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SMR. 767 - 2

MINIWORKSHOP ON STRONG CORRELATIONS
AND QUANTUM CRITICAL PHENOMENA
(4 - 22 July 1994)

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METHODS OF CONFORMAL FIELD THEORY IN STRONG COUPLING PROBLEMS OF CONDENSED MATTER PHYSICS

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These are preliminary lecture notes, intended only for distribution to participants.

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METHODS OF CONFORMAL FIELD THEORY IN STRONG COUPLING PROBLEMS OF CONDENSED MATTER

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REF's:

- NUCL. PHYS. B352 ('91) 849
- NUCL. PHYS. B360 ('91) 641
- PHYS. REV. LETT. 67 ('91) 161
- PHYS. REV. LETT. 67 ('91) 3160
- PHYS. REV. LETT. 68 ('91) 1046
- PHYS. REV. B45 ('92) 7918
- PHYS. REV. B48 ('93) 7297
- INT. J. MOD. PHYS. B8 ('94) 347
- Review: Trieste Summer School [to appear]
- NUCL. PHYS. B [in print]
- PHYS. REV. LETT. 72 ('94) 1064
- NUCL. PHYS. B [in print]
- Phys. Lett. B

⋮

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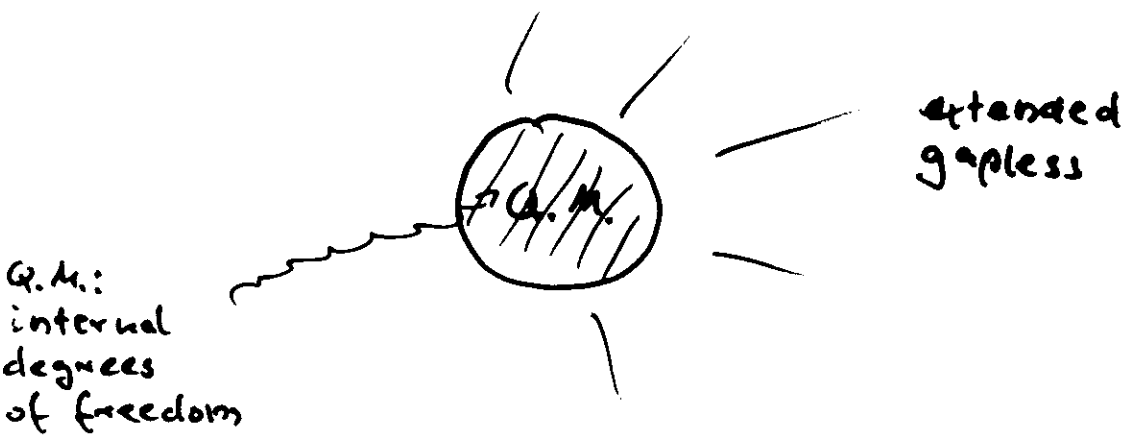
(*) FALL 1994: Santa Barbara

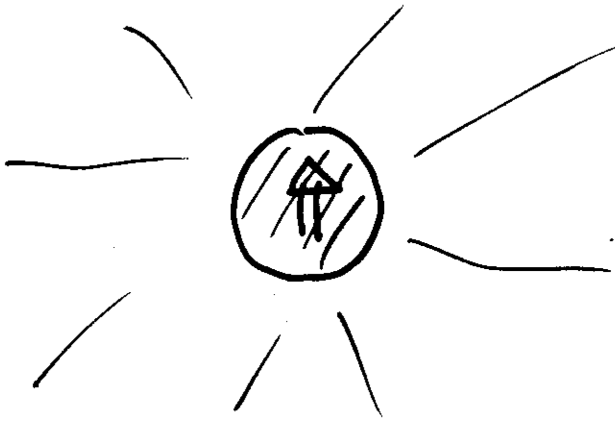
QUANTUM CRITICAL BEHAVIOR IN (QUANTUM) IMPURITY MODELS

QUITE GENERAL CLASS OF PROBLEMS:

A local quantum mechanical degree of freedom interacts with other, gapless (extended) degrees of freedom



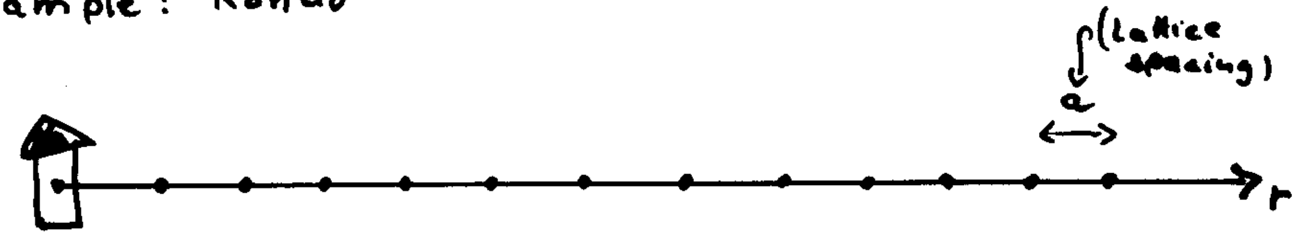
PROTOTYPE MODEL: KONDO



gapless degrees of freedom: low energy excitations of conduction electrons (host metal) about Fermi Surface

Generically: QIM can be mapped on 1D models on half-infinite line (using some appropriate "partial waves")

Example: Kondo



$i = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \ \ldots$

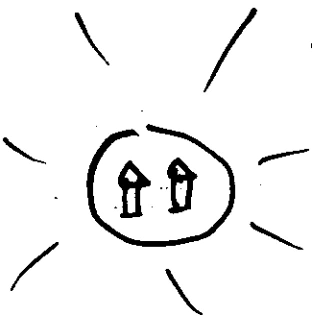
$$H = -t \sum_{i=0}^{\infty} \left[c_i^{\dagger \alpha j} c_{i+1\, \alpha j} + h.c. \right] + \lambda_K \, \vec{S} \cdot \left[c_0^{\dagger \alpha j} \left(\frac{\vec{\sigma}}{2} \right)_\alpha^{\ \beta} c_{0 \beta j} \right]$$

$\alpha, \beta = \uparrow, \downarrow$

k channels: $j = 1, 2, \ldots, k$

Some other examples of Q.I.M.:

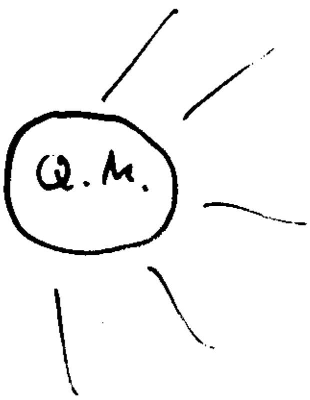
·: 2-Impurity Kondo



conduction electrons

Q.M. = a two-spin object

·: Dissipative Quantum Mechanics



enviroment

Caldeira-Leggett approach

Q.M. = arbitrary system

simple example:

particle in periodic potential

[Callan, Ludwig, Klebanov, Maldacena]

·: Tunneling between Quantum Hall edges

[Kane/Fisher]

·: Callan-Rubakov Effect

⋮

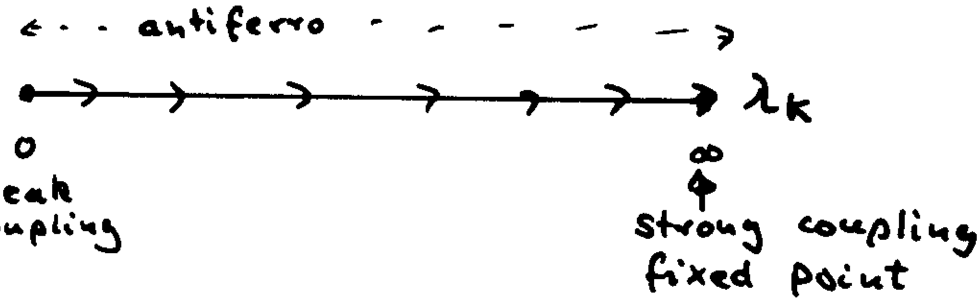
We [with I. Affleck] have observed that the following hypothesis is true quite generally:

The Q.M. degrees of freedom disappear under renormalization (and strong interactions) and turn into a boundary condition (b.c.) on the extended degrees of freedom.

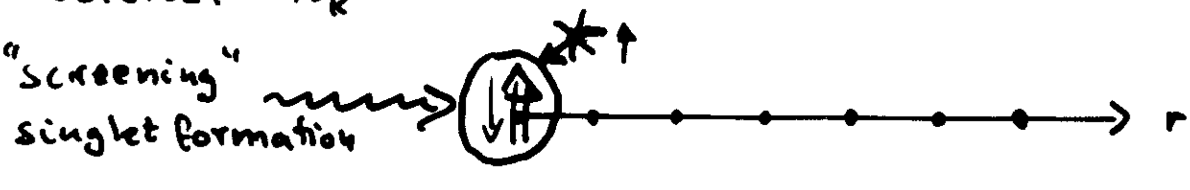
[The b.c. describes the $T=0$ Quantum Critical Point at strong coupling]

→ Illustrate this for (1-channel) Kondo model:

R.G. flow:



Nozières: $\lambda_K \to \infty$



Dirichlet b.c.: $c_0 \equiv 0$

Strong coupling fixed pt. is just a modified b.c.

→: Nozières: Singlet scatters remaining electrons like a potential → phase shift "δ"

$$c^{+}_{in}|0\rangle \to e^{2i\delta}\, c^{+}_{out}|0\rangle$$

No production of particle-hole pairs at $T=0$ near Fermi-level

$$\not\Rightarrow \quad c^{+}(c^{+}c)|0\rangle + \dots$$

A local FERMI-LIQUID !

