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SMR. 767 - 11

MINIWORKSHOP ON STRONG CORRELATIONS
AND QUANTUM CRITICAL PHENOMENA
(4 - 22 July 1994)

ANYON SUPERCONDUCTIVITY

AND

THE GAUGE THEORY OF THE t-J MODEL

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These are preliminary lecture notes, intended only for distribution to participants.

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Anyon Superconductivity + The Gauge Theory of the t-J model

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- subtitles

what are anyons + why do I care?

what are spinons + holons really?

doesn't ~~C. Varma~~ Piers say that slave bosons are nonsense?

don't the experiments fail to find T-violation?

Outline:

High-Tc caused by Antiferromagnetism

Spin + Charge Separate

Separation → Fractional Statistics

Anyon Superconductivity (T-violation)

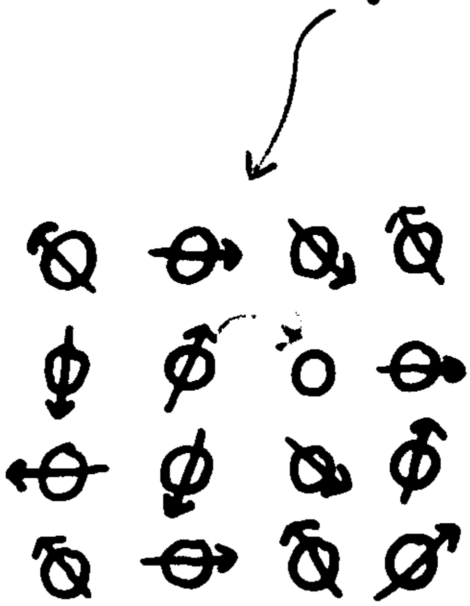
Calculational Accomplishments

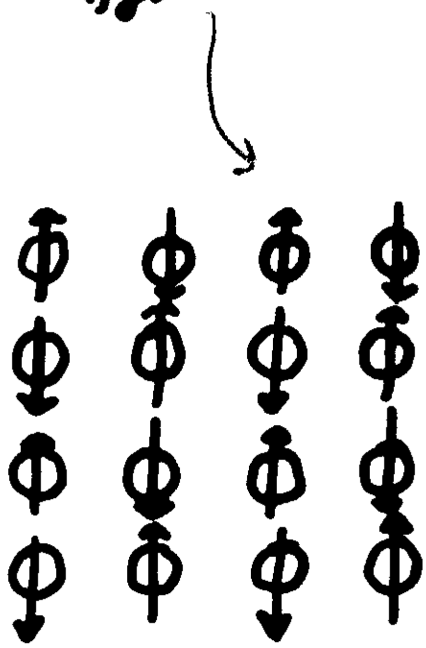
T-violation experiments (?)

Future Outlook

High-Tc caused by Antiferromagnetism !!!

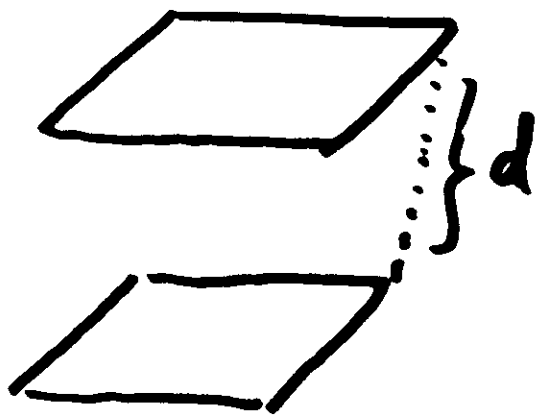
$$\mathcal{H} = -t \sum_{\langle i,j \rangle, \sigma} c^\dagger_{i\sigma} c_{j\sigma} + \frac{J}{2} \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

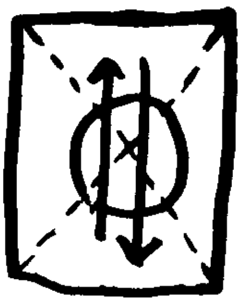




$J = .1 eV$

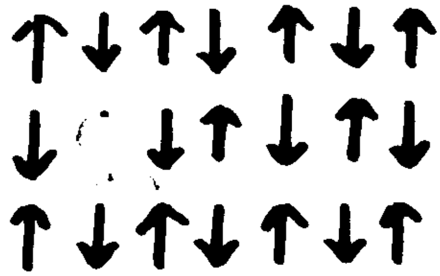
$t = .5 eV$



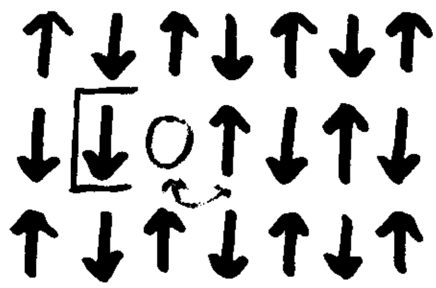


Hole Motion Discourages Antiferromagnetic Order

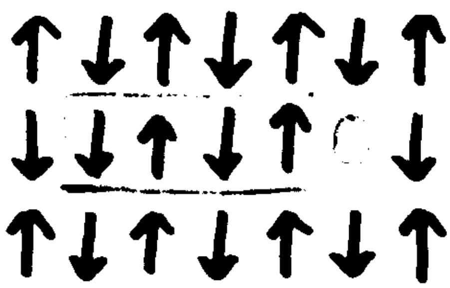
- Single Hole



1 hole



1 hop



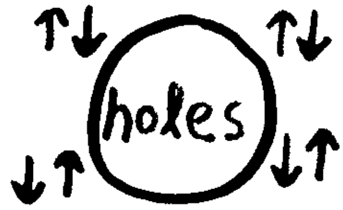
4 hops

$V(r) \sim r$

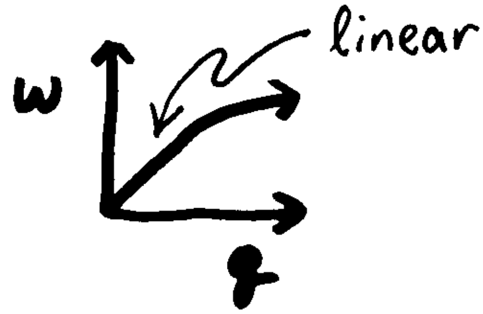
Doping with Holes Destroys Antiferromagnetic Order

