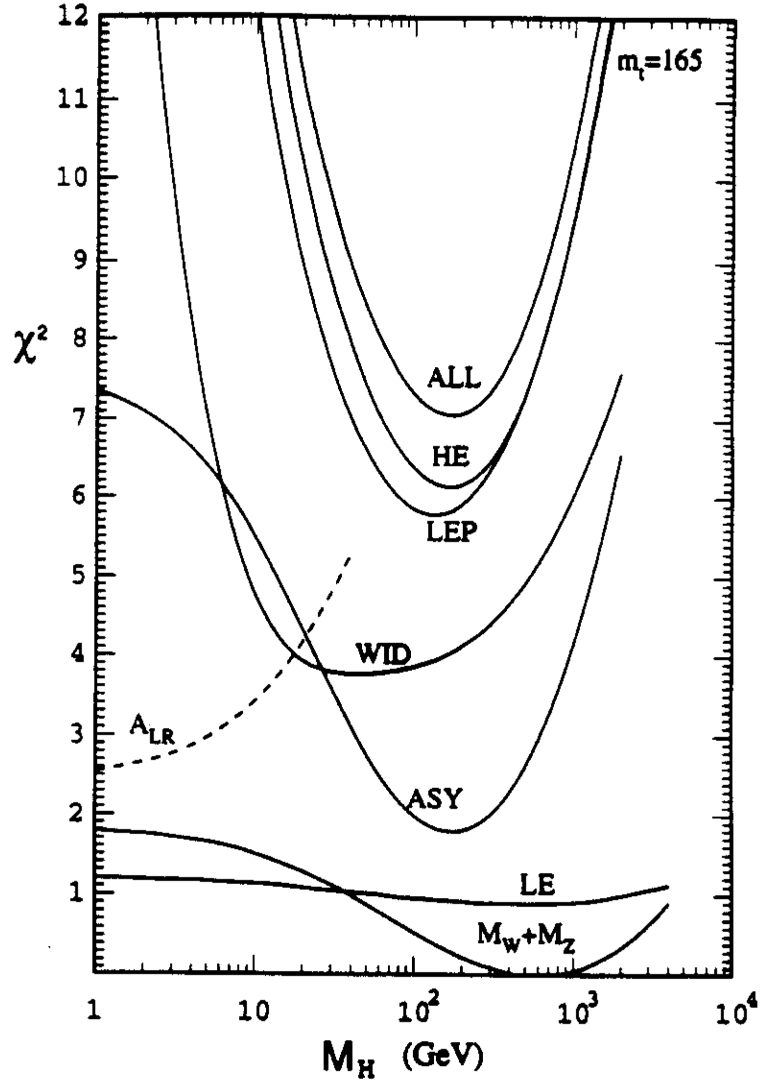

Fig. 3



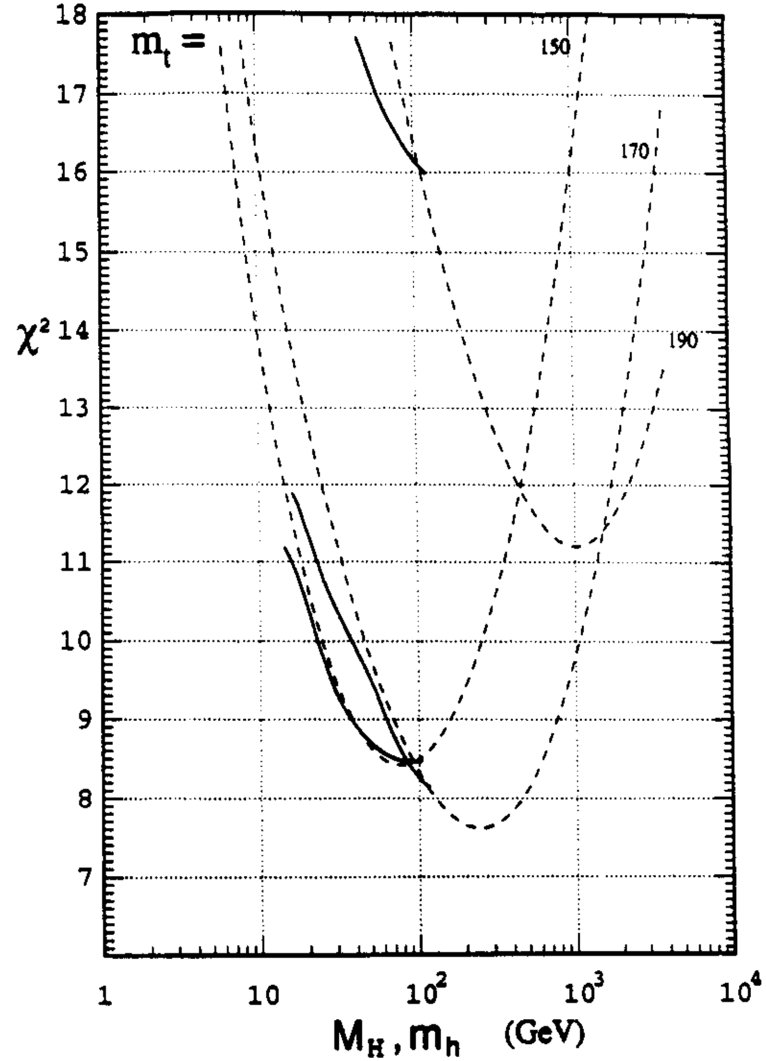
3
2
1
0
Events/10 GeV/$c^2$
100
150
200
250
300
Top Mass (GeV/$c^2$)
36
35
34
33
32
-log(likelihood)
150
160
170
180
190
Top Mass (GeV/$c^2$)