→: According to our hypothesis, this phenomenon is extremely general

[Q.M. disappears ⇝ turns into a b.c. at strong coupling]

: But: b.c. can be quite complicated,

in particular: scattering off the impurity may not be described by phase shift

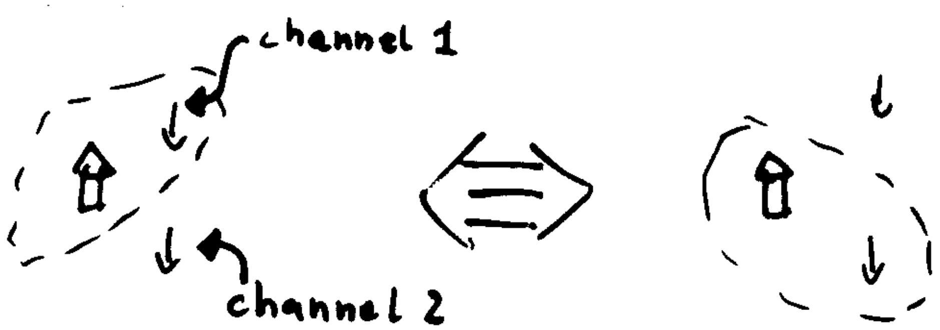
⇝ particle production: most dramatic signature of Non Fermi Liquid ground state

•: The simplest example where this occurs

is the 2-channel Kondo model:

complication easily understoody physically:

No simple singlet formation possible



system cannot decide which channel to screen

some signatures of this complication:

–: T = 0 entropy

$$S_{imp}(T=0) = \ln \sqrt{2} > 0$$

–: Scattering off the 2-channel impurity:

Fermi Liquid is "maximally violated"

A single particle excitation decays immediatedly, even at $T=0$ and right at Fermi level:

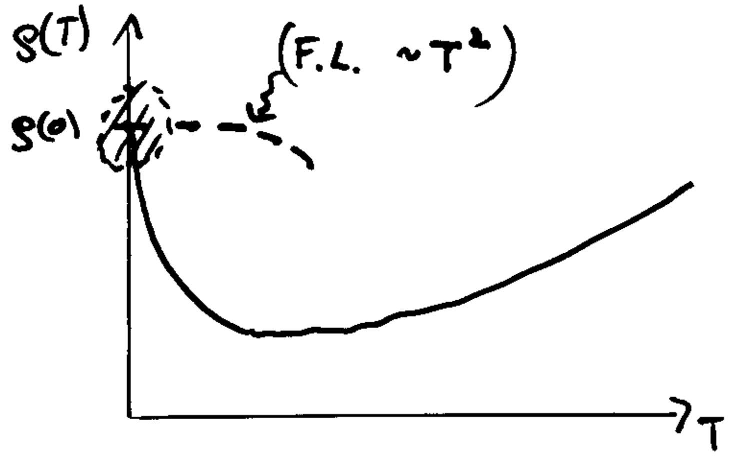
$$c^{+}_{in}|0\rangle \longrightarrow S^{(1)}\, c^{+}_{out}|0\rangle + $$

$$+ \quad c^{+}_{out}\,(c^{+}_{out}\, c_{out})\,|0\rangle + $$

$$+ \ldots$$

$$\boxed{S^{(1)} \equiv 0}$$

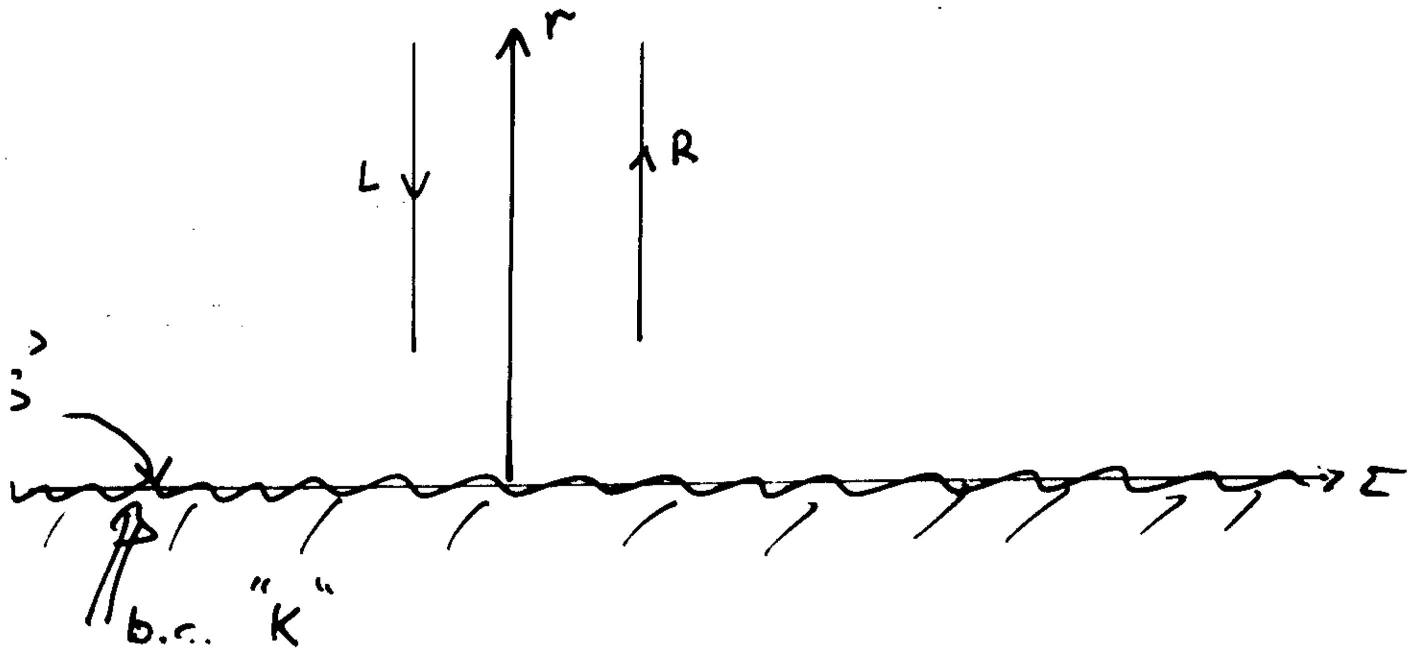
–: Singular Transport properties



$$\frac{\rho(0)-\rho(T)}{\rho(0)} \propto \left(T/T_K\right)^{\frac{1}{2}}$$

How to think of non-trivial b.c. :

space-time geometry



Consider first the most general b.c. which is linear in the Fermions

$$\psi_{L\alpha j} = M_{\alpha j}^{\alpha' j'} \, \psi_{R\alpha' j'}$$

$u(1) \times SU(2) \times SU(2)$ - symmetry

spin, channel

$$\Rightarrow \quad M_{\alpha j}^{\alpha' j'} = e^{2i\delta} \, \delta_{\alpha}^{\alpha'} \, \delta_{j}^{j'} \qquad \left[\begin{array}{l}\text{always a} \\ \text{phase shift}\end{array}\right]$$

$\to$ is even $u(1) \times SU(4)$ symmetric

$\Rightarrow$ b.c. linear in Fermions always has (higher) $u(1) \times SU(4)$ symmetry

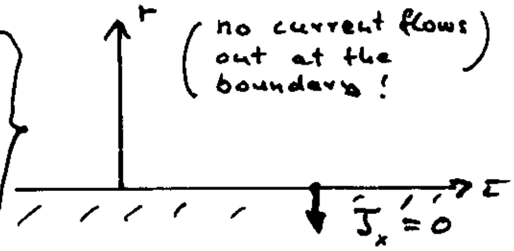
To find more general b.c., consider symmetries:

$$\begin{array}{cccccc} & U(1) & \times & SU(2) & \times & SU(2) \\ & \downarrow & & \downarrow & & \downarrow \\ \text{Noether-currents} & J(r,\tau) & & \vec{J}(r,\tau) & & \vec{I}(r,\tau) \end{array}$$

Boundary preserves symmetry $\Rightarrow$

all spatial components vanish at boundary

$$\left\{\begin{array}{l} J_x(x=0) = J_L(x=0) - J_R(x=0) = 0 \\ \vec{J}_x(x=0) = \vec{J}_L(x=0) - \vec{J}_R(x=0) = 0 \\ \vec{I}_x(x=0) = \vec{I}_L(x=0) - \vec{I}_R(x=0) = 0 \end{array}\right\}$$



(no current flows out at the boundary!)