- Hopping Energy t
- Spin-flip Energy $J/2$

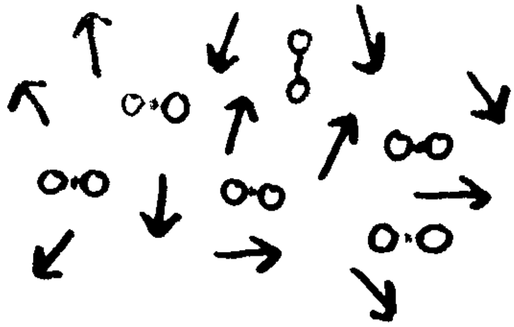
$J/2 \gg t$



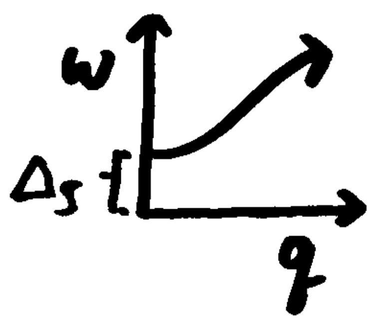
phase separation



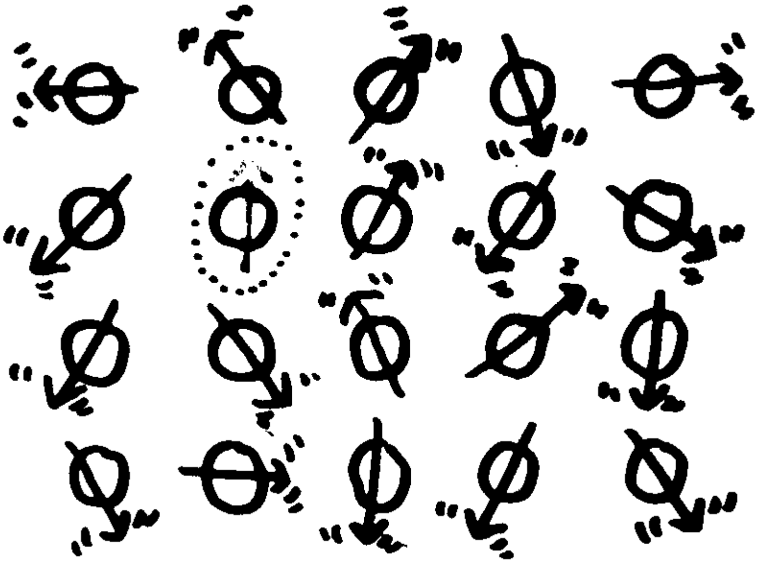
$t > J$



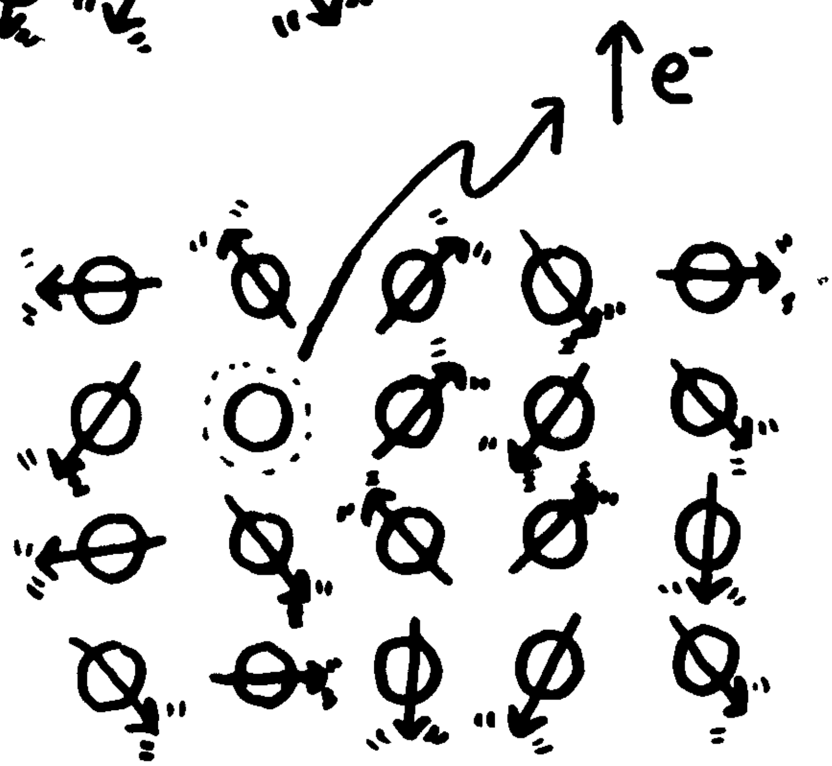
holes move
[? Cooper Pairs]



Spin and Charge Separate



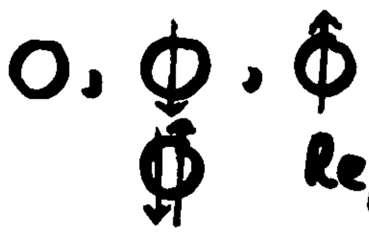
$$c_{i\uparrow} = b_i^\dagger f_{i\uparrow}$$



$$b_i^\dagger b_i + \sum_\sigma f_{i\sigma}^\dagger f_{i\sigma} = 1$$

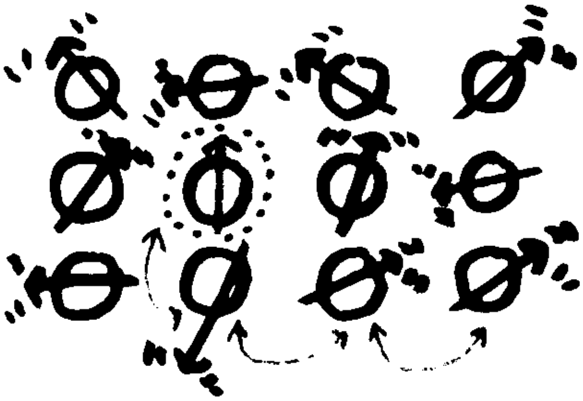
A Spinon has Spin 1/2 and no Charge

- Onsite



Repulsive Energy
$U \Rightarrow \infty$

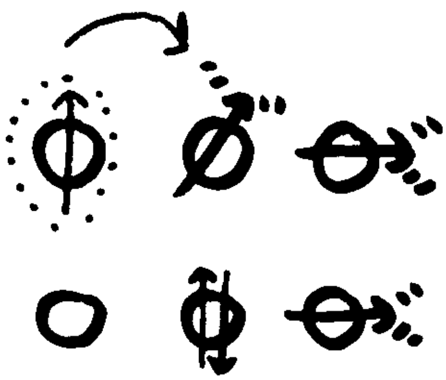
- Spinon



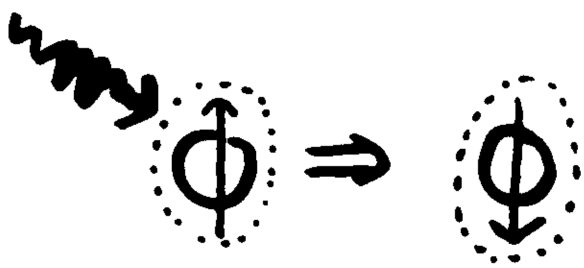
spinon motion $\not\Rightarrow$ charge motion