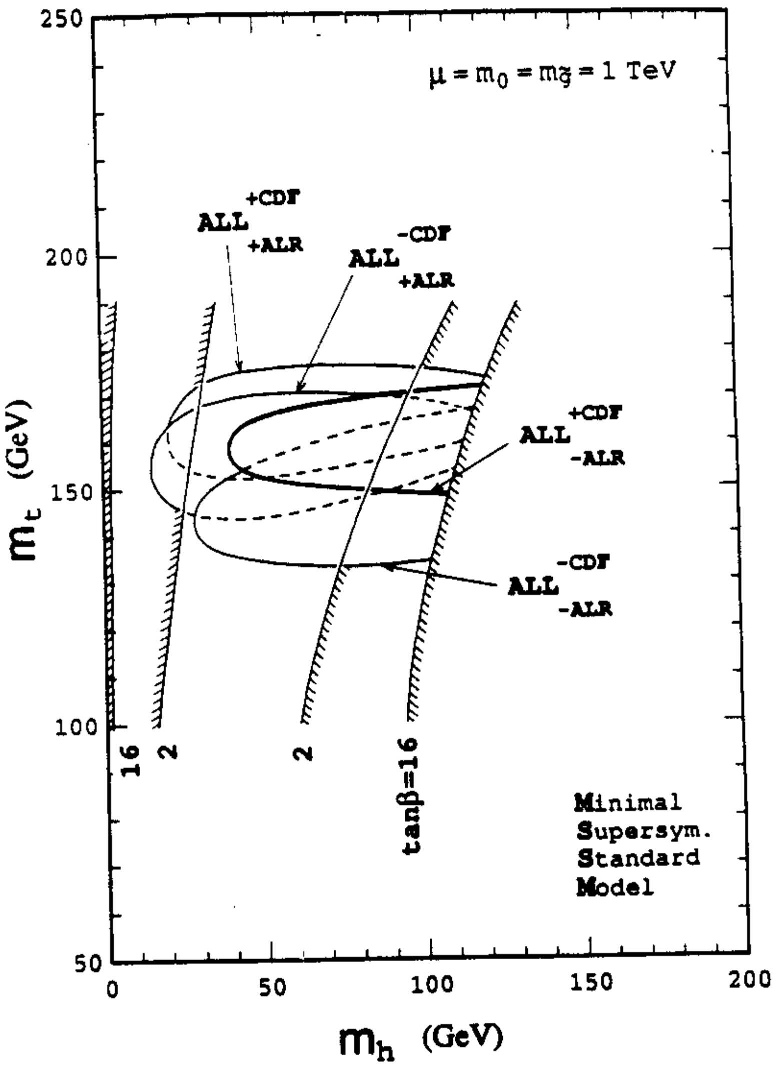
250
200
150
100
50
10
$10^2$
$10^3$
$m_t$ (GeV)
$M_H$ (GeV)
$A_{LR}$
$ALL^{+CDF}_{-ALR}$
CDF
$ALL^{+CDF}_{+ALR}$
$ALL^{-CDF}_{-ALR}$
$ALL^{-CDF}_{+ALR}$
Minimal
Standard
Model