[allows for b.c. more general than phase shift]

This is a b.c. <u>bilinear</u> in Fermions:

$$J_L = \psi_L^{+\alpha j} \psi_{L\alpha j}$$

$$\vec{J}_L = \psi_L^{+\alpha j} \left(\frac{\vec{\sigma}}{2}\right)_\alpha^\beta \psi_{L\beta j}$$

$$\vec{I}_L = \psi_L^{+\alpha i} \left(\frac{\vec{\tau}}{2}\right)_i^j \psi_{L\alpha j}$$

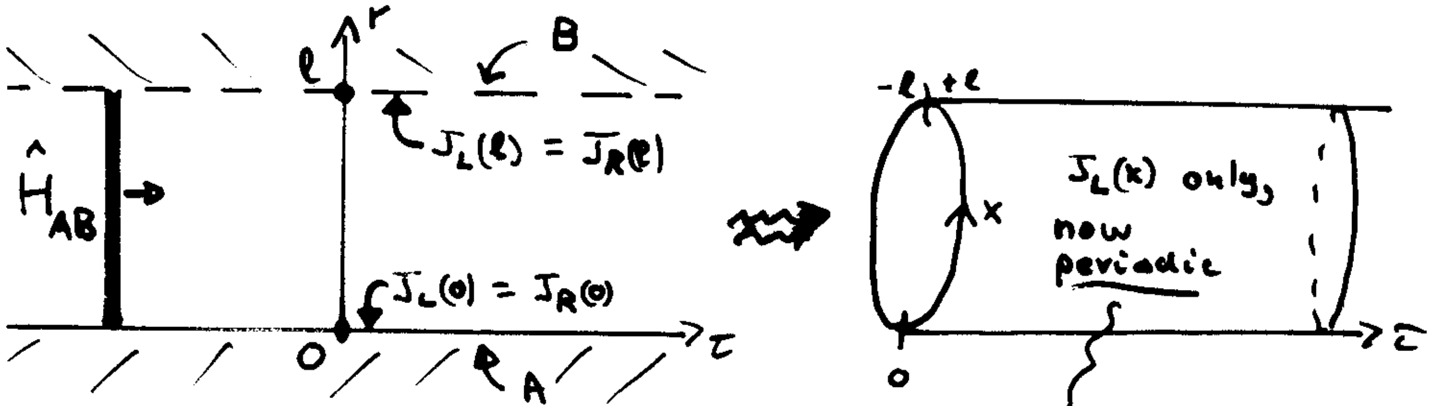
⇒: Symmetry [$U(1) \times SU(2) \times SU(2)$] ⇒

⇒ Ward Identities

These severely constrain the possible b.c.'s up to a few possibilities

This perhaps best illustrated by considering the spectrum

⇒: Finite Size Spectra



can express Hamiltonian in terms of the Noether currents
["Tomanaga-like" construction]

To illustrate this, I consider 3 systems of increasing complexity:

3 systems of increasing complexity:

- (I): Free Fermions without spin:

  trivial b.c. ($\psi_L = \psi_R$) at both ends A, B

  symmetry: $\mathcal{U}(1)$ current: $J_L = \psi_L^+ \psi_L$ (charge)

- (II): Free Fermions with spin and 2 channels:

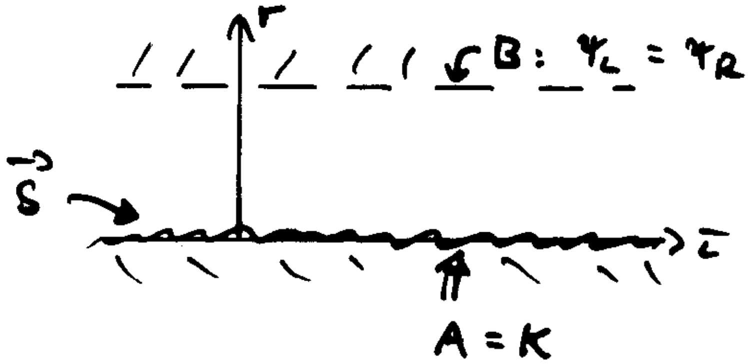
  trivial b.c. $\psi_{L\alpha j} = \psi_{R\alpha j}$ at A and B

  (no impurity spin)

  symmetry: $\mathcal{U}(1) \times SU(2) \times SU(2)$

  currents: J_L, $\vec{J}_L$, $\vec{I}_L$

- (III): Same as (II), but impurity spin at A:



(I): Spinless Fermions (trivial b.c.)

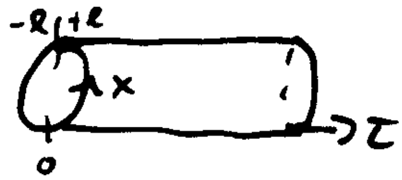
– Tomonaga basis of Hilbert space

$$H = \sum_q v_F\, q\, \Psi^\dagger(q)\, \Psi(q) = v_F \sum_q J(q)\, J(-q)$$

Fourier modes of charge density = $U(1)$ Noether current

–: q's are discrete

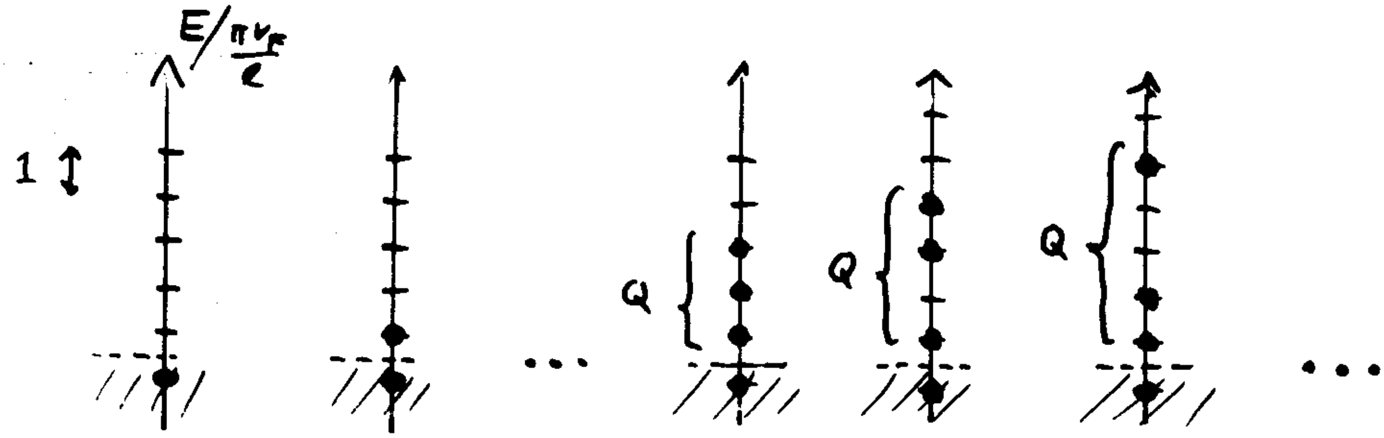
$[q = q_n = \frac{\pi}{\ell} n]$



–: $J(q=0)$ measures Q = # of electrons

→: spectrum of free $e^{\ominus}$'s on finite space $(0 \le r \le \ell)$:

[no spin-index, no channel-index, no impurity spin]

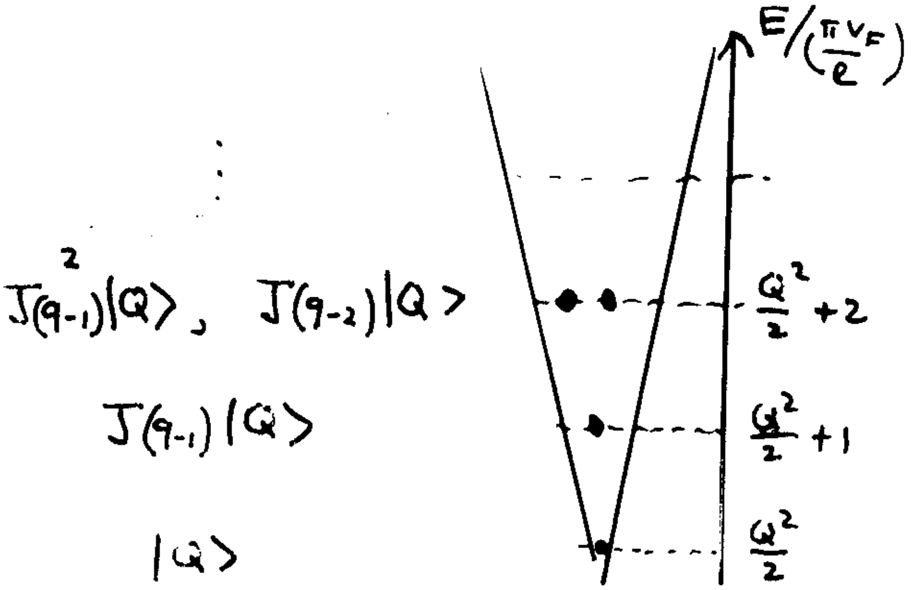


Q	0	1	Q	Q	Q
$E/\frac{\pi v_F}{\ell}$	0	$\frac{1}{2}$	$\frac{Q^2}{2}$	$\frac{Q^2}{2}+2$	$\frac{Q^2}{2}+2$

$\underbrace{\qquad\qquad\qquad\qquad}_{\text{"particle-hole excitations"}}$

→: Different way of writing free-Fermion spectrum generalizes to Non-Fermi-liquids:

Towers of particle-hole excitations



$$\boxed{E = \frac{\pi v_F}{\ell}\left\{\frac{Q^2}{2} + \text{integer}\right\}}$$

→: This spectrum follows simply from energy raising- and lowering- operators:

$$\boxed{\begin{aligned} \left[J(q_n),\ J(q_m)\right] &= \frac{\ell}{\pi}\, q_n\, \delta_{n+m,0} \\ \left[H_0,\ J(q_n)\right] &= -v_F\, q_n\, J(q_n) \end{aligned}}$$

$q_n = \frac{\pi}{\ell} n$, $n = 0, \pm 1, \pm 2, \dots$

(recall: $H_0 = v_F \sum_q \frac{1}{2} J(q) J(-q)$)

This (Tomonaga-like) structure is more general than free Fermion theory:

(II): Separation of spin- and charge- (and channel-) excitations

Consider: free Fermions with spin, $k=2$ channels

[no impurity spin yet]

Define:

particle-density: $$J(q) = \sum_{q'} \sum_{\substack{\alpha=\uparrow,\downarrow \\ j=1,2}} \psi^{+\alpha j}_{(q+q')} \psi_{\alpha j}(q')$$

spin-density: $$\vec{J}(q) = \frac{1}{2} \sum_{q'} \sum_{\substack{\alpha,\beta \\ j}} \psi^{+\alpha j}_{(q+q')} \vec{\sigma}_{\alpha}^{\ \beta} \psi_{\beta j}(q')$$

channel spin-density: $$\vec{I}(q) = \frac{1}{2} \sum_{q'} \sum_{\substack{\alpha \\ i,j}} \psi^{+\alpha i}_{(q+q')} \vec{\tau}_{i}^{\ j} \psi_{\alpha j}(q')$$