- Loss of Spinon Identity

Double Occupancy
Energy U



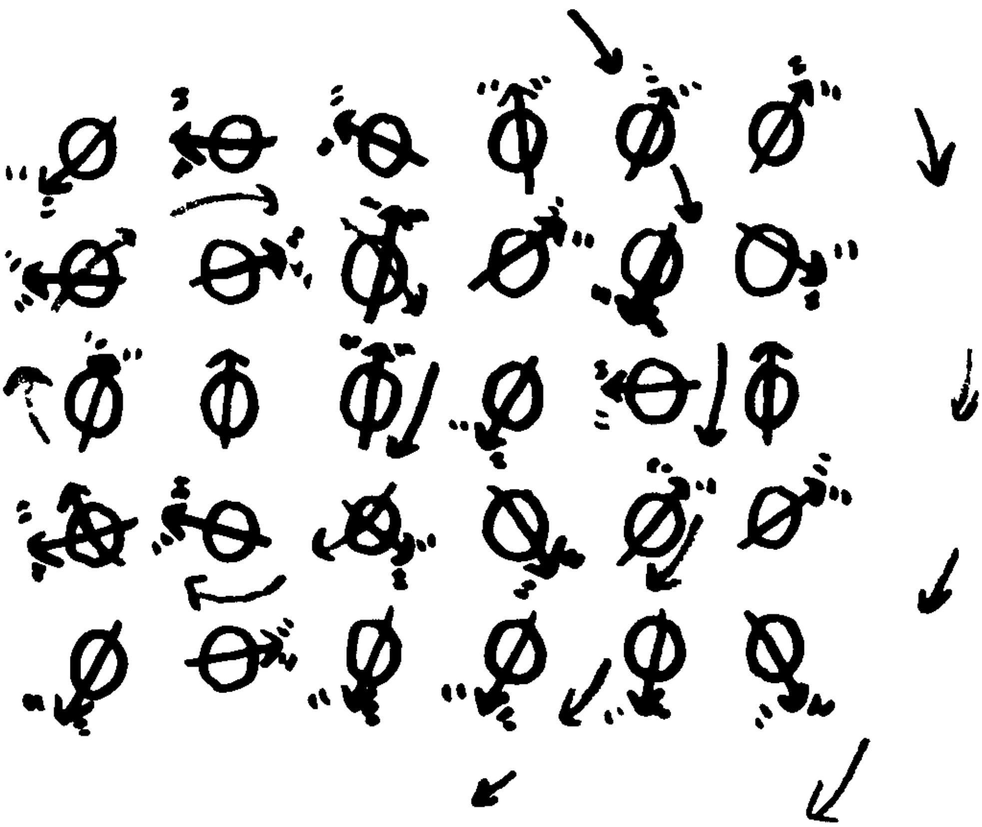
Spin Flip
Energy Δ_s



- $\tau \gg \hbar/\Delta_s, \hbar/U$

Spin and Charge Separation produces Gauge Fields

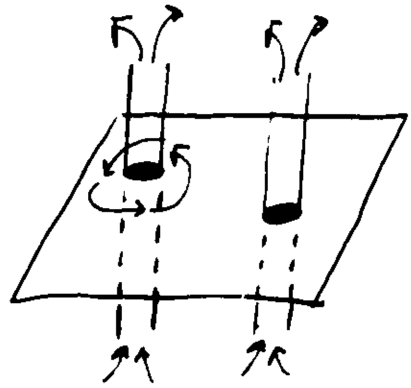
$$c_\uparrow = (e^{-i\theta} b^+)(e^{i\theta} f_\uparrow)$$



Gauge Field Implements Semionic Statistics

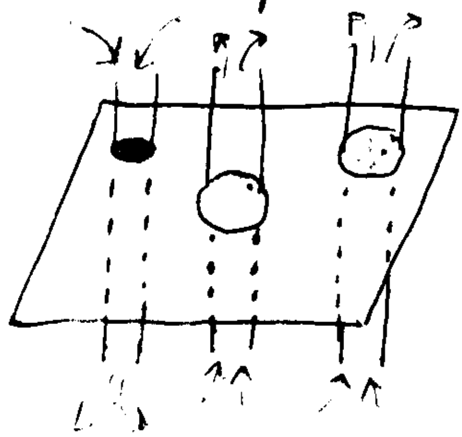
$e^{i\pi\nu}$

$\nu = 1$ Fermions
$\nu = 1/2$ Semions
$\nu = 0$ Bosons



$$e^{i\oint \vec{a}\cdot d\vec{l}} = e^{i\pi(\nu-1)}$$

(Flux Phase Description of Gauge Field)



Anyon Superconductivity

The Gauge Theory description of Spin-Charge Separation

$$\mathcal{H}_{t-J} \Leftrightarrow \mathcal{L}_{t-J} = \mathcal{L}_b + \mathcal{L}_f + \sum_j \phi_j - \frac{J}{4}\sum_{\langle i,j\rangle} |\chi_{ij}|$$

$$\boxed{- \sum_{\langle i,j\rangle} \left[\frac{t^2}{J} - \frac{J}{8}\right](b_i^+ b_i)(b_j^+ b_j)}$$

$$\mathcal{L}_b = \sum_j b_j^+ (i\hbar\partial_t - \phi_j) b_j + t \sum_{\langle i,j\rangle} \chi_{ij}\, b_i^+ b_j$$

$$\mathcal{L}_f = \sum_{j,\sigma} f_{j\sigma}^+ (i\hbar\partial_t - \phi_j) f_{j\sigma} + \frac{J}{2} \sum_{\langle i,j\rangle,\sigma} \chi_{ij}\, f_{i\sigma}^+ f_{j\sigma}$$

$$\chi_{ij} = \chi_{ij}^0 e^{i\theta_{ij}}$$

U(1) Gauge Field (ϕ_j, θ_{ij})

$$f_{i\sigma} \to e^{i\rho_i} f_{i\sigma} \qquad \theta_{ij} \to \theta_{ij} + \rho_i - \rho_j$$

$$b_i \to e^{i\rho_i} b_i \qquad \phi_j \to \phi_j - \hbar\partial_t \rho_j$$

Gauge Field has no Dynamics!!

The Spin-Gap Saddle Point

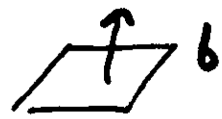
MF Fermi Spectrum determined by $\mathcal{H}_F \Big|_{(\phi_i, \theta_{ij}) = (0, \theta^0_{ij})}$

MF Bose Spectrum determined by $\mathcal{H}_b \Big|_{(\phi_i, \theta_{ij}) = (0, \theta^0_{ij})}$

Ansatz: $(\phi_i, \theta_{ij}) = (0, \theta^0_{ij})$

$\chi^0_{ij} = \chi \equiv$ fixed

$\theta^0_{ij} \Longrightarrow$ constant background field



Equations: $\dfrac{\partial h_{t-J}}{\partial (\chi^0_{ij})^*} = 0 \longrightarrow$ value of χ

$\dfrac{\partial \langle \mathcal{H}_F + \mathcal{H}_b \rangle_{MF}}{\partial b} = 0 \Longrightarrow$