12
11
10
9
8
7
6
5
4
3
2
1
1
10
$10^2$
$10^3$
$10^4$
$\chi^2$
$M_H$ (GeV)
$m_t$=165
ALL
HE
LEP
WID
$A_{LR}$
ASY
LE
$M_W$+$M_Z$

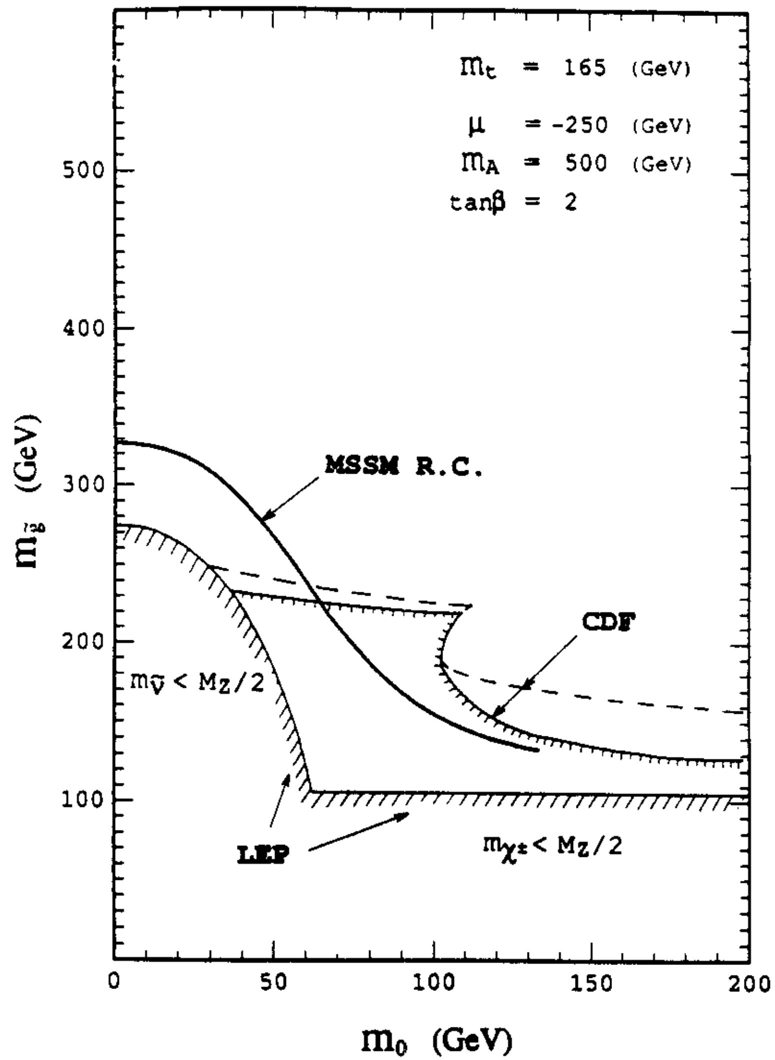


$m_t$ =
150
170
190
$\chi^2$
18
17
16
15
14
13
12
11
10
9
8
7
1
10
$10^2$
$10^3$
$10^4$
$M_H, m_h$ (GeV)

