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SMR. 767 - 15

MINIWORKSHOP ON STRONG CORRELATIONS
AND QUANTUM CRITICAL PHENOMENA
(4 - 22 July 1994)

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A SYSTEMATIC UNDERSTANDING OF THE MAGNETIC AND TRANSPORT PHENOMENA IN CUPRATES BASED ON THE t-J MODEL

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These are preliminary lecture notes, intended only for distribution to participants.

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A Systematic Understanding of the Magnetic and Transport Phenomena in Cuprates Based on the $t-J$ Model

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Collaborator : D.N. Sheng

- Introduction
- Framework: Flux-binding saddle-point state of the $t-J$ model
- Magnetic Properties

  1d: Exact results

  2d: Experimental results

  – commensurate vs. incommensurate magnetic fluctuations

  – "spin gap" behavior

  – NMR spin relaxation rate

  – pairing instability
- Charge Properties:

  1d: Luttinger liquid behaviors

  2d: Transport phenomena

  – linear-T resistivity

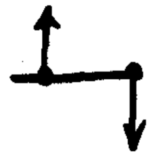
  – Hall effect; Hall angle phenomena; magneto-resistance

  – thermopower
- Conclusions

- Introduction

The t-J model $H_{t-J} = H_t + H_J$

hopping term $H_t = -t \sum_{\langle ij \rangle \sigma} c_{i\sigma}^{\dagger} c_{j\sigma} + \text{H.c.}$ 

superexchange $H_J = J \sum_{\langle ij \rangle} \left(\vec{S}_i \cdot \vec{S}_j - \frac{n_i n_j}{4} \right)$ 

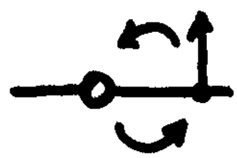
The Hilbert space is restricted by $\sum_{\sigma} c_{i\sigma}^{\dagger} c_{i\sigma} \leq 1$

Slave-boson formulation

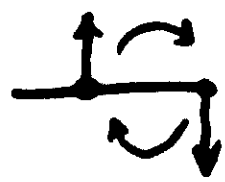
$$c_{i\sigma} = h_i^{\dagger} f_{i\sigma}$$

no double occupancy constraint $h_i^{\dagger} h_i + \sum_{\sigma} f_{i\sigma}^{\dagger} f_{i\sigma} = 1$

$$H_t = -t \sum_{\langle ij \rangle \sigma} h_i^{\dagger} h_j f_{i\sigma}^{\dagger} f_{j\sigma} + \text{H.c.}$$



$$H_J = -\frac{J}{2} \sum_{\langle ij \rangle \sigma\sigma'} f_{i\sigma}^{\dagger} f_{j\sigma} f_{j\sigma'}^{\dagger} f_{i\sigma'}$$



The no double occupancy constraint is not as crucial as in the original case

Mean-field decoupling

$$H_h^{MF} = -t \sum_{\langle ij \rangle \sigma} \chi_{ji}^{\sigma} \, h_i^+ h_j + \text{H.c.}$$

$$H_S^{MF} = -\frac{J}{2} \sum_{\langle ij \rangle \sigma\sigma'} \left(\chi_{ji}^{\sigma'} + \frac{t}{J} H_{ji} \right) f_{i\sigma}^+ f_{j\sigma} + \text{H.c.}$$

Mean-fields: $\chi_{ji}^{\sigma} = \langle f_{j\sigma}^+ f_{i\sigma} \rangle$, $H_{ji} = \langle h_j^+ h_i \rangle$

1. Uniform RVB state

$$\chi_{ji}^{\sigma} = \chi_0, \quad H_{ji} = H_0$$

phase fluctuation around the saddle-point:

Gauge field description — Ioffe & Larkin, Nagaosa & Lee, Kotliar, Wiegmann ...

$\delta \to 0$, AF may underestimated

$\delta = 0$, Variational energy 21% higher

NO AF long-range order

2. Flux-phase

$$\chi_{ji}^{\sigma} = \chi_0 e^{i\theta_{ij}} \qquad (\delta=0,\ H_0=0)$$

Affleck & Marston

$$\phi_{\square} = \sum_{\square} \theta_{ij} = \pi$$

variational energy by VMC: only 5% higher

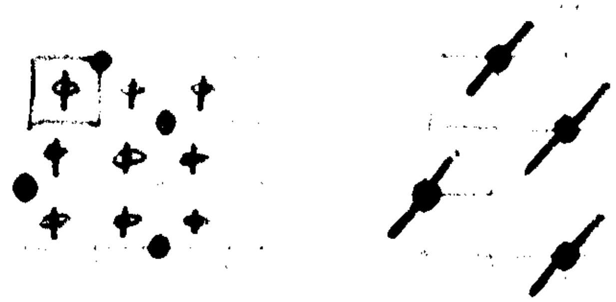
AF long range order is still missing

3. Flux binding states

Exact diagonalization

(Canright, Girvin & Brass)

phys. Rev. Lett. 63 (1989)



Scheme one All spinons,

no matter spins, are bound to the same flux quanta.

$$\sum_{\square} \theta_{ij} = \overset{\hbar_1}{\pi} \sum_{\substack{\sigma \\ l\in\square}} n_{l\sigma}^{f} \qquad \langle n_{l\sigma}^{f}\rangle = 1/2$$

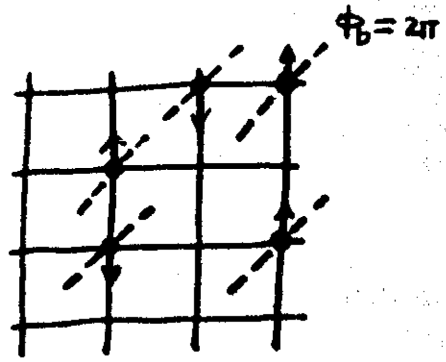
$$\delta = 0$$

Away from half-filling, $\sum_{\square} \langle \theta_{ij} \rangle = \pi(1-\delta) \rightarrow$ Commensurate flux phase

T=0, Superfluid Z.Y. Wang, et al. Phys. Rev. B49, 607(1994

$T>T_c$, anomalous transport properties Z.Y. Wang et al.