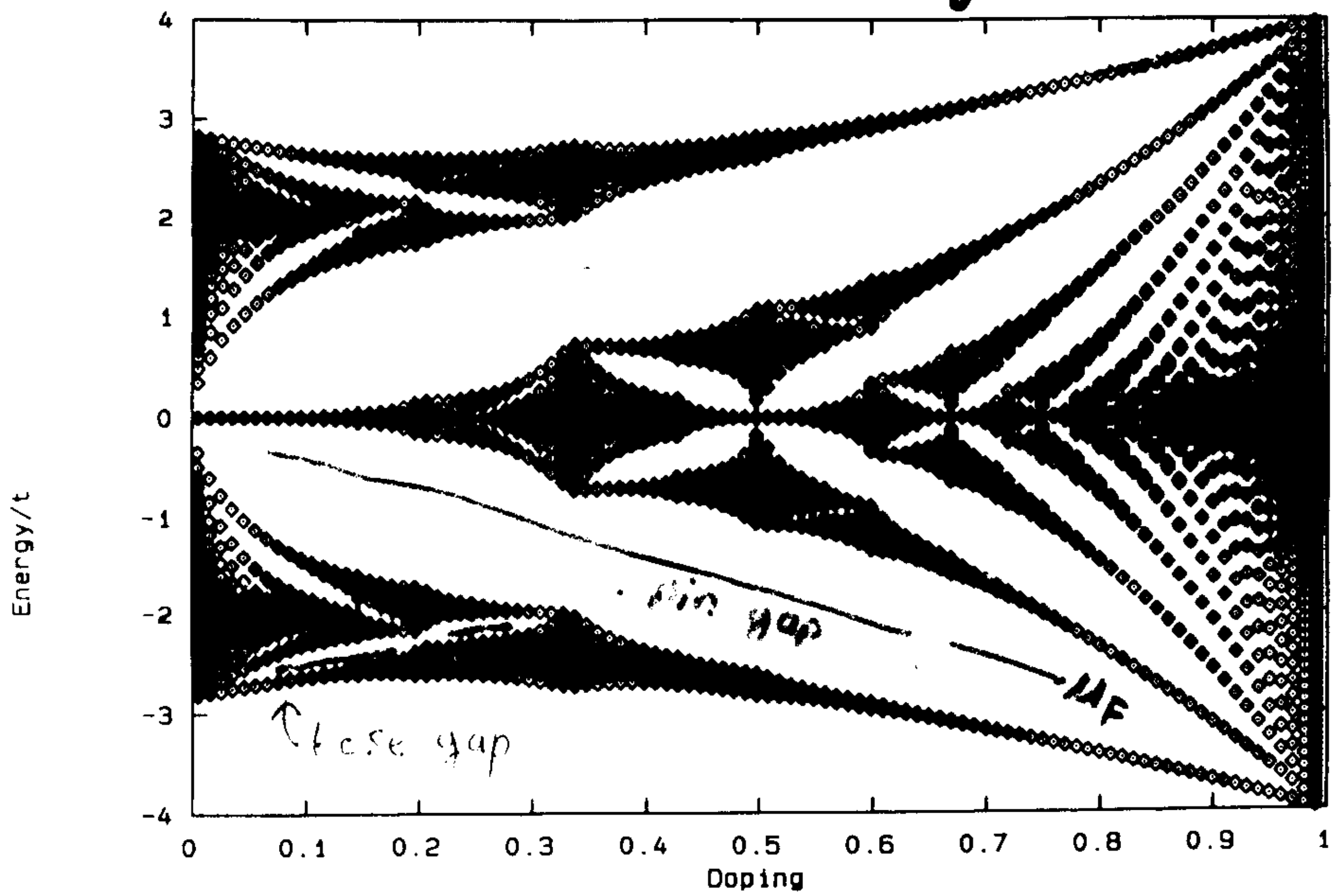
Commensurate Flux Phase

$\boxed{\pi(1-\delta)}$ **Flux π per electron**

P. Lederer, T.M. Rice, P. Wiegmann; PRL 63, 907 (98..

Broken $T+P$

Hofstadter Butterfly



Spectrum of $-t \sum_{\langle i,j\rangle} b_i^+ b_j \, e^{i\theta_{ij}^0}$

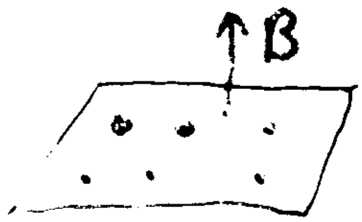
θ_{ij}^0 : $\boxed{\pi(1-\delta)}$

Why don't $T=0$ Bosons Bose Condense

Boom!

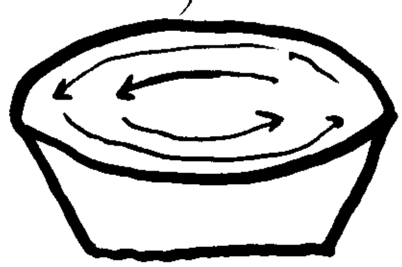
- hardcore bosons with long-range interaction

- prevent Bose condensation by breaking T-reversal invariance

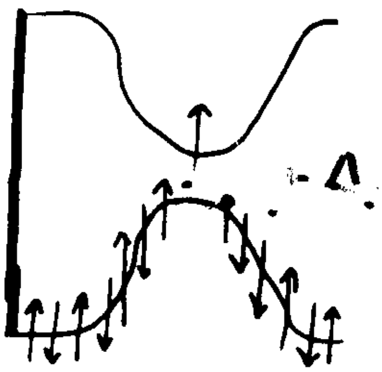


stable at $T=0$

analogy: rotating ^4He bucket



The Spin-Gap Δ_s breaks a Symmetry



- Background Potential, Spin-Dependent
  Breaks Translational Symmetry

$$\begin{array}{cccccc} + & - & + & - & + & - \\ - & + & - & + & - & + \\ + & - & + & - & + & - \\ - & + & - & + & - & + \end{array}$$

∴ Orders Spins - disagrees with experiment

- Background Magnetic Field
  Breaks T-reversal

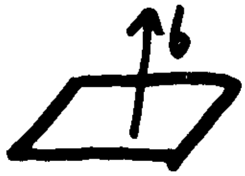
Spins are Disordered
$\Delta_s \simeq J\sqrt{2\pi\delta}$

? T-violation

Anyon Basics ($\nu = 1/2$)

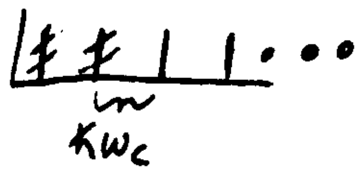
$\mathcal{H} = \mathcal{H}_0 +$ gauge field dynamics + fermion-gf coupling

$\mathcal{H}_0$:



$b = 2\pi(1-\nu)\rho$

π flux per anyon

$D(\varepsilon)$ 

<u>dynamics</u>

 $= -\left(\frac{2}{h}\Pi_{cs}\right)^{-1} \equiv$ gapped pole

$[\Pi_{cs} = O(q,\omega)]$

<u>response kernel</u>:

$\Pi_0 =$  $= -\frac{2}{h}\Pi_{cs} + O(q^2,\omega^2)$

<u>Lagrangian</u>:

$$\mathcal{L} = a^+ \underbrace{\left(\frac{2}{h}\Pi_{cs}\right)}_{\text{initial dynamics}} a - \underbrace{(a+A)^+\Pi_0(a+A)}_{\text{real } \varepsilon + m}$$

sum over a

$\Rightarrow$ <u>Screened Kernel</u> $\quad \Pi = \left[\Pi_0^{-1} + \left[\frac{2}{h}\Pi_{cs}\right]^{-1}\right]^{-1}$

$\Rightarrow$ <u>Screened Dynamics</u>  $= -\left[\Pi_0 + \frac{2}{h}\Pi_{cs}\right]^{-1}$

$\Rightarrow$ Chern-Simons terms cancel, massless pole!

The Gauge Field Calculation is a $\nu = 1/2$ Anyon Gas Calculation

Both have M-F Saddle points with magnetic gaps and π **flux per particle**

Response kernels:

$\Pi_f =$  $= \frac{2}{h}\Pi_{cs} + O(q^2, \omega^2)$

$\Pi_b =$  $= -\frac{2}{h}\Pi_{cs} + O(q^2, \omega^2)$

<u>**Lagrangian**</u>

$$\mathcal{L}_{anyon} = -a^+[\text{dynamics}]a - (a+A)^+\Pi_0(a+A)$$

$$\mathcal{L}_{gf} = -a^+\Pi_F a - (a+A)^+\Pi_b(a+A)$$

bosons have real charge

$\Longrightarrow$ initial dynamics $= -\Pi_F^{-1}$

$= -\left[\frac{2}{h}\Pi_{cs} + O(q^2, \omega^2)\right]^{-1}$

, induced by Π_F, turns the bosons into semions

$= -(\Pi_b + \Pi_F)^{-1}$, **CS terms cancel**

$K = (\Pi_b^{-1} + \Pi_F^{-1})^{-1}$, Ioffe-Larkin Eq.