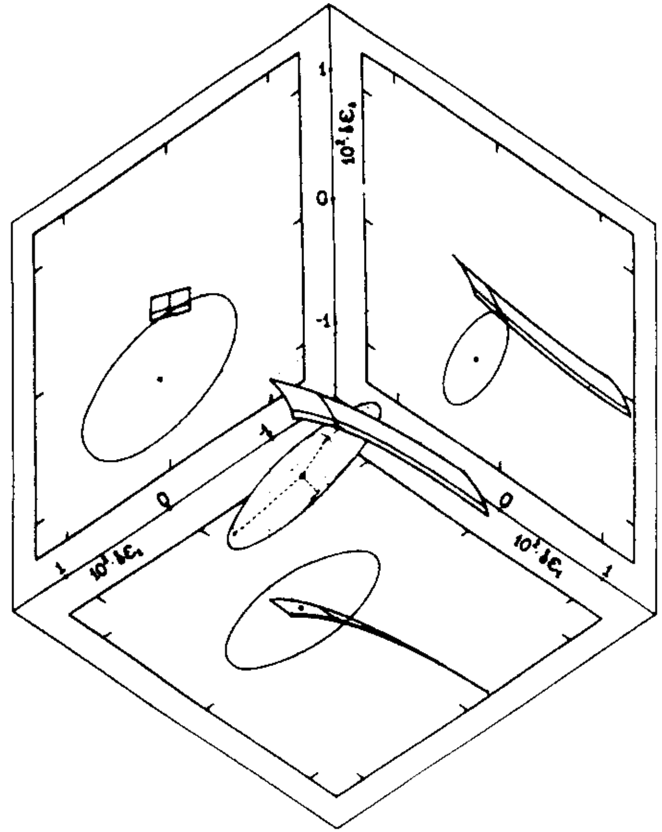



$\mu = m_0 = m_{\tilde{g}} = 1$ TeV
$ALL^{+CDF}_{+ALR}$
$ALL^{-CDF}_{+ALR}$
$ALL^{+CDF}_{-ALR}$
$ALL^{-CDF}_{-ALR}$
$m_t$ (GeV)
250
200
150
100
50
0
50
100
150
200
16
2
2
$\tan\beta=16$
Minimal
Supersym.
Standard
Model
$m_h$ (GeV)