Scheme two Spinons with different spins cannot see each other's flux tubes



$$\sum_{\square} \theta_{ij}^{\sigma} = \underbrace{2\pi}_{\Phi_b} \sum_{l \in \square} n_l^f$$

each flux tube : 2π flux

Normal state is easily accessible in Scheme two

- **Mathematical Framework**

Identical transformation

$$H_J = -\frac{J}{2} \sum_{<ij>\sigma\sigma'} \left(e^{iA_{ij}^{\sigma}} f_{i\sigma}^{+} f_{j\sigma} \right) \left(e^{iA_{ji}^{\sigma'}} f_{j\sigma'}^{+} f_{i\sigma'} \right)$$

$$A_{ij}^{\sigma} = \sigma \sum_{l \neq i,j} (\theta_i(l) - \theta_j(l))[n_{l\sigma}^f + \frac{n_l^h}{2} - \delta_{\sigma,\uparrow}]$$

$$\theta_i(l) = Im \ln(z_i - z_l)$$

$$A_{ij}^{\sigma} \Leftrightarrow \theta_{ij}$$

Mean-field $\qquad \chi_0 = < e^{iA_{ij}^{\sigma}} f_{i\sigma}^{+} f_{j\sigma} >$

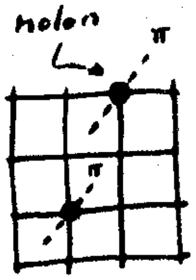
Saddle-point Hamiltonian

$$H_{t-J}^{MF} = H_h + H_s$$

Spinon:

$$H_s = -J_s \sum_{<ij>\sigma} e^{iA_{ij}^{\sigma}} f_{i\sigma}^{+} f_{j\sigma} + H.c.$$

$$\boxed{H_s = -J_s \sum_{<ij>\sigma} \left(e^{i\sigma A_{ij}^{h}}\right) \quad b_{i\sigma}^{+} b_{j\sigma} + H.c.}$$



$$A_{ij}^{h} = \frac{1}{2} \sum_{l \neq i,j} Im \ln \left(\frac{z_i - z_l}{z_j - z_l}\right) n_l^h$$

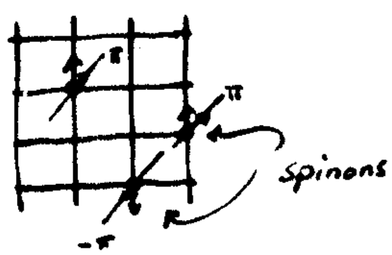
$$b_{i\sigma} = f_{i\sigma} e^{-i\sigma \sum_{l \neq i} \theta_i(l)(n_{l\sigma}^{f} - \delta_{\sigma,1})}$$

Jordan-Wigner transformation

Holon:

$$\boxed{H_h = -t_h \sum_{<ij>} \left(e^{iA_{ij}^{b}}\right) \quad h_i^{+} h_j + H.c.}$$

$$A_{ij}^{b} = \frac{1}{2} \sum_{l \neq i,j} Im \ln \left(\frac{z_i - z_l}{z_j - z_l}\right) \left(\sum_{\sigma} \sigma n_{l\sigma}^{b} - 1\right)$$



Half-filling: $\qquad A^h_{ij} = 0 \quad (\delta = 0)$

$$S_i^+ = (-1)^i \, b_{i\uparrow}^+ b_{i\downarrow}$$

2d case: Bose condensation of $b_{i\sigma}$

$$\langle S_i^+ \rangle = (-1)^i \langle b_\uparrow^+ \rangle \langle b_\downarrow \rangle \neq 0$$

- AF LRO
- upper bound of GSE: 5% higher

1d case:

$$\langle S_i^+(t) S_j^-(0) \rangle = (-1)^i \langle b_{i\uparrow}^+(t) b_{j\uparrow}(0) \rangle \langle b_{i\downarrow}(0) b_{j\downarrow}^+(t) \rangle$$

$$\propto \frac{\cos \frac{\pi}{2}(x_i - x_j)}{\left[(x_i - x_j)^2 - v_f^2 t^2 \right]^{1/2}}$$

Luther & Peshel (1975)

• Magnetic Properties Away From Half-Filling

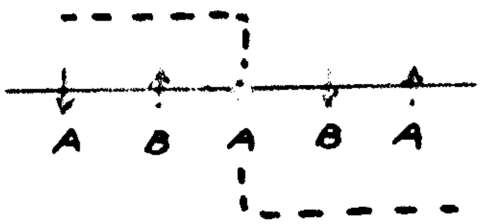
- One-dimensional case

In bosonic representation

$$S_i^+ = (-1)^i b_{i\uparrow}^+ b_{i\downarrow} \quad e^{i\pi \sum_{l<i} n_l^h}$$

Z. Y. Weng et al. Phys. Rev. Lett. 67, 3318 (1991)

Semiclassical picture:



$$< S_i^+(t) S_j^-(0) > = (-1)^{i-j} < b_{i\uparrow}^+(t) b_{j\uparrow}(0) >< b_{j\downarrow}(t) b_{j\downarrow}^+(0) >$$

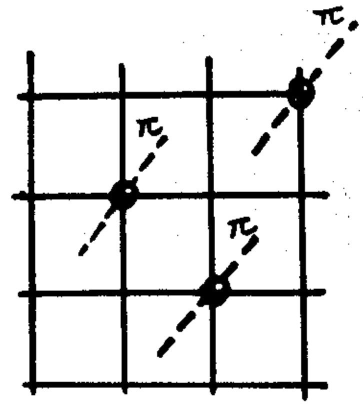
$$\times < e^{i\pi \sum_{l<i} n_l^h(t)} e^{-i\pi \sum_{l<j} n_l^h(0)} >$$

$$\propto \frac{cos(2k_f x)}{(x^2 - v_s^2 t^2)^{1/2} (x^2 - v_h^2 t^2)^{1/4}}$$

$k_f = \frac{\pi}{2a}(1-\delta)$

$x = x_i - x_j$

• Spin Dynamics in Two-dimension Doped Case



Saddle-point Hamiltonian

$$H_s = -J_s \sum_{<ij>\sigma} \left(e^{i\sigma A^h_{ij}}\right) b^+_{i\sigma} b_{j\sigma} + H.c.$$

$$A^h_{ij} = \frac{1}{2} \sum_{l \neq i,j} Im \ln \left(\frac{z_i - z_l}{z_j - z_l}\right) n^h_l$$

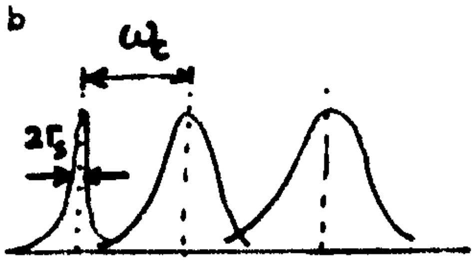
$$n^h_l = < n^h_l > + \delta n^h_l \quad \Longrightarrow \quad A^h_{ij} = \bar{A}^h_{ij} + \delta A^h_{ij}$$

$\hookrightarrow$ fluctuating flux

$B_h = \pi\delta$

Levy & Laughlin

...



• Basic energy scales: $\omega_c = 2\pi J_s \delta \sim \delta J$

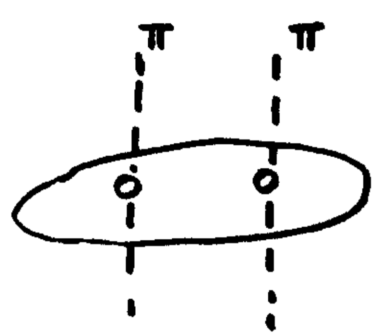
$\Gamma_s(n)$ —— broadening

• Basic length scale: $l_c = 1/\sqrt{B_h} = a/\sqrt{\pi\delta}$ cyclotron length

• Basic temperature scale:

Bose condensation temperature T_c^*

Pairing instability at $T < T_c^*$:



Spin Susceptibility Function

$$\chi''(\mathbf{q},\omega) = -\pi\sum_{l} K_{ll}(\mathbf{q})\int\frac{d\omega'}{2\pi}\rho_b(l,\omega')\rho_b(l,\omega+\omega')[n(\omega+\omega')-n(\omega')]$$
$$-\pi\sum_{l\neq l'} K_{ll'}(\mathbf{q})\int\frac{d\omega'}{2\pi}\rho_b(l,\omega')\rho_b(l',\omega+\omega')[n(\omega+\omega')-n(\omega')]$$

$$K_{ll'}(\mathbf{q}) = \frac{\delta}{16\pi}\int_0^\infty dy L_l(y)L_{l'}(y)e^{-y}J_0(\sqrt{2l_c^2 y}|\mathbf{q}-\mathbf{Q}|)$$

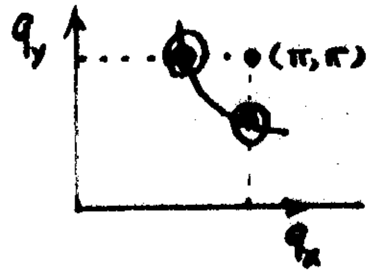
Small T, ω (< ω_c)

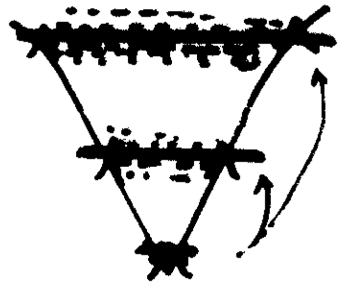
$$\chi''(\mathbf{q},\omega) \propto K_{00}(\mathbf{q}) = \frac{\delta}{16}\exp\left(-\frac{|\mathbf{q}-\mathbf{Q}|^2}{2l_c^2}\right) \qquad \vec{Q} = \left(\frac{\pi}{a}, \frac{\pi}{a}\right)$$

Spin correlation length $l = \sqrt{2}l_c \propto a/\sqrt{\delta}$

Increase of T: width is not changed

Increase of ω: $K_{01}(\mathbf{q})$ involved $\longrightarrow$ incommensurate structure