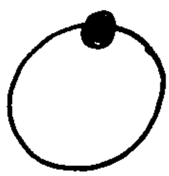
Gauge Field Calculation and
Anyon Gas Calc., continued

$$K = [\,\pi_b^{-1} + \pi_f^{-1}\,]^{-1} \qquad \{\text{gauge fixing is implied}\}$$

$$D = \quad = -[\,\pi_b + \pi_f\,]^{-1}$$

democratic equations $\Rightarrow$ bosons turn **fermions** into semions as well

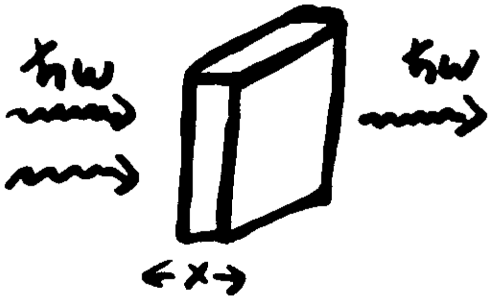
<u>Note:</u> only studied Gaussian Fluctuation about θ_{ij}^{0}



– "guilty until proven innocent"

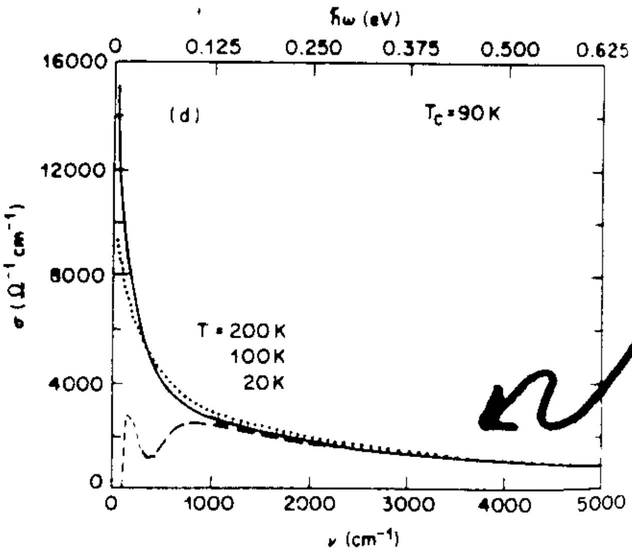
Produces Accurate Calculations

Optical Absorption

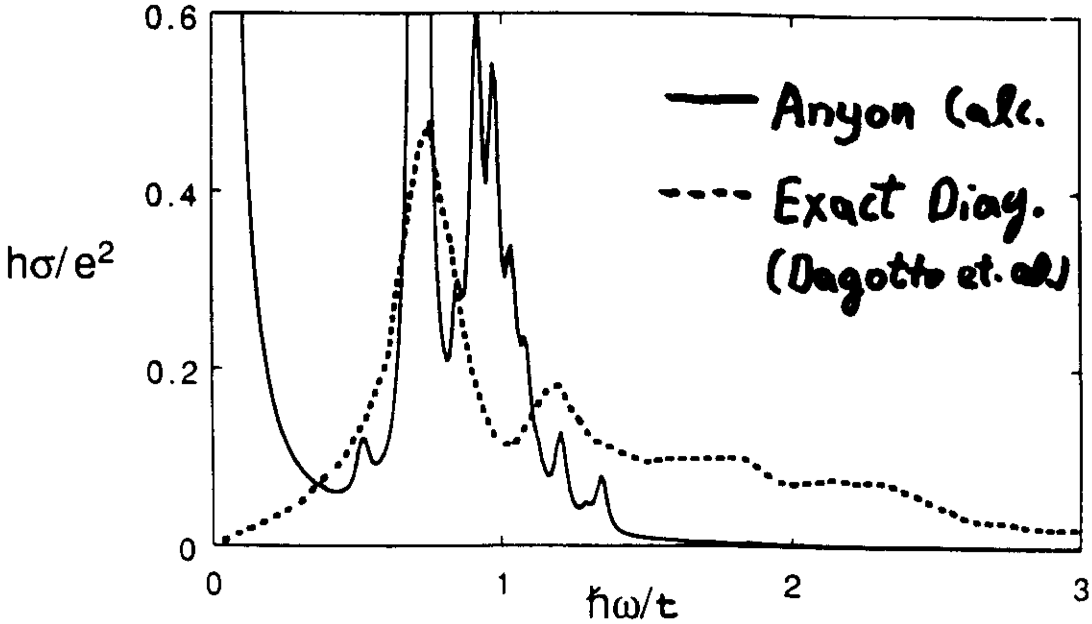