More general Tomonaga-like expression:

$$\boxed{H_0 = \sum_q v_F\, q\; \psi^{+\overbrace{\alpha j}^{\text{(summed)}}}(q)\, \psi_{\alpha j}(q) =}$$

$$\boxed{= H_{charge} + H_{spin} + H_{channel} =}$$

$$\boxed{= \frac{v_F \pi}{\ell} \sum_q \frac{1}{8} J(q) J(-q) + \frac{v_F \pi}{\ell} \sum_q \frac{1}{4} \vec{J}(q) \cdot \vec{J}(-q) + \frac{v_F \pi}{\ell} \sum_q \frac{1}{4} \vec{I}(q) \cdot \vec{I}(-q)}$$

→ H_{charge}, H_{spin}, $H_{channel}$ mutually commuting ←

→ Spectrum of individual Hamiltonians

e.g.: $$\boxed{H_{spin} = \frac{v_F \pi}{\ell} \sum_q \frac{1}{4}\, \vec{J}(q) \cdot \vec{J}(-q)}$$

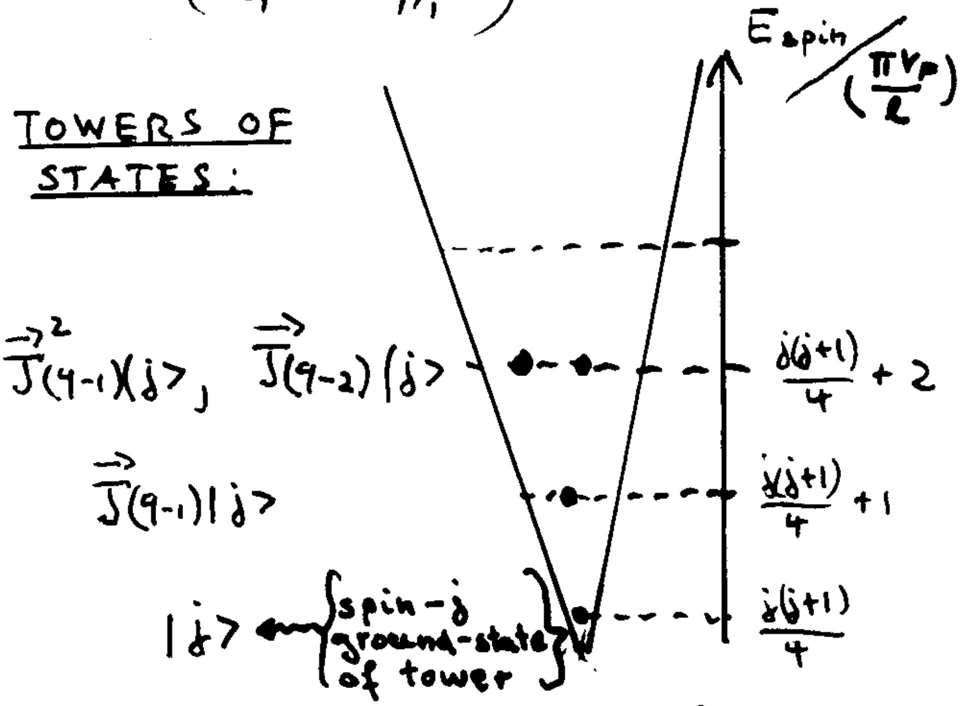
–: The key to Non-Fermi-liquid behavior :
H_{spin} is <u>not</u> a free theory !

–: But easy to analyze using Tomonaga-like energy raising- and lowering- operators $\vec{J}(q_n)$:

$$\boxed{\begin{aligned} \left[J^a(q_n), J^b(q_m) \right] &= i\,\epsilon^{abc}\, J^c(q_n+q_m) + \frac{\ell}{\pi}\, q_n\, \delta^{ab}\, \delta_{n+m,0} \\ \left[H_{spin}, \vec{J}(q_n) \right] &= -v_F\, q_n\, \vec{J}(q_n) \end{aligned}}$$

$(a, b = x, y, z)$

TOWERS OF STATES:



NOT A SPECTRUM OF FREE THEORY !

DESCRIBES INTERACTING "SPINON" THEORY

→: Explicitly:

<u>Basis of Hilbert space:</u>

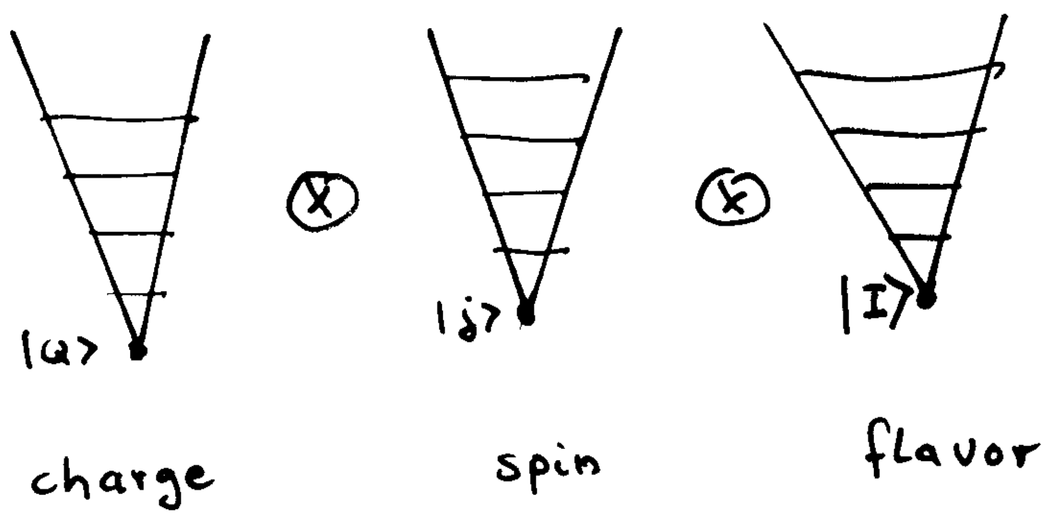
$$\ldots \left[J(q_{-2})\right]^{n_2}\left[J(q_{-1})\right]^{n_1} |Q\rangle \otimes$$

$$\otimes \ldots \left[\vec{J}(q_{-2})\right]^{m_2}\left[\vec{J}(q_{-1})\right]^{m_1} |j\rangle \otimes$$

$$\otimes \ldots \left[I(q_{-2})\right]^{k_2}\left[I(q_{-1})\right]^{k_1} |I\rangle$$

$$\left\{\begin{matrix} n_1, n_2, \ldots \\ m_1, m_2, \ldots \\ k_1, k_2, \ldots \end{matrix}\right\} = 0, 1, 2, \ldots$$

= Product of towers



▶: In order to reproduce free Fermions as product of interacting theories ⇝

⇝ Need a constraint on allowed combinations of charge-, spin-, and channel excitations

SPECTRUM:

Free Fermions with spin, $k=2$ channels [no impurity spin]

$SU(2)_{spin}$ j	$U(1)$ Q	$SU(2)_{channel}$ I	$E/(\frac{2\pi v_F}{\ell})$	State
0	0	0	0	Fermi-Sea $\|0\rangle$
$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$	$\psi_k^{\dagger\alpha j}\|0\rangle$
1	2	0	1	$\psi_k^{\dagger\alpha i}\psi_k^{\dagger\beta i}\|0\rangle$
0	2	1	1	
⋮				

▶: A pure "spinon" $(j,Q,I)=(\frac{1}{2},0,0)$ cannot occur !
("confined"!)

(III): Effect of Impurity Spin

-: in (II) we defined a basis of Hilbert space which is adapted to the symmetry of strong coupling

-: The strong coupling fixed point is described using the same basis. What changes [as compared to (II)] is the way how the charge-, spin-, and channel- excitation towers are put together.

Physically, this can be viewed as a sort of "deconfinement"

→ : CRUCIAL POINT : EFFECT OF KONDO INTERACTION

– : Due to spin-, charge- (channel-) separation

Electron can be thought of

INTERACTING

$$\Psi = (\text{holon}) \times (\text{spinon}) \times (\underset{\text{field}}{\text{channel}})$$

carries : charge spin channel excitations

$$\left(\text{recall: } H_0 = H_{charge} + H_{spin} + H_{channel}\right)$$

– : Without Kondo interaction:
Interacting "particles" confined to within Fermio
(cannot show up freely)