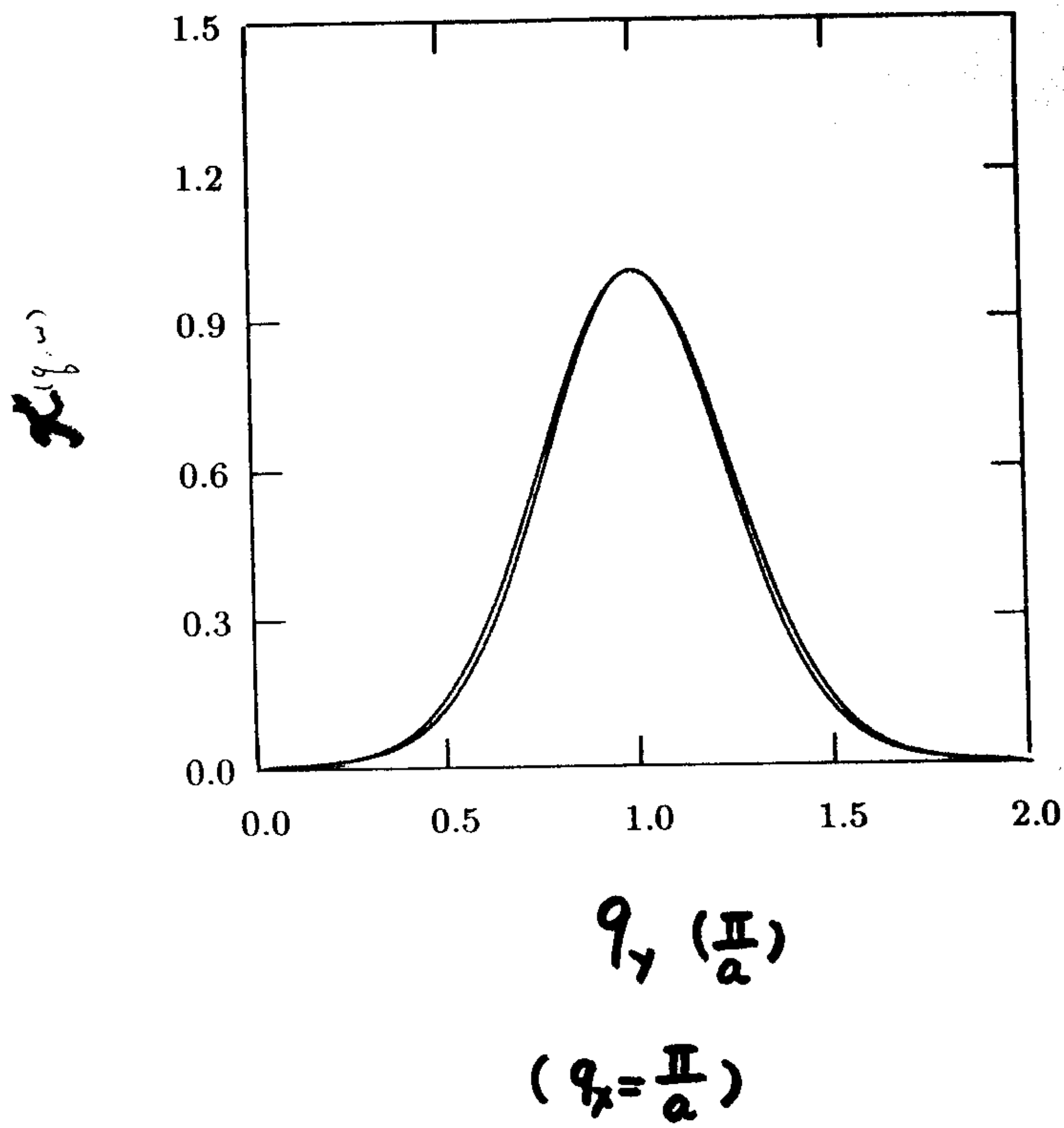




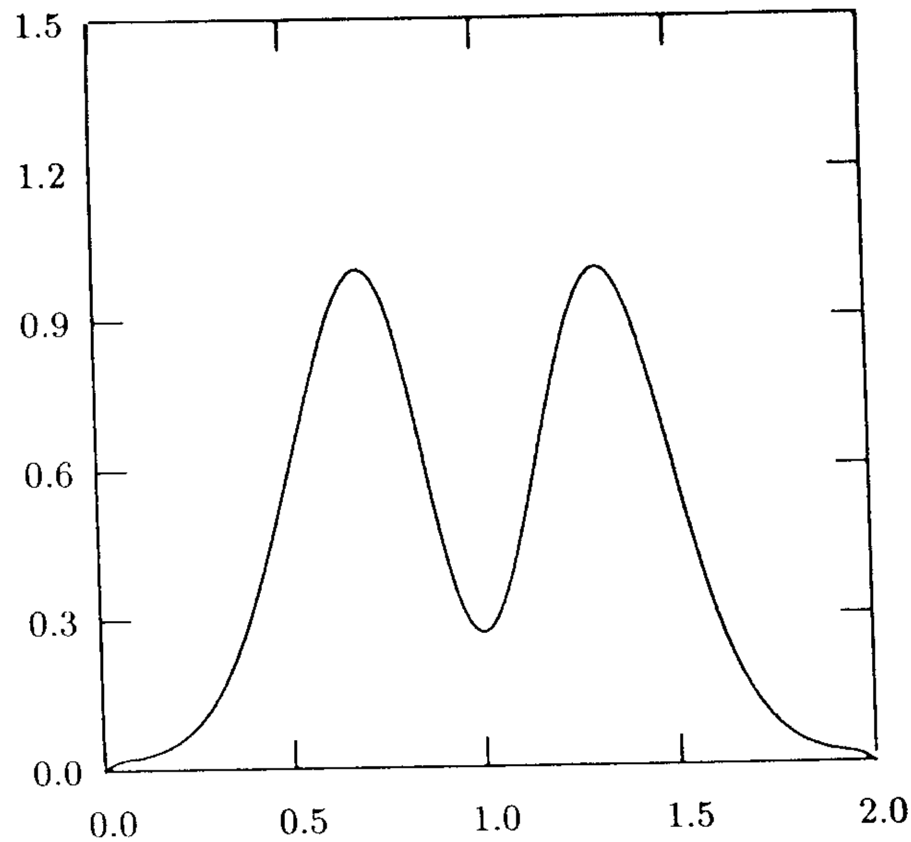
inter-Landau level transition

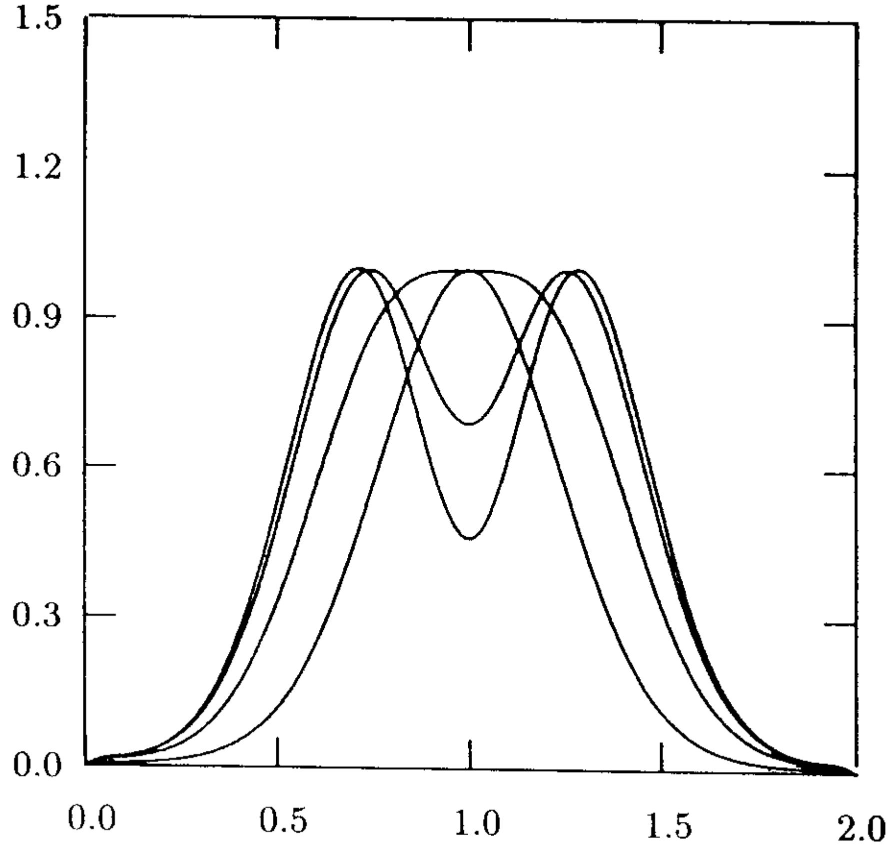
↓

incommensurate peaks



1.5
1.2
0.9
0.6
0.3
0.0
0.0
0.5
1.0
1.5
2.0
$\chi(q,\omega)$
$q_y\ (\frac{\pi}{a})$
$(q_x = \frac{\pi}{a})$


1.5
1.2
0.9
0.6
0.3
0.0
0.0
0.5
1.0
1.5
2.0


1.5
1.2
0.9
0.6
0.3
0.0
0.0
0.5
1.0
1.5
2.0

ω- dependence of the susceptibility function

$$\chi''(\mathbf{Q},\omega) = -\frac{\delta}{16}\sum_l \int \frac{d\omega'}{2\pi}\rho_b(l,\omega')\rho_b(l,\omega+\omega')[n(\omega+\omega')-n(\omega')]$$

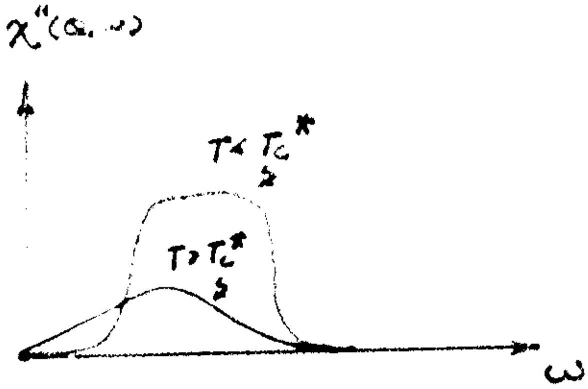
$T > T_c^*, \quad \chi''(\mathbf{Q},\omega) \propto \frac{\omega}{T} \qquad \left(\frac{\omega}{T} \ll 1\right)$

$T = 0, \quad \chi''(\mathbf{Q},\omega) \propto \rho_b(0,\omega)$

B.E.-condensation

"Spin gap"