$$I(t) = I_0 e^{-4\pi t\sigma/d}$$

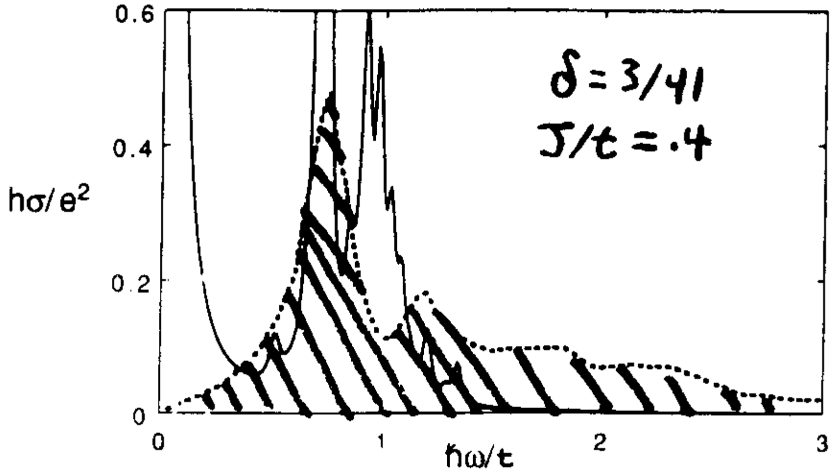
$d \equiv$ dist. between planes

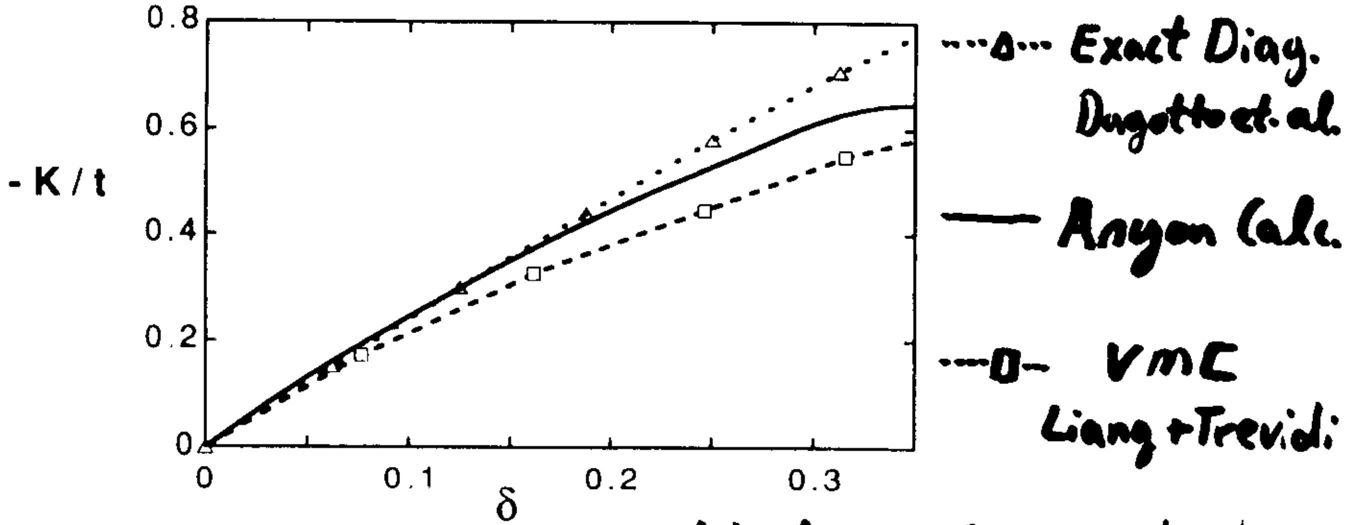


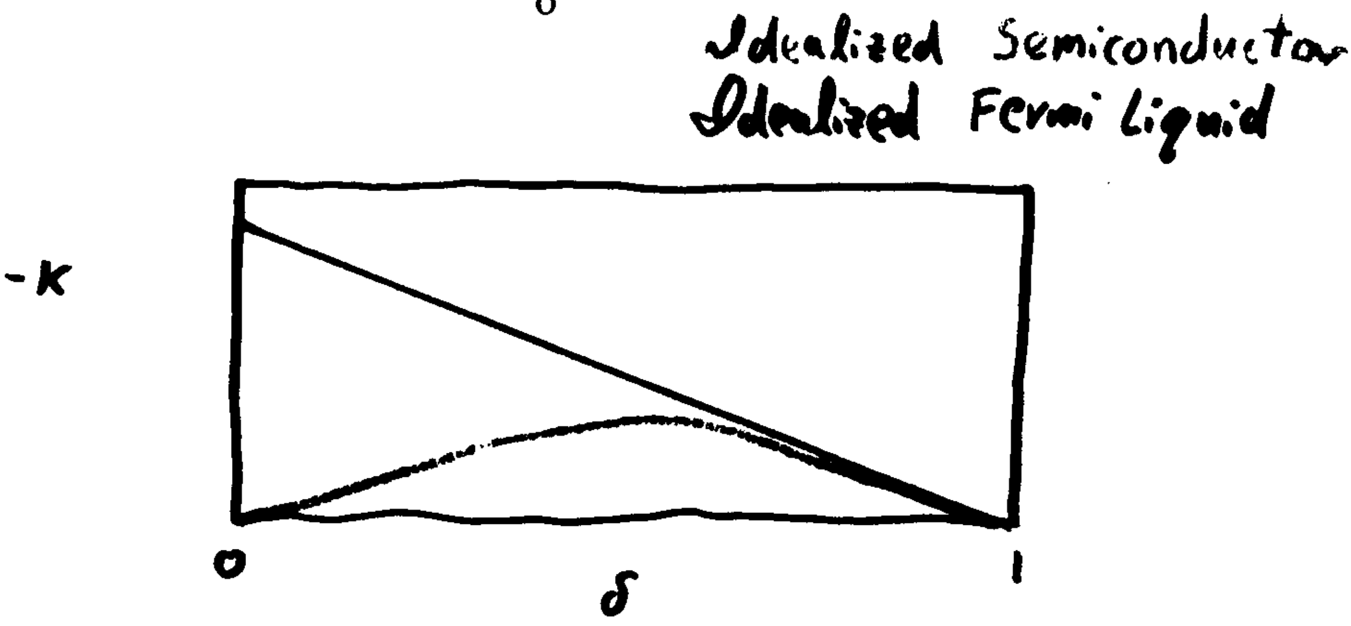
Large Background (G.A. Thomas et. al.)



Accurate Calculations, cont.



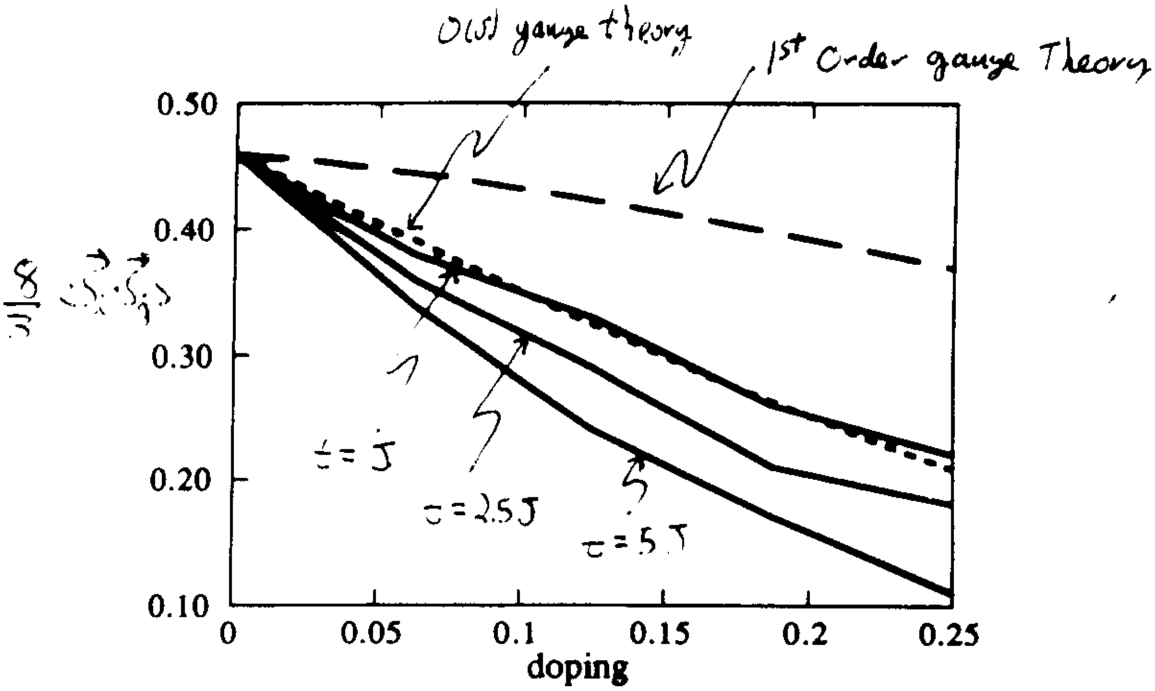


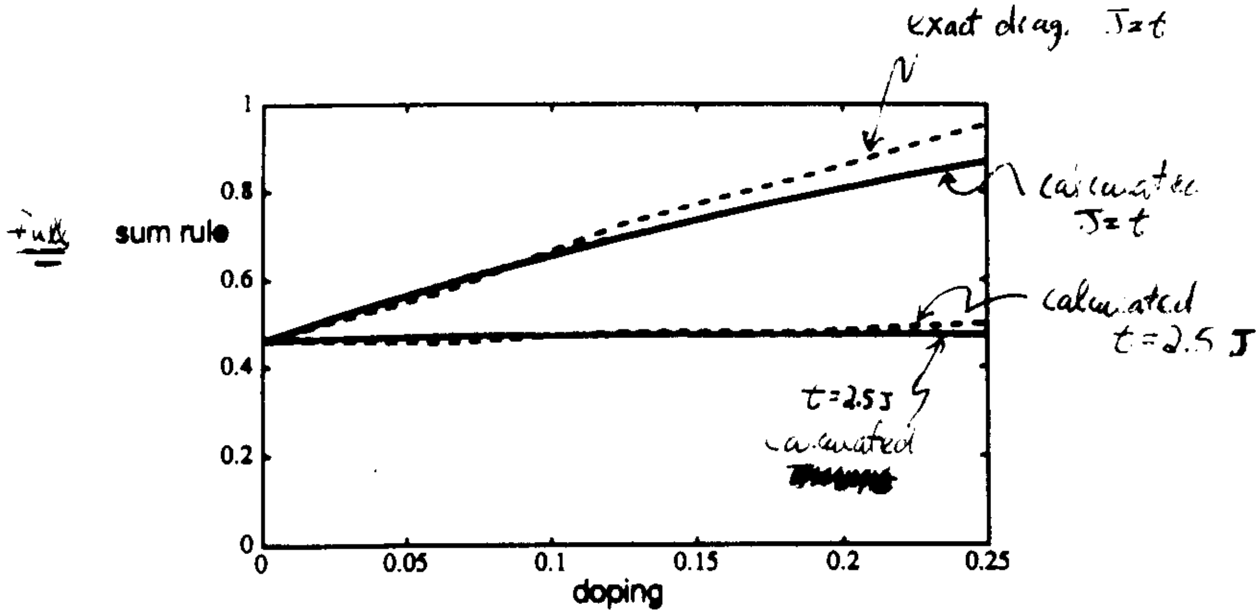


Get Magnetic Energy from a Sum Rule

Sum Rule measures $\langle \frac{8}{3}\hat{J} + \hat{T} \rangle$

$$= -2JN \langle \frac{8}{3}\vec{S}_i \cdot \vec{S}_j + \frac{2t}{J}\sum_\sigma c_{i\sigma}^+ c_{j\sigma} \rangle$$