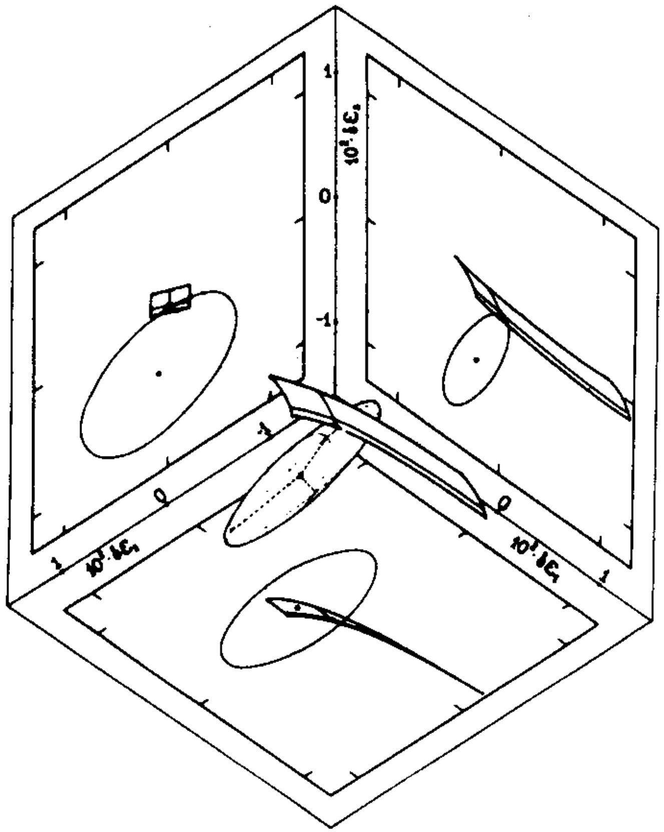
Fig. 18

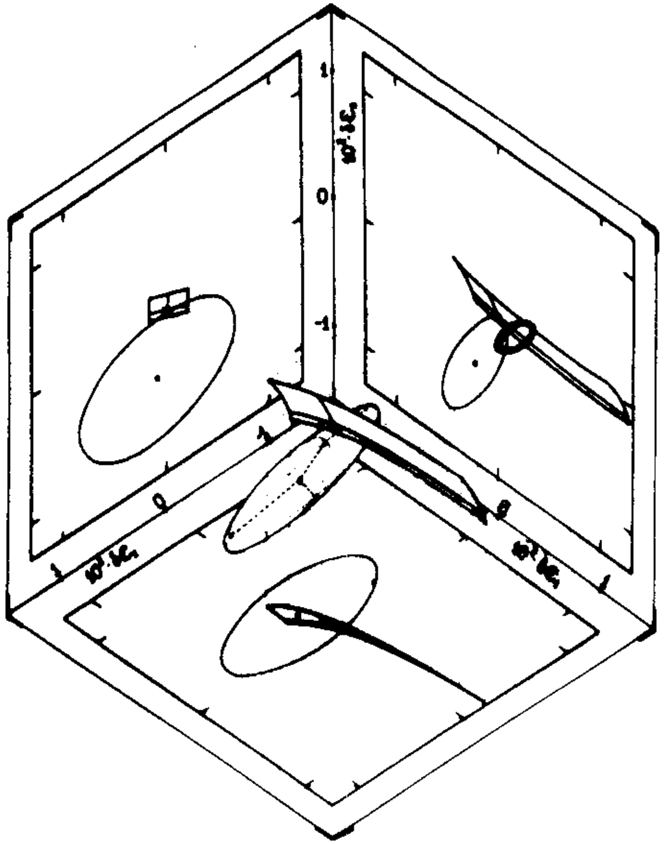


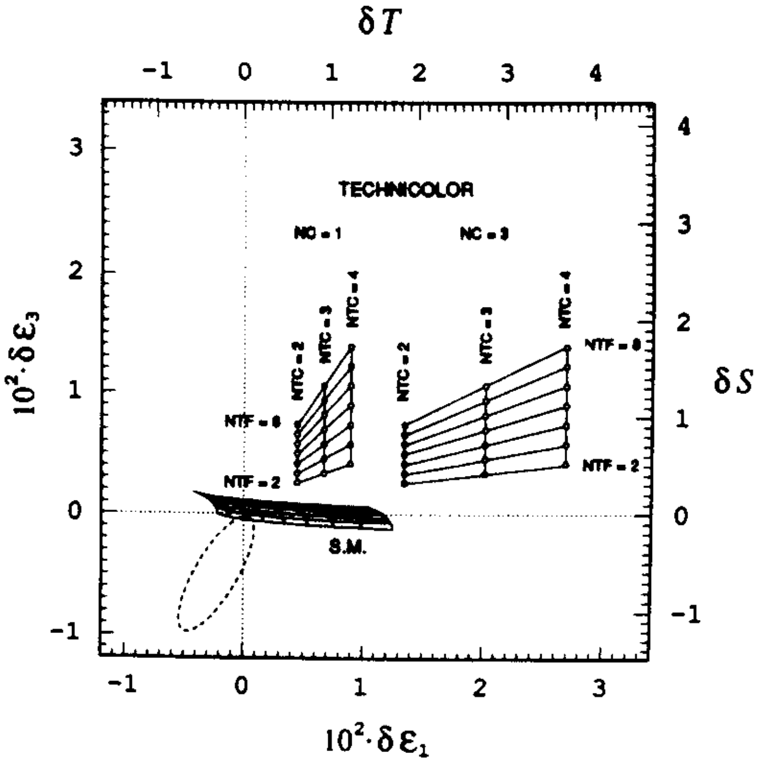
$m_t$ = 165 (GeV)
$\mu$ = -250 (GeV)
$m_A$ = 500 (GeV)
$\tan\beta$ = 2
MSSM R.C.
CDF
$m_{\tilde{\nu}} < M_Z/2$
LEP
$m_{\chi^\pm} < M_Z/2$
$m_{\tilde{g}}$ (GeV)
$m_0$ (GeV)
0
50
100
150
200
100
200
300
400
500