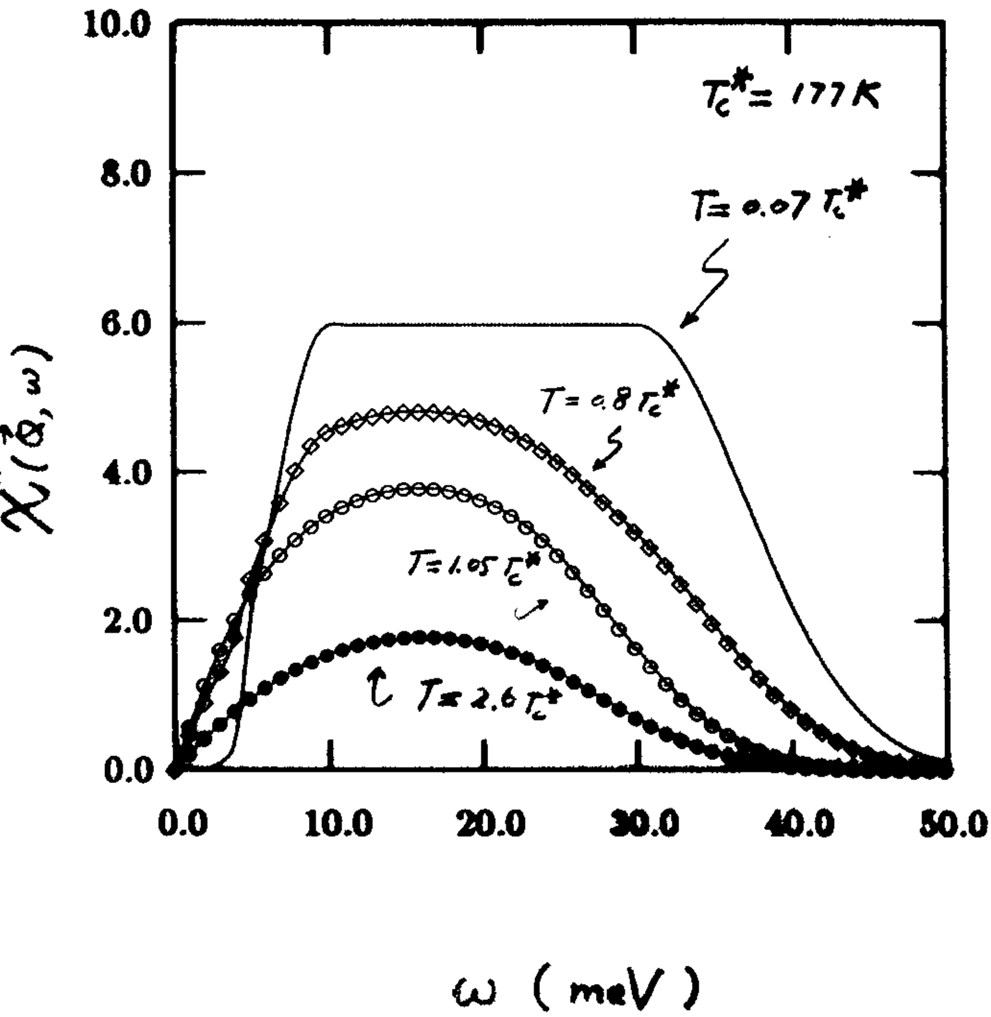
behavior



Neutron data $\xrightarrow{\text{low } T}$ $\rho_b(0,\omega)$ $\xrightarrow[T]{\text{arbitrary}}$ $\chi''(Q,\omega)$

NMR $\frac{1}{T_1 T}$

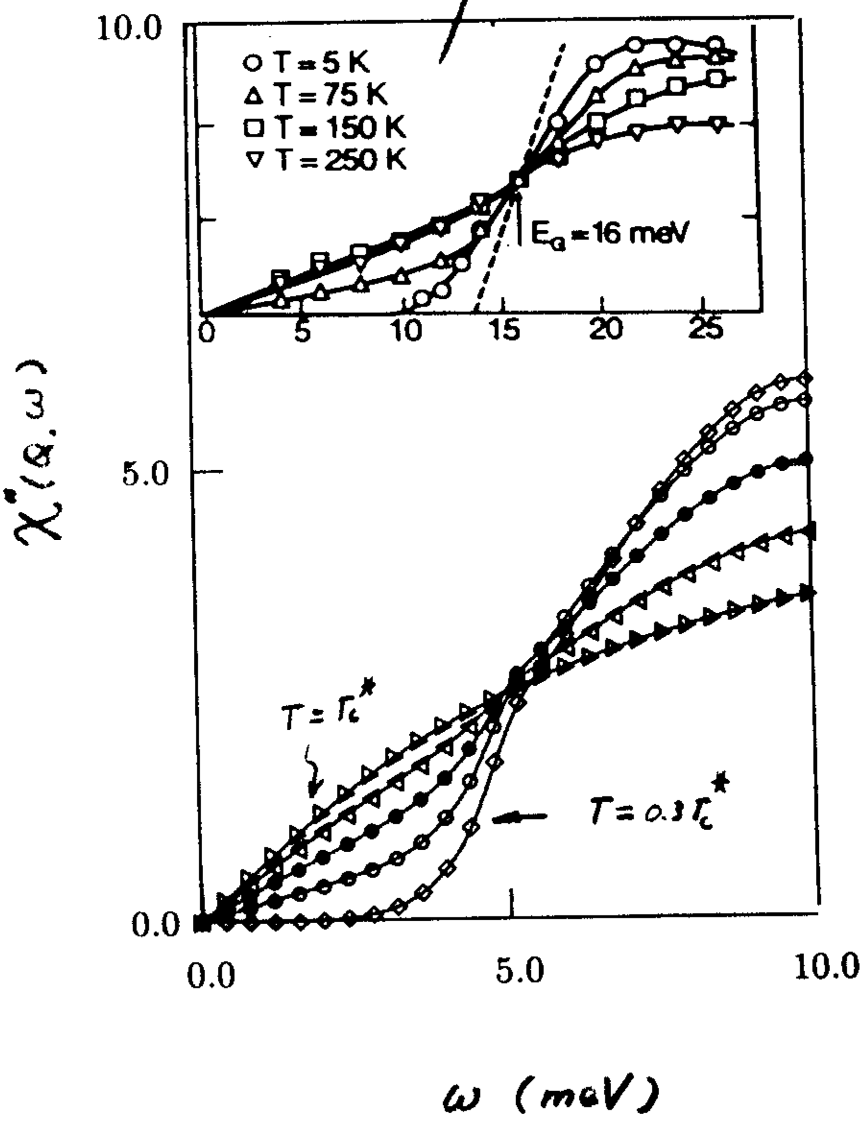
$YBa_2Cu_3O_{6.6}$: Tranquada et al. Phys. Rev. B $\underline{46}$, 5561 (1992)