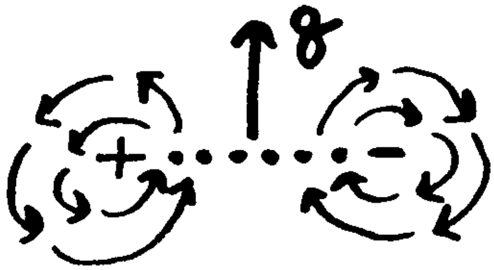




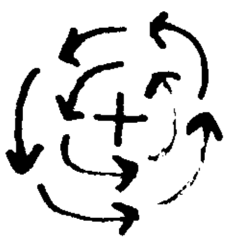
Screened Gauge Field mediates Coulombic interaction

$V(r) \sim \ln r$

- vortex binding,
  anyon superconductivity



- vortex unbinding transition,
  Kosterlitz - Thouless



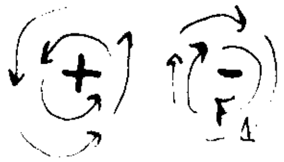
$$E_V = \alpha^2 \ln(R)$$

$$S_V = k_B \ln(R^2)$$

$$F_V = E_V - T S_V \leq 0 \quad \text{for}$$

$$k_B T \geq \alpha^2/2$$

Gauge Theory has a Kosterlitz-Thouless T_{KT}

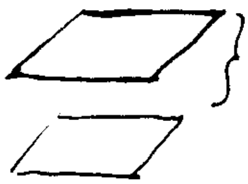


vortex pairing

$$T_c \le T_{KT} \le T_{KT}^{(0)} = \frac{\pi\hbar^2}{8}\chi_{yy}(\omega=0)$$

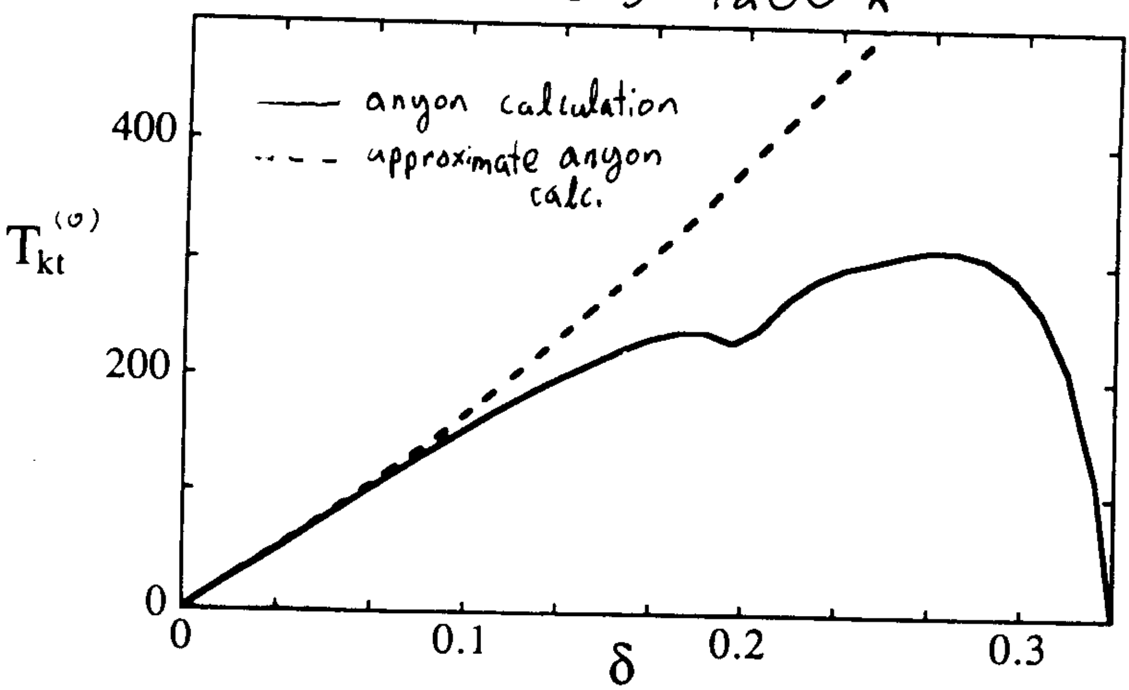
$$\Rightarrow \quad T_c < \frac{\pi\hbar^2}{8}\chi_{yy}(\infty) = \frac{-\pi}{16}\,\frac{\langle 0|\hat{T}|0\rangle}{N}$$

$1/\lambda^2 \propto T_{KT}^{(0)}$ even w/o K-T transition



$d = 10\,\text{Å}$ $\qquad \lambda = 1800\,\text{Å} \Rightarrow T_{KT}^{(0)} = 300\,°K$

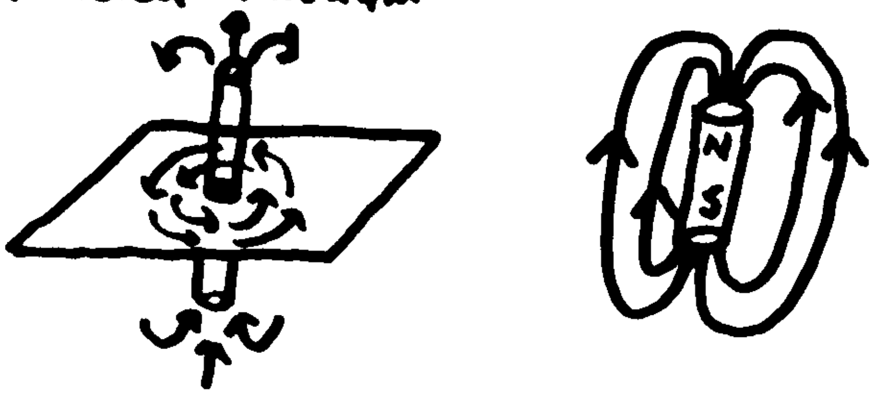
$J = t/5 = 1200\,°K$



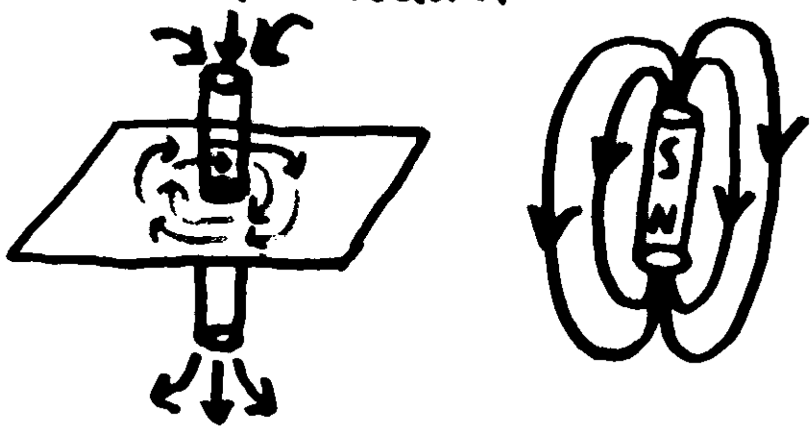
Semions violate P + T

Spontaneous violation of P+T

Right-handed Vacuum



Left-handed Vacuum



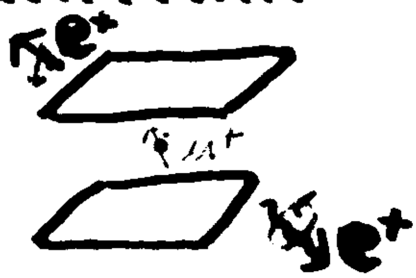
- Natural T-violation
  A-phase of superfluid ^3He
  Complex Order Parameter in UPt_3