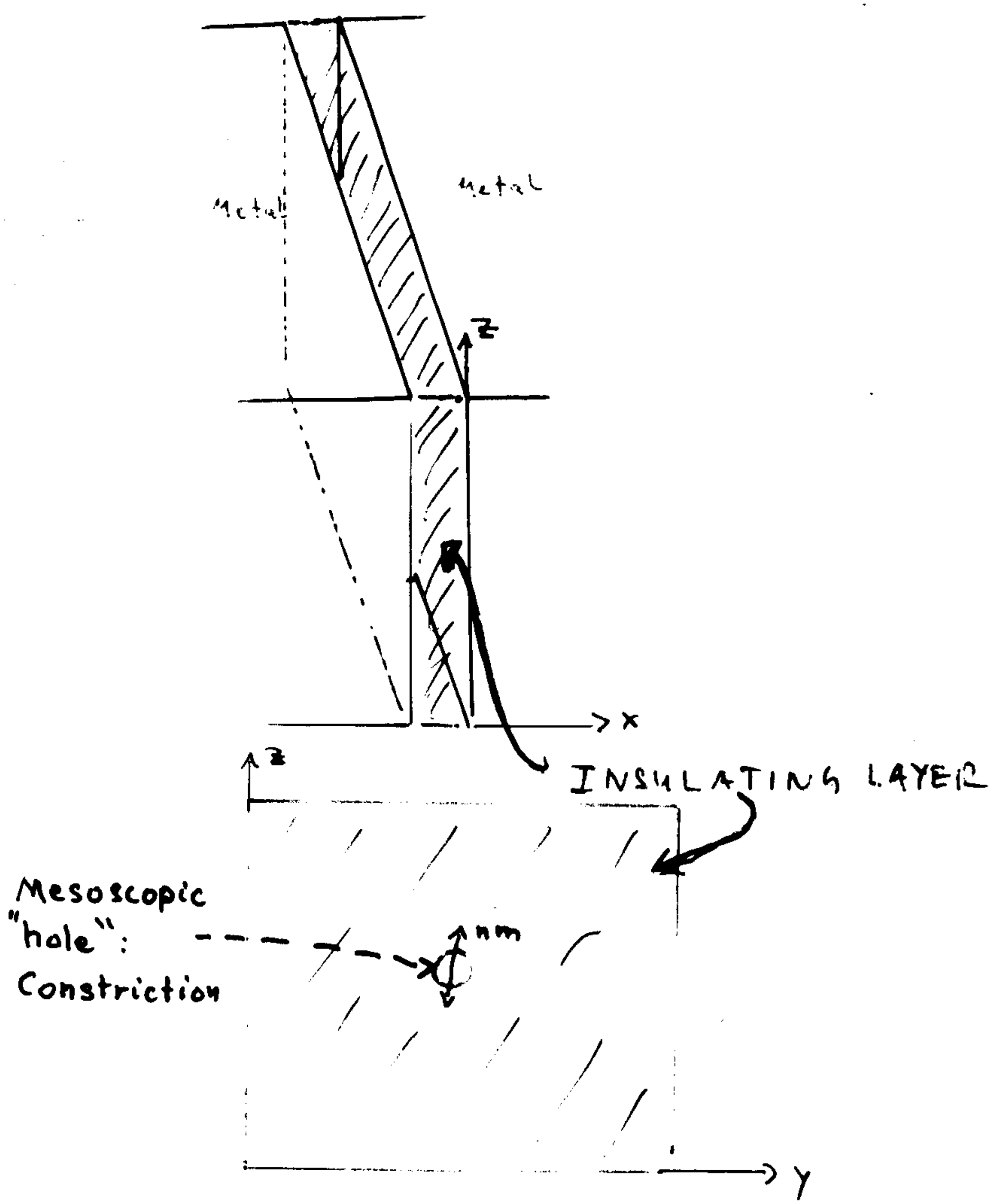
– : Kondo interaction acts only on "spinon"-
-component
Upon scattering of electron off impurity spin:
electron Ψ gets ripped apart
⇒ UNCONFINED SPINONS (INTERACTING!) SHOW UP

NON-FERMI-LIQUID PROPERTIES
ARE DUE TO "DECONFINEMENT"
OF INTERACTING "SPINON-PARTICLES"
WHICH ARE "CONFINED" TO WITHIN FRE
FERMIONS IN ABSENCE OF KONDO
INTERACTION.

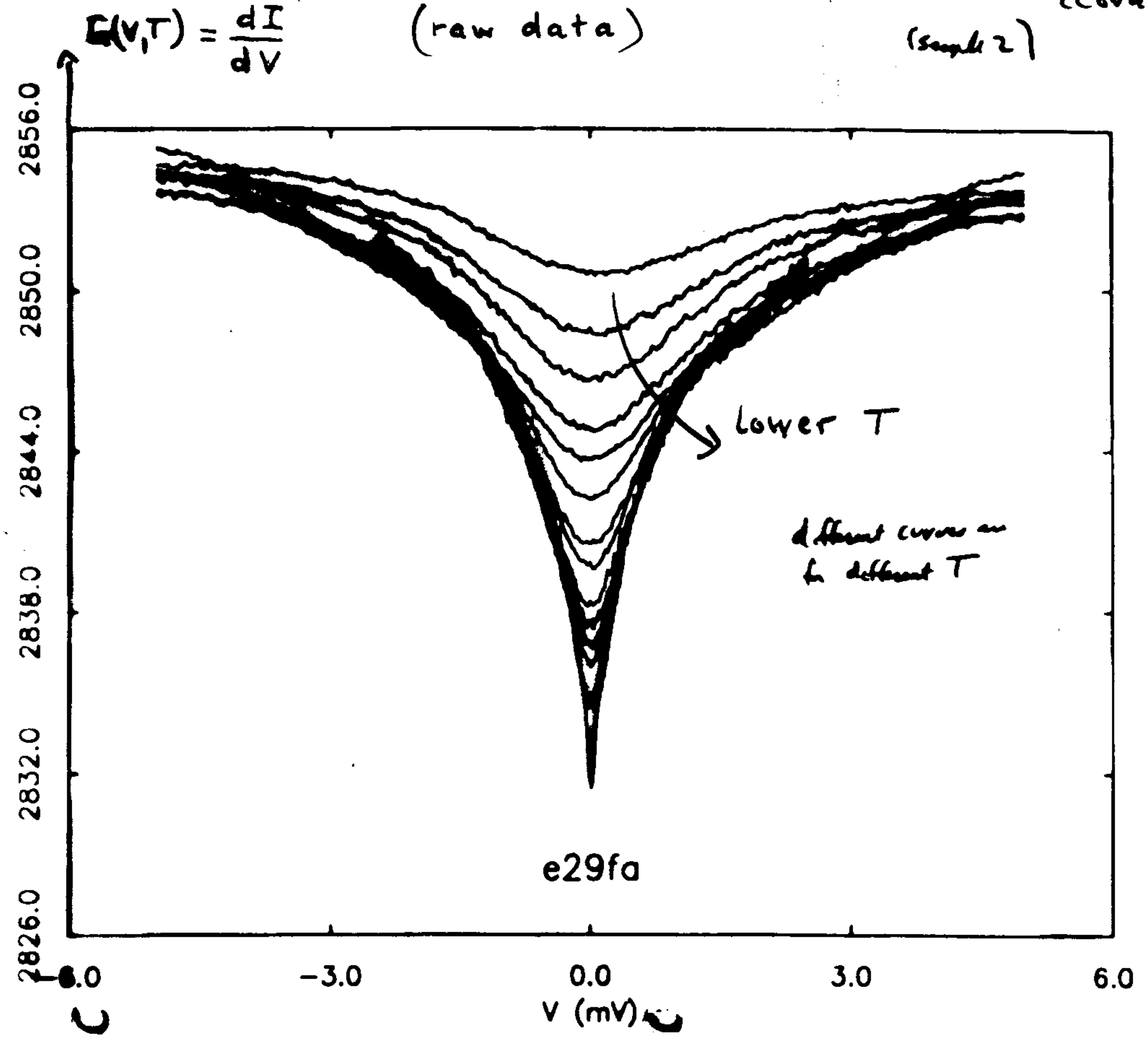
Experiments

E: Experiments on Mesoscopic Metal Point Contact Devices

Buhrman + Ralph (Cornell)
Ralph, Ludwig, v. Delf, Buhrman (PRL, 1994)



Ralph + Buhrman
(Cornell)
(sample 2)



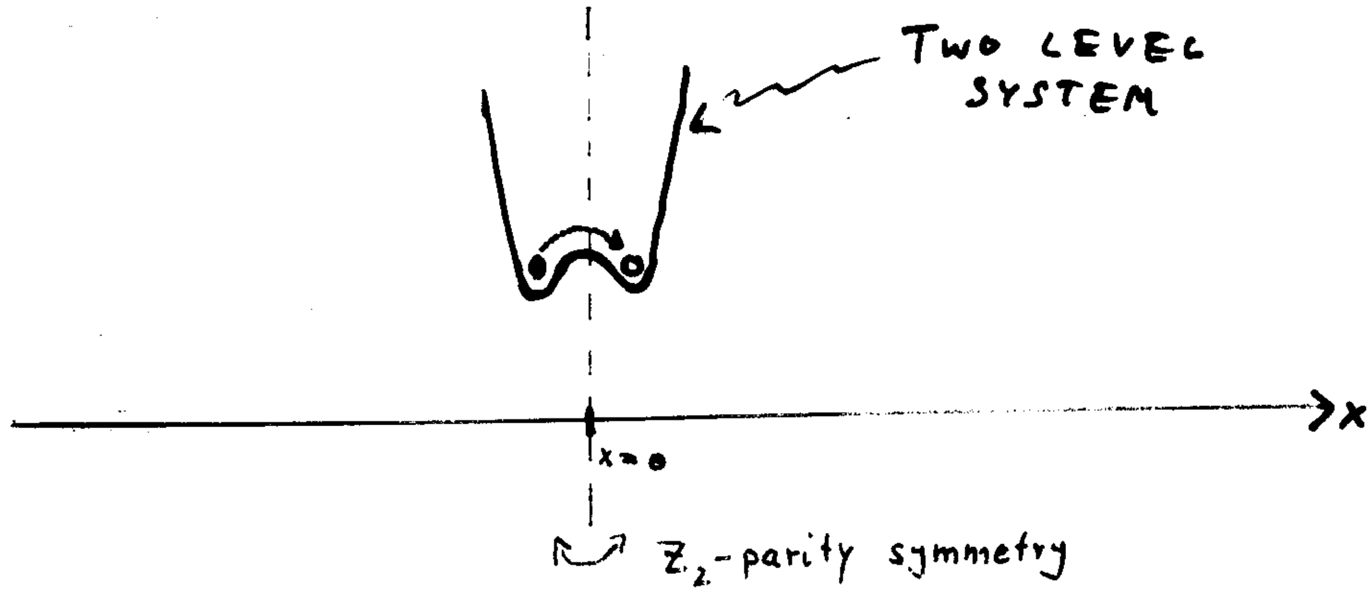
conductance signal: $G(V,T) = \frac{dI}{dV}$

→: Raw data: show zero-bias anomaly
at $V = 0$
("non-ohmic resistor")

→: Ralph + Buhrman suggest:
2-level systems form in constriction
(due to defects)

If correct, this should be a realization
of the $(k=2)$-channel Kondo model,
due to symmetry.