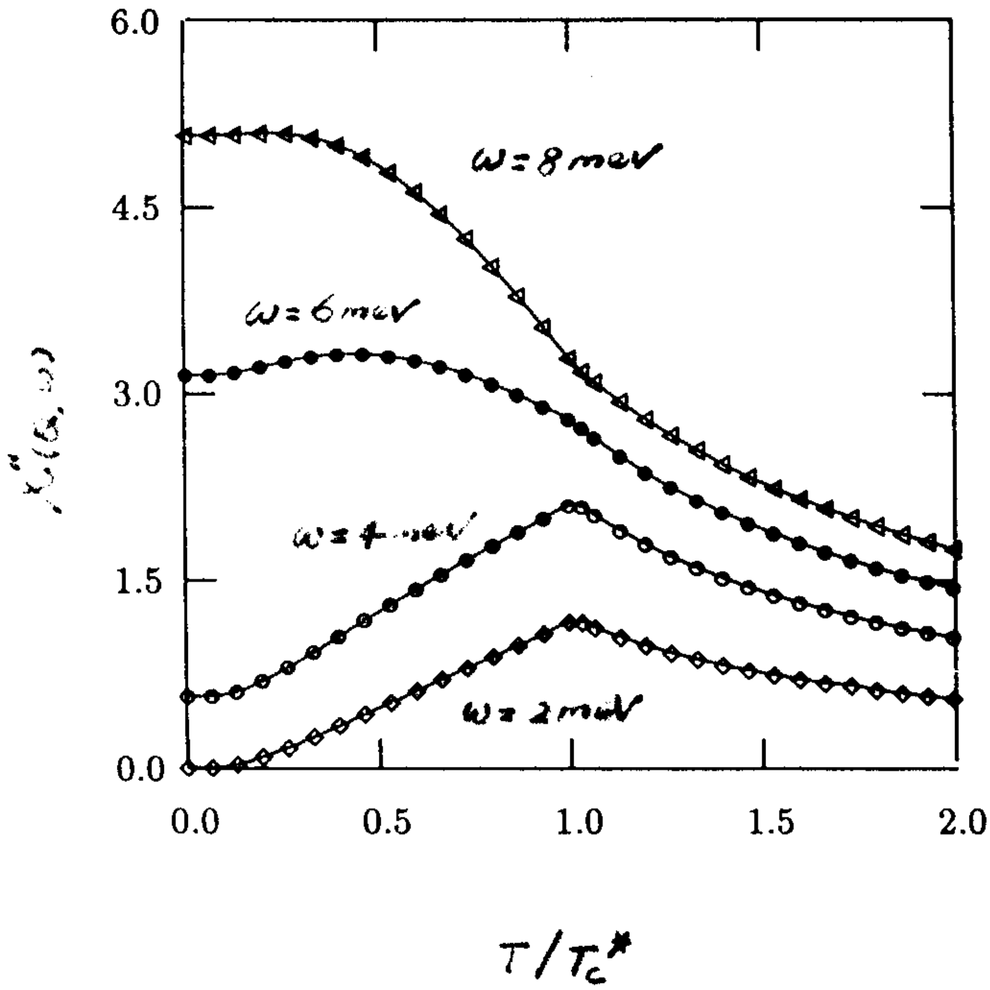


$T_c^* = 177K$
$T = 0.07\,T_c^*$
$T = 0.8\,T_c^*$
$T = 1.05\,T_c^*$
$T = 2.6\,T_c^*$
$\chi''(\vec{Q},\omega)$
ω (meV)
0.0
2.0
4.0
6.0
8.0
10.0
0.0
10.0
20.0
30.0
40.0
50.0

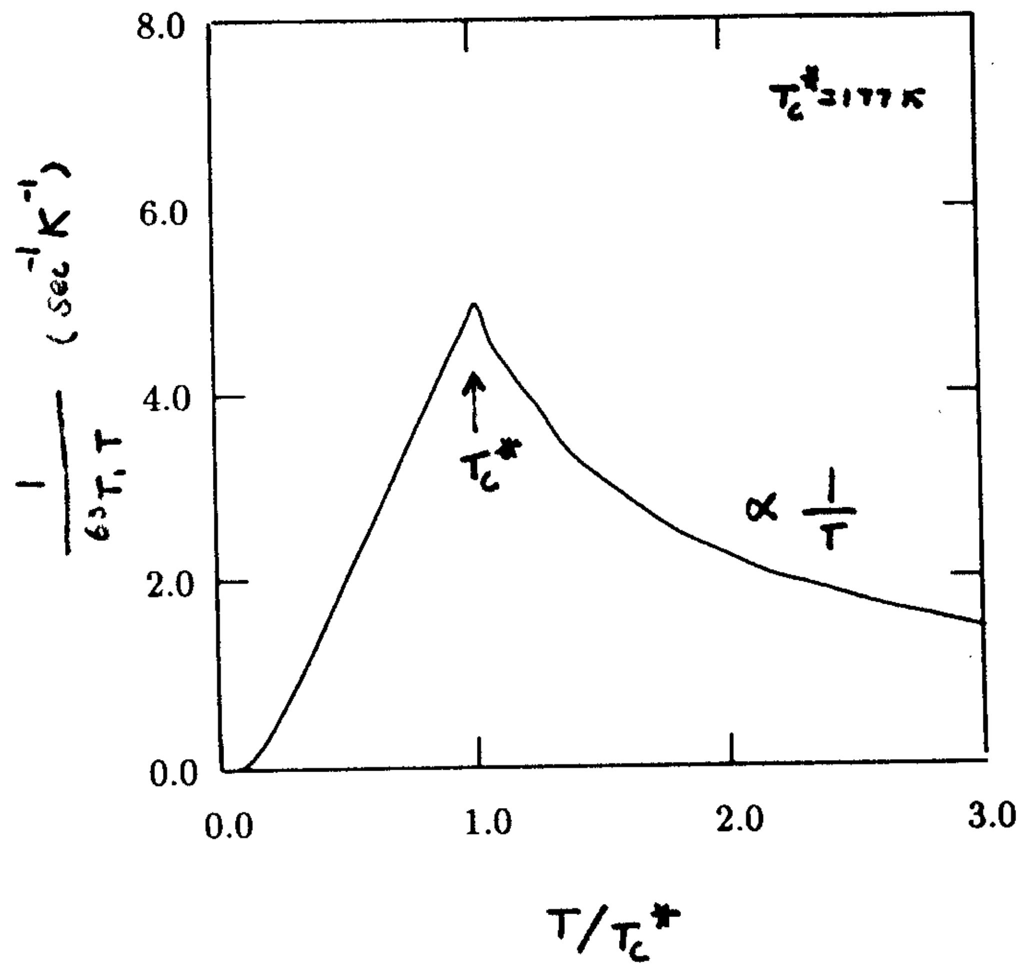
Rossat-Mignod et al. Physica B 185-189 86 (1991)

$YBa_2Cu_3O_{6.69}$



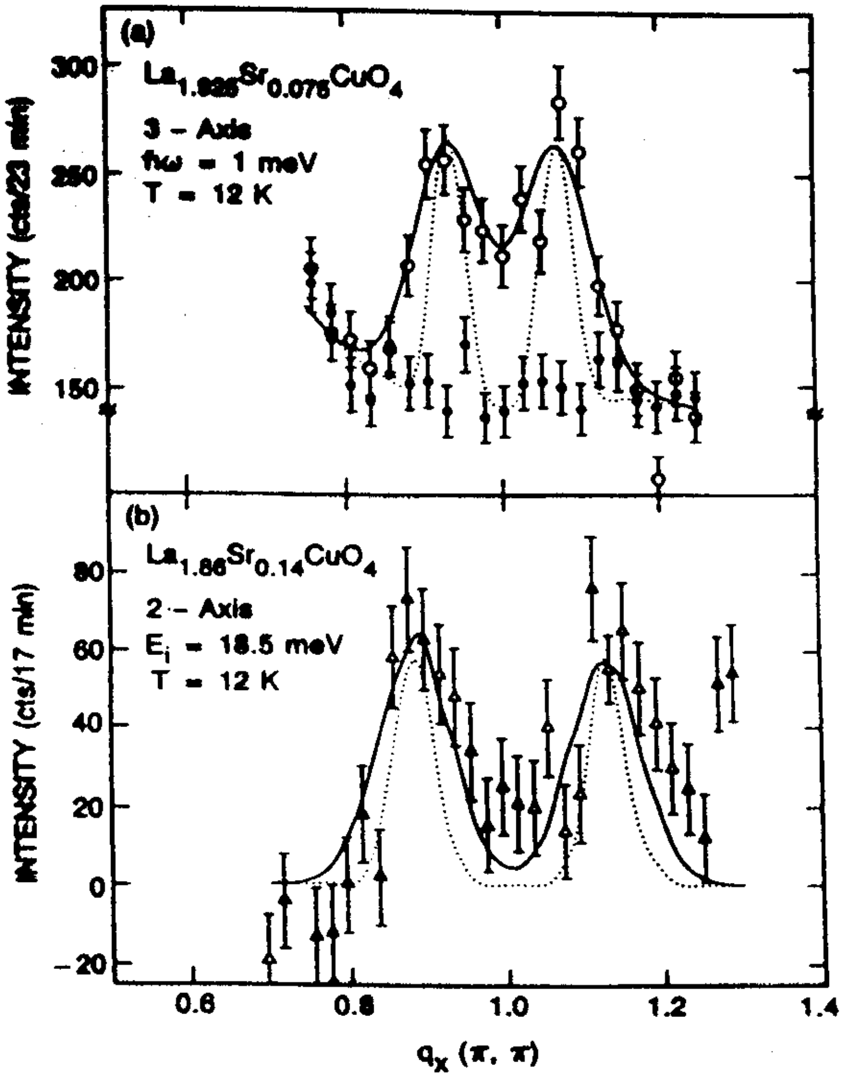


6.0
4.5
3.0
1.5
0.0
0.0
0.5
1.0
1.5
2.0
ω = 8 meV
ω = 6 meV
ω = 4 meV
ω = 2 meV
T / Tc*



$$^{17}T_1^{-1} / ^{63}T_1^{-1} \approx \frac{1}{20}$$

S-W. Cheong, et.al., Phys. Review Lett.
67, 1792 (1991)