T-violation has not yet been observed

Optical Activity

$\sigma_{xy}(\omega) \neq 0$

Muon Spin Rotation



$\sigma_{xy}(q) \neq 0$

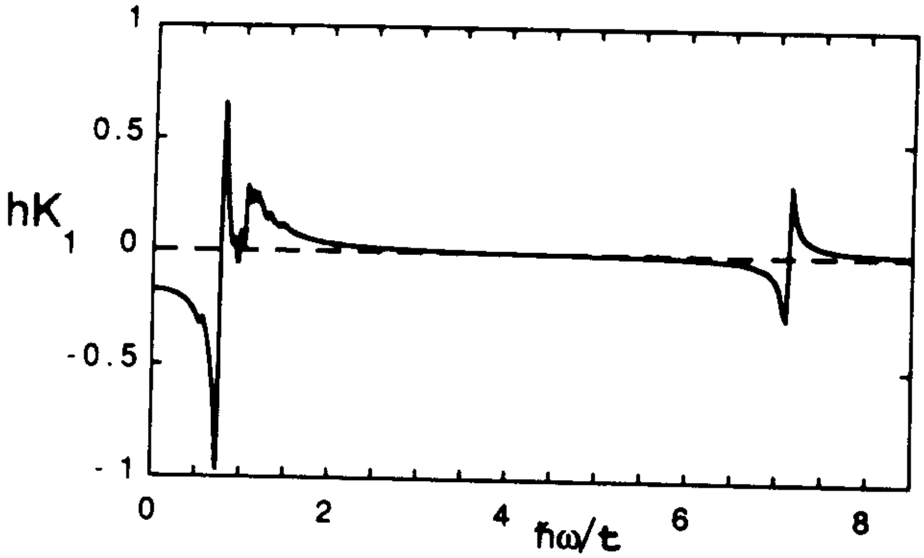
NMR

...... Can not do Quantitative calc. of σ_{xy}

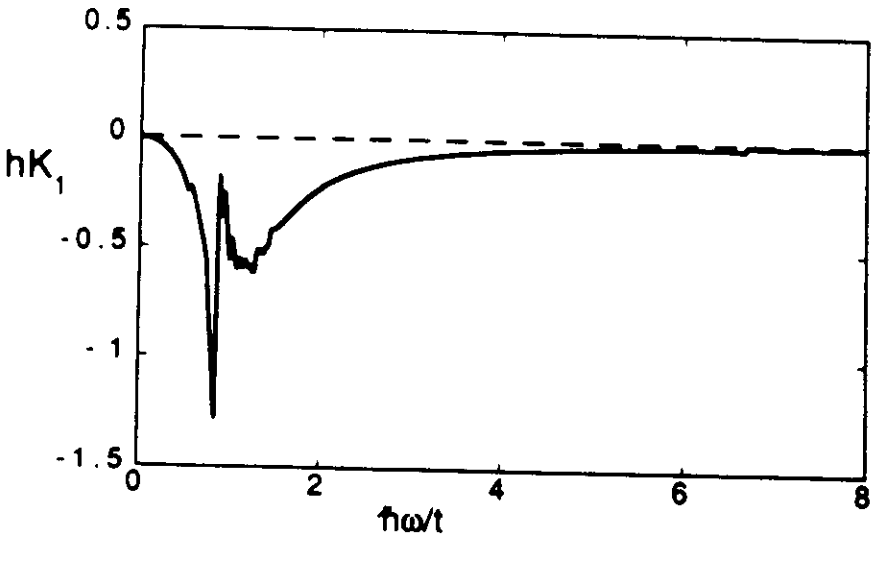
Calculated Hall Conductivity is unreliable

extremely sensitive to

$$\mathcal{L}_{g.f.} \to \mathcal{L}_{g.f.} + (?)(\partial^\tau \partial_\tau)\epsilon^{\alpha\beta\gamma} a_\alpha \partial_\beta a_\gamma$$

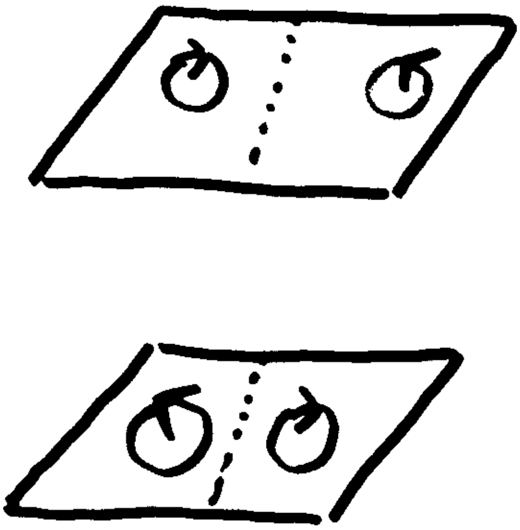


$J/t = .4$
$\delta = 3/41$



"small correction"

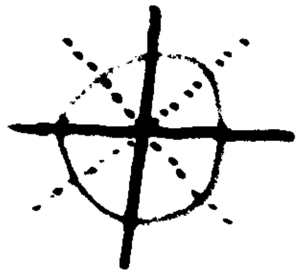
Difficult to Observe T-violation



Antiferromagnetic Stacking
Domain Formation

The Order Parameter Violates T-Reversal

- Normal D-Wave $\Delta_1 = \Delta_0 [\cos k_x b - \cos k_y b]$



$\Delta_1 = 0$ on Fermi Surface $\Rightarrow$ gapless quasiparticles

- D+iD $\Delta = \Delta_0 [\cos k_x b - \cos k_y b + i\epsilon \sin k_x b \sin k_y b]$
$= \Delta_1 \pm i\Delta_2$ <u>2 Order Parameters</u>

$|\Delta| \neq 0$ on FS

- Dirt Reduces both Order Parameters
$|\Delta_1| \gg |\Delta_2|$; $\epsilon \doteq 3\delta$
Δ_1 reduced, $\Delta_2 = 0$

- Most Experiments that average over whole sample see only Δ_1

- STM can see both O.Params in clean parts of sample
<u>T. Hasegawa et. al.</u> – <u>quasiparticle gap</u>

Conclusion  or  ?

<u>Existing Calculations</u>:

Quantitatively Successful Optical Properties
Kosterlitz-Thouless T_{KT}
D+iD Order Parameter - violates T
Electron Spectral Function (only big energies)
Magnetic Response Function

<u>To Calculate</u>:

Effect of finite $\Updownarrow$ U
Phase Diagram - charge + spin instabilities
Small Energy Scales - Transport
Accurate σ_{xy}

<u>Open ?'s</u>

Justify weak-coupling expansion of $g = \infty$ theory
Does T-violation occur or just comput. tool?

<u>Quantum # Fractionalization</u> $\Longrightarrow$ Gauge Field

- generalize principle by testing in condensed matter systems

- make working version of
$\varphi = 1/3 \Longrightarrow SU(3)$