INTERACTION WITH TWO-LEVEL SYSTEM (schematically):
[more details: Zawadowski, ...]



	(pseudo-)spin index	channel-index
conduction electrons $c_{\alpha j}$	$\alpha = \begin{Bmatrix} \text{even} \\ \text{odd} \end{Bmatrix} =$ pseudospin $\begin{Bmatrix} \text{up} \\ \text{down} \end{Bmatrix}$	j = actual spin of electrons = 1,2
2-level system	$\alpha = \begin{Bmatrix} \text{even} \\ \text{odd} \end{Bmatrix} =$ pseudospin $\begin{Bmatrix} \text{up} \\ \text{down} \end{Bmatrix}$ is the IMPURITY SPIN $\vec{S}$ of KONDO MODEL	has no j-index since non-magnetic (experimentally verified by applying magn. field)

spin anisotropy irrelevant: CFT (Affleck, Ludwig, Pang, Cox)

→ If this is a realization of the ($k=2$)-channel Kondo model,

then the singular behavior of Non-Fermi-Liquid should show up in conductance G

THIS IS WHAT THE EXPERIMENTAL DATA SHOW:

Kondo model has zero temperature fixed point [described by non-trivial boundary condition]

⇒: EXPECT SCALING FORM OF CONDUCTANCE

non-universal constants

$$\boxed{G(V,T) - G(0,0) = B \cdot T^{\alpha} \cdot \Gamma\left[A\,\frac{V}{T}\right]}$$

universal scaling function

$\boxed{\text{EXPECT EXPONENT } \alpha = \frac{1}{2}}$ (in contrast: $\alpha = 2$ for Fermi-Liquid)

($k=2$ channels)

PREDICTIONS:

→: If this is correct, then:

1.) $$\frac{G(V,T) - G(0,0)}{B.\ T^{\alpha}} \quad \text{vs.} \quad \left(\frac{V}{T}\right)$$

must collapse onto a single scaling curve

2.) collapse occurs only when exponent α is chosen correctly (namely: $\alpha = \frac{1}{2}$)

3.) The so-obtained scaling function $\Gamma[x]$ must be the same function for different samples (since universal)

4.) Magnetoconductance

prediction from CFT {"H" is a channel-splitting field}

$G(H, T=0)\big|_{\text{at } V=0} \quad \alpha \quad |H|^{\alpha}$ (non-analytic)

.... $\alpha \quad H^2$ for Fermi-Liquid usc 31

Scaling Analysis: (Ralph, Ludwig, v. Delft, Buhrman) PRL '94

$$G(V,T) - G(0,T) = B.T^{\alpha}\ \Gamma\left[A\frac{V}{T}\right]$$

universal scaling function

→: V = 0 data (versus temperature T):

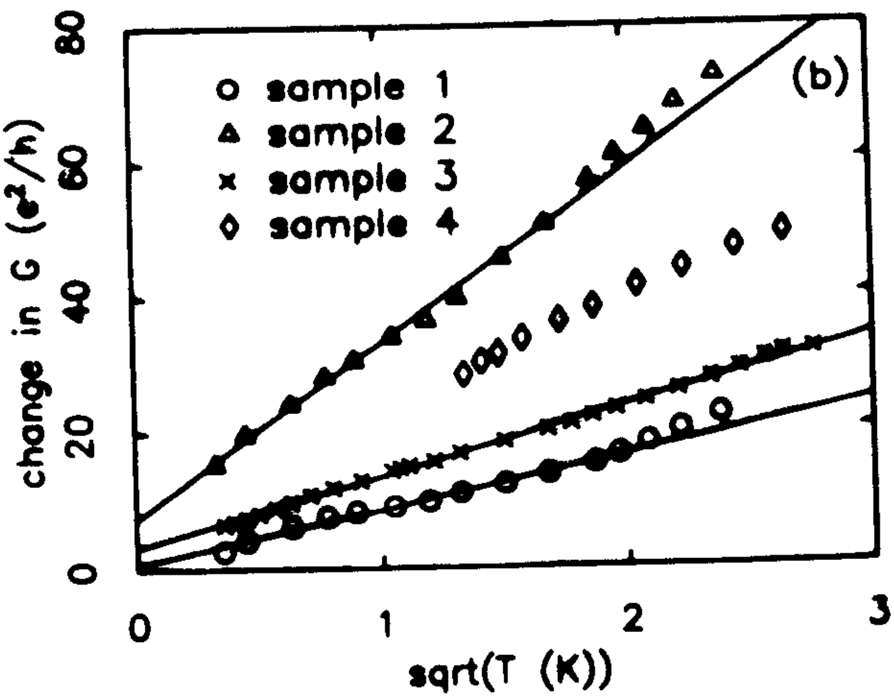
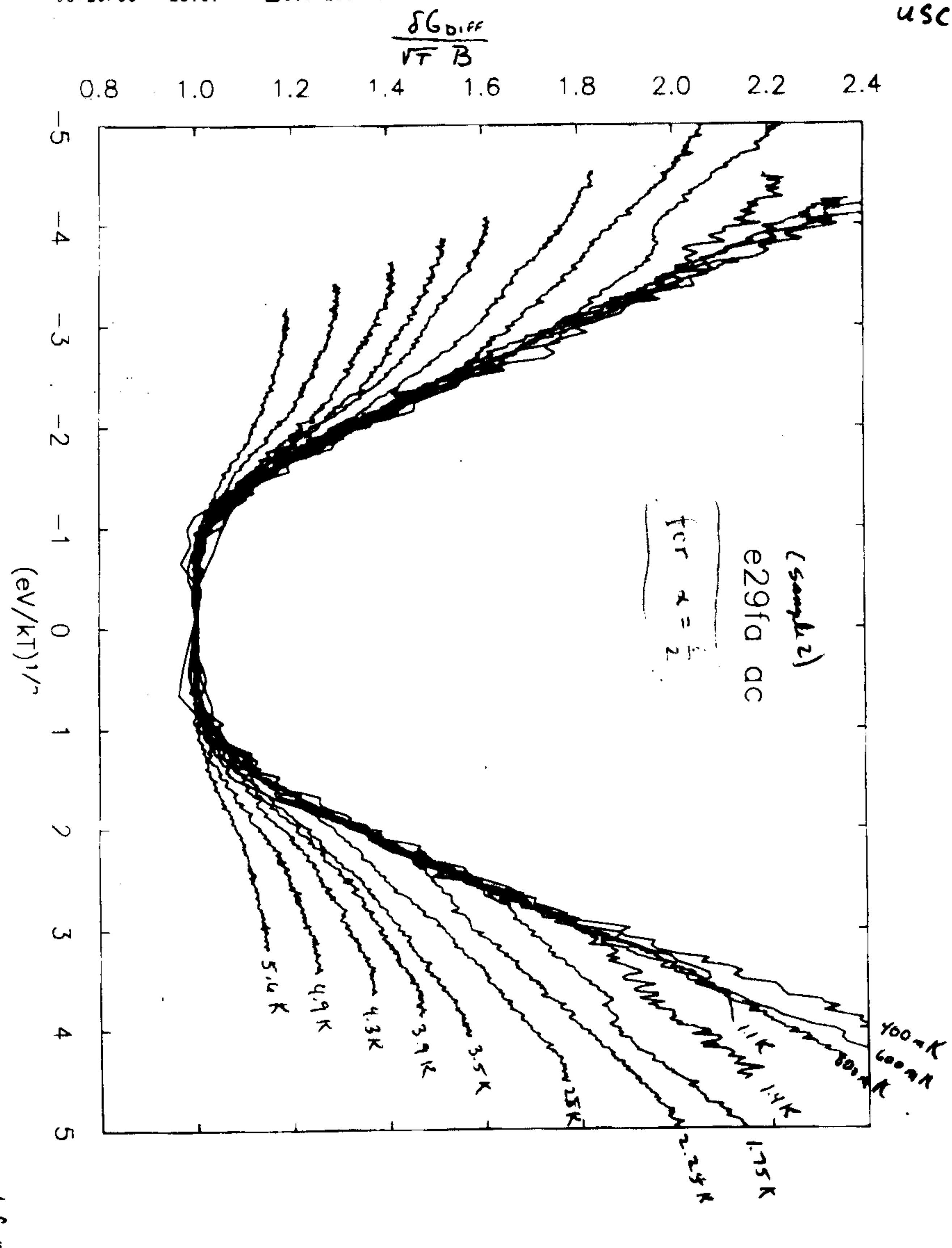