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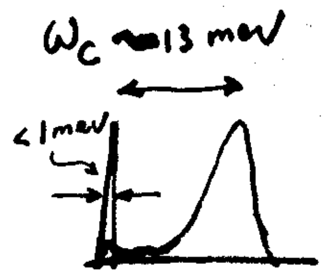
$La_{2-x}Sr_xCuO_4$



A scenario for the spectral function

Consequences :

- NMR spin relaxation rate ($\omega \to 0$ case)

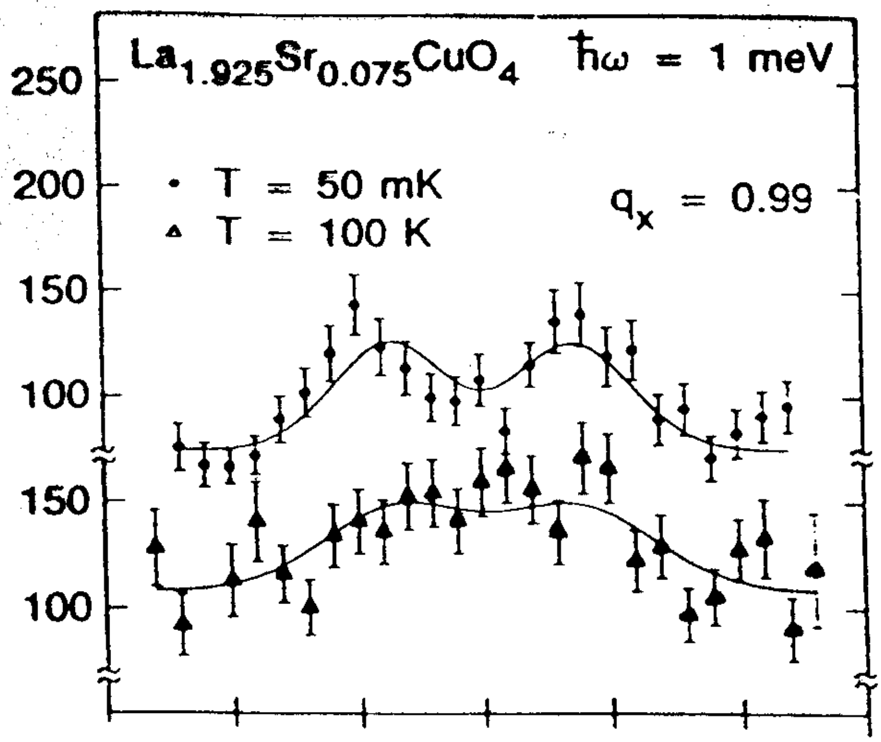
$$\left.\frac{1}{T_1 T}\right|_{^{63}Cu} \sim \frac{C}{T} \qquad \text{persists to lower } T \; (T > T_c^*)$$

$$C = \frac{B^2 c_0(\delta)}{\Gamma_s}(1-\delta)^*$$

- High temperature $\chi''(q, \omega)$:

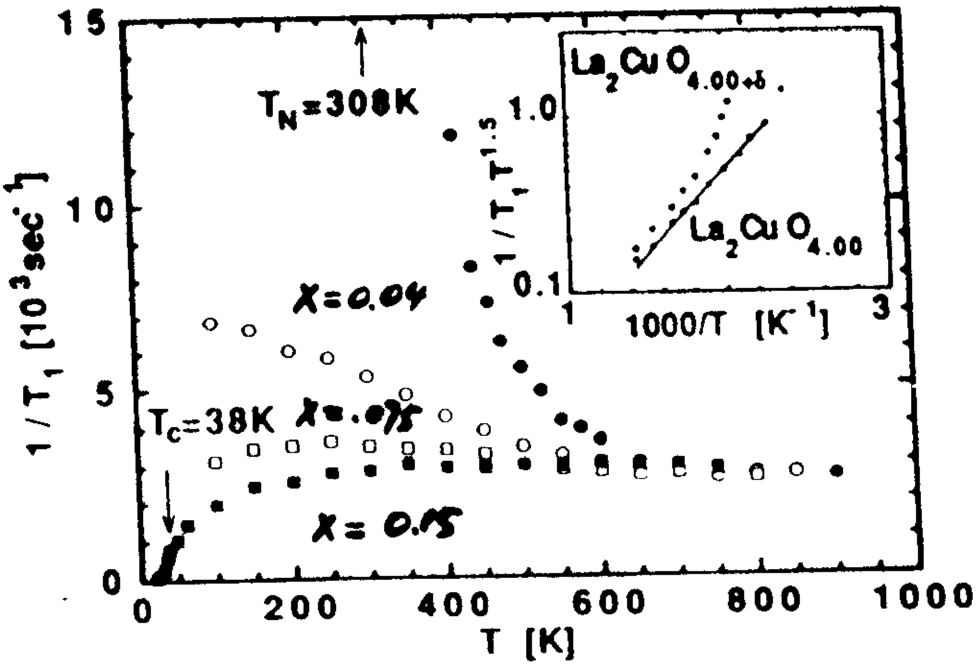
  incommensurate vs. commensurate

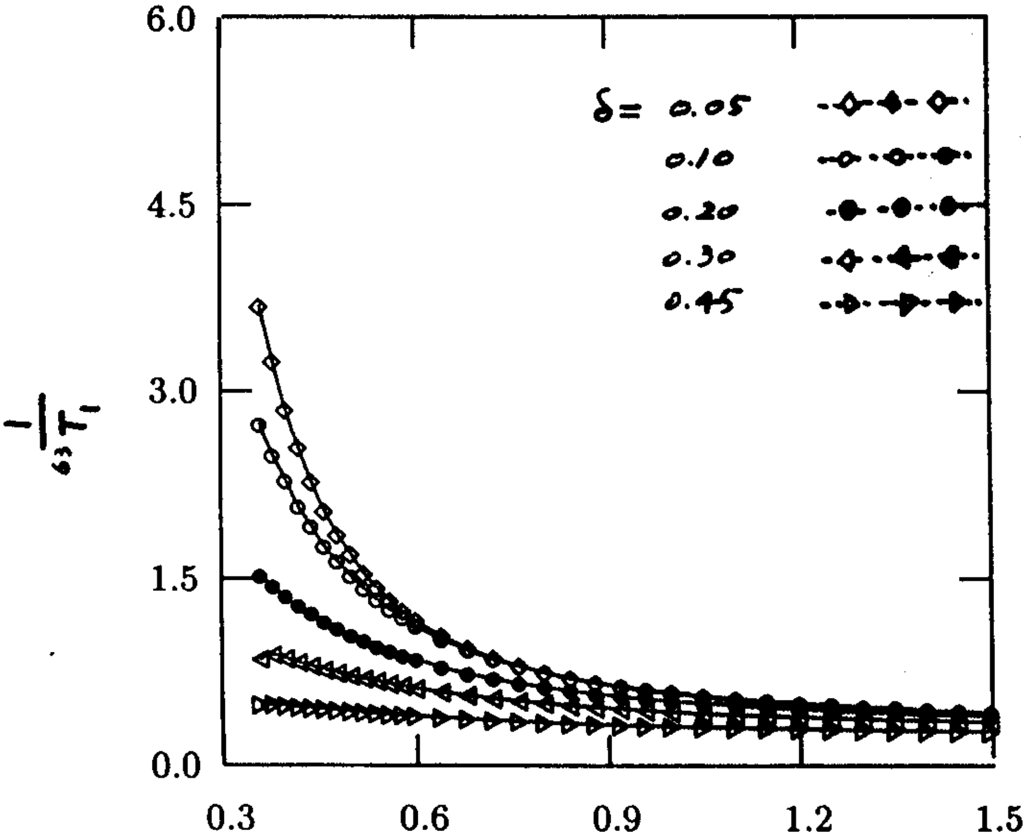
S.-W. Cheong, et. al. Phys. Rev. Lett. 67, 1791(1991)



T. Imai et al. Phys. Rev. Lett. 70, 1002 (1993)

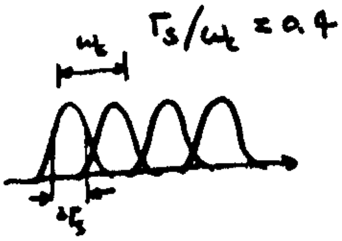
$La_{2-x}Sr_xCuO_4$





$T\ (\omega_b)$

$\hookrightarrow \frac{2\pi J}{S} \sim J$



A Brief Summary of the Magnetic Properties

- Correct spin-spin correlation in 1d
- A consistent explanation of
  - "spin gap" behavior
  - commensurate & incommensurate structure
  - NMR spin relaxation rate
  - Others
- Open question:

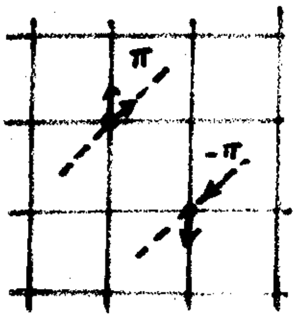
  why spectral functions are different in

  YBCO & LSCO compounds ?

• Charge Degree Of Freeedom (2D)

Holon saddle-point Hamiltonian

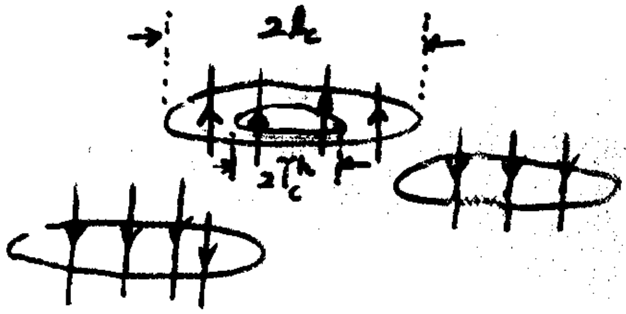
$$H_h = -t_h \sum_{<ij>} e^{i\tilde{A}^b_{ij}} \; e^{-i\phi^0_{ij}} \; h_i^+ h_j + H.c.$$



$$\Sigma_\Box \phi^0_{ij} = \pi$$

$$\tilde{A}^b_{ij} = \frac{1}{2} \sum_{l \neq i,j} Im \ln \left(\frac{z_i - z_l}{z_j - z_l} \right) \sum_\sigma \sigma n^b_{l\sigma}$$

$\langle \tilde{A}^b_{ij} \rangle = 0$ random-flux ?



two length scales:

- length scale of the fluctuating flux ~ $l_c \propto \frac{a}{\sqrt{\delta}}$
- cyclotron length of holon under such a fluctuation flux $\tilde{l}^h_c \propto \frac{a}{\sqrt{1-\delta/2}}$

$$l_c > \tilde{l}^h_c$$

flux tubes

holon = $(h)\,(p)(q)$ → flux tubes carried by down spinons

external fields

Effective Lagrangian for the 2d Holon Degree of Freedom

$$L_h = \int d^2\mathbf{r}\{h^+(\mathbf{r})[\partial_\tau + a_0^{ext}]h(\mathbf{r}) + p^+(\mathbf{r})\partial_\tau p(\mathbf{r}) + q^+(\mathbf{r})\partial_\tau q(\mathbf{r})\}$$

$$+ \int d^2\mathbf{r}\{h^+\frac{(-i\nabla - \mathbf{a}^{ext})^2}{2m_h}h + p^+\frac{(-\nabla - \mathbf{A}^+)^2}{2m_p}p + q^+\frac{(-\nabla - \mathbf{A}^-)^2}{2m_q}q\}$$

$$+ \int d^2\mathbf{r}\{\lambda(\mathbf{r})[p^+p - h^+h] + \beta(\mathbf{r})[q^+q - h^+h]\}$$

density constraint: $p^+(r)\,p(r) = q^+(r)\,q(r) = h^+(r)\,h(r)$ $\quad$ (scale $\gtrsim \ell_c^{-1} = \frac{1}{\sqrt{B_s}}$)

$$\vec{A}^{\pm} = \pm\frac{B_s}{2}\,\hat{z}\times\vec{r} \qquad B_s = \frac{\pi}{2a^2}(1-\delta)$$

Current constraint: $\quad J_h^{\ell} = J_p^{\ell} = J_q^{\ell}$

longitudinal gauge fluctuations of λ, β

→ Scattering mechanism

A Brief Summary

Normal state transport properties of the flux-binding phase:

- Resistivity

$$\rho \propto T$$

$$\rho = a\left(\frac{T}{T_h}\right)T + bT$$

$$\frac{\hbar}{\tau_s} \simeq \eta k_B T, \quad \eta \simeq 2.1$$

- Hall effect

$$\cot\theta = \frac{1}{\omega_H \tau_h} = \alpha T^2 + C$$

$$\cot\theta = \frac{\rho_{xx}}{\rho_{yx}}$$

$$R_H = \frac{1}{nec}\left(\frac{m_p + m_q}{m_H}\right)\frac{\tau_h}{\tau_s} \propto \frac{1}{T}$$

prediction $$\frac{\Delta\rho}{\rho} \propto B^2 T^{-4}$$ observed!

Harris, Yan, Ong
& Anderson
(Princeton)

- Thermal power

$$S = \frac{k_B}{e}\ln\left(\frac{1-2\delta}{2\delta}\right) + f(T)$$