$10^2 \cdot \delta\varepsilon_1$
$10^2 \cdot \delta\varepsilon_3$
1
0
-1

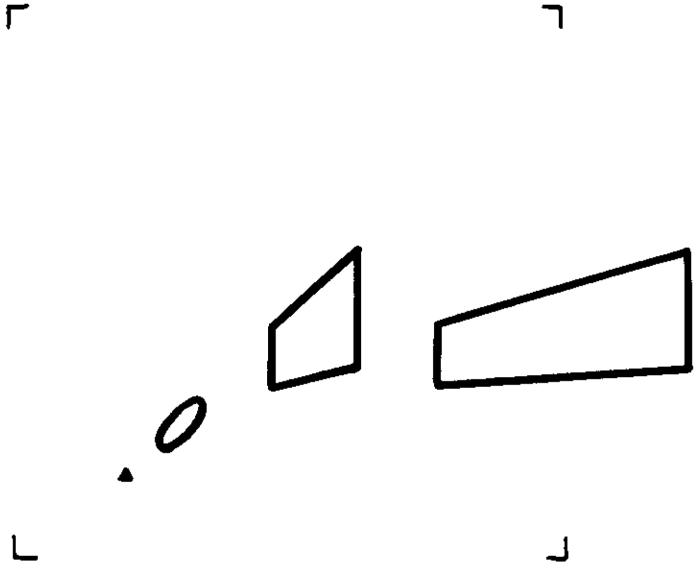

1
0
-1
$10^2 \cdot \delta\varepsilon_3$
$10^2 \cdot \delta\varepsilon_1$
$10^2 \cdot \delta\varepsilon_2$



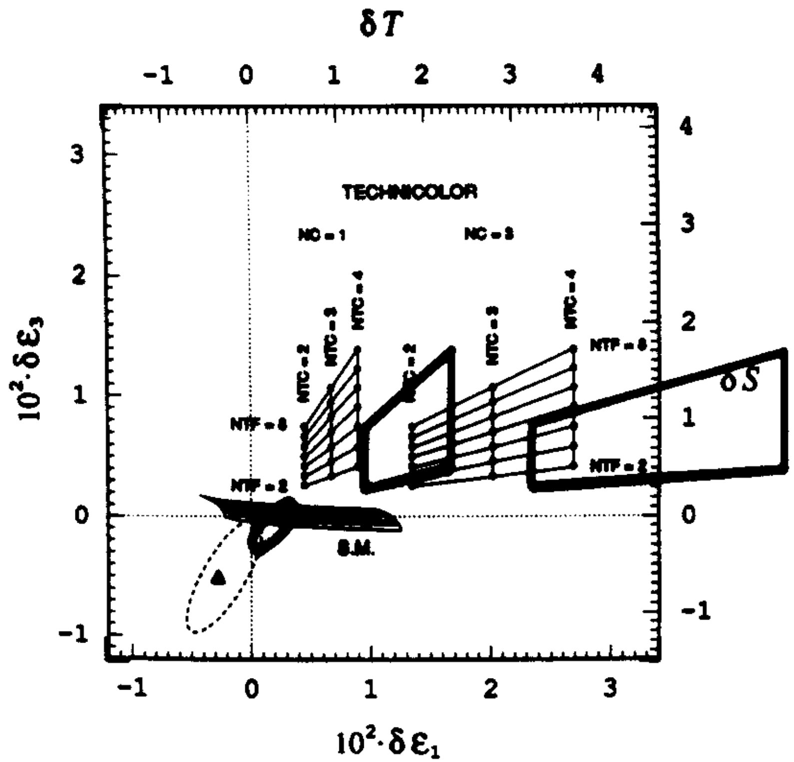


δT
TECHNICOLOR
NC = 1
NC = 3
NTC = 2
NTC = 3
NTC = 4
NTF = 8
NTF = 2
S.M.
$10^2 \cdot \delta\varepsilon_3$
$10^2 \cdot \delta\varepsilon_1$
δS

Fig 10



▲ : BORN




▲ : BORN