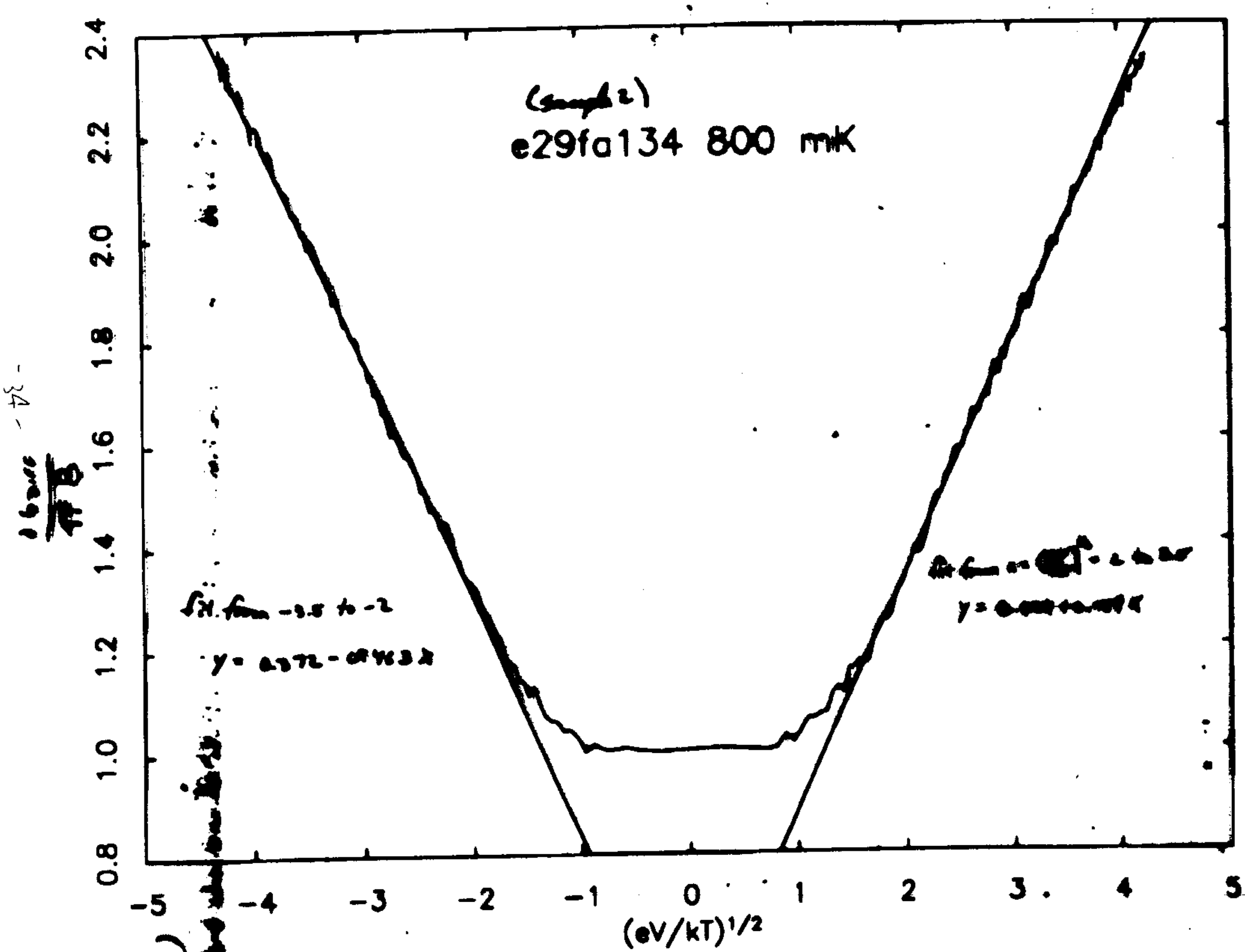
Fig. 6.76

USC (32)



acscalefa4

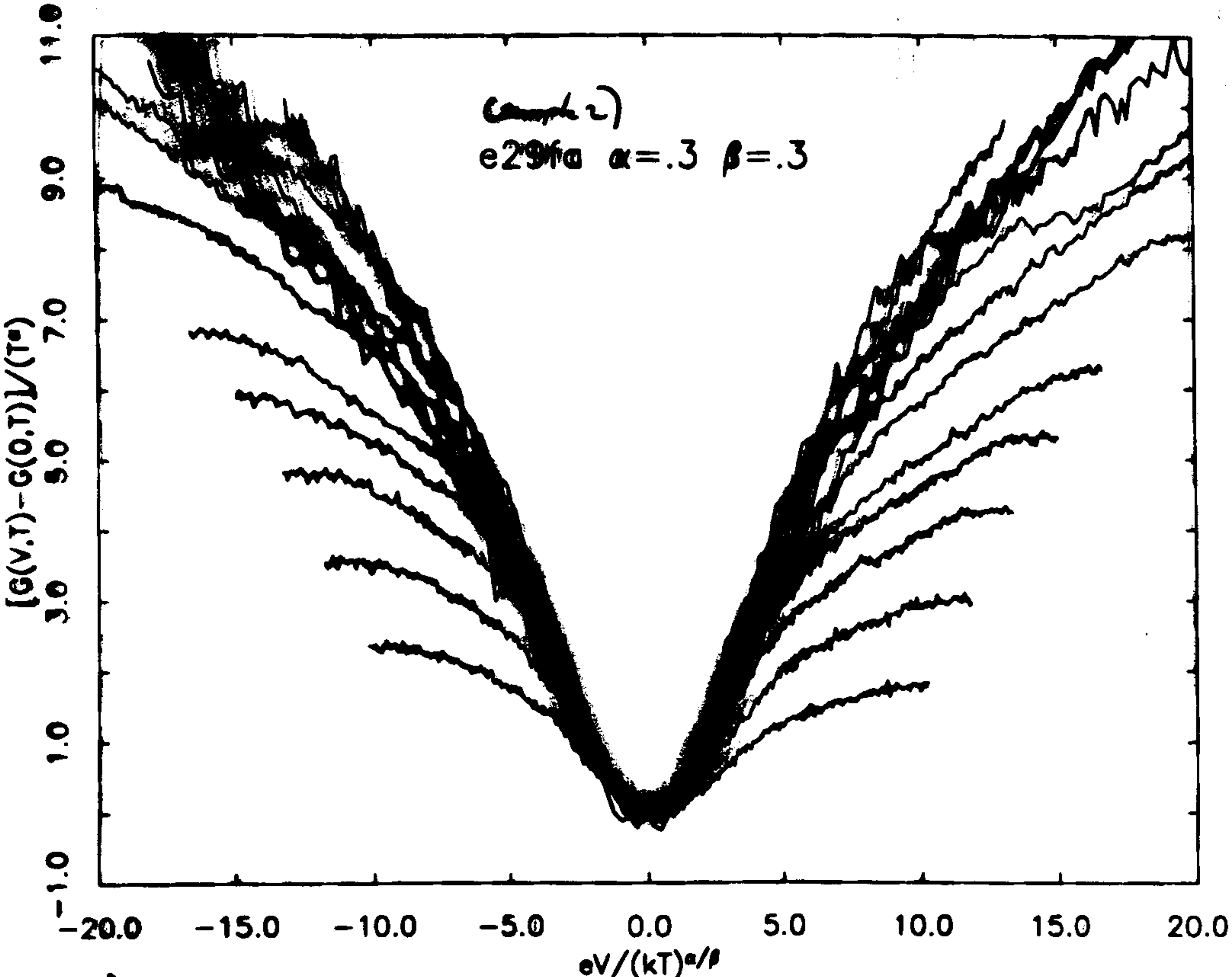
(i)



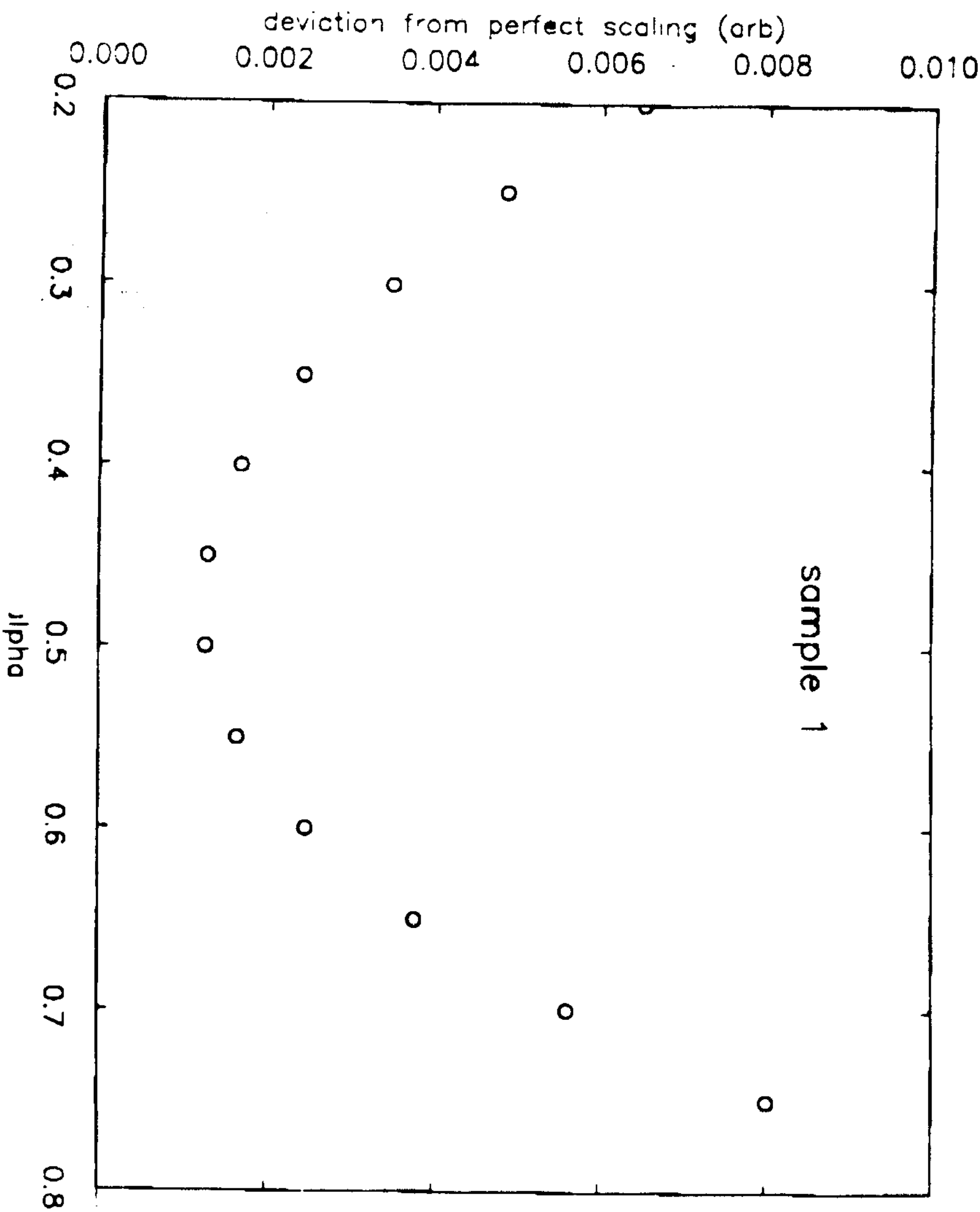
(sample 2)
e29fa134 800 mK
fit from -3.5 to -2
$(eV/kT)^{1/2}$
-5
-4
-3
-2
-1
0
1
2
3
4
5
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4

USC (33)

Scaling plot: different exponent

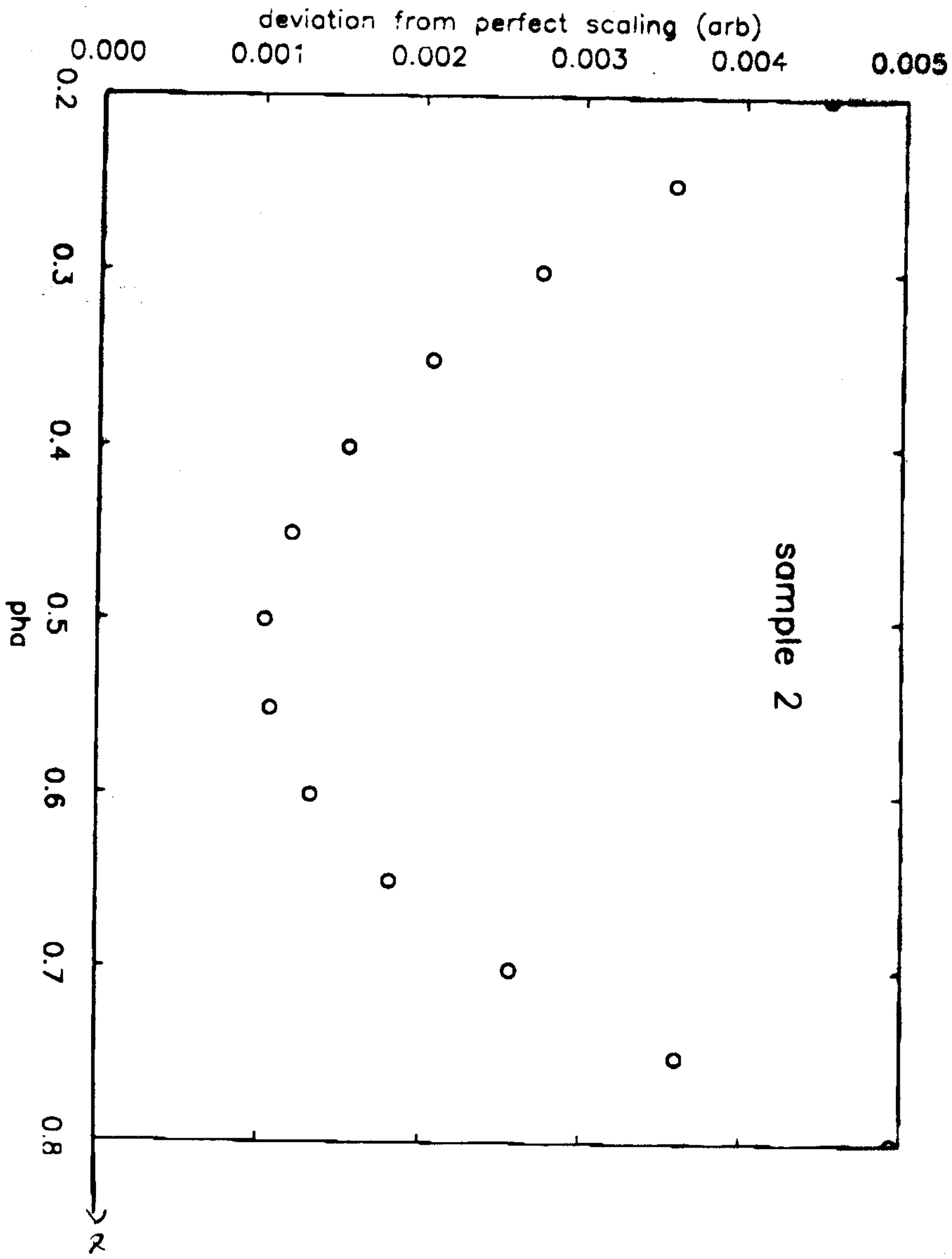


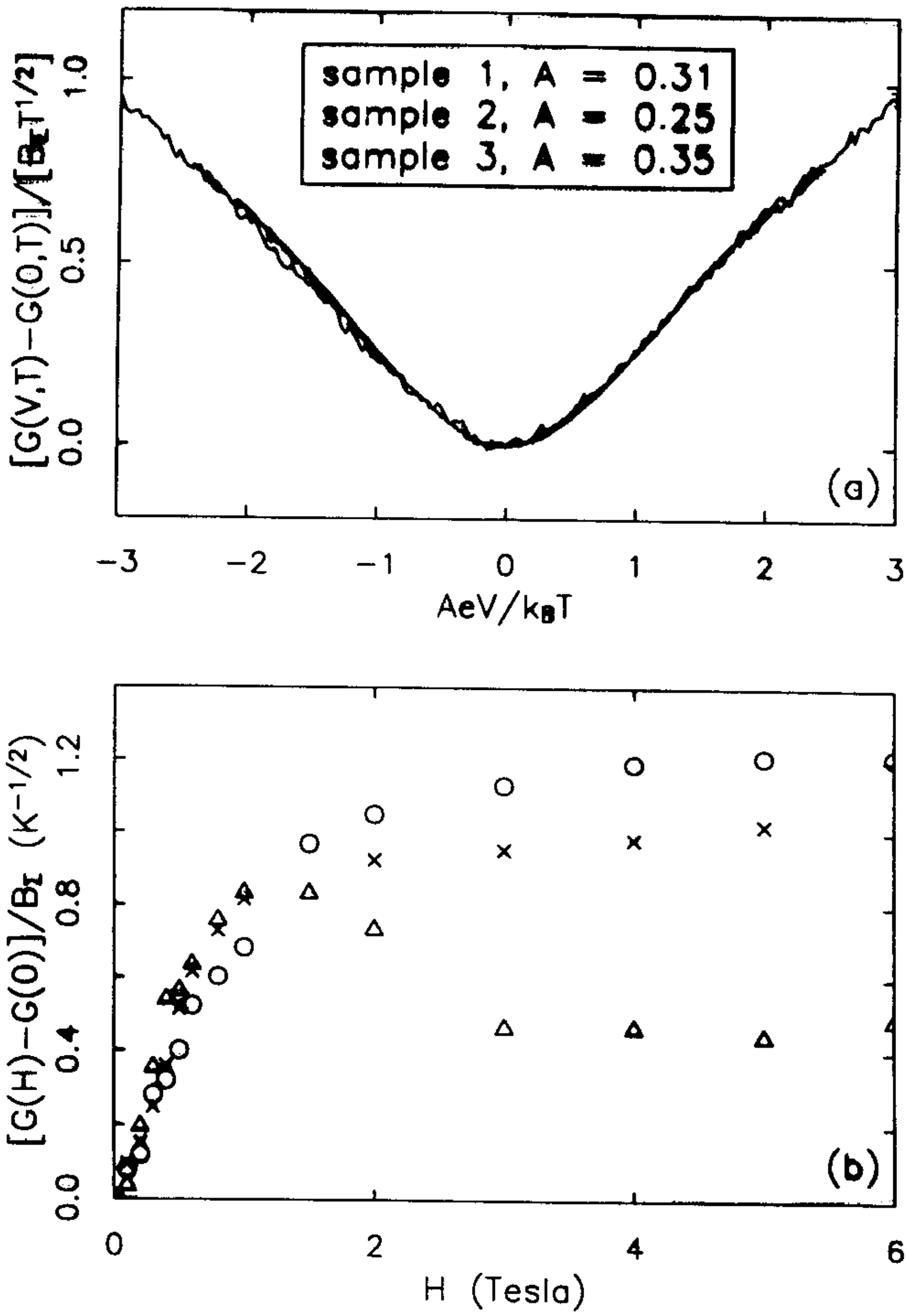
usc 35



1/2

Post-it™ fax transmittal memo 7671 | # of pages ▸ 2 usc


deviation from perfect scaling (arb)
0.000
0.001
0.002
0.003
0.004
0.005
0.2
0.3
0.4
0.5
0.6
0.7
0.8
pha
sample 2


sample 1, A = 0.31
sample 2, A = 0.25
sample 3, A = 0.35
[G(V,T)−G(0,T)]/[B_I T^{1/2}]
AeV/k_B T
(a)
[G(H)−G(0)]/B_I (K^{−1/2})
H (Tesla)
(b)